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📘 1. Introduction to Cubic Functions

🌟 AIDA Framework Breakdown

- **A – Attention**: Ever wondered why roller coaster tracks or water fountains arch and dip like wild waves? 🤔 The path they follow often mirrors a **cubic function**—a powerful mathematical tool that lets us model curves with multiple turns. 🎢 💧
- **I – Interest**: Cubic functions don't just go up or down—they twist, bounce, and intersect the axis up to three times! Perfect for representing real-world motion or behaviors. 📈 📉
- **D – Desire**: By learning to recognize and graph cubic functions, you'll unlock the ability to **analyze patterns**, **predict behaviors**, and **design smooth curves** in everything from physics to animation. 🧠 💡
- **A – Action**: Let's begin by exploring the basics of cubic functions, starting with their general form, their many shapes, and how their coefficients guide the graph's journey. 🚀

🔹 General Form: $y = ax^3 + bx^2 + cx + d$

A **cubic function** is a **third-degree polynomial**, meaning its highest power of x is 3. This is what gives the graph its characteristic "**S**" **shape** (or a flipped version).

- The standard form is:

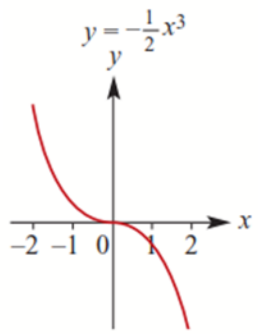
$$y = ax^3 + bx^2 + cx + d$$

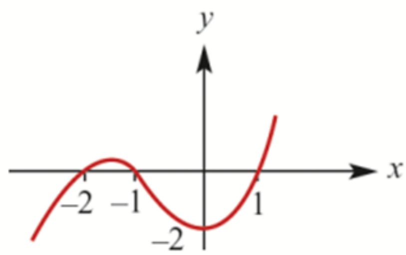
- Here:
  - a = leading coefficient (affects direction + width)
  - b, c = influence the curve's turning points
  - d = y-intercept (where graph crosses y-axis)

🔹 Range of Graph Shapes

Cubic graphs can have **multiple shapes**, depending on the coefficients:

- They can have:
  - **3 x-intercepts (distinct real roots)**
  - **2 x-intercepts (with a repeated root)**
  - **1 x-intercept (1 real root + 2 complex roots)**





🌈 Graph Shapes:

1. **Upward S-curve** (increasing overall) when $a > 0$
2. **Downward S-curve** (decreasing overall) when $a < 0$
3. **"Bounce" at root** if root is repeated (e.g. $(x-2)^2$)

4. **Flat dip or peak** if multiple turning points exist

🔹 Role of Coefficients

Coefficient	Role
a	Determines end behavior and vertical stretch/compression
b, c	Influence position and nature of turning points
d	The y-intercept (value of y when $x = 0$)

🧪 Example:

For $y = 2x^3 - 3x^2 + 5x - 4$:

- $a = 2$: S-curve opens upward
- $b = -3, c = 5$: Affect curve shape
- $d = -4$: y-intercept is -4

🔹 Minimum Number of x-Intercepts

A cubic function **always has at least one real x-intercept**. Why?

Because a **degree 3 polynomial must cross the x-axis at least once**—it can't float entirely above or below the axis like some quadratics.

🧠 Key Point:

- It might **touch** the axis once (repeated root), or
- Cross it up to **three times** if all roots are real and distinct

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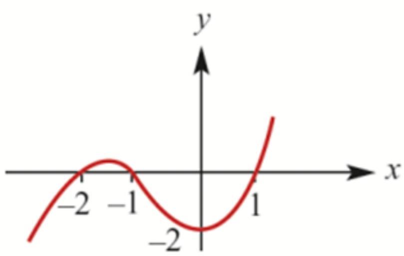
📊 Visual Examples:

1. **Three real roots**:

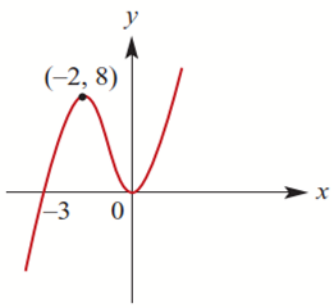
$$y = (x-2)(x+1)(x+$$



➤ Crosses x-axis at -3, -1, 2

2. **Repeated root**:

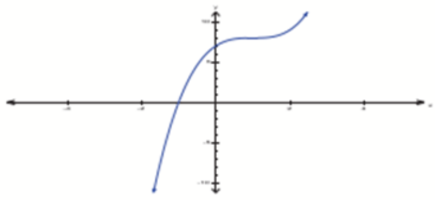
$$y = (x+1)^2(x-2)$$



➤ Bounces at -1, crosses at 2

3. **Only one real root**:

$$y = (x-1)(x^2+4x+5)$$



➤ Only x-intercept is at 1

🧠💭 Concept Check Questions:

1. **Remembering**: What is the highest power of x in a cubic function?
2. **Understanding**: What does the leading coefficient tell us about the graph?
3. **Applying**: Given $y = -x^3 + 3x$, predict whether the graph increases or decreases from left to right.
4. **Analyzing**: How does the graph of $y = (x-2)^2(x+1)$ differ from one with three distinct roots?
5. **Evaluating**: Which function would cross the x-axis the most:
   a) $y = x^3 - x$,
   b) $y = (x+2)^2(x-1)$, or
   c) $y = (x-3)^3$?
6. **Creating**: Can you construct a cubic function that touches the x-axis only once?
7. **Socratic Method**: If every cubic must cross the x-axis at least once, what does that imply about its roots?

🧮 2. Polynomial vs. Factorised Form

🌟 AIDA Framework Breakdown

- **A – Attention**: Ever tried solving a puzzle without knowing where the pieces fit? That's what it's like graphing a cubic from its expanded form. But the moment you see it factorised, the picture becomes clearer! 🧩 ✨
- **I – Interest**: Just like knowing the ingredients of a recipe makes it easier to cook, knowing the **factors** of a cubic makes it easier to graph. 🔍 📈
- **D – Desire**: Mastering the difference between polynomial and factorised forms helps you go from equations to **precise, accurate, and fast graphs** with confidence. 🎓 ⚡
- **A – Action**: Let's compare the two forms, explore how factorised form helps with graphing, and understand how transformations apply. 🧠 ✏️

🔹 Standard Polynomial Form

This is the expanded version of a cubic equation:

$$y = ax^3 + bx^2 + cx + d$$

- **Pros**:
  - Shows general shape and end behavior
  - Useful for finding **y-intercept** easily (just plug in $x = 0$)
- **Cons**:
  - Doesn't directly show the **x-intercepts**
  - Harder to sketch by hand without factoring

🧪 Example:

Given:

$$y = x^3 + 2x^2 - 5x - 6$$

This tells us:

- Cubic with leading coefficient $a = 1$ (graph opens up)
- y-intercept is $d = -6$

But we **don't know where it crosses the x-axis** unless we factor it.

🔹 What is Factorised Form?

This form shows the roots of the equation directly:

$$y = a(x - r_1)(x - r_2)(x - r_3)$$

Where r_1, r_2, r_3 are the **x-intercepts** (roots).

- **Pros**:
  - You can immediately see where the graph touches or crosses the x-axis.
  - Helps you draw sign diagrams.
  - Easier to sketch the shape accurately.
- **Cons**:
  - Doesn't directly show the y-intercept unless you calculate it.

🧪 Example:

$$y = (x + 1)(x - 2)(x + 3)$$

- x-intercepts: $-3, -1, 2$
- y-intercept: Plug $x = 0 \rightarrow y = (1)(-2)(3) = -6$
- This is the **same** function as the one above:
  $y = x^3 + 2x^2 - 5x - 6$

🔹 How Factorised Form Helps in Graphing

📉 Step-by-Step Benefits:

1. **Instant x-intercepts** ✅
2. **Use sign diagram** to determine whether the curve is above or below the x-axis ✅
3. **Shape recognition**: repeated roots → bounce, single roots → cross ✅
4. **Predict end behavior** from the sign of the leading coefficient ✅
5. **Easy turning point estimation** (especially with tech like TI-Nspire) ✅

🔹 Understanding Graph Transformations

Transformations modify how the base cubic $y = x^3$ looks:

Transformation	Function Form	Description
Vertical stretch	$y = 2x^3$	Steeper graph
Vertical flip	$y = -x^3$	Flips upside down

Transformation	Function Form	Description
Horizontal shift	$y = (x-1)^3$	Moves graph right by 1
Vertical shift	$y = x^3 + 4$	Moves graph up by 4

In **factorised form**, transformations look like:

- $y = a(x-r)^3 + k$: Single repeated root
- $y = a(x-r_1)(x-r_2)(x-r_3)$: General graph with 3 real roots

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🧪 Examples:

1. $y = (x-2)(x+3)^2$
   ➤ Touches x-axis at -3 (repeated root), crosses at 2
2. $y = -2(x-1)(x+2)(x-4)$
   ➤ Negative a-value flips it downward
3. $y = (x-1)^3$
   ➤ One real root, flattens at x = 1 (no turning points)

🧠 💭 Concept Check Questions:

1. **Remembering**: What is the general form of a cubic function?
2. **Understanding**: What can we immediately see from a function in factorised form?
3. **Applying**: Factorise $y = x^3 + x^2 - 6x$ and identify the intercepts.
4. **Analyzing**: Compare the graph of $y = (x-1)(x+2)(x+3)$ with $y = (x+2)^2(x-1)$.
5. **Evaluating**: Which form—polynomial or factorised—is more helpful for graphing, and why?
6. **Creating**: Construct a cubic with x-intercepts at -3, 0, and 2.
7. **Socratic Method**: Why might a graph touch the x-axis but not cross it?

3. x-axis and y-axis Intercepts

🌟 AIDA Framework Breakdown

- **A – Attention**: Want to know where your cubic graph hits the axes? That's where the magic begins! 💥 The intercepts are the **anchors** of your graph.
- **I – Interest**: Once you find the x- and y-intercepts, you can start sketching without a calculator. 🎯 It's like solving the first part of a mystery! 🕵️
- **D – Desire**: Mastering intercepts lets you create **accurate graphs**, **interpret real-world problems**, and understand how the graph moves.
- **A – Action**: Time to learn how to find intercepts step by step using factorised cubic functions! ✏️

🔹 Finding x-intercepts by Solving $y = 0$

To find the x-intercepts (roots), we solve:

$$y = a(x - r_1)(x - r_2)(x - r_3) = 0$$

Since any number multiplied by 0 equals 0, set each factor equal to 0:

🧠 Rule:

If $ab = 0$, then either $a = 0$ or $b = 0$

🧪 Example:

$$y = (x - 2)(x + 3)(x + 1)$$

To find x-intercepts, let $y = 0$:

$$0 = (x - 2)(x + 3)(x + 1) \Rightarrow x = 2, -3, -1$$

✅ These are the points where the graph **crosses the x-axis**.

🔹 Finding y-intercepts by Substituting $x = 0$

To find the y-intercept, plug $x = 0$ into the function:

$$y = a(0 - r_1)(0 - r_2)(0 - r_3)$$

This gives the point where the graph cuts the **y-axis**.

🧪 Example:

$$y = (x - 1)(x + 2)(x + 1)$$

Substitute $x = 0$:

$$y = (-1)(2)(1) = -2$$

✅ So the y-intercept is $(0, -2)$

🔹 Interpreting Intercepts on the Graph

- **x-intercepts** = Where the graph touches or crosses the x-axis
  - Root of the equation
  - If root is **repeated** (like $(x-1)^2$), graph **touches** the axis but doesn't cross
  - If all roots are real and distinct → graph **crosses** at each root
- **y-intercept** = Where the graph crosses the y-axis
  - Only **one y-intercept** in a function
  - Often helps to sketch the vertical position of the graph at $x = 0$

📌 Real Graph Example (from textbook Example 24):

Given:

$$y = (x-1)(x+2)(x+1)$$

- **x-intercepts**: Set $y = 0$
  $\rightarrow x = 1, -2, -1$
- **y-intercept**: Set $x = 0$
  $\rightarrow y = (-1)(2)(1) = -2$

So the graph:

- Crosses x-axis at 1, -1, -2
- Crosses y-axis at -2

🧪 More Practice Examples:

1. $y = (x-3)^2(x+2)$
   - x-intercepts: $x = 3$ (repeated), $x = -2$
   - y-intercept: $x = 0 \rightarrow y = (0-3)^2(0+2) = 9 \times 2 = 18$
2. $y = -x(x-2)(x+4)$
   - x-intercepts: $x = 0, 2, -4$
   - y-intercept: $y = -0(0-2)(0+4) = 0$

🎯 Summary Table

Type	How to Find	What It Tells You
x-intercepts	Solve $y = 0$	Where graph crosses or touches x-axis
y-intercept	Evaluate y when $x = 0$	Where graph crosses y-axis
Repeated root	Same x-value shows up more than once	The graph **touches** the axis at that root

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🧠💭 Concept Check Questions:

1. **Remembering**: How do you find the x-intercepts of a factorised cubic?
2. **Understanding**: Why do we set $y = 0$ when finding x-intercepts?
3. **Applying**: Find the x- and y-intercepts of $y = (x + 1)(x - 2)(x + 3)$
4. **Analyzing**: Compare the graphs of $y = (x - 1)(x + 2)(x + 3)$ and $y = (x - 1)^2(x + 3)$
5. **Evaluating**: Is it possible for a cubic function to have only one x-intercept? Explain.
6. **Creating**: Write a factorised cubic with x-intercepts at -2, 0, and 3, and a y-intercept of 6
7. **Socratic Method**: Why does plugging in $x = 0$ always give the y-intercept?

✏️ 4. Sketching Graphs from Factorised Form

🌟 AIDA Framework Breakdown

- **A – Attention**: Ever wonder how you can sketch a beautiful, accurate cubic graph without needing a graphing calculator? 🎯
- **I – Interest**: Knowing just the **factors** and a few smart steps, you can **predict** exactly how the graph looks! 🚀
- **D – Desire**: Mastering this lets you sketch fast and impressively — **essential for exams**, assignments, and competitions! 🏆
- **A – Action**: Let's walk through a complete example, learn about intervals and signs, and understand graph shapes. 🧠 💡

🔹 Step-by-Step Example Walkthrough (Example 24)

Given Function:

$$y = (x - 1)(x + 2)(x + 1)$$

🧩 Step 1: Find x-intercepts

Set $y = 0$:

$$(x - 1)(x + 2)(x + 1) = 0$$

Solutions:

$$x = 1, \quad x = -2, \quad x = -1$$

✅ These are the points where the graph crosses the x-axis.

🧩 Step 2: Find y-intercept

Substitute $x = 0$:

$$y = (0 - 1)(0 + 2)(0 + 1) = (-1)(2)(1) = -2$$

✅ So the y-intercept is $(0, -2)$.

🧩 Step 3: Arrange the x-intercepts on a number line

Order: $-2, -1, 1$

This helps create intervals:

- $x < -2$
- $-2 < x < -1$
- $-1 < x < 1$
- $x > 1$

🧩 Step 4: Determine signs in each interval

Pick test points in each interval and plug into y.

Interval	Test Point	Sign Check	Overall Sign	Graph Behavior
$x < -2$	$x = -3$	$(-)(-)(-) = -$	Negative	Below x-axis
$-2 < x < -1$	$x = -1.5$	$(+)(-)(-) = +$	Positive	Above x-axis
$-1 < x < 1$	$x = 0$	$(-)(+)(+) = -$	Negative	Below x-axis
$x > 1$	$x = 2$	$(+)(+)(+) = +$	Positive	Above x-axis

🧩 Step 5: Sketch the general shape

- Starts **below** x-axis (negative) before $x = -2$
- Crosses to **above** x-axis between -2 and -1
- Dips **below** x-axis between -1 and 1
- Rises **above** x-axis after $x = 1$

✅ Classic **S-curve** behavior!

🎨 Quick Sketch Description:

- Curve **below** before $x = -2$
- **Crosses up** at $x = -2$
- **Crosses down** at $x = -1$
- **Crosses up** again at $x = 1$
- **y-intercept** at $(0, -2)$

🔹 Intervals and Signs of y-values

Using sign analysis:

- Between each root, the graph is **either entirely above or entirely below** the x-axis.
- The **sign diagram** helps predict these sections.

Summary Table:

Interval	Sign	Behavior
$x < -2$	Negative	Below axis
$-2 < x < -1$	Positive	Above axis
$-1 < x < 1$	Negative	Below axis
$x > 1$	Positive	Above axis

🔹 Graph Shape Based on Signs Between Roots

- Graph **crosses** x-axis at every **single root** (no repeated factors).
- It **alternates** between above and below the axis.
- Curve bends smoothly at turning points (local max/min).

Turning points happen somewhere between each pair of intercepts.

🔹 "Beyond the Intercepts" Behavior

Look at **leading coefficient** a:

- If positive → Left ↓, Right ↑
- If negative → Left ↑, Right ↓

In our example:

- $a = 1$ (positive)
- So **left side down, right side up** ✅

Graph **goes to negative infinity** as $x \to -\infty$,
and **goes to positive infinity** as $x \to +\infty$.

🎯 Full Sketch Summary:

✅ Roots: $x = -2, -1, 1$
✅ y-intercept: $(0, -2)$
✅ Left arm down, right arm up
✅ Crosses x-axis at each root
✅ "S" shape with smooth bends at turning points

🧠💭 Concept Check Questions:

1. **Remembering**: What is the first step when sketching a cubic in factorised form?
2. **Understanding**: Why does the graph alternate between being above and below the x-axis?
3. **Applying**: Sketch the graph of $y = (x - 2)(x + 1)(x - 4)$ by following these steps.
4. **Analyzing**: How does the number of x-intercepts affect the number of bends in the graph?
5. **Evaluating**: If a graph only touches the x-axis at a root instead of crossing, what does it mean?
6. **Creating**: Create a factorised cubic with roots at -3, 0, and 2, then sketch the graph.
7. **Socratic Method**: Why does the graph cross the x-axis at a single root but bounce off at a repeated root?

🧲 5. Effects of the Leading Coefficient (a)

🌟 AIDA Framework Breakdown

- **A – Attention**: Ever noticed how some graphs "rise up" on both ends, while others "fall and dive"? 🌊 It's not random—it's controlled by a single number: the **leading coefficient**!
- **I – Interest**: By just glancing at the sign of a, you can **predict** the graph's ultimate behavior without even sketching. 😎
- **D – Desire**: Knowing how the leading coefficient affects the graph lets you draw, analyze, and understand cubic functions lightning fast! ⚡
- **A – Action**: Let's dive in and explore reflections, end behavior, and real examples! 🎯

🔹 Positive vs. Negative Cubic Functions

In a cubic function:

$$y = ax^3 + bx^2 + cx + d$$

the **leading coefficient** a determines the graph's **end behavior** and whether the graph **rises** or **falls**.

If $a > 0$	If $a < 0$
Left side ↓	Left side ↑
Right side ↑	Right side ↓
S-curve rises overall	S-curve falls overall
"Natural" cubic look	Flipped cubic look

🧪 Examples:

1. $y = x^3 - 4x$
   - $a = 1$ (positive)
   - **Left ↓, Right ↑**
2. $y = -2x^3 + x^2 - x$
   - $a = -2$ (negative)
   - **Left ↑, Right ↓**

🔹 Reflections and End Behavior

When a is **negative**, it **reflects** the graph across the x-axis — just like flipping a pancake 🍳.

End Behavior Summary:

a	As $x \to -\infty$	As $x \to +\infty$
Positive	$y \to -\infty$	$y \to +\infty$
Negative	$y \to +\infty$	$y \to -\infty$

🔔 *Think: the graph always heads opposite directions at each end.*

🔹 Example: Graph of $y = -x^3 - 2x^2 + x + 2$

Let's break it down:

- Leading coefficient $a = -1$ → **Graph is reflected**.
- As $x \to -\infty, y \to +\infty$
- As $x \to +\infty, y \to -\infty$

Now find intercepts:

✅ **x-intercepts**: Solve $-x^3 - 2x^2 + x + 2 = 0$
(Factoring may be needed, or use technology if messy!)

✅ **y-intercept**: Substitute $x = 0$

$$y = -(0)^3 - 2(0)^2 + (0) + 2 = 2$$

So the graph crosses the y-axis at $(0, 2)$.

✅ **Shape**: S-curve flipped — starts high on the left, ends low on the right.

🎨 Quick Sketch Summary:

- Left arm goes **up** ⬆️
- Right arm goes **down** ⬇️
- x-intercepts found by solving
- y-intercept at $(0, 2)$
- Reflected S-curve!

🧠 💭 Concept Check Questions:

1. **Remembering**: What effect does a negative a have on the graph?

2. **Understanding**: Why does a positive a mean the graph rises to the right?
3. **Applying**: Describe the end behavior of $y = -2x^3 + 3x^2 - 5x$
4. **Analyzing**: How would the graph of $y = x^3 + 2x^2 - 3x$ change if a were negative?
5. **Evaluating**: Which graph rises both left and right: $y = x^3$ or $y = -x^3$? Explain.
6. **Creating**: Write an equation for a cubic graph that falls to the right and has x-intercepts at -2, 1, and 3.
7. **Socratic Method**: If a cubic graph rises to the left and falls to the right, what can you immediately say about the sign of a?

🧩 6. Sign Diagrams

🌟 AIDA Framework Breakdown

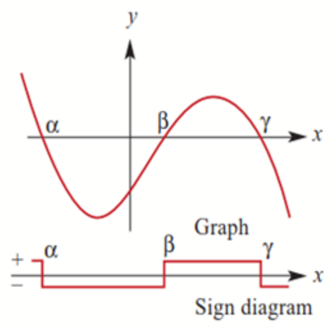
- **A – Attention**: Want a secret shortcut to know whether your cubic graph is above or below the x-axis? 🧙 ✨ Sign diagrams reveal the answer without guessing!
- **I – Interest**: Instead of plotting dozens of points, you can use sign diagrams to **predict** the entire shape based just on the roots. 🚦
- **D – Desire**: Mastering sign diagrams makes your graphing faster, more accurate, and way more powerful, especially for complex cubics. 📊 💪
- **A – Action**: Let's break down how to create and interpret sign diagrams for factorised cubics! 🎯

🔹 What is a Sign Diagram?

A **sign diagram** is a **number line** showing whether the value of the function is **positive** or **negative** between its x-intercepts.

- **Above the axis (positive)**: Curve is above x-axis
- **Below the axis (negative)**: Curve is below x-axis

It helps you predict **where** the graph is **up** and **where** it is **down**. 🧠 📈



🔹 Using Number Lines to Predict Function Signs

Steps to Create a Sign Diagram:

1. **Find the x-intercepts** by solving $y = 0$.
2. **Mark intercepts** on a number line.
3. **Pick test points** between each intercept and plug into the equation.
4. **Record the sign** (+ or -) for each interval.

🧪 Example:

Given:

$$y = (x + 2)(x - 1)(x - 3)$$

x-intercepts: $x = -2, 1, 3$

➡️ Intervals:

- Region 1: $x < -2$

- Region 2: $-2 < x < 1$
- Region 3: $1 < x < 3$
- Region 4: $x > 3$

➡️ Test Points:

- $x = -3 \rightarrow$ plug into y
- $x = 0 \rightarrow$ plug into y
- $x = 2 \rightarrow$ plug into y
- $x = 4 \rightarrow$ plug into y

➡️ Check Signs:

Interval	Test Point	Sign	Behavior
$x < -2$	$x = -3$	(-)(-)(-) → Negative	Below x-axis
$-2 < x < 1$	$x = 0$	(+)(-)(-) → Positive	Above x-axis
$1 < x < 3$	$x = 2$	(+)(+)(-) → Negative	Below x-axis
$x > 3$	$x = 4$	(+)(+)(+) → Positive	Above x-axis

🔹 Connecting Sign Diagrams to Graph Shapes

Now that you know where the graph is **positive** or **negative**, you can:

- Sketch the general "up" and "down" parts of the graph
- Know when the graph **crosses** the x-axis
- Predict **local maxima** and **local minima**

📍 **Positive region**: Graph is above x-axis
📍 **Negative region**: Graph is below x-axis

🎨 Graph Sketch:

- Starts below x-axis before -2
- Rises above x-axis between -2 and 1
- Dips below x-axis between 1 and 3
- Rises above x-axis after 3

A classic **S-curve**! 🎢

🔹 Example: Sign Diagram for a 3-factor Cubic

Given:

$$y = (x - 1)(x + 2)(x + 4)$$

- x-intercepts: -4, -2, 1

Test points:

- $x = -5, x = -3, x = 0, x = 2$

Checking signs:

Interval	Test Point	Sign	Behavior
$x < -4$	$x = -5$	(-)(-)(-) → Negative	Below
$-4 < x < -2$	$x = -3$	(+)(-)(-) → Positive	Above
$-2 < x < 1$	$x = 0$	(+)(+)(-) → Negative	Below
$x > 1$	$x = 2$	(+)(+)(+) → Positive	Above

Thus, the graph:

- Starts below, rises above between -4 and -2, dips below between -2 and 1, and rises again after 1. ✅

🔥 Summary

Concept	Meaning
Sign Diagram	A tool showing positive/negative regions of graph
Number Line	Place roots on number line, test between them
Interpret	Where positive → above x-axis, negative → below

🧠💭 Concept Check Questions:

1. **Remembering**: What is a sign diagram?
2. **Understanding**: Why do we pick test points between x-intercepts?
3. **Applying**: Create a sign diagram for $y = (x - 1)(x + 2)(x - 3)$
4. **Analyzing**: How does the number of sign changes relate to x-intercepts?
5. **Evaluating**: Which is faster: using sign diagrams or plotting lots of points? Why?
6. **Creating**: Make a cubic with x-intercepts at -1, 0, and 4, and sketch its sign diagram.

🎛️ 7. Using Technology to Find Turning Points

🌟 AIDA Framework Breakdown

- **A – Attention**: Struggling to find those tricky peaks and dips on cubic graphs? 📈 🙃 Technology like the TI-Nspire and Casio ClassPad can **instantly** reveal them!
- **I – Interest**: Turning points show you where your graph **changes direction** — super important in physics, economics, and optimization problems. 📉 📈
- **D – Desire**: Learning to quickly find maxima and minima **will** boost your speed, accuracy, and understanding in math tests and real-life projects! 🏆 🧠
- **A – Action**: Let's walk through how to graph cubics and find turning points step-by-step using tech! 📱 🖱️

🔹 Using the TI-Nspire to Find Local Maxima/Minima

The **TI-Nspire** CAS calculator can:

- Plot cubic graphs 📈
- Find **local maximum** (highest point in an interval)
- Find **local minimum** (lowest point in an interval)

➡️ Steps to Find Turning Points:

1. **Enter the function**:
    - Press `Home` ➔ `Graphs` ➔ Type your cubic equation, e.g., $y = (x-1)(x+2)(x+3)$
2. **Adjust graph view**:
    - Zoom out if necessary (menu ➔ window settings)
3. **Analyze Graph**:
    - Press `Menu` ➔ `Analyze Graph` ➔ Choose:
        - **Maximum** to find peaks
        - **Minimum** to find dips
4. **Set bounds**:
    - Move cursor to left of turning point ➔ Press `Enter`
    - Move cursor to right of turning point ➔ Press `Enter`
    - TI-Nspire calculates the exact point! 🎯

🧪 Example:

Graph $y = (x-2)(x+1)(x+3)$.

- Find two turning points:
  - A **maximum** (local peak)
  - A **minimum** (local dip)

Answer:
Approximate maximum at $(-2, 6)$, minimum at $(1, -4)$.

🔹 Using the Casio ClassPad to Graph and Solve

The **Casio ClassPad** offers powerful graphing features too! 🌟

➡️ Steps to Find Turning Points:

1. **Graph function**:
   - Tap `Graph & Table`
   - Enter your cubic equation
2. **Draw the graph**:
   - Press `Draw`
3. **Analyze**:
   - Tap `G-Solv` (Graph Solve)
   - Choose:
     - **Max** for local maximum
     - **Min** for local minimum
4. **Read the coordinates**:
   - ClassPad displays exact values on the graph!

🧪 Example:

Graph $y = -x^3 + 2x^2 + x - 2$.

- Find turning points:
  - Maximum and minimum values.

Answer:
Approximate maximum at $(0.5, 0.375)$, minimum at $(1.5, -2.375)$.

🔹 Steps for Using "Analyze Graph" Functions

Both calculators follow a similar general method:

Step	Action
1. Graph the function	Enter the cubic function
2. Zoom as needed	Adjust to see the whole curve
3. Analyze Graph	Use "Analyze Graph" ➔ Select "Maximum" or "Minimum"
4. Set bounds	Pick points to the left and right of turning point
5. Record coordinates	Read exact locations of turning points (both x and y values)

✅ **Pro Tip**: Always check that you're correctly selecting left and right bounds — this makes sure you find the correct turning point!

🔹 Example 25: Graphing and Interpreting Turning Points

Given:

$$y = (x + 2)(x - 1)^2$$

1. **Graph the function** using TI-Nspire or Casio ClassPad.
2. **Analyze the graph**:
   - Find where the maximum or minimum points occur.
3. **Interpret**:
   - Maximum: the highest point the graph reaches locally.
   - Minimum: the lowest dip of the graph locally.

📍 **Interpretation**:

- The turning point at $x = 1$ is a **touching point** (since $(x - 1)^2$) — graph bounces off the x-axis there.
- No real maximum, graph rises and falls gradually.

🎯 Summary:

Device	Key Button/Tool	Main Task
TI-Nspire	`Analyze Graph`	Find max/min by setting left/right bounds
Casio ClassPad	`G-Solv` ➔ Max/Min	Find max/min directly after graphing
Common Goal	Graph, Analyze, Read!	Quick and accurate turning points

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🧠💭 Concept Check Questions:

1. **Remembering**: Which button on TI-Nspire lets you find a maximum?
2. **Understanding**: What do maxima and minima represent on a cubic graph?
3. **Applying**: Graph $y = (x+2)(x-3)^2$ and find the minimum.
4. **Analyzing**: Why does setting bounds (left/right) matter when using Analyze Graph?
5. **Evaluating**: Compare finding turning points manually versus using a calculator.
6. **Creating**: Sketch a cubic graph based on given turning points: maximum at (-2,5), minimum at (1,-3).
7. **Socratic Method**: If a cubic graph has two turning points, what does that suggest about its general shape?

🔄 8. Repeated Factors in Cubic Graphs

🌟 AIDA Framework Breakdown

- **A – Attention**: Have you seen a cubic graph **touch** the x-axis without crossing it? That's the magic of **repeated roots**! 🎩 ✨
- **I – Interest**: Repeated factors give the graph a **special bounce** or flattening effect — making it look softer and more curved at that point. 🎢
- **D – Desire**: Recognizing repeated roots means you can **predict graph behavior** more accurately and **sketch faster**! 🖍️ ✅
- **A – Action**: Let's learn to identify repeated roots and see how they affect the curve's behavior! 🎯

🔹 Identifying Repeated Roots

Repeated roots happen when a factor is raised to a power greater than 1.

- $(x-r)^2$ → repeated twice (double root)
- $(x-r)^3$ → repeated thrice (triple root)

🧠 What does it mean?

- The root (x-intercept) **is the same value more than once**.
- The graph either **bounces off** the axis or **flattens** there, instead of cleanly crossing.

🧪 Example:

$$y = (x+2)^2(x-3)$$

- Repeated root at $x = -2$ (appears twice)
- Single root at $x = 3$

Thus:

- At $x = -2$, the graph **touches** the x-axis, but **does not cross**.
- At $x = 3$, the graph **crosses** the x-axis.

🔹 Effect on the Graph: Bounce vs. Cross

Type of Root	Graph Behavior
Single Root	Crosses x-axis
Double Root	Touches x-axis ("bounces" off)
Triple Root	Crosses x-axis but flattens (point of inflection)

🎨 Visual Behavior:

- **Single root**: Sharp crossing (linear-like)
- **Double root**: Smooth "kiss" or bounce
- **Triple root**: Crosses but with a flattened bend

📍 How to recognize:

- If a factor is squared → graph **bounces** at that root.
- If a factor is cubed → graph **crosses** but flattens.

🔹 Example: $y = x^2(x - 1)$

Let's break it down:

Given:

$$y = x^2(x - 1)$$

- Roots:
  - $x = 0$ (repeated root, double root because x^2)
  - $x = 1$ (single root)

➡️ Graph Behavior:

- At $x = 0$: Graph **bounces off** the x-axis (since it's a double root)
- At $x = 1$: Graph **crosses** the x-axis normally

➡️ Sketch Summary:

- Left of 0: graph is positive or negative? (Test $x = -1$)
  - $(-1)^2(-2) = 1 \times (-2) = -2$ → Negative

- Between 0 and 1: graph is positive or negative? (Test $x = 0.5$)
  - $(0.5)^2(-0.5) = 0.25 \times (-0.5) = -0.125$ → Negative
- After 1: graph is positive or negative? (Test $x = 2$)
  - $(2)^2(1) = 4 \times 1 = 4$ → Positive

Thus:

- **Left of 0**: Below x-axis
- **Touches** at 0 but stays below (because of bounce)
- **Crosses** at 1 and goes above

🎯 Key Takeaways:

Root Type	Behavior
Single	Crosses x-axis normally
Double	Touches (bounces) at x-axis
Triple	Crosses but flattens

🧠 Remember: **Power of factor tells you behavior!**

🧠💭 Concept Check Questions:

1. **Remembering**: What happens at a double root?
2. **Understanding**: Why does the graph "bounce" at a repeated root?
3. **Applying**: Sketch the behavior of $y = (x + 1)^2(x - 3)$ at $x = -1$ and $x = 3$.
4. **Analyzing**: How does the graph of $y = (x - 2)^3(x + 1)$ behave at $x = 2$?
5. **Evaluating**: Would a graph with a triple root at $x = 0$ have the same behavior as a double root? Why or why not?
6. **Creating**: Create a cubic equation that has a repeated root at $x = 2$ and a single root at $x = -3$.
7. **Socratic Method**: If a graph only touches the x-axis but never crosses it, what can you conclude about the nature of that root?