Excel Coaching Classes

Time: 60min	Gravitation	Marks: 20

Q.1(A) **Choose the correct alternative.** **(2)**

i
According to Newton's law of gravitation, for two objects kept at a distance d from each other, if the distance between the objects is doubled, then the force between the two objects _______.

- [ ] (A) increases 4 times
- [x] (B) decreases 4 times
- [ ] (C) decreases 2 times
- [ ] (D) increases 2 times

Ans: ["B"]

Explanation:

F= $\frac{Gm_1m_2}{r^2}$

$F_1= \frac{Gm_1m_2}{(2r)^2} = \frac{Gm_1m_2}{4r^2} = \frac{F}{4}$

ii
When a feather and a heavy stone are dropped from a same height at the same time in vacuum,

- [x] (A) they both will reach the earth at the same time
- [ ] (B) the feather will reach the earth first
- [ ] (C) the stone will reach the earth first
- [ ] (D) it cannot be predicted as the given data is insufficient

Ans: ["A"]

Explanation:

In a vacuum, there is no air resistance. Therefore, both the feather and the heavy stone will experience the same acceleration due to gravity (g=9.8m/s^2). Since they are dropped from the same height at the same time, they will reach the earth at the same time.

Q.1(B) **Answer the following.** **(3)**

i Complete the analogy:

Increase in altitude: value of g decreases :: at the centre of the earth: _______.

Value of g increases with increase in altitude while the value of g becomes zero at the centre of the earth.

ii State True or False:

As we go above the earth's surface, the value of acceleration due to gravity increases.

Ans: ["B"]

Explanation:

As we go above the earth's surface, the value of acceleration due to gravity decreases.

iii Name the following:

Amount of matter present in an object

Mass

Q.2(A) Give scientific reasons. (Attempt any 1) (2)

i **If the value of g suddenly becomes twice its value, it will become two times more difficult to pull a heavy object along the floor. Why?**

i. In order to pull an object along the floor, it is necessary to do work against the force of friction.

ii. The force of friction is directly proportional to the weight (mg).

iii. If the value of g doubles, the weight of an object doubles, and therefore, the force of friction also doubles.

iv. Thus, it will become two times more difficult to pull a heavy object along the floor.

ii **Explain why the value of g is zero at the centre of the earth.**

i. The acceleration due to gravity (g) on earth's surface is given as, $g = \frac{GM}{R^2}$

The value of g depends on the mass M of the earth and the radius R of the earth.

ii. As we go inside the earth, the value of R decreases.

iii. The part of the earth which contributes towards the gravitational force felt by the object also decreases. It means that the value of M decreases.

iv. As a combined result of change in R and M, the value of g decreases as we go deep inside the earth.

v. At the centre of the earth, the value of R would be zero. Hence, the value of g would be zero too.

Q.2(B) Answer the following. (Attempt any 1) (2)

i(a) **What is escape velocity?**

Escape velocity:

When a body is thrown vertically upwards, at a particular value of initial velocity, the body will be able to overcome the downward pull by the earth and can escape the earth forever and will not fall back on the earth. This velocity is called escape velocity.

(b) **How would be the motion of the moon in absence of the constant force acting on it?**

If there was no force acting on the moon, it would have moved along a straight line which is the tangent to the circle.

ii **Let the period of revolution of a planet at a distance R from a star be T. Prove that if it was at a distance of 2R from the star, its period of revolution will be $\sqrt{8}T$.**

i. According to Kepler's third law, the square of orbital period of revolution T of a planet around a star is directly proportional to the cube of the mean distance R of the planet from the star.

$T^2 \propto R^3$

$\therefore\ T^2 = k(R)^3$ ….(1)

Where, k is constant of proportionality.

ii. When the planet is at a distance of 2R from the star, then its period of revolution T′ will be,

$T'^2 \propto (2R)^3$

$\therefore\ T'^2 = k(2R)^3$ ….(2)

iii. Dividing equations (1) and (2), we get,

$$\frac{T^2}{T'^2} = \frac{(R)^3}{(2R)^3}$$

$$\therefore\ \frac{T^2}{T'^2} = \frac{1}{8}$$

$\therefore\ T' = \sqrt{8}T$

Thus, for a planet at a distance of 2R from the star, its period of revolution will be $\sqrt{8}T$.

iii **There is one body in space which has mass thrice as that of the earth and a radius four times as that of the earth. If the weight of a book on earth is 80 N, what will be its weight on that body?**

Given: Mass of the body (M) = $3M_E$, Radius of the body (R) = $4R_E$, Weight of a book on earth (W_E) = 80 N

To find: Weight of the book on the body (W)

Formula: W = mg = $\frac{GMm}{R^2}$

Calculations: From formula,

Weight of the book on earth,

$80 = \frac{GM_E m}{R_E^2}$ ….(i)

For weight of the book on the body is,

$W = \frac{G(3M_E)m}{(4R_E)^2} = \frac{3}{16} \times \left(\frac{GM_E m}{R_E^2}\right) = \frac{3}{16} \times 80$ ….[From (i)]

$\therefore$ W = **15 N**

Q.3 Answer the following. (Attempt any 2) (6)

i **An object thrown vertically upwards reaches a height of 500 m. What was its initial velocity? How long will the object take to come back to the earth? Assume g = 10 m/s^2.**

Given: Height (s) = 500 m, acceleration due to gravity (g) = 10 m/s^2

To find: i. Initial velocity (u),

ii. Total time taken

Formulae: i. $v^2 = u^2 + 2as$

ii. $s = ut + \frac{1}{2}at^2$

Calculation: For upward motion of the ball, (v) = 0.

$a = -g = -10$ m/s^2

From formula (i),

$0 = u^2 + 2(-10) \times 500$

$\therefore$ $u^2 = 10000$

$\therefore$ u = **100 m/s**

For downward motion of the ball, (u) = 0.

$a = g = 10$ m/s^2

From formula (ii),

$500 = 0 + \frac{1}{2} \times 10\, t^2$

$\therefore \quad t^2 = \frac{500}{5} = 100$

$\therefore \quad t = 10$ s

Time for upward journey of the ball will be the same as time for downward journey i.e., 10 s.

$\therefore$ Total time taken = 2 × t = 2 × 10 = **20 s**

ii(a) **The mass and weight of an object on earth are 5 kg and 49 N respectively. What will be their values on the moon? Assume that the acceleration due to gravity on the moon is 1/6th of that on the earth.**

Given: Mass on earth (m_e) = 5 kg, weight on earth (W_e) = 49 N,

acceleration due to gravity on moon (g_m) = 9.8/6 = 1.63 m/s^2

To find: Mass (m_m), weight (W_m) on moon

Formula: $W_m = m_m g_m$

Calculation: The mass of the object is independent of gravity and remains unchanged i.e., **5 kg**.

From formula,

$W_m = 5 \times 1.63$

$\therefore$ W_m = **8.15 N**

(b) **What is centripetal force?**

Centripetal force:

The force acting on any object moving in a circle and directed towards the centre of the circle is called as centripetal force.

iii **Explain the factors affecting the value of gravitational acceleration of the earth ‘g’.**

The following factors affect the value of gravitational acceleration of the earth ‘g’:

i. Shape of the earth:

a. The shape of the earth is not perfectly spherical. It is slightly flattened at the poles and bulged at the equator.

b. As a result, the radius of the earth at the poles is less than that at the equator.

c. Hence, <u>the value of ‘g’ is the highest at the poles (9.832 m/s^2) and decreases slowly with decreasing latitude</u>. It is the lowest at the equator (9.78 m/s^2).

ii. Height:

a. As the height of an object from the surface of the earth increases, the distance between the object and the centre of the earth (r) increases.

b. As a result, the value of 'g' decreases with increase in height. However, the decrease is rather small for heights which are small in comparison to the earth's radius.

iii. Depth:

a. The value of g is maximum on the surface of the earth.

b. As depth of an object increases, the distance between the object and the centre of the earth (r) decreases.

c. Along with the distance, the part of the earth which contributes towards the gravitational force felt by the object (M) also decreases.

d. As a combined result of change in r and M, the value of g decreases as we go deep inside the earth.

e. At the centre of the earth, the value of 'g' becomes zero.

Q.4 Answer the following. (Attempt any 1) (5)

i(a) **Calculate the maximum height attained by the body thrown vertically upward with a velocity of 9.8 m/s.**

($g = 9.8\ m/s^2$)

Given: Initial velocity (u) = 9.8 m/s, acceleration due to gravity (g) = $9.8\ m/s^2$

To find: Maximum height attained (s)

Formulae: i. $v = u + at$ ii. $s = ut + \frac{1}{2} at^2$

Calculation: For upward motion of the ball, (v) = 0.

$a = -g = -9.8\ m/s^2$

From formula (i),

$0 = 9.8 + (-9.8)\ t$

$\therefore\ t = 1\ s$

Now, from formula (ii),

$$s = 9.8 \times 1 + \frac{1}{2} \times (-9.8) \times (1)^2$$

$$= 9.8 - \frac{9.8}{2} = \frac{9.8}{2}$$

$= \mathbf{4.9\ m}$

(b) **The radius of planet A is half the radius of planet B. If the mass of A is M_A, what must be the mass of B so that the value of g on B is half that of its value on A?**

Given: For planet A: mass = M_A,

radius $R_A = \frac{R_B}{2}$,

acceleration due to gravity = g_A

For planet B: radius = R_B,

acceleration due to gravity $g_B = \frac{g_A}{2}$

To find: Mass of planet B (M_B)

Formula: $g = \frac{GM}{R^2}$

Calculation: From formula,

$$\frac{M_A}{M_B} = \frac{g_A R_A^2}{G} \times \frac{G}{g_B R_B^2} = \frac{g_A R_B^2}{4} \times \frac{2}{g_A R_B^2} \quad \text{....[Substituting for } R_A \text{ and } g_B]$$

$$\therefore \quad \frac{M_A}{M_B} = \frac{1}{2}$$

$\therefore \quad M_B = \mathbf{2M_A}$

(c) **What is free fall?**

Free fall:

Whenever an object moves under the influence of the force of gravity alone, then the object is said to be under free fall.

ii **Write the three laws given by Kepler. How did they help Newton to arrive at the inverse square law of gravity?**

The three laws given by Kepler are:

i. **Kepler's first law:** The orbit of a planet is an ellipse with the Sun at one of the foci.

ii. **Kepler's second law:** The line joining the planet and the Sun sweeps equal areas in equal intervals of time.

iii. **Kepler's third law:** The square of orbital period of revolution of a planet around the Sun is directly proportional to the cube of the mean distance of the planet from the Sun.

Derivation of inverse square law of gravity with the help of Kepler's law:

a. The centripetal force acting on a planet of mass m and velocity v, revolving at a distance of r from the Sun, is given as,

$$F = \frac{mv^2}{r} \qquad ....(1)$$

b. Let T be the period of revolution of the planet and 2πr be the distance travelled by the planet in one revolution. Then the speed of the planet is given as,

$$\text{speed } (v) = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T} \quad ....(2)$$

c. Substituting equation (2) in (1), we get,

$$F = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4m\pi^2 r}{T^2}$$

d. Multiplying and dividing by r^2, we get,

$$F = \frac{4m\pi^2 r^3}{r^2 T^2}$$

$$= \frac{4m\pi^2}{r^2}\left(\frac{r^3}{T^2}\right) \qquad ....(3)$$

e. But, according to Kepler's third law,

$$T^2 \propto r^3$$

$$\therefore \quad \frac{T^2}{r^3} = \text{constant } (K) \qquad ....(4)$$

Substituting equation (4) in (3), we get,

$$F = \frac{4m\pi^2}{r^2 K}$$

$$\text{But, } \frac{4m\pi^2}{K} = \text{constant}$$

$$\therefore \quad F \propto \frac{1}{r^2}$$

Thus, with the help of Kepler's third law, Newton concluded that the centripetal force which is the force acting on the planet and is responsible for its circular motion, must be inversely proportional to the square of the distance between the planet and the Sun. Newton identified this force with the force of gravity and hence postulated the inverse square law of gravitation.