PVC - WAVES SOLUTION (ENGLISH MEDIUM)

1. **(a)** $v_{max} = a\omega = a \times 2\pi n = 0.1 \times 2\pi \times 300 = 60\pi$ cm/sec

2. **(d)** $v = \frac{1}{\sqrt{M}}$

$\Rightarrow \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}}$

$\Rightarrow \frac{4}{1}$

3. **(c)** $y = A\sin(at - bx + c)$ represents a wave, where a may correspond to ω and b may correspond to k.

4. **(c)** Compare the given equation with standard form

$y = r\sin\left[\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right]$

$\frac{2\pi}{\lambda} = 3, \lambda = \frac{2\pi}{3}$ and $\frac{2\pi}{T} = 15$

$T = \frac{2\pi}{15}$

Speed of propagation, $v = \frac{\lambda}{T} = \frac{2\pi/3}{2\pi/15} = 5$

5. **(a)** Given, $y = A\sin(kx - \omega t)$

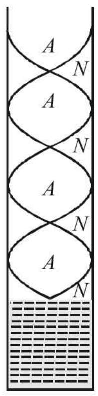
$\Rightarrow v = \frac{dy}{dt} = -A\omega\cos(kx - \omega t)$

$\Rightarrow v_{max} = A\omega$

6. **(a)** $m = \frac{0.035}{5.5}$ kg / m, T = 77 N

$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{77 \times 5.5}{0.035}} = 110$ m / s

7. **(d)** The end in contact with water is a node while the open end is and antinode.



In the present case, tube is in seventh harmonic.

8. **(c)** Frequency of 2^{nd} overtone

$n_3 = 5n_1 = 5 \times 50 = 250$ Hz

9. **(b)** For production of beats different frequencies are essential. The different amplitudes affect the minimum and maximum amplitude of the beats and different phases affect the time of occurrence of minimum and maximum.

10. **(d)** (n –1) and (n + 1) suppose to form frequency n and n will be at resonance.

(n –1) and n → produce 1 beat

(n + 1) and n → produce 1 beat

Number of beats formed re '2'

11. **(b)** $a_1 = 5, a_2 = 10 \Rightarrow \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{5+10}{5-10}\right)^2 = \frac{9}{1}$

12. **(d)** $n' = \left(\frac{v}{v - v_s}\right) n = \frac{330}{330 - 30} \times 500 = 110 \times 5 = 550$ Hz

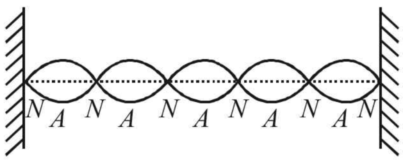
13. **(a)** $L_0 = 60$ cm $\qquad n_0 = 256$ Hz

$n = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad \therefore \quad n \propto \frac{1}{L}$

$\frac{n_1}{n_0} = \frac{L_0}{L_1}$

$n_1 = n_0 \frac{L_0}{L_1} = 256 \times \frac{60}{15} = 1024$ Hz

14. **(b)**



From figure,

Total nodes = 6

Total antinodes = 5

15. **(a)** Newton assumed that sound propagation in a gas takes under isothermal condition.

16. **(a)** $v = \sqrt{\frac{K}{\rho}} \quad \therefore K = v^2\rho = 2.86 \times 10^{10}\ N/m^3$

17. **(a)** $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$\frac{\pi}{5} = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \Delta x = 0.05$ m

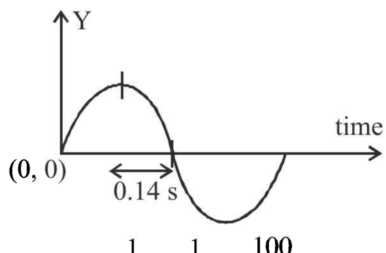
18. **(a)** Frequency (n) = 4.2 MHz = 4.2×10^6 Hz

speed of sound (v) = 1.7 km/s = 1.7×10^3 m/s.

Wave length of sound in tissue;

$\lambda = \frac{v}{n} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4 \times 10^{-4}$ m

19. **(c)** Period, $T = 0.14 \times 4 = 0.56$ s



Frequency $= \frac{1}{T} = \frac{1}{0.56} = \frac{100}{56} = 1.79$ Hz

20. (b) The standard equation of a progressive wave is

$$y = a\sin\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)+\phi\right]$$

The given equation can be written as

$$y = 4\sin\left[2\pi\left(\frac{t}{10}-\frac{x}{18}\right)+\frac{\pi}{6}\right]$$

$\therefore$ a = 4 cm, T = 10 s, λ = 18 cm and $\phi = \pi/6$

21. (b) $y = 60\cos(180t - 6x)$ ....(i)

$$\omega = 180, k = 6 \Rightarrow \frac{2\pi}{\lambda} = 6$$

$$v = \frac{\omega}{k} = \frac{2\pi}{T}\times\frac{\lambda}{2\pi} = \frac{180}{6} = 30\ m/s$$

Differentiating (i) w.r.t. t,

$$v = \frac{dy}{dt} = -60\times 180\ \sin(180t - 6x)$$

$$v_{max} = 60\times 180\ \mu m/s$$

$$= 10800\ \mu m/s$$

$$= 0.0108\ m/s$$

$$\frac{v_{max}}{v} = \frac{0.0108}{30} = 3.6\times 10^{-4}$$

22. (a) $y(x,t) = 0.005\cos(\alpha x - \beta t)$ (Given)

Comparing it with the standard equation of wave $y(x,t) = a\cos(kx - \omega t)$ we get

$k = \alpha$ and $\omega = \beta$

But $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

$\Rightarrow \frac{2\pi}{\lambda} = \alpha$ and $\frac{2\pi}{T} = \beta$

Given that $\lambda = 0.08$ m and $T = 2.0$ s

$\therefore\ \alpha = \frac{2\pi}{0.08} = 25\pi$ and $\beta = \frac{2\pi}{2} = \pi$

23. (a) The given equation is $y = 10\sin(0.01\pi x - 2\pi t)$

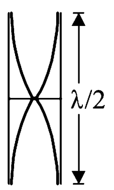
Hence ω = coefficient of t = 2π

Maximum speed of the particle $v_{max} = a\omega = 10\times 2\pi$ $= 10\times 2\times 3.14 = 62.8 \cong 63$ cm/s

24. (d) $y = 0.021\sin(x + 30t) \Rightarrow v = \frac{\omega}{k} = \frac{30}{1} = 30\ m/s$

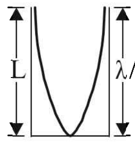
using, $v = \sqrt{\frac{T}{m}} \Rightarrow 30 = \sqrt{\frac{T}{1.3\times 10^{-4}}} \Rightarrow T = 0.117$ N

25. (c) The fundamental frequency of an organ pipe open at both ends is



$$n_0 = \frac{v}{2L} \qquad ....(i)$$

The fundamental frequency of an organ pipe closed at one end is



$$n_c = \frac{v}{4L} \ ....(ii)$$

Dividing equation (i) by (ii)

$$\frac{n_0}{n_c} = \frac{v}{2L}\times\frac{4L}{v} = \frac{2}{1}$$

26. (a) According to problem

$$\frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{v}{4L} \qquad ...(i)$$

and $\frac{1}{2L}\sqrt{\frac{T+8}{m}} = \frac{3v}{4L}$ ...(ii)

Dividing equation (i) by (ii), $\sqrt{\frac{T}{T+8}} = \frac{1}{3}$

$\Rightarrow T = 1N$

27. (b) For first pipe, $n_1 = \frac{v}{4l_1}$ and for second pipe $n_2 = \frac{v}{4l_2}$

So, number of beats $= n_2 - n_1 = 4$

$$\Rightarrow 4 = \frac{v}{4}\left(\frac{1}{l_2}-\frac{1}{l_1}\right) \Rightarrow 16 = 300\left(\frac{1}{l_2}-\frac{1}{1}\right) \Rightarrow l_2 = 94.9\ cm$$

28. (c) $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$

Given, $\frac{I_{max}}{I_{min}} = 25$

$\therefore\ \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = 5 \quad \Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{3}{2}$

$\Rightarrow \frac{I_1}{I_2} = \frac{9}{4}$

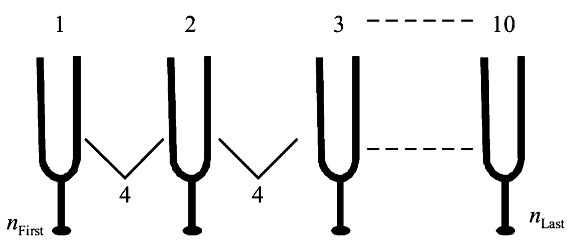
29. (b) $\omega = 2\pi f \quad \therefore\ f = \frac{\omega}{2\pi}$

$$f_1 = \frac{646\pi}{2\pi} = 323\ s^{-1}$$

$$f_2 = \frac{652\pi}{2\pi} = 326\ s^{-1}$$

No. of beats/sec = $f_2 - f_1 = 326 - 323 = 3$

30. (d)



Using $n_{Last} = n_{First} + (N-1)x$
where N = Number of tuning forks in series
x = beat frequency between two successive forks
$\Rightarrow 2n = n + (10-1)\times 4$
$\Rightarrow n = 36$ Hz

31. (d) $n' = n\left(\dfrac{v+v_D}{v-v_S}\right)$

Here, n = 600 Hz, $v_D = 15$ m/s
$v_s = 20$ m/s, v = 340 m/s

$\therefore \quad n' = 600\left(\dfrac{355}{320}\right) \approx 666$ Hz

32. (b) After 2 sec the pulses will overlap completely. The string becomes straight and therefore does not have any potential energy and it entire energy must be kinetic.

33. (a) Given : Wavelength of first wave $(\lambda_1) = 50$ cm = 0.5 m
Wavelength of second wave $(\lambda_2) = 51$ cm = 0.51m
frequency of beats per sec (n) = 12.
We know that the frequency of beats,

$$n = 12 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} \quad \text{[where, v = velocity of sound]}$$

$$\Rightarrow 12 = v\left[\frac{1}{0.5} - \frac{1}{0.51}\right] = v[2-1.9608] = v\times 0.0392$$

$$\text{or, } v = \frac{12}{0.0392} = 306 \text{ m/s}$$

34. (c) Given : Length $(l) = 7$ m
Mass (M) = 0.035 kg and tension (T) = 60.5 N
We know that mass of string per unit length (m)

$$= \frac{0.035}{7} = 0.005 \text{ kg/m}$$

and speed of wave $= \sqrt{\dfrac{T}{m}} = \sqrt{\dfrac{60.5}{0.005}} = 110$ m/s.

35. (c) Let f' be the frequency of sound heard by cliff.

$$\therefore f' = \frac{vf}{v - v_c} \qquad ....(1)$$

Now, for the reflected wave, cliff acts as a source,

$$\therefore 2f = \frac{f'(v+v_c)}{v} \qquad ....(2)$$

$$2f = \frac{(v+v_c)f}{v-v_c} \Rightarrow 2v - 2v_c = v + v_c \text{ or } \frac{v}{3} = v_c$$

36. (c) Intensity $= \dfrac{\text{Energy}}{\text{time}\times\text{area}}$

$$\text{Area} \propto r^2 \Rightarrow I \propto 1/r^2 \Rightarrow \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$$

37. (b) Beat frequency, $f = \dfrac{18}{3} = 6$ Hz

Let f_2 be the frequency of other source

$\therefore \ f_2 = f_1 \pm f = (341 \pm 6)\text{Hz} = 347$ Hz

or 335 Hz

38. (d) $n = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}} \Rightarrow n \propto \dfrac{\sqrt{T}}{l}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{l_2}{l_1}\right)^2 = (2)^2\left(\frac{3}{4}\right)^2 = \frac{9}{4}$$

39. (a) Fundamental frequency of open tubes is,

$$f = \frac{v}{2L}$$

where v is the velocity of sound in air and L is the length of the tube,

$$\therefore \ f = \frac{330 \text{ ms}^{-1}}{2\times 0.25\text{m}} = 660 \text{ Hz}$$

The emitted frequencies are $f, 2f, 3f, 4f$, ...,
i.e., 660 Hz, 1320 Hz, 1980 Hz, 2640 Hz

40. (a)

41. (a) $v_{long.} = 100 v_{trans.}$

$$\sqrt{\frac{Y}{d}} = 100\sqrt{\frac{\text{stress}}{d}}$$

$$\sqrt{1\times 10^{11}} = 100\sqrt{\text{stress}}$$

$$\text{Stress} = \frac{10^{11}}{10^4} = 10^7 \text{ N/m}^2$$

42. (b) The distance between two points i.e. path difference between them

$$\Delta = \frac{\lambda}{2\pi}\times\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{3} = \frac{\lambda}{6} = \frac{v}{6n} \quad (\because v = n\lambda)$$

$$\Rightarrow \Delta = \frac{360}{6\times 500} = 0.12\text{m} = 12 \text{ cm}$$

43. (a) The resultant amplitude is given by

$$A_R = \sqrt{A^2 + A^2 + 2AA\cos\theta} = \sqrt{2A^2(1+\cos\theta)}$$

$$= 2A\cos\frac{\theta}{2} \qquad \left(\because 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right)$$

44. (d) Path difference,

$$\Delta x = S_2P - S_1P = \sqrt{(2\sqrt{10})^2 + 3^2} - \sqrt{4^2+3^2} = 7 - 5 = 2\text{m}$$

For constructive interference,

$\Delta x = n\lambda$, where n = 1, 2, 3, ...

$\Rightarrow f = \frac{nv}{\Delta x} = \frac{1 \times 340}{2}, \frac{2 \times 340}{2}, \frac{3 \times 340}{2}, ...$

$= 170$ Hz, 340 Hz, 510 Hz .....

45. (d) Speed of sound in gases is $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow T \propto M$

(Because, v, γ-constant). Hence $\frac{T_{H_2}}{T_{O_2}} = \frac{M_{H_2}}{M_{O_2}}$

$\Rightarrow \frac{T_{H_2}}{(273+100)} = \frac{2}{32} \Rightarrow T_{H_2} = 23.2\text{K} = -249.7^\circ\text{C}$

46. (b) By using $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T}$

$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{T+600}{T}} = \sqrt{3} \Rightarrow T = 300\text{K} = 27^\circ\text{C}$

47. (b) The minimum distance between compression and rarefaction of the wire $l = \frac{\lambda}{2}$ ∴ Wave length $\lambda = 2l$

Now by $v = n\lambda \Rightarrow n = \frac{360}{2 \times 1} = 180\ \text{sec}^{-1}$

48. (b) The amplitude of the resultant wave is

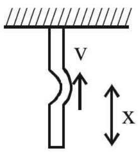
$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

Where A_1 and A_2 are the amplitude and ϕ is the phase difference between two waves.

Here, $A_1 = A_2 = 5\text{mm}, \phi = \frac{\pi}{2}$

$\therefore A = \sqrt{(5)^2 + (5)^2 + 2(5)(5)\cos\frac{\pi}{2}} = 5\sqrt{2}$ mm

49. (b) Speed of pulse at a distance x from bottom, $v = \sqrt{gx}$. While traveling from mid point to the top, frequency remains unchanged.



$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \frac{\sqrt{g(L/2)}}{\lambda_0} = \frac{\sqrt{gL}}{\lambda_2} \Rightarrow \lambda_2 = \sqrt{2}\lambda_0$

50. (c) $\omega_1 = 600\pi, \omega_2 = 604\pi$

$f_1 = 300$ Hz, $f_2 = 302$ Hz

Beat frequency, $f_2 - f_1 = 2$ Hz

⇒ number of beats in three seconds = 6

51. (a) Equation of the harmonic progressive wave given

$y = a\sin 2\pi(bt - cx)$

Here $2\pi n = \omega = 2\pi b \Rightarrow n = b$

$k = \frac{2\pi}{\lambda} = 2\pi c \Rightarrow \frac{1}{\lambda} = c$

(Here c is the symbol given for $\frac{1}{\lambda}$ and not the velocity)

∴ Velocity of the wave $= v = b\frac{1}{c} = \frac{b}{c}$

$\frac{dy}{dr} = a2\pi b\cos 2\pi(bt - cx) = a\omega\cos(\omega t - kx)$

Maximum particle velocity $= a\omega = a2\pi b = 2\pi ab$

given this is $2 \times \frac{b}{c}$

i.e., $2\pi a = \frac{2}{c}$ or $c = \frac{1}{\pi a}$

52. (c) Intensity at A, $I_A = \frac{P}{4\pi r^2}$; intensity at B, $I_B = \frac{P}{4\pi\ (2r)^2}$

Sound level at A, $S_A = 10\log\frac{I_A}{I_0}$

Sound level at B, $S_B = 10\log\frac{I_B}{I_0}$

Difference of sound level at A and B is

$S_A - S_B = 10\log\frac{I_A}{I_0} - 10\log\frac{I_B}{I_0} = 10\log\left(\frac{I_A}{I_B}\right)$

$= 10\log 4 = 20\log 2 \approx 6$ dB

53. (c) Given : Wavelength (λ) = 5000 Å

velocity of star (v) = 1.5×10^6 m/s.

We know that wavelength of the approaching star (λ')

$\lambda' = \lambda\left(\frac{c-v}{c}\right)$ or $\frac{\lambda'}{\lambda} = \left(\frac{c-v}{c}\right) = \left(1 - \frac{v}{c}\right)$

or, $\frac{v}{c} = 1 - \frac{\lambda'}{\lambda} = \frac{\lambda - \lambda'}{\lambda} = \frac{\Delta\lambda}{\lambda}$

Therefore $\Delta\lambda = \lambda \times \frac{v}{c} = 5000 \times \frac{1.5 \times 10^6}{3 \times 10^8} = 25$ Å

[where Δλ = Change in the wavelength]

54. (d) Let the frequency of the tuning fork be n Hz

Then frequency of air column at 15ºC = (n + 4)

Frequency of air column at 10ºC = (n + 3)

According to v = nλ, we have

$v_{15} = (n+4)\lambda$ and $v_{10} = (n+3)\lambda$

$\therefore \frac{v_{15}}{v_{10}} = \left(\frac{n+4}{n+3}\right)$

The speed of sound is directly proportional to the square-root of the absolute temperature.

$$\therefore \frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}} = \sqrt{\frac{288}{283}}$$

$$\therefore \left(\frac{n+4}{n+3}\right) = \sqrt{\frac{288}{283}} = \left(1+\frac{5}{283}\right)^{1/2}$$

$$\Rightarrow 1+\frac{1}{n+3} = 1 + 1/2 \times \frac{5}{283} = 1 + \frac{5}{566}$$

$$\Rightarrow \frac{1}{n+3} = \frac{5}{566}$$

$$\Rightarrow n+3 = 113 \Rightarrow n = 110 \text{ Hz}$$

55. **(a)** Speed of wave on a string $v = \sqrt{\frac{T}{m}}$

$$\Rightarrow v \propto \sqrt{T} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)^2$$

$$\Rightarrow \frac{T_2 - T_1}{T_1} = \frac{v_2^2 - v_1^2}{v_1^2} \quad \text{...(i)}$$

Given, $T = 120$ m, $v_1 = 150$ m/s

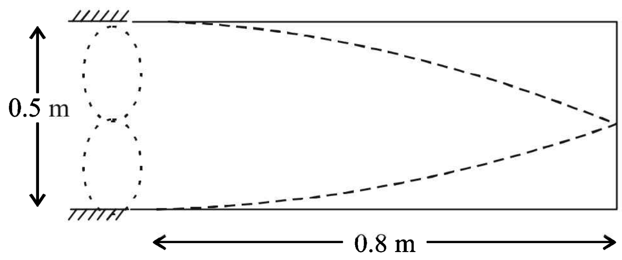
$$v_2 = v_1 + \frac{20}{100}v_1 = \frac{120}{100}v_1 = \frac{6}{5}v_1 = \frac{6}{5}\times 100 = 180 \text{ m/s}$$

Substituting the values in eq. (i), we get,

$$\frac{T_2 - T_1}{T_1} = \frac{(180)^2 - (150)^2}{(150)^2} = \frac{30 \times 330}{150 \times 150} = 0.44$$

Percent increase in tension = 44%

56. **(b)** Frequency of 2nd harmonic of string = Fundamental frequency produced in the pipe



$$\therefore 2 \times \left[\frac{1}{2l_1}\sqrt{\frac{T}{\mu}}\right] = \frac{v}{4l_2}$$

$$\therefore \frac{1}{0.5}\sqrt{\frac{50}{\mu}} = \frac{320}{4 \times 0.8}$$

$$\therefore \mu = 0.02 \text{ kg m}^{-1}$$

The mass of the string = $\mu\, l_1$

$= 0.02 \times 0.5$ kg

$= 10$ g

57. **(c)** Here, bat is a source of sound and the wall is an observer at rest.

$\therefore$ Frequency of sound reaching the wall is

$$f' = \frac{vf}{v - v_s} \quad \text{...(i)}$$

where v is the velocity of sound in air and v_s is velocity of source.

On reflection, the wall is the source of sound of frequency f' at rest and bat is an observer approaching the wall.

$\therefore$ Frequency heard by the bat is

$$f'' = \frac{f'(\upsilon + \upsilon_0)}{\upsilon} = f\left(\frac{\upsilon + \upsilon_0}{\upsilon - \upsilon_s}\right) \quad \text{[Using (i)]}$$

$$= 90 \times 10^3 \left(\frac{330+4}{330-4}\right) = \frac{90 \times 10^3 \times 334}{326}$$

$= 92.1 \times 10^3$ Hz

58. **(d)** $n' = n\left(\frac{v + v_D}{v - v_S}\right)$

Here, $n = 600$ Hz, $v_D = 15$ m/s

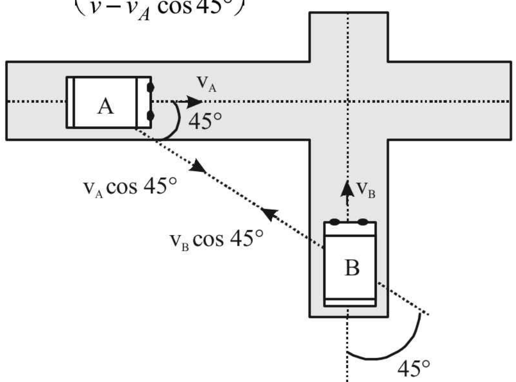
$v_s = 20$ m/s, $v = 340$ m/s

$$\therefore \quad n' = 600\left(\frac{355}{320}\right) \approx 666 \text{ Hz}$$

59. **(b)** Here $v_A = 72$ km / hr $= 20$ m / sec

$v_B = 36$ km/hr $= 10$ m/sec

$$n' = n\left(\frac{v + v_B \cos 45°}{v - v_A \cos 45°}\right)$$



$$\Rightarrow n' = 280\left(\frac{340 + 10/\sqrt{2}}{340 - 20/\sqrt{2}}\right) = 298 \text{ Hz}$$

60. **(d)** The apparent frequency heard by the stationary observer is, $n' = n\frac{v}{v + v_s}$ ... (i)

where, n_0 = frequency of source

v = velocity of sound

v_s = velocity of source

According to problem,

$$n' = n_0 + \frac{50}{100} n_0 \Rightarrow n' = \frac{3}{2} n_0$$

from (i)

$$\Rightarrow \frac{3}{2} n_0 = n_0 \left[\frac{v}{v - vs} \right] \Rightarrow 3v - 3v_s = 2v$$

$$\Rightarrow v = 3v_s$$

$$\Rightarrow vs = \frac{v}{3} = \frac{330}{3} \text{ m/sec}$$

$$\Rightarrow v_s = 110 \text{ m/sec}$$