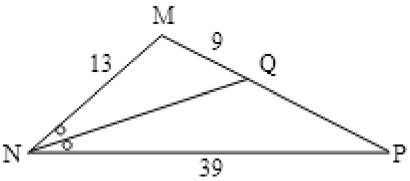
Time: 60min | **Similarity** | **Marks: 20**

Q.1(A) **Choose the correct alternative.** **(4)**

i **Find QP using given information in the figure.**



☐ (A) 26 units ☑ (B) 27 units

☐ (C) 21 units ☐ (D) 18 units

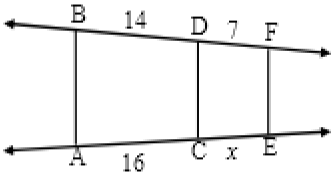
Ans: As in ΔMNP, seg NQ bisects ∠N, we get

$\frac{PN}{MN} = \frac{QP}{MQ}$ …[Property of angle bisector of a triangle]

$\therefore \frac{39}{13} = \frac{QP}{9}$

∴ QP = 3 × 9 = 27 units

ii **In the given figure, if AB || CD || FE, then *x* =**



☑ (A) 8 units ☐ (B) 4 units

☐ (C) 6 units ☐ (D) 7.5 units

Ans: (A)

Explanation:

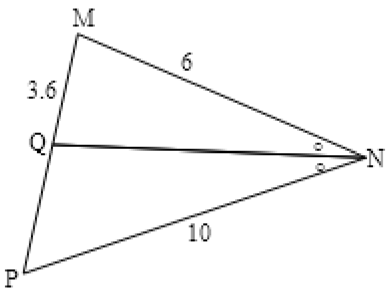
As line AB || line CD || line FE, we get

$\frac{BD}{DF} = \frac{AC}{CE}$ …[Property of three parallel lines and their transversals]

$\frac{14}{7} = \frac{16}{x}$

∴ x = 8 units

iii **In ΔMNP, NQ is a bisector of ∠N. If MN = 6, PN = 10, MQ = 3.6, then QP =**



☐ (A) 4.2 units ☐ (B) 3.6 units

☐ (C) 5 units ☑ (D) 6 units

Ans: (D)

Explanation:

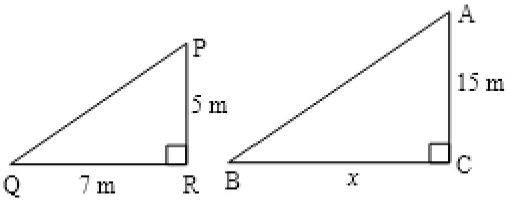
As in ΔMNP, NQ is the bisector of ∠N, we get

$\frac{PN}{MN} = \frac{QP}{MQ}$ …[Property of angle bisector of a triangle]

$\therefore \frac{10}{6} = \frac{QP}{3.6}$

$\therefore$ QP = $\frac{10 \times 3.6}{6}$ = 6 units

iv **As shown in the given figure, two poles of height 15 m and 5 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 7 m, then how long will be the shadow of the bigger pole at the same time?**



(A) 21 (B) 14

(C) 15 (D) 10

Ans: (A)

Explanation:

Here, AC and PR represent the bigger and smaller poles, and BC and QR represents their shadows respectively.

Now, ΔACB ~ ΔPRQ ...[∵ Vertical poles and their shadows form similar figures]

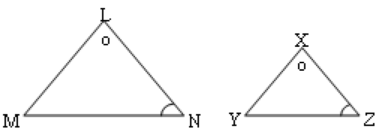
$\therefore \frac{CB}{RQ} = \frac{AC}{PR}$ ...[Corresponding sides of similar triangles]

$\therefore \frac{x}{7} = \frac{15}{5}$

$\therefore x$ = 21 m

Q.1(B) Solve the following questions. (2)

i **State the test by which the given triangles are similar.**



ΔLMN ~ ΔXYZ by AA test of similarity.

ii **ΔABC ~ ΔDEF, ∠A = 45°. and ∠F = 60° then ∠B = ?**

ΔABC ~ ΔDEF … [Given]

$\left.\begin{array}{l}\angle A = \angle D \\ \angle B = \angle E \\ \angle C = \angle F\end{array}\right\}$ ... [Corresponding angles of similar triangles]

$\therefore \angle A = \angle D = 45^\circ$

$\angle C = \angle F = 60^\circ$

In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$ … [Angle sum property of triangle]

$45^\circ + \angle B + 60^\circ = 180^\circ$

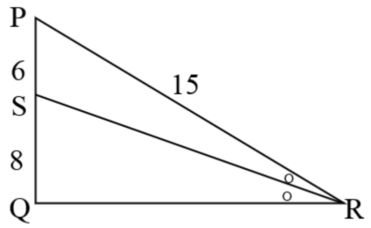
$\therefore \angle B + 105^\circ = 180^\circ$

$\therefore \angle B = 180^\circ - 105^\circ$

$\therefore \angle B = 75^\circ$

Q.2 Solve the following questions.(Any two) (4)

i **In the following figure, in ΔPQR, $seg\ RS$ is the bisector of $\angle PRQ,\ PS = 6,\ SQ = 8,\ PR = 15$. Find QR.**



In $\Delta PQR,\ seg\ RS$ is the bisector of $\angle PRQ$ ...[Given]

$\therefore \frac{PR}{QR} = \frac{PS}{SQ}$ ...[Property of angle bisector of a triangle]

$\therefore \frac{15}{QR} = \frac{6}{8}$

$\therefore QR = \frac{15 \times 8}{6}$

$\therefore$ **The value of QR is 20 units.**

ii **The ratio of the areas of two triangles with common base is $4:3$. Height of the larger triangle is $6\ cm$, then find the corresponding height of the smaller triangle.**

Let the height and area of the larger triangle be h_1 and A_1 respectively.

Let the height and area of the smaller triangle be h_2 and A_2 respectively.

$\frac{A_1}{A_2} = \frac{h_1}{h_2}$ ...[Triangles having common base]

$\therefore \frac{4}{3} = \frac{6}{h_2}$

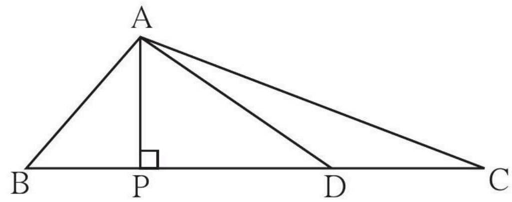
$\therefore h_2 = \frac{6 \times 3}{4}$

$= 4.5\ cm$

$\therefore$ **The height of the smaller triangle is 4.5 cm.**

iii **In $\triangle ABC$ point D on side BC is such that $DC = 6, BC = 15$. Find A (ΔABD): A(ΔABC) and A (ΔABD): A(ΔADC).**

Point A is common vertex of $\triangle ABD, \triangle ADC$ and $\triangle ABC$ and their bases are collinear. Hence, heights of these three triangles are equal



$BC = 15, DC = 6\ \therefore BD = BC - DC = 15 - 6 = 9$

$\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{BD}{BC}$ .... heights equal, hence areas proportional to bases.

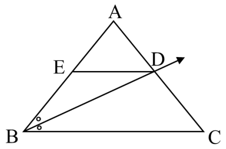
$= \frac{9}{15} = \frac{3}{5}$

$\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{BD}{DC}$ .... heights equal, hence areas proportional to bases.

$= \frac{9}{6} = \frac{3}{2}$

Q.3 **Complete the following activity.** **(3)**

i



In ΔABC, ray BD bisects ∠ABC, A–D–C, seg DE || side BC, A–E–B then for showing prove $\frac{AB}{BC} = \frac{AE}{EB}$., complete the following activity:

Proof:

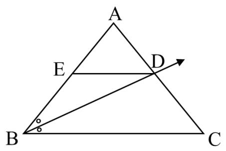
In ΔABC, ray BD bisects ∠B

∴ $\frac{\square}{BC} = \frac{AD}{DC}$ **…(i)** $\left(\boxed{}\right)$

In ΔABC, DE || BC

∴ $\frac{\square}{EB} = \frac{AD}{DC}$ **…(ii)** $\left(\boxed{}\right)$

$\frac{AB}{\square} = \frac{\square}{EB}$ **…[From (i) and (ii)]**



In ΔABC, ray BD bisects ∠B

∴ $\frac{\boxed{AB}}{BC} = \frac{AD}{DC}$ …(i) $\left(\boxed{\textit{Angle bisector theorem}}\right)$

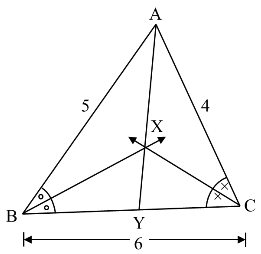
In ΔABC, DE || BC

∴ $\frac{\boxed{AE}}{EB} = \frac{AD}{DC}$ …(ii) $\left(\boxed{\textit{Basic proportionality theorem}}\right)$

$\frac{AB}{\boxed{BC}} = \frac{\boxed{AE}}{EB}$ …[From (i) and (ii)]

Q.4 Solve the following questions.(Any one) (3)

i **In the given figure, bisectors of ∠B and ∠C of ΔABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6, then find $\frac{AX}{XY}$.**



Let the value of BY be *x*.

BC = BY + YC …[B–Y–C]

∴ 6 = x + YC

∴ YC = 6 – x

In ΔBAY, ray BX bisects ∠B. …[Given]

∴ $\frac{AB}{BY} = \frac{AX}{XY}$ …(i) $\left[\begin{array}{l}\text{Property of angle bisector}\\ \text{of a triangle}\end{array}\right]$

Also, in ΔCAY, ray CX bisects ∠C. …[Given]

$\therefore \quad \dfrac{AC}{YC} = \dfrac{AX}{XY}$ ...(ii) $\left[\begin{array}{l}\text{Property of angle bisector}\\ \text{of a triangle}\end{array}\right]$

$\therefore \quad \dfrac{AB}{BY} = \dfrac{AC}{YC}$ ...[From (i) and (ii)]

$\therefore \quad \dfrac{5}{x} = \dfrac{4}{6-x}$

$\therefore \quad 5(6 - x) = 4x$

$\therefore \quad 30 - 5x = 4x$

$\therefore \quad 9x = 30$

$\therefore \quad x = \dfrac{30}{9} = \dfrac{10}{3}$

Now, $\dfrac{AX}{XY} = \dfrac{5}{\left(\frac{10}{3}\right)}$

$= \dfrac{5 \times 3}{10}$ ... $\left[\begin{array}{l}\text{Substituting the value}\\ \text{of } x \text{ in equation } (i)\end{array}\right]$

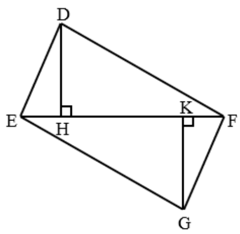
$\therefore \quad \mathbf{\dfrac{AX}{XY} = \dfrac{3}{2}}$

ii **In the following figure,** $seg\ DH \perp seg\ EF$ **and** $seg\ GK \perp seg\ EF$**. If** $DH = 6\ cm,\ GK = 10\ cm$ **and** $A(\Delta DEF) = 150\ cm^2$**, then find:**

i. EF

ii.$A(\Delta GEF)$

iii. $A(\square DFGE)$



i. In ΔDEF, EF is the base and DH is the corresponding height.

$\therefore A(\Delta DEF) = \dfrac{1}{2} \times EF \times DH$

$\therefore \quad 150 = \dfrac{1}{2} \times EF \times 6$

$\therefore EF = \dfrac{150 \times 2}{6}$

$\therefore \mathbf{EF = 50\ cm.}$

ii. In ΔGEF, EF is the base and GK is the corresponding height.

$\therefore A(\Delta GEF) = \dfrac{1}{2} \times EF \times GK$

$= \dfrac{1}{2} \times 50 \times 10$

$\therefore \mathbf{A(\Delta GEF) = 250\ cm^2}$

iii. From the given figure,

$A(\square DFGE) = A(\Delta DEF) + A(\Delta GEF)$

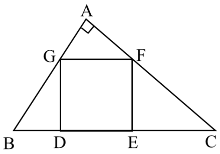
$= 150 + 250$

$\therefore \mathbf{A(\square DFGE) = 400 cm^2.}$

Q.5 **Solve the following questions.(Any one)** **(4)**

i **In the given figure, the vertices of square DEFG are on the sides of ΔABC. If ∠ A = 90°, then prove that DE^2 = BD × EC.**

(Hint: Show that ΔGBD is similar to ΔCFE. Use GD = FE = DE.)



***Proof*:**

□DEFG is a square.

∴ DE = EF = GF = GD …(i) [Sides of a square]

∠GDE = ∠DEF = 90° …[Angles of a square]

∴ seg GD ⊥ side BC, seg FE ⊥ side BC …(ii)

In ΔBAC and ΔBDG,

∠BAC ≅ ∠BDG …[From (ii), each angle is of measure 90°]

∠ABC ≅ ∠DBG …[Common angle]

∴ ΔBAC ~ ΔBDG …(iii) [AA test of similarity]

In ΔBAC and ΔFEC,

∠BAC ≅ ∠FEC …[From (ii), each angle is measure 90°]

∠ACB ≅ ∠ECF …[Common angle]

∴ ΔBAC ~ ΔFEC …(iv) [AA test of similarity]

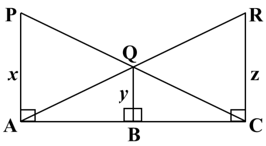
∴ ΔBDG ~ ΔFEC …[From (iii) and (iv)]

∴ $\frac{\text{BD}}{\text{EF}} = \frac{\text{GD}}{\text{EC}}$ …(v) $\begin{bmatrix}\text{Corresponding sides}\\ \text{of similar triangles}\end{bmatrix}$

∴ $\frac{\text{BD}}{\text{DE}} = \frac{\text{DE}}{\text{EC}}$ …[From (i) and (v)]

∴ $\mathbf{DE^2 = BD \times EC}$

ii **In the given figure, seg PA, seg QB and seg RC are perpendicular to seg AC. From the information given in the figure, prove that: $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.**



Proof: Let AB = a, BC = b, AC = p

In ΔRCA and ΔQBA,

∠RCA ≅ ∠QBA …[Each angle is 90°]

∠RAC ≅ ∠QAB …[Common angle]

∴ ΔRCA ~ ΔQBA …[By AA test of similarity]

∴ $\frac{\text{RC}}{\text{QB}} = \frac{\text{CA}}{\text{BA}}$ …[Corresponding sides of similar triangles]

∴ $\frac{z}{y} = \frac{p}{a}$

∴ a = $\frac{py}{z}$ …(i)

In ΔPAC and ΔQBC,

∠PAC ≅ ∠QBC …[Each angle is 90°]

∠PCA ≅ ∠QCB …[Common angle]

∴ ΔPAC ~ ΔQBC …[By AA test of similarity]

∴ $\frac{\text{PA}}{\text{QB}} = \frac{\text{AC}}{\text{BC}}$ …[Corresponding sides of similar triangles]

$\therefore \quad \frac{x}{y} = \frac{p}{b}$

$\therefore \quad b = \frac{py}{x}$ …(ii)

$a + b = \frac{py}{z} + \frac{py}{x}$ …[Adding (i) and (ii)]

$\therefore \quad p = py\left(\frac{1}{z} + \frac{1}{x}\right)$ …[A-B-C]

$\therefore \quad \frac{p}{py} = \frac{1}{z} + \frac{1}{x}$

$\therefore \quad \mathbf{\frac{1}{x} + \frac{1}{z} = \frac{1}{y}}$