Competitive Differential Pricing

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Abstract. This paper analyzes welfare under differential versus uniform pricing across oligopoly markets that differ in costs of service. We establish necessary and sufficient conditions on demand properties—cross/own elasticities and curvature—for differential pricing by symmetric firms to raise aggregate consumer surplus, profit, and total welfare. The analysis reveals intuitively why differential pricing is generally beneficial though not always—including why profit can fall, unlike for monopoly—and why it is more beneficial than oligopoly third-degree price discrimination. When firms have asymmetric costs, however, differential pricing can reduce profit or consumer surplus even with 'simple' demands such as linear.

Keywords: differential pricing, price discrimination, demand curvature, cross-price elasticity, pass-through, oligopoly

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1. Introduction

Distinct consumer groups—or 'markets'—for a product frequently differ in their costs of service or demands. A large literature studies the welfare effects, relative to uniform pricing, of differential pricing across markets based solely on demand elasticities—classic third-degree price discrimination—under monopoly or oligopoly. See, for example, Aguirre, Cowan and Vickers (2010) and references therein for monopoly third-degree price discrimination; and Holmes (1989) and Stole (2007) for the oligopoly case. Very little work has compared uniform pricing (UP) and differential pricing (DP) when, instead, markets vary in costs of service. Yet DP motivated (at least partly) by cost differences is controversial and frequently subject to various constraints in monopoly or oligopoly markets, such as gender-neutral requirements in insurance or pensions, universal-service mandates on utilities, antidumping rules in international trade, and consumer resistance to add-on pricing such as airline bag fees.[1]

This paper analyzes the welfare effects of (cost-based) differential pricing by oligopoly firms. If markets were perfectly competitive, DP obviously would be desirable, as prices would equal marginal costs in each market hence maximize welfare, whereas UP would distort the output allocation. However, the ranking is no longer clear when prices exceed marginal costs, as Chen and Schwartz (2015) show for the case of monopoly: moving from UP to DP there can reduce consumer surplus (aggregated across markets) and—while raising profit—can also reduce total welfare, albeit under rather stringent demand curvature conditions. In oligopoly, the welfare analysis of DP is even richer due to at least two new forces: (1) when firms supply differentiated products, the pricing equilibria will depend also on cross-price elasticities[2]; and (2) firms may differ in their costs within each market, and the pricing equilibria then will depend on the pattern of cost asymmetry even with homogeneous products.

We consider a setting with two competing firms, each selling a single product in two distinct markets. To isolate the role of cost differences, we assume the markets have equal demand

[1] The constraints on cost-based DP can stem from various sources: government policy, contractual restrictions, consumer perceptions of the likely effects, or transaction costs. For further discussion and examples, see Chen and Schwartz (2015), Edelman and Wright (2015), and Nassauer (2017). On add-on pricing generally see Ellison (2005) and Brueckner et al. (2015).

[2] Mrázová and Neary (2017) show that any well-behaved demand function for a single product can be represented by its elasticity and curvature. With differentiated products, cross-price elasticity is additionally needed.

elasticities but different marginal costs of service. Firms sell symmetrically differentiated or homogeneous products and compete in prices under the alternative UP and DP regimes.[3] In our main model, the firms also have symmetric costs—the same cost within a market—and sell differentiated products. This environment serves two purposes. It reveals how the welfare properties of DP under monopoly (Chen and Schwartz 2015) may differ in oligopoly solely due to the cross-elasticity/strategic effect, and permits a natural comparison to *price discrimination* in symmetric oligopoly analyzed by Holmes (1989). In an extension of the model, firms may supply homogeneous or symmetrically differentiated products, but have asymmetric costs.

The pass-through rate from firms' common marginal cost to their symmetric equilibrium price plays a major role in our main model.[4] Under UP each firm sets price based on the average of its marginal costs across markets, whereas under DP it sets prices based on each market's specific marginal cost. Moving to DP effectively lowers firms' marginal cost in one market and raises cost in the other market, with the equilibrium price adjustments determined by the pass-through rate. Our unifying methodology for evaluating the effects of DP thus involves analyzing whether each welfare measure is convex or concave in marginal cost. We obtain necessary and sufficient conditions for DP to raise consumer surplus, profit, or total welfare (Propositions 1-3), based on general properties of the demand system (which also determine the pass-through rate): curvature and own- and cross-price elasticities, and how they vary as firms change price equally. These conditions reduce to their counterparts for monopoly DP—and hence neatly nest the latter—when cross-price effects vanish. The conditions hold for many familiar demand functions; but we also find exceptions where DP is harmful. Throughout, we trace the welfare changes to endogenous forces, such as the change in average price and extent of output reallocation between markets.

Consumer surplus can rise or fall with DP in oligopoly, for the same reason as under monopoly (Chen and Schwartz 2015). Consumers benefit from the price dispersion, but can lose on balance if average price across markets rises sufficiently, as can occur if the pass-through rate increases

[3] Hereafter, unless stated otherwise, DP refers to 'cost-based'. These alternative pricing regimes can be attained through interventions that do not require knowledge of costs: laissez faire yields DP, whereas prohibiting any price differences yields UP. Cowan (2018) analyzes regulatory schemes that constrain a monopolist's price-cost margins, schemes that improve welfare but require cost information for the regulator.

[4] For a general analysis of pass-through in various applications see Weyl and Fabinger (2013).

sufficiently fast. In some such cases, output falls enough that total welfare also declines.

Unlike for monopoly, DP can reduce profits in oligopoly.[5] This can occur in two ways. First, DP induces excessive output reallocation between markets when pass-through exceeds one, as the price difference 'overshoots' the cost difference so the profit margin is smaller in the (low-cost) market that gains output. Under monopoly, DP nevertheless raises profit for any pass-through rate, because a rate above one can occur only when demand is highly convex, in which case moving from UP to DP yields a large output expansion (Chen and Schwartz 2015). In oligopoly, however, pass-through can be high not only due to demand curvature but also due to cross-price effects: with prices as strategic complements, a firm's price response to a cost shock is amplified by the demand feedback from the rival's price response to the same shock. Consequently, DP can reallocate output excessively *and* without expanding total output enough to outweigh this misallocation effect. Second, DP can reduce average price, again without inducing a large (if any) increase in output. If the products are closer substitutes at lower prices than at higher prices, moving to DP can reduce price by more in the low-cost market than it raises price in the other market—and not due to greater demand curvature at lower prices, which again explains why output need not increase. Interestingly, Holmes (1989) finds that classic third-degree *price discrimination* in oligopoly also can reduce average price without expanding output relative to UP, but due to different demand properties as we shall explain.

Overall, our analysis suggests that—while there are exceptions—cost-based DP in symmetric oligopoly is broadly beneficial. Furthermore, DP is more beneficial for consumers and total welfare than oligopoly price discrimination. In both cases, consumers gain from the price dispersion by adjusting quantities. But price discrimination has a bias to raise average price, which harms consumers, whereas cost-based DP does not, for a broad class of demand functions. Regarding total welfare, classic price discrimination misallocates output while UP does not, whereas when markets differ in costs of service, UP misallocates output and DP can improve the allocation.

Do the generally favorable effects of DP persist when firms are asymmetric? We extend our main model to allow cost asymmetries between firms, in addition to cost differences across

[5] In such cases, firms would jointly gain from committing to uniform pricing, but such a commitment would not be unilaterally optimal.

markets, starting with the case where firms sell homogeneous products.[6] In one scenario, the same firm has a cost advantage in both markets. It captures both markets while pricing at the rival's relevant marginal cost—the market-specific cost under DP or the average of marginal costs under UP. Average price under DP then equals the uniform price, hence consumer surplus must rise due to price dispersion, but profit of the lower-cost firm can readily fall if the difference in its costs across markets is lower than for the rival (Proposition 4). Intuitively, the firm suffers from having to set a price differential larger than its cost differential.

In an alternative scenario, each firm has a cost advantage in one market (but under UP each firm must serve both markets). Average price across markets under DP then exceeds the uniform price, because cost dispersion—which determines equilibrium markups—is higher under DP than under UP. If cost heterogeneity between firms is large relative to that across markets, DP reduces consumer surplus as the price-increasing effect dominates the beneficial price dispersion effect (Proposition 5).

When firms have asymmetric costs and supply homogeneous products, therefore, DP can reduce either profit or consumer surplus even for 'simple' demand functions such as linear. As a robustness check, we extend these findings to symmetrically differentiated products with a linear demand system from Shubik and Levitan (1980), showing that DP can reduce either profit or consumer surplus if products are sufficiently close substitutes (Proposition 6).

To our knowledge, the only other analysis addressing cost-based DP in oligopoly is by Adachi and Fabinger (2017). Our contributions are complementary. Adachi and Fabinger add cost differences between markets to Holmes' (1989) symmetric oligopoly setting. They provide sufficient conditions for DP to lower or raise total welfare, using similar techniques as Aguirre, Cowan and Vickers (2010; 'ACV'), who study monopoly price discrimination with no cost differences. Their conditions resemble ACV's, in comparing weighted markups between markets at the equilibrium price(s), but are more complex due to the role of cross-price effects in the weights. Our analysis of symmetric oligopoly assumes equally-elastic demands across markets in order to focus sharply on the role of cost differences. By analyzing when each welfare measure is a convex function of marginal cost, we provide transparent necessary and sufficient conditions on the demand system

[6] Since DP by symmetric firms is always beneficial with homogeneous products (Bertrand competition then yields the same first-best outcome as perfect competition), this case sharply highlights the role of cost asymmetries.

for DP to raise or lower consumer surplus, profit, or total welfare, and decompose the underlying forces. In addition, we highlight the effects of cost asymmetries between firms.

The next section presents the main model and some preliminary results. Section 3 analyses the effects of DP with symmetric firms. Section 4 considers the extension where firms have asymmetric costs. We conclude in Section 5, and gather all proofs in the Appendix.

2. A Model With Symmetric Firms

Two firms, $i = 1, 2$, each produce one product. When firm i's price is p_i, the demand function for firm i, $i = 1, 2 \neq j$, is $D_i(p_1, p_2)$. There are two distinct groups of consumers or markets, L and H. Each firm's constant marginal cost is c_L to serve group L and c_H to serve group H, with $0 \leq c_L < c_H$. Group L's demand for firm i's product is $\lambda D_i(p_1, p_2)$ and group H's demand is $(1-\lambda) D_i(p_1, p_2)$ for $\lambda \in (0, 1)$. Since these demands differ only in scale, they have equal price elasticities at any common price, but the (constant) marginal costs of serving the groups differ.[7]

Firms compete by simultaneously choosing prices, possibly in one of two pricing regimes. Under uniform pricing (UP) each firm can only set a single price for all consumers, whereas under differential pricing (DP) each firm can charge two different prices for the two distinct consumer groups.

As in Holmes (1989), the firms produce symmetrically differentiated substitute products with $D_i(p_1, p_2)$ being a continuous and differentiable function[8] satisfying

$$D_i(x, y) = D_j(y, x) \quad \text{for firm } i \neq j = 1, 2. \tag{1}$$

At equal prices $p_i = p_j \equiv p$, we further define the industry demand as

$$D(p) \equiv 2D_i(p, p) \quad \text{for } i = 1, 2. \tag{2}$$

[7] We could more generally assume that the demands for firm i by the two consumer groups are $D_{iL}(p_1, p_2)$ and $D_{iH}(p_1, p_2)$. Our assumption that demands are proportional allows us to focus on differential pricing motivated purely by cost differences between the groups.

[8] We shall also briefly address the case where the two symmetric firms produce a homogeneous product so that $D_i(p_i, p_j)$ is not continuous at $p_i = p_j$. The analysis is then straightforward.

This setting lets us compare for symmetric oligopoly the welfare properties of differential pricing motivated solely by cost differences between markets versus demand differences analyzed by Holmes. Given firms' symmetry, we will analyze the symmetric equilibria in which both firms charge equal prices under UP or under DP.

Denote the equilibrium price by p^u under UP and by p_k for $k = L, H$ under DP. We assume standard demand conditions such that DP raises price in the high-cost market and reduces price in the low-cost market: $p_L < p^u < p_H$. Let $q^u = D(p^u)$, $q_L = D(p_L)$, and $q_H = D(p_H)$. Then $q_H < q^u < q_L$, and $\Delta q_L \equiv q_L - q^u > 0$ while $\Delta q_H \equiv q_H - q^u < 0$. Define

$$p^d \equiv \lambda p_L + (1 - \lambda) p_H \tag{3}$$

as the *average* price under DP weighted by the relative sizes of the two groups, which equal their relative consumption quantities under UP. If $p^d = p^u$, and consumers in the two groups were to maintain the same consumption quantities as under UP, their total expenditure and welfare would be unchanged. Under monopoly, or a homogeneous-products oligopoly with downward-slopping market demand $D(p)$, $p^d \leq p^u$ is a sufficient condition for DP to raise aggregate consumer surplus because consumers can advantageously adjust quantities to exploit price dispersion—purchasing more where price is lower and less where price is higher (Waugh 1944). With competition and differentiated products, cross-price effects complicate the analysis. But under a regularity condition described below, price dispersion for both goods that does not raise their average price again benefits consumers.

The (aggregate) consumer surplus under uniform pricing is[9]

$$S^u \equiv S(p^u, p^u) = \int_{p^u}^{\infty} D_1(x, p^u)\, dx + \int_{p^u}^{\infty} D_2(\infty, x)\, dx \tag{4}$$

where the second integral is consumer surplus from good 2 at price p^u when good 1 is not available and the first integral is the incremental surplus from good 1 at price p^u when the other

[9] We assume that consumer surplus is uniquely defined, independent of the integration path. This requires the demand functions to exhibit equal cross-partial derivatives, as occurs when demand is derived from a quasi-linear utility function so there are no income effects.

good is also available at the symmetric price p^u. Under differential pricing, consumer surplus is

$$\begin{aligned} S^d &= \lambda \left[\int_{p_L}^{\infty} D_1(x, p_L)\, dx + \int_{p_L}^{\infty} D_2(\infty, x)\, dx \right] \\ &\quad + (1-\lambda) \left[\int_{p_H}^{\infty} D_1(x, p_H)\, dx + \int_{p_H}^{\infty} D_2(\infty, x)\, dx \right]. \end{aligned} \tag{5}$$

We assume the following regularity condition that ensures convexity of the first integral term in (4):

$$-\frac{\partial D_1(y,y)}{\partial p_1} \geq -\int_y^{\infty} \frac{\partial^2 D_1(x,y)}{\partial y^2} dx. \tag{6}$$

The result below then applies when firms' products are homogeneous or differentiated.

Remark 1 *DP increases consumer surplus if average price does not rise ($p^d \leq p^u$).*

Next, let the industry output under DP be $q^d \equiv \lambda q_L + (1-\lambda) q_H$, and also let $\Delta q = q^d - q^u$. Furthermore, let $m_L = p_L - c_L$ and $m_H = p_H - c_H$ be the price-cost margins for the two consumer groups under DP. The difference of industry profit under DP and UP is

$$\Pi^d - \Pi^u = [\lambda(p_L - c_L) q_L + (1-\lambda)(p_H - c_H) q_H] - [p^u - \lambda c_L - (1-\lambda) c_H] q^u.$$

The effect of DP on industry profit can then be decomposed as follows (see Appendix):

$$\Delta\Pi \equiv \Pi^d - \Pi^u = \underbrace{\left(p^d - p^u\right) q^u}_{\text{Average-P Effect}} + \underbrace{\lambda(m_L - m_H)\Delta q_L}_{\text{Reallocation Effect}} + \underbrace{m_H \Delta q}_{\text{Output Effect}}. \tag{7}$$

Since $\Delta q_L > 0$, the reallocation effect will be positive if the margin is higher in market L than in H at the differential prices (as was true under UP) and negative if the reverse holds. Under classic third-degree price discrimination, i.e., markets face different prices but have the same cost of service, the reallocation effect is necessarily *negative*: output shifts to the market where price fell and, hence, where the margin is lower. Thus, profitable price discrimination requires an increase in output or in the average price. In contrast, (cost-based) DP can be profitable even when output and average price fall, because the reallocation effect will be positive as long as the price difference between markets remains less than the cost difference. This distinction will prove useful in section 3.5 and we record it as follows:

Remark 2 *Profitable price discrimination requires an increase in average price or in total output, but profitable cost-based differential pricing does not.*

3. Welfare Analysis

Under uniform pricing, assume that each firm draws customers from groups L and H in proportion to their relative masses λ and $1-\lambda$, as will be true in a symmetric equilibrium. Then a firm's virtual marginal cost under UP is

$$\bar{c} \equiv \lambda c_L + (1-\lambda)\, c_H. \tag{8}$$

If the two firms supply a homogeneous product, then under UP

$$p^u = \bar{c} = \lambda c_L + (1-\lambda)\, c_H,$$

whereas under DP we have $p_L = c_L$ and $p_H = c_H$, with average price

$$p^d = \lambda c_L + (1-\lambda)\, c_H = p^u.$$

Thus, DP obviously is beneficial: Since average price is the same under UP and DP, consumer surplus is higher under DP due to price dispersion, while profit is zero under both regimes with homogeneous products.

However, the results are no longer obvious when products are (symmetrically) differentiated. The remainder of this section addresses that case. Subsections 3.1-3.3 analyze the effects of DP, compared to UP, on consumer surplus, profits, and total welfare, respectively, using general properties of the demand system. Subsection 3.4 provides illustrative examples using specific demand functions. Subsection 3.5 compares the welfare effects of cost-based pricing to classic price discrimination (Holmes 1989).

3.1 Equilibrium Prices and Consumer Surplus

For marginal cost c of both firms ($c = \bar{c}$ under UP, $c = c_L$ or c_H under DP), firm i chooses p_i to maximize

$$\pi_i(p_1, p_2) = (p_i - c) D_i(p_1, p_2),$$

taking as given p_j, for $i = 1, 2, i \neq j$. A sufficient condition for the existence of a unique equilibrium, which we shall maintain, is

$$-\frac{\partial^2 \pi_i}{\partial p_i^2} > \frac{\partial^2 \pi_i(p_1, p_2)}{\partial p_i \partial p_j} > 0. \tag{9}$$

The second inequality implies that firms' prices are strategic complements.

The (symmetric) equilibrium price of both firms, denoted $p^* \equiv p^*(c)$, satisfies the first-order condition

$$\frac{\partial \pi_i(p^*, p^*; c)}{\partial p_i} = D_i(p^*, p^*) + (p^* - c) \frac{\partial D_i(p^*, p^*)}{\partial p_i} = 0. \tag{10}$$

Under UP, given firm 2's price p^u, firm 1 chooses p_1 to maximize

$$(p_1 - c_L)\lambda D_1(p_1, p^u) + (p_2 - c_H)(1 - \lambda) D_1(p_1, p^u) = (p_1 - \bar{c}) D_1(p_1, p^u),$$

while under DP prices are set to maximize profit separately in each market. Therefore, the equilibrium prices in the two pricing regimes are

$$\text{UP:} \quad p^u = p^*(\bar{c}) \qquad\qquad \text{DP:} \quad p_L = p^*(c_L), \quad p_H = p^*(c_H),$$

obtained by substituting the relevant value of c in (10). Moving from UP to DP can thus be analyzed as if marginal cost fell in market L from the virtual level $\bar{c}$ to c_L and rose in market H from $\bar{c}$ to c_H.

Each firm's equilibrium profit under uniform pricing is

$$\pi^u = (p^u - \bar{c}) D_1(p^u, p^u), \tag{11}$$

and under differential pricing, a firm's profit is

$$\pi^d = \lambda [p_L - c_L] D_1(p_L, p_L) + (1-\lambda) [p_H - c_H] D_1(p_H, p_H). \tag{12}$$

By the symmetry of demand, without loss of generality we can conduct our analysis for all variables associated with one firm. For firm 1, say, define

$$\text{own-price elasticity:}\quad \eta_{11} = -\frac{\partial D_1(p,p)}{\partial p_1}\frac{p}{D_1(p,p)} > 0 \tag{13}$$

$$\text{cross-price elasticity:}\quad \eta_{12} \equiv \frac{\partial D_1(p,p)}{\partial p_2}\frac{p}{D_1(p,p)} > 0 \tag{14}$$

$$\text{elasticity ratio:}\quad \eta_r \equiv \frac{\eta_{12}}{\eta_{11}} > 0. \tag{15}$$

As is customary, we assume $-\frac{\partial D_i(p,p)}{\partial p_i} > \frac{\partial D_i(p,p)}{\partial p_j}$, hence $\eta_r \in (0,1)$.[10] A larger η_r reflects greater substitutability between the products.[11] For a firm's demand, define also the

$$\text{(margin-adjusted) curvature:}\quad \alpha(p) \equiv \frac{(p-c)}{p}\left[\frac{p}{-\frac{\partial D_1(p,p)}{\partial p_1}}\frac{d}{dp}\frac{\partial D_1(p,p)}{\partial p_1}\right]. \tag{16}$$

The square-bracketed term is the elasticity of the slope of firm 1's demand with respect to an equal change in both prices. Thus, $\alpha(p) = 0$ if D_1 is linear, and $\alpha(p) > (<)\ 0$ if D_1 is convex (concave) in symmetric price p.

The equilibrium price satisfies the familiar inverse elasticity rule: $\frac{p^*-c}{p^*} = \frac{1}{\eta_{ii}(p^*)}$. Using (10), the pass-through rate from marginal cost to equilibrium price is

$$\begin{aligned} p^{*\prime}(c) &= -\frac{-\frac{\partial D_1(p^*,p^*)}{\partial p_1}}{2\frac{\partial D_1(p^*,p^*)}{\partial p_1} + \frac{\partial D_1(p^*,p^*)}{\partial p_2} + (p^*(c)-c)\frac{d}{dp^*}\frac{\partial D_1(p^*,p^*)}{\partial p_1}} \\ &= \frac{1}{2-[\alpha+\eta_r]}, \end{aligned} \tag{17}$$

[10] For market i and equal prices p by both firms, Holmes (1989) denotes the firm's own-price elastiticy as $e_i^F(p)$, the cross-price elasticity as $e_i^C(p)$, and the elasticity ratio as $e_i^C(p)\ /\ e_i^I(p)$, where $e_i^I(p)$ is the industry demand elasticity when firms change price equally, and $e_i^F(p) = e_i^I(p) + e_i^C(p)$ (with all elasticities > 0). In his notation, our elasticity ratio, instead, is $e_i^C(p)\ /\ e_i^F(p)$.

[11] In symmetric equilibrium, η_r equals the *diversion ratio,* a measure of substitutability commonly used in antitrust (e.g., Chen and Schwartz 2016). The diversion ratio from product 1 to product 2 is $-\frac{(\partial q_2/\partial p_1)dp_1}{(\partial q_1/\partial p_1)dp_1} = \frac{\eta_{12}q_2}{\eta_{11}q_1}$, and $q_2 = q_1$ under symmetry.

where α and η_r (and later also α' and η_r') are evaluated at symmetric equilibrium prices $p = p^*(c)$. Notice that $p^{*'}(c) > 0$ under assumption (9).[12]

Equation (17) shows that a marginal increase in c affects equilibrium price under competition through two channels. One is the (margin-adjusted) curvature of a firm's demand, α: $p^{*'}(c)$ is larger when the firm's demand is convex ($\alpha > 0$) rather than concave ($\alpha < 0$), as under monopoly (Bulow and Pfleiderer 1983). The second channel is the elasticity ratio of firm demand: *ceteris paribus,* a higher η_r raises $p^{*'}(c)$. A common increase in marginal cost c will raise the rival's price, which magnifies the firm's own price increase when prices are strategic complements. This cross-effect increases with η_r and provides a force in favor of a higher pass through under competition than under monopoly.

We now can compare the uniform price, p^u, with the (group-weighted) average price under DP, defined in (3): $p^d \equiv \lambda p^*(c_L) + (1-\lambda) p^*(c_H)$. The difference between p^u and p^d will affect the change in consumer surplus and in profits from a move to DP. Since

$$p^u = p^*(\bar{c}) = p^*(\lambda c_L + (1-\lambda) c_H),$$

we have $p^u \gtreqless p^d$ when

$$p^u = p^*(\lambda c_L + (1-\lambda) c_H) \gtreqless \lambda p^*(c_L) + (1-\lambda) p^*(c_H) = p^d.$$

That is, DP lowers average price if $p^*(c)$ is concave, and raises average price if $p^*(c)$ is convex. From (17),

$$p^{*''}(c) = \left[\alpha'(p^*) + \eta_r'(p^*)\right] \left[p^{*'}(c)\right]^3. \tag{18}$$

Thus, $p^{*''}(c)$ has the same sign as $[\alpha' + \eta_r']$, implying:

Remark 3 *(i) If $\alpha' + \eta_r' > 0$, then DP raises average price, $p^d > p^u$; (ii) if $\alpha' + \eta_r' \leq 0$, then $p^d \leq p^u$.*

[12]For a monopolist with demand $q = D(p)$, its monopoly price $p^m \equiv p^m(c)$ has pass-through $p^{m'}(c) = \frac{1}{2-\sigma}$, where $\sigma \equiv \frac{p^m - c}{p^m} \frac{p^m}{-D'(p^m)} D''(p^m)$ is the curvature of the inverse demand function $P(q) = D^{-1}(q)$ (e.g., Chen and Schwartz 2015). Thus $p^{*'}(c)$ differs from $p^{m'}(c)$ due to the extra term $-\eta_r$ in the denominator and due to the difference in the curvature term for the demand function; with monopoly we would have $\eta_r = 0$ and $\alpha = \sigma$.

Note that if $[\alpha'(p^*(c)) + \eta_r'(p^*(c))]$ has a consistent sign over the relevant range $c \in [c_L, c_H]$, then the condition in (i), i.e., $\alpha' + \eta_r' > 0$, is sufficient and *also necessary* for $p^d > p^u$, and similarly the condition in (ii) is also necessary for $p^d \leq p^u$. This observation also applies to the subsequent Propositions 1 through 3.

If $\alpha' + \eta_r' > 0$, the pass-through increases with price, because the demand curvature term increases ($\alpha' > 0$) and/or the product substitutability increases ($\eta_r' > 0$). Pass-through then will be larger in market H, where a move to DP raises marginal cost (from $\bar{c}$ to c_H) and, hence, price, than in market L where marginal cost falls, explaining why average price rises. Both α and η_r are constant for many familiar demand functions—including linear, CES, and log demands (see section 3.4 below). For these demands, cost-based DP does not raise average price.

The condition $\alpha' + \eta_r' \leq 0$, implying $p^d \leq p^u$, is sufficient for DP to raise consumer surplus (Remark 1) but is not necessary since consumers gain from price dispersion under DP. A tighter sufficient condition is derived next. Define consumer surplus as a function of c as:

$$\begin{aligned} s(c) &\equiv S(p^*, p^*) = \int_{p^*(c)}^{\infty} D_1(x, p^*)\, dx + \int_{p^*(c)}^{\infty} D_2(\infty, y)\, dy \\ &= \int_{p^*(c)}^{\infty} D_1(x, p^*)\, dx + \int_{p^*(c)}^{\infty} D_1(x, \infty)\, dx, \end{aligned}$$

where the second equality is due to the symmetry of D_i. Also, when the two firms charge a symmetric price p, define the price elasticity and the adjusted price elasticity of market (or industry) demand respectively by

$$\eta(p) \equiv -\frac{D'(p)}{D(p)} p; \qquad \hat{\eta}(p) \equiv -\frac{[D'(p) + \delta'(p)]}{D(p) + \delta(p)} p, \tag{19}$$

where

$$\delta(p) = \int_p^{\infty} \left[\frac{\partial D_1(p, x)}{\partial x} - \frac{\partial D_1(x, p)}{\partial p} \right] dx.$$

The adjustment term $\delta(p)$, which is zero for linear demand, reflects the cross-price effect between the two products and the demand curvature. DP raises or lowers consumer surplus as $s(c)$ is convex or concave:

$$S^d = \lambda s(c_L) + (1-\lambda)s(c_H) \gtreqqless s(\bar{c}) = S^u \Leftrightarrow s''(c) \gtreqqless 0.$$

Analyzing the sign of $s''(c)$ yields a tighter condition than $\alpha' + \eta_r' \leq 0$, on the demand parameters $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, and $\hat{\eta} = \hat{\eta}(p)$, for DP to raise consumer surplus:

Proposition 1 *Consumer surplus is higher under differential pricing than under uniform pricing if (20) holds, and is lower if the inequality in (20) is reversed.*

$$\frac{-[\alpha' + \eta_r']}{2 - [\alpha + \eta_r]} p + \hat{\eta} > 0, \quad \text{with } \hat{\eta} > 0. \tag{20}$$

The first term in (20) is the average price effect when moving from UP to DP. Since $\frac{1}{2-[\alpha+\eta_r]} = p^{*'}(c) > 0$ from (17), the first term takes the sign of $-[\alpha' + \eta_r']$. When $\alpha' + \eta_r' \leq 0$, DP does not raise the average price (Remark 3), and consumer surplus increases (Remark 1). The second term, $\hat{\eta} > 0$, reflects the price-dispersion effect. When the adjusted price elasticity of market demand $\hat{\eta}$ is higher, consumers are more capable of making quantity adjustments and thus benefit more from the price dispersion. Therefore, DP raises consumer surplus if it does not raise average price too much, i.e., if $[\alpha(p) + \eta_r(p)]$ does not increase too fast.

Under monopoly, the corresponding condition for DP to raise consumer surplus (Chen and Schwartz 2015) is

$$\frac{\sigma'(q)}{2-\sigma(q)} + \frac{1}{q} > 0 \Longleftrightarrow \frac{-\sigma'(q)\, q'(p)}{2-\sigma(q)} p + \eta > 0.$$

We can consider $\sigma(q)$ as corresponding to $\alpha(p)$ and $-\sigma'(p)\, q'(p)$ to $-\alpha'(p)$. Thus, condition (20) in oligopoly reduces to that under monopoly for $\eta_r' = \eta_r = 0$ and $\hat{\eta} = \eta$. It is not clear whether DP is more favorable to consumers (relative to UP) under competition than under monopoly; but in both cases DP has no tendency to raise average price for a broad class of demand functions and, hence, tends to benefit consumers.

3.2 Profit

Equilibrium profit for firm 1 under marginal cost c is

$$\pi^*(c) = [p^*(c) - c] D_1(p^*(c), p^*(c)).$$

Thus, using the envelope theorem:

$$\pi^{*'}(c) = -\underbrace{D_1(p^*(c), p^*(c))}_{\text{direct effect}} + \underbrace{[p^*(c) - c] \frac{\partial D_1(p^*, p^*)}{\partial p_2} p^{*'}(c)}_{\text{rival's effect}},$$

where the second term, the rival's effect, can be rewritten as

$$D_1(p^*, p^*) \left[\frac{\partial D_1(p^*, p^*)}{\partial p_2} \frac{p^*(c)}{D_1(p^*, p^*)} \right] \left[\frac{p^*(c) - c}{p^*(c)} \right] p^{*'}(c) = D_1(p^*, p^*)(\eta_{12}/\eta_{11}) p^{*'}(c).$$

With $\eta_{12}/\eta_{11} \equiv \eta_r$, we can express $\pi^{*'}(c)$ as

$$\pi^{*'}(c) = -D_1(p^*(c), p^*(c)) \left[1 - p^{*'}(c)\eta_r \right]. \tag{21}$$

For a monopolist, an increase in c lowers profit because its profit margin for each unit of sales is reduced by the increase in c. Under competition, this margin reduction is alleviated by the increase in the rival's price associated with the higher c, as reflected in the additional term $-p^{*'}(c)\eta_r$ in (21). Thus, $\pi^{*'}(c) < 0$ if and only if $p^{*'}(c)\eta_r < 1$. Condition (9) for a unique equilibrium does not ensure $p^{*'}(c)\eta_r < 1$, and we will consider also $p^{*'}(c)\eta_r \geq 1$.

The result below is derived by analyzing when $\pi^*(c)$ is convex or concave.

Proposition 2 *Profit is higher under differential pricing than under uniform pricing if (22) holds, and is lower if the inequality in (22) is reversed.*

$$\frac{\eta_r(\alpha' + \eta_r')}{[2 - \alpha - \eta_r]^2} p + \eta_r' p + \eta \left[1 - \frac{\eta_r}{2 - \alpha - \eta_r} \right] > 0. \tag{22}$$

For a monopolist, in (22) $\eta_r = \eta_r' = 0$, yielding simply $\eta > 0$, which always holds. It represents a monopolist's gain from adjusting outputs across markets in response to mean-preserving cost dispersion[13] which is proportional to demand elasticity, akin to the flexibility gain for consumer surplus. As with consumer surplus, the condition for DP to raise profit in oligopoly embeds and generalizes the condition under monopoly. Next, consider the three terms in (22) for our oligopoly case.

The first term in (22), $\frac{\eta_r(\alpha'+\eta_r')}{[2-\alpha-\eta_r]^2}p$, can be written as $p^{*'}(c)\eta_r\left[\frac{(\alpha'+\eta_r')}{[2-\alpha-\eta_r]}p\right]$. The bracketed term takes the sign of $(\alpha'+\eta_r')$ and tracks the change in average price when moving to DP. It affects the firm's profit via the *rival's* price response to the common cost shocks, and in proportion to the cross-elasticity term, η_r. An increase in average price due to DP boosts industry profit because competition under uniform pricing forces price too low from the standpoint of the industry.

The middle term, $\eta_r'p$, reflects an output externality. Each firm sets its price based on the firm elasticity of demand, η_{11}, but when both firms adjust prices equally, output is determined by market elasticity, $\eta = \eta_{11} - \eta_{12}$. Since $\eta_r \equiv \eta_{12}/\eta_{11} = 1 - \eta/\eta_{11}$, if $\eta_r'(p) > 0$, market elasticity relative to firm elasticity is smaller at higher prices than at lower prices. Moving to DP then induces a positive output externality on the rival firm: each firm ignores that (a) its price increase in market H expands the rival's output and (b) its price decrease in market L reduces the rival's output—but effect (a) exceeds (b) when $\eta_r' > 0$. Oligopoly DP then yields a larger output than predicted based on each firm's own-elasticity, boosting profit.[14]

The last term, $\eta\left[1-\frac{\eta_r}{2-\alpha-\eta_r}\right]$, can be written as $\eta\left[1-p^{*'}(c)\eta_r\right]$: a monopolist's gain ($\eta$) from adjusting prices and outputs across markets in response to cost dispersion, modified in oligopoly by the impact of the rival's symmetric price responses ($\eta p^{*'}(c)\eta_r > 0$). Suppose $p^{*'}(c)\eta_r \in (0,1)$. In market H, where moving to DP effectively raises marginal cost, the firm loses from the cost increase, but less than if it were a monopolist. The reverse occurs in market L, where moving to DP effectively lowers marginal cost. Since the rival's response in each market does not outweigh the own-cost effect, cost dispersion still benefits the firm, in proportion to the elasticity of market demand. However, it is possible to have $p^{*'}(c)\eta_r \geq 1$ so that DP (weakly) decreases profit, unlike for monopoly, as explained next.

The condition $p^{*'}(c)\eta_r \geq 1$ requires $p^{*'}(c) > 1$ (since $\eta_r < 1$), so that a virtual cost decrease

[13] Recall that moving from UP to DP can be analyzed as a virtual decrease in marginal cost from $\bar{c} \equiv \lambda c_L + (1-\lambda)c_H$ to c_L in market L and an increase from $\bar{c}$ to c_H in market H, with respective weights λ and $1-\lambda$.

[14] In addition, when $\eta_r' > 0$ DP tends to raise average price (see first term in (22)), which also boosts profit. Both effects are reversed if $\eta_r' < 0$, and DP then can reduce profit, as we will show.

(in market L) reduces the profit margin; and, furthermore, η_r large enough (though still < 1), so that the price fall is driven sufficiently by strategic interaction that industry output does not rise too much.[15] The reverse pattern occurs in market H, where DP increases profit. Why, then, does overall profit fall? Observe that

$$m_L - m_H = c_H - c_L - \int_{c_L}^{c_H} p^{*'}(c)\, dc \ > (=) < 0 \ \text{ if } \ p^{*'}(c) < (=) > 1. \tag{23}$$

Thus, when $p^{*'}(c) > 1$ the profit margin in market L under DP is lower than in H, hence the output reallocation to L harms profit, by (7). For a monopolist, DP nevertheless raises profit because $p^{*'}(c) > 1$ requires demand to be sufficiently convex that the price dispersion expands output enough to outweigh the harmful misallocation (Chen and Schwartz 2015). In oligopoly, however, $p^{*'}(c) = \frac{1}{2-\alpha-\eta_r} > 1$ can arise not only from demand convexity ($\alpha > 0$), but also from the cross-elasticity term, η_r, so pass-through above 1 need not imply a large output expansion from DP. Example 3 in section 3.4 illustrates $p^{*'}(c)\eta_r = 1$, hence DP fails to raise profits.

A second way DP can reduce profit arises when $\eta_r' < 0$. DP then can lower both average price and total output, and reduce profit due to the first two terms in (22) (tracking the first and third terms in decomposition (7)). See Example 4 in section 3.4.

Although DP by symmetric firms may reduce profit, the required demand conditions seem rather special. In the 'normal' case where a common cost shock moves industry profit in the opposite direction ($p^{*'}(c)\eta_r < 1$),[16] there is a systematic force pushing towards greater profit: the beneficial output reallocation effect captured by the last term in (22). Profit (in the normal case) and consumer surplus both benefit from greater scope for output reallocation under DP, a larger η in (22) or its analogue $\hat{\eta}(p)$ in (20). As we shall illustrate in section 3.4, while there are exceptions, both (20) and (22) are satisfied for many familiar demand functions.

[15] For a given own-price elasticity η_{11}, a larger cross-elasticity η_{12} implies a lower market-demand elasticity η. Thus, for $\eta_r \equiv \eta_{12}/\eta_{11}$ large (but still < 1), η_{11} can be sufficiently high to render a unilateral price increase unprofitable, yet η can be sufficiently low that a joint price increase by all firms, induced by a common cost increase, can be profitable.

[16] Weyl and Fabinger (2013, footnote 16) consider the alternative case "unlikely to be empirically relevant in many symmetric industries."

3.3 Total Welfare

Given marginal cost c and the associated equilibrium price $p^*(c)$, the equilibrium total welfare in a market can be written as

$$\begin{aligned} W(p^*(c)) &\equiv w^*(c) = s^*(c) + 2\left[p^*(c) - c\right] D_1(p^*(c), p^*(c)) \\ &= s^*(c) + \left[p^*(c) - c\right] D(p^*(c)). \end{aligned} \tag{24}$$

Letting $\hat{\eta}_r \equiv \left(\eta_r - \frac{\delta(p^*)}{D(p^*(c))}\right)$, where $\delta(p)$ was defined after (19), and analyzing when $w^*(c)$ is convex, we obtain the following condition for DP to raise or lower total welfare.

Proposition 3 *DP increases total welfare if (25) holds, and DP reduces total welfare if (25) is reversed.*

$$\frac{-\left[\alpha' + \eta_r'\right](1 - \hat{\eta}_r)}{\left[2 - \alpha - \eta_r\right]^2} p + \frac{\hat{\eta}_r' p}{2 - \alpha - \eta_r} + \eta\left[1 + \frac{1 - \hat{\eta}_r}{2 - \alpha - \eta_r}\right] > 0. \tag{25}$$

The first term corresponds to the first terms in (20) and (22), reflecting the net effect of the change in average price on consumer surplus plus profit: when $(\alpha' + \eta_r') \leq 0$, average price does not increase, hence the net effect on total welfare is non-negative, since $\hat{\eta}_r < 1$.[17] In the second term, $\hat{\eta}_r' p$, is similar to the middle term in (22), scaled by the pass-through rate, and captures the net output externality under competition across the two markets, H and L. Together, the first and second terms in (22) reflect how DP affects total welfare through the change in total output. The last term, $\eta\left[1 + \frac{1-\hat{\eta}_r}{2-\alpha-\eta_r}\right]$, corresponds to the output adjustment effect: moving from UP to DP creates price dispersion for consumers and cost dispersion for firms, leading to beneficial output adjustments in total, and a higher η magnifies this effect. DP can raise total welfare due to both the (beneficial) output reallocation and output expansion, but neither of them alone is necessary for DP to raise welfare.[18]

The condition for monopoly DP to raise total welfare (Chen and Schwartz 2015, condition (A1B)) can be written as

[17] Since the demand curve is downward sloping, a change in average price can affect total output, so its effects on consumer surplus and profit do not entirely offset.

[18] Importantly, output reallocation under $p_H - c_H < p_L - c_L$ can improve total welfare even when total output decreases, a key difference between cost-based DP and price discrimination.

$$\eta\left[1+\frac{1}{2-\sigma\left(q^{*}\right)}\right]+p\frac{-\sigma'\left(q^{*}\right)D'\left(p\right)}{\left[2-\sigma\left(q^{*}\right)\right]^{2}}>0,$$

and monopoly DP lowers total welfare if the above inequality is reversed. As with consumer surplus and profit, the condition for DP to raise total welfare under oligopoly, (25), embeds and generalizes that under monopoly: $\eta_r = \eta_r' = \hat{\eta}_r = 0$ for the single-product monopolist, while α in oligopoly corresponds to σ under monopoly and α' corresponds to $\sigma'\left(q^{*}\right)D'\left(p\right)$. And like its counterparts for consumer surplus and profit, (20) and (22), condition (25) is met for a broad class of demands, such as those with constant α and η, but not always; see Example 4 below.

Finally, observe that under monopoly, DP always increases profit, hence total welfare rises under broader conditions than does consumer surplus (compare Propositions 1 and 2 of Chen and Schwartz 2015). In oligopoly, DP can reduce profit, hence consumer surplus may rise yet total welfare fall, as in Example 4, while the reverse occurs in Example 5. Thus, the conditions for DP to raise consumer surplus or total welfare, (20) and (25), are no longer nested.

3.4 Examples

The ensuing examples show that the conditions in Propositions 1-3 for differential pricing to benefit consumers, profits, and overall welfare, are met by many familiar demand functions—though not always—and illustrate the underlying economic forces. To that end, recall from (7) that the output reallocation induced by DP affects profit positively when $m_L > m_H$ and negatively when $m_L < m_H$; and that $m_L > (<)\ m_H$ if $p^{*'}(c) < (>)\ 1$, from (23).[19]

Example 1 *Linear demand (DP increases consumer surplus and profit):*

$$q_i = I - b_1 p_i + b_2 p_j, \qquad I > 0 \text{ and } b_1 > b_2.$$

Then, $\eta_r = \frac{b_2}{b_1}$, $\eta = p\frac{b_1 - b_2}{I-(b_1-b_2)p} = \hat{\eta}$, $\alpha = 0$.

It follows that both (20) and (22) hold, hence DP increases consumer surplus and profit. Average price and total output are the same under UP and DP, so the gains in consumer surplus

[19] More details on the examples in this section are available upon request.

and profit come solely from reallocating output between markets. The gains can be significant. For instance, if $\{I, b_1, b_2, c_L, c_H, \lambda\} = \{8, 2, 1, 3, 5, 0.5\}$, profit and consumer surplus both increases by 6.25% when moving from UP to DP; and if $\{I, b_1, b_2, c_L, c_H, \lambda\} = \{8, 2, 1, 2, 5, 0.4\}$, they increase by 12.24%.

Example 2 *Log demand function (DP increases consumer surplus and profit):*

$$p_i(q_i, q_j) = A - \log[Bq_i + q_j], \qquad B > 1.$$

Then, $\eta_r = \frac{1}{B}$, $\alpha(p) = \frac{B-1}{B} > 0$, $p^*(c) = c + \frac{B-1}{B}$, *and* $p^{*'}(c) = 1$.

Again, both (20) and (22) hold. As in Example 1, DP does not change average price, but now it increases total output, which is the sole reason why profit rises: the output reallocation here is neutral for profit, since $p^{*'}(c) = 1$ implies $m_L = m_H$.

Example 3 *CES Demand (DP increases consumer surplus and total welfare, but profit is unchanged):*

$$q_i = A \frac{p_i^{-1/(1-\rho)}}{p_1^{-\rho/(1-\rho)} + p_2^{-\rho/(1-\rho)}}, \qquad 0 \le \rho < 1 \quad \textit{and} \quad A > 0.$$

Then: $p^*(c) = c\frac{2-\rho}{\rho}$, $\quad p^{*'}(c) = \frac{2-\rho}{\rho} > 1$, $\eta_r = \frac{\rho}{2-\rho}$, $\alpha = 4\frac{\rho-1}{\rho-2}$, *and* $\eta = 1$.

Since η_r and α are constant, average price is unchanged, hence output expands because demand $D(p) = D_i(p, p) = \frac{1}{2}Ap^{-1}$ is convex. The output reallocation is excessive for profit, since $p^{*'}(c) > 1$, but on balance profit is unchanged (as $p^{*'}(c)\eta_r = 1$).

Example 4 *Binomial Logit demand with outside option (DP increases consumer surplus but can reduce total output, profit, and welfare):*

$$q_i = \frac{e^{-p_i}}{e^{-p_i} + e^{-p_j} + A}, \qquad 0 < A < 1.$$

Then, $p^* = c + \frac{A+2e^{-p^*}}{A+e^{-p^*}}$, $p^{*'}(c) = \frac{\left(A+e^{-p^*}\right)^2}{e^{-2p^*}+3Ae^{-p^*}+A^2}$; $\eta_r = \frac{e^{-2p}}{e^{-p}(A+e^{-p})}$, $\eta_r' < 0$; $\alpha = \frac{A^2}{(A+e^{-p})^2} > 0$, $\alpha' > 0$; $\eta = \hat{\eta} = A\frac{p}{A+2e^{-p}}$.

In Example 4, condition (20) always holds and, hence, DP raises consumer surplus. However, conditions (22) and (25) can be reversed, so DP can lower profit and welfare. For instance, let $A = 0.01$, $c_L = 0$, $c_H = 2$, and $\lambda = 0.5$. Then: $m_L^d = 1.94 > m_H^d = 1.71$, so the output reallocation benefits profit and total welfare. But DP lowers average price, from $p^u = 2.85$ to $p^d = 2.82$, and lowers total output, from $q^u = 0.92$ to $q^d = 0.90$, causing profit to fall: $\Pi^d = 1.65 < \pi^u = 1.71$. The output reduction also reduces welfare, albeit slightly (by 0.67%).

The price and output changes can be understood as follows. Demand is more convex at higher prices ($\alpha' > 0$), which pushes the pass-through $p^{*'}(c)$ to be increasing; however, the products' substitutability is smaller at higher prices ($\eta_r' < 0$), which pushes $p^{*'}(c)$ to be decreasing, and this effect dominates since $\alpha' + \eta_r' < 0$. Hence DP lowers average price.[20] Importantly, this reduction in average price is driven solely by firms' profit-maximizing response to the cross-elasticity effect ($\eta_r' < 0$), and runs counter to the demand curvature effect ($\alpha' > 0$), which explains why total output fell: $\alpha' > 0$ implies that demand became more convex in the market where price rose than in the market where price fell.

Example 5 *Modified Logit demand (DP raises average price, and can reduce output and consumer surplus, but total welfare rises):*

$$q_i = \frac{1}{1 + e^{2p_i - p_j - k}}, \qquad k \geq 0.$$

Then, $p^* = c + \frac{1}{2}\frac{e^{-k+p*}+1}{e^{-k+p*}}$, $p^{*'}(c) < 1$, $\eta_r = \frac{1}{2}$, $\alpha = \frac{1}{2}\frac{e^{-k+p}-1}{e^{-k+p}}$, $\alpha' > 0$, $\eta = p\frac{e^{-k+p}}{e^{-k+p}+1}$.

In this example, DP raises average price because $\alpha' + \eta_r' > 0$. Since $\eta_r' = 0$, average price rises solely because $\alpha'(p) > 0$: demand becomes more convex at higher prices. The increase in average price can harm consumers on balance, despite their gain from adjusting outputs. For instance, suppose $c_L = 0$, $c_H = 2$, $\lambda = 0.5$, $\bar{c} = 1$, $k = 4$. Then: $S^d - S^u = -0.06 < 0$; $\Pi^d - \Pi^u = 0.1 > 0$; $W^d - W^u = 0.04 > 0$. Differential pricing here reduces total output, $\Delta q = -6.86 \times 10^{-3}$. Profit and total welfare nevertheless increase due to the output reallocation to market L, where the margin remains higher than in market H under DP ($m_L = 2.58 > m_H = 1.41$).

[20] When A is larger, for instance $A = 0.3$, DP can instead raise average price, but total output still expands and consumer surplus increases due to price dispersion; profit and total welfare also increase.

Summarizing our findings, DP raises both consumer surplus and profit when the demand curvature (α) and the elasticity ratio (η_r) do not change too fast relative to the elasticity of market demand (η). In particular, DP raises both consumer surplus and profit when α and η_r are constant, such as for linear, log, and CES (no effect on profit) demand. There are demands for which, under some parameter values, profit, consumer surplus, or total welfare can be lower under DP than under UPs, but such cases appear to be unusual.

3.5 Comparison to Oligopoly Price Discrimination

For symmetric oligopoly, Holmes (1989) analyzed price discrimination rather than our cost-based differential pricing. Our results exhibit similarities to his findings as well as differences.

In both settings, differential pricing—cost-based or demand-based—may reduce profit relative to uniform pricing, unlike for monopoly. Holmes shows this can occur if the market with the smaller elasticity of demand has the larger cross-price elasticity between firms. Price discrimination then lowers price in the 'wrong market' and can reduce total output. In our setting, markets differ only in costs, but DP still can reduce output and profit if cross-price elasticity relative to the own-price elasticity for the common demand function is greater at lower prices than at higher prices (Example 4). Then price can fall by more in the low-cost market than it rises in the other market—but not due to greater demand curvature at lower prices as under monopoly, hence output falls. (In addition, we showed in Example 3 that cost-based DP can (weakly) reduce profit through a second force: excessive output reallocation between markets when $p^{*\prime}(c) > 1$.)

Turning to differences, cost-based DP is more likely to benefit consumers than is price discrimination. From Remark 2, profitable price discrimination requires an increase in total output or in average price; indeed, price discrimination has a tendency to raise average price, which of itself harms consumers.[21] By contrast, DP can raise profit even if average price does not rise (Examples 1-2), which ensures that consumers also benefit. The cost savings achieved by reallocating output to the lower-cost market provide firms an incentive to adopt DP also under

[21] "There is a sense in which discrimination increases 'average' price; the increase in price in the strong market above the uniform price is 'large' relative to the decrease in the weak-market price." (Holmes 1989, p. 248.) Under monopoly, this price bias of price discrimination is discussed by Chen and Schwartz (2015, pp. 449-451).

demand conditions that do not yield an increase in average price, and consumers benefit from the price dispersion by adjusting their consumption patterns.

Total welfare also is more likely to rise with cost-based DP than with price discrimination. Discrimination misallocates output between markets, hence an increase in total output is necessary but not sufficient for total welfare to rise.[22] In contrast, cost-based DP can increase welfare when output remains constant, as with linear demand (Example 1),[23] or even when output decreases (Example 5), due to the favorable output reallocation to the lower-cost market.

4. Firms With Asymmetric Costs

Do asymmetries between firms introduce new forces, beyond demand-side factors such as pass-through, that alter the welfare properties of DP relative to UP? We extend our model to allow cost asymmetries between firms for a given market, in addition to cost differences across markets: firm i has costs (c_{iL}, c_{iH}), where $c_{iH} > c_{iL}$, for $i = 1, 2$. The firms are still assumed symmetric in demand: they produce either homogeneous products or symmetrically differentiated products.

4.1 Homogeneous Products

Consider two scenarios of cost asymmetries:

(1) *Global Cost Advantage*: the same firm, say firm 1, has a cost advantage in serving both consumer groups: $c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$;

(2) *Local Cost Advantage*: each firm has a cost advantage in serving a different group. Without loss of generality, let $c_{1L} < c_{2L}$ and $c_{1H} > c_{2H}$, with $\bar{c}_1 \leq \bar{c}_2$.

Global Cost Advantage

We adopt the standard assumption for Bertrand competition with asymmetric costs: the lower-cost firm can capture the market by pricing at the rival's marginal cost. Assume also that firms' costs are not too far apart, so the lower-cost firm sets price below its monopoly level, at

[22] Holmes (1989, fn.2) notes that this well-known result under monopoly also "holds for this oligopoly analysis."

[23] In Holmes' setting, where demand differs between markets, price discrimination can increase or decrease output even with linear demand, depending on his elasticity-ratio condition, which compares relative market-demand elasticities to relative cross-price elasticities. When output falls, total welfare also must fall, because price discrimination misallocates output.

the rival's cost. Under DP, competition occurs market-by-market and the equilibrium prices in the two markets are therefore:

$$p_L = \max\{c_{1L}, c_{2L}\} = c_{2L}; \qquad p_H = \max\{c_{1H}, c_{2H}\} = c_{2H}. \tag{26}$$

Under UP, we assume that the firm with the lower average cost can capture both markets by pricing at the other firm's *average cost.* Therefore, the equilibrium uniform price is given by

$$p^u = \max\{\bar{c}_1, \bar{c}_2\} = \bar{c}_2. \tag{27}$$

The next result shows that, while DP benefits consumers, profits can readily fall. The profit comparison depends on the difference in marginal costs of serving the two markets for firm 1 ($\Delta c_1 \equiv c_{1H} - c_{1L} > 0$) relative to firm 2 ($\Delta c_2 \equiv c_{2H} - c_{2L} > 0$).

Proposition 4 *For any given pair of costs $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ with $c_{1L} < c_{2L}$, $c_{1H} < c_{2H}$, and $\Delta c_i \equiv c_{iH} - c_{iL} > 0$, $i = 1, 2$:*
(i) $p^d = p^u$, and hence $S^d > S^u$;
(ii) with linear demand, $\pi^d > \pi^u$ if $\Delta c_1 > \Delta c_2$ and $\pi^d < \pi^u$ if $\Delta c_1 < \Delta c_2$;
(iii) relative to linear demand, $\pi^d - \pi^u$ and $W^d - W^u$ are higher if demand is strictly convex and lower if demand is strictly concave.

Part (*i*) is straightforward. Consider part (*ii*). With linear demand, total output as well as average price are the same under DP and UP, hence the change in firm 1's profit is determined entirely by the reallocation effect in (7). Firm 1's prices are set equal to firm 2's costs: $p_L = c_{2L}$ and $p_H = c_{2H}$. Thus, the difference in firm 1's profit margins under DP between markets L and H is $(c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = \Delta c_1 - \Delta c_2$. The output reallocation under DP raises profit if the margin is higher in market L, which occurs if $\Delta c_1 > \Delta c_2$, and lowers profit if $\Delta c_1 < \Delta c_2$.[24] Intuitively, firm 1 is harmed by being constrained to adopt a price differential larger than the difference in its costs.

[24] With linear demand, total welfare rises if $\Delta c_1 \geq \Delta c_2$, but can fall if $\Delta c_1 < \Delta c_2$. As $\Delta c_1 \to 0$, the output allocation under uniform pricing converges to the first-best, but is inefficient under differential pricing, since $p_{1H} - p_{1L} = \Delta c_2 > 0$, hence $W^d < W^u$.

Turning to part (*iii*), in market H where price rises under DP, output decreases less if demand is strictly convex instead of linear, while in market L where price falls under DP, output increases by more if demand is strictly convex instead of linear. Relative to linear demand, therefore, $\pi^d - \pi^u$ and $W^d - W^u$ are both higher if demand is strictly convex, and the conclusion is reversed if demand is strictly concave.

Local Cost Advantage

Now suppose firm 1 has the cost advantage for market L and firm 2 has the advantage for H: $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, with $\bar{c}_1 \leq \bar{c}_2$. Under DP, firm 1 serves market L at price $p_L = c_{2L} < \bar{c}_2$ and firm 2 serves market H at price $p_H = c_{1H} > c_{2H} > \bar{c}_2$. Under UP, we assume that each firm cannot refuse to serve its higher-cost market; it must be willing to sell in both markets or none. Suppose also that at equal prices ($p_1 = p_2$), if $\bar{c}_1 < \bar{c}_2$, firm 1 can capture both markets at price $\bar{c}_2$, while if $\bar{c}_1 = \bar{c}_2$, the firms split both markets equally. In both cases, the equilibrium uniform price is $p^u = \bar{c}_2$, and moving to DP lowers price in market L and raises price in market H, but raises average price.[25] The next result shows that while profit necessarily rises, consumer surplus can fall without requiring unusual demand conditions.

Proposition 5 *For any given pair of costs* $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ *with* $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, *and* $\bar{c}_1 \leq \bar{c}_2$:

(i) average price is higher under DP than under UP: $p^d \equiv \lambda p_L + (1 - \lambda) p_H > p^u$;

(ii) consumer surplus is higher under DP ($S^d > S^u$) *if cost differences within markets,* $\Delta_L \equiv c_{2L} - c_{1L}$ *and* $\Delta_H \equiv c_{1H} - c_{2H}$, *are small, but* $S^d < S^u$ *if* $c_{2H} - c_{2L}$ *is small;*

(iii) profits are always higher under DP: $\pi^d > \pi^u$.

Result (i) above can be understood as follows. The uniform price is determined by the firm with the higher *average* of the marginal costs across the two markets. Under DP, each market's price is set by the higher of the two firms' marginal costs for that market. Since cost heterogeneity is greater market-by-market than on average, the average price is higher under DP. The effect is similar to one noted by Dana (2012).

[25] We have considered a variant of this scenario, where firm 1 has lower cost to serve market A than market B and the reverse holds for firm 2. Differential pricing then raises price in both markets.

Consumer surplus is subject to opposing effects: it increases due to the price dispersion, but decreases due to the rise in average price. When the cost difference between firms within each market ($c_{2k} - c_{1k}$, $k = L, H$) is sufficiently small, the average price under DP converges to the uniform price, hence the price dispersion effect dominates and DP raises consumer surplus. On the other hand, if the cost difference between markets for firm 2 ($c_{2H} - c_{2L}$) is small, then $p^u = \bar{c}_2$ is close to c_{2L}, so that moving to DP lowers price in market L only slightly but raises price in market H substantially. However, DP can lower consumer surplus even when $c_{2H} - c_{2L}$ is not 'small'. For example, suppose $\lambda = 1/2$, $D(p) = 10 - p$, $c_{1L} = 3, c_{2L} = 4$, $c_{2H} = 6, c_{1H} = 7$. Then $\bar{c}_1 = \bar{c}_2 = 5 = p^u$, $p_L = 4$, $p_H = 7$, $p^d = 5.5 > p^u$, and $S^d = 11.25 < S^u = 12.5$.

Industry profits always rise with DP, for three reasons: as with monopoly, output is reallocated to the lower-cost market; in addition, DP now leads to each market being served by the efficient firm in that market (firm 2 replaces 1 in market H) and, furthermore, DP raises average price by relaxing the competitive constraint.

Summarizing, when a different firm has the cost advantage in each market, DP has opposing effects on consumer surplus. The price dispersion is beneficial, but the increase in average price is harmful. The first effect dominates if the cost heterogeneity across markets is large relative to that between firms in a given market, while the second effect dominates in the reverse case.

4.2 Differentiated Products With Linear Demands

Propositions 4 and 5 showed that DP can reduce profit or consumer surplus even with 'simple' demand functions—such as linear demand—when firms have asymmetric costs and produce homogeneous products, i.e., perfect substitutes. To check whether these findings may extend to imperfect substitutes, we consider differentiated products with the following linear demand system adapted from Shubik and Levitan (1980):

$$q_i = a - p_i + \gamma(p_j - p_i) \quad \text{for } i, j \in \{1, 2\} \ (j \neq i), \tag{28}$$

where $a > 0$, and $\gamma \in (0, +\infty)$ measures the degree of product substitutability. The products become unrelated (highly differentiated) as $\gamma \to 0$ and highly substitutable as $\gamma \to \infty$; with equal marginal costs c, firms' equilibrium price converges to c as $\gamma \to \infty$.

As in subsection 4.1, the two firms may have different costs of serving the same market; c_{1k} may differ from c_{2k}, for $k = L, H$, in arbitrary ways—including the cases of global or local cost advantage.

Proposition 6 *Suppose the demand system is given in (28). There exist critical values γ_1 and γ_2 such that:*
(i) when $\gamma < \gamma_1$, consumer surplus and industry profit are both higher under DP than under UP regardless of the cost asymmetry between firms;
(ii) when $\gamma > \gamma_2$, the following results in Propositions 4 and 5 hold: with global cost advantage ($c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$), DP reduces profit if $c_{1H} - c_{1L} < c_{2H} - c_{2L}$, whereas with local cost advantage ($c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$ and $\bar{c}_1 \leq \bar{c}_2$), DP reduces consumer surplus if $c_{2H} - c_{2L}$ is sufficiently small.

When the products are sufficiently differentiated ($\gamma < \gamma_1$), under both UP and DP the equilibrium is *interior,* with both firms producing positive outputs and the prices determined by the standard first-order conditions. The average prices under DP and under UP are then equal, as in the case of symmetric costs and linear demands (Example 1). Hence, consumer surplus is higher under DP, and equilibrium industry profit also is higher because it is a convex function of (c_1, c_2).

When products are sufficiently close substitutes ($\gamma > \gamma_2$), under both regimes we have a *corner equilibrium.* Under UP, firm 2 (the higher-cost firm here) sets price at marginal cost $\bar{c}_2$ while firm 1 captures the market by setting a limit price below $\bar{c}_2$ that induces zero demand for firm 2, and this limit price $\to \bar{c}_2$ as $\gamma \to \infty$; and similarly under DP the lower-cost firm in each market, L or H, sets a limit price to capture that market. Thus, as the products converge to perfect substitutes, the outcome converges to the homogeneous-products case, described in Proposition 4 for global cost advantage and in Proposition 5 for local cost advantage.[26]

[26] Chen, Li and Schwartz (2017, Proposition 5) considered an alternative linear-demands system: $q_i = a - p_i + \gamma p_j$ for $i \neq j = 1, 2$, $\gamma \in (0, 1)$. The corner solution is absent in that model because the products do not converge to perfect substitutes as $\gamma \to 1$. The equilibrium then is always interior, and DP increases consumer surplus and profits regardless of the cost configuration, consistent with our Proposition 6 for $\gamma < \gamma_1$. The qualitative difference between the two types of equilibria seems to be that in an interior equilibrium prices depend on both firms' costs, whereas in a corner equlibrium the price of the firm that makes positive sales is determined solely by the rival's marginal cost.

5. Conclusion

The welfare properties of uniform versus differential pricing in oligopoly when markets differ in costs of service have gone largely unexplored, despite the prevalence of industries where firms are constrained from adopting cost-based differential pricing. In a standard oligopoly setting where firms face symmetric demands, we showed that the effects of purely cost-based differential pricing on consumer welfare and profits depend on whether products are homogeneous or differentiated and whether firms are symmetric in costs or not.

With symmetric firms, if products are homogeneous then differential pricing obviously maximizes consumer welfare whereas uniform pricing does not, while profits are zero in both regimes. Under differentiated products, differential pricing increases consumer surplus and profits under conditions met by many standard demand functions. The systematic force driving higher profit is cost savings from reallocating output between markets by adjusting prices; consumers benefit from this price dispersion provided average price does not rise too much. Although profit can fall with differential pricing, unlike for monopoly, as can consumer surplus, such outcomes require demand conditions that seem rather stringent.

When firms have asymmetric costs, however, differential pricing can reduce profit or, under an alternative cost configuration, reduce consumer surplus even for standard demand functions such as linear demands.

The welfare effects of cost-based differential pricing in oligopoly markets are therefore quite subtle. By uncovering these effects and the underlying economic forces, this paper advances the understanding of a significant issue in economics—in parallel to the extensive studies on third degree price discrimination—and helps evaluate prevalent constraints on a common business practice.

Appendix

Proof of Remark 1. For homogeneous goods, the result is known from Waugh (1944). So we consider differentiated products. Define $g_2(y)$ as the consumer surplus from good 2 at price

y when good 1 is not available:

$$g_2(y) \equiv \int_y^{\infty} D_2(\infty, x)\, dx;$$

and $g_1(y)$ as the incremental consumer surplus from good 1 at price y when the other good is also available at the symmetric price y:

$$g_1(y) \equiv \int_y^{\infty} D_1(x, y)\, dx.$$

Since $g_2(y)$ is decreasing and convex, we have:

$$\lambda g_2(p_L) + (1-\lambda)\, g_2(p_H) > g_2(\lambda p_L + (1-\lambda)\, p_H) = g_2\left(p^d\right).$$

Since the goods are imperfect substitutes ($0 < \frac{\partial D_1}{\partial p_2} < -\frac{\partial D_1}{\partial p_1}$), $g_1(y)$ is decreasing:

$$\begin{aligned} g_1'(y) &= -D_1(y,y) + \int_y^{\infty} \frac{\partial D_1(x,y)}{\partial y} dx < -D_1(y,y) - \int_y^{\infty} \frac{\partial D_1(x,y)}{\partial x} dx \\ &= -D_1(y,y) - D_1(\infty, y) + D_1(y,y) = 0. \end{aligned}$$

Moreover, given the regularity condition (6), $-\frac{\partial D_1(y,y)}{\partial p_1} \geq -\int_y^{\infty} \frac{\partial^2 D_1(x,y)}{\partial y^2} dx$,

$$g_1''(y) = -\frac{\partial D_1(y,y)}{\partial p_1} - \frac{\partial D_1(y,y)}{\partial p_2} - \frac{\partial D_1(y,y)}{\partial p_1} + \int_y^{\infty} \frac{\partial^2 D_1(x,y)}{\partial y^2} dx > 0.$$

Thus, $\lambda g_1(p_L) + (1-\lambda)\, g_1(p_H) > g_1(\lambda p_L + (1-\lambda)\, p_H) = g_1\left(p^d\right)$. Therefore,

$$\begin{aligned} \Delta S &\equiv S^d - S^u \\ &= \lambda\left[g_1(p_L) + g_2(p_L)\right] + (1-\lambda)\left[g_1(p_H) + g_2(p_H)\right] - \left[g_1(p^u) + g_2(p^u)\right] \\ &= \left[\lambda g_1(p_L) + (1-\lambda)\, g_1(p_H) - g_1(p^u)\right] + \left[\lambda g_2(p_L) + (1-\lambda)\, g_2(p_H) - g_2(p^u)\right] \\ &> 0 \qquad \text{if} \quad p^d \leq p^u. \end{aligned} \tag{29}$$

■

Proof of Equation (7). The profit decomposition in Equation (7) is obtained as follows:

$$\begin{aligned}\Pi^d - \Pi^u &= [\lambda (p_L - c_L) q_L + (1-\lambda)(p_H - c_H) q_H] - [p^u - \lambda c_L - (1-\lambda) c_H] q^u \\ &= \lambda (p_L - c_L)(q_L - q^u) + \lambda (p_L - c_L) q^u + (1-\lambda)(p_H - c_H)(q_H - q^u) \\ &\quad + (1-\lambda)(p_H - c_H) q^u - (p^u - \lambda c_L - (1-\lambda) c_H) q^u \\ &= \left(p^d - p^u\right) q^u + \lambda (m_L - m_H)(q_L - q^u) + \lambda m_H (q_L - q^u) + (1-\lambda) m_H (q_H - q^u).\end{aligned}$$

■

Proof of Proposition 1. First, from the proof of Remark 1, consumer surplus when both markets have price p is

$$S(p,p) = \int_p^\infty D_1(x,p)\,dx + \int_p^\infty D_1(x,\infty)\,dx = g_1(p) + g_2(p),$$

$S'(p,p) < 0$, and $S''(p,p) > 0$ because, from (29) and letting $p^u = p^d = \lambda p_L + (1-\lambda) p_H$, for any $p_L < p_H$ we have

$$\lambda S(p_L, p_L) + (1-\lambda) S(p_H, p_H) > S(\lambda p_L + (1-\lambda) p_H).$$

Therefore

$$\begin{aligned} S'(p,p) &= -D_1(p,p) + \int_p^\infty \frac{\partial D_1(x,p)}{\partial p} dx - D_1(p,\infty) \\ &= -2D_1(p,p) - \int_p^\infty \left[\frac{\partial D_1(p,x)}{\partial x} - \frac{\partial D_1(x,p)}{\partial p}\right] dx = -D(p) - \delta(p) < 0, \end{aligned}$$

and $-\left[D'(p) + \delta'(p)\right] = S''(p,p) > 0$. It follows that

$$\hat{\eta} = -\frac{\left[D'(p^*) + \delta'(p^*)\right]}{\left[D(p^*) + \delta(p^*)\right]} > 0.$$

Furthermore,

$$s'(c) = S'(p^*, p^*)\, p^{*'}(c) = -\left[D(p^*) + \delta(p^*)\right] p^{*'}(c),$$

$$s''(c) = -\left[D'(p^*) + \delta'(p^*)\right]\left[p^{*'}(c)\right]^2 - \left[D(p^*) + \delta(p^*)\right] p^{*''}(c).$$

Next, from (18):

$$s''(c) = -\left[D'(p^*) + \delta'(p^*)\right]\left[p^{*'}(c)\right]^2 - \left[D(p^*) + \delta(p^*)\right]\left[\eta_r'(p^*) + \alpha'(p^*)\right]\left[p^{*'}(c)\right]^3 > 0$$

$$\Longleftrightarrow -\left[D'(p^*) + \delta'(p^*)\right] - \left[D(p^*) + \delta(p^*)\right]\left[\eta_r'(p^*) + \alpha'(p^*)\right]p^{*'}(c) > 0$$

$$\Longleftrightarrow -\frac{\left[D'(p^*) + \delta'(p^*)\right]p^*(c)}{\left[D(p^*) + \delta(p^*)\right]} - \left[\alpha'(p^*) + \eta_r'(p^*)\right]p^*(c)\,p^{*'}(c) > 0$$

$$\Longleftrightarrow -\left[\alpha'(p^*(c)) + \eta_r'(p^*(c))\right]\frac{p^*(c)}{2 - \eta_r(p^*(c)) - \alpha(p^*(c))} + \hat{\eta}(p^*(c)) > 0.$$

Therefore, if (20) holds, $s(c)$ is convex and

$$\begin{aligned} S^u &= S(p^u(\bar{c}), p^u(\bar{c})) = s(\bar{c}) = s(\lambda c_L + (1-\lambda)c_H) \\ &< \lambda s(c_L) + (1-\lambda)s(c_H) = \lambda S(p^*(c_L), p^*(c_L)) + (1-\lambda)S(p^*(c_H), p^*(c_H)) = S^d. \end{aligned}$$

Similarly, if (20) is reversed, then $s(c)$ is concave and DP lowers consumer surplus. ■

Proof of Proposition 2. We derive the condition for equilibrium profit $\pi^*(c)$ to be convex as follows:

$$\pi^{*''}(c) = \frac{dD_1(p^*, p^*)}{dp^*}p^{*'}(c)\left[p^{*'}(c)\eta_r - 1\right] + D_1(p^*, p^*))\left[p^{*''}(c)\eta_r + p^{*'}(c)\eta_r'\right] > 0$$

$$\underset{\text{since } D_1 = \frac{1}{2}D}{\Longleftrightarrow} \frac{D'(p^*(c))}{D(p^*(c))}p^{*'}(c)\left[p^{*'}(c)\eta_r - 1\right] + \left[\left(\eta_r' + \alpha'\right)\left[p^{*'}(c)\right]^3\eta_r + p^{*'}(c)\eta_r'\right] > 0$$

$$\Longleftrightarrow \frac{-D'(p^*(c))}{D(p^*(c))}\left[1 - p^{*'}(c)\eta_r\right] + \left[\left(\eta_r' + \alpha'\right)\left[p^{*'}(c)\right]^2\eta_r + \eta_r'\right] > 0$$

$$\Longleftrightarrow \eta(p^*)\left[1 - \frac{\eta_r(p^*)}{2 - \eta_r(p^*) - \alpha(p^*)}\right] + p^*\left[\frac{\eta_r(p^*)\left[\eta_r'(p^*) + \alpha'(p^*)\right]}{\left[2 - \eta_r(p^*) - \alpha(p^*)\right]^2} + \eta_r'(p^*)\right] > 0.$$

Denote the equilibrium profits of each firm under uniform pricing by $\pi^u = \pi^*(\bar{c})$, and under differential pricing by $\pi^d = \lambda\pi^*(c_L) + (1-\lambda)\pi^*(c_H)$. Then, when $\pi^{*''}(c) \gtreqless 0$,

$$\pi^u = \pi^*(\bar{c}) \lesseqgtr \lambda\pi^*(c_L) + (1-\lambda)\pi^*(c_H) = \pi^d.$$

Therefore, for $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, and $\eta = \eta(p)$: $\pi^{*''}(c) > 0$ or $\pi^d > \pi^u$ if (22) holds, and

$\pi^d < \pi^u$ if (22) is reversed. ■

Proof of Proposition 3. From (24),

$$w^{*\prime}(c) = -\left[D\left(p^*\right) + \delta\left(p^*\right)\right] p^{*'}(c) + \left[p^{*'}(c) - 1\right] D(p^*(c)) + \left[p^*(c) - c\right] D'(p^*(c))p^{*'}(c)$$
$$= p^{*\prime}(c)\left[D'(p^*(c))(p^*(c) - c) - \delta\left(p^*\right)\right] - D(p^*(c)).$$

But because $p^* \frac{D'(p^*(c))}{D(p^*(c))} = -\eta_{11} + \eta_{12}$ and $\frac{(p^*(c)-c)}{p^*} = \frac{1}{\eta_{11}}$, we have

$$\begin{aligned} w^{*\prime}(c) &= -D(p^*(c))\left[p^{*\prime}(c)\left(1 - \eta_r + \frac{\delta\left(p^*\right)}{D(p^*(c))}\right) + 1\right] \\ &= -D(p^*(c))\left[1 + p^{*\prime}(c)\left(1 - \hat{\eta}_r\right)\right], \end{aligned} \tag{30}$$

where $\hat{\eta}_r \equiv \left(\eta_r - \frac{\delta(p^*)}{D(p^*(c))}\right)$. It follows that

$$\begin{aligned} w^{*''}(c) &= -D'(p^*(c))p^{*'}(c)\left[1 + p^{*\prime}(c)\left(1 - \hat{\eta}_r\right)\right] - D(p^*(c))\left[p^{*''}(c)\left(1 - \hat{\eta}_r\right) - \left[p^{*\prime}(c)\right]^2 \hat{\eta}_r'\right] \\ &= -D'(p^*(c))p^{*'}(c)\left[1 + p^{*\prime}(c)\left(1 - \hat{\eta}_r\right)\right] - D(p^*(c))\left[\left(\eta_r' + \alpha'\right)\left[p^{*\prime}(c)\right]^3\left(1 - \hat{\eta}_r\right) - \left[p^{*\prime}(c)\right]^2 \hat{\eta}_r'\right] \\ &\gtreqless 0 \end{aligned}$$

$$\Longleftrightarrow \frac{-D'(p^*(c))}{D(p^*(c))}\left[1 + p^{*\prime}(c)\left(1 - \hat{\eta}_r\right)\right] - \left[\left(\eta_r' + \alpha'\right)\left[p^{*'}(c)\right]^2\left(1 - \hat{\eta}_r\right) - p^{*'}(c)\hat{\eta}_r'\right] \gtreqless 0 \Longleftrightarrow$$

$$\eta\left(p^*\right)\left[1 + \frac{1 - \hat{\eta}_r\left(p^*\right)}{2 - \eta_r\left(p^*\right) - \alpha\left(p^*\right)}\right] + p^*\left[\frac{-\left[\alpha'\left(p^*\right) + \eta_r'\left(p^*\right)\right]\left(1 - \hat{\eta}_r\left(p^*\right)\right)}{\left[2 - \eta_r\left(p^*\right) - \alpha\left(p^*\right)\right]^2} + \frac{\hat{\eta}_r'\left(p^*\right)}{2 - \eta_r(p^*) - \alpha(p^*)}\right] \gtreqless 0.$$

When $w^{*''}(c) \gtreqless 0$, we have

$$\begin{aligned} W\left(p^*\left(\bar{c}\right)\right) &= w^*(\bar{c}) = w^*(\lambda c_L + (1-\lambda)c_H) \\ &\lesseqgtr \lambda w^*(c_L) + (1-\lambda)w^*(c_H) = \lambda W\left(p^*\left(c_L\right)\right) + (1-\lambda)\, W\left(p^*\left(c_H\right)\right). \end{aligned}$$

■

Proof of Proposition 4. (i) From (26) and (27):

$$p^d \equiv \lambda p_L + (1-\lambda)p_H = \lambda c_{2L} + (1-\lambda)c_{2H} \equiv p^u.$$

Then $S^d > S^u$ holds, from Remark 1.

(ii) For profit we only need to consider firm 1, since the higher-cost rival earns no profit under either pricing regime. Given $p^d = p^u$, linear demand implies that total output also remains unchanged:

$$q^d = \lambda D(p_L) + (1-\lambda)D(p_H) = D(\lambda p_L + (1-\lambda)p_H) = D(p^u) = q^u.$$

With $p^d = p^u$ and $q^d = q^u$, (7) implies that $sign(\Pi^d - \Pi^u) = sign(m_L - m_H)$. Since $p_L = c_{2L}$ and $p_H = c_{2H}$, we have

$$m_L - m_H = (c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = c_{1H} - c_{1L} - (c_{2H} - c_{2L}) \equiv \Delta c_1 - \Delta c_2.$$

(iii) From (26) and (27), the prices p_L, p_H and p^u are determined by firm 2's marginal costs independent of the curvature of $D(p)$. Suppose $D(p)$ is strictly convex. Consider the linear demand $L(p)$ that is tangent to $D(p)$ at p^u.[27] Uniform pricing yields the same price and output with $L(p)$ or $D(p)$, hence the same profit and welfare. But under differential pricing, since $p^d = p^u$, outputs in both markets will be greater with $D(p)$ than with $L(p)$. Since firm 1's margins in both markets are positive ($p_L = c_{2L} > c_{1L}$, $p_H = c_{2H} > c_{1H.}$), profit and total welfare will be higher with $D(p)$ than with $L(p)$. The reverse holds if $D(p)$ is strictly concave.

■

Proof of Proposition 5. (i) Price. $p^d \equiv \lambda c_{2L} + (1-\lambda)\, c_{1H} > \lambda c_{2L} + (1-\lambda)\, c_{2H} \equiv \bar{c}_2 = p^u$.

(ii) Consumer surplus.

$$S^u = s(\bar{c}_2) = \int_{\bar{c}_2}^{\infty} D(p)\, dp,$$

$$S^d = \lambda s(c_{2L}) + (1-\lambda)\, s(c_{1H}) = \lambda \int_{c_{2L}}^{\infty} D(p)\, dp + (1-\lambda) \int_{c_{1H}}^{\infty} D(p)\, dp.$$

When $\Delta_L \equiv c_{2L} - c_{1L} \to 0$ and $\Delta_H \equiv c_{1H} - c_{2H} \to 0$, $\lambda c_{2L} + (1-\lambda)\, c_{1H} \to \bar{c}_2$, and hence, because $s(p)$ is strictly convex,

$$S^u = s(\bar{c}_2) \to s(\lambda c_{2L} + (1-\lambda)\, c_{1H}) < \lambda s(c_{2L}) + (1-\lambda)\, s(c_{1H}) = S^d.$$

[27] The ensuing argument is inspired by Malueg (1994).

On the other hand, when $c_{2L} \to c_{2H}$ so that $c_{2L} \to \bar{c}_2$,

$$S^d - S^u = \lambda \int_{c_{2L}}^{\bar{c}_2} D(p)\, dp - (1-\lambda) \int_{\bar{c}_2}^{c_{1H}} D(p)\, dp < 0.$$

(iii). Profits**.** Under UP, firm 2's profit is zero, but under DP, each firm earns positive profit. Total profits under the two regimes are

$$\begin{aligned} \Pi^u &= (\bar{c}_2 - \bar{c}_1) D(\bar{c}_2), \\ \Pi^d &= \pi_1^d + \pi_2^d = \lambda(c_{2L} - c_{1L}) D(c_{2L}) + (1-\lambda)(c_{1H} - c_{2H}) D(c_{1H}). \end{aligned}$$

Thus,

$$\begin{aligned} \Pi^d - \Pi^u &= \lambda(c_{2L} - c_{1L}) D(c_{2L}) - (\bar{c}_2 - \bar{c}_1) D(\bar{c}_2) + (1-\lambda)(c_{1H} - c_{2H}) D(c_{1H}) \\ &> \lambda(c_{2L} - c_{1L}) D(c_{2L}) - \lambda(c_{2L} - c_{1L}) D(\bar{c}_2) - (1-\lambda)(c_{2H} - c_{1H}) D(\bar{c}_2) \\ &= \lambda(c_{2L} - c_{1L}) \left[D(c_{2L}) - D(\bar{c}_2) \right] + (1-\lambda)(c_{1H} - c_{2H}) D(\bar{c}_2) > 0. \end{aligned}$$

■

Proof of Proposition 6. Following Section 4.1, suppose $\bar{c}_1 < \bar{c}_2$. Under UP, firm $i's$ profit function is:

$$\pi_i = (p_i - \bar{c}_i)\left(a - p_i + \gamma(p_j - p_i)\right).$$

Suppose $\gamma \leq \gamma^u \equiv \frac{3a + \bar{c}_1 - 4\bar{c}_2 + \sqrt{9a^2 - 2a\bar{c}_1 + \bar{c}_1^2 - 16a\bar{c}_2 + 8\bar{c}_2^2}}{2(\bar{c}_2 - \bar{c}_1)}$. Using the first order conditions, the equilibrium prices and outputs of firms $i \neq j = 1, 2$ are:

$$\begin{aligned} p_i^u &= \frac{a(2+3\gamma) + (1+\gamma)\left(\bar{c}_j\gamma + 2\bar{c}_i(1+\gamma)\right)}{4 + 8\gamma + 3\gamma^2}, \\ q_i^u &= \frac{(1+\gamma)\left(\bar{c}_j\gamma(1+\gamma) + a(2+3\gamma) - \bar{c}_i(2 + 4\gamma + \gamma^2)\right)}{4 + 8\gamma + 3\gamma^2} \geq 0. \end{aligned}$$

Note that if $\gamma > \gamma^u$, then $q_2^u < 0$ and the above p_i^u and q_i^u no longer form an equilibrium. Instead, the equilibrium will be a corner solution, described shortly.

Similarly, under DP, for $k = L, H$, there exists γ_k such that if $\gamma \leq \gamma_k$, the equilibrium prices and quantities of firms $i \neq j = 1, 2$ are:

$$\begin{aligned} p_{ik} &= \frac{a(2+3\gamma) + (1+\gamma)\left(c_{jk}\gamma + 2c_{ik}(1+\gamma)\right)}{4+8\gamma+3\gamma^2}, \\ q_{ik} &= \frac{(1+\gamma)\left(c_{jk}\gamma(1+\gamma) + a(2+3\gamma) - c_{ik}(2+4\gamma+\gamma^2)\right)}{4+8\gamma+3\gamma^2} \geq 0, \end{aligned}$$

while if $\gamma > \gamma_k$, and $c_{ik} > c_{jk}$, then $q_{ik} < 0$ and the equilibrium instead will be a corner solution.

Hence, when $\gamma \leq \gamma_1 \equiv \min\{\gamma^u, \gamma_L, \gamma_H\}$, the average prices under DP and UP are equal:

$$p_1^d = \lambda p_{1L} + (1-\lambda)\, p_{1H} = p_1^u; \qquad p_2^d = \lambda p_{2L} + (1-\lambda)\, p_{2H} = p_2^u.$$

Furthermore, it is straightforward to verify that when $\gamma \leq \gamma_1$, in equilibrium consumer surplus as a function of (q_1, q_2) and industry profit as a function of (c_1, c_2) are convex, implying that consumer surplus, industry profit and total welfare are all higher under DP than under UP.

Now turn to the case where $\gamma > \gamma_2 \equiv \max\{\gamma^u, \gamma_L, \gamma_H\}$. Suppose $\bar{c}_i < \bar{c}_j$. Under UP, the equilibrium is a corner solution in which $p_j^u = \bar{c}_j$ and firm i captures all consumers in both markets by setting a limit price p_i^u that induces zero demand from firm j:

$$q_j^u = a - p_j^u + \gamma(p_i^u - p_j^u) = 0,$$

where $p_i^u = \frac{(\gamma+1)\bar{c}_j - a}{\gamma}$ and $q_i^u = \frac{(a-\bar{c}_j)(1+2\gamma)}{\gamma}$.

Similarly, under DP, the equilibrium is a corner solution in which the higher cost firm sets price $p_{jk} = c_{jk}$ and the lower cost firm chooses price $p_{ik} = \frac{(\gamma+1)c_{jk} - a}{\gamma}$ that induces zero demand from firm j in market k.

Suppose firm 1 has global cost advantage with $c_{1k} < c_{2k}$ as in Proposition 4. Then, $p_2^u = \bar{c}_2$ and $p_1^u = \frac{(\gamma+1)\bar{c}_2 - a}{\gamma}$ under UP, and $p_{2k} = c_{2k}$ and $p_{1k} = \frac{(\gamma+1)c_{2k} - a}{\gamma}$ in market k under DP. Firm 2 receives zero profit under both DP and UP. For firm 1, we have $p_1^d - p_1^u = 0$ and $\Delta q_1 = 0$.

Thus, using (7), $\Delta\Pi = \Pi^d - \Pi^u = \pi_1^d - \pi_1^u$ has the same sign as $m_{1L} - m_{1H}$. Note that

$$
\begin{aligned}
m_{1L} - m_{1H} = (p_{1L} - c_{1L}) - (p_{1H} - c_{1H}) &= \left[\frac{(\gamma+1)c_{2L} - a}{\gamma} - c_{1L}\right] - \left[\frac{(\gamma+1)c_{2H} - a}{\gamma} - c_{1H}\right] \\
&= (c_{1H} - c_{1L}) - \frac{\gamma+1}{\gamma}(c_{2H} - c_{2L}) < 0
\end{aligned}
$$

holds if $\Delta c_1 < \frac{\gamma+1}{\gamma}\Delta c_2$. Thus, $\pi^d > \pi^u$ if $\Delta c_1 > \frac{\gamma+1}{\gamma}\Delta c_2$ and $\pi^d < \pi^u$ if $\Delta c_1 < \frac{\gamma+1}{\gamma}\Delta c_2$.

Next consider local cost advantage with $c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$, and $\bar{c}_1 \leq \bar{c}_2$. Under UP, $q_2^u = 0$ and $q_1^u = \frac{(a-\bar{c}_2)(1+2\gamma)}{\gamma}$. Consumer surplus can be computed as

$$
S^u = U(q_1^u, q_2^u) - p_1^u q_1^u - p_2^u q_2^u = \frac{(a-\bar{c}_2)(1+\gamma)(1+2\gamma)}{2\gamma^2}.
$$

Similarly, consumer surplus under DP can be computed as

$$
s_L^d = \frac{(a-c_{2H})(1+\gamma)(1+2\gamma)}{2\gamma^2}, \qquad s_H^d = \frac{(a-c_{1H})(1+\gamma)(1+2\gamma)}{2\gamma^2}.
$$

Therefore,

$$
\begin{aligned}
\Delta S &= S^d - S^u = \lambda s_L^d + (1-\lambda)s_H^d - S^u \\
&= \lambda\frac{(a-c_{2H})(1+\gamma)(1+2\gamma)}{2\gamma^2} + (1-\lambda)\frac{(a-c_{1H})(1+\gamma)(1+2\gamma)}{2\gamma^2} - \frac{(a-\bar{c}_2)(1+\gamma)(1+2\gamma)}{2\gamma^2} \\
&= \frac{(1+\gamma)(1+2\gamma)(1-\lambda)\left[(c_{2H}-c_{2L})^2\lambda - (2a - c_{1H} - c_{2H})(c_{1H} - c_{2H})\right]}{2\gamma^2},
\end{aligned}
$$

with $\Delta S < 0$ if $c_{2H} - c_{2L}$ is sufficiently small, and $\Delta S > 0$ if $c_{1H} - c_{2H}$ is sufficiently small. ■

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