

Platform Competition With Cash-back Rebates Under No Surcharge Rules

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Abstract

We analyze competing strategic platforms setting fees to a local monopolist merchant and cash-back rebates to end users, when the merchant may not surcharge platforms' customers, a rule imposed by some credit card networks. Each platform has an incentive to gain transactions by increasing the spread between its merchant fee and user rebate above its rival's spread. This incentive yields non-existence of pure strategy equilibrium in many natural environments. In some circumstances, there is a mixed strategy equilibrium where platforms choose fee structures that induce the merchant to accept only one platform with equal probability, a form of monopolistic market allocation.

Keywords: Platform price competition; rebates; no surcharge; payment networks; credit cards.

JEL Codes: L13, L41, L42, D43.

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1 Introduction

The credit card sector is an important part of a modern economy, with almost eleven trillion dollars of sales globally in 2015 (Carlton and Winter, 2018). It is characterized by competition between platforms that charge fees to both merchants and cardholders (or offer rebates) and is, thus, an archetypal example of a two-sided market. A long-standing provision of card platforms such as American Express and Visa is the no-surcharge rule (NSR): it bars merchants from charging higher prices when consumers pay via a platform’s card instead of specified alternatives, such as a competing platform’s card.

Despite the prevalence of NSR constraints, their full implications for equilibrium pricing by competing platforms to merchants and card users when cash-back rebates are used are not well-understood. In this paper, we demonstrate that, with a general demand structure and under different timing assumptions, the presence of a NSR implies that a pure strategy equilibrium (PSE) where a merchant accepts both cards does not exist. The result is robust to several natural extensions of the model. Under one particular timing, we find a mixed strategy equilibrium with a merchant rejecting each platform with equal probabilities. This equilibrium, however, requires stringent assumptions and, while it may explain a few prominent examples, it is not consistent with industry facts overall.

What should one make of these findings? If one believes that non-existence of PSE implies unstable pricing, this raises empirical questions: Are the fees charged by credit card platforms to merchants and cardholders indeed unstable? With or without equilibrium, is pricing behavior consistent with the incentives revealed by our analysis? Alternatively, if platform fees are stable, our analysis raises the puzzle: are there factors beyond those we considered that might restore a PSE or otherwise explain the stability of prices? We view this paper’s contribution as elucidating the pricing incentives generated for platforms by no-surcharge rules and identifying open questions for future research.

A NSR reduces the elasticity of demand for a platform’s transactions with respect to its merchant fee, for potentially two reasons. First, for a given increase in the platform’s fee the merchant will raise price to consumers by less with an NSR than without because the NSR requires any price increase to apply uniformly to other transactions. Second, if platforms are substitutes, the uniform increase in the merchant’s price diverts some sales back to the first platform. The reduced demand elasticity induces platforms noncooperatively to raise their fees to merchants (Boik and Corts, 2016;

Ramezzana, 2016; Carlton and Winter, 2018).¹ This one-sided reasoning is correct as far as it goes, but yields only a partial understanding of equilibrium pricing when competing platforms additionally can offer cash rebates to their card users. In the one-sided case there is a natural force that ultimately caps fee increases to merchants: loss of transactions volume. When a platform can offer cash rebates, however, it seemingly can increase volume while maintaining its profit margin by raising its merchant fee and rebate equally. Equilibrium fee setting by competing platforms in this case remains an open issue in economic theory.

The issue is important also for public policy. NSRs have long existed in credit cards and more recently have featured in ‘new economy’ sectors such as online travel booking sites, triggering extensive regulatory and antitrust scrutiny (Bender and Fairless, 2014; Assaf and Moskowitz, 2015; Gonzales-Diaz and Bennett, 2015; Mantovani, et. al., 2018).² Understanding the equilibrium pricing implications when cash rebates are feasible is important for a fuller welfare assessment of NSR restraints. Notably, the one-sided reasoning underlay the U.S. district court’s important decision in *United States v. American Express* (2015). The decision barred various provisions imposed by Amex, including a NSR, that prevent merchants from steering customers to use competing cards (with potentially lower merchant fees), arguing that such ‘no-steering’ provisions anti-competitively induce higher fees to merchants and ultimately harm card users as well.³ In overturning this decision, the appellate court

¹Johnson (2017) obtains a similar price-raising effect under an alternative contracting arrangement, the agency model (used, for example, by Amazon Marketplace): each retailer sets the share of its revenue that will go to its suppliers and each supplier (not the retailer) sets the retail price for its product. Retail price-parity clauses, requiring a supplier’s price not to differ across retailers, induce each retailer to demand a higher revenue share, leading suppliers to raise retail prices.

²Early regulatory concerns emphasized that a NSR imposed by a card platform would harm merchants and non-card customers such as cash users (Katz, 2001; Farrell, 2006; Schwartz and Vincent, 2006). More recently, antitrust concerns stress that NSRs adopted by rival card platforms will harm merchants and card customers.

³“By suppressing the incentives of its network rivals to offer merchants, and by extension their customers, lower priced payment options at the point of sale [...] American Express’s merchant restraints harm interbrand competition.” (pp. 100-101.) “American Express’s merchant restraints have allowed all four networks to raise their swipe fees more easily and more profitably than would have been possible were merchants permitted to influence their customers’ payment decisions.” (p. 111.) Amex’s provisions are framed broadly to include various merchant conduct that would discourage payments via Amex credit or charge cards. Most relevant for our purposes, the merchant may not “impose any restrictions, conditions, disadvantages or fees [...] that are not imposed equally on all Other Payment Products, except for electronic funds transfer, or cash and check” (Id., pp. 25-26, emphasis added).

wrote: “The District Court erred in concluding that increases in merchant pricing are properly viewed as changes to the net price charged across Amex’s integrated platform,’ [...] because merchant pricing is only one half of the pertinent equation.” The appellate court added: “Because the two sides of the platform cannot be considered in isolation, it was error for the District Court to discard evidence [of two-sided price’ calculations]” (*United States v. Amex*, 2016, p. 49).⁴

This paper analyzes the pricing incentives of competing card platforms when a merchant cannot charge differential prices to consumers based on which platform they use, due to contractual restrictions (as in *Amex*) or a merchant’s intrinsic reluctance to charge different prices for a given good. For brevity, we denote all such constraints as NSRs. We proceed under the traditional assumption that rational consumers care about the total cost of purchases – the merchant’s price plus platform’s fee (or rebate). Our model has two symmetric platforms offering intermediary services viewed by end users as differentiated substitutes. We set aside potential efficiency roles of NSRs, such as preventing free riding or hold-up problems (e.g., Wright, 2003; Bourguignon, Gomes and Tirole, 2014), and abstract from downstream competition by assuming a local monopolist merchant. Each platform (card network for concreteness) sets per-transaction fees to the merchant and to its cardholders, potentially with NSRs, and the merchant decides which platform(s) to accept.

We examine two timing structures. In the first, both platforms simultaneously set merchant and cardholder fees; the merchant accepts both platforms, one, or none; and lastly, the merchant sets its price(s). We also consider alternative timing: cardholder terms are set after the merchant set its price(s). Our main result holds in both cases: under NSR pricing restraints, there exists no pure strategy equilibrium with the merchant accepting both platforms. This non-existence result derives squarely from two-sided pricing: starting from any candidate equilibrium, at least one platform can profitably deviate by suitably adjusting the *spread* (or gap) between its fee to the merchant and its fee to card users and gain sales at the other’s expense.

Consider platform fees at which the merchant accepts both platforms and the NSR binds: the merchant would prefer to charge a higher price for platform j than for k . Let platform k expand its fee spread: raise its merchant fee by a small amount and

⁴Economically, the impact of such provisions on the fee to card users is clearly relevant for a full welfare analysis. We take no position on the overall merits of the appellate court’s decision, which raises also legal issues such as evidential considerations and the appropriate burden of proof. For a critique of the appellate court’s position see Carlton and Winter (2018). In June, 2018, the U.S. Supreme Court affirmed the appellate court’s decision (*Ohio v. Amex*, 2018).

cut its card user fee (or increase the rebate) equally, maintaining its total fee constant. This induces an upward shift in the merchants inverse demand from platform k users and in its marginal cost for their transactions, so the merchant would optimally raise price only to k s users by the same amount. However, with a binding NSR the merchant raises price (to all cardholders) by less, hence platform k 's users obtain a net price reduction and platform k gains volume and profit. Moreover, and less obvious, the merchant also benefits from this deviation: its (unconstrained) optimal price for platform k rises, coming closer to its platform j optimal price, thereby mitigating the harm from the NSR. Starting with a binding NSR, therefore, at least one platform can profitably expand its fee spread and maintain merchant acceptance. Starting from a non-binding NSR, either platform could profitably expand its spread until the NSR binds slightly without being dropped, since platforms are differentiated.

The above logic of our non-existence proof emphasizes platforms' incentives to increase their fee spreads when their fees are not too different. When fee profiles are quite different, however, a platform can prefer to *decrease* its fee spread. Suppose the merchant's unconstrained optimal price is higher for platform j than for k , platform k charges a lower total fee than j , and the NSR is onerous for the merchant because the platforms' fee structures are very different. By decreasing its fee spread platform k aggravates the NSR's impact, leading the merchant to drop platform j (whose total fee is higher) since the NSR becomes irrelevant if only one platform is accepted. Such a deviation can be more profitable for platform k than an increase in its spread because it gains a discontinuous increase in sales when the merchant switches from accepting both platforms (multi-homing) to dropping the rival.

This incentive to induce single-homing also plays a role in the timing structure where platforms set cardholder fees last. For that case, we show that if the platforms are sufficiently close substitutes, there exists an interesting mixed strategy equilibrium where the merchant single-homes and randomizes its acceptance decision. Specifically, platforms offer NSRs and equal total fees but sufficiently different spreads so that the merchant accepts only one platform with equal probabilities. Viewed in this light, setting sufficiently disparate merchant fees along with a NSR can serve in certain settings as a profitable mechanism to achieve probabilistic market allocation among platforms.

Our work is related to two broad literatures: Two-Sided Markets, and Most Favored Nation (MFN) clauses. Our analysis is two-sided in the key sense that a platform's pricing structure matters, but we ignore some central issues in that literature, such as the role of platform fixed fees in attracting participation on both sides (e.g.,

Armstrong, 2006; Rochet and Tirole, 2006; Rysman, 2009). Articles on competing two-sided platforms under NSRs, notably Rochet and Tirole (2003), Guthrie and Wright (2007) and Edelman and Wright (2015), have not encountered our equilibrium non-existence or resolved it under assumptions that are not entirely satisfactory in the context of cash-back rebates. MFN clauses are contractual provisions requiring non-discriminatory’ terms between various agents, often uniform pricing.⁵ This literature mostly considers retail MFNs, involving a firm and its direct customers. There is less formal work on MFNs imposed by firms at different vertical stages. Carlton and Winter (2018) show how vertical MFNs can induce suppliers to raise prices to downstream merchants, but their formal analysis does not consider user rebates. Boik and Corts (2016) briefly consider rebates and provide an informal argument for non-existence based on a similar logic as ours. We discuss their work further after presenting our analysis.

The paper is organized as follows. Section 2 describes the setting. Section 3 proves that pure strategy equilibria with both platforms accepted cannot exist under NSRs when each network sets its fees to the merchant and card users simultaneously. Section 4 shows the same result in the alternative case where merchant fees are set first and user fees are set after the merchant sets its price(s). It also establishes the mixed-strategy equilibrium with probabilistic market allocation and notes its shortcomings. Section 5 explains why prior work on competing platforms with NSRs obtains equilibrium existence under assumptions that are not entirely convincing. It also discusses several extensions of our basic model and why those additional factors do not restore a PSE. The conclusion identifies some questions for future research. All proofs are in the Appendix. The extensions are in the Online Appendix.

2 The Model With Simultaneous Platform Fees

2.1 Agents, Prices and Payoffs

We examine an economic environment where two classes of complementary products are required in fixed proportions to generate a transaction for final consumption; the providers interact with each other through pricing; and each provider also interacts with the end-users through pricing. The motivating context is credit cards. In order

⁵ They can arise in diverse settings and perform efficient roles (e.g., reduce transaction costs or delays in purchasing) or anti-competitive roles (e.g., prevent selective discounts that undermine collusion). For a comprehensive survey see LEAR (2012).

for a merchant to complete a transaction, it often must combine its services with those of a credit card offered by a platform. We consider two differentiated competing platforms and a local monopolist merchant. Other papers, such as Rochet and Tirole (2003), assume a continuum of merchants (and uniform pricing by platforms to merchants) while Guthrie and Wright (2007) assume duopoly merchants. Our market structure lets us focus on strategic interaction between platforms while abstracting from strategic interaction among downstream merchants who, nevertheless, possess some power over price.

Platforms 1 and 2 offer differentiated payment services to a single downstream merchant. Let P_i be the total per unit price paid by a consumer for a purchase via platform i . Demand for sales made on platform i , $D^i(P_1, P_2)$, is twice continuously differentiable and satisfies the properties:

$$-\frac{\partial D^i(P_1, P_2)}{\partial P_i} > \frac{\partial D^i(P_1, P_2)}{\partial P_j} \geq 0, \quad D^1(x, y) = D^2(y, x).$$

The first condition implies that the two platforms are (imperfect) gross substitutes with demand varying strictly with price. The second condition indicates that we restrict attention to symmetric platforms. This representation of demand is a ‘reduced form’ characterization that abstracts from the micro-structure underlying consumers’ choice of payment mode. The assumption $\partial D^i / \partial P_i < 0$ can reflect substitution between platforms in response to i ’s price increase or, as in Schwartz and Vincent (2006), elastic demand for the merchant’s good.

Assume that for every price, P_1 , there is a choke price, $P_2(P_1)$ such that

$$D^2(P_1, P_2(P_1)) = 0.$$

Symmetry implies a similar property for platform 2. The assumption of gross substitutes implies that $P_2(\cdot)$ is weakly increasing. Define

$$\bar{D}^1(P_1) \equiv D^1(P_1, P_2(P_1))$$

to be the demand for platform 1 sales when platform 2 is not available. Gross substitutes implies that for all (P_1, P_2) such that $D^2(P_1, P_2) > 0$,

$$\bar{D}^1(P_1) > D^1(P_1, P_2)$$

and similarly for platform 2.

Each platform i sets a per transaction fee f_i to cardholders (typically negative, i.e., rebates) and m_i to the merchant.⁶ If the merchant accepts a platform, the merchant sets a price p_i for a purchase through that platform and the cardholder total price is

$$P_i \equiv p_i + f_i$$

and define platform i 's total fee as

$$t_i \equiv f_i + m_i.$$

Marginal costs for both platforms and merchant are assumed constant and normalized to zero. Platform profits are

$$(f_i + m_i)q_i = t_i q_i.$$

The merchant's outside option – its profit from carrying no platform – is also normalized to zero. If the merchant adopts both platforms, using $m_i = t_i - f_i$, merchant profits can be expressed in terms of (t_1, t_2) and total cardholder prices, (P_1, P_2) :

$$\begin{aligned} \Pi(P_1, P_2; t_1, t_2) &= (p_1 - m_1)D^1(p_1 + f_1, p_2 + f_2) + (p_2 - m_2)D^2(p_1 + f_1, p_2 + f_2) \\ &= (P_1 - t_1)D^1(P_1, P_2) + (P_2 - t_2)D^2(P_1, P_2). \end{aligned} \quad (1)$$

Here and in Section 3, we consider the following *Simultaneous Fees Game*:

- 1) Both platforms simultaneously select cardholder and merchant fees, (f_i, m_i) .
- 2) The merchant observes all fees and accepts both platforms, one or neither.
- 3) For each accepted platform, i , the merchant sets a price, p_i , potentially subject to restrictions, for its product and the cardholder's price is $P_i = p_i + f_i$.
- 4) For each accepted platform, i , consumers observe (f_i, p_i) : If both platforms are accepted, transactions via platform i are given by $D^i(p_1 + f_1, p_2 + f_2)$; If only platform i is accepted, its transactions are given by $\bar{D}^i(p_i + f_i)$ and none occur via platform j .

This timing captures a sense in which a merchant is able to change its consumer prices more rapidly than platforms can alter their fees either to merchants or consumers. In Section 4 we examine an alternative timing where platforms set cardholder fees *after* merchants set consumer prices.

⁶In the literature on payment systems, the merchant fee is often termed ‘the merchant discount’.

2.2 Unrestricted Merchant Pricing

Start with the benchmark case where the merchant is free to set differential prices, (p_1, p_2) . Using (1), the merchant’s profit maximization problem can be equivalently expressed as, given platform fees, selecting total cardholder prices, (P_1, P_2) , rather than merchant prices, (p_1, p_2) . This representation illustrates the well-known ‘neutrality’ property that, with no restriction on merchant pricing (including no NSR), equilibrium cardholder prices, P_i , depend solely on total fees, (t_1, t_2) , and are independent of the platform’s fee *structure* – the split between cardholder fees and merchant fees. (Rochet and Tirole, 2002; Gans and King, 2003). The merchant’s optimal quantities, therefore, depend solely on t_i :

$$q_i(t_1, t_2) \equiv D^i(P_1(t_1, t_2), P_2(t_1, t_2)).$$

Since platform i ’s profit margin also depends only on t_i and not on f_i, m_i separately, the neutrality property follows.⁷

Neutrality thus implies that, with no pricing constraints on the merchant, rival platforms can be thought of as playing a strategic game solely in total fees, t_1, t_2 . For a platform i , define an induced best response function from the neutrality environment as

$$r^i(t_j) \equiv \operatorname{argmax}_{t_i} t_i q_i(t_1, t_2),$$

and denote the partial derivatives of the merchant’s profit function with respect to price as

$$\Pi_i \equiv \frac{\partial \Pi}{\partial P_i}, \quad \Pi_{ij} \equiv \frac{\partial \Pi_i}{\partial P_j}.$$

For the remainder of this Section and Section 3, we assume:

- A1) Platform profits are strictly quasi-concave in t_i for all t_j and smoothly supermodular in (t_1, t_2) , and $r^i(t_j)$ is continuously differentiable with $r^{i'}(t_j) \in [0, 1)$.
- A2) For all t_1, t_2 , Π is strictly quasi-concave in (P_1, P_2) and there is a unique $(\hat{P}_1(t_1, t_2), \hat{P}_2(t_1, t_2))$ such that $\Pi_i(\hat{P}_1(t_1, t_2), \hat{P}_2(t_1, t_2)) = 0, i = 1, 2$.

⁷In our setting, where platforms charge only usage (i.e., per-transaction) fees, neutrality requires that a shift in a platform’s fee structure – say, an increase of Δ to the merchant and decrease of Δ to the cardholder – will yield an equal and offsetting change in the merchant’s price to the cardholder. Neutrality thus implies that the merchant price can adjust freely, unimpeded by contractual restrictions or other factors. In environments where platforms charge fixed fees, neutrality can break down even if the merchant’s price can adjust freely (Rochet and Tirole, 2006).

A3) For all $t_1, t_2, P_1, P_2, \Pi_{ii} + \Pi_{ij} < 0$.

A4) For any fixed (t_1, t_2) , if a merchant accepts only one platform, that platform's sales are (weakly) higher than its sales when the merchant accepts both platforms.⁸

Assumption A1) implies that this is a game in strategic complements and there is a unique equilibrium in t_1, t_2 (see Vives, 2001, p. 47). The remaining assumptions allow us to focus primarily on first order conditions to conduct the proofs.

Under mild conditions, assumptions A1)-A4) are satisfied for two commonly used demand systems:

Linear Demand System (LDS): Demand for platform 1 given by

$$D^1(P_1, P_2) = \frac{1 - \gamma - P_1 + \gamma P_2}{1 - \gamma^2}, \quad (2)$$

and symmetrically for platform 2. This differentiated products system can be generated by a representative consumer with quadratic/quasi-linear consumer preferences where $\gamma = 0$ implies that demands are unrelated and $1 > \gamma > 0$ corresponds to platforms as imperfect substitutes (see Vives, 2001). Direct calculations show that assumptions A1)-A4) hold (see Schwartz and Vincent, 2017).

Independent Demands (ID): Consumer preferences are quasi-linear

$$u(q_1, q_2, y) = V(q_1) + V(q_2) + y, \quad V' > 0, V'' < 0,$$

and demand for transactions on platform i is given by

$$D^i(P_1, P_2) = V'^{-1}(P_i).$$

In the **ID** case, if merchant profits are concave in prices for each platform use, then A2) is satisfied and, since $\Pi_{ij} = 0$, A3) is also satisfied. Since $r^{ij}(t_j) = 0$, A1) holds trivially. A4) also holds trivially since demand for good i is unaffected by good j .

A variant of the **ID** case corresponds to a model examined in Schwartz and Vincent (2006) where one platform is interpreted as cash and its corresponding fees are fixed at 0 (so one platform is non-strategic).⁹ They examine the fee-setting behavior of the other (card) platform when the merchant must charge equal prices for both means of payments. In the next section, we examine the effects of a similar constraint where *both* platforms can set fees, to the merchant and to consumers.

⁸Chen and Riordan (2015) prove that a similar property holds quite generally when demand is generated by a discrete choice model.

⁹That model is not exactly nested in this one, though, since the quantity of cash sales at equal prices is not required to equal those of the other platform, that is, demands can be asymmetric.

3 No Equilibrium Under No Surcharge Rule(s)

If the merchant accepts both platforms and the NSR applies to both, the merchant's prices for purchases on either platform must be equal, $p_1 = p_2$. We take this restriction as exogenous to the environment.¹⁰ It only has force if a merchant accepts both platforms. Later in this section we consider a NSR effective only for a single platform, i , so the restriction is tantamount to the inequality, $p_i \leq p_j$.

Fix any f_1, f_2 . Given $p_1 = p_2$, total consumer prices can differ solely because of different platform fees to consumers:

$$P_2 = P_1 + f_2 - f_1. \quad (3)$$

Since $m_i = t_i - f_i$, we can represent the strategic choice of a platform equivalently as setting (f_i, m_i) or setting (f_i, t_i) . In what follows, we use the latter representation.

Assumptions A2) and A3) on the merchant's profit function can now be used to derive the impact of a platform's fee structure on sales given a NSR. As long as the merchant continues to accept both platforms, an increase in spread between a platform's cardholder and merchant fees, holding the total fee constant, will increase that platform's sales and benefit that platform.

Lemma 1. *If Platform 1 lowers f_1 by (small) $\delta > 0$ holding t_1 fixed and the merchant accepts the NSR, then sales on platform 1 rise and sales on platform 2 fall.*

Will a merchant accept an increase in a platform fee spread? Under a NSR, the merchant's profit maximization problem can be expressed as the constrained problem

$$\begin{aligned} \max_{P_1, P_2} \quad & \Pi(P_1, P_2; t_1, t_2) & (CP) \\ \text{s.t.} \quad & P_1 + f_2 - f_1 - P_2 = 0, \end{aligned}$$

where $\Pi(P_1, P_2; t_1, t_2)$ is defined in (1). Let the lagrangian associated with (CP) be

$$L(P_1, P_2, \lambda; t_1, t_2, f_1, f_2) = \Pi(P_1, P_2; t_1, t_2) + \lambda(P_1 + f_2 - f_1 - P_2) \quad (4)$$

and denote the solution to (CP) (including the associated lagrange multiplier on the constraint) by $(\hat{P}_1, \hat{P}_2, \hat{\lambda})$. Since the NSR here is an equality constraint, the

¹⁰Our analysis therefore considers sub-games conditional on a NSR being in place (for one platform or both), either contractually or due to transaction costs. As we show, a NSR yields non-existence of pure strategy equilibrium, making it difficult to evaluate platforms' incentives to impose a contractual NSR.

lagrange multiplier can take either sign.¹¹ In the formulation above, $\hat{\lambda} > 0$ implies that, (locally) given (t_1, t_2, f_1, f_2) , the merchant would prefer to charge a higher total price for platform 2 but is prevented from doing so by the NSR.¹² When fees are such that $\hat{\lambda} > 0$, Lemma 2 exploits the envelope theorem to show that, holding t_1, t_2, f_2 fixed, merchant profits increase as f_1 falls. Intuitively, with t_1 fixed, a fall in cardholder fee f_1 requires an equal rise in merchant fee m_1 . These changes raise demand for transactions on platform 1 and also raise the merchant's marginal cost of these transactions. Both effects increase the merchant's unconstrained optimal price for platform 1, relaxing the effect of the constraint and thus benefitting the merchant. The same logic implies that a fall in cardholder fee and rise in merchant fee of platform 2 exacerbate the constraint on the merchant.

Lemma 2. *Suppose $\hat{\lambda}$ is strictly positive (at current fees, the merchant prefers to charge a higher price to platform 2). Holding (t_1, t_2, f_2) fixed, merchant profits increase as f_1 falls. Holding (t_1, f_1, t_2) fixed, merchant profits decrease as f_2 falls.*

For a given profile of fees, (t_1, t_2, f_1, f_2) , if a merchant accepts only a single platform i , a NSR has no force and neutrality implies that the merchant's maximal profit is given by

$$\Pi^S(t_i) \equiv \max_P (P - t_i)\bar{D}^i(P). \quad (5)$$

The envelope theorem implies that stand-alone profits are decreasing in t_i , therefore, if a merchant avoids a NSR by accepting only one platform, it will accept the platform with the lower total fee. Thus, when offered fees (t_1, t_2, f_1, f_2) and a NSR, the merchant's best alternative to accepting both platforms is either its outside option, 0, or $\Pi^S(\min\{t_1, t_2\})$. This feature along with Lemmas 1 and 2 imply that at any pure strategy equilibrium where the NSR is accepted, the NSR cannot bind on the merchant:

Lemma 3. *In a pure strategy equilibrium in platform fees when the merchant accepts a NSR from both platforms, the lagrange multiplier in the solution to (CP) satisfies $\hat{\lambda} = 0$.*

The forces at work in Lemmas 1 through 3 are as follows. Whenever a NSR binds on the merchant, two properties will hold. First, conditional on merchant acceptance,

¹¹Since the constraint is linear, the usual constraint qualification in the Lagrange Theorem is satisfied.

¹²Strict quasi-concavity of Π , from assumption A2) also implies the merchant's optimal p_2 without the NSR is strictly higher than the constrained merchant price, $\hat{P}_2 - f_2$.

either platform gains by raising its merchant fee and cutting its user fee equally. With this change, platform i 's total fee remains constant, but expanding the spread ($m_i - f_i$) increases transactions on platform i : its users' willingness to pay increases by the cut in f_i whereas the merchant raises price by less since the price rise must apply also to the other platform's users. Second, expanding the fee spread will raise the merchant's preferred price for i 's transactions and, hence, *mitigate* the NSR constraint if and only if the merchant initially preferred to charge a lower price for i than for the other platform because the merchant's preferred prices would move closer. An increase in the fee spread of platform i therefore relaxes the NSR constraint, benefitting the merchant. It follows that in any pure strategy equilibrium a NSR cannot bind; if it did, the platform with the lower fee spread could profitably deviate by increasing its fee spread while retaining merchant acceptance.

Lemmas 1 through 3 can now be combined to obtain our main result.

Proposition 1. *In the Simultaneous Fees Game under Assumptions A1) through A4), if an NSR is present for both platforms, there is no equilibrium in pure strategies.*

To understand this result, observe first that in any candidate pure strategy equilibrium both platforms must be accepted. If only platform j were accepted, platform i could deviate by mimicking j 's fees, hence the NSR would not bind. The merchant then would strictly prefer to accept both platforms since they are differentiated, and this differentiation implies positive profit for the newly accepted platform.

Next, consider a candidate equilibrium with both platforms accepted. Given the platform fees, the NSR again cannot bind on the merchant (Lemma 3). Therefore, the fee structure is neutral and only the total fees (t_1, t_2) matter. Suppose total fees are equal. Since the platforms are symmetric and differentiated, the merchant strictly prefers to accept both instead of just one. Thus, either platform could profitably deviate by raising its fee spread slightly until the NSR starts to bind (Lemma 1) without being dropped by the merchant, breaking the candidate equilibrium. Now consider unequal total fees, say $t_2 > t_1$. In a candidate equilibrium, platform 1's fee must weakly exceed its best-response to 2's fee, $t_1 \geq r_1(t_2)$: if $t_1 < r_1(t_2)$, then 1 could profitably raise t_1 and the merchant would continue accepting it given $t_2 > t_1$. In turn, $t_1 \geq r_1(t_2)$ implies $t_2 > r_2(t_1)$, so that platform 2 would strictly prefer to cut its fee:

$$t_2 - r_2(t_1) > t_1 - r_2(t_1) \geq t_1 - r_1(t_2) \geq 0,$$

where the first inequality follows given $t_2 > t_1$, the second follows since $r_2(t_1) \leq$

$r_1(t_2)$ given $t_2 > t_1$, platform symmetry and the assumption $r^{j'}(\cdot) \geq 0$, and the final inequality was explained earlier.

To this point, we assumed a NSR for both platforms. Suppose instead only one platform operates under a NSR. For example, Visa and MasterCard dropped their no-steering rules in a 2010 settlement with the Department of Justice, while Amex litigated and retained its rules. The merchant's profit maximization problem is an obvious modification of (CP) where the constraint becomes the inequality constraint (assuming the NSR is present only for platform 2):

$$P_2 \leq P_1 + f_2 - f_1$$

and the lagrange multiplier in (4) must be non-negative. Proposition 2 demonstrates that with a single NSR, equilibrium existence continues to fail.

Proposition 2. *In the Simultaneous Fees Game under Assumptions A1) through A4), if the NSR is present for only one platform, there is no equilibrium in pure strategies.*

The argument is similar to the logic underlying Proposition 1. If a NSR binds in equilibrium, it must clearly bind on the price of the platform with the NSR, say, platform 2. Platform 1 then has the incentive described by Lemma 1 to increase its fee spread and thereby increase transactions, and the merchant would accept such a change (Lemma 2). This implies that the NSR cannot bind in equilibrium (Lemma 3) and the proof from Proposition 1 now proceeds in the same fashion.

Propositions 1 and 2 show that pricing restraints such as no surcharge rules raise important questions about the stability of a pricing game with competing platforms. In both cases, the logic of the argument is that, with the NSR, no matter the size of the gap between merchant and cardholder fees, if competing platforms have equal gaps (so the NSR is not locally binding) each platform wants to increase its gap and divert sales from the rival.

The non-existence proof emphasizes the incentives of platforms to increase the gaps in their fees relative to their rivals. However, this property does not imply one should expand an ever-expanding cycle of increased merchant fees and increased customer rebates. For some fee profiles, a platform may actually gain more by *decreasing* its gap when a rival's gap becomes too large. Consider a case where platform 1 has both a higher total fee and a higher gap relative to platform 2. By Lemmas 1 and 2, Platform 1 has an incentive to increase this gap until the merchant is just indifferent between accepting both platforms with the NSR or rejecting platform 1 and serving

only customers on platform 2 in which case the NSR has no bite. At that profile of fees, however, platform 2 would typically prefer to *reduce* its fee gap slightly since doing so exacerbates the NSR constraint and induces the merchant to reject platform 1 (since it has the higher total fee) and induce single-homing. Were this to occur, because of neutrality, platform 2's fee gap becomes irrelevant. Because platform 2 has a strictly lower gap, the effect of this slight deviation results in a discrete increase in platform 2's sales and therefore a discrete increase in profits. While this countervailing incentive does not restore equilibrium, of course, it indicates that non-existence does not necessarily imply a never-ending race by platforms to raise their fee gaps.

4 Carduser Fees Set After Merchant Prices

One candidate explanation for non-existence of pure strategy equilibrium is that the strategic structure has been misspecified. We explore this possibility by considering a related game where the timing is modified so that platforms set fees to card users after setting fees to the merchant. The underlying logic is robust to this new specification: the incentive remains for each platform to exploit a NSR by increasing its fee spread so as to shift transactions to itself from the rival and non-existence of pure strategy equilibrium persists. However, we also demonstrate that a mixed-strategy equilibrium can exist where the merchant single-homes but randomizes over which platform it accepts.

Beyond providing a robustness check on the results of Section 3, the alternative timing structure where platforms choose cardholder fees *after* the merchant sets its price may better reflect certain economic situations. If contracts between platforms and merchants and platforms and consumers are relatively long-term, de facto or de jure, while merchant prices can be altered more quickly and easily, the simultaneous pricing game may be a close representation of the interactions among agents. In other circumstances, however, contracts between merchants and platforms may extend over a longer period than contracts between platforms and cardholders and, furthermore, the true fees between cardholders and platforms are not likely to be known (and credibly committed) to merchants before they set their prices. If so, then any initial cardholder fees are not generally sequentially rational.

A timing structure that more accurately captures these features is reflected in the following *Sequential Fees Game*:

- 1) Both platforms simultaneously select merchant fees, m_i .

- 2) The merchant observes both fees and accepts both, one or neither platform.
- 3) For each accepted platform, i , the merchant sets a price, p_i , potentially subject to restrictions.
- 4) Merchant price(s) are observed and each accepted platform, i , sets cardholder fee, f_i .
- 5) For each accepted platform, i , consumers observe (f_i, p_i) : If both platforms are accepted, transactions via platform i are given by $D^i(p_1 + f_1, p_2 + f_2)$; If only platform i is accepted, its transactions are given by $\bar{D}^i(p_i + f_i)$ and none occur via platform j .

In this alternative pricing game, given the merchant fees set by the platform and the subsequent merchant prices, the two platforms play a subgame in cardholder fees, f_1, f_2 . In order to apply the natural solution concept, subgame perfection, we need to determine how the equilibria of these subgames vary with the the prior selected fees and prices, (m_1, m_2) and (p_1, p_2) . This requirement restricts our ability to provide general results; however, the case of linear demands (**LDS**) is tractable and offers useful insights. In particular, merchant and platform best responses with a NSR mirror in many ways those of the simultaneous structure and indicate that non-existence again emerges.

4.1 Equilibrium Behavior in the Continuation Game

With or without the NSR, the subgame perfect equilibrium is obtained by first finding the equilibrium of the subgame where platforms select cardholder fees given merchant prices and merchant fees, (m_1, m_2) . Anticipating these equilibrium fees and given any pair of merchant fees: if there is no NSR and both platforms are accepted, the merchant then selects (p_1, p_2) ; if there is an NSR and both platforms are accepted, the merchant selects a single price, p ; if the merchant accepts only a single platform, i , the merchant selects p_i . Anticipating this behavior, the platforms select (m_1, m_2) .

In the **LDS** model, platform demand is given by (2). Using $P_i = p_i + f_i$, we can generate platform best responses in f_i as

$$f_i(f_j) = (1 - \gamma + \gamma p_j - p_i - m_i + \gamma f_j)/2$$

This is a familiar linear game in strategic complements (see for example Vives, 2001

pp. 159-160) and the equilibrium in cardholder fees satisfies

$$f_i(p_1, p_2, m_1, m_2) = \frac{2A_i + \gamma A_j}{4 - \gamma^2}, \quad (6)$$

where

$$A_i \equiv 1 - \gamma + \gamma p_j - p_i - m_i.$$

Setting $p_1 = p_2$ yields the equilibrium cardholder fees with a NSR in place and setting $\gamma = 0$ yields the fees with a single platform. With this equilibrium behavior, a merchant then selects prices to maximize profits. This analysis enables us to characterize the continuation equilibrium in any subgame given merchant fees, (m_1, m_2) and a merchant's acceptance decision over platforms.

No NSR. In a market with no NSR, neutrality implies that quantities and profits depend only on the total fees, (t_1, t_2) . To see this, fix any m_1, m_2 . Suppose p_1, p_2, f_1, f_2 are the equilibrium prices that maximize merchant profits given that the subsequent f_i are determined by (6). The corresponding quantities are $q_i = D^i(p_1 + f_1, p_2 + f_2)$. Now suppose a different pair of merchant fees, \tilde{m}_1, \tilde{m}_2 , are offered. If the merchant sets prices $\tilde{p}_i = p_i + (\tilde{m}_i - m_i)$ and the platforms each set cardholder fees $\tilde{f}_i = f_i - (\tilde{m}_i - m_i)$, then quantities and platform and merchant profits are the same as in the original equilibrium and therefore, this new profile of price and fees forms an equilibrium with the same outcome. This implies that even though platforms set merchant fees first, the outcome ultimately mirrors a vertical chain where the merchant acts as an upstream price-setter, setting p_1, p_2 and platforms then react in an imperfectly competitive way. The equilibrium margins and quantities are then¹³

$$p_i - m_i = \frac{1}{2}, \quad q_i = \frac{1}{2(1 + \gamma)(2 - \gamma)}.$$

The optimal merchant pricing then yields platform margins as

$$f_i + m_i = \frac{1 - \gamma}{2(2 - \gamma)}.$$

Observe that, consistent with neutrality, platform margins are independent of the initial stage offer of merchant fees, (m_1, m_2) , and platform margins vanish as the degree of differentiation between platforms vanishes (γ approaches one).

Single Platform. If the merchant accepts only a single platform i , the NSR is irrelevant and once again neutrality implies that quantities and profits depend only

¹³All calculations for this section are available from the authors.

on the total fee, t_i . The resulting game is one with an upstream supplier, in this case, the merchant, and a downstream firm, platform i . This is a familiar vertical chain with linear demand and double marginalization because of linear pricing. The merchant's subsequent profits are independent of the merchant fee, m_i , agreed to in the first stage. For the **LDS** model, merchant profits are $\frac{1}{8}$ while the profits of the accepted platform i are $\frac{1}{16}$.

NSR in Force, Two Platforms. The equilibrium of the merchant pricing subgame is computed by setting $p_1 = p_2 = p$, determining equilibrium f_i and solving the merchant's maximization problem in p . This yields an equilibrium common price

$$p(m_1, m_2) = \frac{1 + m_1 + m_2}{2},$$

and quantity:

$$q_i = \max\left\{0, \frac{1}{2(2 - \gamma)(1 + \gamma)} + \frac{m_i - m_j}{2(2 + \gamma)(1 - \gamma)}\right\}. \quad (7)$$

As in the initial game with alternative timing, when a NSR is in place, sales on platform i increase in the difference between platform i 's merchant fee and that of its rival (Lemma 1).¹⁴

The equilibrium prices imply that, under a NSR, merchant profits are *decreasing* in the difference in merchant fees (by the same logic as in Lemma 2 of Section 3):

$$\frac{1}{2(2 - \gamma)(1 + \gamma)} - \frac{(m_1 - m_2)^2}{2(2 + \gamma)(1 - \gamma)}. \quad (8)$$

Using the equilibrium price and *cardholder* fees, the profit margin of each platform for a given profile of merchant fees is

$$f_i + m_i = \frac{1 - \gamma}{2 - \gamma} + (m_i - m_j) \frac{1 + \gamma}{2 + \gamma}. \quad (9)$$

Platform i 's margin and sales increase in $m_i - m_j$, therefore, its profits increase in this difference as well.

4.2 Behavior in the Full Game

These properties of the continuation game immediately imply a result mirroring Propositions 1 and 2 for the *Simultaneous Fees Game*.

¹⁴ Intuitively, if $m_i > m_j$, then platform i will have a greater incentive to increase sales, leading it to subsequently choose $f_i < f_j$. With this profile of fees, the NSR constraint binds on the merchant for platform i 's transactions.

Proposition 3. *In the Sequential Fees Game with linear demands, if an NSR is present, there is no pure strategy equilibrium.*

We can, however, construct a mixed strategy equilibrium in this game. With a single accepted platform, the Sequential Pricing Game collapses to effectively a two-stage pricing game where the merchant sets price and the platform then sets an optimal cardholder fee (and therefore optimal total fee, t_i). Define Δ^* to be the maximum difference in platform fees to the merchant such that the merchant is just willing to accept both platforms with a NSR compared to accepting a single platform (yielding profit $\frac{1}{8}$). Using (8), this implies

$$\Delta^* = \left(\frac{(1-\gamma)(2+\gamma)}{(2-\gamma)(1+\gamma)} - \frac{(1-\gamma)(2+\gamma)}{4} \right)^{1/2}$$

Note that as γ approaches 1 (perfect substitutes), Δ^* approaches zero. Economically, as the platforms become closer substitutes, the merchant's incremental profit from accepting a second platform decreases. Therefore, to maintain the merchant's willingness to accept a second platform under a binding NSR, the burden of the NSR must be eased, requiring a smaller gap between the platforms' merchant fees.

As platforms become close enough substitutes, there is a mixed strategy equilibrium with a NSR such that the merchant accepts only a single platform, each with equal probability. In such an equilibrium, the platforms exploit the NSR to weaken the strong competition between them by inducing the merchant to single-home:

Proposition 4. *Suppose $\hat{m}_i > \hat{m}_j + \Delta^*$. In the Sequential Fees Game with linear demands and a NSR present for both platforms, as γ approaches 1, it is an equilibrium for platforms to offer (\hat{m}_1, \hat{m}_2) along with a NSR. The merchant adopts a single platform, rejecting each platform with equal probability.*

The logic for Proposition 4 is as follows. By accepting a single platform, the merchant renders a NSR irrelevant. Moreover, if a single platform is accepted, neutrality implies that only that platform's total fee matters, and platform symmetry implies the total fee would be the same whichever platform is accepted. (Regardless of a platform's *merchant fee*, the platform's unique equilibrium *total fee* is determined by its subsequent choice of cardholder fee, set after the merchant's price to consumers.) Thus, the merchant is indifferent between the platforms, justifying randomization over acceptance.¹⁵ The merchant prefers this to accepting both platforms with a

¹⁵By contrast, in the Simultaneous Fees Game each platform commits initially to both fees, hence the mixed-strategy equilibrium is ruled out: either platform would undercut the other's total fee slightly and be accepted with probability one.

NSR when their merchant fees differ enough, because the merchant’s preferred prices to consumers will then differ sufficiently that satisfying the NSR becomes too onerous. By adopting a NSR with sufficiently disparate merchant fees, the platforms therefore can ensure that only one of them will be accepted. Finally, when platforms are sufficiently close substitutes, their margins will be arbitrarily small if both are accepted (which could be achieved by offering similar merchant fees), whereas if only one is accepted, its profit is positive and independent of the degree of substitutability.

Some anecdotal evidence is consistent with the mixed-strategy outcome, insofar as several large merchants accept only a single card. One example is the major retailer Costco, that historically accepted only a single card, originally Discover, then American Express prior to 2014, and more recently Visa.¹⁶ However, many merchants still accept multiple cards even under NSR restrictions – placing the broader empirical plausibility of the mixed-strategy equilibrium into question.

Additionally, the mixed-strategy equilibrium requires strong modelling assumptions for it to hold. It requires platforms to be only very slightly differentiated, it requires a very rigid timing structure in platform pricing (a similar type of equilibrium is not supported in the simultaneous pricing version of Section 3), and it assumes that a platform ignores the sequentially rational option of re-pricing its fees if it ends up not being selected by a single-homing merchant. While this equilibrium is intriguing and may apply in certain settings, we do not think it offers a convincing explanation for behavior by card platforms more generally. In the next section we return to the issue of pure strategy equilibrium, including its treatment in prior work and potential extensions.

5 Discussion

To our knowledge, the equilibrium existence issue has been raised only in a brief treatment by Boik and Corts (2016). Their core model assumes that platforms set

¹⁶ Another example is Neiman Marcus: prior to 2011 it accepted only American Express out of the four major card platforms. <http://www.wsj.com/articles/SB10001424052970204505304577000103355671444>. The Second Circuit Court opinion noted that almost one-third of all merchants that accept cards do not accept American Express. Details on these single-homing examples can be found at <https://consumerist.com/2014/11/06/costco-may-finally-start-accepting-something-other-than-american-express/> and <http://www.usatoday.com/story/money/2016/06/13/walmart-canada-will-stop-accepting-visa-cards/85826704/>.

fees only to the merchant, a simplification that lets them tackle various issues we ignore, including a NSR’s effect on entry by a lower-quality, lower-priced platform. In an appendix, however, they provide a linear demand example where platforms also offer rebates to their end users, and posit that a pure strategy equilibrium does not exist, for the same basic force we identify: each platform vies to increase the spread between its merchant fee and user rebate. We prove the non-existence result in a more general environment, show it is robust to alternative timing, and, most importantly, we explicitly incorporate the merchant’s acceptance behavior.¹⁷

Why have some other analyses of platform pricing under a NSR not encountered our equilibrium non-existence? Rochet and Tirole (2002) and Schwartz and Vincent (2006) consider a single card platform, with the NSR applying to a ‘passive’ alternative service, cash. Competing platforms with two-sided pricing under a NSR are analyzed by Rochet and Tirole (2003), Guthrie and Wright (2007), and Edelman and Wright (2015).¹⁸ In those models, there are, effectively, brakes on each platform’s incentive to expand its fee spread – that is, to increase its merchant fee and offer higher rebates to its cardholders.

In Rochet and Tirole (2003), merchants’ decisions to accept a payment card depend solely on the card network’s merchant fee and are unaffected by the fee to cardholders, so raising the former while lowering the latter equally will cause some merchants to refuse the card and platform transactions will not necessarily rise. Networks therefore lack the persistent incentive to increase their fee spread that arises in our model.

Guthrie and Wright (2007) assume that merchants do internalize buyers’ benefits from card use, which biases platforms to tilt the fee structure against merchants as in our model. However, there is a limit on the maximal rebate to card users, driven the assumption that the merchant must charge the same price to cash users and card users. Consumers have unit demands and are heterogeneous in their value for card use, hence to increase a card platform’s transactions requires convincing more customers to convert from using cash to cards. If the merchant must charge the same price not only to all card users but to cash users as well, it becomes increasingly

¹⁷One might conjecture that the merchant’s acceptance constraint puts a lid on platforms expanding their fee spread. The logic in Lemmas 1 and 2 refutes this intuition, however. The merchant’s acceptance behavior also is central for our second main result, on the mixed-strategy equilibrium under sequential timing.

¹⁸Caillaud and Julien (2003), another pioneering analysis of competing platforms, assumes that each platform can only charge a total fee, not separate fees to each side.

costly to draw in cash users, and simultaneously increasingly costly for platforms to induce merchants to raise the common price to remaining cash customers.

This explanation is not entirely convincing. Non-card payment modes have been exempted from card networks' no-surcharge rules for some time (e.g., Amex exempted electronic funds transfer, cash, and check. See Footnote 3.) Potentially, exogenous factors might limit merchants' ability to set differential prices for non-card customers. But such exogenous price coherence is at odds with significant efforts by card networks to maintain their contractual no-surcharge rules, and with the fact that surcharging of card transactions has often occurred once permitted (Bourguignon, Gomes and Tirole, 2014). When, instead, the NSR constraint governs only the merchant's prices to users of the same payment mode (cards), leaving the merchant free to charge separate prices to users of other means of payment, this limiting effect is absent and the source of our equilibrium non-existence emerges.

Edelman and Wright (2015) assume that increased expenditure on cardholder rewards by a platform delivers benefits to its users at a diminishing rate. Thus, a platform cannot increase its transaction volume while maintaining its profit margin by raising its merchant fee and rewards equally. This caps the benefit it will offer to users and the fee to merchants. The diminishing returns feature may hold for certain cardholder rewards, such as airline miles, but does not seem plausible for cash-back rebates. Indeed, Edelman and Wright explicitly rule out cash rebates.

We briefly investigated three separate extensions to our underlying model to see if some plausible modifications to the environment would help restore pure strategy equilibrium (see Online Appendix). For conciseness, all three models restricted attention to the special case of our original model with independent platforms each facing linear demand (see Section 2, **LDS** with $\gamma = 0$) and assume simultaneous fee setting as in Section 3 of the paper. In all three extensions, the logic underlying Lemmas 1 and 2 of the paper continues to apply and destroys the possibility of symmetric pure strategy equilibria. At any symmetric profile of fees, the NSR does not bind on the merchant, hence a slight increase in the fee gap by one platform imposes a negligible cost on the merchant but yields a significant gain in profits to the platform, thereby prompting a deviation from the candidate equilibrium.

Costly Rebates. The first extension follows the spirit of Edelman and Wright (2015) by assuming that a platform incurs additional costs when providing rebates. However, in keeping with the nature of cashback rebates, the costs are assumed to be linear rather than increasing: each dollar of cashback costs a platform $1 + a$ dollars, with $a \geq 0$. For a sufficiently large, no cash rebates would be offered, which contradicts

observed practice. Whereas for a below some threshold, it remains profitable for each platform to increase its fee spread starting from a symmetric equilibrium. This extension highlights that costly rebates alone are not sufficient to restore equilibrium – to put a brake on rebates the marginal cost would have to be increasing.

Multiple Merchants and a Common Consumer Rebate. In practice, card platforms can charge different fees to merchants based on observed characteristics such as their specific industries, but may be less able to vary the size of consumer rebates based on where purchases are made, for marketing reasons such as reducing complexity for consumers.¹⁹ We model this scenario by considering two separate monopolized merchant markets with each platform able to charge different fees to these merchants but constrained to offer uniform cashback rebates to its cardholders for all their transactions, e.g. 2% cash-back anywhere. The logic of our paper directly rules out an equilibrium where both platforms charge the same fees in each market. We also examined the possibility of a ‘complementary symmetric’ equilibrium where the NSR binds for a different platform in each market. In this case, the candidate equilibrium is broken by invoking the other best response incentive identified in the paper. In the market where the NSR is binding on its rival, a platform has the incentive to reduce its fee gap and induce that merchant to single-home to it, thus generating a discrete increase in sales in that market.

Merchants with Heterogeneous Benefits. We adopted a feature used in some literature on two-sided markets by assuming the merchant has some privately known benefit from card transactions. Merchant participation in a platform is now determined by a distribution of these benefits – a lower bound on such benefits represents the lowest type of merchant that would join. Intuitively, no symmetric pure strategy equilibrium can exist in this case either. At a symmetric equilibrium, the NSR does not bind on any merchant, including the merchant on the margin between participating and not. A slight increase in the fee spread by one platform, holding the total fee fixed, yields a first-order gain in revenue for the platform but imposes only a second-order cost on a merchant thus having a negligible impact on merchant participation.

6 Conclusion

The factors invoked in prior literature to yield a pure strategy equilibrium (PSE) with competing symmetric platforms under NSRs are not entirely convincing when plat-

¹⁹ We thank a referee for suggesting this extension.

forms offer cash-back rebates, and the extensions we considered also do not resolve the issue. Our apparently robust non-existence result poses a challenge to economists who believe that a PSE exists. Of course, we may have missed some key component that would induce a PSE. A potential area to explore is merchant competition (instead of our monopoly case), though it is not obvious why that would eliminate platforms’ incentives to increase their fee spreads. Additionally, we assumed symmetric platforms, whereas platforms such as Amex and Visa appear asymmetric. Richer models could explore whether platform asymmetries could yield a PSE.

Our analysis also suggests some avenues for empirical studies. For example, the mixed-strategy equilibrium where the merchant is induced to accept a single platform is driven by platforms adopting very different fee profiles in conjunction with no surcharge rules. A study of the incidence of single-homing versus multi-homing and their relationship to total fees as well as the gaps in fees would be informative.

Additionally, several commenters on our paper have claimed that, in fact, credit card fees are much more stable than a model with no pure strategy equilibrium would predict. This assertion should be empirically confirmed – we do not know of any direct study of the stability or variability of platform fees and, in principle, the question could be examined. Our model also provides some empirical guidance. The logic of our proofs focuses on platforms’ best responses. Platform best responses indicate that, in or out of equilibrium, when the gap between merchant fees and card-holder fees is small, platforms have an incentive to leap-frog each other’s gap. On the other hand, when these same gaps are quite different, the platform with the smaller gap (and smaller total fee) has an incentive to reduce its gap further and induce merchants to single-home. Should a rich time series of competing platform fees be available, these reactive responses could be looked for.

Finally, while our analysis is motivated most closely by the credit card sector, its relevance may extend to other multi-sided platforms, such as online booking sites, insofar as they offer cash rebates to their users while merchants are subject to no surcharge restraints.

7 Appendix

7.1 Proof of Lemma 1

Proof. Fix t_1, t_2, f_1, f_2 . Set $d = f_1 - f_2$ and note from (3) that $(\hat{P}_1, \hat{P}_1 - d)$ are thus the merchant’s optimal price(s) under the NSR at these fees. This pair is unique by

A2). By definition,

$$\Pi_1(\hat{P}_1, \hat{P}_1 - d) + \Pi_2(\hat{P}_1, \hat{P}_1 - d) = 0. \quad (10)$$

Now suppose platform 1 changes its cardholder fee to $f_1 - \delta$ and suppose the merchant chose to raise the common price by δ so that the new cardholder prices become $(\hat{P}_1, \hat{P}_1 - d + \delta)$. That is, the total cardholder price for platform 1 stays constant, while the total price for platform 2 goes up by δ because the common merchant price is raised by δ . Observe that by the fundamental theorem of calculus,

$$\Pi_1(\hat{P}_1, \hat{P}_1 - d + \delta) = \Pi_1(\hat{P}_1, \hat{P}_1 - d) + \int_0^\delta \Pi_{12}(\hat{P}_1, \hat{P}_1 - d + \tau) d\tau$$

and

$$\Pi_2(\hat{P}_1, \hat{P}_1 - d + \delta) = \Pi_2(\hat{P}_1, \hat{P}_1 - d) + \int_0^\delta \Pi_{22}(\hat{P}_1, \hat{P}_1 - d + \tau) d\tau$$

Summing the two expressions and using the first order conditions from (10) to eliminate the first term on the right side of each equation, gives

$$\Pi_1(\hat{P}_1, \hat{P}_1 - d + \delta) + \Pi_2(\hat{P}_1, \hat{P}_1 - d + \delta) = \int_0^\delta \Pi_{12}(\hat{P}_1, \hat{P}_1 - d + \tau) + \Pi_{22}(\hat{P}_1, \hat{P}_1 - d + \tau) d\tau.$$

Assumption A3) implies this is negative. Since the left side is the derivative of merchant profits with respect to price, this means the merchant prefers to lower prices from this point.

Consider the price profile, $(\hat{P}_1 - \delta, \hat{P}_1 - d)$ so that the cardholder price for platform 2 remains the same but for platform 1 falls by δ . A similar argument to above yields

$$\begin{aligned} \Pi_1(\hat{P}_1 - \delta, \hat{P}_1 - d) + \Pi_2(\hat{P}_1 - \delta, \hat{P}_1 - d) &= \int_0^{-\delta} \Pi_{11}(\hat{P}_1 + \tau, \hat{P}_1 - d) + \Pi_{21}(\hat{P}_1 + \tau, \hat{P}_1 - d) d\tau \\ &= - \int_{-\delta}^0 \Pi_{11}(\hat{P}_1 + \tau, \hat{P}_1 - d) + \Pi_{21}(\hat{P}_1 + \tau, \hat{P}_1 - d) d\tau. \end{aligned}$$

The second line just changes the direction of integration and therefore is multiplied by -1 . Assumption A3) implies this is positive. Since under the NSR, the derivative of merchant profits with respect to P_1 is positive at the lower end of the interval, $[\hat{P}_1 - \delta, \hat{P}_1]$ and negative at the upper end and since strict quasi-concavity implies that the merchant profits are single-peaked along any ray in (P_1, P_2) , this implies that the new optimal cardholder fees involve a lower total price for platform 1 and a higher total price for platform 2, so the gross substitutes property implies that platform 1 use rises and platform 2 use falls.

□

7.2 Proof of Lemma 2

Proof. Partially differentiate the expression in (4) with respect to f_1 and apply the envelope theorem. \square

7.3 Proof of Lemma 3

Proof. Suppose, for example, $\hat{\lambda} > 0$. Platform 1 can lower its cardholder fee f_1 while keeping t_1 fixed (that is, raise m_1 by the same amount). This has no effect on the merchant's outside option, $\Pi^S(t_i)$, and by Lemma 2, this raises merchant profits, so the merchant would continue to accept both platforms. By Lemma 1, platform 1 profits rise. A parallel argument holds for platform 2 if $\hat{\lambda} < 0$. \square

7.4 Proof of Proposition 1

Proof. Observe that it cannot be an equilibrium for the merchant to accept a single platform. Suppose it was – the merchant accepted Platform 1 at fees (m, f) and total fee $t = m + f$. Let the resulting price be p with consumer price $P = p + f$ generating merchant profits

$$(P - t)\bar{D}(P) = (P - t)D^1(P, P_2(P)).$$

Suppose Platform 2 also offered fees (m, f) . If the merchant accepted and responded with consumer prices (P, P) , then its margins would remain the same but total demand would rise. To see this, note that

$$D^1(P, P_2(P)) = D^1(P, P_2(P)) + D^2(P, P_2(P))$$

by definition of $P_2(P)$. At consumer prices (P, P) , total demand is $D^1(P, P) + D^2(P, P)$. The fundamental theorem of calculus implies that

$$D^1(P, P) + D^2(P, P) = D^1(P, P_2(P)) + D^2(P, P_2(P)) + \int_{P_2(P)}^P \left(\frac{\partial D^1(P, x)}{\partial x} + \frac{\partial D^2(P, x)}{\partial x} \right) dx.$$

Since $P < P_2(P)$, and imperfect substitutes implies $\frac{\partial D^1(P, x)}{\partial x} + \frac{\partial D^2(P, x)}{\partial x} < 0$, this yields

$$D^1(P, P) + D^2(P, P) = D^1(P, P_2(P)) + D^2(P, P_2(P)) - \int_P^{P_2(P)} \left(\frac{\partial D^1(P, x)}{\partial x} + \frac{\partial D^2(P, x)}{\partial x} \right) dx.$$

The second term is strictly positive, so demand strictly increases. Thus, if $t > 0$, Platform 2 could mimic Platform 1, the merchant would strictly prefer to accept

both, and Platform 2's profits would be strictly positive. Continuity implies that merchant profits are also strictly higher for t_2 slightly higher than t , so even if $t = 0$, Platform 2 could offer a slightly higher total fee and the merchant would also accept.

We next establish an additional lemma showing that if a platform charges strictly lower total fees under a NSR than its rival, the merchant would continue to accept if the platform raised its total fees slightly:

Lemma 4. *Suppose the profile of fees (t_1, t_2, f_1, f_2) are such that $\hat{\lambda} = 0$ and $t_1 < t_2$. If the merchant accepts the NSR and there are positive sales through both platforms, merchant profits decline in t_i . For a small increase in t_1 , the merchant prefers the NSR to rejecting platform 2.*

Proof. Clearly if the merchant were to reject a platform, it would reject the high total fee platform 2. Recall that $(\hat{P}_1, \hat{P}_2, \hat{\lambda})$ is the solution to (CP) using the lagrangian in (4). The assumption that $\hat{\lambda} = 0$, implies the NSR does not bind and that the derivative of the lagrangian with respect to P_i equals Π_i at (\hat{P}_1, \hat{P}_2) and must equal zero. Assumption A2) then implies that (\hat{P}_1, \hat{P}_2) also represents the optimal prices given (t_1, t_2, f_1, f_2) under no NSR. The envelope theorem implies that the change in merchant profits with respect to t_i is (by partially differentiating (4) with respect to t_i)

$$-D^i(\hat{P}_1, \hat{P}_2) < 0.$$

Let \bar{P}_1 be the optimal price offered by the merchant if it rejected platform 2 and sold only through platform 1. Again, the envelope theorem implies that the change in merchant profits with respect to a small increase in t_1 is

$$-\bar{D}^1(\bar{P}_1).$$

Assumption A4) implies that this decline in profits is more than the decline in profits under the NSR, therefore if the low total fee platform 1 raised t_1 slightly (which in this case implies raising m_1 as we are *partially* differentiating) the merchant would continue to accept both platforms and the NSR instead of accepting only platform 1. \square

Lemma 3 implies that in any equilibrium, $\hat{\lambda} = 0$, so the NSR does not bind.

First, suppose $t_1 = t_2$ in an equilibrium and let p be the uniform merchant price. Since the NSR does not bind, this must imply $f_1 = f_2$. Suppose not. Consider the equilibrium prices under no NSR. A2) implies this is unique in (t_1, t_2) and neutrality implies that equilibrium quantities and, therefore, consumer prices depend only

(t_1, t_2) . Symmetry and the hypothesis that $t_1 = t_2$ imply that $p + f_1 = P_1 = P_2 = p + f_2$. This implies $f_1 = f_2$. Since the platforms are not perfect substitutes, then the merchant does strictly better accepting the NSR and both platforms than rejecting one platform. But, then Lemma 1 implies each platform would increase profits by lowering f_i holding t_i fixed so this cannot be an equilibrium.

Therefore, suppose $t_1 < t_2$ and $\hat{\lambda} = 0$. Lemma 1 implies that the merchant's participation constraint must bind, otherwise, each platform has an incentive to raise m_i and lower f_i . We show that at least one platform will wish to change its t_i thus contradicting the assumption of an optimum.

Lemma 4 implies that $t_1 \geq r^1(t_2)$. To see this, note that if $t_1 < r^1(t_2)$, Lemma 4 shows that platform 1 can raise t_1 and the merchant will still accept the NSR. Since platform profits are strictly quasi-concave in t_1 , platform 1 will prefer the higher t_1 so we cannot be at an equilibrium. Therefore, $t_1 \geq r^1(t_2)$.

Symmetry and A1) imply that the equilibrium of the platform game with no NSR is symmetric, say (\hat{t}, \hat{t}) . Assumption A1) implies $t_2 > t_1$ and $t_1 \geq r^1(t_2)$ if and only if $t_1 > \hat{t}$. This is because the inverse of platform 1's best response function, $r^1(\cdot)$ has slope greater than one in (t_1, t_2) space and, so, crosses the line $t_1 = t_2$ from below at \hat{t} .

Assumption A1) also implies that $t_1 \geq \hat{t}$ if and only if $t_1 \geq r^2(t_1)$ ($r^2(\cdot)$ crosses the line $t_1 = t_2$ exactly once and does so at \hat{t} from above since $r^{2'}(\cdot) < 1$). Thus, $t_1 > \hat{t}$ if and only if $r^2(t_1) < t_1$. But this then yields, $t_2 > t_1 \geq r^2(t_1)$.²⁰ Since platform 2 profits are strictly quasi-concave, this implies platform 2 would like to lower t_2 and Lemma 4 implies the merchant would continue to accept the NSR with the lower t_2 since the merchant's single-homing option (accepting only platform 1) is unchanged and merchant profits under an NSR increase with a decline in t_2 . \square

7.5 Proof of Proposition 2

Proof. Note that the proof of Proposition 1 applies in this case as well if it can be shown that with a single NSR, $\hat{\lambda} = 0$ is necessary for a pure strategy equilibrium. Therefore, suppose $\hat{\lambda} > 0$ at an equilibrium profile of fees such that the NSR from platform 2 is accepted by the merchant. Lemma 2 implies that merchant profits under an NSR rise as f_1 falls holding all other fees constant, thus the merchant continues to accept both platforms (since t_i is held fixed, its outside option has not changed).

²⁰In Section 3 we obtained the same conclusion exploiting platform symmetry rather than the condition $r^{2'}(\cdot) < 1$.

Lemma 1 implies that since the NSR strictly binds, platform 1 sales rise and platform 2 sales fall as f_1 falls. Thus, it is feasible for platform 1 to lower f_1 and raise m_1 keeping t_1 fixed and its profits would rise. \square

7.6 Proof of Proposition 3

Proof. Equation (7) implies that, assuming the merchant accepts a NSR, profits of platform i increase in $m_i - m_j$. Suppose a NSR equilibrium exists with, say, $m_1 < m_2$. Merchant profits must be weakly greater under the NSR than what could be earned by rejecting one platform, $(\frac{1}{8})$. Equation (8) illustrates that merchant profits with a NSR strictly increase if platform 1 raises m_1 slightly (and therefore, the merchant would continue to accept the NSR) and equation (9) implies that platform 1 profits rise as well. So $m_1 < m_2$ cannot be a best response. Similarly for the case $m_2 < m_1$. If $m_1 = m_2$, equations (8) and (??) imply that the merchant gets strictly higher profits with both platforms than with one (assuming that the platforms are not perfect substitutes, $\gamma < 1$) so each platform could raise its m_i slightly, increase its profits and still have the merchant accept. \square

7.7 Proof of Proposition 4

Proof. If the merchant single-homes, neutrality implies that the continuation game is independent of m_i and merchant profits are the same $(\frac{1}{8})$ no matter which platform it selects to single-home with. Therefore, (conditional on single-homing) randomizing over platforms is a best response for the merchant. By definition of Δ^* and equation (8), a merchant will never accept both platforms with this profile of merchant discounts and a NSR. As γ approaches 1, (9) shows that platform profits approach zero when a NSR with both platforms is accepted for any $|m_i - m_j| \leq \Delta^*$, while platform profits under single-homing, $\frac{1}{16}$, are bounded above zero. The proposed equilibrium, then, offers platforms higher equilibrium profits than either a market with no NSR, or one in which merchant discounts are such that the merchant would accept both platforms and a NSR. \square

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Extensions to ‘Platform Competition With
Cash-back Rebates Under No Surcharge Rules’ –
Costly Rebates and Heterogeneous Merchants

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Online Appendix

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1 Outline

This appendix provides a brief analysis of three extensions to the model of competing platforms with a no-surcharge rule (NSR). It specializes to the case of linear and independent demand for platforms and zero costs for platforms and merchants. Section 2 describes the **common** framework that applies to the extensions. Section 3 reproduces the non-existence of symmetric pure strategy equilibrium result for the case of small to moderate, linear costs of consumer rebates to a platform with a single merchant. Section 4 reproduces the non-existence result for the case where there are two independent monopoly merchant markets (for example, geographic or product markets) and the platforms can charge different merchant fees but are constrained to charge the same consumer fees regardless of which market the consumer purchases in. Section 5 extends the original model to allow for private benefits from card transactions for merchants.

2 Model and Preliminaries

Locally monopolistic merchant(s) are denoted by $j = A, B$. Each platform $i = 1, 2$ simultaneously offers merchants and cardholders fees:

$$\{(m_i^j, f_i^j)\}$$

potentially subject to constraints. For convenience, denote the total fee and fee gap per merchant transaction as

$$t_i^j = m_i^j + f_i^j, \Delta_i^j = m_i^j - f_i^j, i = 1, 2.$$

Thus, this game can be equivalently represented as a game in which each platform simultaneously presents the pair

$$(t_i^j, \Delta_i^j)$$

to each merchant. Merchants then choose whether to accept either or both platforms, choose prices p_i^j for selected platforms, and cardholders then choose quantities q_i^j knowing all prices and fees.

A NSR imposed by both platforms requires a merchant j to charge its customers the same price independent of which platform the customer uses. This implies the constraint,

$$p_1^j = p_2^j$$

on each merchant, j .

Restrict attention to the special case of independent linear demand ($\gamma = 0$):

$$q_i^j \equiv D^i(P_1^j, P_2^j) = 1 - P_i^j,$$

where $P_i^j = p^j + f_i^j$ and assume $c = 0$ for both merchants.

If a merchant chooses to accept a single platform, then equilibrium implies that prices, quantities and merchant profits are given by:

$$\begin{aligned} p_i^j(\Delta_i^j) &= \frac{1 + \Delta_i^j}{2} \\ q_i^j(t_i^j) &= \frac{1 - t_i^j}{2} \\ \pi_i^j(t_i^j) &= \frac{(1 - t_i^j)^2}{4} \end{aligned} \quad (1)$$

reflecting the neutrality result that with a single merchant, an NSR has no bite and total output and profits depend solely on the total fee, t_i^j .

Under a NSR, if both platforms are accepted by merchant j , its profit margin and sales on platform i is given by

$$\begin{aligned} p^j - m_i^j &= \frac{1 - t_i^j}{2} - \frac{\Delta_i^j - \Delta_{i'}^j}{4} \\ q_i^j &= \frac{1 - t_i^j}{2} + \frac{\Delta_i^j - \Delta_{i'}^j}{4}. \end{aligned} \quad (2)$$

Thus, merchant j 's profit margin on platform i sales is decreasing in the difference in the fee gap between platform i and i' and its sales on platform i are increasing in that difference.

Merchant profit with both platforms is

$$\frac{(1 - t_1^j)^2}{4} - \frac{(\Delta_1^j - \Delta_2^j)^2}{8} + \frac{(1 - t_2^j)^2}{4}.$$

If a merchant were to reject a platform, it would reject the platform with the highest total fee (by (1)). Thus, for any profile of fees, $((t_1^j, \Delta_1^j), (t_2^j, \Delta_2^j))$, applying (1), a merchant would accept an NSR and serve both platforms if and only if

$$\frac{(1 - t_i^j)^2}{4} - \frac{(\Delta_1^j - \Delta_2^j)^2}{8} \geq 0 \quad (3)$$

for platform, i such that $t_i^j \geq t_{i'}^j$.

3 Costly Rebates

Suppose there is a single market, so the j superscripts can be dropped. Suppose, further, that for the case of rebates ($f < 0$), in order for a platform to induce its cardholders to behave as if they had a discount of f dollars on a merchant price of p , the platform endured the cost $(1+a)|f|$, $a > 0$. The parameter a could be literally interpreted as a marketing cost incurred by the platform to make its customers aware of the rewards from using its card. It might also approximate an environment where, after a change in rebate, there is a short period of time where customers continue to behave as if they had the original rebates. Therefore, platform i profits are $t_i q_i$ if $f_i > 0$ and $(t_i + a f_i) q_i$ if $f_i \leq 0$. In this section it is demonstrated that no symmetric pure strategy equilibrium with positive sales exists whenever $a < 2$.

Consider a symmetric equilibrium with $f_i > 0$. In this case, Proposition 1 applies. Each platform i has an incentive and ability to increase its sales by holding t_i fixed and increasing Δ_i . Equation (2) implies its sales and profits rise and (3) implies that at a profile of fees $\Delta_1 = \Delta_2$ the merchant would continue to accept both platforms under a NSR. Thus, no such equilibrium is possible.

Suppose, therefore, there is a symmetric equilibrium, $(\bar{t}, \bar{\Delta})$, with $f_i \leq 0$ (rebates). Note that

$$f_i = \frac{t_i - \Delta_i}{2}. \quad (4)$$

The assumption of rebates implies then that $\bar{t} \leq \bar{\Delta}$. If the equilibrium has positive sales then (2) implies $\bar{t} < 1$ and symmetry and (3) imply that the merchant strictly prefers to accept both platforms at the candidate equilibrium. Using (4) and (2), at the proposed equilibrium, platform i profits are

$$(\bar{t} + a \frac{\bar{t} - \bar{\Delta}}{2}) (\frac{1 - \bar{t}}{2}).$$

Consider the following deviation. Let $t_i = \bar{t} + \epsilon$ and $\Delta_i = \bar{\Delta} + 2\epsilon$. (This is accomplished by raising m_i by $3\epsilon/2$ and decreasing f_i by $\epsilon/2$.) Equation (2) yields

$$q_i = \frac{1 - \bar{t} - \epsilon}{2} + \frac{\bar{\Delta} + 2\epsilon - \bar{\Delta}}{4} = \frac{1 - \bar{t}}{2}$$

so this change induces no change in sales on platform i . Platform i 's margin, however, is now

$$\bar{t} + \epsilon + a \frac{\bar{t} - \bar{\Delta} - \epsilon}{2} = \bar{t} + a \frac{\bar{t} - \bar{\Delta}}{2} + (1 - \frac{a}{2})\epsilon.$$

The first expression uses (4) for f_i . (Alternatively, we can see the new margin as

$$\bar{m} + 3\epsilon/2 + \bar{f} - \epsilon/2 - a(\bar{f} - \epsilon/2) = \bar{m} + \bar{f} - a\bar{f} + \epsilon(2 - a)/2.$$

Recall, $a < 2$. This expression implies that for small ϵ , either platform can profitably deviate from the candidate equilibrium. Since $\bar{t} < 1$, (3) is satisfied with strict inequality at $(\bar{t}, \bar{\Delta})$. Continuity then implies there is an ϵ small enough such that the merchant would accept both platforms following the deviation, thus eliminating $(\bar{t}, \bar{\Delta})$ as an equilibrium.

4 Heterogeneous Merchants with a Common Consumer Fee

In this section we return to the case $a = 0$ and analyse the case where there are two markets each with a locally monopolistic merchant, $j = A, B$. Each platform can charge different fees to merchants in each market but is constrained to charge the same fee to consumers no matter which market they buy from. This implies that platform i selects a triple,

$$(m_i^A, m_i^B, f_i)$$

indicating the property that a platform can charge each merchant a specific fee but cannot charge cardholders a fee that varies depending on the merchant that is visited.

Observe that the constraint that only a single cardholder fee can be charged by platform i is captured by the condition:

$$\Delta_i^A - \Delta_i^B = t_i^A - t_i^B. \quad (5)$$

That is, the requirement that card-holder fees be the same across all markets implies that the difference in total fees across markets is the same as the difference in the gap in fees across markets.

The logic from the paper indicates that there cannot be an equilibrium where both platforms charge the same fees in each market: in any such candidate equilibrium, $\Delta_1^B = \Delta_1^A = \Delta_2^B = \Delta_2^A$ and the NSR does not bind on either merchant. Furthermore, the merchant participation constraint (3) is satisfied with strict inequality in each market. Platform i 's profits in market j are given by $t_i^j q_i^j$ (and by symmetry are the same in each market). By (5), platform i can raise Δ_i^A and Δ_i^B by the same small amount, holding t_i^A and t_i^B fixed, induce an increase in sales in both markets on its platform by (2) and the merchants will continue to accept. (This follows the logic of Lemmas 1 and 2). This profitable deviation would break such an equilibrium.

Therefore, posit a profile of fees such that $\Delta_1^A > \Delta_2^A$ and $\Delta_1^B < \Delta_2^B$ with complementary symmetry implying that $\Delta_1^A = \Delta_2^B$ and $\Delta_2^A = \Delta_1^B$ and $t_1^A = t_2^B > t_1^B = t_2^A$.

This implies the NSR binds for platform 1 on merchant A and for platform 2 on merchant B . Also, combined with (5), $\Delta_1^A > \Delta_2^A = \Delta_1^B$ implies

$$t_1^A > t_1^B = t_2^A. \quad (6)$$

Platform 1's profit is given by

$$t_1^A q_1^A + t_1^B q_1^B$$

where q_1^j depends on the profile of fees given in (2) and increases in Δ_1^j holding other fees fixed as long as both platforms are accepted.

Suppose

$$\frac{(1 - t_1^A)^2}{4} < \frac{(\Delta_1^A - \Delta_2^A)^2}{8}.$$

Then, from (3), the merchant in A strictly prefers double homing and recall that since $\Delta_1^B < \Delta_2^B$ the NSR for platform 1 does not bind in B . Therefore, (5) implies that platform 1 can increase both Δ_1^B and Δ_1^A equally without affecting its total fees and still keep a uniform cardholder fee.

This observation implies that platform 1 will increase Δ_1^A up until (3) holds with equality. That is, until

$$\frac{(1 - t_1^A)^2}{4} = \frac{(\Delta_1^A - \Delta_2^A)^2}{8}. \quad (7)$$

However, (6) implies that if merchant A rejects a platform it would reject platform 1. If platform 2 *reduces* its own gap, Δ_2^A slightly so that

$$\frac{(1 - t_1^A)^2}{4} < \frac{(\Delta_1^A - \Delta_2^A)^2}{8}$$

it would induce the merchant to reject platform 1 and platform 2's sales in market A would go up discontinuously by the amount $(\Delta_1^A - \Delta_2^A)/4 > 0$ using (2) and the assumption that $\Delta_1^A > \Delta_1^B = \Delta_2^A$. Thus (7) and (6) are inconsistent with both t_1^A and Δ_2^A as mutual best responses in a complementary symmetric equilibrium.

5 Merchants with Heterogeneous Benefits

Consider a single market but with a merchant who is privately informed about the per transaction benefit b it enjoys when either platform is used, with $b \in [0, \bar{b}]$ and merchant types distributed according to a continuous distribution function $G(b)$ with density $g(b)$. The following is a heuristic argument why, again, no symmetric pure strategy equilibrium exists.

The equations in Section 2 become

$$\begin{aligned}
p_i(\Delta_i) &= \frac{1 - b + \Delta_i}{2} \\
q_i(t_i) &= \frac{1 + b - t_i}{2} \\
\pi_i(t_i) &= \frac{(1 + b - t_i)^2}{4}
\end{aligned} \tag{8}$$

for a single-homing merchant and

$$\begin{aligned}
p - m_i &= \frac{1 + b - t_i}{2} - \frac{\Delta_i - \Delta_{i'}}{4} \\
q_i^j &= \frac{1 + b - t_i}{2} + \frac{\Delta_i - \Delta_{i'}}{4}.
\end{aligned} \tag{9}$$

for a multi-homing merchant under a NSR.

Merchant profit with both platforms for a merchant of type b is

$$\frac{(1 + b - t_1)^2}{4} - \frac{(\Delta_1 - \Delta_2)^2}{8} + \frac{(1 + b - t_2)^2}{4}.$$

Suppose a symmetric equilibrium exists with $t_1 = t_2 = \bar{t}$, $\Delta_1 = \Delta_2 = \bar{\Delta}$. It is immediate that at this equilibrium, no merchant of type b can be on the margin between single- and double-homing and earn strictly positive profits. The only margin, therefore, that could be in force at an equilibrium would be for some merchant type $b \in [0, \bar{b})$ who is exactly earning zero profits in equilibrium accounting for its card benefits. (If no such merchant exists, then all merchant types participate and the argument in main paper follows.)

Consider a small increase in fee gap by platform 1, $\Delta_1 = \bar{\Delta} + \epsilon$ while holding total fee fixed. The impact on profits of a merchant of type b is of second order around $\epsilon = 0$:

$$\frac{d}{d\epsilon} \left(\frac{(1 + b - \bar{t})^2}{4} - \frac{(\bar{\Delta} + \epsilon - \bar{\Delta})^2}{8} + \frac{(1 + b - \bar{t})^2}{4} \right) = -\frac{(\bar{\Delta} - \bar{\Delta})}{4} = 0.$$

On the other hand, (9) implies transactions on platform 1 rise by $\frac{\epsilon}{4}$. Thus for small such changes, the impact on the profit of a merchant of any type is negligible so participation by such a merchant is virtually unaffected while platform profits increase by a first order eliminating the profile as a potential equilibrium.