Excel Coaching Classes

Time: 60min	Electrostatics	Marks: 20

SECTION A

Q.1 **Select and write the correct answer for the following multiple choice type of questions:** **(3)**

i **If the charge on the condenser of 10 μF is doubled, then the energy stored in it becomes ______.**

- [ ] (A) zero
- [ ] (B) twice that of initial energy
- [ ] (C) half the initial energy
- [x] (D) four times the initial energy

["D"]

ii **The flow of electrons which does not pass through the battery is known as _______ current.**

- [x] (A) conduction
- [ ] (B) charging
- [ ] (C) leakage
- [ ] (D) displacement

Ans: ["A"]

Explanation:

The type of current which flows from the positive plate of the capacitor to the negative plate of the capacitor without passing through the battery is conduction current.

iii **A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential, capacitance respectively is**

- [ ] (A) constant, decreases, decreases
- [ ] (B) increases, decreases, decreases
- [ ] (C) constant, decreases, increases
- [x] (D) constant, increases, decreases

Ans: ["D"]

Explanation:

For a parallel plate capacitor:

Charge is conserved. Hence, remains constant.

i. $C = \frac{A\varepsilon_0}{d} \Rightarrow C \propto \frac{1}{d}$

ii. $V = Ed \Rightarrow V \propto d$

Q.2 **Answer the following questions:** **(3)**

i **State formula for work done against electrostatic force while displacing charge +q_0 at distance r_1 towards charge +Q at distance r_2.**

Work done against electrostatic force = Change in electrostatic potential energy

W = ΔU = $\frac{Qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$

ii **A parallel plate air condenser has a capacity of 15 μF. What will be the new capacity if a marble slab of dielectric constant 6 is introduced between the two plates such that entire space between plates is filled by marble slab?**

The capacity of parallel plate condenser with dielectric is, C′ = kC_0

∴ C′ = 15 × 6 = **90 μF**

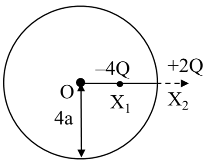
iii **What is capacitor?**

Capacitor is a system consisting of two conductors having equal and opposite charges separated by an insulator or dielectric.

SECTION B

Attempt any TWO questions of the following: **(4)**

Q.3 **Two charges of magnitudes –4Q and +2Q are located at points (2a, 0) and (5a, 0) respectively. What is the electric flux due to these charges through a sphere of radius 4a with its centre at the origin?**



Given: d(OX_1) = 2a

d(OX_2) = 5a

Radius, r = 4a

A Gaussian surface is of radius 4a and contains charge –4Q inside.

As the flux depends only upon charge enclosed by Gaussian surface, required flux according to Gauss' law would be,

$\phi = \frac{q}{\varepsilon_0} = \frac{-4Q}{\varepsilon_0}$

Q.4 **Explain energy storing in a capacitor.**

i. A capacitor is a device used to store energy.

ii. Charging a capacitor means transferring electron from one plate of the capacitor to the other. Hence work will have to be done by the battery in order to remove the electrons against the opposing forces.

iii. These opposing forces arise as the electrons are being pushed to the negative plate which repels them and electrons are removed from the positive plate which tends to attract them.

iv. In both the cases, the coulombian forces oppose the transfer from one plate to another.

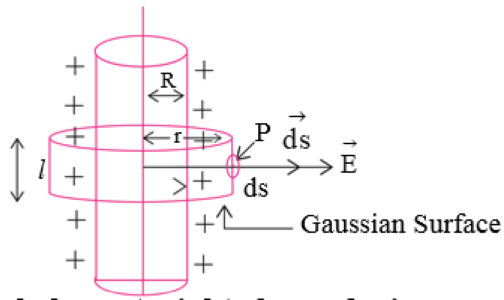
v. As the charges on the plate increases, its opposing force also increases.

vi. This work done is stored in the form of electrostatic energy in the electric field between the plates, which can later be recovered by discharging the capacitor.

Q.5 Obtain an expression for electric field intensity due to an infinitely long straight charged wire or charged conducting cylinder.

i. Consider a uniformly charged wire of infinite length having a constant linear charge density λ (charge per unit length), kept in a medium of permittivity $\varepsilon(\varepsilon = \varepsilon_0 k)$.

ii. To find the electric field intensity at P, at a distance r from the axis of the charged wire, imagine a coaxial Gaussian cylinder of length *l* and radius r (closed at each end by plane caps normal to the axis) passing through the point P as shown in the figure. Consider a very small area ds at the point P on the Gaussian surface.



Infinitely long straight charged wire (cylinder)

iii. By Gauss' theorem, the net flux through a closed surface,

$\phi = \frac{q}{\varepsilon_0}$ (for air/vacuum k = 1) ….(1)

where q is the total charge inside the closed surface.

iv. By symmetry, the magnitude of the electric field will be the same at all the points on the curved surface of the cylinder and will be directed radially outward. The angle between the direction of $\vec{E}$ and the normal to the curved surface of the cylinder $\left(\vec{ds}\right)$ is zero. Similarly the angle is (π/2) for the flat surface of the cylinder.

i.e., cos θ = 1 (for curved surface)

or cos θ = 0 (for flat surface)

$\therefore \quad \vec{E} \cdot \vec{ds}$ = Eds cosθ = 0 ....(For flat surface)

$\therefore \quad \vec{E} \cdot \vec{ds}$ = Eds cosθ = Eds ....(For curved surface)

Flux dφ through the area ds, dφ = Eds

Total electric flux through the Gaussian surface

$\phi = \oint \vec{E} \cdot \vec{ds} = \oint E ds = E \oint ds$

$\therefore \quad \phi = E \times 2\pi r l$ ….(2)

v. From equations (1) and (2)

$\frac{q}{\varepsilon_0} = E \times 2\pi r l$

Since $\lambda = \frac{q}{l}$, $q = \lambda l$

$\therefore \quad \frac{\lambda l}{\varepsilon_0} = E 2\pi r l$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

The direction of the electric field E is directed outward if λ is positive and inward if λ is negative.

Q.6 (A) How is capacitor represented symbolically?

Symbol: —| |—

(B) Determine the work done in bringing a charge of 5 μC from infinity to the point A. The potential at point A is 400 kV.

Work done, $W = qV = 5 \times 10^{-6} \times 400 \times 10^{3} = 2000 \times 10^{-3} = $ **2 J**

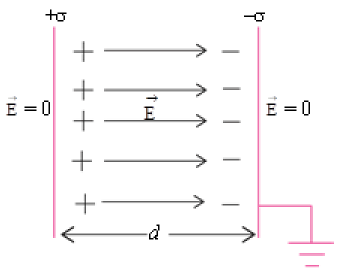
SECTION C

Attempt any TWO questions of the following: (6)

Q.7 Obtain an expression for capacitance of a parallel plate capacitor without a dielectric.

i. A parallel plate capacitor consists of two thin conducting plates each of area A, held parallel to each other, at a suitable distance d apart.

ii. The plates are separated by an insulating medium like paper, air, mica, glass etc. One of the plates is insulated and the other is earthed as shown in figure below.



iii. When a charge +Q is given to the insulated plate, then a charge –Q is induced on the inner face of earthed plate and +Q is induced on its farther face. But as this face is earthed the charge +Q being free, flows to earth.

iv. In the outer regions the electric fields due to the two charged plates cancel out. Making net field is zero.

$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

v. In the inner regions between the two capacitor plates the electric fields due to the two charged plates add up. The net field is thus

$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ ….(1)

The direction of E is from positive to negative plate.

vi. Let V be the potential difference between the two plates. Then electric field between the plates is given by

$E = \frac{V}{d}$ or $V = Ed$ ….(2)

Substituting equation (1) in equation (2),

$V = \frac{Q}{A\varepsilon_0} d$

vii. Thus, capacitance of the parallel plate capacitor is given by,

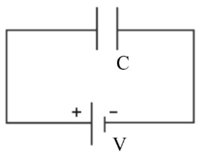
$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{A\varepsilon_0}{d}$$

This is the required expression.

Q.8

Obtain an expression for energy of a charged capacitor and express it in different forms.

i. Consider a capacitor of capacitance C being charged by a DC source of V volts as shown in figure below.



Capacitor charged by a DC source

ii. During the process of charging, let q′ be the charge on the capacitor and V be the potential difference between the plates. Hence $C = \frac{q}{V}$

iii. A small amount of work is done if a small charge dq is further transferred between the plates.

$\therefore$ $dW = V\,dq = \frac{q'}{C}\, dq$

iv. Total work done in transferring the charge

$$W = \int dW = \int_0^Q \frac{q'}{C}\, dq = \frac{1}{C}\int_0^Q q'\, dq$$

$$= \frac{1}{C}\left[\frac{(q')^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

v. This work done is stored as electrical potential energy U of the capacitor. This work done can be expressed in different forms as follows.

$\therefore$ $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$ ….($\because$ Q = CV)

Q.9 **A parallel plate air condenser has a capacity of 20 μF. What will be the new capacity if**

i. the distance between two plates is doubled?

ii. a marble slab of dielectric constant 8 is introduced between the two plates?

Given: C_1 = 20 μF, d_2 = 2d, $k_1 = k_2$ = k (air),
k_3 = 8

To find: i. The new capacity if distance between two plates is doubled (C_2)

ii. The new capacity if a marble slab is introduced between two plates (C_3)

Formula: $C = \frac{A\varepsilon_0 k}{d}$

Calculation:

i. From formula,

$C_1 = \frac{A\varepsilon_0 k}{d_1}$ and

$C_2 = \frac{A\varepsilon_0 k}{d_2} = \frac{A\varepsilon_0 k}{2d_1}$

$\therefore \frac{C_2}{C_1} = \frac{A\varepsilon_0 k/2d_1}{A\varepsilon_0 k/d_1}$

$\therefore \frac{C_2}{C_1} = \frac{1}{2}$

$\therefore C_2 = C_1 \times \frac{1}{2} = 20 \times \frac{1}{2}$

$\therefore$ C_2= **10 μF**

ii. $C_3 = \frac{A\varepsilon_0 k_3}{d}$

$= \frac{A\varepsilon_0 \times 8}{d}$ [$\because$ Distance between plates remains same]

$= 8C_1 = 8 \times 20 = 160$ μF

$\therefore$ C_3= **160 μF**

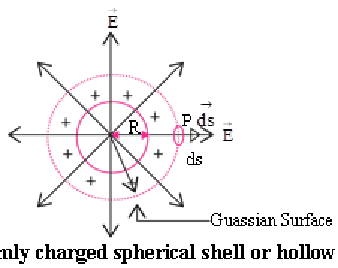
SECTION D

Attempt any ONE question of the following: (4)

Q.10 Obtain expression for electric field intensity due to uniformly charged spherical shell or hollow sphere.

i. Consider a sphere of radius R with its centre at O, charged to a uniform surface charge density σ placed in a dielectric medium of permittivity ε ($\varepsilon = \varepsilon_0 k$). The total charge on the sphere, $q = \sigma \times 4\pi R^2$.

ii. To find the electric field intensity at a point P, at a distance r from the centre of the charged sphere, imagine a concentric Gaussian sphere of radius r passing through P as shown in the figure below. Let ds be a small area around the point P on the Gaussian surface.



Uniformly charged spherical shell or hollow sphere

iii. By Gauss' theorem, the net flux through a closed Gaussian surface,

$\phi = \frac{q}{\varepsilon_0}$ (for air/vacuum k = 1) ….(1)

where q is the total charge inside the closed surface.

iv. Due to symmetry and spheres being concentric, the electric field at each point on the Gaussian surface has the same magnitude E and it is directed radially outward. Also, the angle between the direction of E and the normal to the surface of the sphere (ds) is zero i.e., cos θ = 1

$\therefore \quad \vec{E} \cdot \vec{ds}$ = E ds cosθ = E ds

$\therefore$ Flux $d\phi$ through the area ds, $d\phi$ = E ds

Total electric flux through the Gaussian surface

$$\phi = \oint \vec{E} \cdot \vec{ds} = \oint E ds = E \oint ds$$

$\therefore \quad \phi = E\, 4\pi r^2 \quad ....(2)$

v. From equations (1) and (2),

$$\frac{q}{\varepsilon_0} = E\, 4\pi r^2$$

$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 r^2} \quad ....(3)$

Substituting $q = \sigma \times 4\pi R^2$ in equation (3),

$$E = \sigma \times \frac{4\pi R^2}{4\pi\varepsilon_0 r^2}$$

$\therefore \quad E = \frac{\sigma R^2}{\varepsilon_0 r^2}$

From equation (3) it can be seen that, the electric field at a point outside the shell is the same as that due to a point charge.

vi. Direction of electric field is outward if shell or sphere is positively charged and inward if it is negatively charged.

vii. Thus, it can be concluded that a uniformly charged sphere is equivalent to a point charge at its centre.

viii. **Special cases:**

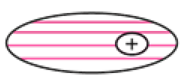
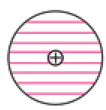
Case (a): If point P lies on the surface of the charged sphere, r = R

$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0}$

Case (b): If point P lies inside the sphere Since there are no charges inside, σ = 0

$\therefore \quad E = 0$

Q.11 (A) Distinguish between polar and non-polar dielectrics.

Sr. No.	Polar dielectric	Non-polar dielectric
i.	A dielectric molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons), because of the asymmetric shape of the molecules is called polar dielectric.	A dielectric in which the centre of mass of the positive charges coincides with the centre of mass of the negative charges is called a non-polar dielectric.
ii.	Representation: 	Representation: 

iii.	They have permanent dipole moments of the order of 10^{-30} Cm. They act as tiny electric dipoles, as the charges are separated by a small distance.	These have symmetrical shapes and have zero dipole moment in the normal state.
iv.	Examples: HCl, water, alcohol, NH_3	Examples: H_2, N_2, O_2, CO_2, benzene, methane

(B) A wire is bent in a circle of radius 10 cm. It is given a charge of 250 μC which spreads on it uniformly. What is the electric potential at the centre?

Given: $q = 250\ \mu C = 250 \times 10^{-6}$ C

$r = 10$ cm $= 10^{-1}$ m

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$

To find: Electric potential (V)

Formula: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Calculation: From formula

$$V = \frac{9 \times 10^9 \times 250 \times 10^{-6}}{0.1}$$

$$= 225 \times 10^{9-6+1+1}$$

$$= \mathbf{2.25 \times 10^7\ V}$$