Topic Name : Unit & Measurement Assignment
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Target: JEE Main & Advanced | NEET
Submission Format: 50 Seconds Video
Assignment Alloted: During LIVE Sessions

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Exercise 1

Q.1 The force acting on an object of mass m, travelling at velocity V in a circle of radius r is given by $F = \frac{mv^2}{r}$

The measurements are recorded as

$m = (3.5 \pm 0.1)$ kg; $v = (20 \pm 1)$ ms^{-1} and

$r = (12.5 \pm 0.5)$ m.

Find the maximum possible (i) relative error and (ii) percentage error in the measurement of force.

Q.2 The side of a cube is measured as

(75 ± 0.1) cm. Find the volume of the cube.

Q.3 In the formula $g = \frac{4\pi^2 L}{T^2}$, (ℓ) has 2% uncertainty and (T) has 5% uncertainty. What is the maximum uncertainty in the value of g?

Q.4 The length and breadth of a rectangular field are measured as length, $\ell = (250 \pm 5)$ m; breadth $b = (150 \pm 4)$ m. What is the area of the field?

Q.5 The initial temperature of a body is $(15 \pm 0.5)^0C$ and the final temperature is $(17 \pm 0.3)^0C$. What is the rise in temperature of the body?

Q.6 The error in measurement of radius of a sphere is 0.4%. What is the permissible error in the measurement of its surface area?

Q.7 5.74 gm of a substance occupies 1.2cc. Find the density of the substance to correct significant figures.

Q.8 The diameter of a circle is 1.06 m. Calculate its area with regard to significant figures.

Q.9 A substance of mass 5.74 g, occupies a volume of 1.2 cm^3 . Find its density with due regard to significant figures.

Q.10 If $m_1 = 1.2$ kg and $m_2 = 5.42$ gm. Find $(m_1 + m\ )$ with due regard to significant figures.

Q.11 Assuming force (F), length (L) and time (T) as fundamental units, what should be the dimensions of mass?

Q.12 The velocity (v) of a particle depends on time (t) according to the relation: $v = At^2 + Bt + C$ where V is in m/s and t is in s. Write the units and dimensions of constants A, B and C.

Q.13 A calorie is a unit of heat or energy and it equals about 4.2 J, where J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals (α)

kilogram, the unit of length equals (β) meter and unit of time (γ) seconds. Show that a calorie has a magnitude $4.2\,\alpha^{-1}\beta^{-2}\gamma^{2}$ in terms of new units.

Q.14 The centripetal force (F) acting on a particle moving in the circumference of a circle depends upon its mass (m), linear velocity (v) and radius (r) of the circle. Use method of dimensions to find the expression for centripetal force.

Q.15 Show by method of dimensions:

(i) Joule = 10^7 Erg (ii) 10^5 dyne/cm^2 = 10^4 N/m^2

Q.16 The latent head of ice is 80 cal/ gm. Express it in J/kg.

Q.17 A satellite is revolving around the earth in a circular orbit. The period of revolution (T) depends on

(i) Mass of earth (M)

(ii) Radius of orbit (r) and

(iii) Gravitational constant (G)

Use the method of dimensions to prove that $T\propto\sqrt{\left(\frac{r^3}{GM}\right)}$

Q.18 The pressure (P), volume (V) and temperature (T) of a real gas are related through Van der Waals equation:

$$\left(P+\frac{q}{V^2}\right)(V-b)=RT$$

Find the dimensions of constants a and b and also write the units of a and b in the SI system.

Q.19 If the dimensions of length are expressed as $\left[G^xC^yh^z\right]$ where G, C and h are universal gravitational constant, speed of light in vacuum and Plank's constant respectively, then what are the values x, y and z?

Q.20 Laplace corrected Newton's calculation for the velocity of sound. Laplace said that speed of sound in a solid medium depends upon the coefficient of elasticity of the medium under adiabatic conditions (E) and the density of the medium (ρ).

Prove that $v=k\sqrt{\frac{E}{\rho}}$

Q.21 The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula

$$\eta=\frac{\pi R^4 P}{8\,\ell\,Q}$$

Where R= radius of the capillary tube,

ℓ =length of the tube, P= pressure difference between its ends, and Q=volume of liquid flowing per second.

Which measurement needs to be made most accurately and why?

Q.22 Consider a planet of mass (m), revolving round the sun. The time period (T) of revolution of the planet depends upon the radius of the orbit (r), mass of the sun (M) and the gravitational constant (G). Using dimensional analysis, verify Kepler's third law of planetary motion.

Exercise 2

Q.1 A research worker takes 100 careful readings in an experiment. If he repeats the same experiment by taking 400 readings, then by what factor will be the probable error be decreased?

Q.2 The length, breadth and thickness of a rectangular sheet of metal are 4.234m, 1.005m and 2.01 cm respectively. Find the area and volume of the sheet to correct significant figures.

Q.3 The intensity of X- rays decreases exponentially according to the law $I = i\,e^{-\mu x}$, where i is the initial intensity of X-rays and I is the intensity after it penetrates a distance X through lead. If μ be the absorption coefficient, then find the dimensional formula for μ.

Q.4 Two resistors have resistance $R_1 = (24 \pm 0.5)\Omega$ and $R_2 = (8 \pm 0.3)\Omega$. Calculate the absolute error and the percentage relative error in calculating the combination of two resistors when they are in (a) Series (b) Parallel

Q.5 In an electrical set up, the following readings are obtained.

Voltmeter reading (V)=6.4V

Ammeter reading (I) = 2.0A

The respective least counts of the instruments used in these measurements are 0.2V and 0.1A. Calculate the value of resistance of the wire with maximum permissible absolute error and relative percentage error.

Q.6 The radius of a proton is 10^{-9} micron and that of universe is 10^{27}m. Identify an object whose size lies approximately midway between these two extremes on the logarithmic scale.

Q.7 If the velocity of light (c), gravitational constant (G) and the plank's constant (h) are selected as the fundamental units, find the dimensional formulae for mass, length and time in this new system of units.

Q.8 The critical velocity (V_c) of flow of a liquid through a pipe depends upon the diameter (d) of the pipe, density (ρ), and the coefficient of viscosity (η) of the liquid. Obtain an expression for the critical velocity.

Q.9 The mass m of the heaviest stone that can be moved by the water flowing in a river varies with the speed of water (V), density of water (d) and the acceleration due to gravity. Prove that the heaviest mass moved is proportional to the sixth power of speed. Also find the complete dependence.

Q.10 The frequency (f) of a stretched string of linear mass density (m), length (ℓ) depends (in addition to quantities specified before) on the force of stretching (F). Prove that $f = \frac{k}{\ell}\sqrt{\frac{F}{m}}$ where k is a dimensionless constant.

Q.11 Find out the maximum percentage error while the following observations were taken in the determination of the value of acceleration of the value of acceleration due to gravity. Length of thread=100.2cm; radius of bob=2.34cm; Time of one oscillation=2.3s. Calculate the value of maximum percentage error up to the required significant figures. Which quantity will be measured more be measured more accurately?

Q.12 Determine the focal length of the lens from the following readings:

Object distance, $u = 20.1 \pm 0.2\text{cm}$;

Image distance, $v = 50.1 \pm 0.5\text{cm}$.

Q.13 The specific gravity of the material of a body is determined by weighing the body first in air and then in water. If the weight in air is 10.0 ± 0.1 gw and weight in water is $50. \pm 0.1$ gw, then what is the maximum possible percentage error in the specific gravity?

Q.14 The following observation were actually made during an experiment to find the radius of curvature of a concave mirror using a spherometre: ℓ = 4.4 cm; h=0.085 cm. The distance ℓ between the legs of the spherometre was measured with a metre rod and the least count of the spherometre was 0.001 cm. Calculate the maximum possible error in the radius of curvature.

Q.15 It has been observed that the rate of flow (V) of a liquid of viscosity η through a capillary tube of radius (r) depends upon η, r and the pressure gradient P maintained across the length (ℓ) of the tube. Assuming a power law dependence, prove that the rate of flow of liquid is proportional to r^4. Also find the exact expression up to a constant.

Q.16 The height h to which a liquid rises in a tube of radius (r) depends upon the density of the liquid (d), surface tension (T), and acceleration due to gravity (g). Show that it would not be possible to derive the relation without the additional information that h is inversely proportional to r. Also find the relation.

Q.17 The viscosity η of gas depends upon its mass m, the effective diameter D and the mean speed v of the molecules present in the gas. Assuming a power law, find dependence of η on all these quantities.

Q.18 The distance moved by a particle in time from the center of a ring under the influence of its gravity is given by x=a sin ωt where a and ω are constant. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitation constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

Q.19 The centripetal force is given by $F=\frac{mv}{r}$. The mass, velocity and radius of the circular path of an object are 0.5kg, 10m/s and 0.4 m respectively. Find the percentage error in the force. Given: m,v and are measured to accuracies of 0.005 kg, 0.01m/s and 0.01 m respectively.

Q.20 An experiment to determine the specific resistance ρ of a metal wire provided the following observations.

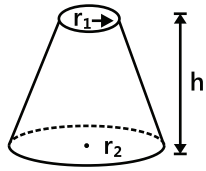
Resistance of R= (64 ± 2) ohm; Length $\ell = (156 \pm 0.1)$ cm; Radius r= (0.26 ± 0.02) cm

If s is expressed as: $\rho = \frac{\pi r^2 R}{\ell}$ Find the percentage error in ρ.

Q.21 The consumption of natural gas by a company satisfied the empirical equation $V = 1.50\,t + 0.008t^2$, where 'V' is the volume in millions of cubic metre and 't' is the time in months. Expressed this equation in units of cubic metre and seconds, put the proper units on the coefficients. Assume a month is of 30 days.

Q.22 As part of their introduction of the metric system the national convention made an attempt to introduce decimal time. In this plan, which was not successful, the day-starts at midnight into 10 decimal hours consisting of 100 decimal minutes each. The hands of a surviving decimal pocket watch are stopped at 8 decimal hours, 22.8 decimal minutes. What time is it representing in the usual system?

Q.23 Figure shows a frustum of a cone



Match the following dimensionally:

(a) Total circumference of the flat circular faces	(i) $\pi(r_1+r_2)\left[h^2+(r_1-r_2)^2\right]^{1/2}$
(b) Volume	(ii) $2\pi(r_1+r_2)$
(c) Area of the curved surface	(iii) $\pi h\left(r_1^2+r_1r_2+r_2^2\right)$

Q.24 Suppose that a man defines a unit of force as that which acts due to gravitation between two point masses each of 1 kg and 1 m apart. What would be the value of 'G' in this new system? What would be the value of one newton in this new system?

Given: G (in SI unit system) $=6.6\times10^{-11}$.

$\left[\text{Use}\left(F=\frac{Gm_1m_2}{r^2}\right)\right]$

Q.25 The distance between neighbouring atoms or molecules, in a solid substance can be estimated by calculating twice the radius of a sphere with volume equal to the volume per atoms of the material. Calculate the distance between neighboring atoms in the following: (a) iron (b) sodium

Given: The densities of iron and sodium are 7870 kg/m^3 and 1013 kg/m^3 respectively, the mass of an iron atom is 9.27×10^{-26} kg and the mass of sodium atom is 3.82 $\times10^{-26}$ kg.

Q.26 If force 'F' and density'd' are related as $F=\frac{\alpha}{\beta+\sqrt{d}}$, then find out the dimensions of α & β.

Q.27 If the velocity of light 'c' Gravitational constant 'G' & Plank's constant 'h' be chosen as fundamental units, find the dimensions of mass, length & time in this new system.

Q.28 In the formula; $\rho = \frac{nRT}{v-b} e^{-\frac{a}{RTV}}$, find the dimensions of 'a' and 'b' where P=pressure, n=no. of moles, T=temperature, V=volume and R=universal gas constant.

Q.29 A ball thrown horizontally from a height 'H' with speed 'v' travels a total horizontal distance 'R'. From dimensional analysis, find a possible dependence of 'R' on H, v and g. It is known that 'R' is directly proportional to 'v'.

Exercise-1

Sol 1: $F = \frac{mv^2}{r}$

$$\frac{\Delta F}{F} = \left(\frac{\Delta m}{m}\right) + 2\left(\frac{\Delta v}{v}\right) + \left(\frac{\Delta r}{r}\right) \rightarrow \quad \text{... (i}$$

Generally any measured physical quantity is noted in the form of $(S \pm \Delta S)$. So, here ΔS is the absolute error.

$\therefore$ When we look at $m = 3.5 \pm 0.1$;

$\Delta m = 0.1$ and $m = 3.5$

Now using formula (i)

$$\frac{\Delta F}{F} = \left(\frac{0.1}{3.5}\right) + 2\left(\frac{1}{20}\right) + \left(\frac{0.5}{12.5}\right) = 0.168$$

$$\frac{\Delta F}{F} = 0.17$$

Hence the relative error is 0.17 and the percentage would be $(0.17) \times 100 = 17\%$

Sol 2: Volume of the cube for side of length 'L' is L^3 Cu. Units.

$\therefore V = L^3$

$$\left(\frac{\Delta V}{V}\right) = 3 \cdot \left(\frac{\Delta L}{L}\right) \rightarrow \quad \text{... (i}$$

Here we have to write the volume in standard form i.e. $V + \Delta V$

$V = (75)^2 \text{ cm}^3$

$V = 421875 \text{ cm}^3$

$\Rightarrow V = 422000 \text{ cm}^3 \rightarrow \quad$ ... (ii

Now $\frac{\Delta V}{V} = 3.\left(\frac{\Delta L}{L}\right).V$

$$\Delta V = 3\left(\frac{0.1}{75} \times 421875\right)$$

$\Delta V = 1687.5$

$\Rightarrow \Delta V = 1700 \rightarrow \quad$ ... (iii

$\therefore$ Volume of the cube $= (422+1.7) \times 10^3 \text{cm}^3$

Sol 3: $g = \frac{4\pi^2 \ell}{T^2}$

$$\left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right)$$

Now what we mean by 2% uncertainty in ℓ is $\left(\frac{\Delta L}{L}\right) \times 100$.

Accordingly;

$$\left(\frac{\Delta g}{g}\right) \times 100 = \left(\frac{\Delta L}{L}\right) \times 100 + 2\left(\frac{\Delta T}{T}\right) \times 100$$

$= 2\% + 2 \times 5\% = 12\%$

Sol 4: $\ell = (250 \pm 5)\text{m}$ and $b = (150 \pm 4)\text{m}$

Area of the rectangle = $\ell \times b$ sq.units

$\therefore A = Lb \rightarrow \quad$ ... (i

$$\frac{\Delta A}{A} = \left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta b}{b}\right) \rightarrow \quad \text{... (ii}$$

Now let us first find the area,

$A = 250 \times 150 \text{m}^2$

$A = 375 \times 10^2 \text{ m}^2$

Now $\frac{\Delta A}{A} = \left(\frac{5}{250}\right) + \left(\frac{4}{150}\right)$

$\frac{\Delta A}{A} = 0.046$

$\Delta A = 1750 \text{ m}^2$

$\therefore$ Area $= (375 \times 10^2 + 1750)\text{m}^2$

Area $= (375 + 0.17) \times 10^4 \text{m}^2$

Sol 5: $T_{\text{initial}} = (15 \pm 0.5)^{\theta}\text{C}$

$T_{\text{final}} = (17 \pm 0.3)^{\theta}\text{C}$

Rise in temperature $= = (2 \pm 0.8)$.

Wondering why it's not 0.2.....99

There we not the error value such that in the given reading will not fluctuate above the error value. So let us say $T_1 = (15 - 0.5)^{\theta}\text{C}$ and $T_f = (17 + 0.3)^{\theta}\text{C}$ then in this case we get $\Delta T = (2 \pm 0.8)°\text{C}$!

Now by no means we can get ΔT more then $2.8^0 c$ this is the inner meaning of error.

Sol 6: Surface area of the sphere = $4\pi r^2$ Sq. units

$$\frac{\Delta S}{S} = 2.\left(\frac{\Delta r}{r}\right) \Rightarrow \left(\frac{\Delta S}{S}\right) \times 100 = 2\left(\frac{\Delta r}{r}\right) \times 100$$

$$\Rightarrow \left(\frac{\Delta S}{S}\right) \times 100 = 2(0.4\%) = 0.8\ \%$$

Sol 7: Density $= \frac{\text{mass}}{\text{volume}}$

Mass = 5.74 gm $\rightarrow$ 3 significant digits

Volume = 1.2 cc $\rightarrow$ 2 significant digits

$\therefore\ d = \frac{5.74}{1.2} = 4.78 = 4.8$ gm/cc

$\because$ Final result should be in 2 significant digits.

Sol 8: Diameter given

= 1.06 m $\rightarrow$ 3 significant digits.

And now Area $= \frac{\pi . d^2}{4}$

$A = 0.88206 \Rightarrow A = 0.882 m^2$

Sol 9: Solution similar to Q.7

Sol 10: $m_1 = 1.2 kg \rightarrow 2$ Significant digits

$m_2 = 5.42 g \rightarrow 3$ Significant digits

$\therefore$ $(m_1 + m_2)$ would be of 2 significant digit ... (i

$m_1 + m_2 = (1.2 \times 10^3 + 5.42) gms = 1205.42 gm$

$\Rightarrow 1200 gm[$ [using(i)]

$\therefore m_1 + m_2 = 1.2 kg$

[This is what happens when we add 10 to a million]

Sol 11: All the problems of this kind; can be solved by the following method.

$M = F^a\ L^b\ T^c$

But we know that;

$F = [M\ L\ T^{-2}]$

$\therefore\ M = [ML\,T^{-2}]^a [L]^b [T]^c$

$M = [M^a\ L^{a+b}\ T^{-2a+c}]$

Comparing the corresponding coefficients

$a = 1;\ a + b = 0;\ -2a + c = 0$

$\Rightarrow\ a = 1;\ b = -1;\ c = 2.$

$\therefore\ M = [F\ L^{-1}\ T^2]$

Sol 12: $V = At^2 + Bt + C$

Now using the concept of Dimensions; all the individual terms i.e. At^2, Bt, C should have the dimension of velocity v.

$\therefore\ C = [L\ T^{-1}] = m / s.$

And $Bt = [L\ T^{-1}]$

$\Rightarrow B[T] = [L\,T^{-1}] \Rightarrow B = [L\ T^{-2}] = m / s^2$

And $At^2 = [L\,T^{-1}]$

$A[t^2] = [L\ T^{-1}]$

$A = [L\ T^{-3}] \Rightarrow A = [L\ T^{-3}] = m / s^3$.

Calorie = $4.2[M^2\ L^2\ T^{-2}]$

Now we change the system of units to m', L', T'

$m' = \alpha m$

$L' = \beta \ell$

$t' = r t.$

$$\text{Hence } C = 4.2\left[\frac{m'}{\alpha} \cdot \frac{L'^2}{\beta^2} \cdot \left(\frac{T'}{\gamma}\right)^{-2}\right]$$

$$C = \frac{4.2}{\alpha \beta^2 \gamma^{-2}}[m'\ L'^2\ T'^{-2}]$$

$$\Rightarrow C = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2\ [m'\ L'^2\ T'^{-2}]$$

Hence the magnitude in new units is 4.2 $\alpha^{-1}\ \beta^{-2}\ \gamma^2$.

(*) Do the same procedure for any question on change in the units.

Sol 14: This is again a very standard problem.

$F = m^a\ v^b\ r^c$

And $F = [M L\,T^{-2}]$

$\Rightarrow [M\ L\ T^{-2}] = [M]^a \cdot [L\,T^{-1}]^b\ [L]^c$

$\Rightarrow [M\ L\ T^{-2}] = [M^a\ L^{b+c}\ T^{-b}]$

Comparing the powers;

$a = 1,\ b = 2,\ c = -1$

$$\therefore F = \frac{mv^2}{r}$$

Sol 15: (i) Dimensions of energy = $[M L^2 T^{-2}]$

Let M_1, L_1, T represent mass in gram, length in cm and time in second.

And M_2, L_2, T_2 represents mass in kilogram, length in meters and time in second.

Now $n_1 [M_1 L_1^2 T^{-2}] = n_2 [M_2 L_2^2 T_2^{-2}]$

$$n_1 = n_2 \left[\frac{M_2}{M_1} \left(\frac{L_2}{L_1} \right)^2 \left(\frac{T_2}{T_1} \right)^{-2} \right]$$

$$\Rightarrow n_1 = n_2 \left(10^3 (10^2)^2 1 \right) \Rightarrow n_1 = n_2 \times \left[10^{3+4} \right]$$

$$\Rightarrow n_1 = n_2 \times 10^7$$

$\Rightarrow$ 1 Joule = 10^7 erg. $[\because n_2 = 1]$

(ii) Similar method for (ii).

Sol 16: L = 80 cal/gm. Now we have to express it in J/kg.

We know that 1 cal = 4.2 J and 1 gm = 10^{-3} Kg.

$$\Rightarrow L = 80 \times \left[\frac{4.2}{10^{-3}} J/kg \right] = 80 \times 4.2 \times 10^3 \ J/kg$$

$$L = 336 \times 10^3 \ J/kg$$

$$\Rightarrow L = 3.3 \times 10^5 \ J/kg$$

Sol 17: Method is explained in detail in the solution of 11. Try this yourself.

[Hint: - $G = [M^{-1} L^3 T^{-2}]$]

Sol 18: $\left(p + \frac{q}{V^2} \right)(V - b) = RT$

Using the concept of dimensional analysis;

$\frac{q}{V^2}$ must have dimension of P (pressure)

$\Rightarrow q[L^{-6}] = [m L^{-1} T^{-2}] \Rightarrow q = [ML^5 T^{-2}] = kg m^5 s^{-2}$

And for b; it will have dimension of V,

$b = [L^3] = m^3$

Sol 19: $L = G^x C^y h^z$

$G = [M^{-1} L^3 T^{-2}]$ $C = [LT^{-1}]$

$h = [ML^2 T^{-1}]$

$L = [M^{-1} L^3 T^{-2}]^a [L T^{-1}]^b [ML^2 T - 1]^c$

$L = [M^{-a+c} L^{3a+b+2c} T^{-2a-b-c}]$

Comparing the corresponding component;

$c - a = 0$

$3a + b + 2c = 1c$

$-2a - b - c = 0$

Solve for a,b,c.

∗ This is a typical question from this chapter. So keep practicing problems of this type.

Sol 20: $V \propto (k)^a (E)^b (\rho)^c$

$k = [ML^{-1} T^{-2}]$, $E = [ML^2 T^{-2}]$, $\rho = [ML^{-3}]$, $V = [L T^{-1}]$

Now follow the same procedure as above to find a=1, $b = 1/2$, $c = -1/2$

Sol 21: $n = \frac{\Pi}{8} . \frac{R^4}{\ell} . \frac{pP}{qQ}$

$$\left(\frac{\Delta n}{n} \right) = 4. \left(\frac{\Delta R}{R} \right) + \left(\frac{\Delta L}{L} \right) + \left(\frac{\Delta P}{P} \right) + \left(\frac{\Delta Q}{Q} \right)$$

From this it is evident that an error in R gets magnified by four times. So we have to be careful in measuring R.

Sol 22: $T \propto r^a M^b G^c$

$G = [M^{-1} L^3 T^{-2}]$

Use the standard method followed above to derive

$$T \propto \left(\frac{r^3}{GM} \right)^{1/2}$$

Infact Keplers third law is $\frac{4\pi^2}{T^2} = \frac{GM}{r^3}$

This is the real application of dimensional Analysis. One can derive the body of any formula. Constant are then found or performing a couple of experiments.

Exercise 2

Sol 1: Increasing the number of readings reduces the errors. This is because, we have more chances to get the mean closer to the actual value.

So increasing reading from 100 to 400; reduce the problem error by a factor of four.

Sol 2: Length $(L) = 4.234\,m \rightarrow 4$ significant digits

Breadth $(B) = 1.005\,m \rightarrow 4$ significant digits

Thickness $(H) = 2.01 \times 10^{-2}\,m \rightarrow 3$ significant digits

$\therefore$ The volume = Lbh will have 3 significant digits

$\Rightarrow\ V = (4.234 \times 1.005 \times 2.01) \times 10^{-2}\,m^3$

$\Rightarrow\ V = 8.5528 \times 10^{-2}\ m^3$

$\Rightarrow\ V = 8.55 \times 10^{-2}\ m^3$

Sol 3: $I = I_0\ e^{-\mu x}$

Now μx should have the dimension $[M^0 L^0 T^0]$

$\Rightarrow\ \mu.[L] = [M^0 L^0 T^0]$

$\mu = [M^0 L^{-1} T^0]$

Sol 4: $R_1 = (24 \pm 0.5)\Omega$

$R_2 = (8 \pm 0.3)\Omega$

(a) Series :-

$R_{eff.} = R_1 + R_2$

$R_{eff.} = 32$

$\Delta R_{eff} = \Delta R_1 + \Delta R_2 = 0.5 + 0.3 = 0.8$

$\therefore R_{eff} = (32 \pm 0.8)\Omega$

Now absolute error = 0.8 and

Relative error $= \dfrac{0.8}{32} \times 100 = 2.5\%$

(b) Parallel:-

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow \qquad \text{...(i)}$$

$$R_{eff} = \frac{24 \times 8}{24 + 8} = \frac{24 \times 8}{32} = 6\Omega$$

And for R_{eff}; using (i)

$$\frac{\Delta R_{eff}}{R^2_{eff}} = \frac{\Delta R_1}{R_2^1} + \frac{\Delta R_2}{R_2^2} \rightarrow$$

Remember this formula !

$$\left[\text{Hint}: \frac{d}{dx}\left(\frac{1}{\theta}\right) = -\frac{1}{\theta^2}.\frac{d\theta}{dx} = \frac{\Delta\theta}{\theta^2}!\right]$$

$$\Delta R_e = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right).R_{eff}{}^2$$

$$\Delta R_e = \left(\frac{0.5}{24 \times 24} + \frac{0.3}{8 \times 8}\right) \times (6)^2$$

$\Delta R_e = 0.2$

$\Delta R_e = (6 \pm 0.2)\Omega$

And for relative error;

$$\left(\frac{\Delta R_e}{R_e}\right) = \frac{0.2}{6} = \left(\frac{1}{30}\right)$$

$$\%\ \text{relative error} = \frac{1}{30} \times 100 = \frac{10}{3} = 3.33\%$$

Sol 5: Ohm's Law: V=IR $\Rightarrow\ R = \dfrac{V}{I}$

$$R = \frac{6.4}{2} = 3.2\Omega \ \text{ and } \ \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\Delta R = \left[\frac{0.2}{6.4} + \frac{0.1}{2}\right]3.2$$

$\Delta R = 0.26$

Resistance $R = (3.2 \pm 0.26)\ \Omega$

And relative error $= \dfrac{\Delta R}{R} = \left(\dfrac{0.26}{3.2}\right)$

% Relative error $= \dfrac{0.26}{3.2} \times 100 = 8.1\%$

Sol 6: Radius of proton

$= 10^{-9}\mu = 10^{-9} \times 10^{-6} m = 10^{-15} m$

Size of universe $= 10^{27} m$

Now let us use $\log_{10}(r)$ as an operator.

$\log(r_p) = \log_{10}(10^{-15}) = -15$

and

$$\log(r_4) = \log_{10}(10^{27}) = 27\ \ \log(r) = \frac{27 + (-15)}{2} = \frac{12}{2} = 6$$

$\Rightarrow r = 10^6\,m$

Sol 7: Refer to the solution of Q_{11} (Ex – 1) and Q_{19} (Ex – 2) and y it yourself.

Sol 8: $V_c \propto [d]^a (\rho)^b (\eta)^c$

$V_c = [L\ T^{-1}]$

$\rho = [M\ L^{-3}]$

$d = [L]$

$\eta = [ML^{-1}\ T^{-1}]$

$[L\ T^{-1}] = [L]^a\ [ML^{-3}]^b\ [ML^{-1}T^{-1}]^c$

$[L\ T^{-1}] = [M^{b+c}\ L^{a-3b-c} T^{-c}]$

$b + c = 0$

$a - 3b - c = 1$

$-c = -1$

We get $c = 1,\ b = -1,$ and $a = -1$

$\therefore V_c \propto \frac{\eta}{d\rho}$

Sol 9: $m \propto V^a\ (d)^b\ (g)^c$

$m = [M]$

$V = [L\ T^{-1}]$

$d = [M\ L^{-3}]$

$g = [L\ T^{-2}]$

$[M] = [LT^{-1}]^a\ [ML^{-3}]^b [L\ T^{-2}]^c$

$[M] = [M^6\ L^{a-3b+c}\ T^{-a-2c}]$

$b = 1;\ a - 3b + c = 0;\ -a - 2c = 0$

$\Rightarrow c = -3$ and $a = 6$

$\therefore m \propto V^6.\ d\ g^{-3}$

$\Rightarrow m = \frac{kV^6 d}{g^3}$

Sol 10: $f \propto m^a\ \ell^6\ F^c$

$[f] = [T^{-1}]$

$m \equiv [M]$

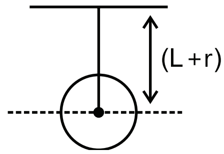
$\ell \equiv [L]$

$F \equiv [M\ L\ T^{-1}]$

Now proceeding the same way as we did in Q11 – (Ex – 1)

Sol 11: Now if there is an error, the next possible value of L would be 100.3 or 100.4 cm.

i.e least count for r=2.34 cm, L.C=0.01



[$\therefore$ 2.35 or 2.36]

and for $t = 2.3s$, $L.C = 0.1s$

$$\therefore \left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta r}{r}\right) + 2\left(\frac{\Delta T}{T}\right)$$

$$\therefore \left(\frac{\Delta g}{g}\right) = \left(\frac{0.1}{100.2}\right) + \left(\frac{0.01}{2.34}\right) + 2.\left(\frac{0.1}{2.3}\right)$$

$$\left(\frac{\Delta g}{g}\right) = 0.092$$

$$\left(\frac{\Delta g}{g}\right) \times 100 = 9.2\%$$

'T' has to be measured more accurately because each error gets double magnified in calculating g.

Sol 12: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow f = 14.3$

$$\frac{1}{f} = \frac{1}{50.1} + \frac{1}{20.1}$$

And then $\frac{\Delta f}{f^2} = \frac{\Delta V}{v^2} + \frac{\Delta 4}{u^2}$

$$\Delta f = \left(\frac{\Delta V}{v^2} + \frac{\Delta u}{u^2}\right) f^2$$

$$\Delta f = \left(\frac{0.5}{(50.1)^2} + \frac{(0.2)}{(20.1)^2}\right)(14.3)^2$$

$\Delta f = 0.4cm$

Focal length $= (14.3 \pm 0.4)cm$

Sol 13: Specific gravity (s) $= \frac{w_{air}}{w_{water}}$

$$\left(\frac{\Delta s}{s}\right) = \left(\frac{\Delta w_{air}}{w_{air}}\right) + \left(\frac{\Delta w_{water}}{w_{water}}\right)$$

$$\left(\frac{\Delta s}{s}\right) = \left(\frac{0.1}{10}\right) + \left(\frac{0.1}{5}\right)$$

$$\left(\frac{\Delta s}{s} \times 100\right) = 3\%$$

Sol 14: $R = \left(\frac{h}{2}\right) + \left(\frac{L^2}{6h}\right)$

$R = \frac{h}{2} + \frac{L^2}{6h}$

$h = (0.085 \pm 0.001)\text{cm}$

$L = (4.4 \pm 0.1)\text{cm}$

$\frac{\Delta R}{R} = \frac{\Delta h}{h} + 2\frac{\Delta L}{L} + \frac{\Delta h}{h}$

$\frac{\Delta R}{R} = 2\left(\frac{\Delta h}{h} + 2\frac{\Delta L}{L}\right)$

$\left(\frac{\Delta R}{R}\right) = 2\left(\frac{0.001}{0.085} + \frac{0.1}{4.4}\right)$

$\left(\frac{\Delta R}{R}\right) = 0.068$

Sol 15: $V \propto n^a r^b \left(\frac{p}{\ell}\right)^c$

V-Rate of flow $\Rightarrow$ Volume/time $\equiv [L^3\ T^{-1}]$

$\eta \equiv [m\,L^{-1}\ T^{-1}]$

$r \equiv [L]$

$p \equiv [m\,L^{-1}\ T^{-2}] * \frac{p}{L} \equiv [m\,L^{-2}\,T^{-2}]$

Now solving for the constants a, b, c gives us the result.

$[L^3\,T^{-1}] = [M\,L^{-1}\ T^{-1}]^a\,[L]^b\,[M\,L^{-2}\ T^{-2}]^c$

$[L^3\ T^{-1}] = [M^{a+c}\,L^{-a+b-2c}T^{-a-2c}]$

$a + c = 0$

$-a + b - 2c = 3$

$-a - 2c = -1$

$a = -1\quad c = 1\ \text{and}\ b = 4$

Sol 16: $h \propto (\rho)^a\ (T)^b\,(g)^c\,(r)^d$

$h \equiv [L]$

$\delta \equiv [ML^{-3}]$

$T \equiv [M\,L\,T^{-2}]$

$g = [L\ T^{-2}]$

$r = [L]$

Now proceeding;

$[L] \equiv [ML^{-3}]^a[M\ L\ T^{-2}]^6[L\,T^{-2}]^c[L]^d$

$[L] \equiv [M^{a+b}\,L^{-3a+b+c+d}\ T^{-2b-2c}]$

$\Rightarrow\ a + b = 0;\ -2(b + c) = 0;\ -3a + b + c + d = 1$

Here we have 3 equations and four variables to solve $\therefore$ Not possible.

And now;

Given that h is inversely proportional to 'r'

$\therefore d = -1$

Now we can solve for a, b, c

Sol 17: $\eta \propto m^a\ D^b\ v^c$

$\eta \equiv [M\,L^{-1}\ T^{-1}]$

$m \equiv [M]$

$D \equiv [L]$

$V \equiv [L\,T^{-1}]$

Solve in a similarly way to Q.16

Sol 18: $\omega \propto r^a\,m^b\,G^c$

$[\omega] \equiv [m^0\ L^0\ T^0]$

$[r] \equiv [L]$

$m \equiv [M]$

$G = [M^{-1}\ L^3\ T^{-2}]$

And solve for a, b, c

Sol 19: $F = \frac{mv^2}{r}$

$m = (0.5 \pm 0.005)\text{kg}$

$v = (10 \pm 0.01)\text{m / s}$

$r = (0.4 \pm 0.01)\text{m}$

$\frac{\Delta F}{F} = \left(\frac{\Delta m}{m}\right) + 2\left(\frac{\Delta v}{v}\right) + \left(\frac{\Delta r}{r}\right)$

$= \left(\frac{0.005}{0.5}\right) + 2\left(\frac{0.01}{10}\right) + \left(\frac{0.01}{0.4}\right)$

$\left(\frac{\Delta F}{F}\right) = 0.037$

$\left(\frac{\Delta F}{F} \times 100\right) = 3.7\%$

Sol 20: $\rho = \frac{\pi r^2 R}{\ell}$

$$\left(\frac{\Delta\rho}{\rho}\right) = 2\cdot\left(\frac{\Delta r}{r}\right)+\left(\frac{\Delta R}{R}\right)+\left(\frac{\Delta L}{L}\right)$$

$$\left(\frac{\Delta\rho}{\rho}\times 100\right) = 2\cdot\left(\frac{0.02}{0.26}\times 100\right)$$

$$+\left(\frac{2}{64}\times 100\right)+\left(\frac{0.1}{156}\times 100\right) = 18.6\%$$

Sol 21: $V = \underset{(2)}{\frac{+.50t}{\downarrow}} + \underset{(2)}{\frac{0.008}{\downarrow}}t^2$

Now let us examine the units of (1) and (2) for (1); unit is m^2/s and dimension is $[L^3\ T^{-1}]$. And for 2; unit is m^3/s^2 and dimension is $[L^3\ T^{-2}]$.

$$\therefore\ V = (1.50\ m^3/s_1)t + \left(0.008\cdot\frac{m_1^3}{s_1^2}\right)t^2$$

By changing the unit system;

The value of coefficients of 't' and 't^2' change.

$$n_1\left[L_1^3\ T_1^{-1}\right] = n_2\left[L_2^3\ T_2^{-1}\right]$$

Using this we can find the values of new coefficients.

Sol 22: 24 hours $\equiv$ 10 Decimal hours

$\Rightarrow$ 1 D h = 2.4 hrs. $\rightarrow$ (1)

and 1 Dh = 100 D mins

$\Rightarrow$ 1 D min $= \frac{2.4}{100} \rightarrow (2)$

Given time = 8 Dh and 22.8 Dmin

$= 8(2.4) + 22.8\left(\frac{2.4}{100}\right)$hours. $= 19.747$ hours

$\Rightarrow$ 19 Hr, 10 minutes, 50 seconds

Sol 23: Aim of the question is to do dimensional analysis. Circumference will have dimension [L]

Volume $[L^3]$

Area $[L^2]$

Verify the options for correct choices

Sol 24: For $m_1 = m_2 = 1kg$ and $r = 1\ m$,

The force $F = \frac{G.(1)^2}{1} = GN$

$\therefore\ G = 6.6\times 10^{-11}$ N.

But according to the problem, the force is 1 unit.

$\Rightarrow F = 6.6\times 10^{-11}\ N \equiv 1$ unit

$\therefore\ 1\ N \equiv \frac{1}{6.6\times 10^{-11}}$unit

We call this unit as Gunit.

$\therefore\ 1\ N = 1.5\times 10^{10}$ Gunit

Sol 25: For each atom, we have,

$$v = \frac{m}{\rho} = \frac{9.27\times 10^{-26}}{7870} = 1.178\times 10^{-29} m^3 / \text{atom}$$

Now, $\frac{4\pi r^3}{3} = 1.178\times 10^{-29} m^3 \Rightarrow r = 1.41\times 10^{-10} m$

Hence, the distance between atoms is d = 2r= 2.82 × 10^{-10}m

Sol 26: $F = \frac{\alpha}{\beta+\sqrt{d}}$

Here the dimension of β and $\sqrt{d}$ should be same.

Hence $[m^{1/2} L^{-3/2}] = \beta$

And now the dimension of $\frac{\alpha}{\sqrt{d}}$ should be

Same as F.

$$\frac{\alpha}{[m^{-1/2}L^{-3/2}]} = [MLT^{-2}]$$

$$\Rightarrow \alpha = [m^{3/2}L^{-1/2}T^{-2}]$$

Sol 27: $c = [L\ T^{-1}]\quad G = [M^{-1}\ L^3\ T^{-2}]$

$h = [M\,L^2\ T^{-1}]$

Now for mass $M = c^x G^y h^z$

Finding the value x, y, by following the method described in Q11 (Ex1)

Sol 28: $p = \left(\frac{nRT}{v-b}\right)e^{\frac{-a}{RTV}}$

There we can use the ideal gas equation;

$pv = nRT$

In solving question involving RT; we can replace RT by PV and then proceed.

So, now $\frac{a}{RTV}$ will have a dimensionally $[M^0 L^0 T^0]$

$\frac{a}{pv^2} = [M^0 L^0 T^0]$

$a \equiv [M L^{-1} T^{-2}][L^6]$

$a = [M L^5 T^{-2}]$

And now for b;

$\frac{nRT}{v-b}$ should be dimension equal to P.

$\therefore \ \frac{pv}{v-b} \equiv p$

$\Rightarrow \ v - b \equiv v$

$\Rightarrow$ b should be dimensionally equal to V

$\therefore \ b = [L^3]$

Sol 29: $R \propto H^a v^b g^c$

$r \equiv [L]$

$H \equiv [L]$

$v \equiv [L\ T^{-1}]$

$g \equiv [L\ T^{-2}]$

$[L] \equiv [L]^a [L\ T^{-1}]^b [L\ T^{-2}]^c$

$[L] \equiv [L^{a+b+c}\ T^{-b-2c}]$

$a + b + c = 1 \rightarrow$ ... (i

$b + 2c = 0 \rightarrow$ ... (ii

And also given that, $R \propto v$, $a = 1$, $b = 1$

So $a + c = 0$

$1 + 2c = 0$

$\Rightarrow c = -1/2$ and $a = 1/2$

$\therefore R \propto \sqrt{\frac{H}{g}}\, v$

$\therefore R = k\sqrt{\frac{H}{g}}\, v$

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