Excel Coaching Classes

Date: 19-Jul-2026
Time: 60min

Pair of Lines

Marks: 20

SECTION A

Q.1 **Select and write the correct answer for the following multiple choice type of questions:** **(4)**

i **If the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product, then the value of 'k' is _______.**

- [ ] (A) 2
- [ ] (B) 1
- [ ] (C) –1
- [x] (D) –2

Ans: ["D"]

Explanation:

According to the given condition,

$m_1 + m_2 = 2m_1m_2$ ...(i)

Here, a = 1, h = $\frac{k}{2}$, b = –3

$m_1 + m_2 = \frac{-2h}{b} = \frac{-k}{-3} = \frac{k}{3}$

$m_1m_2 = \frac{a}{b} = \frac{1}{-3}$

$\therefore$ (i) $\Rightarrow$ k = –2

ii **If the lines represented by $3y^2 + 9xy + kx^2 = 0$ are perpendicular to each other, then k =**

- [ ] (A) 1
- [ ] (B) –2
- [x] (C) –3
- [ ] (D) 0

Ans: ["C"]

Explanation:

Given equation of pair of lines is

$3y^2 + 9xy + kx^2 = 0$

i.e. $kx^2 + 9xy + 3y^2 = 0$

$\therefore$ a = k, b = 3

The lines are perpendicular

$\therefore$ a + b = 0

$\Rightarrow$ k + 3 = 0 $\Rightarrow$ k = –3

Q.2 **Answer the following questions:** **(2)**

i **Show that lines represented by $x^2 + 6xy + 9y^2 = 0$ are coincident.**

Given equation of the lines is $x^2 + 6xy + 9y^2 = 0$

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get a = 1, h = 3, b = 9

Consider $h^2 - ab = (3)^2 - 1 \times 9$

$= 9 - 9 = 0$

$\therefore$ Lines represented by $x^2 + 6xy + 9y^2 = 0$ are coincident.

ii **Find the value of product of the slopes for pair of lines $2x^2 + 8xy + 3y^2 = 0$.**

Given equation of the lines is $2x^2 + 8xy + 3y^2 = 0$

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get

a = 2, 2h = 8, b = 3

Let m_1 and m_2 be the slopes of the lines represented by $2x^2 + 8xy + 3y^2 = 0$

$\therefore m_1.m_2 = \frac{a}{b} = \frac{2}{3}$

SECTION B

Attempt any TWO of the following questions: **(4)**

Q.3 **Find the measure of the acute angle between the lines represented by $2x^2 + 7xy + 3y^2 = 0$**

Given equation of the lines is $2x^2 + 7xy + 3y^2 = 0$.

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get

a = 2, h = $\frac{7}{2}$ and b = 3

Let θ be the acute angle between the lines.

$\therefore \quad \tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\left(\frac{7}{2}\right)^2-2\left(3\right)}}{2+3}\right|$

$$= \left|\frac{2\sqrt{\frac{49}{4}-6}}{5}\right|$$

$$= \left|\frac{2\sqrt{\frac{49-24}{4}}}{5}\right| = \left|\frac{2.\frac{5}{2}}{5}\right|$$

$\therefore \quad \tan\theta = 1$

$\therefore \quad \theta = \tan^{-1}(1)$

$\therefore \quad \theta = 45°$

Q.4 **Find the joint equation of lines $x + y - 3 = 0$ and $2x + y - 1 = 0$.**

Given equation of the lines are $x + y - 3 = 0$ and $2x + y - 1 = 0$

$\therefore$ Joint equation of these lines is $(x + y - 3)(2x + y - 1) = 0$

$\therefore \quad 2x^2 + xy - x + 2xy + y^2 - y - 6x - 3y + 3 = 0$

$\therefore \quad 2x^2 + 3xy + y^2 - 7x - 4y + 3 = 0$

Q.5 **Find k if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$.**

Given equation of the lines is $6x^2 + kxy + y^2 = 0$.

Its auxiliary equation is $m^2 + km + 6 = 0$

Since $2x + y = 0$ is one of the lines given by $6x^2 + kxy + y^2 = 0$

Slope of the line $2x + y = 0$ is -2.

$\therefore$ -2 is one of the roots of the auxiliary equation $m^2 + km + 6 = 0$

$\therefore \quad (-2)^2 + k(-2) + 6 = 0$

$\therefore \quad 4 - 2k + 6 = 0$

$\therefore \quad 2k = 10$

$\therefore \quad k = 5$

SECTION C

Attempt any TWO of the following questions: **(6)**

Q.6 **Find k, if slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ exceeds the slope of the other by 8.**

Given equation of the lines is $kx^2 + 4xy - y^2 = 0$.

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get a = k, 2h = 4, b = –l.

Let m_1 and m_2 be the slopes of the lines represented by $kx^2 + 4xy - y^2 = 0$.

$\therefore$ $m_1 + m_2 = -\frac{2h}{b} = 4$ and $m_1 m_2 = \frac{a}{b} = -k$

According to the given condition,

$m_2 = m_1 + 8$

Now, $m_1 + m_2 = 4$

$\therefore$ $m_l + (m_1 + 8) = 4$

$\therefore$ $2m_l = -4$

$\therefore$ $m_1 = -2$ …(i)

Also, $m_1 m_2 = -k$

$\therefore$ $m_1(m_l + 8) = -k$

$\therefore$ $(-2)(-2 + 8) = -k$ …[From (i)]

$\therefore$ $-2(6) = -k$

$\therefore$ $-12 = -k$

$\therefore$ $k = 12$

Q.7 **If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ bisects an angle between the co-ordinate axes, then show that $(a + b)^2 = 4h^2$.**

Auxiliary equation of $ax^2 + 2hxy + by^2 = 0$ is $bm^2 + 2hm + a = 0$.

Since one of the lines bisects an angle between the co-ordinate axes, the line will make an angle of 45° or 135° with the positive direction of X-axis.

$\therefore$ Slope of the line = tan 45° or tan 135°

$\therefore$ m = tan 45° or tan(180° – 45°)

$\therefore$ m = 1 or –1

$\therefore$ m = ±1 are the roots of the auxiliary equation.

$bm^2 + 2hm + a = 0$.

$\therefore\quad b(\pm 1)^2 + 2h(\pm 1) + a = 0$

$\therefore\quad b \pm 2h + a = 0$

$\therefore\quad a + b = \mp 2h$

$\therefore\quad (a + b)^2 = 4h^2$

Q.8 **Show that equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of lines.**

Given equation of the lines is

$2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$

Comparing with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 2$, $h = -\frac{1}{2}$, $b = -3$, $g = -3$, $f = \frac{19}{2}$, $c = -20$

Consider $abc + 2fgh - af^2 - bg^2 - ch^2$

$= (2)(-3)(-20) + 2\left(\frac{19}{2}\right)(-3)\left(-\frac{1}{2}\right) - (2)\left(\frac{19}{2}\right)^2 - (-3)(-3)^2 - (-20)\left(-\frac{1}{2}\right)^2$

$= 120 + \frac{57}{2} - \frac{361}{2} + 27 + 5$

$= 152 - \frac{304}{2}$

$= 0$

Now, $h^2 - ab = \left(-\frac{1}{2}\right)^2 - (2)(-3)$

$= \frac{1}{4} + 6$

$= \frac{25}{4}$

$\therefore\quad h^2 - ab \geq 0$

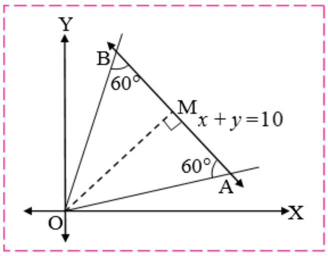
$\therefore$ The given equation represents a pair of lines.

SECTION D

Attempt any ONE of the following questions: **(4)**

Q.9 **Show that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 10$ contain the sides of an equilateral triangle. Find the area of the triangle.**

Let OA and OB be the lines passing through the origin, each making an angle of 60° with $x + y = 10$. Slope of $x + y - 10 = 0$ is -1



Let m be the slope of OA (or OB)

$\therefore$ its equation is $y = mx$ …(i)

Also, $\tan 60° = \left|\frac{m-(-1)}{1+m(-1)}\right|$

$\therefore$ $\sqrt{3} = \left|\frac{m+1}{1-m}\right|$

By taking square of both sides, we get

$3 = \frac{(m+1)^2}{(1-m)^2}$

$\therefore$ $3(1 - 2m + m^2) = m^2 + 2m + 1$

$\therefore$ $3 - 6m + 3m^2 = m^2 + 2m + 1$

$\therefore$ $2m^2 - 8m + 2 = 0$

$\therefore$ $m^2 - 4m + 1 = 0$

$\therefore$ $\left(\frac{y}{x}\right)^2 - 4\left(\frac{y}{x}\right) + 1 = 0$ …[From (i)]

$\therefore$ $y^2 - 4xy + x^2 = 0$

$\therefore$ $x^2 - 4xy + y^2 = 0$ is the joint equation of the two lines through the origin, each making an angle of 60° with the origin.

$\therefore$ $x^2 - 4xy + y^2 = 0$ and $x + y = 10$ form an equilateral triangle OAB.

To find the area of the equilateral triangle, let seg OM $\perp$ AB.

$\therefore$ $OM = \left|\frac{-10}{\sqrt{1+1}}\right| = 5\sqrt{2}$

$\therefore$ Area of $\Delta OAB = \frac{(OM)^2}{\sqrt{3}} = \frac{\left(5\sqrt{2}\right)^2}{\sqrt{3}}$

$\therefore$ Area of $\Delta OAB = \frac{50}{\sqrt{3}}$ sq. units

Q.10 **Find p and q if the equation $2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$ represents a pair of parallel lines.**

Given equation is $2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$

Comparing with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 2, h = 4, b = p, g = \frac{q}{2}, f = 1, c = -15$

The given equation represents a pair of lines parallel to each other

$\therefore \quad h^2 = ab$

$\therefore \quad (4)^2 = 2p$

$\therefore \quad p = 8 \qquad \ldots(i)$

Also, the given equation represents a pair of lines

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\therefore \quad (2)(p)(-15) + 2(1)\left(\frac{q}{2}\right)(4) - (2)(1)^2 - (p)\left(\frac{q}{2}\right)^2 - (-15)(4)^2 = 0$

$\therefore \quad (2)(8)(-15) + 4q - 2 - (8)\frac{q^2}{4} + (15)(16) = 0 \quad \ldots$[From (i)]

$\therefore \quad -(15)(16) + 4q - 2 - 2q^2 + (15)(16) = 0$

$\therefore \quad q^2 - 2q + 1 = 0$

$\therefore \quad (q - 1)^2 = 0$

By taking square root of both sides, we get

$q - 1 = 0$

$\therefore \quad q = 1$

$\therefore \quad p = 8$ and $q = 1$