

Differential Pricing When Costs Differ: A Welfare Analysis*

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Abstract. This article analyzes the welfare effects of monopoly differential pricing in the important, but largely neglected, case where costs of service differ across consumer groups. Cost-based differential pricing is shown to increase total welfare and consumer welfare relative to uniform pricing for broad classes of demand functions, even when total output falls or the output allocation between consumers worsens. We discuss why cost-based differential pricing tends to be more beneficial for consumers than its demand-based counterpart, third-degree price discrimination. We also provide sufficient conditions for welfare-improving differential pricing when both costs and demands differ across consumer groups.

Keywords: differential pricing, price discrimination, demand curvature, pass-through rate.

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1. INTRODUCTION

A firm often wishes to charge different prices for its product to distinct consumer groups that differ in demand elasticities or marginal costs of service. How does such differential pricing affect total welfare and consumer welfare compared to uniform pricing? An extensive economics literature, dating to Pigou (1920) and advanced recently by Aguirre, Cowan, and Vickers (2010) and Cowan (2012), addresses this question under monopoly when consumer groups differ only in demand elasticity—classic (third-degree) price discrimination.¹ There has been scant welfare analysis, however, of differential pricing when costs of service differ. Yet the issue has significant policy relevance because, in many industries, firms are constrained to price uniformly despite heterogeneous costs of serving diverse consumer groups, as with geographically averaged pricing in traditional utilities, gender neutral pricing in insurance markets, and further examples discussed below.

This article presents a welfare analysis of monopoly differential pricing when marginal costs differ across consumer groups. Our focus is on the case where *only* costs differ, but we also compare the results to those under price discrimination and consider the mixed case with both cost and demand differences, thereby providing a unified treatment of the problem. To facilitate the comparison with classic price discrimination, we adopt the standard setting of that literature: under uniform pricing, the firm serves two consumer groups or markets, and moving to differential pricing raises price in one market but lowers it in the other. The extant literature suggests that price discrimination—except where it opens new markets—is tilted against aggregate consumer welfare and is more likely to reduce than to increase total welfare, which includes profit. By contrast, we find that differential pricing motivated (only) by cost differences will raise aggregate consumer welfare under broad demand conditions, and increase overall welfare more generally still.

Differential pricing obviously has the *potential* to increase total welfare when marginal

¹By “classic” price discrimination we mean different prices when marginal costs of service are equal. Price discrimination, of course, can occur more broadly. Indeed, uniform pricing under different costs entails discrimination as commonly defined, because the (zero) price difference does not reflect cost differences.

costs differ: uniform pricing will misallocate output, so a small output reallocation to the low-cost market will raise total welfare. Not surprisingly, welfare can rise even if total output falls, unlike with classic price discrimination.² But these arguments do not address *profit maximizing* cost-based pricing, which we show can worsen the output allocation compared to uniform pricing. This can occur if the pass-through rate from marginal cost to the monopoly price exceeds one—for example, with constant-elastic demands.³ Differential pricing then yields a larger price-cost margin in the high-cost market, and an excessive output reallocation away from that market.

Nevertheless, we show that cost-based differential pricing will raise total welfare for all demand functions satisfying a mild condition: that the curvature (“degree of convexity”) of inverse demand not decrease too fast as output increases; or, equivalently, the pass-through rate not increase too fast as price rises. Interestingly, the condition depends not on the level of the pass-through rate, but only on its rate of change. The intuition is as follows. If pass-through does not exceed one, differential pricing improves the output allocation and—under our demand conditions—raises welfare even if total output falls. The output allocation between groups can worsen if pass-through exceeds one, but this requires demand to be highly convex — in which case total output will increase by enough to outweigh any output-misallocation effect. Therefore, as long as the pass-through rate does not increase too fast, the positive effect will dominate (if the other effect is negative) and welfare will rise no matter how high is the pass-through rate.

Aggregate consumer welfare also rises under broad conditions, including cases where output falls. The mechanism is subtle, because the cost savings from output reallocation do not flow to consumers. The reason consumers benefit is that, in order to reallocate

²With a common marginal cost, uniform pricing allocates a given output optimally whereas differential pricing does not, because marginal values will differ between consumers that pay different prices. Thus, an increase in total output is necessary but not sufficient for third-degree price discrimination to raise total welfare (Schmalensee, 1981; Varian, 1985; Schwartz, 1990).

³Pass-through by firms with market power was first analyzed by Cournot (1838), and recently by Weyl and Fabinger (2013). It is relevant here because we will show that a move from uniform pricing to differential pricing can be analyzed as if marginal cost rose in one market and fell in the other.

output, the firm must vary its prices, and consumers gain from the resulting price dispersion. Importantly, and in contrast to classic price discrimination, cost-motivated price dispersion does not entail an upward bias in the average price across markets. Even if demand exhibits an increasing pass-through rate, so that average price rises, the beneficial effect of price dispersion will dominate as long as pass-through does not increase too fast. The relevant curvature condition is qualitatively similar to, albeit somewhat tighter than, the condition for total welfare. The condition is met by demand functions commonly assumed in industrial organization, including those with constant pass-through and several whose pass-through is increasing.

As further motivation for the analysis, we now provide several examples where government policy, private contracts, or customer perceptions have constrained firms to price uniformly across customer groups that impose different costs.⁴ *Universal service* regulation in the U.S. requires local phone companies to set uniform residential rates across large geographic areas within which the costs of connecting premises can vary widely with customer density (Nuechterlein and Weiser, 2007). Similar geographic averaging provisions apply in other utility industries such as electricity, water, and postal service, in the U.S. and elsewhere.⁵ *Gender-neutral* rules provide another illustration. U.S. federal rules bar employer-offered health insurance plans from charging different premiums based on gender or age (Geruso, 2012). And the European Court of Justice ruled that, effective December 21, 2012, premiums or benefits for certain private insurance and private pension services in EU member states may not differ based on gender (ECJ, 2011). Because costs vary by gender, but differently across services, price uniformity will benefit women in some cases and harm them in

⁴Uniformity constraints also arise in sales of inputs to competing firms. For instance, the U.S. Robinson-Patman Act, which prohibits price discrimination that may substantially reduce competition, allows cost-justified price differentials. But the standard of proof was often stringent, leading manufacturers to charge uniform prices to highly diverse distributors (Schwartz, 1986).

⁵These examples involve regulated industries, but are relevant to our analysis of a profit-maximizing monopolist insofar as price levels are not always tightly regulated, only the price structure (uniform versus differential). For a detailed treatment of third-degree price discrimination under regulation see Armstrong and Vickers (1991).

others.⁶ Although all the uniform-price mandates discussed above may reflect social goals, it is nevertheless important to understand their welfare implications.

Anti-dumping rules in international trade can also induce firms to set uniform prices despite cost differences. Dumping sometimes may be found when an exporter’s price to one foreign country is lower than to another foreign country (WTO, 2013), yet the costs of serving different foreign countries can vary (and in ways that cannot be precisely documented). Uniform price constraints have also arisen in payment card networks, such as Amex, Mastercard and Visa, that imposed *no-surcharge rules* barring merchants from surcharging when customers pay with a card instead of other means (Prager et al., 2009).

Another broad class of examples involves resistance to *add-on pricing*. Sellers commonly offer a base good and optional add-on services that can only be consumed with the base good (Ellison, 2005): airlines sell a ticket and offer options such as checking a bag; hotels offer a room and extras such as breakfasts. Importantly, not all consumers use the optional items. Charging an all-inclusive price (bundled pricing) represents uniform pricing across consumer groups that impose different costs depending on whether they use the add-ons or not. Pricing the add-ons separately can be used to implement cost-based pricing. At the same time, such unbundled pricing is often controversial (Trejos, 2012) because the add-on prices may substantially exceed the incremental costs. Moreover, they appear to be partly motivated by demand differences across the customer groups—add-on pricing may implement indirect price discrimination (Brueckner et al., 2014). Our analysis provides insights into the welfare properties of these pricing practices.

2. PRICING REGIMES AND WELFARE BOUNDS

Consider two markets, H and L , with strictly decreasing demand functions $D_H(p)$, $D_L(p)$ and inverse demands $P_H(q)$, $P_L(q)$. When not needed, we omit the subscripts. The markets

⁶The lifetime cost of providing the same annual pension is higher for women due to their longer life expectancy; but the cost of car insurance coverage is lower for women than men (Hyde and Evans, 2011). Since the EU’s uniformity directive took effect, the average pension annuity and the average car insurance premium in the U.K. reportedly have fallen for men and risen for women (Wall, 2013).

can be supplied at constant marginal costs c_H and c_L , with $c_H \geq c_L$.

Denote the prices in the two markets by p_H and p_L . Let

$$\pi_i(p_i) \equiv (p_i - c_i) D_i(p_i)$$

denote profit in market i , $i = H, L$. Assume $\pi_i(\cdot)$ has a unique maximum for the relevant domain of prices.

Under differential pricing, maximum profit in each market is achieved when $p_i = p_i^*$, where p_i^* satisfies

$$\pi'_i(p_i^*) = D_i(p_i^*) + (p_i^* - c_i) D'_i(p_i^*) = 0.$$

We assume $p_H^* > p_L^*$. In Robinson's (1933) taxonomy, H is the "strong" market whereas L is "weak," although here the prices may also differ for cost reasons.⁷

If the firm is constrained to charge a uniform price, we assume parameter values are such that both markets will be served (obtain positive outputs) at the unique optimal uniform price. That price, \bar{p} , solves

$$\pi'_H(\bar{p}) + \pi'_L(\bar{p}) = 0.$$

Because $\pi_i(p)$ is single peaked and $p_H^* > p_L^*$, it follows that $p_H^* > \bar{p} > p_L^*$, $\pi'_H(\bar{p}) > 0$, and $\pi'_L(\bar{p}) < 0$. Let $\Delta p_L \equiv p_L^* - \bar{p} < 0$ and $\Delta p_H \equiv p_H^* - \bar{p} > 0$. Also, let $\Delta q_L = D_L(p_L^*) - D_L(\bar{p}) \equiv q_L^* - \bar{q}_L > 0$ and $\Delta q_H = D_H(p_H^*) - D_H(\bar{p}) \equiv q_H^* - \bar{q}_H < 0$.

Aggregate consumer surplus across the two markets, which we take as the measure of consumer welfare, is

$$S^* = \int_{p_L^*}^{\infty} D_L(p) dp + \int_{p_H^*}^{\infty} D_H(p) dp \quad (1)$$

under differential pricing. Aggregate consumer surplus under uniform pricing, \bar{S} , is obtained by replacing p_i^* with \bar{p} in (1). The change in consumer surplus due to differential pricing is

$$\Delta S \equiv S^* - \bar{S} = \int_{p_L^*}^{\bar{p}} D_L(p) dp - \int_{\bar{p}}^{p_H^*} D_H(p) dp, \quad (2)$$

⁷Thus, under differential pricing the markets are ranked unambiguously as "high-price" or "low-price." In Section 4 we discuss briefly the alternative case where the lower-cost market has the less elastic demand.

which, together with $p_H^* > \bar{p} > p_L^*$, $\Delta p_L < 0$ and $\Delta p_H > 0$, immediately implies the following lower and upper bounds for ΔS , which will be used in Section 3:

$$-\bar{q}_L \Delta p_L - \bar{q}_H \Delta p_H < \Delta S < -q_L^* \Delta p_L - q_H^* \Delta p_H. \quad (3)$$

The lower bound applies the price changes to the initial outputs (i.e., at the uniform price) and thereby ignores consumers' gain from substitution, whereas the upper bound uses the new outputs and thereby overstates the gain from substitution.

The result below follows immediately from (3), and provides sufficient conditions for differential pricing to raise or lower aggregate consumer surplus:

Lemma 1 (i) $\Delta S > 0$ if $\bar{q}_L \Delta p_L + \bar{q}_H \Delta p_H \leq 0$, and (ii) $\Delta S < 0$ if $q_L^* \Delta p_L + q_H^* \Delta p_H \geq 0$.

The condition in Lemma 1(i) can be expressed as

$$\Delta S > 0 \text{ if } \left(\frac{\bar{q}_L}{\bar{q}_L + \bar{q}_H} \right) p_L^* + \left(\frac{\bar{q}_H}{\bar{q}_L + \bar{q}_H} \right) p_H^* \leq \bar{p}. \quad (4)$$

That is, differential pricing raises consumer surplus if the average of the new prices weighted by each market's share of total output at the uniform price is no higher than the uniform price. Increased price dispersion that does not raise the weighted average price will benefit consumers overall because they can advantageously adjust the quantities purchased.⁸

Next consider total welfare, consumer surplus plus profit: $W = S + \Pi$. Because differential pricing increases profit (by revealed preference), total welfare must rise if consumer surplus does not fall; but if consumer surplus falls the welfare change is ambiguous. It proves useful to analyze welfare directly as willingness to pay minus cost. Under differential pricing

$$W^* = \int_0^{q_L^*} [P_L(q) - c_L] dq + \int_0^{q_H^*} [P_H(q) - c_H] dq. \quad (5)$$

Welfare under uniform pricing, \bar{W} , is obtained by replacing q_L^* and q_H^* in (5) with \bar{q}_L and \bar{q}_H . The change in total welfare from moving to differential pricing is

$$\Delta W = W^* - \bar{W} = \int_{\bar{q}_L}^{q_L^*} [P_L(q) - c_L] dq + \int_{\bar{q}_H}^{q_H^*} [P_H(q) - c_H] dq, \quad (6)$$

⁸This point, which follows because demand curves slope down, dates back to Waugh (1944).

which, together with $\Delta q_L = q_L^* - \bar{q}_L > 0$ and $\Delta q_H = q_H^* - \bar{q}_H < 0$, immediately implies the following lower and upper bounds for ΔW :

$$(p_L^* - c_L) \Delta q_L + (p_H^* - c_H) \Delta q_H < \Delta W < (\bar{p} - c_L) \Delta q_L + (\bar{p} - c_H) \Delta q_H. \quad (7)$$

The lower bound weights the sum of the output changes by the price-cost margins at the new (differential) prices, whereas the upper bound weights instead by the markups at the original (uniform) price.⁹

From (7), we obtain sufficient conditions for differential pricing to raise or lower total welfare. As with Lemma 1, these conditions arise because demands are negatively sloped:

Lemma 2 (i) $\Delta W > 0$ if $(p_L^* - c_L) \Delta q_L + (p_H^* - c_H) \Delta q_H \geq 0$, and (ii) $\Delta W < 0$ if $(\bar{p} - c_L) \Delta q_L + (\bar{p} - c_H) \Delta q_H \leq 0$.

The insight from the price discrimination literature, that discrimination reduces welfare if total output does not increase, obtains as a special case of Lemma 2(ii) when $c_H = c_L$. When costs differ ($c_L < c_H$), part (i) of Lemma 2 implies:

Remark 1 *If differential pricing does not reduce total output compared to uniform pricing ($\Delta q_L \geq -\Delta q_H > 0$), then total welfare increases if the price-cost margin under differential pricing is greater in the lower-cost than in the higher-cost market ($p_L^* - c_L \geq p_H^* - c_H$).*

Intuitively, under uniform pricing the price-cost margin (the marginal social value of output) is higher in the lower-cost market than in the higher-cost market ($\bar{p} - c_L > \bar{p} - c_H$), so welfare can be increased by reallocating some output to market L . Differential pricing induces such a reallocation, and if the margin in L remains no lower than in H then the entire reallocation is beneficial; hence, welfare must increase if total output does not fall.

To distinguish the effects of output reallocation and a change in total output, we use the mean value theorem to rewrite (6) as

$$\Delta W = [P_L(\xi_L) - c_L] \Delta q_L + [P_H(\xi_H) - c_H] \Delta q_H,$$

⁹Varian (1985) provides a similar expression for the case where marginal costs are equal.

where $\xi_L \in (\bar{q}_L, q_L^*)$ and $\xi_H \in (q_H^*, \bar{q}_H)$ are constants, with $P_L(\xi_L) < \bar{p}$ and $P_H(\xi_H) > \bar{p}$ representing the *average* valuation for the output increments Δq_L and Δq_H , respectively.

Let $\Delta q \equiv \Delta q_L + \Delta q_H$. Using $\Delta q_H = \Delta q - \Delta q_L$ gives the following decomposition:

$$\Delta W = \underbrace{[P_L(\xi_L) - P_H(\xi_H)] \Delta q_L}_{\text{consumption misallocation}} + \underbrace{(c_H - c_L) \Delta q_L}_{\text{cost saving}} + \underbrace{[P_H(\xi_H) - c_H] \Delta q}_{\text{output effect}}, \quad (8)$$

where the first term is negative and represents the reduction in consumers' total value from reallocating output between markets starting at the efficient allocation under uniform pricing, the second term is positive and represents the cost savings from the same output reallocation to the lower-cost market, and the last term is the welfare effect from the change in total output (which takes the sign of Δq because price exceeds marginal cost).

We can combine the first two terms in (8) and call it the (output) reallocation effect:

$$\Delta W = \underbrace{[(P_L(\xi_L) - c_L) - (P_H(\xi_H) - c_H)] \Delta q_L}_{\text{reallocation effect}} + \underbrace{[P_H(\xi_H) - c_H] \Delta q}_{\text{output effect}}. \quad (9)$$

When output does not decrease ($\Delta q \geq 0$), differential pricing increases welfare if the *average* value net of cost of the reallocated output is higher in market L : $P_L(\xi_L) - c_L > P_H(\xi_H) - c_H$. This is a weaker condition than $p_L^* - c_L \geq p_H^* - c_H$ in Remark 1 (because $P_L(\xi_L) > p_L^*$ and $P_H(\xi_H) < p_H^*$), but the latter condition may be more observable.

3. WELFARE COMPARISONS WHEN ONLY COSTS DIFFER

To isolate the role of pure cost differences, in this section we consider demand functions in the two markets that have equal elasticities at any common price. This requires that demands be proportional, which we express as $D_L(p) = \lambda D(p)$ and $D_H(p) = (1 - \lambda) D(p)$ so that $D_L(p) = \frac{\lambda}{1-\lambda} D_H(p)$, for $\lambda \in (0, 1)$.¹⁰ A natural interpretation is that all consumers have identical demands $D(p)$ whereas λ and $(1 - \lambda)$ are the shares of all consumers represented by market L and H , respectively. The function $D(\cdot)$ can take a general form. For

¹⁰For two demand functions $f(p)$ and $g(p)$, equal elasticities at any common price p imply $f'(p)p/f(p) = g'(p)p/g(p)$, hence $d(\ln f(p) - \ln g(p))/dp = 0$, so $\ln f(p) - \ln g(p)$ is constant, implying $f(p)/g(p)$ is constant (demands must be proportional).

constant marginal cost c , the monopolist's profit under demand $D(p)$ with inverse demand $P(q)$ is $\pi = q[P(q) - c]$. The monopoly price $p^*(c)$ satisfies $P(q) + qP'(q) - c = 0$. Let $q^* = D(p^*(c))$; and let $\sigma = -qP''(q)/P'(q)$ be the curvature (elasticity of the slope) of inverse demand, which takes the sign of $P''(q)$.

As shown by Bulow and Pfleiderer (1983), the pass-through rate from marginal cost to the monopoly price, $p^{*'}(c)$, equals the ratio of the slope of inverse demand to the slope of marginal revenue. Thus,

$$p^{*'}(c) = \frac{P'(q^*)}{2P'(q^*) + q^*P''(q^*)} = \frac{1}{2 - \sigma(q^*)}. \quad (10)$$

Given the standard assumption that marginal revenue is downward sloping, $2P'(q^*) + q^*P''(q^*) < 0$; hence, $p^{*'}(c)$ and $2 - \sigma(q^*)$ are both positive. Our analysis will utilize both the pass-through rate and how this rate changes along the demand curve. The pass-through rate depends on the curvature of inverse demand:

$$p^{*'}(c) \begin{cases} \leq \\ \geq \end{cases} \frac{1}{2} \text{ as } \sigma(q^*) \begin{cases} \leq \\ \geq \end{cases} 0 \text{ i.e., as } P''(q^*) \begin{cases} \leq \\ \geq \end{cases} 0. \quad (11)$$

The *change* in the pass-through rate depends on the change in curvature, with

$$p^{*''}(c) = \frac{\sigma'(q^*)}{[2 - \sigma(q^*)]^2} D'(p^*) p^{*'}(c) \leq 0 \quad (12)$$

if and only if

$$\sigma'(q) \geq 0. \quad (13)$$

The pass-through rate will be non-increasing in marginal cost if and only if the curvature of inverse demand is not decreasing in output—inverse demand is neither less convex nor more concave at higher quantities.

Consumer Welfare

We first investigate the effects of cost-based differential pricing on consumer welfare. With proportional demands, the monopolist's differential prices are given by the same function $p^*(c)$ but evaluated at the different costs: $p_L^* \equiv p^*(c_L)$, $p_H^* \equiv p^*(c_H)$. Let $\bar{c} \equiv$

$\lambda c_L + (1 - \lambda) c_H$. The optimal uniform price \bar{p} maximizes

$$\pi(p) = \lambda(p - c_L)D(p) + (1 - \lambda)(p - c_H)D(p) = [p - \bar{c}]D(p).$$

Thus, $\bar{p} \equiv p^*(\bar{c})$: the monopolist chooses its uniform price as though its marginal cost in both markets were \bar{c} , the average of the actual marginal costs weighted by each market's share of all consumers. It follows that if $p^*(c)$ is concave; that is, if the pass-through rate is non-increasing ($p^{*''}(c) \leq 0$), then

$$\lambda p_L^* + (1 - \lambda) p_H^* \leq \bar{p}, \tag{14}$$

or differential pricing does not raise average price across the two markets.¹¹ Aggregate consumer surplus then rises by Lemma 1(i). This reasoning highlights why cost-motivated differential pricing tends to benefit consumers even though the cost savings from output reallocation accrue only to the firm: cost differences give the firm an incentive to reallocate output by varying prices without raising the average price.

Due to the gain from price dispersion, consumer welfare will increase with differential pricing even if the average price rises somewhat, as occurs when $\sigma'(q) < 0$ (hence $p^{*''}(c) > 0$), provided $\sigma(q)$ does not decrease too fast. In particular, the result below uses the following demand-curvature condition which we will further interpret shortly:

$$\sigma'(q) > -\frac{2 - \sigma(q)}{q}, \tag{A1}$$

where the right hand side is negative because $2 - \sigma(q) > 0$ from (10).¹²

Proposition 1 *Assume $D_L(p) = \lambda D(p)$ and $D_H(p) = (1 - \lambda) D(p)$ for $\lambda \in (0, 1)$. If (A1) holds, differential pricing increases consumer surplus relative to uniform pricing.*

¹¹Because $\bar{q}_L = \lambda D(\bar{p})$ and $\bar{q}_H = (1 - \lambda) D(\bar{p})$, inequality (14) is equivalent to $\bar{q}_L \Delta p_L + \bar{q}_H \Delta p_H \leq 0$ (as stated in (4)). We shall explain shortly that under classic price discrimination this inequality is likely to be reversed, indicating a bias for average price to rise.

¹²A recent paper that uses a condition equivalent to (A1) for analyzing double marginalization is Adachi and Ebina (2014).

Proof. First, we show that, if and only if (A1) holds, aggregate consumer surplus is a strictly convex function of constant marginal cost c . With demand $D(p)$, aggregate consumer surplus under $p^*(c)$ is

$$s(c) \equiv S(p^*(c)) = \int_{p^*(c)}^{\infty} D(p) dp.$$

Thus, $s'(c) = -D(p^*(c))p^{*'}(c)$ and $s''(c) = -D'(p^*(c))\left[p^{*'}(c)\right]^2 - D(p^*(c))p^{*''}(c)$. Using expressions (10) and (12) for $p^{*'}(c)$ and $p^{*''}(c)$, we have $s''(c) > 0$ if and only if (A1) holds.

Second, consumer surplus under differential pricing (S^*) and under uniform pricing (\bar{S}) are ranked as follows:

$$\begin{aligned} S^* &= \lambda S(p_L^*) + (1 - \lambda) S(p_H^*) \\ &= \lambda s(c_L) + (1 - \lambda) s(c_H) \\ &> s(\lambda c_L + (1 - \lambda) c_H) \quad (\text{by the convexity of } s(\cdot)) \\ &= S(p^*(\bar{c})) = \bar{S}. \end{aligned}$$

■

Condition (A1) is a fairly tight sufficient condition for differential pricing to raise consumer surplus, insofar as it is the sufficient *and* necessary condition for consumer surplus to be a strictly convex function of constant marginal cost.¹³ It can be reinterpreted as follows to highlight the role of pass-through. Strictly convex $s(c)$ requires $-s'(c) = D(p^*(c))p^{*'}(c)$ to decrease in c . Because $D(p^*(c))$ is decreasing, the condition will be met provided $p^{*'}(c)$ does not increase too fast. Condition (A1) bounds the change in curvature of demand and, hence, how fast the pass-through rate may increase as one moves up the demand curve: recalling $p^{*'}(c) = \frac{1}{2-\sigma(q^*)}$, we have that $D(p^*(c))p^{*'}(c)$ decreases in c if $\frac{q}{2-\sigma(q)}$ increases in q , or $\sigma'(q) > -\frac{2-\sigma(q)}{q}$, condition (A1).

¹³If $s''(c)$ has a consistent sign over the relevant range of c , then (A1) will also be the necessary condition for differential pricing to increase consumer welfare. In general $s''(c)$ may not have a consistent sign, and thus (A1) is sufficient but may not be necessary.

Condition (A1) is satisfied by many common demand functions. The class of demand functions with constant pass-through rates, for which (A1) is satisfied, was identified by Bulow and Pfleiderer (1983): (i) $P = a - bq^\delta$ for $\delta > 0$, with pass-through rate $p^{*'}(c) = 1/(1 + \delta) \in (0, 1)$; (ii) constant-elasticity demand functions $P = \beta q^{-1/\eta}$ for $\beta > 0, \eta > 1$, hence $p^{*'}(c) = \eta/(\eta - 1) > 1$; and (iii) $P = a - b \ln q$ for $a, b > 0$ and $q < \exp(a/b)$, which reduces to exponential demand ($D(p) = e^{-\alpha p}$) if $a = 0$ and $\alpha = 1/b$, with $p^{*'}(c) = 1$. Decreasing pass-through holds for the AIDS demand function, $D(p) = [a + b \log(p)]/p$ with $b < 0$ (Fabinger and Weyl, 2012).

Fabinger and Weyl (2012) point out, however, that *increasing* pass-through is likely when market demand is derived from individual unit demands with valuations drawn from a unimodal distribution.¹⁴ They identify common unimodal distributions that yield globally increasing pass-through rates. Importantly, condition (A1) admits also increasing pass-through rates, as, for example, if $D(p) = 1 - F(p)$, with $F(p)$ being the standard logistic, normal, or log-normal distribution, where the corresponding logistic and normal distributions have zero mean and the log-normal distribution's mean is 1.¹⁵

Contrast With Classic Price Discrimination

Cowan (2012) analyzes demand functions under which classic price discrimination will raise aggregate consumer surplus. Beyond a condition equivalent to our (A1), he provides an additional (sufficient) condition; and he notes that, together, these conditions are satisfied by only two demand functions.¹⁶ In contrast, when differential pricing is motivated solely by different costs, our Proposition 1 shows that (A1) *alone* is sufficient for consumer welfare

¹⁴They note that any unimodal distribution will generate a market demand that is concave at prices below the mode and convex above, implying pass-through less than 1/2 at prices below the mode and greater than 1/2 at prices above the mode. Thus, pass-through will increase at least over some price ranges.

¹⁵These correspond respectively to the standard logit, probit, and log-normal demand functions. The pass-through rate increases for the standard logit and probit demands, and for the standard log-normal demand when price is close to zero. But for all these three familiar demand functions (A1) is satisfied.

¹⁶The additional condition requires that the ratio of pass-through to demand elasticity at the uniform price be no lower in market L than in H . The demand functions whose shapes guarantee this are logit with pass-through above one half, and demand based on the Extreme Value distribution. (Cowan, pp. 340-1.)

to rise.

Classic price discrimination is less favorable for consumer surplus than cost-based differential pricing due to the different effects on average price.¹⁷ For linear demands, we show, in Section 4, that cost-based pricing leaves the average price unchanged whereas elasticity-based pricing raises it. But the bias to raise price is more general, as shown indirectly by the fact that price discrimination—when it does not serve additional markets—tends to reduce consumer surplus, which is possible only if it tends to raise average price. Below we provide a direct explanation for this upward price bias.

Suppose that the marginal cost in both markets is c , and $\eta_H(\bar{p}) < \eta_L(\bar{p})$, where $\eta(p) = -pD'(p)/D(p)$ denotes the price elasticity of demand in absolute value. Recall that, in a given market, $\pi'(p) = D(p) \left(1 - \frac{p-c}{p}\eta\right)$, and the monopolist's optimal uniform price \bar{p} satisfies $\bar{q}_H \left(1 - \frac{\bar{p}-c}{\bar{p}}\eta_H(\bar{p})\right) = \bar{q}_L \left(\frac{\bar{p}-c}{\bar{p}}\eta_L(\bar{p}) - 1\right)$, with $\frac{\bar{p}-c}{\bar{p}}\eta_H(\bar{p}) < 1 < \frac{\bar{p}-c}{\bar{p}}\eta_L(\bar{p})$, whereas under discrimination $\frac{p_H^*-c}{p_H^*}\eta_H = 1 = \frac{p_L^*-c}{p_L^*}\eta_L$. Defining

$$\theta(p) \equiv \frac{p-c}{p}\eta(p), \quad (15)$$

we have $0 = \bar{q}_H [\theta_H(p_H^*) - \theta_H(\bar{p})] - \bar{q}_L [\theta_L(\bar{p}) - \theta_L(p_L^*)]$. The mean value theorem implies the existence of $\tilde{p}_H \in (\bar{p}, p_H^*)$ and $\tilde{p}_L \in (p_L^*, \bar{p})$ such that

$$0 = \bar{q}_H \theta'_H(\tilde{p}_H) \Delta p_H + \bar{q}_L \theta'_L(\tilde{p}_L) \Delta p_L. \quad (16)$$

Let $\tilde{q}_i = D_i(\tilde{p}_i)$ for $i = L, H$. We next provide sufficient conditions, satisfied by many common demands, under which classic price discrimination raises average price.¹⁸

Claim 1 *Suppose that $\eta'(p) \geq 0$ and*

$$\theta_L(\tilde{p}_L) [1 + \theta_L(\tilde{p}_L) (1 - \sigma_L(\tilde{q}_L))] \geq \theta_H(\tilde{p}_H) [1 + \theta_H(\tilde{p}_H) (1 - \sigma_H(\tilde{q}_H))]. \quad (17)$$

Then $\bar{q}_H \Delta p_H + \bar{q}_L \Delta p_L > 0$.

¹⁷Recall that we are considering situations where the same set of markets would be served under uniform or differential pricing.

¹⁸From Lemma 1, a rise in the average price weighted by the initial (uniform-price) quantities is not sufficient for consumer surplus to fall, since total output may expand. Nevertheless, comparing this average price under classic discrimination versus cost-based pricing helps explain why the latter is more favorable for consumers.

Proof. First, because $\theta'(p) = \frac{c}{p^2}\eta(p) + \frac{p-c}{p}\eta'(p)$, from $\eta'(p) \geq 0$ we have $\theta'(p) > 0$.

Next, because $\theta(p) = (p-c) \left(-\frac{D'(p)}{D(p)}\right)$, we also have

$$\begin{aligned}\theta'(p) &= -\frac{D'(p)}{D(p)} + (p-c) \frac{-D''(p)D(p) + [D'(p)]^2}{D(p)^2} \\ &= -\frac{D'(p)}{D(p)} - \frac{D'(p)}{D(p)} \frac{(p-c)}{p} \eta \left(1 - \frac{\alpha}{\eta}\right) = \frac{\theta}{p-c} [1 + (1-\sigma)\theta],\end{aligned}$$

where $\alpha \equiv -pD''(p)/D'(p)$ is the curvature of demand. Therefore, from (17) and the fact that $\tilde{p}_H > \bar{p} > \tilde{p}_L$, we have:

$$\begin{aligned}\theta'_L(\tilde{p}_L) &= \frac{\theta_L(\tilde{p}_L)}{\tilde{p}_L - c} \{1 + \theta_L(\tilde{p}_L) [1 - \sigma_L(\tilde{q}_L)]\} \\ &\geq \frac{\theta_H(\tilde{p}_H)}{\tilde{p}_L - c} \{1 + \theta_H(\tilde{p}_H) [1 - \sigma_H(\tilde{q}_H)]\} \\ &> \frac{\theta_H(\tilde{p}_H)}{\tilde{p}_H - c} \{1 + \theta_H(\tilde{p}_H) [1 - \sigma_H(\tilde{q}_H)]\} = \theta'_H(\tilde{p}_H).\end{aligned}$$

Finally, from $0 < \theta'_H(\tilde{p}_H) < \theta'_L(\tilde{p}_L)$, $\bar{q}_H > 0$, $\Delta p_H > 0$, and (16), we have

$$0 = \bar{q}_H \theta'_H(\tilde{p}_H) \Delta p_H + \bar{q}_L \theta'_L(\tilde{p}_L) \Delta p_L < \bar{q}_H \theta'_L(\tilde{p}_L) \Delta p_H + \bar{q}_L \theta'_L(\tilde{p}_L) \Delta p_L.$$

It follows that $\bar{q}_H \Delta p_H + \bar{q}_L \Delta p_L > 0$. ■

Intuitively, a move from uniform to differential pricing means the firm lowers $\theta_L(p_L)$ and raises $\theta_H(p_H)$ until they both equal to 1, which, because $\theta'(p) > 0$ from the assumption $\eta'(p) > 0$, implies p_L and p_H will (monotonically) fall and rise, respectively. From the proof of the Claim, $\theta'(p) = \frac{\theta}{p-c} [1 + (1-\sigma)\theta]$. With $\tilde{p}_H > \bar{p} > \tilde{p}_L$, condition (17) then suggests that $\theta'_L(p_L)$ is higher than $\theta'_H(p_H)$, so that it takes a smaller reduction in p_L but a larger increase in p_H to equalize $\theta_L(p_L)$ and $\theta_H(p_H)$. Hence, the average price is likely to rise.

The condition $\eta'(p) \geq 0$ is satisfied by many commonly assumed demand functions. A sufficient condition for (17) is that $\sigma_L(\tilde{q}_L) \leq \sigma_H(\tilde{q}_H) \leq 1 + \frac{1}{2\theta_L(\tilde{p}_L)}$.¹⁹ Thus (17) is also satisfied by many common demand functions, including those with a constant pass-through rate not exceeding $\frac{1}{1-2\theta}$ (i.e., a constant σ not exceeding $1 + \frac{1}{2\theta}$). We thus conclude that classic price discrimination tends to raise the average market price. By contrast, recalling

¹⁹This is because $\theta_L(\tilde{p}_L) > 1 > \theta_H(\tilde{p}_H)$, and $\theta[1 + \theta(1-\sigma)]$ is non-decreasing in θ if $\sigma \leq 1 + \frac{1}{2\theta}$.

(14), with cost-based differential pricing $\bar{q}_H \Delta p_H + \bar{q}_L \Delta p_L \leq 0$, so there is no tendency for average price to rise, for any demand that exhibits a decreasing or constant pass-through rate (a subset of the class that satisfy (A1)).

Total Welfare

Now consider total welfare, which increases with cost-based differential pricing in more situations than does consumer surplus, because total welfare includes profits which necessarily rise. The comparison of total welfare uses the following sufficient condition:

$$\sigma'(q) \geq -\frac{[3 - \sigma(q)][2 - \sigma(q)]}{q}. \quad (\text{A1}')$$

Note that $3 - \sigma(q) > 1$ because $2 - \sigma(q) > 0$ from (10), so condition (A1') relaxes (A1). Condition (A1') is the necessary and sufficient condition for welfare to be a strictly convex function of marginal cost (as (A1) is for consumer surplus). This yields the next result, the proof of which is similar to that of Proposition 1 and therefore relegated to the Appendix.

Proposition 2 *Assume $D_L(p) = \lambda D(p)$ and $D_H(p) = (1 - \lambda) D(p)$ for $\lambda \in (0, 1)$. If (A1') holds, differential pricing increases total welfare.*

The connection between (A1) and (A1') can be further understood as follows. Condition (A1) for strictly convex $s(c)$ is equivalent to $-s'(c) = D(p^*(c))p^{*'}(c)$ decreasing in c . Because total welfare is $w(c) = s(c) + \pi(c)$ and $\pi'(c) = -D(p^*(c))$ by the envelope theorem, $w(c)$ is strictly convex when $-w'(c) = -s'(c) + D(p^*(c)) = D(p^*(c))(p^{*'}(c) + 1)$ decreases in c , which holds if $[\frac{q}{2 - \sigma(q)} + q]$ increases in q . The latter is equivalent to (A1').

Given that (A1') relaxes (A1), it also holds for all demands whose pass-through is non-decreasing in c and for many common demands with increasing pass-through that satisfy (A1), including the standard logit, probit, and log-normal demand functions (see footnote 15 in Section 3). Furthermore, it can hold for demands with increasing pass-through that violates (A1). For instance, consider the general logit demand corresponding to logistic distributions with a non-zero mean. Then (A1) is violated when $q > 1/2$; but (A1'), which

can be equivalently written as

$$\frac{p^{*''}(c)}{p^{*'}(c)} \leq \frac{-D'(p^*(c))}{D(p^*(c))} [p^{*'}(c) + 1],$$

is satisfied by the general logit demand, for which the pass-through is $p^{*'}(c) = 1 - q$.²⁰

We now discuss the forces driving the increase in welfare. When costs differ, uniform pricing misallocates output because the price-cost margin is lower in the higher-cost market by the size of the cost gap, $c_H - c_L$. If the pass-through rate does not exceed one ($p^{*'}(c) \leq 1$), the price-cost margin under differential pricing will remain (weakly) lower in the higher-cost market: $p^{*'}(c) \leq 1$ implies

$$p_H^* - p_L^* = \int_{c_L}^{c_H} p^{*'}(c) dc \leq \int_{c_L}^{c_H} dc = c_H - c_L,$$

hence the output reallocation from market H is beneficial. This case tracks the common intuition for why differential pricing is desirable when only costs differ.

But if $p^{*'}(c) > 1$, then $p_H^* - c_H > p_L^* - c_L$, implying that differential pricing diverts too much output away from the higher-cost market. In some such cases the output reallocation can be harmful on balance. Indeed, because (A1') allows any constant pass-through rate, including much larger than 1, it encompasses cases where differential pricing creates a severe output misallocation, yet welfare still rises (see Example 1 below). What then prevents welfare from falling? Recall from (11) that $p^{*'}(c) \lesseqgtr \frac{1}{2}$ as $\sigma(q^*) \lesseqgtr 0$; that is, as $P''(q^*) \lesseqgtr 0$. The output allocation *may* worsen only if the pass-through exceeds 1, which requires inverse demand to be highly convex ($\sigma > 1$). And with convex demand, differential pricing will increase total output if average price does not rise too much: when $P(q)$ is strictly convex ($\sigma > 0$), so is $D(p)$, and if $\lambda p^*(c_L) + (1 - \lambda)p^*(c_H) \leq \bar{p}$, then $\lambda D(p^*(c_L)) + (1 - \lambda)D(p^*(c_H)) > D(\lambda p^*(c_L) + (1 - \lambda)p^*(c_H)) \geq D(\bar{p})$.²¹ Condition

²⁰We note that the equivalent condition for (A1') holds if $p^{*'}(c)$ is increasing but $\frac{p^{*'}(c)}{p^*(c)}$ is non-increasing, or $\frac{p^{*''}(c)}{p^{*'}(c)} \leq \frac{p^{*'}(c)}{p^*(c)}$ (i.e., $p^*(c)$ is log-concave). This is because $1 = \frac{-D'(p^*(c))}{D(p^*(c))} [p^*(c) - c] \leq \frac{-D'(p^*(c))}{D(p^*(c))} p^*$, implying that $\frac{p^{*'}(c)}{p^*(c)} \leq \frac{-D'(p^*(c))}{D(p^*(c))} [p^{*'}(c) + 1]$.

²¹Under classic price discrimination total output can increase only if demand is less convex in the market where price rises (Robinson 1933; Shih, Mai, and Liu, 1988; Aguirre, Cowan, and Vickers, 2010). Malueg (1993) obtains bounds on the change in welfare based on the concavity or convexity of demands.

(A1') on the demand curve—that the pass-through rate does not increase too fast with marginal cost—limits the rise in average price (if any); given (A1'), Proposition 2 implies that whenever demand is convex enough for differential pricing to cause a harmful output reallocation, total output will expand and by enough that total welfare still rises.

Differential pricing can reduce output when the pass-through rate is constant (or even somewhat decreasing) if demand is strictly *concave*. But in that case the pass-through is less than 1, which ensures that the reallocation effect is beneficial, and under (A1') is strong enough that total welfare will rise even if output decreases (as in Example 2 below). In short, the reallocation and the output effects are naturally connected through the profit-maximizing pass-through rate, so that in general at least one effect will be positive and, under (A1'), will dominate the other effect if the latter is negative.

Contrast with Classic Price Discrimination

Aguirre, Cowan, and Vickers (2010) analyze the effects of classic price discrimination on total welfare. They assume an increasing ratio condition (IRC): $z(p) = (p - c) / [2 - \theta(p)\sigma]$ strictly increases. Under the IRC, they show in Proposition 1 that price discrimination *reduces* welfare if the direct demand function in the strong market (our H) is at least as convex as in the weak market at the uniform price. One can verify that $z'(p) > 0$ is equivalent to

$$\sigma'(q) < \frac{1}{-D'(p)} \left[\frac{2 - \theta(p)\sigma}{p - c} + \theta'(p)\sigma \right] \frac{1}{\theta(p)},$$

which, provided $\theta'(p) \geq 0$, is satisfied if $\sigma'(q)$ is not too positive.²² Therefore, the IRC condition and our (A1') both can be satisfied if $\sigma(q)$ neither increases nor decreases too fast, which encompasses the important class of demand functions with constant σ . For these demand functions differential pricing that is purely cost based will *increase* welfare.²³

²²From Aguirre, Cowan, and Vickers, condition $z'(p) > 0$ holds for many common demand functions, including linear, constant-elasticity, and exponential. IRC neither implies nor is implied by our (A1').

²³Aguirre, Cowan, and Vickers show in Proposition 2 that welfare is higher with discrimination if prices are not far apart and inverse demand is locally more convex in the weak market. Our Proposition 2 shows that cost-driven differential pricing increases welfare also when market demands have the same curvature.

Examples

In the first example, the pass-through rate exceeds one and differential pricing worsens the output allocation, but is still beneficial due to the large output expansion.

Example 1 (*Differential pricing worsens allocation but raises consumer and total welfare.*)

Consider constant-elasticity demands: $D_H(p) \equiv D_H(p) = p^{-\eta}$. Then $p^*(c) = c \frac{\eta}{\eta-1}$. Suppose $c_L = 0.1$, $c_H = 0.3$, $\eta = \frac{5}{4}$. Then $\bar{p} = 1$, $\bar{q}_L = \bar{q}_H = 1$; $p_L^* = 0.5$, $q_L^* = 2.38$; $p_H^* = 1.5$, $q_H^* = 0.60$. Using (9), the average value of the reallocated output in markets L and H respectively is $P_L(\xi_L) = 0.69$, $P_H(\xi_H) = 1.21$, hence $P_L(\xi_L) - c_L = 0.59 < 0.91 = P_H(\xi_H) - c_H$, so the output reallocation is quite harmful. But this demand satisfies (A1), so differential pricing raises both consumer and total welfare. Total welfare rises because the output expansion dominates the negative and large reallocation effect. Consumer welfare increases due to the price dispersion because average price is unchanged.

In Example 2, differential pricing is beneficial only due to improved output allocation.

Example 2 (*Differential pricing reduces output but raises consumer and total welfare.*)

Suppose $P(q) = a - bq^\delta$, with $D(p) = \left(\frac{a-p}{b}\right)^{1/\delta}$ and $\delta > 1$. For $c < a$, we have $p^*(c) = a - \frac{a-c}{\delta+1}$, $q^*(c) \equiv D(p^*(c)) = \left(\frac{1}{b} \frac{a-c}{\delta+1}\right)^{1/\delta}$, so $q^*(c)$ is strictly concave when $\delta > 1$. Hence

$$\Delta q = (q_L^* + q_H^*) - (\bar{q}_L + \bar{q}_H) = \lambda q^*(c_L) + (1 - \lambda) q^*(c_H) - q^*(\lambda c_L + (1 - \lambda) c_H) < 0,$$

so differential pricing reduces total output. However, this demand function satisfies (A1). Thus, differential pricing increases consumer surplus and, hence, also total welfare.

Consumer surplus increases here because the weighted-average price is equal to the uniform price (because $p^{*''}(c) = 0$) and the pure price dispersion benefits consumers. Welfare increases due to the reallocation effect, which is beneficial because the pass-through rate is less than one, $p^{*'}(c) = 1/(\delta + 1)$, and in this case dominates the negative output effect.²⁴

²⁴The reallocation is beneficial for any $\delta > 0$. If $\delta \leq 1$ (instead of > 1 as assumed thus far), then differential pricing would not lower total output, and the two effects would reinforce each other to increase total welfare.

For equally-elastic demands, although unusual, there are cases where (A1) does not hold and differential pricing *reduces* consumer surplus, as in the example below.

Example 3 (*Differential pricing reduces consumer welfare.*) Assume $c_L = 0$, $c_H = 0.5$, $\lambda = 1/2$, and logit demand $D_L(p) = \frac{1}{1+e^{p-a}} = D_H(p)$; $P_L(q) = a - \ln \frac{q}{1-q} = P_H(q)$. Whereas (A1) is satisfied by the standard logit with $a = 0$, with $a > 0$ (A1) may be violated. For instance, let $a = 8$. Then $p_L^* = 6.327$, $p_H^* = 6.409$, $\bar{p} = 6.367$; $q_L^* = 0.842$, $q_H^* = 0.831$, $\bar{q} = 0.837$. Differential pricing now raises the average price and lowers output (slightly). Consumer welfare decreases: $\Delta S = -8.59 \times 10^{-4}$; but total welfare increases: $\Delta W = 4.87 \times 10^{-4}$. Notice that in this example, (A1) is violated when $q > 0.5$, but (A1') is satisfied for $q < 1$ (which is always true).

We have not found examples where differential pricing reduces total welfare for demand functions that are everywhere differentiable. However, if demand is a step function then $W^* < \bar{W}$ is possible, as shown in the example below, where $p_i^* = \arg \max_p \pi_i(p)$ and $\bar{p} = \arg \max_p [\pi_H(p) + \pi_L(p)]$.

Example 4 (*Differential pricing reduces total welfare.*) Assume $c_L = 0.6$, $c_H = 1.4$, $\lambda = 1/2$, and demand

$$D_L(p) = D_H(p) = \frac{1}{2} \begin{cases} (2 - 0.5p) & \text{if } 0 \leq p \leq 2 \\ (3 - p) & \text{if } 2 < p \leq 3 \end{cases}.$$

Then, $p_L^* = 2$, $p_H^* = 2.2$, $q_L^* = 0.5$, $q_H^* = 0.4$; $\bar{p} = 2$, $\bar{q}_L = 0.5 = \bar{q}_H$; and $\Delta W = -0.07$. Notice that (A1') is not satisfied at $p = 2$, where the demand has a kink.

In Example 4, due to the kink which makes the demand function concave, switching to differential pricing does not increase sales in the low-cost market but reduces sales in the high-cost market. Consequently, differential pricing reduces total welfare.

4. WELFARE COMPARISONS WHEN COSTS AND DEMANDS DIFFER

We now let markets differ *both* in costs of service and demand elasticities. We first provide necessary and sufficient conditions for beneficial differential pricing if demands are linear,

and then provide sufficient conditions when demands have general curvature.

Linear Demands

Suppose that

$$P_i(q) = a_i - b_i q, \text{ where } a_i > c_i \text{ for } i = H, L.$$

Note that the demand elasticity in market i equals $p/(a_i - p)$, which depends only on the choke price a_i (the vertical intercept) and not on the slope. Under differential pricing,

$$p_i^* = \frac{a_i + c_i}{2}; \quad q_i^* = \frac{a_i - c_i}{2b_i}; \quad \pi_i^* = \frac{(a_i - c_i)^2}{4b_i},$$

and $p_H^* > p_L^*$ requires that $(a_H - a_L) + (c_H - c_L) > 0$. Under uniform pricing, provided that both markets are served:

$$\bar{p} = \frac{(a_H + c_H)b_L + (a_L + c_L)b_H}{2(b_L + b_H)}; \quad \bar{q}_i = \frac{1}{b_i} \left[a_i - \frac{(a_H + c_H)b_L + (a_L + c_L)b_H}{2(b_L + b_H)} \right].$$

Straightforward algebra shows that:

$$q_H^* + q_L^* = \bar{q}_H + \bar{q}_L.$$

Pigou (1920) proved this equal-outputs result for linear demands when marginal cost depends only on the level of total output and not its allocation between markets. We showed that the result holds also when marginal costs differ across markets but are constant.

As discussed in Section 3, classic price discrimination tends to raise the weighted average price, whereas cost-based differential pricing does not. Linear demands illustrate this point. If only demand elasticities differ ($a_H > a_L$, $c_H = c_L$), the (weighted) average price rises :

$$\bar{q}_H \Delta p_H + \bar{q}_L \Delta p_L = \frac{1}{2} (a_H - a_L) \frac{a_H - a_L + c_H - c_L}{b_H + b_L} > 0,$$

whereas if only costs differ ($a_H = a_L$, $c_H > c_L$), the average price is unchanged:

$$\bar{q}_H \Delta p_H + \bar{q}_L \Delta p_L = \frac{(a_H - a_L + c_H - c_L)(a_H - a_L)}{2(b_H + b_L)} = 0.$$

From (2):

$$\Delta S = \frac{(a_H - a_L + c_H - c_L)[(c_H - c_L) - 3(a_H - a_L)]}{8(b_H + b_L)}, \quad (18)$$

which takes the sign of $[(c_H - c_L) - 3(a_H - a_L)]$. Furthermore, because $\Delta\Pi = \frac{(a_H - a_L + c_H - c_L)^2}{4(b_H + b_L)}$, we have

$$\Delta W = \Delta S + \Delta\Pi = \frac{(a_H - a_L + c_H - c_L) [3(c_H - c_L) - (a_H - a_L)]}{8(b_H + b_L)}. \quad (19)$$

which takes the sign of $[3(c_H - c_L) - (a_H - a_L)]$.

Expressions (18) and (19) yield the ensuing necessary and sufficient conditions for differential pricing to increase consumer surplus and total welfare based on the difference in costs relative to the difference in the demand elasticity parameter a_i (market i 's choke price):

Proposition 3 *If demand curves are linear, a move from uniform to differential pricing has the following effects. (i) Total welfare increases if the difference between markets in their choke prices is less than three times the difference in marginal costs ($a_H - a_L < 3(c_H - c_L)$) and decreases if the inequality is reversed. (ii) Consumer surplus increases if the difference in choke prices is less than one third of the cost difference ($a_H - a_L < \frac{c_H - c_L}{3}$) and decreases if the inequality is reversed.*

Proposition 3 demonstrates the contrasting welfare effects of classic price discrimination versus cost-based differential pricing under linear demands. When only demand elasticities differ, $a_H - a_L > 3(c_H - c_L) = \frac{c_H - c_L}{3} = 0$, hence classic price discrimination lowers both consumer and total welfare; whereas, when only costs differ, $0 = a_H - a_L < \frac{c_H - c_L}{3} < 3(c_H - c_L)$, differential pricing motivated only by cost differences raises both consumer and total welfare.²⁵ Between these polar cases, differential pricing is beneficial when the difference in demand elasticities is not too large relative to the difference in costs.

For linear demands and constant costs Valletti (2006) and Bertolotti (2009) extend some of the above results to n (≥ 2) markets: Uniform or differential pricing yield the same total output if all markets are served under both regimes; and differential pricing raises

²⁵With overall output remaining constant, total welfare changes due to the reallocation, which is harmful under classic discrimination. When only costs differ, uniform pricing misallocates output and differential pricing improves the allocation since pass-through with linear demands is below one. The consumer surplus intuition follows the behavior of average price.

consumer and total welfare when markets differ only in costs and lowers both if only demand elasticities differ.²⁶ Our Proposition 3 considers only two markets but provides the necessary and sufficient conditions for beneficial differential pricing in that case.

Until now we assumed that the market with the (weakly) higher cost has the (weakly) lower demand elasticity, so that markets under differential pricing can be ranked unambiguously as “high-price” or “low-price.” If the lower-cost market has the less elastic demand, differential pricing will still be beneficial if the cost difference is sufficiently large, the same qualitative finding as in Proposition 3. To see this, continue assuming $a_H > a_L$, so demand is less elastic in market H , but suppose $c_H < c_L$. If the cost difference is small relative to $a_H - a_L$, differential pricing will raise price in market H and lower price in L , thereby shifting output away from the lower-cost market and increasing the distortion, so total welfare and consumer surplus will fall. However, if the cost difference is large enough ($a_H - a_L < c_L - c_H$), price will fall in market H and rise in L , yielding cost savings. From (18) and (19), differential pricing will then raise consumer and total welfare.

General Demands

When demand is linear in both markets Proposition 3 showed that differential pricing is beneficial if the cost difference is sufficiently large relative to the demand difference. It is not clear whether this result would extend to general demands, because as the cost difference grows the average price under differential pricing may rise faster than that under uniform pricing (as in Example 6 of Chen and Schwartz, 2013). To address the mixed case where there are differences both in general demand functions and in costs, we develop an alternative analytical approach that more clearly disentangles their roles, and use it to derive sufficient conditions for differential pricing to be beneficial.

²⁶When both costs and demands differ, the authors present welfare comparisons depending on the variance of costs and demands and their correlation. Bertoletti provides sufficient conditions for welfare gains based on Laspeyre or Paasche price variations. Valletti (2006) shows how the alternative pricing regimes affect incentives to invest in quality.

Without loss of generality, let

$$c_H = c + t, \quad c_L = c - t.$$

Then, $c_H - c_L = 2t$, which increases in t , and $c_H = c_L$ when $t = 0$. Thus, c is the average of the marginal costs and t measures the cost differential. For $i = H, L$, the monopoly price under differential pricing $p_i(t)$ satisfies $\pi'_i(p_i(t)) = 0$, from which we obtain:

$$p'_i(t) = \frac{D'_i(p_i(t))}{\pi^{i''}(p_i(t))} \frac{dc_i}{dt} = \frac{1}{2 + \frac{[p_i(t) - c_i]}{p_i(t)} p_i(t) \frac{D'_i(p_i(t))}{D'_i(p_i(t))}} \frac{dc_i}{dt}.$$

because $\alpha \equiv -pD''/D'$ is the curvature of the direct demand function, $\frac{p^*(c)-c}{p^*(c)} = \frac{1}{\eta(p^*(c))}$, and $\sigma = \alpha/\eta$, we have, with $dc_L/dt = -dc_H/dt = -1$:

$$p'_H(t) = \frac{1}{2 - \sigma_H(q_H(t))} > 0; \quad p'_L(t) = -\frac{1}{2 - \sigma_L(q_L(t))} < 0, \quad (20)$$

where for $i = L, H$, $q_i(t) \equiv D_i(p_i(t))$, and $2 - \sigma_i(q_i(t)) > 0$ from (10).

Let $\bar{p}(t)$ be the monopoly uniform price, which solves $\pi'_H(\bar{p}(t)) + \pi'_L(\bar{p}(t)) = 0$, from which we obtain

$$\bar{p}'(t) = \frac{D'_L(\bar{p}(t)) - D'_H(\bar{p}(t))}{-\pi''_H(\bar{p}(t)) - \pi''_L(\bar{p}(t))}. \quad (21)$$

Thus $\bar{p}'(t) \geq (<) 0$ if $D'_L(\bar{p}(t)) \geq (<) D'_H(\bar{p}(t))$. Intuitively, an increase in the cost difference t leads the monopolist to raise the output-mix ratio $q_L(t)/q_H(t)$. This requires increasing the uniform price if $D_L(p)$ is steeper than $D_H(p)$ and lowering price if $D_L(p)$ is flatter.

From (1), the change in consumer welfare under differential pricing due to a marginal change in t is

$$S^{*'}(t) = -q_L(t)p'_L(t) - q_H(t)p'_H(t). \quad (22)$$

Under uniform pricing,

$$\bar{S}'(t) = -[D_L(\bar{p}(t)) + D_H(\bar{p}(t))] \bar{p}'(t), \quad (23)$$

which takes the sign opposite of $\bar{p}'(t)$ as determined by the relative slopes of the demand functions in (21). From (2), the difference in the changes of consumer welfare due to a

marginal increase in t under the two pricing regimes is equal to

$$\Delta S'(t) = S^{*'}(t) - \bar{S}'(t). \quad (24)$$

Consumer welfare will increase faster under differential than under uniform pricing with a marginal increase in t if $\Delta S'(t) > 0$, and, for any given $t > 0$, consumer welfare will be higher under differential pricing if $\Delta S(t) > 0$.

The result below provides a sufficient condition under which consumer and total welfare are higher under differential than under uniform pricing if the cost difference is large.

Proposition 4 *Suppose that there exist some $t_1 \geq 0$ and $\hat{t} \geq t_1$ such that*

$$\bar{p}'(t) > \frac{q_H(t)p'_H(t) + q_L(t)p'_L(t)}{D_H(\bar{p}(t)) + D_L(\bar{p}(t))} \text{ for } t \geq t_1 \quad (A2)$$

and $\Delta S(\hat{t}) \geq 0$. Then, differential pricing increases consumer welfare (i.e., $\Delta S(t) > 0$) if $t > \hat{t}$.

Proof. From (22), (23), and (24), we have

$$\Delta S'(t) = -[q_L(t)p'_L(t) + q_H(t)p'_H(t)] + [D_L(\bar{p}(t)) + D_H(\bar{p}(t))]\bar{p}'(t).$$

Thus $\Delta S'(t) > 0$ when (A2) holds with $t \geq t_1$. Therefore, with $\Delta S(\hat{t}) \geq 0$ and $t > \hat{t} \geq t_1$,

$$\Delta S(t) = \Delta S(\hat{t}) + \int_{\hat{t}}^t \Delta S'(t) dt > 0.$$

■

Intuitively, (A2) is based on a comparison of the pass-through from t to the prices under uniform and differential pricing. Under (A2) and when $t \geq t_1$, as t rises the uniform price increases relative to a weighted average of the differential prices, causing consumer welfare under differential pricing to rise relative to that under uniform pricing, or $\Delta S'(t) > 0$. Thus, in both (A1) and (A2), properties of pass-through determine how differential pricing may affect consumer welfare. We note that (A2) is tight as part of a sufficient condition, in the sense that if it is violated, differential pricing in general, with both demand and

cost differences, cannot improve consumer welfare unless classic price discrimination does. Notice also that if $\Delta S(t_1)$ is too negative, (A2) may not guarantee that $\Delta S(t) > 0$ even for a large t . The condition that $\Delta S(\hat{t}) \geq 0$ for some $\hat{t} \geq t_1$ ensures that $\Delta S(t_1)$ cannot be too negative.

The conditions in Proposition 4 can hold under direct, but more restrictive, sufficient conditions on demand functions:

Corollary 1 *Assume: (i) (A1) holds, (ii) $D'_L(\cdot) \geq D'_H(\cdot)$, (iii) $\sigma_L(q_L(0)) \geq \sigma_H(q_H(0))$, and (iv) $q_H(0) - q_L(0)$ is sufficiently small. Then, there is some $\hat{t} \geq 0$ such that $\Delta S(t) > 0$ when $t > \hat{t}$.*

Proof. Notice that (A2) holds if $\bar{p}'(t) \geq 0$ and

$$q_H(t)p'_H(t) + q_L(t)p'_L(t) < 0.$$

Under (ii), $\bar{p}'(t) \geq 0$. From (20), $q'_H(t) = D'_H(p_H(t))p'_H(t) < 0$, $q'_L(t) = D'_L(p_L(t))p'_L(t) > 0$, and

$$q_H(t)p'_H(t) + q_L(t)p'_L(t) = \frac{q_H(t)}{2 - \sigma_H(q_H(t))} - \frac{q_L(t)}{2 - \sigma_L(q_L(t))}$$

decreases in t if (A1) holds. Under (iii) and (iv),

$$\frac{q_H(0)}{2 - \sigma_H(q_H(0))} - \frac{q_L(0)}{2 - \sigma_L(q_L(0))}$$

is not too positive. Hence, from (i), (iii), and (iv),

$$\frac{q_H(t_1)}{2 - \sigma_H(q_H(t_1))} - \frac{q_L(t_1)}{2 - \sigma_L(q_L(t_1))} < 0$$

for some relatively small $t_1 \geq 0$. Thus (A2) is satisfied under (i)-(iv). Furthermore, under (iii) and (iv), $\Delta S(0)$ is close to zero, and hence there exists some $\hat{t} \geq t_1 > 0$ such that $\Delta S(t) > 0$ for $t > \hat{t}$. ■

The demand conditions of Proposition 4 can hold in numerous settings where classic price discrimination ($c_H = c_L$) would reduce consumer welfare, as in many of the cases identified in Proposition 1 of Aguirre, Cowan, and Vickers (2010) and Cowan (2007). For instance,

linear demands are covered by Proposition 4.²⁷ Proposition 4 also applies when $D_H(p)$ is an affine transformation of $D_L(p)$: $D_H(p) = a + bD_L(p)$, under some parameter restrictions. (If $a = 0$, this reduces to proportional demands as in Section 3.) The example below applies Proposition 4 to a setting where $D_H(p)$ is neither an affine transformation of $D_L(p)$ nor linear. In this example, consumer welfare is lower under differential pricing when there is no cost difference ($t = 0$), but is higher when the cost difference is large enough.

Example 5 *Suppose that $D_L(p) = \frac{2}{3}(1-p)$, $D_H(p) = (1-p)^{1/2}$, $c = 0.4$, $t \in [0, 0.3]$. Then, $p_L(t) = \frac{1.4-t}{2}$, $p_H(t) = \frac{2.4+t}{3}$, and both markets are served under uniform pricing. When $t = 0$, $\bar{p} = 0.770$ and $\Delta S = -0.002 < 0$, so that classic price discrimination ($c_H = c_L$) reduces consumer welfare. However, (A2) is satisfied with some $t_1 < 0.1$, and $\hat{t} = 0.1$. Therefore $\Delta S > 0$ for all $t > \hat{t} = 0.1$. As expected, differential pricing increases total welfare for an even larger set of parameter values. In fact, in this example $\Delta W > 0$ for all $t \geq 0$.*

If the conditions in Proposition 4 are not met, differential pricing can reduce total welfare, hence also consumer surplus, even as $c_H - c_L$ becomes arbitrarily large (subject to the constraint that both markets will still be served under uniform pricing). See Example 6 in Chen and Schwartz (2013).

5. CONCLUSION

Prevailing economic analysis of third-degree price discrimination by an unregulated monopolist paints an ambivalent picture of its welfare effects relative to uniform pricing. In order for overall welfare to rise total output must expand. Without specific knowledge of the shapes of demand curves the literature yields no presumption about the change in output unless discrimination leads the firm to serve additional markets. Moreover, because

²⁷Recall that $D_H = \frac{a_H - p}{b_H}$ and $D_L = \frac{a_L - p}{b_L}$, with $a_H > a_L$. Then, (A2) holds for $t > t_1 = \frac{a_H - a_L}{2}$, and $\hat{t} = \frac{3}{2}(a_H - a_L)$. Thus, if $t > \hat{t}$ —implying $(c_H - c_L) > 3(a_H - a_L)$, the condition in part (ii) of Proposition 3—then differential pricing increases consumer welfare, even though for linear demands classic price discrimination reduces consumer welfare.

discrimination raises profits, an increase in overall welfare is necessary but not sufficient for aggregate consumer surplus to rise.

This article showed that judging differential pricing through the lens of classic price discrimination understates its beneficial role when price differences are motivated at least in part by differences in the costs of serving various markets. Differential pricing then saves costs by reallocating output to lower-cost markets, and benefits consumers in the aggregate under broad demand conditions by creating price dispersion which—unlike classic price discrimination—does not come with a systematic bias for average price to rise.

Our analysis formalizes the intuition that price uniformity mandated in pursuit of social goals likely comes at a cost to aggregate consumer welfare. It also cautions against hostility in unregulated settings to differential pricing that is plausibly cost based, such as the common and growing practice of add-on pricing that unbundles the pricing of various elements from the price of the base good. An important extension would be to analyze whether and how the beneficial aspects of differential pricing under different costs might extend beyond monopoly to imperfect competition, building on the analyses of oligopoly price discrimination (e.g., Stole, 2007).

APPENDIX

Proof of Proposition 2. First, we show that if and only if (A1') holds, total welfare is a strictly convex function of constant marginal cost c . Total welfare under $p^*(c)$ is

$$w(c) \equiv W(p^*(c)) = \int_0^{D(p^*(c))} [P(x) - c] dx.$$

Thus, $w'(c) = [p^*(c) - c] D'(p^*(c)) p^{*'}(c) - D(p^*(c))$. From the first-order condition for $p^*(c)$, we have $[p^*(c) - c] D'(p^*(c)) = -D(p^*(c))$. Hence

$$w'(c) = -D(p^*(c)) p^{*'}(c) - D(p^*(c)) = -D(p^*(c)) [p^{*'}(c) + 1],$$

$$w''(c) = -D'(p^*(c)) p^{*'}(c) \left[\frac{1}{2 - \sigma(q^*)} + 1 \right] - D(p^*(c)) \frac{\sigma'(q^*)}{[2 - \sigma(q^*)]^2} D'(p^*) p^{*'}(c).$$

Therefore, $w''(c) > 0$ if and only if

$$3 - \sigma(q^*) + D(p^*(c)) \frac{\sigma'(q^*)}{2 - \sigma(q^*)} > 0,$$

or if and only if (A1') holds. Next,

$$\begin{aligned} W^* &= \lambda W(p^*(c_L)) + (1 - \lambda) W(p^*(c_H)) \\ &= \lambda w(c_L) + (1 - \lambda) w(c_H) \\ &> w(\lambda c_L + (1 - \lambda) c_H) \text{ (by the convexity of } w(c)) \\ &= W(p^*(\bar{c})) = \bar{W}. \end{aligned}$$

■

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