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Assigning Default Position for Digital Goods: Competition, Regulation, and Welfare

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ABSTRACT

We analyze alternative ways to assign the default position for digital goods like search engines. When two competing firms vie for the default through bidding, the higher-quality firm typically wins but delivers lower utility than the rival due to heightened monetization from exploiting consumer switching costs. The distribution of consumer switching costs plays a crucial role in driving the bidding outcome and welfare results. Paradoxically, increasing via regulation the rival's default share tends to raise profit and harm consumers, at least in the short run. Letting consumers choose the default benefits them in the short run, but harms the weaker firm.

JEL Classification: L1, L4

1 | Introduction

This paper addresses an important controversy regarding key digital products: How to choose the supplier whose product will be preset as the default for consumers? In many situations where competing products vie for the default position, the selection is made by a third party that supplies a different good to the consumers. For example, the manufacturer of a PC or a mobile device may choose the browser, search engine, or other software that will be pre-installed. Although consumers may switch to a non-default product, doing so can entail switching costs that create significant inertia; many consumers lack the technical savvy to switch or the willingness to incur the hassle. Thus, the firm whose product obtains default status can gain a substantial competitive edge over rivals.

A striking example comes from the landmark case brought by the U.S. Department of Justice (DOJ) vs. Microsoft in the late 1990s for exclusionary practices against Netscape's Navigator browser, the main competitor to Microsoft's Internet Explorer browser. A key piece of the DOJ's evidence was the much larger growth in Explorer's market share at Internet Service Providers (ISPs) that agreed to distribute Explorer as the default browser: From 20% to 90% vs. from 20% to 30% at other ISPs Dunham [1]. Even where switching appears easy—"just a click away" for some digital products—the default position can be valuable, as evidenced by the large payments that firms are willing to make for this position.[1] Google reportedly pays hundreds of millions of dollars annually to be the default search engine on Mozilla's Firefox browser; billions annually to be the default search engine on Apple's Safari browser; and considerable sums to other parties such as wireless carriers DOJ [2], Ostrovsky [3]. The European Commission's [4] Android decision, finding that Google foreclosed distribution outlets to competing search engines, flagged such default payments to third parties, as did the DOJ's [2] lawsuit.[2]

The Google search controversy offers a useful springboard for addressing some questions of broader interest. Often, as with Google in search, one of the firms vying for the default position enjoys a quality advantage, e.g., due to initially superior technology. Critics worry that the leading firm may prolong its dominance by paying for default positions at key distribution outlets to deprive rivals of the scale needed to compete effectively. Google, or a similarly situated leading firm, might plausibly counter that its willingness to outbid rivals for default status derives only from its product superiority (e.g., Walker [5]), because a firm that expects to retain more consumers than would a lower-quality rival typically is willing to spend more to attract consumers. For example, in Milgrom and Roberts [6] a higher-quality firm is willing to spend more on advertising, which acts as a signal of quality. A similar finding occurs in the search models of Athey and Ellison [7] and Chen and He [8], where competing firms bid for higher ad positions that will be searched earlier by consumers.[3] Although firms charge equal prices in those models, a higher-quality firm outbids lower-quality rivals for a higher position because increased exposure yields it more product matches than such rivals and hence greater sales.

The greater-sales argument, however, may be less applicable to bidding for default position for search engines or some other major digital products. Consider this toy model: A unit mass of consumers demand the product and are initially assigned to the default firm. Firm A provides higher quality than firm B, and both firms earn the same revenue per consumer, normalized to one. If firm A wins the default, it retains all consumers and firm B gets none. If firm B wins, a share q of consumers (those with lower switching costs) quit and move to firm A. Each firm's maximum bid equals the difference between the number of its consumers with and without the default position: Firm B's maximum bid is $(1 - q) - 0$ and firm A's is $1 - q$, the same amount regardless of firm A's quality advantage proxied by q.[4] Thus, it is not obvious whether superior quality alone would induce Google to outbid a rival for default.

Moreover, Google's view that it offers a superior product is disputed by some critics. They argue that while Google may deliver more relevant search results, it offers a worse overall consumer experience than some other search engines because it engages in excessive monetization, e.g., through intrusive tracking or by prioritizing ads over natural search results. They contend that Google's enduring market share dominance is attributable to its ubiquitous default position, not to superior quality. This, in turn, raises the question: If Google offers an inferior product, how can it outbid rivals for the default position?

Our paper addresses two broad issues. First, what are the characteristics of equilibrium when the default position is assigned through competitive bidding? In particular, does the high-quality firm necessarily win? Is consumer welfare higher in this case than it would be if the default were awarded to the lower-quality rival? Second, compared to the high-quality firm winning, what are the welfare effects of alternative regulatory schemes? Specifically, we consider assigning the default position for some share of consumers to one firm and the rest to the rival, or letting consumers choose their preferred default.

We tackle these issues using a parsimonious model that captures salient features of the search engine environment. Consumers choose between two competing suppliers of a given product that differ only in product quality.[5] Their valuations for the products are high enough so the market is fully covered. Each firm sets the level of a monetization activity, "charge" for brevity, which harms a consumer but generates revenue as a general function of the charge. This formulation admits broad interpretations of the charge, monetary and/or non-monetary (see also de Cornière and Taylor [10] discussed later), as many digital products have zero price but firms can monetize them through other methods, including (unwanted) targeted advertising, selling consumer data to a third party, or using consumer information to engage in price discrimination for a related product. Consumers are presented with one product as the default and can switch to the other product by incurring a private switching cost randomly drawn from a known probability distribution. They decide whether to switch by considering the firms' known qualities and their (simultaneously-chosen) observed charges, as well as the private switching cost.

Under competitive bidding, a third party selects the default product and assigns the default position to the highest-bidding firm. In equilibrium, conditional on the default position being assigned to either firm, the default firm will exploit its sticky consumers by setting its charge high enough that consumers obtain lower utility than from the rival product—even when the default product has higher quality (Proposition 1).[6] This pattern is consistent with claims by some Google critics noted earlier. Since the default product yields lower utility in equilibrium, some consumers will switch to the non-default product.

Firms bid for the default position, anticipating the equilibrium outcomes under the two alternative default assignments. We first observe that a firm wins if and only if assigning the default to it rather than the rival results in higher *industry profit*. It is not obvious which default assignment yields higher industry profit in this asymmetric duopoly setting; hence, which firm will win the default? Indeed, we provide an example where the lower-quality firm wins. Nevertheless, for broad classes of the revenue function and switching cost distribution, the higher-quality firm wins (Proposition 2 and Corollary 1). Henceforth, we take this outcome as the benchmark case under competitive bidding. Interestingly, consumer surplus can be higher if the default instead is awarded to the lower-quality firm (Corollary 2), due to reduced industry monetization.

The shape of the switching cost distribution is crucial for which default assignment yields higher industry profit, and for the consumer surplus comparison, because—for a given charge by the rival—each firm's residual demand and its slope are fully determined by the switching cost distribution. In particular, the density function of the switching cost distribution is the absolute value of the slope of each firm's (residual) demand, and whether the density function is increasing or decreasing determines whether the distribution is "skewed" towards high or low values. This in turn determines whether moving the default position from firm B to firm A raises the sum of charges from the two firms (Lemma 1), with crucial implications for the rankings of industry profit and consumer surplus.

An alternative to competitive bidding is to assign the default position through regulation. One possibility is equal shares, i.e., assign to each firm the default for half the consumers, and we characterize the resulting equilibrium (Proposition 3). The high-quality firm A now provides higher utility than its rival firm B—unlike when firm A wins the default for all consumers (Proposition 1): Now that firm A competes for B's default consumers, A's equilibrium charge will exceed B's charge by less than A's quality advantage. Interestingly, industry profit is higher and consumer surplus is lower than under competitive bidding. Industry profit rises because when firm B holds the default for half the consumers instead of none, it raises its charge substantially, inducing firm A to raise its charge as well. Thus, greater symmetry in firms' installed bases of sticky consumers softens competition, unlike greater symmetry in costs or quality, which intensifies competition. Consumer surplus falls due to the softened competition, and because additional consumers are diverted to the lower-quality product. Total welfare can rise or fall. Extending this analysis, we consider setting A as the default to a share of consumers between one half and one, i.e., between equal shares and the competitive bidding outcome. Consumer surplus again tends to be lower—while industry profit can be higher or lower—than under competitive bidding; and both welfare measures can vary non-monotonically with firm A's share (Proposition 4).

Departing from exogenous qualities, we briefly consider a scenario where quality can be improved by serving more consumers. This scenario is at the heart of the DOJ's [2] complaint against Google. DOJ argues that search algorithms improve with the number of users due to learning via experimentation, and that by obtaining default status at leading distribution outlets, Google deprives rival search engines of users, impairing their quality without necessarily raising its own quality as much. Evaluating this foreclosure argument in an equilibrium model is complex and beyond the scope of this paper. Nevertheless, using our basic model, we illustrate conditions such that transferring a minority share of default positions to the weaker firm can raise consumer surplus.

Instead of assigning the default, a leading alternative approach is to let individual consumers choose their preferred default, as required by the European Union's [11] Digital Markets Act. In our setting, such a "choice screen" remedy, paradoxically, is worse for the weaker firm than even the bidding outcome where the higher-quality rival obtains the default position everywhere. This stark result hinges on the specifics of our model (including no consumer heterogeneity except in switching costs) but the basic message is fairly robust: For consumer welfare, choice screen is likely to dominate regulatory assignment in the short run but not necessarily in the long run if learning effects are important, because the weaker firm's quality may not improve as much.

The rest of this paper proceeds as follows. After discussing related literature, Section 2 presents the model. Section 3 analyzes assignment of the default position through competitive bidding, while Section 4 considers assignment through regulation. Section 5 concludes.

1.1 | Related Literature

To our knowledge, there is minimal work directly on our topic, but a large literature on various related themes. Here we only discuss some of the closest work.

Switching costs play a central role in our analysis. An extensive literature has studied competition in markets with consumer switching costs (e.g., Klemperer [12], Farrell and Klemperer [13]).[7] Switching costs may arise due to the time and effort needed to find a new supplier, learn about a new product, or set up a new product.[8] They are likely to vary across consumers, e.g., due to different time values or technical savvy. The literature often considers firms with equal product quality and has shown that even for firms that offer ex ante homogeneous products, switching costs can create market power and soften price competition. In our model, firms differ in product quality, and we consider alternative assignments of the default position. Notably, consumers' switching patterns will depend on which firm holds the default—in addition to the firms' charges—and these foreseen switching patterns will themselves affect the firms' equilibrium bids.

Bidding for default is conceptually similar to an issue studied in some literature on ordered search—bidding for prominence, where a more prominent firm is searched earlier (e.g., Armstrong et al. [17], Armstrong and Zhou [18], Athey and Ellison [7], Chen and He [8]). There, a higher-quality firm is willing to outbid a lower-quality rival for the most prominent position, akin to our default position, partly because it attains a greater gain in sales:[9] Obtaining the top position attracts consumers who otherwise would not search that firm but instead would "leak" to other options, and this increased exposure yields greater sales to a higher-quality firm because of its greater likelihood of delivering a successful product match. In our setting, all consumers who do not buy from the low-quality firm if it has the default will buy from the high-quality firm—there is no leakage—hence, obtaining the default will yield the same extra sales to both firms.[10] In practice, leakage is likely to be substantial for the goods typically envisioned in the prominence papers, where consumers choose among many alternatives and lack information about important attributes. Whereas for important digital goods such as search engines, consumers may have good information about the leading alternatives and are likely to purchase one of them (the market is covered). As an approximation, our model considers such a case of no leakage. Differences in the firms' bids for default are then driven entirely by how the default assignment affects industry monetization, and conceivably, the lower-quality firm may win.

The welfare effects also vary across the two settings. Assigning the prominent position to the highest-quality firm tends to maximize total welfare and consumer welfare because the order of search is chosen by consumers. Prominence acts as a signal of high quality and guides consumers to search for earlier products that, in equilibrium, offer a better deal, benefiting both them and industry profits. In our setting, consumers are assigned to a default product, and the welfare effects of alternative assignments depend entirely on the resulting monetization choices of both firms, which admits a wide range of possible outcomes depending on the distribution of consumer switching costs.

Our paper shares some features with work on biased intermediation by de Cornière and Taylor [10]. They consider an intermediary integrated with one of two horizontally-differentiated symmetric sellers, that can shift demand by providing a biased recommendation to uninformed consumers. One policy they analyze ("neutrality") requires the intermediary to send half of the consumers to each seller. This policy eliminates bias but nevertheless harms consumers by softening competition because the non-integrated seller is now guaranteed half the uninformed consumers. The softening competition effect also arises in our setting when firms are assigned equal shares of default positions. (In our case, consumers suffer further harm because consumption shifts to the lower-quality product, whereas equal shares are efficient in their setting.) Another similarity involves the modeling of how sellers compete for consumers of digital goods, which often involves instruments other than the usual product price. Our "charge" imposes equal disutility on each consumer, while generating a general revenue function per consumer. Their formulation is more general by allowing two instruments, a monetary price and a quality variable. However, their demands (as functions of utility levels offered by the sellers) are linear due to the standard Hotelling framework, while we consider general demand functions (stemming from general distributions of switching costs), which can yield different outcomes.

Though not our main focus, we illustrate in a "reduced form" manner that improving the weaker firm's quality by awarding it the default position at some share of consumers can benefit consumers if learning effects are increasing in the number of users. Relatedly, Hagiu and Wright [19] provide a rich dynamic model of competition with data-enabled learning and show that mandated data sharing—whereby the leading firm in a given period must share its data with the laggard rival—can benefit consumers. Interestingly, this only occurs if the laggard is at a sufficiently large quality disadvantage.[11]

Closest to our work in focusing on the default position is the contemporaneous paper by Hovenkamp [20], whose basic setting is similar. Our contributions are complementary. Hovenkamp includes elements absent from our paper, such as explicit treatment of advertisers as the source of revenue (linear in the number of consumers) and horizontal product differentiation between the firms. But, his analysis is more restrictive in other respects, especially by assuming that consumers have identical switching costs and their demands are linear (as functions of advertising levels, our "charges"). Correspondingly, some of the findings differ, for example, the higher-quality firm always outbids the rival for the default position in his setting, but not always in ours.

2 | Model

The market contains two firms, A and B, that may provide a product to a unit mass of consumers via a third party. To reduce notation, we denote the firms' products also by $i = A, B$. Each consumer demands one unit of the product from firm $i = A$ or B and obtains utility $u_i = v_i - x_i$, where $x_i \geq 0$ is i's action to monetize its product ("charge"), and $v_A - v_B \equiv \Delta > 0$ so that A has higher quality. Firm i earns revenue $r(x_i)$ per consumer from its charge x_i, where $r'(x) > 0$, and production cost is normalized to zero. If x is the usual price, then $r(x) = x$; but as noted in the Introduction, our formulation allows general monetization activities, such as unwanted advertising, common for digital products.

One of the two products is set as the default option for consumers by the third party. For instance, a PC manufacturer will preset the default search engine from among competing providers. If product i is the default, denoted by $D = i$, a consumer who wishes to use product $j \neq i$ will need to incur a switching cost s. Each consumer's switching cost is the realization of a random variable that has distribution function $F(s)$ and density function $f(s) > 0$ on $s \in [0, 1]$. We assume $\Delta \in (0, 1)$, which, given $s \in [0, 1]$, helps ensure that firm B will choose a positive charge even when A is the default.

In our base model, the default position is allocated through competitive bidding. In a variant of the model, we shall examine regulated assignments of the default position. The game with competitive bidding proceeds as follows:

First, the firm that bids higher is assigned the default position and pays the lower bid. Next, either $D = A$ or $D = B$ starts a subgame in which A or B is the default for all consumers and the two firms simultaneously choose x_A and x_B. Finally, consumers choose which product to patronize, after observing D, x_A, and x_B.[12] If $D = i$ and a consumer chooses product $j \neq i$, she needs to incur her personal switching cost.

A strategy of firm i specifies its bid for the default position and its choice of x_i conditional on the assignment of the default position. A consumer's strategy specifies her decision on which product to use, based on D, x_A, x_B, and her realized s. We study the subgame perfect Nash equilibrium of this market game, where the strategies of the firms and consumers induce a Nash equilibrium in every subgame.

We assume the outside option of consuming nothing is arbitrarily low relative to v_A and v_B so that all consumers will purchase the good in equilibrium. Unless stated otherwise, we further assume condition (C) below:

$$\begin{aligned} &(i) \quad \frac{d^2}{ds^2} \ln f(s) \leq 0, \\ &(ii) \quad r(x) = a\, x^m, \quad a > 0,\ m > \Delta\, f(0) \end{aligned} \tag{C}$$

where (i) says that $f(s)$ is log-concave, which is a familiar assumption in the literature, while under (ii) $r(x)$ takes the form of a power function, which is also log-concave, and the assumption $m > \Delta f(0)$ is made to improve the tractability of our analysis. Notice that both $f(s)$ and $r(x)$ can be either convex or concave functions. Condition (C) guarantees an interior equilibrium with positive charges by both firms under either default assignment.[13] In addition, condition (C) ensures that in our model x_A and x_B will be strategic complements, as in Bulow et al. [22].

3 | Market Equilibrium

We first characterize the equilibrium choices of x_A and x_B under a given assignment of the default position, $D = A$ or $D = B$, and then analyze the firms' equilibrium bidding incentives and equilibrium default assignment. Later, we provide welfare results.

3.1 | Equilibrium When A or B is the Default

First, consider the subgame where $D = A$. In this case, a consumer with switching cost (or "type") s will remain with firm A if

$$v_A - x_A \geq v_B - x_B - s$$

and will switch to B otherwise. The consumer who is indifferent between A and B is $s = x_A - x_B - \Delta$, hence the profit functions of the two firms under $D = A$ are

$$\begin{aligned} \pi_A &= r\left(x_A\right)\ \left[1 - F(x_A - x_B - \Delta)\right] \\ \pi_B &= r\left(x_B\right)\ F(x_A - x_B - \Delta) \end{aligned} \tag{1}$$

Next, consider the subgame where $D = B$. A consumer with type s will remain with B if

$$v_B - x_B \geq v_A - x_A - s$$

and will switch to A otherwise. The indifferent consumer is $s = x_B - x_A + \Delta$, hence the profit functions under $D = B$ are

$$\begin{aligned} \pi_A &= r\left(x_A\right)\ F\left(x_B - x_A + \Delta\right), \\ \pi_B &= r\left(x_B\right)\ \left[1 - F(x_B - x_A + \Delta)\right] \end{aligned} \tag{2}$$

Denote the equilibrium charges by $\hat{x}_A$, $\hat{x}_B$ when $D = A$, and $\tilde{x}_A$, $\tilde{x}_B$ when $D = B$. Denote also the marginal switching consumer by $\hat{\sigma} \equiv \hat{x}_A - \hat{x}_B - \Delta$ when $D = A$ and $\tilde{\sigma} \equiv \tilde{x}_B - \tilde{x}_A + \Delta$ when $D = B$. The result below references Equations (3) and (4), which are based on the first-order conditions for the equilibrium charges (see proof of Proposition 1):[14]

$$\frac{m}{\hat{x}_A} = \frac{f(\hat{\sigma})}{1 - F(\hat{\sigma})},\quad \frac{m}{\hat{x}_B} = \frac{f(\hat{\sigma})}{F(\hat{\sigma})} \tag{3}$$

$$\frac{m}{\tilde{x}_A} = \frac{f(\tilde{\sigma})}{F(\tilde{\sigma})},\quad \frac{m}{\tilde{x}_B} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \tag{4}$$

Proposition 1. *Assume condition (C). Under either default assignment, there exists a unique equilibrium, where both firms set positive charges, the default product yields lower utility than the other product, some consumers switch to the other product, and there is less switching when the high-quality product is the default. Formally*:

i. *When* $D = A$, $\left(\hat{x}_A, \hat{x}_B\right)$ *uniquely solve* (3), $\hat{x}_A - \hat{x}_B > \Delta$, *and* $F(\hat{\sigma}) < \frac{1}{2}$.

ii. *When* $D = B$, $\left(\tilde{x}_A, \tilde{x}_B\right)$ *uniquely solve* (4), *and* $\tilde{x}_A - \tilde{x}_B < \Delta$; *if* $F(\Delta) \leq \frac{1}{2}$ *then* $\tilde{x}_A \leq \tilde{x}_B$ *and* $F(\tilde{\sigma}) \leq \frac{1}{2}$, *but if* $F(\Delta) > \frac{1}{2}$ *then* $\tilde{x}_A > \tilde{x}_B$ *and* $F(\tilde{\sigma}) > \frac{1}{2}$.

iii. $0 < \hat{\sigma} < \tilde{\sigma} < 1$; $\hat{x}_A > \tilde{x}_B$ *and* $\tilde{x}_A > \hat{x}_B$.

For a given assignment of the default position, the equilibrium has several noteworthy features. First, the default product yields lower consumer surplus than the rival product: (i) when $D = A$, $\hat{x}_A - \hat{x}_B > \Delta \implies v_A - \hat{x}_A < v_B - \hat{x}_B$; and (ii) when $D = B$, $\tilde{x}_A - \tilde{x}_B < \Delta \implies v_B - \tilde{x}_B < v_A - \tilde{x}_A$. The default firm clearly will not offer a higher surplus than the rival because, starting from such a case, it could raise its charge while retaining all customers. At equal surplus, the default firm would still retain all consumers since switching is costly, but under Assumption (C) it prefers to raise its charge, while ceding to the rival some consumers with low switching costs.[15] The property that product A offers lower utility in equilibrium when it holds the default, even though it has higher quality, differs from many other settings. It is consistent with perceptions of some critics, discussed in the Introduction, that Google delivers a worse consumer experience due to high monetization.

Second, while switching costs deter some consumers from moving to the non-default product, other consumers do switch in equilibrium and receive a higher surplus (net of their switching costs) than non-switchers. As might be expected, fewer consumers will switch when A is the default, $\hat{\sigma} < \tilde{\sigma}$. The difference between these thresholds can be expressed as $\hat{\sigma} - \tilde{\sigma} = (\hat{x}_A - \hat{x}_B) - (\tilde{x}_B - \tilde{x}_A) - 2\Delta$. Although $\hat{x}_A - \hat{x}_B > \tilde{x}_B - \tilde{x}_A$, i.e., the default product's charge exceeds the rival's charge by more when $D = A$ than when $D = B$, this effect is outweighed by A's quality advantage, hence $\hat{\sigma} < \tilde{\sigma}$. Consequently, the deadweight loss from switching costs is lower when the high-quality product is the default.

The equilibrium charges under each assignment, together with the distribution of switching costs, determine the allocation of consumers (via $\hat{\sigma}$ and $\tilde{\sigma}$), hence firms' profits. Denote the equilibrium profits of A and B by (i) $\hat{\pi}_A$ and $\hat{\pi}_B$ when $D = A$ and (ii) $\tilde{\pi}_A$ and $\tilde{\pi}_B$ when $D = B$. These foreseen profits determine each firm's gain g_i from winning the default position and, hence, its maximum bid:

$$g_A = \hat{\pi}_A - \tilde{\pi}_A, \qquad g_B = \tilde{\pi}_B - \hat{\pi}_B$$

The maximum bids satisfy

$$g_A - g_B \gtreqless 0 \iff \hat{\pi}_A - \tilde{\pi}_A \gtreqless \tilde{\pi}_B - \hat{\pi}_B \iff \hat{\pi}_A + \hat{\pi}_B \gtreqless \tilde{\pi}_A + \tilde{\pi}_B$$

We thus immediately have the following:

Remark 1. Firm $i \in \{A, B\}$ is willing to bid more than the rival, and hence in equilibrium $D = i$, if and only if industry profit $\Pi \equiv \pi_A + \pi_B$ is higher when $D = i$.

We will use Remark 1 to analyze which firm will win the bidding. The role of industry profit can be grasped as follows. Moving from $D = B$ to $D = A$ increases firm A's profit but decreases firm B's profit, and firm A outbids B if and only if its gain from this move exceeds B's loss, i.e., if industry profit is higher under $D = A$.[16] The same logic underlies Gilbert and Newbery's [23] result that an incumbent monopolist would outbid a potential entrant for a single innovation or, more generally, for any single asset needed to enter (See also Tirole [24].) The market remains a monopoly if the incumbent wins the bidding, but becomes a duopoly if the entrant wins, and the incumbent wins under the fairly weak condition that industry profit is higher under monopoly (over both technologies/products). Our comparison is more complex, as it involves alternative asymmetric duopoly regimes.

Before addressing that comparison in Section 3.2. below, consider the hypothetical case where $x_A = x_B \equiv x_E$ so that $r(x_A) = r(x_B) = r(x_E) = \bar{r}$ is a constant under either $D = A$ or $D = B$. This could be the case, for example, if there is a regulation that limits the firms' monetization actions to some common level. Then, from Equations (1) and (2), $\widehat{\Pi} = \bar{r} = \widetilde{\Pi}$. The result below follows immediately from Remark 1.

Remark 2. If x_A and x_B are constrained to take an equal value $x_E \geq 0$, so that each firm earns the same revenue per consumer $r(x_E)$, then their maximum bids are equal, $g_A = g_B$.

Intuitively, if firms earn the same revenue per consumer, then industry profit is unaffected when consumers are redistributed between the two firms, given that all consumers would purchase in all cases—the market always is covered. This "no leakage" property explains why both firms would *gain equal sales* from obtaining the default (hence will bid equally if monetization is equal), as noted in the Introduction. To see the role of leakage, consider a simple extension of the toy model from the Introduction. Of the share q that quit firm B if it wins the default, a fraction l will leave the market ("leak") and $1 - l$ will switch to firm A. If firm A wins the default, it retains all consumers as before. Let $n_i|j$ denote sales of firm i when firm j wins the default, i and $j \in \{A, B\}$. The mass of consumers is 1, hence: $n_A|A = 1$, $n_B|A = 0$; $n_A|B = q(1 - l)$, $n_B|B = 1 - q$. The extra sales each firm gains by winning the default are $G_A = n_A|A - n_A|B = 1 - q(1 - l)$, $G_B = n_B|B - n_B|A = 1 - q - 0$. Thus, $G_A - G_B = ql$, where $q > 0$ represents firm A's quality advantage. Hence $G_A > G_B$ if $l > 0$; but $G_A = G_B$ for any q if $l = 0$ as assumed in our toy model.

The same possibilities can arise in a search context discussed in the Introduction. Consider the following special case of the model in Chen and He [8], adapted to track our setting: There is only one placement (the default position), all consumers search that firm first, and consumers who do not find a product match will search other firms randomly. There are $m \geq 1$ firms that do not win the default, where $m = 1$ if only firms A and B are in the market. For simplicity, consumers conduct only one random search, so any non-default firm is searched with probability $1/m$ by any consumer who did not purchase from the default firm. All firms charge the same (equilibrium) price. Firm A has a higher match probability than firm $B : 1 \geq \alpha > \beta$. The extra sales each firm gains by winning the default are $G_A = \alpha \left[1 - (1 - \beta)\frac{1}{m}\right]$ and $G_B = \beta \left[1 - (1 - \alpha)\frac{1}{m}\right]$. Thus, $G_A - G_B = (\alpha - \beta)l$ where $l \equiv (m - 1)/m$ is the *leakage ratio*: Of those consumers who did not buy from the default firm j, the fraction that did not search firm i but instead "leaked" to other firms. If $m > 1$ (as in Chen and He [8]), then $G_A - G_B > 0$ and the difference rises with the quality gap $\alpha - \beta$. However, with no leakage ($m = 1$), all consumers who do not buy from the default firm will search the rival, hence $G_A = G_B$ regardless of the quality gap.

3.2 | Equilibrium Assignment of the Default Position

With endogenous choices of x, equilibrium industry profits when $D = A$ and $D = B$ are

$$\widehat{\Pi} = r\left(\hat{x}_A\right)\ [1 - F(\hat{\sigma})] + r\left(\hat{x}_B\right)\ F(\hat{\sigma}),$$
$$\widetilde{\Pi} = r\left(\tilde{x}_B\right)\ [1 - F(\tilde{\sigma})] + r\left(\tilde{x}_A\right)\ F(\tilde{\sigma}) \qquad (5)$$

It is not obvious which default assignment generates higher industry profit, hence, which firm will win the bidding. The default firm earns higher revenue under $D = A$ than under $D = B$, as its charge is higher ($\hat{x}_A > \tilde{x}_B$) and so is its share of consumers ($\hat{\sigma} < \tilde{\sigma} \implies [1 - F(\hat{\sigma})] > [1 - F(\tilde{\sigma})]$), but the non-default firm's revenue is lower under $D = A$ (since $\hat{x}_B < \tilde{x}_A$ and $F(\hat{\sigma}) < F(\tilde{\sigma})$). Nevertheless, we will provide a sufficient condition for $\widehat{\Pi} > \widetilde{\Pi}$ by comparing the arithmetic sum of per-consumer charges of the two firms ("total charge") under the alternative default assignments: $\hat{x}_A + \hat{x}_B$ under $D = A$ vs. $\tilde{x}_B + \tilde{x}_A$ under $D = B$. (Equivalently, we compare the simple average of the charges.) Even though $\hat{x}_A > \tilde{x}_B$ while $\hat{x}_B < \tilde{x}_A$, with our maintained condition (C) a clear comparison of total charges can be made:

Lemma 1. *In equilibrium,* $\hat{x}_A + \hat{x}_B > \tilde{x}_B + \tilde{x}_A$ *if* $f'(s) > 0$, $\hat{x}_A + \hat{x}_B = \tilde{x}_B + \tilde{x}_A$ *if* $f'(s) = 0$, *and* $\hat{x}_A + \hat{x}_B < \tilde{x}_B + \tilde{x}_A$ *if* $f'(s) < 0$.

Hence, the ranking of equilibrium total charges is determined entirely by the sign of the derivative of the density function of the switching cost distribution, $f'(s)$, i.e., whether the distribution is "skewed" towards high values ($f'(s) > 0$) or low values ($f'(s) < 0$). If $f'(s) > 0$, moving the default to firm A from firm B raises the total charge—firm A's charge rises more than B's charge falls—and the reverse occurs if $f'(s) < 0$. The intuition is subtle and will help explain some of our ensuing results.

Observe that $f(s)$ is also the absolute value of the slope of each firm's demand with respect to its own charge evaluated at the marginal consumer in the switching cost distribution (i.e., the consumer who is indifferent between the two firms): $s = \hat{\sigma}$ when $D = A$ and $s = \tilde{\sigma}$ when $D = B$.[17] The marginal consumer is determined by the equilibrium differential in firms' charges and, from Proposition 1iii, is located lower in the distribution when firm A has the default: $\hat{\sigma} < \tilde{\sigma}$. Thus, moving the default to firm A will reduce the density at the marginal consumer if $f'(s) > 0$, thereby steepening each firm's demand curve and motivating each firm to raise its charge. For firm A, this incentive is reinforced by its demand increase from obtaining the default, hence its charge rises; for firm B, the demand reduction effect dominates, hence its charge falls, but by less than A's rise. This explains why the total charge is higher when firm A has the default if $f'(s) > 0$, but lower if $f'(s) < 0$.

The result below further highlights the key role that the distribution of switching costs plays in the comparison of industry profit:

Proposition 2. *When* $f'(s) \geq 0$, *industry profit is higher under* $D = A$ *if the revenue function* $r(x) = ax^m$ *is convex or not too concave; but when* $f'(s) < 0$, *it is possible that industry profit is lower under* $D = A$ *if* $r(x)$ *is linear. Formally: When* $f'(s) \geq 0$,

$\widehat{\Pi} > \widetilde{\Pi}$ *if* m *is not too much below* 1; *but when* $f'(s) < 0$, *it is possible that* $\widehat{\Pi} < \widetilde{\Pi}$ *if* $m = 1$.

The proof (in the appendix) first establishes that the ranking of charges satisfies $\hat{x}_B < \max\{\tilde{x}_B, \tilde{x}_A\} < \hat{x}_A$. Then, when $f'(s) \geq 0$, because $\hat{x}_A + \hat{x}_B > \tilde{x}_B + \tilde{x}_A$ and because $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$ and $F(\hat{\sigma}) < F(\tilde{\sigma})$, it follows that $\{\hat{x}_B, \hat{x}_A\}$ is a mean-increasing spread of $\{\tilde{x}_B, \tilde{x}_A\}$, which implies that

$$[1 - F(\hat{\sigma})]\, \hat{x}_A + F(\hat{\sigma})\hat{x}_B > [1 - F(\tilde{\sigma})]\, \tilde{x}_B + F(\tilde{\sigma})\tilde{x}_A$$

Therefore, with $r(x) = ax^m$, clearly $\widehat{\Pi} > \widetilde{\Pi}$ if $m \geq 1$ and also if m is not too much below 1, so that $r(x)$ is convex or not too concave. Whereas if $f'(s) < 0$, then $\hat{x}_A + \hat{x}_B < \tilde{x}_B + \tilde{x}_A$. In this case, it is possible that $\widehat{\Pi} < \widetilde{\Pi}$, as illustrated shortly in Example 2.

A special case of (C), which we call the power functions, is

$$F(s) = s^n \quad \text{and} \quad r(x) = ax^m \tag{C1}$$

where $n \geq 1$, $m \geq \Delta$, and $n = 1$ if $m < 1$. We have:

Corollary 1. *Suppose that* $F(s)$ *and* $r(x)$ *are the power functions given by (C1). Then* $\widehat{\Pi} > \widetilde{\Pi}$.

When $n \geq 1$, $f'(s) \geq 0$, and it then follows immediately from Proposition 2 that industry profit is higher under $D = A$ if $m \geq 1$. But if $\Delta \leq m < 1$, $r(x)$ allows for a wide range of strict concavity, and yet still $\widehat{\Pi} > \widetilde{\Pi}$ if $n = 1$. To understand this, notice that the comparison of industry profit depends both on the dispersion of charges $\{\hat{x}_B, \hat{x}_A\}$ relative to $\{\tilde{x}_B, \tilde{x}_A\}$ and on the degree of concavity of the revenue function $r(x)$. In the case here, it appears that when $r(x)$ becomes more concave, $\{\hat{x}_B, \hat{x}_A\}$ also become less dispersed relative to $\{\tilde{x}_B, \tilde{x}_A\}$, which offsets the effect of increasing concavity so that $\widehat{\Pi} > \widetilde{\Pi}$. In particular, when $F(s) = s$: $\hat{x}_A - \hat{x}_B = \frac{2}{3}\Delta = \tilde{x}_A - \tilde{x}_B$ if $m = 1$, while $\hat{x}_A - \hat{x}_B = \frac{1}{2}\Delta = \tilde{x}_A - \tilde{x}_B < \frac{2}{3}\Delta$ if $m = \frac{1}{2}$, which partly explains why $\widehat{\Pi} > \widetilde{\Pi}$ for all $m \geq \Delta$ in this case.

For convenience, we have assumed $r(x) = ax^m$ in Equations (C) and (C1). However, $\widehat{\Pi} > \widetilde{\Pi}$ is also possible when $r(x)$ takes other functional forms, even when it is concave and when $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, as illustrated in the next example:

Example 1. Suppose $F(s) = s$ and $r(x) = e^{-\frac{1}{x}}$. Then $r(x)$ may be concave, and $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$. Despite this, numerical analysis indicates that $\widehat{\Pi} > \widetilde{\Pi}$ for various values of Δ. For instance, if $\Delta = 0.5$, then $\hat{x}_A = 0.943$, $\hat{x}_B = 0.333$, $\hat{\sigma} = 0.11$; $\tilde{x}_A = 0.707 = \tilde{x}_B$, $\tilde{\sigma} = 0.5$; and $\widehat{\Pi} = 0.314 > \widetilde{\Pi} = 0.243$.

In this example, although $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, $\hat{x}_A$ is much higher than $\hat{x}_B$ and $r(\hat{x}_A)$ is weighted more than $r(\hat{x}_B)$, i.e., applies to more consumers ($\hat{\sigma} = 0.11 \implies [1 - F(\hat{\sigma})] = 0.89 > 0.11 = F(\hat{\sigma})$), vs. equal weights under $D = B$ for $r(\tilde{x}_A)$ and $r(\tilde{x}_B)$ (since $F(\tilde{\sigma}) = \frac{1}{2}$). (The different weighting arises because less switching occurs when $D = A$.) As a result, $\widehat{\Pi}$ is higher than $\widetilde{\Pi}$ even though $r(x)$ is concave. The ranking $\widehat{\Pi} > \widetilde{\Pi}$ holds also for all other values of Δ that we checked in Example 1 and, for instance, if $r(x) = e^{-\frac{1}{2x}}$.

Proposition 2, Corollary 1, and Example 1 suggest that industry "normally" is higher if the default position goes to the high-quality firm, $\widehat{\Pi} > \widetilde{\Pi}$. However, somewhat surprisingly, industry profit can be higher when the *low-quality* firm holds the default if $f'(s) < 0$, violating condition (C), as in the next example:

Example 2. Suppose $F(s) = s^{0.7}$ and $r(x) = ax$. Then, $\widehat{\Pi} < \widetilde{\Pi}$ if, for instance, $\Delta = 0.3$, where $\hat{x}_A = 0.5832$, $\hat{x}_B = 0.1666$, $\hat{\sigma} = 0.1166$; $\tilde{x}_A = 0.4964$, $\tilde{x}_B = 0.5439$, $\tilde{\sigma} = 0.3475$; and $\widehat{\Pi} = 0.4906a < \widetilde{\Pi} = 0.5213a$.

In Example 2, because $f'(s) < 0$, the total charge is lower under $D = A$ than under $D = B$: $\hat{x}_A$ is only slightly higher than $\tilde{x}_B$ while $\hat{x}_B$ is much lower than $\tilde{x}_A$. The intuition was explained after Lemma 1: Moving the default to firm A shifts the marginal consumer to a lower value in the switching cost distribution; then, with $f'(s) < 0$ the density of the distribution at the marginal consumer increases, hence the slope of each firm's demand curve flattens, incentivizing lower charges.[18] Industry profit could still be higher under $D = A$ because $\hat{\sigma} < \tilde{\sigma}$ so that $r(\hat{x}_A)$ is weighted more heavily than $r(\tilde{x}_B)$. However, the mass of consumers with $s < \hat{\sigma}$ is greater when $f(s)$ is decreasing than when it is increasing, and hence for a given $\hat{\sigma}$ more consumers will switch to B (under $D = A$) when $f(s)$ is decreasing—reducing the weight on $r(\hat{x}_A)$ sufficiently to yield a lower weighted average of $r(\hat{x}_A)$ and $r(\hat{x}_B)$ than under $D = B$. Therefore, when $f'(s) < 0$, it is possible that industry profit is lower when the higher-quality firm A has the default:

$$\begin{aligned}\widehat{\Pi} &= r\left(\hat{x}_A\right)\ [1 - F(\hat{\sigma})] + r\left(\hat{x}_B\right)\ F(\hat{\sigma}) \\ &< r\left(\tilde{x}_B\right)\ [1 - F(\tilde{\sigma})] + r\left(\tilde{x}_A\right)\ F(\tilde{\sigma}) = \widetilde{\Pi}\end{aligned}$$

In the rest of the paper, we shall assume $F(s)$ and $r(x)$ are given by (C1) so that A will be assigned the default position in equilibrium.

3.3 | Consumar Surplus and Total Welfare

Which default assignment will result in higher consumer surplus? In each case, some switching occurs because, from Proposition 1i and ii, the default product offers lower utility in equilibrium than the rival product. In the Appendix (see proof of Corollary 2), we show that consumer surplus takes the form in Equation (6):

$$\begin{aligned}\widehat{S} &= v_A - \hat{x}_A + \int_0^{\hat{\sigma}} F(s)ds \\ \widetilde{S} &= v_B - \tilde{x}_B + \int_0^{\tilde{\sigma}} F(s)ds\end{aligned} \tag{6}$$

Thus, consumer surplus can be expressed as the surplus that all consumers would get if they stayed with the default product ($v_A - \hat{x}_A$ or $v_B - \tilde{x}_B$), plus the integral term denoting the gain to those consumers who switch.[19] The difference in consumer surplus under the two default assignments is

$$\widehat{S} - \widetilde{S} = [\Delta - (\hat{x}_A - \tilde{x}_B)] - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)ds \tag{7}$$

The square-bracketed term is the difference in utilities of all consumers had they stayed with the default product under $D = A$

compared to $D = B$: A's quality advantage, $\Delta \equiv v_A - v_B$, minus A's charge premium when A holds the default position compared to B's charge when B holds the default. The integral term is the extra gain to switchers under regime $D = B$ compared to $D = A$, where $\tilde{\sigma} > \hat{\sigma}$ from Proposition 1iii.

Total welfare—the sum of industry profit and consumer surplus—under the alternative default assignments is

$$\widehat{W} = \widehat{\Pi} + \widehat{S}, \qquad \widetilde{W} = \widetilde{\Pi} + \widetilde{S}$$

To obtain clear welfare comparisons under the alternative assignments, we now consider some special cases of $F(s)$ and $r(x)$ that satisfy (C1), hence $\widehat{\Pi} > \widetilde{\Pi}$ (Corollary 1) so the default position would go to firm A under competitive bidding. We provide sufficient conditions for consumer surplus and for total welfare also to be higher under this default assignment, or the alternative $D = B$:

Corollary 2. *Suppose (C1) holds so that $F(s) = s^n$ and $r(x) = ax^m$. (i) If $n = 1$, then $\widehat{S} > \widetilde{S}$ for any $m \geq \Delta$. (ii) If $n = 2$ and $m = 1$, then $\widehat{S} < \widetilde{S}$, but $\widehat{W} > \widetilde{W}$ if $a \geq 0.45$.*

Consumer surplus tends to be higher under $D = A$ than under $D = B$ when the sum of the firms' charges ("total charge" $x_A + x_B$) is not (much) higher under $D = A$.[20,21] However, if the total charge is sufficiently higher when A is the default, then $\widehat{S} < \widetilde{S}$ is possible. As Corollary 2ii shows, when $F(s) = s^2$ and $r(x) = ax$, we have $\widehat{S} < \widetilde{S}$. In this case, conditional on holding the default position, firm A's charge exceeds B's by more than A's quality advantage: $\hat{x}_A - \tilde{x}_B = \frac{10}{8}\Delta$ (see Proof of Corollary 2ii), so in Equation (7) $[\Delta - (\hat{x}_A - \tilde{x}_B)] < 0$. Thus, consumers who stay with the default product are better off under $D = B$ than under $D = A$. (And the gain to switchers always is greater under $D = B$.) By contrast, replacing $F(s) = s^2$ with uniformly distributed switching costs $F(s) = s$ yields $\hat{x}_A - \tilde{x}_B = \frac{2}{3}\Delta \implies [\Delta - (\hat{x}_A - \tilde{x}_B)] > 0$ (see Proof of Corollary 2i), so consumers who stay with the default product are better off under $D = A$.

Thus, while industry profit is typically higher under $D = A$, hence firm A wins the default position, in such cases, consumer surplus can be higher *or lower* than under the reverse assignment $D = B$. It is perhaps surprising that there are plausible situations where consumer surplus is lower if the default is assigned to the higher-quality firm. This possibility arises because in the default position, the higher-quality firm may set a charge that exceeds the rival's charge, if instead it held the default, by more than the quality advantage. The distribution of consumers' switching costs $F(s)$ plays a key role—as explained after Lemma 1—and more so than the curvature of $r(x)$: With the same linear revenue function $r(x) = ax$, we obtained $\widehat{S} > \widetilde{S}$ if $F(s) = s$ but $\widehat{S} < \widetilde{S}$ if $F(s) = s^2$.

With this linear revenue function, it is helpful to recap how the slope of the switching-cost density function, $f'(s)$, affects our various results. Consider shifting the default to the higher-quality firm A. In equilibrium, there will be less switching, i.e., the consumer who is indifferent between switching from the default firm ("marginal consumer") will be located lower in the distribution. If $f'(s) = 0$ (uniform distribution), the density at the marginal consumer remains unchanged, hence the slope of firms' demands is unchanged. Firm A then sets a higher charge than B would set with the default, but A's charge premium is less than its quality advantage, hence, consumer surplus rises. Industry profit also rises. However, if $f'(s) > 0$ then shifting the default will lower the density at the marginal consumer, rendering demands less elastic and inducing the firms' average charge to rise (Lemma 1); if this effect is strong enough, consumer surplus falls, while industry profit still rises. Conversely, if $f'(s) < 0$, the average charge falls. In this case, it is possible, though unlikely, that industry profit is lower if firm A has the default.

Regarding total welfare, two forces push it to be higher if the default is assigned to firm A. First, the deadweight loss from switching costs is then lower since less switching occurs when $D = A$ than when $D = B$ (Proposition 1iii). Second, while total output is the same under either regime given that the market is always covered, the share of consumers using the higher-quality product A is likely to be larger when $D = A$.[22] However, whereas revenue is a pure transfer in standard environments, here the monetization activity x may be weighted differently by firms and consumers, hence can directly affect total welfare. These considerations are reflected in Corollary 2ii, where the revenue function is $r(x) = ax$ and the switching cost distribution is $F(s) = s^2$, hence $\widehat{\Pi} > \widetilde{\Pi}$ and $\widehat{S} < \widetilde{S}$. The ranking of total welfare then depends on the size of a. For $a = 1$, the expenditure ax is a pure transfer from consumer surplus to profit, implying $\widehat{W} > \widetilde{W}$ due to the aforementioned two forces. Thus, $\widehat{W} > \widetilde{W}$ also for a not too far below 1 (specifically, for $a > 0.45$), i.e., if the contribution of the charge x to profit is not too far lower than its disutility to consumers (recall that $u'(x) = -1$, hence $a = -r'(x)/u'(x)$). (In Corollary 2 part (i), $\widehat{W} > \widetilde{W}$ obviously holds since $\widehat{S} > \widetilde{S}$ and assumption (C1) ensures $\widehat{\Pi} > \widetilde{\Pi}$.)

4 | Welfare-Improving Regulation?

We now investigate how regulations governing default-position assignment may affect firms, consumers, and efficiency in this market.

4.1 | Equal Shares of Default Position

Suppose regulation requires that A and B are each assigned as the default for half of the consumers. We next examine the equilibrium in this case and compare it with that under competitive bidding when firm A wins, as occurs under condition (C1).

If $v_A - x_A \geq v_B - x_B$, then the only consumers who may switch are those with $D = B$, from B to A, with the marginal switching consumer type being $s = \sigma = x_B - x_A + \Delta$. The profit functions of the two firms would then be

$$\pi_A = \frac{r(x_A)}{2}\left[1 + F(x_B - x_A + \Delta)\right]$$
$$\pi_B = \frac{r(x_B)}{2}\left[1 - F(x_B - x_A + \Delta)\right] \tag{8}$$

Instead, if $v_A - x_A < v_B - x_B$, then the only consumers who may switch are those with $D = A$, from A to B, with the marginal switching consumer being $s = \sigma = x_A - x_B - \Delta$. However, we will show that switching from A to B will not occur in equilibrium.

With equal shares of the default, denote A's and B's equilibrium choices by x_A^e and x_B^e, and similar notation is adopted for other outcome variables. The result below references the following equations, derived from Equation (8) in the Appendix (Proof of Proposition 3):

$$\frac{m}{x_A^e} = \frac{f(\sigma^e)}{1 + F(\sigma^e)}; \qquad \frac{m}{x_B^e} = \frac{f(\sigma^e)}{1 - F(\sigma^e)} \tag{9}$$

where $\sigma^e = \Delta - x_A^e + x_B^e \in (0, 1)$, $0 \le x_A^e - x_B^e < \Delta$, and $v_A - x_A^e > v_B - x_B^e$. The result also establishes that the equilibrium industry profit and consumer surplus are given by

$$\Pi^e = r\left(x_A^e\right) \frac{1 + F(\sigma^e)}{2} + r\left(x_B^e\right) \frac{1 - F(\sigma^e)}{2},$$
$$S^e = \frac{v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)\, ds}{2} \tag{10}$$

Proposition 3. *Under equal shares of the default position, the equilibrium x_A^e and x_B^e satisfy (9). Consumers with $D = B$ and $s < \sigma^e$ switch to A, and there is no equilibrium where consumers with $D = A$ switch to B. Equilibrium industry profit and consumer surplus are given by (10). Moreover, $\Pi^e > \hat{\Pi}$ and $S^e < \hat{S}$ either if $\sigma^e \le \hat{\sigma}$, or if one of the following holds for $F(s) = s^n$ and $r(x) = ax^m$:*

i. *$n = 1$ and $m \ge 2\Delta$; or (ii) $n = m = 1$; or (iii) $n = 2$ and $m = 1$.*

Notice that under equal shares of the default, $x_A^e - x_B^e < \Delta$ (from Equation (9)), in contrast to $\hat{x}_A - \hat{x}_B > \Delta$ when $D = A$ for all consumers (Proposition 1i). That is, A's equilibrium charge now exceeds B's charge by less than A's quality advantage, hence consumers switch only from B to A, instead of the reverse direction when A is the default for all consumers.

Since industry profit is "typically" higher under $D = A$ than under $D = B$, one might have expected that shifting half the consumers from $D = A$ to $D = B$ would reduce industry profit. However, such a move will often *raise* industry profit. The reason is *softened competition*, resulting in higher charges.[23] In fact, if $\sigma^e \le \hat{\sigma}$, then $x_A^e > x_B^e \ge \hat{x}_A > \hat{x}_B$, so both firms set higher charges when they have equal shares of the default position than when firm B has none. Firm B is now motivated to raise its charge to exploit some of its default consumers, those with high switching costs. This *installed base effect* causes x_B^e to be substantially above $\hat{x}_B$. Although firm A's installed base falls by the same amount as B's rises (i.e., by half of the market), this constitutes a smaller proportional change than for firm B, hence, it exerts less downward pressure on x_A. The foreseen increase in x_B pushes firm A to raise its charge as well, because x_A and x_B are strategic complements, and this strategic effect tends to dominate A's installed base effect. Consequently, both charges rise relative to the bidding equilibrium (though A's charge rises by less).

As an illustration, consider the case in part (ii) of Proposition 3, $n = m = 1$. This is a subcase of (C1) that we will reference occasionally, where switching costs are uniformly distributed and the revenue function is linear:

$$F(s) = s, \qquad r(x) = a\, x \tag{Uniform-Linear}$$

In this Equation (*Uniform-Linear*) case,

$$x_A^e = 1 + \frac{\Delta}{3}, \quad x_B^e = 1 - \frac{\Delta}{3};$$
$$\hat{x}_A = \frac{2}{3} + \frac{\Delta}{3}, \quad \hat{x}_B = \frac{1}{3} - \frac{\Delta}{3}$$

so charges are uniformly higher under equal shares of the default than when firm A has the default for all consumers ($x_A^e - \hat{x}_A = \frac{1}{3}$, $x_B^e - \hat{x}_B = \frac{2}{3}$) for all $\Delta < 1$, even though $\sigma^e = \frac{\Delta}{3} \le \hat{\sigma} = \frac{1-\Delta}{3}$ only if $\Delta \le \frac{1}{2}$.

Consumer surplus under equal assignment, S^e, can be lower than under competitive bidding, $\hat{S}$, even when $\hat{S} < \tilde{S}$, i.e., even when the competitive bidding outcome yields lower consumer surplus than would obtain if B were the default for all consumers. For example, if $F(s) = s^2$ and $r(x) = ax$, then $S^e < \hat{S}$ even though $\hat{S} < \tilde{S}$ (from Corollary 2ii), because the total charge when default shares are equal is higher than under $D = A$, which in turn is higher than under $D = B$.

The comparison of total welfare W is generally ambiguous, due to the typically opposite changes in profit and consumer surplus. The proof of Proposition 3 in the appendix also establishes for the Equation (*Uniform-Linear*) case the following welfare rankings, where the thresholds a_1^e and a_2^e depend on the quality difference Δ:

Example 3. If $F(s) = s$ and $r(x) = ax$, then $\widehat{W} \gtreqless W^e$ when $a \lesseqgtr a_1^e \in \left(\frac{7}{8}, \frac{17}{4}\right)$; while if $F(s) = s^2$ and $r(x) = ax$, then $\widehat{W} \gtreqless W^e$ when $a \lesseqgtr a_2^e \in (0.035, 1)$.

Thus, when switching costs are uniformly distributed and the revenue function is linear, total welfare is higher under competitive bidding than under equal default shares when a is below a threshold $a_1^e \in \left(\frac{7}{8}, \frac{17}{4}\right)$; whereas if the switching cost distribution is quadratic, the threshold is $a_2^e \in (0.035, 1)$. In both these cases, $\hat{\Pi} < \Pi^e$ while $\hat{S} > S^e$ (Proposition 3), hence $\widehat{W} > W^e$ if the weight on profit relative to consumer surplus, a, is sufficiently low.

4.2 | Other Shares of Default Position

Consider regulation that assigns $D = A$ for a portion $\lambda \in \left(\frac{1}{2}, 1\right)$ of consumers. The higher-valued product is then assigned as the default for more than half of the consumers, but not all. Earlier, we analyzed the cases $\lambda = \frac{1}{2}$ (equal assignment) and $\lambda = 1$ (competitive bidding outcome when A wins), which are limiting cases of this more general setting. We next establish that for λ close to $\frac{1}{2}$ there exists a unique equilibrium similar to that when $\lambda = \frac{1}{2}$, whereas for λ close to 1 the unique equilibrium is similar to that when $\lambda = 1$. We will also discuss how profits and consumer surplus may change as λ varies in both ranges.

At the candidate equilibrium for λ close to $\frac{1}{2}$, where consumer switching occurs only from B to A, the marginal switching consumer is

$$\sigma = x_B - x_A + \Delta \ge 0$$

Then, the profits of the two firms are

$$\pi_A = r(x_A)\left[\lambda + (1-\lambda)F(x_B - x_A + \Delta)\right],$$
$$\pi_B = r(x_B)(1-\lambda)\left[1 - F(x_B - x_A + \Delta)\right]$$

In this case, denote the equilibrium charges of the two firms by x_A^- and x_B^-, the marginal consumer by σ^-, and the other outcome variables by Π^-, S^-, and W^-.

At the candidate equilibrium for λ close to 1, where consumer switching occurs only from A to B, the marginal switching consumer is

$$\sigma = x_A - x_B - \Delta > 0$$

Then, the profit functions of the two firms are

$$\pi_A = r(x_A)\,\lambda\left[1 - F(x_A - x_B - \Delta)\right],$$
$$\pi_B = r(x_B)\left[1 - \lambda + \lambda F(x_A - x_B - \Delta)\right]$$

In this case, denote the equilibrium charges of the two firms by x_A^+ and x_B^+; and similarly for σ^+, Π^+, S^+, and W^+.

Proposition 4. *There exist $\lambda^- \in \left(\frac{1}{2}, 1\right)$ and $\lambda^+ \in (\lambda^-, 1)$ such that if $\lambda < \lambda^-$, then $\Delta > x_A^- - x_B^- > 0$ and there is consumer switching only from B to A; while if $\lambda > \lambda^+$, then $x_A^+ - x_B^+ > \Delta$ and there is consumer switching only from A to B. Moreover, if $F(s) = s$ and $r(x) = ax$, the Uniform-Linear case, then $\lambda^- = \frac{\Delta+1}{\Delta+2} < \frac{1}{2-\Delta} = \lambda^+$, and*

i. *for $\lambda < \lambda^-$, $\Pi^- > \hat{\Pi}$, $S^- < \hat{S}$, $\frac{d\Pi^-}{d\lambda} > 0$, and $\frac{dS^-}{d\lambda} < 0$;*

ii. *for $\lambda > \lambda^+$, $\Pi^+ \gtreqless \hat{\Pi}$ if $\lambda \lesseqgtr \frac{5/2}{(\Delta+1)^2}$, $S^+ < \hat{S}$, $\frac{d\Pi^+}{d\lambda} \gtreqless 0$ if $\lambda \gtreqless \frac{\sqrt{5/2}}{(\Delta+1)}$, and $\frac{dS^+}{d\lambda} > 0$.*

As λ changes from $\frac{1}{2}$ to 1, equilibrium industry profit and consumer surplus can vary non-monotonically, as occurs in the Equation (*Uniform-Linear*) case. First, for $\lambda < \lambda^-$, as λ rises, industry profit also rises, but consumer surplus falls. A higher λ raises A's installed base (consumers with A as the default), which induces a rise in x_A^- and a smaller rise in x_B^-. (The latter reflects the strategic response to the rise in x_A^-, which outweighs the effect on x_B of the reduction in B's installed base.) Consequently, industry profit rises but consumer surplus falls. Thus, for all $\lambda \in \left(\frac{1}{2}, \lambda^-\right)$ we have $\Pi^- > \Pi^e > \hat{\Pi}$ and $S^- < S^e < \hat{S}$ (where the second inequalities follows from Proposition 3), so profit is higher but consumer surplus is lower than in the competitive bidding outcome ($D = A$).

Next, for $\lambda > \lambda^+$, as λ rises consumer surplus now rises $\left(\frac{dS^+}{d\lambda} > 0\right)$, but remains below $\hat{S}$. The behavior of profit is more complex. As shown in the proof of Proposition 4, if the quality difference $\Delta \leq 0.581$, then profit decreases in λ but remains above $\hat{\Pi}$ for all $\lambda \in (\lambda^+, 1)$. If $\Delta > 0.581$, profit may decrease or increase with λ, and can be lower or higher than $\hat{\Pi}$. These patterns are roughly explained as follows. For λ near 1, B's customer base is small, and as λ rises the large proportional decrease in B's customer base exerts a powerful downward effect on x_B, so that both x_B^+ and x_A^+ can fall, though the latter by less (since A's customer base rose). However, with a higher λ, more consumers may patronize A, which has a higher charge. As a result, industry profit tends to—but not always—fall.

Recall from Proposition 3 that if the default position is distributed equally between the two firms $\left(\lambda = \frac{1}{2}\right)$, then industry profit tends to be higher, but consumer surplus tends to be lower than in the competitive bidding outcome ($\lambda = 1$): $\Pi^e > \hat{\Pi}$ and $S^e < \hat{S}$. Compared to that outcome, a regulated assignment with $\lambda^- \in \left(\frac{1}{2}, 1\right)$ also tends to increase industry profit and decrease consumer surplus.

Total welfare can rise or fall relative to $\widehat{W}$, the level under competitive bidding where $\lambda = 1$. Continuing with the Equation (*Uniform-Linear*) case, when a is above some threshold, the profit effect tends to dominate, resulting in higher total welfare than when $\lambda = 1$; and conversely, if a is below the threshold.

4.3 | Regulated Assignment With Endogenous Product Quality

Suppose there is learning by the firms, so that when more consumers use product B, its quality v_B can increase. This scenario is at the heart of the DOJ's complaint against Google DOJ [2]. DOJ argues that search algorithms improve with experimentation and, hence, improve with a search engine's number of users. By obtaining default status at leading distribution outlets for search engines, Google deprives rival search engines of users and, hence, impairs their ability to improve their quality through learning. We take no position on the merits of the DOJ's argument,[24] but will attempt to capture its essence and the potential welfare effects of reducing Google's share of default positions.

Before proceeding to that analysis, we offer brief observations on an alternative way to strengthen the weaker firm: Mandatory data sharing. This seemingly is an obvious remedy, because sharing historical data may be "easy" and is non-rivalrous—it can improve the efficiency of the receiving firm without impeding the sharing firm's efficiency. However, the required data is often complex and rapidly changing, which can hamper regulatory enforcement of data-sharing obligations. Additionally, and less obviously, there can be a rivalrous aspect in the use of data because two firms may choose different interactions with a user in response to the same raw data. (We credit an oral remark by Michael Katz for this point.) For example, faced with a given search query, firm A will place a certain ad whereas firm B might have preferred to experiment with a different ad for learning purposes, depending on its existing knowledge. Thus, firm B would not learn as much from obtaining A's raw data, including the ad's outcome, as it would if it served that customer as the default and controlled ad placement.[25]

To formally model the quality improvement issues under alternative default assignments, one would need a dynamic model. For instance, one might consider a "simple" setting with two periods, where v_A and v_B are exogenously given in period 1, but may improve in period 2 due to learning in period 1, and greater improvement occurs when a firm serves more consumers in period 1. Under competitive bidding, $D = A$ for all consumers (i.e., $\lambda = 1$) in period 1, while under the regulated assignment, $D = A$ for some portion $\lambda \in \left(\frac{1}{2}, 1\right)$ of consumers. Then, with regulation, more consumers would use B in period 1, which could increase v_B and result in more consumers patronizing B in period 2.

There are significant complexities, however, to analyze even a two-period scenario in an equilibrium model. The number of consumers that firms serve in period 1 will depend not only on λ but also on their endogenous choices of the charges x_A and x_B in period 1. Also, the product qualities in period 2 will be influenced by these numbers, which could in turn affect the firms' equilibrium charges in both periods. Moreover, the switching decisions of consumers in period 1 may also depend on their expectations about the second-period equilibrium charges. Since our purpose is mainly to illustrate possibilities—of whether and when consumer surplus can be higher under regulation with endogenous product quality—we adopt a "reduced-form" approach with some simplifying assumptions.

Specifically, for $i = A, B$, denote firm i's first-period equilibrium market share and second-period equilibrium product quality by q_i^c and v_{2i}^c under competitive bidding, and by q_i^r and v_{2i}^r under regulation. Let $\Delta_2^c = v_{2A}^c - v_{2B}^c$ and $\Delta_2^r = v_{2A}^r - v_{2B}^r$. Given $\Delta = v_A - v_B > 0$ and λ, the choices of the firms and consumers in period 1 will determine the second-period equilibrium qualities. Assume that firm i's second-period product quality v_{2i}^j under policy regime $j = c, r$ will be higher if firm i has more users in period 1. Moreover, assume that quality increases with scale at an increasing rate initially but a decreasing rate eventually, and that regulation reduces A's quality lead but does not eliminate it, with $\Delta \geq \Delta_2^c > \Delta_2^r > 0$.[26]

For convenience, we also assume that each period contains a separate group of consumers, so consumers face no intertemporal choice, and the default assignment in period 2 is determined as in period 1—by competitive bidding or regulation.

Under regulation, denote the equilibrium consumer surplus by S_1^r for period 1 and S_2^r for period 2, with the (overall) equilibrium consumer surplus being

$$S^r = S_1^r + \phi S_2^r$$

where ϕ is the weight on consumer surplus in period 2 relative to that in period 1, and ϕ can be larger or smaller than 1 to allow for possible differences in consumer population size or time length for periods 1 and 2. Similarly denote the consumer surplus under competitive bidding by S_1^c, S_2^c, and S^c. Then, the change in consumer surplus due to the regulation is:

$$\Lambda_S = S^r - S^c = \Lambda_{S_1} + \phi \Lambda_{S_2}$$

where $\Lambda_{S_1} = S_1^r - S_1^c$ and $\Lambda_{S_2} = S_2^r - S_2^c$.

We consider the Equation (*Uniform-Linear*) case, $F(s) = s$ and $r(x) = ax$, and pre-learning qualities $v_A = 5$ and $v_B = 4.5$, hence $\Delta = 0.5$. Although v_{2A}^r, v_{2B}^r, v_{2A}^c, and v_{2B}^c are all endogenous, depending on the firms' number of users in period 1, once these quality values are determined we can use Proposition 4 to evaluate Λ_{S_2}. For illustration purposes, Corollary 3 below will use the following quality values for the second period:

$$v_{2A}^c = 5.1, \quad v_{2B}^c = 4.6, \quad \Delta_2^c = 0.5;$$
$$v_{2A}^r \in [5,\ 5.1], \quad v_{2B}^r = 4.85, \quad \Delta_2^r = v_{2A}^r - 4.85 \in [0.15,\ 0.25] \tag{11}$$

where we have normalized the quality values for $v_{2A}^c = 5.1$, $v_{2B}^c = 4.6$, and $v_{2B}^r = 4.85$ but allowed a range of values for v_{2A}^r. In Equation (11), both firms' product qualities are higher in period 2 than in period 1 (due to learning) under both policy regimes, but under regulation A's quality improves less while B's quality improves more: $v_{2A}^c - v_A = 0.1$ and $v_{2B}^c - v_B = 0.1$, while $v_{2A}^r - v_A \in [0, 0.1]$ and $v_{2B}^r - v_B = 0.35$. As we show in the proof of Corollary 3, if firms are myopic and choose their first-period charges to maximize current profits, their equilibrium first-period market shares will be $q_A^c = 0.833$ and $q_B^c = 0.167$ under competitive bidding but $q_A^r = 0.733$ and $q_B^r = 0.267$ when $\lambda = 0.8$ under regulation, with $\Lambda_{S_1} = -0.078 < 0$. In the second period, the equilibrium is described by Proposition 4 for $\lambda \geq \lambda^+$ (since $\lambda^+ = \frac{1}{2-\Delta} = 2/3$). Then, the second-period quality values in Equation (11) are consistent with situations where the quality benefit of learning is first increasing and then decreasing with a firm's number of users: $v_{2B}^c - v_B = 0.1$ with a change in the number of users equal to $q_B^c - 0 = 0.167$; $v_{2B}^r - v_{2B}^c = 0.25$ with $q_B^r - q_B^c = (0.267 - 0.167) = 0.10$, and $v_{2A}^c - v_{2A}^r \in [0, 0.1]$ with $q_A^c - q_A^r = 0.1$.

Corollary 3. *Assume $F(s) = s$, $r(x) = ax$, $v_A = 5.0$, $v_B = 4.5$, $\lambda = 0.8 \geq \lambda^+ = \frac{1}{2-\Delta}$, and $\Delta \geq \Delta_2^c > \Delta_2^r$. Then, Λ_{S_2} increases in q_A^r, q_B^r, and Δ_2^c, but Λ_{S_2} decreases in v_{2A}^c and v_{2B}^c. Furthermore, when (11) holds and $\Lambda_{S_1} < 0$, $\Lambda_{S_2} \gtreqless 0$ when $v_{2A}^r \gtreqless 5.045$, implying that $\Lambda_S > 0$ if v_{2A}^r and ϕ are sufficiently high, but $\Lambda_S < 0$ if $v_{2A}^r \leq 5.045$.*

Thus, if regulation endows firm B with the default position for portion $1 - \lambda$ of consumers, which improves v_B possibly due to learning, then consumers can indeed benefit compared to the bidding outcome where firm A obtains the default position for all consumers, provided A's quality does not suffer too much. Consumers may gain in the second period through several channels. A greater increase in v_B under the regulation *directly benefits* consumers who use product B. Additionally, it has strategic effects: A higher v_B, which reduces the quality asymmetry between the two products (i.e., $\Delta_2^r < \Delta_2^c$) can lower the charges by both firms due to *intensified competition* when they are more symmetric in quality. Also, x_A and x_B will be closer to each other under the regulation, which would reduce the amount of switching, hence *reduce the switching costs* incurred by consumers.

However, by increasing the installed base for firm B, the regulation also softens competition and negatively impacts consumer surplus (as discussed after Proposition 3). This can lead to $\Lambda_{S_1} < 0$ in period 1 and also to a lower Λ_{S_2} in period 2. The smaller increase in v_A under the regulation also negatively impacts consumer surplus in period 2, and it is possible that $\Lambda_{S_2} < 0$ despite the greater increase in v_B. In our numerical example, if $v_{2A}^c - v_{2A}^r \geq 0.055$ (i.e., if $v_{2A}^r \in [5,\ 5.045]$), which represents a decrease in v_A due to the regulation that is at least $\frac{5.1-5.045}{4.85-4.6} = 22\%$ of the increase in v_B, then $\Lambda_{S_2} < 0$ and hence $\Lambda_S < 0$ as well; while if

$v^c_{2A} - v^r_{2A}$ is sufficiently small (e.g., $v^c_{2A} - v^r_{2A} \leq 0.04$) and ϕ sufficiently large, then the regulation will increase consumer surplus. In sum, product quality improves for both firms even without the regulation, and the regulation will lead to higher charges by both firms under given product qualities. Hence, the regulation can increase consumer surplus only if its positive (negative) impact on firm B's (firm A's) quality improvement is sufficiently large (small).[27]

4.4 | Choice Screen

Instead of assigning a default product to consumers, an alternative policy known as the "choice screen" allows consumers to choose their preferred default from a set of displayed options. This policy was first adopted by the European Commission in 2009: Microsoft was required to display alternative web browsers along with its own Internet Explorer instead of presetting Explorer as the default. A choice screen was also adopted in the Commission's Android [4] case, where Google was required to display other search engines in addition to its own.[28] The Digital Markets Act adopted by the European Union [11] requires large online platforms designated as "gatekeepers" to provide a choice screen for users to select their default apps for online search engines, virtual assistants, or web browsers.

We analyze a choice screen policy under the same informational assumptions of our core model: Consumers know both qualities and observe both firms' charges before choosing between these products. However, instead of being assigned a default product, consumers now choose their preferred product without having to incur a switching cost. Since consumers differ only in their switching costs, which are now rendered moot, and are identical in their product valuations, the equilibrium resembles Bertrand competition with asymmetric product qualities: The weaker firm B sets its charge x_B equal to marginal cost (that we normalized to zero), and firm A captures the entire market while charging a premium equal to its quality advantage: $x_A = x_B + \Delta$. Ironically, firm B would attract no customers in such a scenario, unlike the bidding-for-default outcome even when firm A wins.[29]

This stark pattern—that a choice screen policy would lead all consumers to choose the stronger product—emerges in our setting because of some special assumptions. Notably, consumers are heterogeneous only in their switching costs, and have perfect information about qualities and firms' charges. Relaxing either assumption could result in some consumers forgoing the stronger product under a choice screen. In fact, Decarolis et al. [31] found that Google incurred modest decreases in its search market share after the introduction of a choice screen. Such a pattern could be explained by factors outside our model, notably richer consumer heterogeneity, that would allow both firms to attract consumers under Bertrand competition with no preassigned defaults. For instance, consumers may differ in their valuation of quality (as with standard vertical differentiation) and/or in their "location" (horizontal differentiation à la Hotelling, e.g., the weight placed on accuracy of search results vs. invasion of privacy).[30] Imperfect information also would open up a range of possibilities.[31] Therefore, we are not suggesting that a choice screen would necessarily reduce the weaker firm's market share. Nevertheless, our analysis offers the following robust insights.

There is a strong presumption that a choice screen would be the superior policy for consumer welfare in the short run if consumers face de minimis cost to set up the default themselves through the choices presented.[32] Consumers would then obtain their preferred choice. Additionally, they would benefit from lower monetization charges because competition is intensified when firms must compete for a larger share of the market instead of having a base of default consumers. From a longer-run standpoint, however, a choice screen may be inferior to some regulatory default assignments. If product quality improves with a firm's share of consumers at a diminishing rate, then shifting some consumers to the weaker firm will increase the latter's quality more than it reduces the leader's quality, and ultimately, consumers can benefit, directly and from stronger competition. Under a choice screen, too few consumers would choose the lower-quality product because consumers individually ignore the positive competition externality they generate by enabling the weaker firm to improve its quality. Thus, if the predominant policy concern is to enable improvement by the weaker firm, a choice screen approach can be problematic.

5 | Concluding Remarks

We analyzed several methods of assigning the default position for a product supplied by two competing firms with exogenously different qualities, when consumers face heterogeneous costs of switching from the default product to the rival. The default firm enjoys market power over its inframarginal consumers, those with higher switching costs, which it exploits through greater monetization, such as unwanted advertising. Consequently, when the default position is assigned through competitive bidding for all consumers, the default winner provides lower utility than the rival, even when the winner is the higher-quality firm. That firm indeed tends to win (though we show a counter-example), not due to its quality advantage directly, but because industry monetization is greater when it, rather than the rival, holds the default. Interestingly, the shape of the switching cost distribution plays an important role in determining whether the higher-quality firm wins the bidding and whether consumer surplus is higher or lower under this default assignment.

Our analysis also yields some policy insights. Compared to the stronger firm winning the default everywhere, assigning via regulation the default to the rival for some minority share of consumers tends to increase profit and harm consumers. Profit rises because competition is softened when both firms have sticky (default) consumers. All consumers lose from the softened competition, and those who are assigned the lower-quality product suffer additional harm directly. We briefly considered another scenario where product quality is not fixed but instead improves at a decreasing rate with the firm's share of users, possibly due to learning. Assigning the default position to the weaker firm for some share of consumers may then benefit consumers in the long run, but this must be weighed against the short-run harm. An alternative approach is to let consumers select their preferred option from a choice screen. This approach will likely benefit consumers in the short run, but can be problematic for longer-term competition and consumer welfare if learning effects are paramount. Too few consumers will choose the weaker product because they ignore the beneficial externality they would generate by helping the weaker firm improve its quality.

Finally, we note that our model omits some features that could yield a better alignment between consumer welfare and the default assignment under competitive bidding. The leading firm may have an advantage not only in quality but also in monetization efficiency (e.g., better targeting of ads), which would yield it greater revenue than the rival per dollar harm to consumers. Alternatively, or in addition, it may enjoy greater utilization of its product by consumers than would the rival, instead of our assumption of fixed aggregate consumption. Lastly, the third-party may assign the default position not solely based on the highest bid, but also weighing its customers' utility from the competing products (e.g., Apple claims it selects the default search engine that is best for iPhone buyers). Therefore, it may award the default to the second-highest bidder if that assignment is better for consumers.

Our model also abstracts away from some additional considerations that can be relevant in practice. For instance, consumers may have imperfect information about product quality, and they may also place different values on the quality increase (as in models of vertical differentiation). Moreover, rather than a single lump-sum fee for the default position, the payment to the third party could include a per-unit royalty component. It would be interesting to consider these possibilities in future research.

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Endnotes

[1] Switching between browsers in the late 1990s was more difficult than switching between search engines today, notably due to the slowness of downloading a second browser via a narrowband Internet connection.

[2] In August 2024, U.S. federal judge Amit Mehta found Google's default agreements anti-competitive, a ruling Google has vowed to appeal.

[3] Similarly, in the literature on auctions of advertising positions by an online platform (e.g., Edelman et al. [9]), an ad placed at a higher position will be seen by more consumers.

[4] We will provide a simple extension of this toy model in Section 3.1. to show when the greater-sales argument holds and connect the logic to search settings, before proceeding to our main model with variable revenue per consumer.

[5] Our main model has fixed qualities, hence abstracts from concerns that Google maintains a quality advantage partly by denying rivals the scale needed to improve their quality. We will address this issue in a "reduced form" manner in Section 4.3.

[6] We assume no regulation constrains firms' monetization charges. This allows us to focus on the assignment of default position and the resulting equilibrium characteristics. In practice, there could be regulations, especially for non-price charges, which would change the equilibrium outcomes.

[7] Our formulation of heterogeneous consumer switching costs follows the approach in Chen [14]. This approach has been used to analyze a variety of competition issues, including exclusionary contracts, e.g., Bedre-Defolie and Biglaiser [15].

[8] Much of our analysis would also apply if (heterogeneous) switching costs were replaced by (varying degrees of) status quo bias. Such bias has been shown to be important (see Fletcher [16] and the references cited therein). We adopt the switching costs formulation primarily for purposes of welfare analysis. To illustrate the distinction, letting consumers choose their preferred default at the outset benefits them in our setting by avoiding switching costs, but may be detrimental if decision making is onerous.

[9] Greater sales is the sole reason for different bids in Athey and Ellison [7] and Chen and He [8] since all firms charge the same price. In Armstrong et al. [17] (Section 3), a higher-quality firm enjoys both greater sales and a higher price.

[10] We will flesh out in Section 3.1. The role of leakage for the greater-sales argument in both settings.

[11] Although data sharing improves the laggard's quality without reducing the leader's quality, it can harm consumers by reducing the laggard's incentive to improve its quality via aggressive pricing to attract consumers. This disincentive effect is weak if the laggard is far behind (hence does not price aggressively), but dominates when qualities are close, a finding that derives from the explicitly dynamic analysis.

[12] We can allow the possibility that a certain part of a firm's monetizing activities is not observed by consumers before they decide to use its product. Both firms would then set the maximum level for such unobservable activities, and consumers would rationally expect this. We may thus view the variable x in our model as the observable monetizing charge beyond the unobservable level.

[13] In an earlier version of the paper Chen and Schwartz [21], we considered more general forms of $r(x)$, but the analysis involved more complications without substantial gain in insights. In particular, if $r(0) > 0$ were high, firm A may optimally charge Δ when $D = A$ to retain all the consumers, so that $x_B = 0$. We could extend our analysis to allow $x_i < 0$, without the need to impose parameter restrictions that ensure $x_i > 0$ in equilibrium. However, to reduce the number of cases, we focus on the more relevant scenarios where there are monetization charges that are undesirable to consumers in equilibrium.

[14] Unless stated otherwise, proofs for formally-presented results are contained in the Appendix.

[15] Since switching costs are heterogeneous, each firm faces a downward-sloping demand, yielding the familiar outcome for Bertrand competition with imperfect substitutes where both firms earn positive margins.

[16] Observe that this logic does not generally extend beyond duopoly: Although firm A will outbid B only if their combined profits are higher if A wins, each firm's bid does not incorporate the effects on other firms, which could differ between the two assignments.

[17] Under $D = A$ the marginal switching consumer is $s = \hat{\sigma} = (x_A - x_B - \Delta)$, and the demand functions are $Q_A(\hat{\sigma}) = 1 - F(\hat{\sigma})$, $Q_B(\hat{\sigma}) = F(\hat{\sigma})$, hence $-\frac{\partial Q_A}{\partial x_A} = f(\hat{\sigma}) = -\frac{\partial Q_B}{\partial x_B}$. Similarly, under $D = B : \tilde{\sigma} = (x_B - x_A + \Delta)$, $Q_A(\tilde{\sigma}) = F(\tilde{\sigma})$, $Q_B(\tilde{\sigma}) = 1 - F(\tilde{\sigma})$, hence, again, $-\frac{\partial Q_A}{\partial x_A} = f(\tilde{\sigma}) = -\frac{\partial Q_B}{\partial x_B}$.

[18] Interestingly, and thematically related, Hagiu and Julien [25] show that putting an inferior option first—an intermediary directs consumers to their inferior option, "search diversion"—can affect the level of firms' charges to benefit the intermediary. (Their mechanism is different: Search diversion increases the proportion of low-demand consumers faced by each firm, which drives down prices and increases total sales.)

[19] The integral term is the difference in consumer utility from the two products minus the switching costs. When, say, $D = A$, $\int_0^{\hat{\sigma}} F(s)ds = \hat{\sigma} F(\hat{\sigma}) - \int_0^{\hat{\sigma}} s f(s) ds$, where $\hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta$ is the gross gain to any

consumer from switching to B (hence also denotes the consumer indifferent between remaining at A or incurring $s = \hat{\sigma}$ to switch), while $\int_0^{\hat{\sigma}} s f(s) ds$ is the total switching costs.

[20] This holds, for instance, under (C1) with $n = 1$ and any $m \geq \Delta$, where the total charges under $D = A$ and $D = B$ are equal. $\widehat{S} > \widetilde{S}$ also in Example 1 where $n = 1$ and $r(x) = e^{-\frac{1}{x}}$, and in Example 2 where $n = 0.7$ and $m = 1$, in both of which $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$.

[21] If the total charge is higher under $D = A$, then $v_A - \hat{x}_A - \hat{x}_B - (v_B - \tilde{x}_B - \tilde{x}_A)$ is reduced, which makes it more likely that $\widehat{S} < \widetilde{S}$; but $\tilde{\sigma} - \hat{\sigma} = v_A - \tilde{x}_A - (v_B - \tilde{x}_B) - [v_B - \hat{x}_B - (v_A - \hat{x}_A)]$ may also be lower, which would reduce the difference in the mass of consumers who switch to benefit from the non-default product's higher utility; so, it is still possible that $\widehat{S} > \widetilde{S}$.

[22] The share of consumers that will use product A is $1 - F(\hat{\sigma})$ when $D = A$ and $F(\tilde{\sigma})$ when $D = B$. From Proposition 1, $1 - F(\hat{\sigma}) > \frac{1}{2}$ while $F(\tilde{\sigma}) \leq \frac{1}{2}$ if $F(\Delta) \leq \frac{1}{2}$. But if $F(\Delta) > \frac{1}{2}$, then $F(\tilde{\sigma}) > \frac{1}{2}$, rendering the comparison ambiguous.

[23] Katz [26] (in pp. 23–25) provides another example where competition is softened by a "neutrality" policy that creates some captive consumers for each firm, akin to a finding of de Cornière and Taylor [10] discussed earlier.

[24] Note that Gilbert and Newbery's [23] result, that an incumbent monopolist would outbid a potential entrant for a single vital asset, does not immediately extend to the Google case because there are *multiple* distribution outlets for which firms can bid. The profitability of sustaining monopoly through bidding for multiple assets is an open question Kamien and Zang [27], Malueg and Schwartz [28], Krishna [29].

[25] On the nuances of the role of data in competition generally see Crémer et al. [30].

[26] Initially, with no users, A's quality advantage is Δ, and under competitive bidding firm A has more users than firm B in the first period, $q_A^c > q_B^c$, which could expand A's quality advantage in period 2, $\Delta < \Delta_2^c$. The inequality $\Delta \geq \Delta_2^c$ therefore requires the assumption that quality increases with scale at an increasing rate initially but a decreasing rate eventually, so that firm A improves less in period 2 than does firm B due to A's larger scale. This assumption also plays a role in the welfare analysis, as discussed later.

[27] We have explored alternative values of v_{2i}^c and v_{2i}^r. The regulation appears more likely to increase consumer welfare if the learning benefit is first increasing and then decreasing in the number of users, because $v_{2B}^r - v_{2B}^c$ would then increase more relative to $v_{2B}^c - v_B$ and $v_{2A}^c - v_{2A}^r$. In our two-period setting, the regulation is also more likely to increase consumer welfare if adopted in period 1 but not in period 2.

[28] In the Android case, unlike in Microsoft, the rival products displayed in the choice screen (for both search engines and web browsers) were determined through auctions conducted by Google starting in 2020. Ostrovsky [3] shows that the identity of the winning bidders will depend on whether a bidder pays a flat fee for the right to be displayed in the choice screen, or a fee per user that installs its product ("per install").

[29] There, firm A exploits its customers' heterogeneous switching costs, setting $\hat{x}_A > \hat{x}_B + \Delta$, which in turn allows firm B to attract some (switching) customers in equilibrium. Essentially, firm A behaves like a "fat cat" Tirole [24], exploiting its installed base by raising price, which allows the weaker firm to survive.

[30] Modeling such additional differentiation would make the analysis much more complicated, because, together with heterogeneous switching costs, the model would essentially become one of multi-dimensional product differentiation.

[31] For example, suppose firms adjusted their charges to the new equilibrium levels after the introduction of a choice screen policy, but only a fraction of consumers observed these new charges while the rest based their product choices on the historical charges under the default regime. Those charges resulted in lower utility from the default product than from the rival (Proposition 1), hence the fraction of consumers who observe only the historical charges will select the weaker product under a choice screen.

[32] For search engines, with a properly designed choice screen the cost may well be de minimis (and we are setting aside any psychic costs of making a choice). However, in other situations, a consumer may need to incur costs if (s)he chooses the default. Whereas switching costs are avoided, the consumer may need to incur costs to find the relevant alternatives or to install the default option.

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Appendix A

The appendix contains proofs for Propositions 1–4, Lemma 1, and Corollaries 1–3.

Proof of Proposition 1.

i. When $D = A$, the equilibrium $\hat{x}_A$ and $\hat{x}_B$, if they are strictly positive, satisfy the following first-order conditions obtained from Equation (1):

$$\frac{\partial \pi_A}{\partial x_A} = amx_A^{m-1}\left[1 - F(x_A - x_B - \Delta)\right] - ax_A^m f(x_A - x_B - \Delta) = 0 \quad \text{(A1)}$$

$$\frac{\partial \pi_B}{\partial x_B} = amx_B^{m-1} F(x_A - x_B - \Delta) - ax_B^m f(x_A - x_B - \Delta) = 0 \quad \text{(A2)}$$

First, we show that $\hat{x}_B > 0$. If, to the contrary, $\hat{x}_B = 0$, then $\hat{x}_A > \Delta$ because $\left.\frac{\partial \pi_A}{\partial x_A}\right|_{x_A=\Delta} = a\, m\, \Delta^{m-1} - a\, \Delta^m\, f(0) > 0$ by Assumption (C), so some consumers will switch to B. But then B could increase its profit by slightly raising x_B above 0 while still attracting some switching consumers, contradicting $\hat{x}_B = 0$.
Next, $\hat{x}_A - \hat{x}_B > \Delta$, because if $\hat{x}_A - \hat{x}_B < \Delta$, A could increase its profit by raising x_A. Also, if $\hat{x}_A - \hat{x}_B = \Delta$, we would have $\left.\frac{\partial \pi_B}{\partial x_B}\right|_{\hat{x}_B} < 0$, contradicting $\hat{x}_B$ being optimal. Therefore $\hat{x}_A - \hat{x}_B > \Delta$.

Observe that (A1) and (A2) can be rewritten as the two equations in Equation (3). With $\sigma_A = x_A - x_B - \Delta$, let

$$\mu(x_A) \equiv \frac{m}{x_A}, \quad h(\sigma_A) \equiv \frac{f(\sigma_A)}{1 - F(\sigma_A)}, \quad \textit{and} \quad g(\sigma_A) \equiv \frac{f(\sigma_A)}{F(\sigma_A)}$$

where $\mu'(x_A) < 0$, while $h'(\sigma_A) > 0$ and $g'(\sigma_A) < 0$ because $f(s)$ is logconcave from Equation (C). We show that the $\hat{x}_A$ and $\hat{x}_B$ that satisfy (3) are unique. Each equation in Equation (3) implicitly defines x_A as a function of x_B, and the curves in the (x_B, x_A)-space for the two functions, where x_A is on the vertical axis, respectively have the following slopes:

$$\frac{dx_A}{dx_B} = \frac{h'(\sigma_A)}{h'(\sigma_A) - \mu'(x_A)} \in (0, 1),$$

$$\frac{dx_A}{dx_B} = \frac{g'(\sigma_A) + \mu'(x_A)}{g'(\sigma_A)} > 1$$

Thus, the two curves intersect only once, implying that $\hat{x}_A$ and $\hat{x}_B$ exist uniquely. Notice that the positive slopes imply that the two firms' choices are strategic complements.
Finally, because $\hat{x}_A - \hat{x}_B > \Delta$, $\hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta > 0$, and x/m increases in x, we have

$$\frac{\hat{x}_A}{m} - \frac{\hat{x}_B}{m} = \frac{1 - 2F(\hat{\sigma})}{f(\hat{\sigma})} > 0$$

and hence $F(\hat{\sigma}) < \frac{1}{2}$.

ii. When $D = B$, the equilibrium $\tilde{x}_A$ and $\tilde{x}_B$, if they are strictly positive, satisfy the following first-order conditions obtained from Equation (2):

$$\frac{\partial \pi_A}{\partial x_A} = amx_A^{m-1} F(x_B - x_A + \Delta) - r(x_A) f(x_B - x_A + \Delta) = 0 \quad \text{(A3)}$$

$$\frac{\partial \pi_B}{\partial x_B} = amx_B^{m-1}\left[1 - F(x_B - x_A + \Delta)\right] - r(x_B) f(x_B - x_A + \Delta) = 0 \quad \text{(A4)}$$

For any $\tilde{x}_B \geq 0$, firm A will choose $\tilde{x}_A > 0$ to profit from the switching consumers. It follows that $\tilde{x}_B > 0$ as well.
Next, since both $\tilde{x}_A > 0$ and $\tilde{x}_B > 0$, we must have $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A > 0$, and hence $\tilde{x}_A - \tilde{x}_B < \Delta$. Equations (A3) and (A4) can be rewritten as the two equations in Equation (4), each of which implicitly defines x_A as a function of x_B, and the curves in the (x_B, x_A)-space for the two functions, where x_A is on the vertical axis, respectively have the following slopes:

$$\frac{dx_A}{dx_B} = \frac{g'(\sigma_B)}{\mu'(x_A) + g'(\sigma_B)} \in (0, 1),$$

$$\frac{dx_A}{dx_B} = \frac{h'(\sigma_B) - \mu'(x_B)}{h'(\sigma_B)} > 1$$

where $\sigma_B = \Delta + x_B - x_A$. Thus, the two curves intersect only once, implying that $\tilde{x}_A$ and $\tilde{x}_B$ exist uniquely. Notice that this also implies that the two firms' choices are strategic complements.
If $F(\Delta) \leq \frac{1}{2}$, we show that $\tilde{x}_A \leq \tilde{x}_B$, and thus $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A \geq \Delta$, which further implies $F(\tilde{\sigma}) \leq \frac{1}{2}$. Suppose to the contrary that $\tilde{x}_A > \tilde{x}_B$, then $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A < \Delta$ and $\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} = \frac{m}{\tilde{x}_A} < \frac{m}{\tilde{x}_B} = \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)}$, which implies

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} < \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \implies 1 - F(\tilde{\sigma}) < F(\tilde{\sigma}) \implies \frac{1}{2} < F(\tilde{\sigma}) < F(\Delta)$$

a contradiction. Thus $\tilde{x}_A \leq \tilde{x}_B$. It follows that

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} = \frac{m}{\tilde{x}_A} \geq \frac{m}{\tilde{x}_B} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \implies 1 - F(\tilde{\sigma}) \geq F(\tilde{\sigma}) \implies F(\tilde{\sigma}) \leq \frac{1}{2}$$

On the other hand, if $F(\Delta) > \frac{1}{2}$, we show that $\tilde{x}_A > \tilde{x}_B$ and hence $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A < \Delta$. If, to the contrary, $\tilde{x}_A \leq \tilde{x}_B$, then $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A \geq \Delta$ and $\frac{m}{\tilde{x}_A} \geq \frac{m}{\tilde{x}_B}$, which implies

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} \geq \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \implies 1 - F(\tilde{\sigma}) \geq F(\tilde{\sigma}) \implies \frac{1}{2} \geq F(\tilde{\sigma}) \geq F(\Delta)$$

a contradiction. Hence $\tilde{x}_A > \tilde{x}_B$. It follows that

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} = \frac{m}{\tilde{x}_A} < \frac{m}{\tilde{x}_B} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \implies 1 - F(\tilde{\sigma}) < F(\tilde{\sigma}) \implies F(\tilde{\sigma}) > \frac{1}{2}$$

iii. Suppose, to the contrary, that $\tilde{\sigma} \leq \hat{\sigma}$. Then

$$\frac{\hat{x}_A}{m} = \frac{1 - F(\hat{\sigma})}{f(\hat{\sigma})} \leq \frac{1 - F(\tilde{\sigma})}{f(\tilde{\sigma})} = \frac{\tilde{x}_B}{m} \implies \hat{x}_A \leq \tilde{x}_B,$$
$$\frac{\hat{x}_B}{m} = \frac{F(\hat{\sigma})}{f(\hat{\sigma})} \geq \frac{F(\tilde{\sigma})}{f(\tilde{\sigma})} = \frac{\tilde{x}_A}{m} \implies \hat{x}_B \geq \tilde{x}_A$$

Hence

$$\tilde{\sigma} - \hat{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A - \left[\hat{x}_A - \hat{x}_B - \Delta\right] = 2\Delta + \tilde{x}_B - \hat{x}_A + \hat{x}_B - \tilde{x}_A > 0$$

which produces a contradiction. Therefore $\tilde{\sigma} > \hat{\sigma}$. It follows that $\hat{x}_A > \tilde{x}_B$ and $\tilde{x}_A > \hat{x}_B$.

Moreover, for future reference, if $F(\Delta) \leq \frac{1}{2}$ so that $F(\tilde{\sigma}) \leq \frac{1}{2}$, then $\tilde{x}_A \leq \tilde{x}_B$ and hence $\hat{x}_A > \tilde{x}_B \geq \tilde{x}_A > \hat{x}_B$. If $F(\Delta) > \frac{1}{2}$ so that $F(\tilde{\sigma}) > \frac{1}{2}$, then $\tilde{x}_A > \tilde{x}_B$. □

Proof of Lemma 1. From Equations (3) and (4), because $\tilde{\sigma} > \hat{\sigma}$,

$$\frac{\hat{x}_A}{m} + \frac{\hat{x}_B}{m} - \left[\frac{\tilde{x}_A}{m} + \frac{\tilde{x}_B}{m}\right] = \frac{1}{m}\left[\hat{x}_A + \hat{x}_B - (\tilde{x}_A + \tilde{x}_B)\right] = \frac{1 - F(\hat{\sigma})}{f(\hat{\sigma})} + \frac{F(\hat{\sigma})}{f(\hat{\sigma})} - \left[\frac{1 - F(\tilde{\sigma})}{f(\tilde{\sigma})} + \frac{F(\tilde{\sigma})}{f(\tilde{\sigma})}\right] = \frac{1}{f(\hat{\sigma})} - \frac{1}{f(\tilde{\sigma})} \gtreqless 0 \iff f'(s) \gtreqless 0$$

□

Proof of Proposition 2. From Proposition 1: $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B, \hat{\sigma} < \tilde{\sigma}, \hat{x}_A > \tilde{x}_B$ and $\tilde{x}_A > \hat{x}_B$. Thus, if $\tilde{x}_A \leq \tilde{x}_B$, then $\hat{x}_A > \tilde{x}_B \geq \tilde{x}_A > \hat{x}_B$. Suppose instead $\tilde{x}_A > \tilde{x}_B$. If $\tilde{x}_B \leq \hat{x}_B$, then $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$ implies $\hat{x}_A > \tilde{x}_A$; while if $\tilde{x}_B > \hat{x}_B$, then if $\hat{x}_A \leq \tilde{x}_A$, we would have $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, contradicting the result that $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$. Hence, $\hat{x}_A > \tilde{x}_A$ if $\tilde{x}_A > \tilde{x}_B$. Thus $\hat{x}_A > \max\{\tilde{x}_B,\ \tilde{x}_A\} > \hat{x}_B$. This, together with $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$, implies that $(\hat{x}_B,\ \hat{x}_A)$ is more dispersed than $(\tilde{x}_B,\ \tilde{x}_A)$.

When $f'(s) \geq 0$, $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$. Therefore, since $\hat{\sigma} < \tilde{\sigma}$, the pair $\{\hat{x}_B,\ \hat{x}_A\}$ is a mean-increasing spread of $\{\tilde{x}_B,\ \tilde{x}_A\}$; that is:

$$[1 - F(\hat{\sigma})]\,\hat{x}_A + F(\hat{\sigma})\hat{x}_B > [1 - F(\tilde{\sigma})]\,\tilde{x}_B + F(\tilde{\sigma})\tilde{x}_A$$

Notice that for given Δ, $\widehat{\Pi} = [1 - F(\hat{\sigma})]\,r(\hat{x}_A) + F(\hat{\sigma})r(\hat{x}_B)$ exceeds $\widetilde{\Pi} = [1 - F(\tilde{\sigma})]\,r(\tilde{x}_B) + F(\tilde{\sigma})r(\tilde{x}_A)$ by a strictly positive number if $r(x) = ax$, or $m = 1$. By continuity, there exists some $\underline{m} < 1$ such that $\widehat{\Pi} > \widetilde{\Pi}$ if $m \geq \underline{m}$.

When $f'(s) < 0$, $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$. It is then possible that $\widehat{\Pi} > \widetilde{\Pi}$, and we prove this by Example 2 where $F(s) = s^{0.7}$ and $r(x) = ax$. □

Proof of Corollary 1. Under (C1), $F(s) = s^n$ and $r(x) = ax^m$, where $n \geq 1$, $m \geq \Delta$, and $n = 1$ if $m < 1$. If $n \geq 1$ and $m \geq 1$, then it follows directly from Proposition 2 that $\widehat{\Pi} > \widetilde{\Pi}$. On the other hand, if $n = 1$ and $m \in (\Delta, 1)$, then from Equations (3) and (4) we obtain:

$$\hat{x}_A = \frac{m + m\Delta + m^2}{2m+1}, \quad \hat{x}_B = \frac{m(m-\Delta)}{2m+1}, \quad \hat{\sigma} = \frac{m-\Delta}{2m+1};$$

$$\tilde{x}_A = \frac{m\Delta + m^2}{2m+1}, \quad \tilde{x}_B = \frac{m - m\Delta + m^2}{2m+1}, \quad \tilde{\sigma} = \frac{m+\Delta}{2m+1}$$

where $\hat{x}_i > 0$ and $\tilde{x}_i > 0$. Hence

$$\widehat{\Pi} = a\left(\frac{m + m\Delta + m^2}{2m+1}\right)^m\left(1 - \frac{m-\Delta}{2m+1}\right) + a\left(\frac{m(m-\Delta)}{2m+1}\right)^m\frac{m-\Delta}{2m+1},$$

$$\widetilde{\Pi} = a\left(\frac{m - m\Delta + m^2}{2m+1}\right)^m\left(1 - \frac{m+\Delta}{2m+1}\right) + a\left(\frac{m\Delta + m^2}{2m+1}\right)^m\frac{m+\Delta}{2m+1}$$

But because $\widehat{\Pi} - \widetilde{\Pi} = 0$ when $\Delta = 0$ and

$$\frac{\partial\left(\widehat{\Pi} - \widetilde{\Pi}\right)}{\partial\Delta} = (m+1)\frac{\left(\frac{m - m\Delta + m^2}{2m+1}\right)^m - \left(\frac{m^2 - m\Delta}{2m+1}\right)^m + \left(\frac{m(m+\Delta+1)}{2m+1}\right)^m - \left(m\frac{m+\Delta}{2m+1}\right)^m}{2m+1} > 0$$

for all $\Delta \in [0, m]$, we have $\widehat{\Pi} - \widetilde{\Pi} > 0$ for all $\Delta \in (0, m]$. □

Proof of Corollary 2. First,

$$\widehat{S} = (v_A - \hat{x}_A)\,[1 - F(\hat{\sigma})] + \int_0^{\hat{\sigma}} (v_B - \hat{x}_B - s) f(s) ds, \quad \text{and}$$

$$\widetilde{S} = (v_B - \tilde{x}_B)\,[1 - F(\tilde{\sigma})] + \int_0^{\tilde{\sigma}} (v_A - \tilde{x}_A - s) f(s) ds$$

We can rewrite

$$\widehat{S} = (v_A - \hat{x}_A) - (v_A - \hat{x}_A)F(\hat{\sigma}) + (v_B - \hat{x}_B)F(\hat{\sigma}) - \hat{\sigma}F(\hat{\sigma}) + \int_0^{\hat{\sigma}} F(s)ds$$
$$= (v_A - \hat{x}_A) + [-\Delta + \hat{x}_A - \hat{x}_B - \hat{\sigma}]F(\hat{\sigma}) + \int_0^{\hat{\sigma}} F(s)ds$$
$$= (v_A - \hat{x}_A) + \int_0^{\hat{\sigma}} F(s)ds$$

Similarly,

$$\tilde{S} = v_B - \tilde{x}_B + \int_0^{\tilde{\sigma}} F(s)ds$$

Thus,

$$\widehat{S} \gtreqless \widetilde{S} \iff \Delta - \hat{x}_A + \tilde{x}_B \gtreqless \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)ds$$

Next, we have:

i. If $F(s) = s$ and $r(x) = ax^m$, then

$$\hat{x}_A = \frac{m + m\Delta + m^2}{2m+1} > 0, \qquad \hat{x}_B = \frac{m(m-\Delta)}{2m+1} > 0,$$
$$\hat{\sigma} = \frac{m-\Delta}{2m+1},$$
$$\tilde{x}_A = \frac{m\Delta + m^2}{2m+1} > 0, \qquad \tilde{x}_B = \frac{m - m\Delta + m^2}{2m+1} > 0,$$
$$\tilde{\sigma} = \frac{m+\Delta}{2m+1}$$

$$\begin{aligned}\hat{S} - \tilde{S} &= \Delta - (\hat{x}_A - \tilde{x}_B) - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)ds \\ &= \Delta - \left(\frac{m + m\Delta + m^2}{2m+1} - \frac{m - m\Delta + m^2}{2m+1}\right) - \int_{\frac{m-\Delta}{2m+1}}^{\frac{m+\Delta}{2m+1}} s ds \\ &= \frac{\Delta}{(2m+1)^2} > 0\end{aligned}$$

For illustration and later reference, if $F(s) = s$ and $r(x) = ax$, then,

$$\begin{aligned}&\hat{x}_A = \frac{2+\Delta}{3} > 0, \quad \hat{x}_B = \frac{1-\Delta}{3} > 0 \\ &\hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta = \frac{1}{3}(1-\Delta); \\ &\tilde{x}_A = \frac{1+\Delta}{3} > 0, \quad \tilde{x}_B = \frac{2-\Delta}{3} > 0 \\ &\tilde{\sigma} = \tilde{x}_B - \tilde{x}_A + \Delta = \frac{1}{3}(1+\Delta)\end{aligned} \tag{A5}$$

$$\begin{aligned}&\widehat{S} = v_A - \frac{2+\Delta}{3} + \int_0^{\frac{1}{3}(1-\Delta)} s ds = v_A - \frac{1}{18}\left(11 + 8\Delta - \Delta^2\right); \\ &\widetilde{S} = v_B - \frac{2-\Delta}{3} + \int_0^{\frac{1}{3}(1+\Delta)} s ds = v_B - \frac{1}{18}\left(11 - 8\Delta - \Delta^2\right) \\ &\widehat{S} - \widetilde{S} = \frac{1}{9}\Delta > 0\end{aligned} \tag{A6}$$

ii. Suppose $F(s) = s^2$ and $r(x) = ax$. Then

$$\begin{aligned}&\hat{x}_A = \frac{5}{8}\Delta + \frac{3}{8}\sqrt{\Delta^2+4}, \quad \hat{x}_B = \frac{1}{8}\sqrt{\Delta^2+4} - \frac{1}{8}\Delta, \\ &\hat{\sigma} = \frac{1}{4}\left(\sqrt{\Delta^2+4} - \Delta\right); \\ &\tilde{x}_A = \frac{1}{8}\Delta + \frac{1}{8}\sqrt{\Delta^2+4}, \quad \tilde{x}_B = \frac{3}{8}\sqrt{\Delta^2+4} - \frac{5}{8}\Delta, \\ &\tilde{\sigma} = \frac{1}{4}\left(\sqrt{\Delta^2+4} + \Delta\right)\end{aligned}$$

$$\begin{aligned}\widehat{S} - \widetilde{S} &= [\Delta - (\hat{x}_A - \tilde{x}_B)] - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)ds \\ &= -\frac{1}{24}\Delta\left(\Delta^2+9\right) < 0\end{aligned}$$

The comparison of profits (and, for later reference, also of total welfare) is as follows:

$$\begin{aligned}\widehat{\Pi} &= a\hat{x}_A\left(1-\hat{\sigma}^2\right) + a\hat{x}_B\hat{\sigma}^2 \\ &= \frac{1}{64}a\Big(40\Delta + 24\sqrt{\Delta^2+4} - \left(\Delta^2+4\right)^{\frac{3}{2}} - 3\Delta^3 \\ &\quad + 5\Delta^2\sqrt{\Delta^2+4} - \Delta\left(\Delta^2+4\right)\Big), \\ \widetilde{\Pi} &= a\tilde{x}_A\tilde{\sigma}^2 + a\tilde{x}_B\left(1-\tilde{\sigma}^2\right) \\ &= \frac{1}{64}a\left(-8\Delta + 8\sqrt{\Delta^2+4} + \Delta^3 + 3\Delta\left(\Delta^2+4\right)\right),\end{aligned}$$

$$\begin{aligned}\widehat{\Pi} - \widetilde{\Pi} &= \frac{1}{64}a\Big(32\Delta + 16\sqrt{\Delta^2+4} - \left(\Delta^2+4\right)^{\frac{3}{2}} \\ &\quad - 8\Delta^3 + 5\Delta^2\sqrt{\Delta^2+4}\Big) > 0\end{aligned}$$

$$\begin{aligned}\widehat{W} - \widetilde{W} &= \frac{1}{64}a\Big(32\Delta + 16\sqrt{\Delta^2+4} - \left(\Delta^2+4\right)^{\frac{3}{2}} \\ &\quad - 8\Delta^3 + 5\Delta^2\sqrt{\Delta^2+4}\Big) \\ &\quad - \frac{1}{24}\Delta\left(\Delta^2+9\right) \gtreqless 0 \\ &\iff a \gtreqless \frac{8}{3}\left(\frac{\Delta\left(\Delta^2+9\right)}{32\Delta + 16\sqrt{\Delta^2+4} - \left(\Delta^2+4\right)^{\frac{3}{2}} - 8\Delta^3 + 5\Delta^2\sqrt{\Delta^2+4}}\right) \\ &\in (0, 0.45)\end{aligned}$$

Thus $\widehat{W} > \widetilde{W}$ if $a \geq 0.45$. □

Proof of Proposition 3. First, suppose

$$v_A - x_A > v_B - x_B$$

so that some of the consumers with $D = B$ will switch to A, but no consumer whose $D = A$ will switch to B. The marginal switching consumer with $D = B$ is

$$\sigma = \Delta - x_A + x_B$$

From Equation (8), the equilibrium x_A^e and x_B^e satisfy the first-order conditions

$$\begin{aligned}&\frac{\partial \pi_A}{\partial x_A} = amx_A^{m-1}\left[1 + F\left(\Delta - x_A + x_B\right)\right] - ax_A^m f\left(\Delta - x_A + x_B\right) = 0, \\ &\frac{\partial \pi_B}{\partial x_B} = amx_B^{m-1}\left[1 - F\left(\Delta - x_A + x_B\right)\right] - ax_B^m f\left(\Delta - x_A + x_B\right) = 0\end{aligned}$$

which can be rewritten as (9) if $x_A^e > 0$ and $x_B^e > 0$.

Note that $x_A^e - x_B^e \geq 0$, because otherwise $x_B^e > x_A^e > 0$, which implies

$$\frac{m}{x_A^e} = \frac{f(\sigma^e)}{1 + F(\sigma^e)} > \frac{f(\sigma^e)}{1 - F(\sigma^e)} = \frac{m}{x_B^e} \implies -F(\sigma^e) > F(\sigma^e)$$

a contradiction.

Next, $\sigma^e = \Delta - x_A^e + x_B^e > 0$, because if $\sigma^e < 0$, B can increase π_B by raising x_B; and if $\sigma^e = 0$, we would have $x_A^e = x_B^e$ (from Equation (9) since F would be 0), which implies $\sigma^e = \Delta - x_A^e + x_B^e = \Delta > 0$, a contradiction. Hence $x_A^e - x_B^e < \Delta$. And $\sigma^e < 1$, because $\frac{\partial \pi_B}{\partial x_B}\Big|_{x_B = 1 + x_A^e - \Delta} < 0$.

The only other potential equilibrium may arise when $v_A - x_A < v_B - x_B$, in which case the marginal switching consumer whose $D = B$ is $\sigma = -\Delta + x_A - x_B > 0$, and the two firms' profit functions are

$$\begin{aligned}&\pi_A = r\left(x_A\right)\frac{1}{2}\left[1 - F(-\Delta + x_A - x_B)\right], \\ &\pi_B = r\left(x_B\right)\frac{1}{2}\left[1 + F(-\Delta + x_A - x_B)\right]\end{aligned}$$

We now show there can be no such equilibrium. Suppose, to the contrary, that the equilibrium exists. Then at such an equilibrium, (x_A, x_B) satisfy the first-order conditions

$$\begin{aligned}&amx_A^{m-1}\left[1 - F\left(-\Delta + x_A - x_B\right)\right] - ax_A^m f\left(-\Delta + x_A - x_B\right) = 0, \\ &amx_B^{m-1}\left[1 + F\left(-\Delta + x_A - x_B\right)\right] - ax_B^m f\left(-\Delta + x_A - x_B\right) \leq 0\end{aligned}$$

where

$$\sigma = -\Delta + x_A - x_B > 0 \implies x_A - x_B > \Delta \implies x_A > x_B \geq 0$$

Hence,

$$\frac{m}{x_A} = \frac{f(-\Delta + x_A - x_B)}{1 - F(-\Delta + x_A - x_B)} < \frac{m}{x_B} \leq \frac{f(-\Delta + x_A - x_B)}{1 + F(-\Delta + x_A - x_B)}$$
$$\implies 1 + F(\sigma) < 1 - F(\sigma) \implies 2F(\sigma) < 0$$

a contradiction.

We next establish the expressions for S^e and Π^e: In equilibrium, consumers whose $D = B$ will switch to A if $s < \sigma^e$. Hence, consumer surplus is

$$S^e = \frac{1}{2}(v_A - x_A^e) + \frac{1}{2}(v_B - x_B^e)[1 - F(\sigma^e)] + \frac{1}{2}\int_0^{\sigma^e}(v_A - x_A^e - s)f(s)ds$$
$$= \frac{1}{2}\left[(v_A - x_A^e) + (v_B - x_B^e) + \int_0^{\sigma^e} F(s)ds\right]$$

The expression for Π^e follow directly from Equation (8).

Moreover, suppose $\sigma^e \leq \hat{\sigma}$. From Equations (4) and (9),

$$\frac{m}{x_B^e} = \frac{f(\sigma^e)}{1 - F(\sigma^e)} \leq \frac{f(\hat{\sigma})}{1 - F(\hat{\sigma})} = \frac{m}{\hat{x}_A}$$

and hence $x_B^e \geq \hat{x}_A$. It follows that $x_A^e > x_B^e \geq \hat{x}_A > \hat{x}_B$. Therefore,

$$\Pi^e = a(x_A^e)^m \frac{[1 + F(\sigma^e)]}{2} + a(x_B^e)^m \frac{[1 - F(\sigma^e)]}{2} > a(x_B^e)^m$$

while

$$\widehat{\Pi} = a\hat{x}_A^m[1 - F(\hat{\sigma})] + a\hat{x}_B^m F(\hat{\sigma}) < a\hat{x}_A^m$$

Thus $\Pi^e > \widehat{\Pi}$. Also, because $\hat{x}_A \leq x_B^e < x_A^e$, $v_B < v_A$, and $\sigma^e \leq \hat{\sigma}$,

$$\hat{S} = v_A - \hat{x}_A + \int_0^{\hat{\sigma}} F(s)ds = \frac{2(v_A - \hat{x}_A) + 2\int_0^{\hat{\sigma}} F(s)ds}{2}$$
$$> \frac{v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)ds}{2} = S^e$$

Next, suppose (C1) is satisfied: $r(x) = ax^m$ and $F(s) = s^n$.

i. If $n = 1$, then

$$x_A^e = \frac{(m + m\Delta + 2m^2)}{2m + 1}, \quad x_B^e = \frac{(m - m\Delta + 2m^2)}{2m + 1},$$
$$\sigma^e = \frac{\Delta}{2m + 1};$$
$$\Pi^e = a\left(\frac{(m + m\Delta + 2m^2)}{2m + 1}\right)^m \frac{1 + \frac{\Delta}{2m+1}}{2} + a\left(\frac{(m - m\Delta + 2m^2)}{2m + 1}\right)^m \frac{1 - \frac{\Delta}{2m+1}}{2};$$
$$S^e = \frac{1}{2}\left(v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)ds\right); \quad \text{and}$$
$$\sigma^e - \hat{\sigma} = \frac{\Delta}{2m + 1} - \frac{m - \Delta}{2m + 1} = \frac{2\Delta - m}{2m + 1} \leq 0 \iff m \geq 2\Delta$$

Thus, $m \geq 2\Delta \implies \sigma^e \leq \hat{\sigma}$, which we showed was sufficient for $\widehat{\Pi} < \Pi^e$ and $\hat{S} > S^e$.

ii. If $n = m = 1$, then

$$x_A^e = 1 + \frac{1}{3}\Delta, \quad x_B^e = 1 - \frac{1}{3}\Delta > 0, \quad \sigma^e = \frac{1}{3}\Delta;$$
$$S^e = \frac{1}{2}(v_A + v_B - 2) + \frac{\Delta^2}{36}, \quad \Pi^e = \frac{1}{9}a(\Delta^2 + 9), \quad \text{and}$$
$$W^e = \frac{1}{9}a(\Delta^2 + 9) + \frac{1}{2}(v_A + v_B - 2) + \frac{\Delta^2}{36}.$$
$$\hat{S} - S^e = \frac{1}{36}(2\Delta + \Delta^2 + 14) > 0;$$
$$\widehat{\Pi} - \Pi^e = \frac{1}{9}a(2\Delta + \Delta^2 - 4) < 0.$$
$$\widehat{W} - W^e = \frac{1}{36}(14 - 16a + 2\Delta + 8a\Delta + \Delta^2 + 4a\Delta^2) \gtreqless 0$$
$$\iff a \lesseqgtr a_1^e \equiv \frac{\Delta^2 + 2\Delta + 14}{4(4 - 2\Delta - \Delta^2)}, \quad \text{with}$$
$$a_1^e \in \left(\frac{7}{8}, \frac{17}{4}\right) \text{ for } \Delta \in (0, 1)$$

iii. If $n = 2$ and $m = 1$, then

$$x_A^e = \frac{(\Delta^2 + 4)}{4\Delta}, \quad x_B^e = -\frac{(\Delta^2 - 4)}{4\Delta}, \quad \sigma^e = \frac{\Delta}{2}.$$
$$\sigma^e - \hat{\sigma} = \frac{\Delta}{2} - \frac{\left(\sqrt{\Delta^2 + 4} - \Delta\right)}{4} < 0$$
$$\iff 3\Delta < \sqrt{\Delta^2 + 4} \iff \Delta^2 < \frac{1}{2}$$

But for any $\Delta < 1$,

$$\Pi^e = r(x_A^e)\frac{1 + F(\sigma^e)}{2} + r(x_B^e)\frac{1 - F(\sigma^e)}{2} = \frac{a}{16}\frac{\Delta^4 + 16}{\Delta},$$
$$S^e = \frac{1}{2}\left(v_A - \frac{(\Delta^2 + 4)}{4\Delta} + v_B + \frac{(\Delta^2 - 4)}{4\Delta} + \int_0^{\frac{\Delta}{2}} F(s)ds\right).$$
$$\widehat{\Pi} - \Pi^e = \frac{a}{64}\frac{-36\Delta^2 + 5\Delta^3\sqrt{\Delta^2 + 4} + 24\Delta\sqrt{\Delta^2 + 4} - \Delta(\Delta^2 + 4)^{\frac{3}{2}} - 64}{\Delta} < 0$$

$$\hat{S} - S^e = -\frac{132\Delta^2 + 8\Delta^4 - 3\Delta^3\sqrt{\Delta^2 + 4} - 96\Delta^2 + 72\Delta\sqrt{\Delta^2 + 4} - \Delta(\Delta^2 + 4)^{\frac{3}{2}} - 192}{192\Delta} > 0$$

$$\widehat{W} \gtreqless W^e \iff a \lesseqgtr a_2^e$$
$$\equiv \frac{36\Delta^2 + 8\Delta^4 - 3\Delta^3\sqrt{\Delta^2 + 4} + 72\Delta\sqrt{\Delta^2 + 4} - \Delta(\Delta^2 + 4)^{\frac{3}{2}} - 192}{(3)\left(-36\Delta^2 + 5\Delta^3\sqrt{\Delta^2 + 4} + 24\Delta\sqrt{\Delta^2 + 4} - \Delta(\Delta^2 + 4)^{\frac{3}{2}} - 64\right)}$$
$$\in (0.035, 1)$$

□

Proof of Proposition 4. First, at the potential equilibrium where some consumers with $D = B$ switch to A, the marginal consumer is $\sigma = \Delta - (x_A - x_B) > 0$. The equilibrium x_A^- and x_B^- solve the first-order conditions

$$\frac{\partial \pi_A}{\partial x_A} = r'(x_A)[\lambda + (1 - \lambda)F(\Delta - x_A + x_B)] - r(x_A)(1 - \lambda)f(\Delta - x_A + x_B) = 0,$$

$$\frac{\partial \pi_B}{\partial x_B} = r'(x_B)\left[1 - F(\Delta - x_A + x_B)\right] - r(x_B) f(\Delta - x_A + x_B) = 0$$

which can be rewritten as

$$\frac{m}{x_A} = \frac{(1-\lambda) f(\sigma)}{\lambda + (1-\lambda) F(\sigma)}, \qquad \frac{m}{x_B} = \frac{f(\sigma)}{1 - F(\sigma)} \tag{A7}$$

Since $\left(x_A^- - x_B^-\right) < \Delta$ if λ is sufficiently close to $\frac{1}{2}$, there is some $\lambda^- \in \left(\frac{1}{2}, 1\right)$ such that if $\lambda \le \lambda^-$, in equilibrium $x_A^- > x_B^- > 0$, $\sigma^- = \Delta - \left(x_A^- - x_B^-\right) > 0$, and there is consumer switching only from B to A.

Next, consider the potential equilibrium where some consumers with $D = A$ switch to B. In this case, the equilibrium x_A^+ and x_B^+ solve the first-order conditions

$$r'(x_A)\left[1 - F(-\Delta + x_A - x_B)\right] - r(x_A) f(-\Delta + x_A - x_B) = 0,$$
$$r'(x_B)\left[1 - \lambda + \lambda F(-\Delta + x_A - x_B)\right] - r(x_B) \lambda f(-\Delta + x_A - x_B) = 0$$

which can be written as

$$\frac{m}{x_A^+} = \frac{f(\sigma^+)}{1 - F(\sigma^+)}, \qquad \frac{m}{x_B^+} = \frac{\lambda f(\sigma^+)}{1 - \lambda + \lambda F(\sigma^+)} \tag{A8}$$

where $\sigma^+ = x_A^+ - x_B^+ - \Delta > 0$, or $x_A^+ - x_B^+ > \Delta$. As $\lambda \to 1$, this equilibrium exists as in Proposition 1. On the other hand, as $\lambda \to \frac{1}{2}$,

$$\frac{m}{x_A^+} - \frac{m}{x_B^+} = \frac{f(\sigma^+)}{1 - F(\sigma^+)} - \frac{\lambda f(\sigma^+)}{1 - \lambda + \lambda F(\sigma^+)}$$
$$\Longrightarrow \frac{r'(x_A^+)}{r(x_A^+)} - \frac{r'(x_B^+)}{r(x_B^+)} = \frac{f(\sigma^+)}{1 - F(\sigma^+)} - \frac{f(\sigma^+)}{1 + F(\sigma^+)} > 0$$

which cannot hold if $x_A^+ > \Delta + x_B^+$. Hence, there is some $\lambda^+ \in (\lambda^-, 1)$ such that (A8) holds if and only if $\lambda > \lambda^+$.

Now suppose $r(x) = ax$ and $F(s) = s$. Then

$$x_A^- = \frac{1}{3}\left(\frac{1 + \Delta + \lambda - \Delta\lambda}{(1-\lambda)}\right), \qquad x_B^- = \frac{1}{3}\left(\frac{2 - \Delta - \lambda + \Delta\lambda}{(1-\lambda)}\right)$$

$$\sigma^- = \Delta - \left(x_A^- - x_B^-\right) = \frac{1}{3}\left(\frac{1 + \Delta - 2\lambda - \Delta\lambda}{1 - \lambda}\right) > 0$$
$$\Longleftrightarrow \lambda < \lambda^- \equiv \frac{\Delta + 1}{\Delta + 2}$$

$$\Pi^- = \pi_A^- + \pi_B^- = \frac{1}{9} a \frac{5 - 2\Delta + 2\Delta^2 - 2\lambda(1-\Delta)(1 - 2\Delta - \lambda + \Delta\lambda)}{1 - \lambda} \tag{A9}$$

$$\frac{d\Pi^-}{d\lambda} = \frac{1}{9} a \frac{3 + 4\Delta - 2\Delta^2 + 2\lambda(\Delta - 1)^2(2 - \lambda)}{(\lambda - 1)^2} > 0.$$

$$\Pi^- - \hat{\Pi} = \frac{1}{9} a \frac{4\Delta(2\lambda - 1) + \lambda(3 - 2\Delta^2) + 2\lambda^2(\Delta - 1)^2}{1 - \lambda} > 0.$$

$$S^- = v_B + \frac{b}{a} + \frac{8\left(\Delta + \lambda - \lambda^2\right) + \Delta^2 + \Delta\lambda(-2\Delta - 2\lambda + \Delta\lambda - 6) - 11}{18(1 - \lambda)}.$$

$$\frac{dS^-}{d\lambda} = -\frac{3 - 2\Delta + \Delta^2 + \lambda(\Delta + 2)(\Delta - 4)(\lambda - 2)}{18(\lambda - 1)^2} < 0.$$

$$S^- - \hat{S} = \frac{-2\Delta - 3\lambda - \lambda(4 - \Delta)(-\Delta + 2\lambda + \Delta\lambda)}{18(1 - \lambda)} < 0$$

On the other hand,

$$x_A^+ = \frac{1}{3\lambda}(1 + \lambda + \Delta\lambda), \qquad x_B^+ = \frac{1}{3\lambda}(2 - \lambda - \Delta\lambda)$$

$$\sigma^+ = -\Delta + x_A^+ - x_B^+ = \frac{1}{3}\frac{2\lambda - \Delta\lambda - 1}{\lambda} > 0 \Longleftrightarrow \lambda > \frac{1}{2 - \Delta} \equiv \lambda^+ \tag{A10}$$

$$\lambda^+ - \lambda^- = \frac{1}{2 - \Delta} - \frac{\Delta + 1}{\Delta + 2} = \frac{\Delta^2}{(2 - \Delta)(\Delta + 2)} > 0$$

$$\Pi^+ = \frac{1}{9} a \frac{5 + 2\lambda(\Delta + 1)(\lambda + \Delta\lambda - 1)}{\lambda} \tag{A11}$$

$$\frac{d\Pi^+}{d\lambda} = \frac{1}{9} a \frac{2\lambda^2(\Delta + 1)^2 - 5}{\lambda^2} \gtreqless 0 \quad \text{if} \quad \lambda \gtreqless \frac{\sqrt{5/2}}{(\Delta + 1)}$$

If $\Delta \le 0.581$, $\frac{\sqrt{5/2}}{(\Delta+1)} \ge 1$, and hence $\frac{d\Pi^+}{d\lambda} < 0$ for all $\lambda > \lambda^+$; while if $\Delta > 0.581$, $\lambda^+ = \frac{1}{2-\Delta} > \frac{1}{2-0.581} = 0.70472$, and $\frac{d\Pi^+}{d\lambda} < 0$ if $\lambda \in \left(\lambda^+, \frac{\sqrt{5/2}}{(\Delta+1)}\right)$ and $\frac{d\Pi^+}{d\lambda} > 0$ if $\lambda > \frac{\sqrt{5/2}}{(\Delta+1)}$.

$$\Pi^+ - \hat{\Pi} = \frac{1}{9} a (1 - \lambda) \frac{5 - 2\lambda(\Delta + 1)^2}{\lambda} \gtreqless 0 \quad \text{if} \quad \lambda \lesseqgtr \frac{5/2}{(\Delta + 1)^2}$$

If $\Delta \le 0.581$, $\frac{5/2}{(\Delta+1)^2} \ge 1$, and hence $\Pi^+ - \hat{\Pi} > 0$ for all $\lambda > \lambda^+$; while if $\Delta > 0.581$, $\frac{5/2}{(\Delta+1)^2} < 1$, and hence $\Pi^+ > \hat{\Pi}$ if $\lambda \in \left(\lambda^+, \frac{5/2}{(\Delta+1)^2}\right)$ and $\Pi^+ < \hat{\Pi}$ if $\lambda > \frac{5/2}{(\Delta+1)^2}$. Furthermore,

$$S^+ = \lambda\left(v_A - x_A^+\right) + (1 - \lambda)\left(v_B - x_B^+\right) + \lambda \int_0^{\sigma^+} F(s) ds$$
$$= \lambda\left(v_A - \frac{1}{3a\lambda}(a + a\lambda + a\Delta\lambda)\right)$$
$$+ (1 - \lambda)\left(v_B - \frac{1}{3a\lambda}(2a - a\lambda - a\Delta\lambda)\right) + \lambda \int_0^{\frac{1}{3}\frac{2\lambda - \Delta\lambda - 1}{\lambda}} s ds$$
$$= \frac{8\lambda - 8\lambda^2 + \Delta^2\lambda^2 + 8\Delta\lambda + 18\lambda v_B - 16\Delta\lambda^2 + 18\lambda^2 v_A - 18\lambda^2 v_B - 11}{18\lambda}$$
$$= \frac{8\lambda - 8\lambda^2 + \Delta^2\lambda^2 + 8\Delta\lambda + 18\lambda v_B + 2\lambda^2\Delta - 11}{18\lambda}$$

Hence:

$$S^+ = v_A + \frac{8\lambda - 10\Delta\lambda - \lambda^2(\Delta + 4)(2 - \Delta) - 11}{18\lambda}.$$

$$\frac{dS^+}{d\lambda} = \frac{11 - \lambda^2(\Delta + 4)(2 - \Delta)}{18\lambda^2} > 0 \tag{A12}$$

$$S^+ - \hat{S} = v_A + \frac{8\lambda - 8\lambda^2 + \Delta^2\lambda^2 - 10\Delta\lambda + 2\Delta\lambda^2 - 11}{18\lambda} - \left(v_A - \frac{\left(11 + 8\Delta - \Delta^2\right)}{18}\right)$$
$$= -\frac{1}{18}(1 - \lambda)\frac{11 - 8\lambda + 2\Delta\lambda + \Delta^2\lambda}{\lambda} < 0 \tag{A13}$$

□

Proof of Corollary 3. Suppose $v_A = 5$, $v_B = 4.5$, $\Delta = 0.5$, and $\lambda = 0.8 > \lambda^+ = \frac{1}{2-0.5} = 0.667$. In period 1, if firms are myopic in choosing their first-period charges, then under competitive bidding, from Equation (A5), the marginal switching consumer is $\hat{\sigma} = 0.16667$, and the two firms' market shares are

$$q_A^c = (1 - F(\hat{\sigma})) = 1 - 0.16667 = 0.83333,$$
$$q_B^c = F(\hat{\sigma}) = 0.16667$$

Under regulation, from Equation (A10)

$$\begin{aligned}
\sigma^+ &= \frac{1}{3}\left(\frac{2\lambda - \Delta\lambda - 1}{\lambda}\right) \\
&= \frac{1}{3}\left(\frac{2(0.8) - (0.5)(0.8) - 1}{(0.8)}\right) = 0.08333, \\
q_A^r &= \lambda\left(1 - F\left(\sigma^+\right)\right) = 0.8(1 - 0.083) = 0.73333, \\
q_B^r &= 1 - \lambda + \lambda F\left(x_A - x_B - \Delta\right) = 1 - 0.8 + 0.8(0.083) = 0.26667
\end{aligned}$$

Therefore, regulation decreases A's (increases B's) market share in period 1. Provided that firms are myopic in choosing their first-period charges, the consumer surplus change in period 1 due to regulation is $\Lambda_{S_1} = S^+ - \hat{S}$, where $\hat{S}$ is given in Equation (A6) and S^+ is given in Equation (A12). Thus, using (A13),

$$\begin{aligned}
\Lambda_{S_1} &= -\frac{(1-\lambda)}{18}\,\frac{11 - 8\lambda + 2\Delta\lambda + \Delta^2\lambda}{\lambda} \\
&= -\frac{(1-0.8)}{18}\,\frac{11 - 8(0.8) + 2(0.5)(0.8) + (0.5)^2(0.8)}{0.8} = -0.078
\end{aligned}$$

In period 2, both firms have higher qualities from learning. Since $\lambda \geq \lambda^+ = \frac{1}{2-\Delta}$ and $\Delta \geq \Delta_2^c > \Delta_2^r$, the analysis in Proposition 4 for $\lambda \geq \lambda^+$ applies. Under regulation, from Equation (A12):

$$S_2^r = v_{2A}^r + \frac{8\lambda - 10\Delta_2^r\lambda - \lambda^2\left(\Delta_2^r + 4\right)\left(2 - \Delta_2^r\right) - 11}{18\lambda} \tag{A14}$$

Under competitive bidding, from Equation (A6):

$$S_2^c = v_{2A}^c - \frac{1}{18}\left(11 + 8\Delta_2^c - (\Delta_2^c)^2\right)$$

Thus,

$$\begin{aligned}
\Lambda_{S_2} &= S_2^r - S_2^c \\
&= v_{2A}^r - v_{2A}^c + \frac{8 - 10\Delta_2^r - \lambda\left(\Delta_2^r + 4\right)\left(2 - \Delta_2^r\right)}{18} \\
&\quad - \frac{11}{18\lambda} + \frac{11 + 8\Delta_2^c - (\Delta_2^c)^2}{18}
\end{aligned}$$

where $\Delta_2^r = v_{2A}^r - v_{2B}^r$ and $\Delta_2^c = v_{2A}^c - v_{2B}^c$. Hence,

$$\frac{\partial \Lambda_{S_2}}{\partial q_A^r} = \frac{\partial \Lambda_{S_2}}{\partial v_{2A}^r}\frac{dv_{2A}^r}{dq_A^r} = \left[\frac{1}{9}\left(\lambda + \lambda\Delta_2^r + 4\right)\right]\frac{dv_{2A}^r}{dq_A^r} > 0$$

$$\frac{\partial \Lambda_{S_2}}{\partial q_B^r} = \frac{\partial \Lambda_{S_2}}{\partial v_{2B}^r}\frac{dv_{2B}^r}{dq_B^r} = \left[\frac{1}{9}\left(5 - \lambda - \lambda\Delta_2^r\right)\right]\frac{dv_{2B}^r}{dq_B^r} > 0$$

$$\frac{\partial \Lambda_{S_2}}{\partial q_A^c} = \frac{\partial \Lambda_{S_2}}{\partial v_{2A}^c}\frac{dv_{2A}^c}{\partial q_A^c} = -\frac{1}{9}\left(\Delta_2^c + 5\right)\frac{dv_{2A}^c}{\partial q_A^c} < 0$$

$$\frac{\partial \Lambda_{S_2}}{\partial q_B^c} = \frac{\partial \Lambda_{S_2}}{\partial v_{2B}^c}\frac{dv_{2B}^c}{dq_B^c} = \frac{1}{9}\left(\Delta_2^c - 4\right)\frac{dv_{2B}^c}{dq_B^c} < 0$$

$$\frac{\partial \Lambda_{S_2}}{\partial \Delta_2^c} = \frac{1}{9}\left(4 - \Delta_2^c\right) > 0$$

The second-period quality values in Equation (11) are consistent with situations where the learning rate is non-monotonic in output: The learning rate initially increases and eventually decreases in the number of period-1 users. The consumer surplus change in period 2 is

$$\begin{aligned}
\Lambda_{S_2} &= v_{2A}^r - v_{2A}^c + \frac{8\lambda - 10\Delta_2^r\lambda - \lambda^2(\Delta_2^r + 4)(2 - \Delta_2^r) - 11}{18\lambda} \\
&\quad + \frac{11 + 8\Delta_2^c - \Delta_2^{c2}}{18} \\
&= v_{2A}^r - 5.1 + \frac{\begin{array}{c}8 - 10\left(v_{2A}^r - 4.85\right) \\ -\,0.8\left(v_{2A}^r - 4.85 + 4\right)\left(2 - (v_{2A}^r - 4.85)\right)\end{array}}{18} \\
&\quad - \frac{11}{18(0.8)} + \frac{11 + 8(0.5) - 0.5^2}{18} \\
&= \frac{1}{9000}\left(920.0v_{2A}^r + 400.0v_{2A}^{r2} - 14821\right) \gtreqless 0 \\
&\iff v_{2A}^r \gtreqless 5.0448
\end{aligned}$$

Hence, if v_{2A}^r and ϕ are sufficiently high, then $\Lambda_S > 0$; while if $v_{2A}^r < 5.04$, then $\Lambda_S < 0$. □