TOPIC TEST

ANSWERS

1. Gravitation

Q.1. (A)

i. (B) ***[1 Mark]***

ii. (A) ***[1 Mark]***

Q.1. (B)

i. Centre of Earth
All others have non-zero acceleration due to gravity, whereas acceleration due to gravity at the centre of Earth is zero. ***[1 Mark]***

ii. Mass : kg : : Weight : **Newton (N)** ***[1 Mark]***

iii. Mass ***[1 Mark]***

Q.2. (A)

i.
a. The weight of the body is defined as the force with which Earth attracts the object.
b. It is given by the formula, W = mg
c. From the formula it is clear that weight of an object is directly proportional to the mass of the object and gravitational acceleration (g).
d. Therefore if the g value doubles, the weight of the object also doubles and thus it will be difficult to pull the object along the floor. ***[2 Marks]***

ii.
a. The weight of an object is defined as the force with which the earth attracts the object. It is given as,
W = F = mg
b. The weight of an object depends on the mass of the object and the value of acceleration due to gravity.
c. On the surface of the earth, the value of g is highest at the poles and decreases slowly with decreasing latitude becoming lowest at the equator.

Hence, a 1 kg of gold would weigh more at the poles and less at equator. ***[2 Marks]***

*[**Note:** Students are expected to attempt any one out of two questions]*

Q.2. (B)

i.

	Universal gravitational constant (G)		Gravitational acceleration of Earth (g)
a.	The gravitational force acting between unit masses kept at a unit distance away from each other equals gravitational constant (G).	a.	The acceleration produced in a body under the influence of the force of gravity alone is called acceleration due to gravity (g).
b.	The value of G is constant.	b.	Value of 'g' changes from place to place.
c.	Its unit is Nm^2/kg^2 or dyne cm^2/g^2	c.	Unit of g is m/s^2.
d.	It is a scalar quantity.	d.	It is a vector quantity.

[2 Marks]

ii.
a. According to Kepler's third law, the square of orbital period of revolution T of a planet around a star is directly proportional to the cube of the mean distance R of the planet from the star.
$T^2 \propto R^3$
$T^2 = kR^3$ ....(1)
Where, k is constant of proportionality.
b. When the planet is at a distance of 2R from the star, then its period of revolution T will be,
$T^2 \propto (2R)^3$
$T^2 = 8R^3$ ....(2)

c. When the planet is at a distance of 8R from the star, then its period of revolution T′ will be,

$T'^2 \propto (8R)^3$

$T'^2 = 512R^3$ ....(3)

d. Dividing equation (2) by equation (3), we get,

$$\frac{T^2}{T'^2} = \frac{8R^3}{512R^3}$$

$$\frac{T^2}{T'^2} = \frac{1}{64}$$

$T' = 8T$

Thus, for a planet at a distance of 2R from the star, its period of revolution will be 8T.

[2 Marks]

***[Note:** Students are expected to attempt any one out of two questions]*

Q.3.

i. a. The acceleration produced in a body under the influence of the force of gravity alone is called acceleration due to gravity. ***[1 Mark]***

b. *Given:* Mass of the earth $M' = 2M$, radius of the earth $R' = \frac{R}{2}$

To find: gravitational acceleration (g′)

Formula: $g = \frac{GM}{R^2}$

Calculation: From formula,

$$g' = \frac{G \times M'}{(R')^2} = \frac{G \times 2M}{\left(\frac{R}{2}\right)^2}$$

$$g' = \frac{G \times 2M \times 4}{R^2}$$

$\therefore$ $g' = 8g = 8 \times 9.8$

$\therefore$ $g = \mathbf{78.4\ m/s^2}$ i.e., 8 times the original value of g.

Ans: The value of g would be **78.4 m/s²** on the surface of the earth if its mass was twice as large and its radius half of the present value. ***[2 Marks]***

ii. a. Consider an object of mass m moving with initial velocity equal to escape velocity $v_{esc.}$ on the surface of the earth.

The kinetic energy of the object is given as, $K.E = \frac{1}{2}mv_{esc}^2$

The potential energy of the object is given as,

Potential energy $= -\frac{GMm}{R}$

$\therefore$ Total energy $= E_1 = K.E + P.E = \frac{1}{2}mv_{esc}^2 - \frac{GMm}{R}$ ....(1)

b. The object escapes the gravitational force of the earth and comes to rest at infinite distance from the earth.

The kinetic energy of the object is given as, K.E = 0

The potential energy of the object is given as,

Potential energy $= -\frac{GMm}{\infty} = 0$

$\therefore$ Total energy $= E_2 = K.E + P.E = 0$ ....(2)

c. From the principle of conservation of energy, $E_1 = E_2$

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$\therefore$ $v_{esc}^2 = \frac{2GM}{R}$

$\therefore$ $v_{esc} = \sqrt{\frac{2GM}{R}}$ ....(3)

d. Also, we know, acceleration due to gravity is given as,

$$g = \frac{GM}{R^2}$$

∴ $GM = gR^2$ ....(4)

e. Substituting equation (4) in (3), we get,

$$v_{esc} = \sqrt{\frac{2gR^2}{R}}$$

$$= \sqrt{2gR} \quad ....(5)$$

Equations (3) and (5) represent the equations for escape velocity from the surface of the earth. ***[3 Marks]***

iii. *Given:* Height (s) = 750 m, acceleration due to gravity (g) = 10 m/s^2

To find: i. Initial velocity (u), ii. time taken (t)

Formulae: i. $v^2 = u^2 + 2\,as$ ii. $s = ut + \frac{1}{2}\,at^2$

Calculation: For upward motion of the ball, (v) = 0.

$a = -g = -10$ m/s^2

From formula (i),

$0 = u^2 + 2\,(-10) \times 750$

∴ $u^2 = 50000$

∴ u = **122.47 m/s**

For downward motion of the ball, (u) = 0.

$a = g = 10$ m/s^2

From formula (ii),

$750 = 0 + \frac{1}{2} \times 10\,t^2$

∴ $t^2 = \frac{750}{5} = 150$

∴ t = 12.25 s

Time for upward journey of the ball will be the same as time for downward journey i.e., 12.25s.

∴ Total time taken = 2 × 12.25 = **24.5 s**

Ans: i. The initial velocity of the object is **122.47 m/s**.

ii. The total time taken by the object to reach the height and come down is **24.5 s**. ***[3 Marks]***

***[Note:** Students are expected to attempt any two out of three questions]*

Q.4.

i. a. From the given diagram, we understand Kepler's laws of planetary motion. ***[1 Mark]***

b. **Kepler's second law:** The line joining the planet and the Sun sweeps equal areas in equal intervals of time. ***[1 Mark]***

c. **Kepler's third law:** The square of orbital period of revolution of a planet around the Sun is directly proportional to the cube of the mean distance of the planet from the Sun. ***[1 Mark]***

d. At point P, the velocity of the planet will be maximum. ***[1 Mark]***

e. According to Kepler's second law, the line joining the planet and the Sun sweeps equal areas in equal intervals of time.

As area CF_1D = 2 × area AF_1B, the time taken to go from C to D is equal to twice the time taken to go from A to B. ***[1 Mark]***

∴ $t_1 = 2t_2$

ii. a. 1. Consider a stone thrown vertically upwards with initial velocity 'u'. It reaches a height 'h' before coming down.

2. The kinematical equations of motion are given as,

$v = u + at$ ....(i)

$s = ut + \frac{1}{2}at^2$ ....(ii)

$v^2 - u^2 = 2as$ ....(iii)

3. For upward motion of the stone,
$a = -g$ (negative sign indicates the direction of force is opposite to that of velocity.)
$v = 0$ ($\because$ at the highest point velocity becomes zero).
Substituting this in equation (i), the time t_1 taken by the stone to reach the maximum height is given as,

$\therefore$ $0 = u - gt_1$

$\therefore$ $t_1 = \frac{u}{g}$ ....(iv)

Similarly, substituting the values of a and v in equation (iii), the maximum height h which the stone reaches is given as,
$0^2 - u^2 = -2gh$

$\therefore$ $h = \frac{u^2}{2g}$ ....(v)

4. For downward motion of the stone,
$a = g$
$u = 0$ ($\because$ at maximum height, initial velocity is zero.)
Substituting this in equation (ii), the time t_2 taken by the stone to reach the maximum height is given as,

$h = 0 + \frac{1}{2}gt_2^2$

$\therefore$ $t_2^2 = \frac{2h}{g}$

$\therefore$ $t_2 = \sqrt{\frac{2h}{g}}$ ....(vi)

5. Substituting equation (v) in (vi),

$t_2 = \sqrt{\frac{2}{g} \times \frac{u^2}{2g}}$

$\therefore$ $t_2 = \frac{u}{g}$ ....(vii)

Thus, from equations (iv) and (vii), we can conclude that the time taken by the stone to go up is same as the time taken to come down. ***[3 Marks]***

b. *Given:* Time (t) = 5 s, height (s) = 5 m
To find: Gravitational acceleration (g)
Formula: $s = ut + \frac{1}{2}gt^2$
Calculation: From formula,

$5 = 0 \times t + \frac{1}{2}g(5)^2$

$\therefore$ $5 = \frac{1}{2}g \times 25$

$\therefore$ $g = \frac{2}{5}$

$\therefore$ $g = \mathbf{0.4\ m/s^2}$

Ans: The gravitational acceleration of the planet is $\mathbf{0.4\ m/s^2}$. ***[2 Marks]***

*[**Note:** Students are expected to attempt any one out of two questions]*