Assigning Default Position for Digital Goods: Competition, Regulation, and Welfare

Yongmin Chen[†] Marius Schwartz[‡]

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Abstract. We analyze alternative ways to assign the default position for digital goods such as search engines. When two competing firms vie for the position through bidding, the higher-quality firm typically wins but delivers lower utility than the rival due to heightened monetization from exploiting consumer switching costs. However, increasing via regulation the rival's default share tends to raise profit and harm consumers, at least in the short run. Letting consumers choose the default benefits them in the short run but harms the weaker firm. Our findings highlight the subtle welfare tradeoffs in default assignment, an important and controversial policy issue.

Keywords: Default Position, Digital Goods, Competition, Regulation

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[†]University of Colorado Boulder; yongmin.chen@colorado.edu

[‡] Georgetown University; mariusschwartz@mac.com

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1. INTRODUCTION

This paper addresses an important controversy regarding key digital products: how to choose the supplier whose product will be preset as the default for consumers? In many situations where competing products vie for the default position, the selection is made by a third party that supplies a different good to the consumers. For example, the manufacturer of a PC or a mobile device may choose the browser, search engine, or other software that will be pre-installed. Although consumers may switch to a non-default product, doing so can entail switching costs that create significant inertia; many consumers lack the technical savvy to switch or the willingness to incur the hassle. Thus, the firm whose product obtains default status can gain a substantial competitive edge over rivals.

A striking example comes from the landmark case brought by the U.S. Department of Justice (DOJ) against Microsoft in the late 1990s for exclusionary practices against Netscape's Navigator browser, the main competitor to Microsoft's Internet Explorer browser. A key piece of the DOJ's evidence was the much larger growth in Explorer's market share at Internet Service Providers (ISPs) that agreed to promote Explorer exclusively: from 20% to 90% vs. from 20% to 30% at other ISPs (Dunham, 2006). Even where switching appears easy—"just a click away" for some digital products—the default position can be valuable, as evidenced by the large payments that firms are willing to make for this position.[1] Google reportedly pays hundreds of millions of dollars annually to be the default search engine on Mozilla's Firefox browser; billions annually to be the default search engine on Apple's Safari browser; and considerable sums to other parties such as wireless carriers (DOJ, 2020; Ostrovsky, 2023). The European Commission's (2018) Android decision, finding that Google foreclosed distribution outlets to competing search engines, flagged such payments to third parties, as did the DOJ's (2020) lawsuit.[2]

The Google search controversy offers a useful springboard for addressing some questions

[1] Switching between browsers in the late 1990s was more difficult than switching between search engines today, notably due to the slowness of downloading a second browser via a narrowband Internet connection.

[2] Both agencies also stressed Google's requirement that device manufacturers preinstall its search app and Chrome browser (an important outlet for Google Search) in order to obtain access to the Google Play Store.

of broader interest. Often, as with Google in search, one of the firms vying for the default position enjoys a quality advantage, for example due to initially superior technology. Critics worry that the leading firm may prolong its dominance by paying for default positions at key distribution outlets to deprive rivals of the scale needed to compete effectively. Google, or a similarly situated leading firm, might plausibly counter that its willingness to outbid rivals for default status derives only from its product superiority (e.g., Walker, 2020), because a firm that expects to retain more consumers than would a lower-quality rival typically is willing to spend more to attract consumers. For example, in Milgrom and Roberts (1986) greater expenditure on advertising acts as a signal of quality. Quality signaling also occurs in search models. Athey and Ellison (2011) and Chen and He (2011) analyze auctions of ad positions to competing firms where a higher position will be searched earlier by consumers.[3] Although firms charge equal prices, a higher-quality firm outbids lower-quality rivals for a higher position because increased exposure yields it more product matches than to such rivals and hence greater sales.

The greater-sales argument, however, may be less applicable to bidding for default position for search engines or some other digital products. If the default is won by an inferior firm, some of its customers may switch to the superior firm, which reduces the latter's willingness to pay for default status. Thus, it is not obvious that greater popularity alone would induce Google to outbid a rival for default.[4]

Moreover, Google's view that it offers a superior product is disputed by some critics. They argue that while Google may deliver more relevant search results, it offers a worse overall consumer experience than some other search engines because it engages in excessive

[3] Similarly, in the literature on auctions of advertising positions by an online platform (e.g., Edelman et al. 2007), an ad placed at a higher position will be seen by more consumers.

[4] Consider this simple example: a unit mass of consumers demand the product and are initially assigned to the default firm. Firm A provides higher quality than firm B, and both firms earn equal revenue per consumer, normalized to one. If firm A wins the default, it retains all consumers and firm B gets none. If firm B wins, a share q of consumers quit and move to firm A. Each firm's maximum bid equals the increase in the number of its patronizing consumers if if obtains the default position: firm B's maximum bid is $(1 - q) - 0$ and firm A's is $1 - q$, the same amount.

monetization, e.g., through intrusive tracking or by prioritizing ads over natural search results. They contend that Google's enduring market share dominance is attributable to its ubiquitous default position, not to superior quality. This, in turn, raises the question: If Google offers an inferior product, how can it outbid rivals for the default position?

Our paper addresses two broad issues. First, what are the characteristics of equilibrium when the default position is assigned through competitive bidding? In particular, does the high-quality firm necessarily win? Is consumer welfare higher in this case than it would be if the default were awarded to the lower-quality rival? Second, compared to the high-quality firm winning, what are the welfare effects of alternative regulatory schemes? Specifically, we consider assigning the default position for some share of consumers to one firm and the rest to the rival, or letting consumers choose their preferred default.

We tackle these issues using a parsimonious model that captures salient features of the search engine environment. Consumers choose between two competing suppliers of a given product that differ only in product quality.[5] Their values for the products are high enough so the market is fully covered. Each firm sets the level of a monetization activity, 'charge' for brevity, which harms a consumer but generates revenue as a general function of the charge. This formulation admits broad interpretations of the charge, monetary and/or non-monetary (see also de Cornière and Taylor, 2019, discussed later), as many digital products have zero price but firms can monetize them through other methods, including (unwanted) targeted advertising, selling consumer data to a third party, or using consumer information to engage in price discrimination for a related product. Consumers are presented with one product as the default, and can switch to the other product by incurring a private switching cost randomly drawn from a known probability distribution. They decide whether to switch by considering the firms' (exogenous) qualities and their (simultaneously-chosen) charges, as well as the private switching cost.

Under competitive bidding, a third party selects the default product and assigns the

[5] Our main model has fixed qualities, hence abstracts from concerns that Google maintains a quality advantage partly by denying rivals the scale needed to improve their quality. We will address this issue in a 'reduced form' manner in Section 4.3.

default position to the highest-bidding firm. In equilibrium, conditional on the default position being assigned to either firm, the default firm will exploit its sticky consumers by setting its charge such that consumers obtain lower utility than from the rival product—even when the default product has higher quality (Proposition 1).[6] This pattern is consistent with claims made by some Google critics noted earlier. Since the default product yields lower utility in equilibrium, some consumers will switch to the non-default product.

Firms bid for the default position anticipating the equilibrium outcomes under the two alternative default assignments. We first observe that a firm wins if and only if assigning the default to it rather than the rival results in higher *industry profit*. It is not obvious which default assignment yields higher industry profit in this asymmetric duopoly setting, hence which firm will win the default. Indeed, we provide an example where the lower-quality firm wins. Nevertheless, for broad classes of the revenue function and switching cost distribution, the higher-quality firm wins (Proposition 2 and Corollary 1). Henceforth, we take this outcome as the benchmark case under competitive bidding. Importantly, consumer surplus can be higher or lower than it would be if the default instead were awarded to the lower-quality firm (Corollary 2), in contrast to findings of some search articles discussed later. The shape of the switching cost distribution is crucial for which default assignment yields higher industry profit, as well as for the consumer surplus comparion, because each firm's residual demand and its slope are fully determined by the switching cost distribution.

An alternative to competitive bidding is to assign the default position through regulation. One possibility is equal shares, i.e. assign to each firm the default for half the consumers, and we characterize the resulting equilibrium (Proposition 3). The high-quality firm, firm A, now provides higher utility than its rival firm B—unlike when firm A wins the default for all consumers: now that firm A competes for B's default consumers, A's equilibrium charge will exceed B's charge by less than A's quality advantage. Interestingly, industry

[6] We assume there is no regulation that constrains firms' monetization charges. This allows us to focus on the assignment of default position and the resulting equilibrium characteristics. In practice, there could be regulations, especially for non-price charges, which would change the equilibrium outcomes. We briefly discuss such regulations in the concluding section.

profit is higher and consumer surplus is lower than under competitive bidding. Industry profit rises because when firm B holds the default for half the consumers (instead of none) it raises its charge substantially, inducing firm A to raise its charge as well. Thus, greater symmetry in firms' installed bases of sticky consumers softens competition, unlike greater symmetry in costs or quality which intensifies competition. Consumer surplus falls due to the softened competition, and because additional consumers are diverted to the lower-quality product. Total welfare can rise or fall. Extending this analysis, we consider setting A as the default to a share of consumers between one half and one, i.e. between equal shares and the competitive bidding outcome. Consumer surplus again tends to be lower—while industry profit can be either higher or lower—than under competitive bidding; and both welfare measures can vary non-monotonically with firm A's share (Proposition 4).

Departing from exogenous qualities, we briefly consider a scenario where quality can be improved by serving more consumers. This scenario is at the heart of the DOJ's (2020) complaint against Google. DOJ argues that search algorithms improve with the number of users due to learning via experimentation, and that by obtaining default status at leading distribution outlets Google deprives rival search engines of users, impairing their quality without necessarily raising its own quality as much. Evaluating this foreclosure argument in an equilibrium model is complex and beyond the scope of this paper. Nevertheless, using our basic model we illustrate conditions such that transferring a minority share of default positions to the weaker firm can raise consumer surplus.

Instead of assigning the default, a leading alternative approach is to let individual consumers choose their preferred default, as required by the European Union's (2022) Digital Markets Act. In our setting such a "choice screen" remedy, paradoxically, is worse for the weaker firm than even the bidding outcome where the higher-quality rival obtains the default position everywhere. While this stark result hinges on the specifics of our model (such as no consumer heterogeneity except in switching costs), the basic message is fairly robust. For consumer welfare, choice screen is likely to dominate regulatory assignment in the short run but not necessarily in the long run if learning effects are important, because the weaker firm's quality may not improve as much. Our findings underscore the intricacies in evaluating the welfare consequences of alternative assignment methods.

The rest of this paper proceeds as follows. After discussing related literature, Section 2 presents the model. Section 3 analyzes assignment of the default position through competitive bidding, while Section 4 considers assignment through regulation. Section 5 concludes.

Related Literature

To our knowledge, there is minimal work directly on our topic, but a large literature on various related themes. Here we only discuss some of the closest work.

Switching costs play a central role in our analysis. An extensive literature has studied competition in markets with consumer switching costs (e.g., Klemperer, 1987; Farrell and Klemperer, 2007).[7] Switching costs may arise due to time and effort needed to find a new supplier, learn about a new product, or set up a new product.[8] They are likely to vary across consumers, e.g., due to different time values or technical savvy. The literature often considers firms with equal product quality, and has shown that even for firms that offer ex ante homogeneous products, switching costs can create market power and soften price competition. In our model, firms differ in product quality and we consider alternative assignments of the default position. Notably, consumers' switching pattern will depend on which firm holds the default position—in addition to the firms' charges—and these foreseen switching patterns will themselves affect the firms' equilibrium bids.

Bidding for default is conceptually similar to an issue studied in some literature on ordered search—bidding for prominence, where a more prominent firm is searched earlier (e.g., Armstrong et al., 2009; Armstrong and Zhou, 2011; Athey and Ellison, 2011; Chen and He, 2011). There, a higher-quality firm is willing to pay more than a lower-quality rival for

[7] Our formulation of heterogeneous consumer switching costs follows the approach in Chen (1997). This approach has been used to analyze a variety of competition issues, including exclusionary contracts, e.g., Bedre-Defolie and Biglaiser (2017).

[8] Much of our analysis would also apply if switching costs were replaced by (degrees of) status quo bias. Such bias has been shown to be important (see Fletcher, 2023 and the references cited therein). We adopt the switching costs formulation primarily for purposes of welfare analysis. To illustrate the distinction, letting consumers choose their preferred default at the outset benefits them by avoiding switching costs but may be detrimental if decision making is onerous.

the most prominent position (akin to our default) because expected volume of sales rises with product quality (and the price may rise as well, e.g. Armstrong et al., 2009, section 3). Moreover, awarding the prominent position to the highest-quality firm maximizes total welfare and consumer surplus (even when price rises with quality). Intuitively, because the order of search is endogenous, prominence acts as a signal of high quality and guides consumers toward better products, benefitting both them and industry profits. Our economic environment differs from the search setting,[9] and can yield different outcomes: conceviably the low-quality firm may win the bidding and, more plausibly, when the high-quality firm wins, consumer surplus may be lower than under the alternative assignment.

Our paper shares some features with work on biased intermediation by de Cornière and Taylor (2019). They consider an intermediary that can shift demand between horizontally-differentiated symmetric sellers, not by assigning the default position but by providing a biased recommendation to uniformed consumers. One policy intervention they analyze ("neutrality") requires the intermediary to send half of the consumers to each of the two sellers. This policy eliminates bias but nevertheless harms consumers by softening competition, because sellers now compete only for the informed consumers instead of all consumers. The softening competition effect arises also in our setting when firms are assigned equal shares of default positions. (In our case consumers suffer further harm because consumption shifts to the lower-quality product, whereas equal shares are efficient in their setting.) Another similarity involves the modeling of how sellers compete for consumers of digital goods, which often involves instruments other than the usual product price. Our charge imposes equal disutility on each consumer, while generating a general revenue function per consumer. Their formulation is more general by allowing two instruments, a monetary price and a quality variable, that can impact consumer utility and profits differently. However, their demands (as functions of utility levels offered by the sellers) are linear due to the stan-

[9] In these prominence papers, each consumer faces numerous products online that she may search for a match, hence it is fairly natural to consider product quality as uncertain and that total transactions volume may vary significantly with the order of search. For the ubiquitously-used products we have in mind, we adopt a model of known product quality and where total transactions are fixed (the market is covered).

dard Hotelling framework, while we consider general demand functions (stemming from general distributions of switching costs). Correspondingly, our bidding equilibrium and its welfare effects admit richer possibilities.

Though not our main focus, we illustrate in a 'reduced form' manner that improving the weaker firm's quality by awarding it the default at some share of consumers can benefit consumers if learning effects are increasing and concave in the number of users. Relatedly, Hagiu and Wright (2023) provide a rich dynamic model of competition with data-enabled learning and show that mandated data sharing—whereby the leading firm in a given period must share its data with the laggard rival—can benefit consumers. Interestingly, this only occurs if the laggard is at a sufficiently large quality disadvantage.[10]

Closest to our work is the contemporaneous paper by Hovenkamp (2024), whose basic setting is similar. Our contributions are complementary. Hovenkamp includes elements absent from our paper, such as explicit treatment of advertisers as the source of revenue (linear in the number of consumers) and horizontal product differentiation between the firms. Additionally, he studies an issue that we do not: whether default payments from search engine providers to device manufacturers could induce lower device prices sufficient to reverse the otherwise harmful effect of assigning a default position relative to no default (choice screen). On the other hand, his analysis is more restrictive in some aspects, especially by assuming that consumer demands are linear (as functions of firms' advertising levels, our 'charges').[11] His results are the same as ours when the revenue function is linear and the switching cost distribution is uniform: assigning the default to the higher-quality firm yields higher industry profit and consumer surplus. However, a key message from our analysis is that the nature of switching cost distribution matters greatly for the industry profit and consumer surplus rankings under alternative default assignments.

[10] Although data sharing improves the laggard's quality without reducing the leader's quality, it can harm consumers by reducing the laggard's incentive to improve its quality via aggressive pricing to attract consumers. This disincentive effect is weak if the laggard is far behind (hence does not price aggressively), but dominates when qualities are close, a finding that derives from the explicitly dynamic analysis.

[11] The linear demands arise from the assumptions that consumers and advertisers are uniformly distributed on the Hotelling line and that the switching cost can only take two possible values.

2. MODEL

The market contains two firms, A and B, that may provide a product to a unit mass of consumers through a third party. To reduce notation, we denote the firms' products also by $i = A, B$. Each consumer demands one unit of the product from firm $i = A$ or B and obtains utility $u_i = v_i - x_i$, where $x_i \geq 0$ is i's action to monetize its product ('charge'), and $v_A - v_B \equiv \Delta > 0$ so that A has higher quality. Firm i earns revenue $r(x_i)$ per consumer from its charge x_i, where $r'(x) > 0$, and production cost is normalized to zero. If x is the usual price, then $r(x) = x$; but as noted in the Introduction, our formulation allows general monetization activities, such as unwanted advertising, that are common for digital products. Hence, $r(x)$ can have general forms including, for instance, concave or convex functions. We allow $r(0) \geq 0$, where $r(0)$ can be interpreted as a firm's monetizing revenue from activities that are neutral to consumers.

One of the two products is set as the default option for consumers by the third party. For instance, a PC manufacturer will preset the default search engine from among competing providers. If product i is the default, denoted by $D = i$, a consumer who wishes to use product $j \neq i$ will need to incur a switching cost s. Each consumer's switching cost is the realization of a random variable that has distribution function $F(s)$ and density function $f(s) > 0$ on $s \in [0, 1]$. We assume $\Delta \in (0, 1)$ which, given $s \in [0, 1]$, helps ensure that firm B will choose a positive charge when A is the default.

In our base model, the default position is allocated through competitive bidding. In a variant of the model we shall examine regulated assignments of the default position. The game with competitive bidding proceeds as follows:

First, the firm that bids higher is assigned the default position and pays the lower bid. Next, either $D = A$ or $D = B$ starts a subgame in which A or B is the default for all consumers and the two firms simultaneously choose x_A and x_B. Finally, consumers choose which product to patronize, after observing D, x_A, and x_B.[12] If $D = i$ and a consumer

[12] We can allow the possibility that a certain part of a firm's monetizing activities is not observed by consumers before they decide to use its product. Both firms would then set the maximium level for such

chooses product $j \neq i$, she needs to incur her personal switching cost.

A strategy of firm i specifies its bid for the default position and its choice of x_i conditional on the assignment of the default position. A consumer's strategy specifies her decision on which product to use, based on D, x_A, x_B and her realized s. We study the subgame perfect Nash equilibrium of this market game, where the strategies of the firms and consumers induce a Nash equilibrium in every subgame.

We assume that v_A and v_B are both high enough to ensure full market coverage in equilibrium.[13] Unless stated otherwise, we further assume condition (C) below, where $\rho(x) \equiv r(x)/r'(x)$:

$$\rho'(x) > 0, \quad \rho''(x) \leq 0; \qquad \text{and} \qquad f'(s) \geq 0, \quad f''(s) \leq \left[f'(s)\right]^2 / f(s), \tag{C}$$

with $\rho(\Delta) f(0) < 1$, and $r(0)$ is equal or sufficiently close to zero.

Under condition (C), $\rho(x)$ is increasing and concave, while $f(s)$ is non-decreasing and log-concave. Hence, $r(x)$ and $f(s)$ each can be a convex or concave function. The additional requirement, $\rho(\Delta) f(0) < 1$, ensures that firm A's quality lead is not too large relative to switching costs, which, together with $r(0)$ being small enough, guarantees an interior equilibrium with positive charges by both firms under either default assignment.[14] In addition, condition (C) ensures that in our model x_A and x_B will be strategic complements, as in Bulow et al. (1985).

Condition (C) holds for broad classes of $r(x)$ and $F(s)$ functions, including (see Appendix) the following *power functions*:

unobservable activities and consumers would rationally expect this. We may thus consider the x in our model as the observable monetizing charge beyond the unobservable level.

[13] For example, if $\frac{r'(1)}{r(1)} < \max\left\{\frac{f(\Delta)}{F(\Delta)}, \frac{f(\Delta)}{1-F(\Delta)}\right\}$, then the market coverage condition is satisfied if $v_A > v_B > 1$. The full-market coverage assumption seems reasonable for the type of goods we are interested in, such as the Internet search engine.

[14] If $\rho(\Delta)$ or $r(0)$ were high, firm A may optimally charge Δ when $D = A$ and $x_B = 0$, in order to retain all the consumers. This is ruled out by our assumption. We could extend our analysis to allow $x_i < 0$, without the need to impose parameter restrictions that ensure $x_i > 0$ in equilibrium. However, to reduce the number of cases, we focus on the more relevant scenarios where there are monetization charges that are undesirable to consumers in equilibrium.

$$r(x) = ax^m, \qquad \text{and} \qquad F(s) = s^n, \tag{C1}$$

where $a > 0$, $m > \Delta \in (0,1)$, $n \geq 1$, and $n = 1$ if $m < 1$.[15] We will sometimes use a special case of (C1) with $m = n = 1$, denoted the *Linear-Uniform* case:

$$r(x) = ax, \qquad \textit{and} \qquad F(s) = s.$$

3. MARKET EQUILIBRIUM

We first characterize the equilibrium choices of x_A and x_B under a given assignment of the default position, $D = A$ or $D = B$, and then analyze the firms' equilibrium bidding incentives and equilibrium default assignment. Later we provide welfare results.

3.1 Equilibrium when A or B is the Default

First, consider the subgame where $D = A$. In this case, a consumer with switching cost (or 'type') s will remain with firm A if

$$v_A - x_A \geq v_B - x_B - s$$

and will switch to B otherwise. The consumer who is indifferent between A and B is $s = x_A - x_B - \Delta$, hence the profit functions of the two firms under $D = A$ are

$$\pi_A = r(x_A)[1 - F(x_A - x_B - \Delta)]; \qquad \pi_B = r(x_B) F(x_A - x_B - \Delta). \tag{1}$$

Next, consider the subgame where $D = B$. A consumer with type s will remain with B if

$$v_B - x_B \geq v_A - x_A - s$$

and will switch to A otherwise. The indifferent consumer is $s = x_B - x_A + \Delta$, hence the profit functions under $D = B$ are

$$\pi_A = r(x_A) F(x_B - x_A + \Delta), \qquad \pi_B = r(x_B)[1 - F(x_B - x_A + \Delta)]. \tag{2}$$

[15] We could extend (C1) to allow $r(x) = ax^m + b$, but $b \geq 0$ needs to be small enough to ensure positive charges in equilibrium, depending on m and n. For example, if $m = n = 1$, then the condition on b is $0 \leq b < \frac{1-\Delta}{3a}$. For convenience, we set $b = 0$ in (C1). Chen and Schwartz (2023) allow $b > 0$.

Denote the equilibrium charges by $\hat{x}_A$, $\hat{x}_B$ when $D = A$, and $\tilde{x}_A$, $\tilde{x}_B$ when $D = B$. Denote also the marginal switching consumer by $\hat{\sigma} \equiv \hat{x}_A - \hat{x}_B - \Delta$ when $D = A$ and $\tilde{\sigma} \equiv \tilde{x}_B - \tilde{x}_A + \Delta$ when $D = B$. The result below references equations (3) and (4), which are based on the first-order conditions for the equilibrium charges (see proof of Proposition 1):[16]

$$\frac{r'(\hat{x}_A)}{r(\hat{x}_A)} = \frac{f(\hat{\sigma})}{1 - F(\hat{\sigma})}, \qquad \frac{r'(\hat{x}_B)}{r(\hat{x}_B)} = \frac{f(\hat{\sigma})}{F(\hat{\sigma})}; \tag{3}$$

$$\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} = \frac{f(\tilde{\sigma})}{F(\tilde{\sigma})}, \qquad \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})}. \tag{4}$$

Proposition 1 *Assume condition (C). Under either default assignment, there exists a unique equilibrium, where both firms set positive charges, the default product yields lower utility than the other product, some consumers switch to the other product, and there is less switching when the high-quality product is the default. Formally:*
(i) When $D = A$, $(\hat{x}_A, \hat{x}_B)$ *uniquely solve (3),* $\hat{x}_A - \hat{x}_B > \Delta$, *and* $F(\hat{\sigma}) < \frac{1}{2}$.
(ii) When $D = B$, $(\hat{x}_A, \hat{x}_B)$ *uniquely solve (4), and* $\tilde{x}_A - \tilde{x}_B < \Delta$; *if* $F(\Delta) \leq \frac{1}{2}$ *then* $\tilde{x}_A \leq \tilde{x}_B$ *and* $F(\tilde{\sigma}) \leq \frac{1}{2}$, *but if* $F(\Delta) > \frac{1}{2}$ *then* $\tilde{x}_A > \tilde{x}_B$ *and* $F(\tilde{\sigma}) > \frac{1}{2}$.
(iii) $0 < \hat{\sigma} < \tilde{\sigma} < 1$; $\hat{x}_A > \tilde{x}_B$ *and* $\tilde{x}_A > \hat{x}_B$.

For a given assignment of the default position, the equilibrium has several noteworthy features. First, the default product yields lower consumer surplus than the non-default product: (i) when $D = A$, $\hat{x}_A - \hat{x}_B > \Delta \Rightarrow v_A - \hat{x}_A < v_B - \hat{x}_B$; and (ii) when $D = B$, $\tilde{x}_A - \tilde{x}_B < \Delta \Rightarrow v_B - \tilde{x}_B < v_A - \tilde{x}_A$. The default firm clearly will not offer higher surplus than the rival because starting from such a case it could raise its charge while retaining all customers. At equal surplus, the default firm would still retain all consumers since switching requires some cost, but under Assumption (C) it prefers to raise its charge, while ceding some consumers with low switching costs to the rival.[17] The property that product A offers

[16] Unless stated otherwise, proofs for formally-presented results are contained in the Appendix.

[17] Since switching costs are heterogeneous, each firm faces a downward-sloping (not horizontal) demand, yielding the familiar outcome for Bertrand competition with imperfect substitutes where both firms earn positive margins.

lower utility in equilibrium when it holds the default, even though it has higher quality / product value, differs from many other settings. It is consistent with perceptions of some critics, discussed in the Introduction, that Google delivers a worse consumer experience due to excessive monetization.

Second, while switching costs deter some consumers from moving to the non-default product, other consumers do switch in equilibrium and receive higher surplus (net of their switching costs) than non-switchers. As might be expected, fewer consumers will switch when A is the default, $\hat{\sigma} < \tilde{\sigma}$. The difference between these threshholds can be expressed as $\hat{\sigma} - \tilde{\sigma} = (\hat{x}_A - \hat{x}_B) - (\tilde{x}_B - \tilde{x}_A) - 2\Delta$. Although $\hat{x}_A - \hat{x}_B > \tilde{x}_B - \tilde{x}_A$, i.e. the default product's charge exceeds the rival's charge by more when $D = A$ than when $D = B$, this effect is outweighed by A's quality advantage, hence $\hat{\sigma} < \tilde{\sigma}$. Consequently, the deadweight loss from switching costs is lower when the high-quality product is the default.

The equilibrium charges under each assignment, together with the distribution of switching costs, determine the allocation of consumers (via $\hat{\sigma}$ and $\tilde{\sigma}$), hence firms' profits. Denote the equilibrium profits of A and B by (i) $\hat{\pi}_A$ and $\hat{\pi}_B$ when $D = A$ and (ii) $\tilde{\pi}_A$ and $\tilde{\pi}_B$ when $D = B$. These foreseen profits determine each firm's gain g_i from winning the default position and, hence, its maximum bid:

$$g_A = \hat{\pi}_A - \tilde{\pi}_A \qquad g_B = \tilde{\pi}_B - \hat{\pi}_B.$$

The maximum bids satisfy

$$g_A - g_B \gtreqless 0 \Longleftrightarrow \hat{\pi}_A - \tilde{\pi}_A \gtreqless \tilde{\pi}_B - \hat{\pi}_B \Longleftrightarrow \hat{\pi}_A + \hat{\pi}_B \gtreqless \tilde{\pi}_A + \tilde{\pi}_B.$$

We thus immediately have the following:

Remark 1 *Firm $i \in \{A, B\}$ is willing to bid more than the rival, and hence in equilibrium $D = i$, if and only if industry profit $\Pi \equiv \pi_A + \pi_B$ is higher when $D = i$.*

We will use Remark 1 to analyze which firm will win the bidding. The role of industry profit can be grasped as follows. Moving from $D = B$ to $D = A$ increases firm A's profit but decreases firm B's profit, and firm A outbids B if and only if its gain from this move exceeds

B's loss, i.e., if industry profit is higher under $D = A$. The same logic underlies Gilbert and Newbery's (1982) classic result that an incumbent monopolist would outbid a potential entrant for a single innovation or, more generally, for any single asset needed to enter. (See also Tirole, 1988.) The market remains a monopoly if the incumbent wins the bidding but becomes a duopoly if the entrant wins, and the incumbent wins under the fairly weak condition that industry profit is higher under monopoly (over both technologies/products), because monopoly avoids rent dissipation from duopoly competition. Our comparison of industry profits is more complex, as it involves alternative asymmetric duopoly regimes.

Before addressing that comparison in Section 3.2 below, consider the hypothetical case where $x_A = x_B \equiv x_E$ so that $r(x_A) = r(x_B) = r(x_E) = \bar{r}$ is a constant under either $D = A$ or $D = B$. This could be the case, for example, if there is regulation that limits the firms' monetization actions to some common level. Then, from (1) and (2), $\widehat{\Pi} = \bar{r} = \widetilde{\Pi}$. The result below follows immediately from Remark 1.

Remark 2. *If x_A and x_B are constrained to take an equal value $x_E \geq 0$, so that each firm earns the same revenue per consumer $r(x_E)$, then their maximum bids are equal, $g_A = g_B$.*

Intuitively, if firms earn the same revenue per consumer then industry profit is unaffected when consumers are redistributed between the two firms, given that all consumers would purchase in all cases (the market always is covered). Hence, a higher consumer value for A's product in itself does not imply that A will outbid the rival for the default position—generalizing the simple example from the Introduction.

3.2 Equilibrium Assignment of the Default Position

With endogenous choices of x, equilibrium industry profits when $D = A$ and $D = B$ are

$$\widehat{\Pi} = r(\hat{x}_A)[1 - F(\hat{\sigma})] + r(\hat{x}_B) F(\hat{\sigma}), \qquad \widetilde{\Pi} = r(\tilde{x}_B)[1 - F(\tilde{\sigma})] + r(\tilde{x}_A) F(\tilde{\sigma}). \quad (5)$$

It is not obvious which default assignment generates higher industry profit, hence which firm will win the bidding. The default firm earns higher revenue under $D = A$ than under $D = B$, as its charge is higher ($\hat{x}_A > \tilde{x}_B$) and so is its share of consumers (since $\hat{\sigma} < \tilde{\sigma} \Longrightarrow$

$[1 - F(\hat{\sigma})] > [1 - F(\tilde{\sigma})]$), but the non-default firm's revenue is lower under $D = A$ (since $\hat{x}_B < \tilde{x}_A$ and $F(\hat{\sigma}) < F(\tilde{\sigma})$). Nevertheless, we can establish the following:

Proposition 2 *Assume condition (C). Then industry profit is greater when the higher quality product is the default if the revenue function is not too concave. Formally, there is some (small) $\varepsilon > 0$ such that $\widehat{\Pi} > \widetilde{\Pi}$ if $r''(x) \geq -\varepsilon$.*

The proof (in the appendix) first establishes that the sum of the firms' charges satisfies $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$, and the ranking of charges further satisfies

$$\hat{x}_B < \max\{\tilde{x}_B,\ \tilde{x}_A\} < \hat{x}_A. \tag{6}$$

Then, because $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$ and $F(\hat{\sigma}) < F(\tilde{\sigma})$, it follows that $\{\hat{x}_B, \hat{x}_A\}$ is a mean-increasing spread of $\{\tilde{x}_A, \tilde{x}_B\}$, so that

$$[1 - F(\hat{\sigma})]\,\hat{x}_A + F(\hat{\sigma})\,\hat{x}_B > [1 - F(\tilde{\sigma})]\,\tilde{x}_B + F(\tilde{\sigma})\,\tilde{x}_A.$$

Hence, $\widehat{\Pi} > \widetilde{\Pi}$ if $r(x)$ is not too concave (i.e., $r''(x) \geq -\varepsilon$).

Proposition 2 can be illustrated with classes of $r(x)$ and $F(s)$ that are power functions given by (C1), hence satisfy condition (C).

Corollary 1 *Suppose $r(x)$ and $F(x)$ are given by (C1). Then $\widehat{\Pi} > \widetilde{\Pi}$.*

When $m \geq 1$, $r(x) = ax^m$ is convex, and from Proposition 2 industry profit is higher under $D = A$ than under $D = B$. But if $m < 1$, $r(x)$ is strictly concave, and yet still $\widehat{\Pi} > \widetilde{\Pi}$ for $\Delta < m < 1$, which allows for a wide range of concavity: $r''(x) = am(m-1)x^{m-2} < 0$. To understand this, notice that the comparison of industry profit depends both on the dispersion of $\{\hat{x}_B, \hat{x}_A\}$ relative to $\{\tilde{x}_B, \tilde{x}_A\}$ and on the degree of concavity. In the case here, it appears that when $r(x)$ becomes more concave, $\{\hat{x}_B, \hat{x}_A\}$ also become less dispersed relative to $\{\tilde{x}_B, \tilde{x}_A\}$, which offsets the effect of increasing concavity so that $\widehat{\Pi} > \widetilde{\Pi}$. For instance, when $F(s) = s$: $\hat{x}_A - \tilde{x}_B = \frac{2}{3}\Delta = \tilde{x}_A - \hat{x}_B$ if $m = 1$, while $\hat{x}_A - \tilde{x}_B = \frac{1}{2}\Delta = \tilde{x}_A - \hat{x}_B < \frac{2}{3}\Delta$ if $m = \frac{1}{2}$, which partly explains why $\widehat{\Pi} > \widetilde{\Pi}$ for all $m > \Delta$ in this case.

If condition (C) is violated, e.g. when $F(s) = s$ (uniformly distributed switching costs) but $\rho''(x) > 0$, we can have $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$. However, $\widehat{\Pi}$ can still be higher than $\widetilde{\Pi}$ even when $r(x)$ is concave, as in the example below:

Example 1 *Suppose* $F(s) = s$ *and* $r(x) = e^{-\frac{1}{x}}$*. Then* $r''(x) = -e^{\frac{1}{x}} \frac{2x+1}{x^4}$, $\rho'' = 2 > 0$, *and* $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$*. Despite this, numerical analysis indicates that* $\widehat{\Pi} > \widetilde{\Pi}$ *for various values of* Δ*. For instance, if* $\Delta = 0.5$*, then* $\hat{x}_A = 0.943$, $\hat{x}_B = 0.333$, $\hat{\sigma} = 0.11$; $\tilde{x}_A = 0.707 = \tilde{x}_B$, $\tilde{\sigma} = 0.5$; *and* $\widehat{\Pi} = 0.314 > \widetilde{\Pi} = 0.243$.

In this example, although $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, $\hat{x}_A$ is much higher than $\hat{x}_B$ and $F(\hat{\sigma})$ is much below $F(\tilde{\sigma})$, so that $r(\hat{x}_A)$ is weighted much more than $r(\hat{x}_B)$ under $D = A$, versus equal weights under $D = B$ for $r(\tilde{x}_A)$ and $r(\tilde{x}_B)$ (since $F(\tilde{\sigma}) = \frac{1}{2}$). As a result, $\widehat{\Pi}$ is higher than $\widetilde{\Pi}$ even though $r(x)$ is concave. Specifically, if A holds the default, the sum of firms' charges is lower $(0.943 + 0.333 < 0.707 + 0.707)$ because the non-default firm's charge is much lower ($\hat{x}_B = 0.333 << 0.707 = \tilde{x}_A$); but the default firm's charge is higher ($\hat{x}_A = 0.943 > 0.707 = \tilde{x}_B$) and applies to many more consumers since less switching occurs than when B is the default ($\hat{\sigma} = 0.11 < 0.5 = \tilde{\sigma}$). The ranking $\widehat{\Pi} > \widetilde{\Pi}$ holds also for all other values of Δ that we checked in Example 1 and, for instance, if $r(x) = e^{-\frac{1}{2x}}$.

Proposition 2, Corollary 1, and Example 1 suggest that under fairly mild conditions, industry profit is higher if the default position goes to the high-quaity firm, $\widehat{\Pi} > \widetilde{\Pi}$. However, when the switching-costs distribution is skewed towards low values ($f'(s) < 0$), it is possible that $\widehat{\Pi} < \widetilde{\Pi}$, as in the next example where the revenue function is linear:

Example 2 *Suppose* $r(x) = ax$ *for* $a > 0$ *and* $F(s) = s^{0.7}$*, so* $f'(s) < 0$ *violating condition (C). In this case,* $\widetilde{\Pi} < \widehat{\Pi}$ *if, for instance,* $\Delta = 0.3$*, where* $\hat{x}_A = 0.5832$, $\hat{x}_B = 0.1666$, $\hat{\sigma} = 0.1166$; $\tilde{x}_A = 0.4964$, $\tilde{x}_B = 0.5439$, $\tilde{\sigma} = 0.3475$; *and* $\widehat{\Pi} = 0.4906a < \widetilde{\Pi} = 0.5213a$.

In Example 2, $\rho''(x) = 0$ but $f'(s) < 0$, which leads to a lower total charge under $D = A$ than under $D = B$: $\hat{x}_A$ is only slightly higher than $\tilde{x}_B$ while $\tilde{x}_A$ is much higher than $\hat{x}_B$ ($\hat{x}_A > \tilde{x}_B > \tilde{x}_A >> \hat{x}_B$). Industry profit could still be higher under $D = A$ because $\hat{\sigma} < \tilde{\sigma}$ so that $r(\hat{x}_A)$ is weighted more heavily than $r(\tilde{x}_B)$. However, the mass of consumers with

$s < \hat{\sigma}$ is greater when $f(s)$ is decreasing than when it is increasing, and hence for a given $\hat{\sigma}$ more consumers will switch to B (under $D = A$) when $f(s)$ is decreasing—reducing the weight on $r(\hat{x}_A)$ sufficiently to yield a lower weighted average of $r(\hat{x}_A)$ and $r(\hat{x}_B)$ than under $D = B$. Therefore, when $f'(s) < 0$, it is possible that

$$\widehat{\Pi} = r(\hat{x}_A)[1 - F(\hat{\sigma})] + r(\hat{x}_B) F(\hat{\sigma}) < r(\tilde{x}_B)[1 - F(\tilde{\sigma})] + r(\tilde{x}_A) F(\tilde{\sigma}) = \widetilde{\Pi}.$$

Similar reasoning suggests why industry profit is higher when A is the default, $\widehat{\Pi} > \widetilde{\Pi}$, if (C1) is satisfied, so that $f'(s) \geq 0$ (see Corollary 1). With $f'(s) \geq 0$, fewer consumers will switch from A, increasing the weight on per-consumer revenue from the highest charge, $r(\hat{x}_A)$.

Therefore, whereas typically $\widehat{\Pi} > \widetilde{\Pi}$, the comparison of industry profits is subtle**;** although the three elements of our model—$r(x)$, $F(s)$, and Δ—all matter for the profit ranking, the skewness of switching-cost distribution plays an especially important role.

Consumer Surplus and Total Welfare

Which default assignment will result in higher consumer surplus? In the Appendix (see proof of Corollary 2) we show that consumer surplus takes the forms in (7):

$$\widehat{S} = v_A - \hat{x}_A + \int_0^{\hat{\sigma}} F(s)\, ds; \qquad \widetilde{S} = v_B - \tilde{x}_B + \int_0^{\tilde{\sigma}} F(s)\, ds. \tag{7}$$

In each case, consumer surplus equals the surplus that all consumers would get if they stayed with the default product ($v_A - \hat{x}_A$ or $v_B - \tilde{x}_B$), plus the integral term denoting the gain to those consumers who switch.[18] This gain arises because, from Proposition 1(i) and (ii), the default product offers lower utility in equilibrium than the rival product. The difference in consumer surplus under the two default assignments is

[18] The integral term is the difference in consumer utility from the two products minus the switching costs. When, say, $D = A$, $\int_0^{\hat{\sigma}} F(s)\, ds = \hat{\sigma} F(\hat{\sigma}) - \int_0^{\hat{\sigma}} s f(s)\, ds$, where $\hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta$ is the gross gain to any consumer from switching to B (hence also denotes the consumer indifferent between between remaining at A or incurring $s = \hat{\sigma}$ to switch), while $\int_0^{\hat{\sigma}} s f(s)\, ds$ is the total switching costs.

$$\widehat{S} - \widetilde{S} = [\Delta - (\hat{x}_A - \tilde{x}_B)] - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds. \tag{8}$$

The square-bracketed term is the difference in utilities of all consumers had they stayed with the default product under $D = A$ compared to $D = B$: A's quality advantage, $\Delta \equiv v_A - v_B$, minus A's charge premium when A holds the default position compared to B's charge when B holds the default. The integral term is the extra gain to switchers under regime $D = B$ compared to $D = A$, where $\tilde{\sigma} > \hat{\sigma}$ from Proposition 1(iii).

Total welfare—the sum of industry profit and consumer surplus—under the alternative default assignments is

$$\widehat{W} = \widehat{\Pi} + \widehat{S}; \qquad \widetilde{W} = \widetilde{\Pi} + \widetilde{S}.$$

To obtain clear welfare comparisons under the alternative assignments, we now consider some special cases of $F(s)$ and $r(x)$ that satisfy (C1), hence $\widehat{\Pi} > \widetilde{\Pi}$ (Corollary 1) so the default position would go to firm A under competitive bidding. We provide sufficient conditions for consumer surplus and for total welfare also to be higher under this default assignment, or under the alternative $D = B$:

Corollary 2 *Suppose (C1) holds. (i) If $n = 1$ (i.e. $F(s) = s$), then $\widehat{S} > \widetilde{S}$; but (ii) if $m = 1$ and $n = 2$ (i.e. $r(x) = ax$ and $F(s) = s^2$), then $\widehat{S} < \widetilde{S}$, while $\widehat{W} \gtreqless \widetilde{W} \iff a \gtreqless a(\Delta)$ $\in (0, 0.45)$.*

Consumer surplus tends to be higher under $D = A$ than under $D = B$ when the sum of the firms' charges ("total charge" $x_A + x_B$) is not (much) higher under $D = A$.[19,20] However, if the total charge is sufficiently higher when A is the default, then $\widehat{S} < \widetilde{S}$ is

[19] This holds, for instance, under (C1) with $m = n = 1$, i.e. the *Linear-Uniform* case, where the total charges under $D = A$ and $D = B$ are equal. $\widehat{S} > \widetilde{S}$ also under (C1) if $n = 1$ and $m \neq 1$, as well as in Examples 1-2, where (C1) is violated and $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$.

[20] Notice that if the total charge is higher under $D = A$, then $v_A - \hat{x}_A - \hat{x}_B - (v_B - \tilde{x}_B - \tilde{x}_A)$ is reduced, which makes it more likely that $\widehat{S} < \widetilde{S}$; but $\tilde{\sigma} - \hat{\sigma} = v_A - \tilde{x}_A - (v_B - \tilde{x}_B) - [v_B - \hat{x}_B - (v_A - \hat{x}_A)]$ may also be lower, which would reduce the difference in the mass of consumers who switch to benefit from the non-default product's higher utility; so, it is still possible that $\widehat{S} > \widetilde{S}$.

possible. As Corollary 2(ii) shows, when $r(x) = ax$ and $F(s) = s^2$, we have $\widehat{S} < \widetilde{S}$. In this case, conditional on holding the default position, firm A's charge exceeds B's by more than A's quality advantage: $\hat{x}_A - \tilde{x}_B = \frac{10}{8}\Delta$ (see Proof of Corollary 2(ii)), so in (8) $[\Delta - (\hat{x}_A - \tilde{x}_B)] < 0$. Thus, consumers who stay with the default product are better off under $D = B$ than under $D = A$. (And the gain to switchers always is greater under $D = B$.) By contrast, retaining $r(x) = ax$ but replacing $F(s) = s^2$ with uniformly distributed switching costs $F(s) = s$ yields $\hat{x}_A - \tilde{x}_B = \frac{2}{3}\Delta \Longrightarrow [\Delta - (\hat{x}_A - \tilde{x}_B)] > 0$ (see Proof of Corollary 2(i)), so consumers who stay with the default product are better off under $D = A$.

Thus, while industry profit is typically higher under $D = A$, hence firm A wins the default position, in such cases consumer surplus can be higher *or lower* than under the reverse assignment $D = B$. It is perhaps surprising that in our context there are plausible situations where consumer surplus would be higher if the lower-quality firm were assigned the default. The above discussion suggests an explanation: the higher-quality firm may exploit its default position to set a charge that exceeds the rival's by more than its quality advantage. The distribution of consumers' switching costs $F(s)$ again plays a key role, as illustrated in the above example.

Regarding total welfare, whereas revenue is a pure transfer in standard environments, here the monetization activity x may be weighted differently by firms and consumers, hence can directly affect total welfare. Setting aside this complication, two factors push total welfare to be higher if firm A is assigned the default. First, the deadweight loss from switching costs is then lower since less switching occurs when $D = A$ than when $D = B$ (Proposition 1(iii)). Second, while total output is the same under either regime given our assumption that the market is always covered, the share of consumers using the higher-quality product A is likely to be higher when $D = A$.[21] These two forces are reflected in Corollary 2(ii), where $\widehat{S} < \widetilde{S}$, yet $\widehat{W} > \widetilde{W}$ for some $a < 1$ (specifically, for $a > 0.45$), i.e. if the contribution of the charge x to profit is not too far lower than its disutility to consumers (recall that

[21] The share of consumers that will use product A is $1 - F(\hat{\sigma})$ when $D = A$ and $F(\tilde{\sigma})$ when $D = B$. From Proposition 1, $1 - F(\hat{\sigma}) > \frac{1}{2}$ while $F(\tilde{\sigma}) \leq \frac{1}{2}$ if $F(\Delta) \leq \frac{1}{2}$. But if $F(\Delta) > \frac{1}{2}$, then $F(\tilde{\sigma}) > \frac{1}{2}$, rendering the comparison ambiguous.

$u\prime(x) = -1$, hence $a = -r'(x)/u'(x))$. For $a = 1$, the expenditure ax is a pure transfer from consumer surplus to profit, implying $\widehat{W} > \widetilde{W}$ due to the aforementioned two 'real' forces; thus, $\widehat{W} > \widetilde{W}$ also for a somewhat below 1. (In Corollary 2 part (i), $\widehat{W} > \widetilde{W}$ obviously holds since $\widehat{S} > \widetilde{S}$ and assumption (C1) ensures $\widehat{\Pi} > \widetilde{\Pi}$.)

4. WELFARE-IMPROVING REGULATION?

We now investigate how regulations governing default-position assignment may affect firms, consumers, and efficiency in this market.

4.1 Equal Shares of Default Position

If firms are not permitted to make payments for the default position, then one possibility is that A and B are randomly assigned as the default across consumers, so that $D = A$ or $D = B$ with equal probability ($\frac{1}{2}$). Thus, A and B each is the default for half of the consumers. We next examine the equilibrium in this case and compare it with that under competitive bidding when firm A wins, as occurs under condition (C).

If $v_A - x_A \geq v_B - x_B$, then the only consumers who may switch are those with $D = B$, from B to A, with the marginal switching consumer type being $s = \sigma = x_B - x_A + \Delta$. The profit functions of the two firms would then be

$$\pi_A = \frac{r(x_A)}{2}\left[1 + F(x_B - x_A + \Delta)\right], \qquad \pi_B = \frac{r(x_B)}{2}\left[1 - F(x_B - x_A + \Delta)\right]. \tag{9}$$

Instead, if $v_A - x_A < v_B - x_B$, then the only consumers who may switch are those with $D = A$, from A to B, with the marginal switching consumer being $s = \sigma = x_A - x_B - \Delta$. However, we will show that switching from A to B will not occur in equilibrium.

In this case where the two firms have equal shares of default positions, denote A's and B's equilibrium choices by x_A^e and x_B^e, and similar notation is adopted for other outcome variables. The result below references the following equations, derived from (9) in the Appendix (Proof of Proposition 3):

$$\frac{r'\left(x_A^e\right)}{r\left(x_A^e\right)} = \frac{f(\sigma^e)}{1 + F(\sigma^e)}; \qquad \frac{r'\left(x_B^e\right)}{r\left(x_B^e\right)} = \frac{f(\sigma^e)}{1 - F(\sigma^e)}, \tag{10}$$

where $\sigma^e = \Delta - x_A^e + x_B^e \in (0,1)$, $0 \leq x_A^e - x_B^e < \Delta$, and $v_A - x_A^e > v_B - x_B^e$. The result also establishes that the equilibrium industry profit and consumer surplus are given by

$$\Pi^e = r\left(x_A^e\right)\frac{1+F\left(\sigma^e\right)}{2} + r\left(x_B^e\right)\frac{1-F\left(\sigma^e\right)}{2}; \quad S^e = \frac{v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F\left(s\right)ds}{2}. \tag{11}$$

Proposition 3 *Under equal shares of the default position, the equilibrium x_A^e and x_B^e satisfy (10). Consumers with $D = B$ and $s < \sigma^e$ switch to A, and there is no equilibrium where consumers with $D = A$ switch to B. Equilibrium industry profit and consumer surplus are given by (11). Moreover, $\Pi^e > \widehat{\Pi}$ and $S^e < \widehat{S}$ either if $\sigma^e \leq \hat{\sigma}$, or if (C1) is satisfied and additionally one of the following holds:*
(i) $m > 2\Delta$ and $n = 1$; or (ii) $m = n = 1$; or (iii) $m = 1$ and $n = 2$.

Notice that under equal shares of the default, $x_A^e - x_B^e < \Delta$, in contrast to $\hat{x}_A - \hat{x}_B > \Delta$ when $D = A$ for all consumers (Proposition 1(i)). That is, A's equilibrium charge now exceeds B's charge by less than A's quality advantage, hence consumers switch only from B to A, instead of the reverse direction when A is the default for all consumers.

Since industry profit is 'typically' higher under $D = A$ than under $D = B$, one might have expected that shifting half the consumers from $D = A$ to $D = B$ would reduce industry profit. However, such a move commonly *raises* industry profit. The reason is *softened competition,* resulting in higher charges.[22] In fact, if $\sigma^e \leq \hat{\sigma}$, then $x_A^e > x_B^e \geq \hat{x}_A > \hat{x}_B$, so both firms set higher charges when they have equal shares of the default position than when firm B has none. Firm B now is motivated to raise its charge so as to exploit some of its default consumers, those with high switching costs. This *installed base effect* causes x_B^e to be substantially above $\hat{x}_B$. Although firm A's installed base falls by the same amount as B's rises (i.e. by half of the market), this constitutes a smaller proportional change than for firm B, hence exerts less downward pressure on x_A. The foreseen increase in x_B pushes firm A to raise its charge as well, because x_A and x_B are strategic complements, and this

[22] Katz (2024, pp. 23-25) provides another example where competition is softened by a "neutrality" policy that creates some captive consumers for each firm, akin to a finding of de Cornière and Taylor (2019) discussed earlier.

strategic effect tends to dominate A's installed base effect. Consequently, both charges rise relative to the bidding equlibrium (though A's charge rises by less).

The condition $\sigma^e \leq \hat{\sigma}$ is sufficient for $\Pi^e > \widehat{\Pi}$ and $S^e < \widehat{S}$ (Proposition 3), but clearly is not necessary for these rankings, which hold also, for example, in the Linear-Uniform case, $r(x) = ax$ and $F(s) = s$. Then,

$$x_A^e = 1 + \frac{\Delta}{3}, \quad x_B^e = 1 - \frac{\Delta}{3}; \qquad \hat{x}_A = \frac{2}{3} + \frac{\Delta}{3}, \quad \hat{x}_B = \frac{1}{3} - \frac{\Delta}{3},$$

so charges are uniformly higher under equal shares of the default than when firm A has the default for all consumers ($x_A^e - \hat{x}_A = \frac{1}{3}$, $x_B^e - \hat{x}_B = \frac{2}{3}$) for all $\Delta < 1$, even though $\sigma^e = \frac{\Delta}{3} \leq \hat{\sigma} = \frac{1-\Delta}{3}$ only if $\Delta \leq \frac{1}{2}$.

Consumer surplus under equal assignment, S^e, can be lower than under competitive bidding, $\widehat{S}$, even when $\widehat{S} < \widetilde{S}$, i.e. even when the competitive bidding outcome yields lower consumer surplus than would obtain if B were the default for all consumers. For example, if $m = 1$ and $n = 2$ in (C1), then $S^e < \widehat{S}$ even though $\widehat{S} < \widetilde{S}$ (from Corollary 2(ii)), because the total charge when default shares are equal is higher than under $D = A$, which in turn is higher than under $D = B$.

The comparison of total welfare (W) is generally ambiguous, due to the typically opposite changes in profit and consumer surplus. The proof of Proposition 3 in the appendix also establishes the following for special cases of $F(s)$ and $r(s)$ that satisfy (C1):

Example 3 *If $m = n = 1$ in (C1), then $\widehat{W} \gtreqless W^e$ when $a \lesseqgtr a_1^e(\Delta) \in \left(\frac{7}{8}, \frac{17}{4}\right)$; while if $m = 1$ and $n = 2$ in (C1), then $\widehat{W} \gtreqless W^e$ when $a \lesseqgtr a_2^e(\Delta) \in (0.035, 1)$.*

Thus, if the revenue function is linear ($m = 1$ so $r(x) = ax$) and switching costs are uniformly distributed ($n = 1$ so $F(s) = s$), then total welfare is higher under competitive bidding than under equal default shares when a is below a threshold that depends on the quality difference, $a_1^e(\Delta) \in \left(\frac{7}{8}, \frac{17}{4}\right)$ (and when a is above that threshold the ranking is reversed, $\widehat{W} < W^e$). Instead, if $F(s) = s^2$, the threshold is $a_2^e(\Delta) \in (0.035, 1)$. Under (C1), $\widehat{\Pi} < \Pi^e$ while $\widehat{S} > S^e$ (Proposition 3), hence $\widehat{W} > W^e$ if the weight on profit relative to consumer surplus, a, is sufficiently low.

4.2 Other Shares of Default Position

Consider regulation that assigns $D = A$ for a portion $\lambda \in \left(\frac{1}{2}, 1\right)$ of consumers. The higher-valued product is then assigned as the default for more than half of the consumers but not all. Earlier we analyzed the cases $\lambda = \frac{1}{2}$ (equal assignment) and $\lambda = 1$ (competitive bidding outcome when A wins), which are limiting cases of this more general setting. We next establish that for λ close to $\frac{1}{2}$ there exists a unique equilibrium similar to that when $\lambda = \frac{1}{2}$, whereas for λ close to 1 the unique equilibrium is similar to that when $\lambda = 1$. We will also discuss how profits and consumer surplus may change as λ varies in both ranges.

At the candidate equilibrium for λ close to $\frac{1}{2}$, where consumer switching occurs only from B to A, the marginal switching consumer is

$$\sigma = x_B - x_A + \Delta \geq 0.$$

Then, the profits of the two firms are

$$\pi_A = r(x_A)\left[\lambda + (1-\lambda) F(x_B - x_A + \Delta)\right], \qquad \pi_B = r(x_B)(1-\lambda)\left[1 - F(x_B - x_A + \Delta)\right].$$

In this case, denote the equilibrium charges of the two firms by x_A^- and x_B^-, the marginal consumer by σ^-, and the other outcome variables by Π^-, S^-, W^-.

At the candidate equilibrium for λ close to 1, where consumer switching occurs only from A to B, , the marginal switching consumer is

$$\sigma = x_A - x_B - \Delta > 0.$$

Then, the profit functions of the two firms are

$$\pi_A = r(x_A)\lambda\left[1 - F(x_A - x_B - \Delta)\right], \qquad \pi_B = r(x_B)\left[1 - \lambda + \lambda F(x_A - x_B - \Delta)\right].$$

In this case, denote the equilibrium charges of the two firms by x_A^+ and x_B^+; and similarly for σ^+, Π^+, S^+, and W^+.

Proposition 4 *There exist $\lambda^- \in \left(\frac{1}{2}, 1\right)$ and $\lambda^+ \in \left(\lambda^-, 1\right)$ such that if $\lambda < \lambda^-$, then $\Delta > x_A^- - x_B^- > 0$ and there is consumer switching only from B to A; while if $\lambda > \lambda^+$, then*

$x_A^+ - x_B^+ > \Delta$ *and there is consumer switching only from A to B. Moreover, if (C1) holds with* $m = n = 1$*, then* $\lambda^- = \frac{\Delta+1}{\Delta+2} < \frac{1}{2-\Delta} = \lambda^+$*, and*

(i) for $\lambda < \lambda^-$, $\Pi^- > \widehat{\Pi}$, $S^- < \hat{S}$, $\frac{d\Pi^-}{d\lambda} > 0$, *and* $\frac{dS^-}{d\lambda} < 0$;

(ii) for $\lambda > \lambda^+$, $\Pi^+ \gtreqless \widehat{\Pi}$ *if* $\lambda \lesseqgtr \frac{5/2}{(\Delta+1)^2}$, $S^+ < \widehat{S}$, $\frac{d\Pi^+}{d\lambda} \gtreqless 0$ *if* $\lambda \gtreqless \frac{\sqrt{5/2}}{(\Delta+1)}$, *and* $\frac{dS^+}{d\lambda} > 0$.

In the *Linear-Uniform* case of (C1), where $m = n = 1$ hence $r(x) = ax$ and $F(s) = s$, as λ changes from $\frac{1}{2}$ to 1, equilibrium industry profit and consumer surplus can vary non-monotonically. For $\lambda < \lambda^-$, as λ rises industry profit also rises but consumer surplus falls. A higher λ raises A's installed base (consumers with A as the default), which induces a rise in x_A^- and a smaller rise in x_B^-. (The latter reflects the strategic response to the rise in x_A^-, which outweighs the effect on x_B of the reduction in $B's$ installed base.) Consequently, industry profit rises but consumer surplus falls. Thus, for all $\lambda \in \left(\frac{1}{2}, \lambda^-\right)$ we have $\Pi^- > \Pi^e > \widehat{\Pi}$ and $S^- < S^e < \widehat{S}$ (where the second inequalities follows from Proposition 3), so profit is higher but consumer surplus is lower than in the competitive bidding outcome ($D = A$).

For $\lambda > \lambda^+$, as λ rises consumer surplus now rises ($\frac{dS^+}{d\lambda} > 0$), but remains below $\widehat{S}$. The behavior of profit is more complex. As shown in the proof of Proposition 4, if the quality difference $\Delta \leq 0.581$, then profit decreases in λ but remains above $\widehat{\Pi}$ for all $\lambda \in \left(\lambda^+, 1\right)$. If $\Delta > 0.581$, profit may decrease or increase with λ, and can be lower or higher than $\widehat{\Pi}$. These patterns are roughly explained as follows. For λ near 1, B's customer base is small, and as λ rises the decrease in B's customer base exerts a powerful downward effect on x_B, so that both x_B^+ and x_A^+ can fall, though the latter by less (since $A's$ customer base rose). However, with a higher λ, more consumers may patronize A, which has a higher charge. As a result, industry profit tends to—but not always—fall.

Recall from Proposition 3 that if the default position is distributed equally between the two firms ($\lambda = \frac{1}{2}$), then industry profit tends to be higher but consumer surplus tends to be lower than in the competitive bidding outcome ($\lambda = 1$): $\Pi^e > \widehat{\Pi}$ and $S^e < \widehat{S}$. Compared to that outcome, a regulated assignment with $\lambda^- \in \left(\frac{1}{2}, 1\right)$ also tends to increase industry profit and decrease consumer surplus.

Total welfare can rise or fall relative to $\widehat{W}$, the level under competitive bidding where

$\lambda = 1$. For example, if $r(x) = ax$ and $F(s) = s$, when a is above some threshold, the profit effect tends to dominate, resulting in higher total welfare than when $\lambda = 1$; and conversely if a is below the threshold.

4.3 Regulated Assignment with Endogenous Product Quality

Suppose there is learning by the firms, so that when more consumers use product B its quality v_B can increase. This scenario is at the heart of the DOJ's complaint against Google (DOJ, 2020). DOJ argues that search algorithms improve with experimentation and, hence, improve with a search engine's number of users. By obtaining default status at leading distribution outlets for search engines, Google deprives rival search engines of users and, hence, impairs their ability to improve their quality through learning. We take no position on the merits of the DOJ's argument,[23] but will attempt to capture its essence and the potential welfare effects of reducing Google's share of default positions.

To formally model the quality improvement issues, one would need a dynamic model. For instance, one might consider a setting with two periods, where v_A and v_B are exogenously given in period 1, but may improve in period 2 due to learning in period 1, and greater improvement occurs when a firm serves more consumers in period 1. Then, if λ is reduced from $\lambda = 1$ to some lower value in period 1, potentially more consumers would use B in period 1, which could increase v_B and result in more consumers patronizing B in period 2.

There are significant complexities to analyze this scenario in an equilibrium model. In particular, the numbers of consumers that firms serve in period 1 are endogenous, depending on their choices of x_A and x_B in period 1. Therefore, the number of consumers who have $D = A$ and $D = B$ at the beginning of period 2 are also endogenous, assuming that a consumer who patronized firm i in period 1 will start with $D = i$ in period 2. Also,

[23] Note that Gilbert and Newbery's (1982) result, that an incumbent monopolist would outbid a potential entrant for a single vital asset, does not immediately extend to the Google case because there are *multiple* distribution outlets for which firms can bid. The profitability of sustaining monopoly through bidding for multiple assets is an open question (Kamien and Zang, 1990; Malueg and Schwartz, 1991; Krishna, 1993).

since the firms' choices of x_A and x_B in period 2 will depend on those numbers and the default assignment, the switching decisions of rational consumers in period 1 will hinge on their expectations about how the second-period equilibrium may depend on λ, x_A, and x_B in period 1. Moreover, the incentives to win the default position may also change under competitive bidding. Since our purpose is mainly to gain insight on whether, with endogenous product quality, consumer surplus can be higher under regulation, we adopt a 'reduced-form' approach without analyzing a dynamic model that formally incorporates the intertemporal strategic interactions.

Specifically, consider the following: Under competitive bidding, $D = A$ for all consumers (i.e., $\lambda = 1$) with given v_A and v_B; while under regulated assignment, $D = A$ for some portion $\lambda \in \left(\frac{1}{2}, 1\right)$ of consumers and $A's$ and $B's$ product qualities with learning become $v_A^l \leq v_A$ and $v_B^l > v_B$. This can be viewed as the second period of a two-period model, assuming that (i) quality increases with scale but at a decreasing rate, so that if λ decreases from $\lambda = 1$ by a relatively small amount, then v_A will not be (much) lower in period 2 but v_B could be substantially higher; (ii) each period contains a separate unit mass of consumers, so consumers face no intertemporal choice; and (iii) the first period is much shorter than the second, so the welfare comparisons can be approximated by their comparisons for period 2. We investigate the quality improvement $v_B^l - v_B$ needed to achieve higher consumer surplus for some values of $\lambda < 1$ than under $\lambda = 1$. Our analysis incorporates the different equilibrium values of x_A and x_B under the bidding and regulated assignments.

For this analysis we assume the *Linear-Uniform* case, and define

$$v_A^l = v_A - \delta_A, \quad v_B^l = v_B + \delta_B, \quad \Delta^l = v_A^l - v_B^l = \Delta - \delta_A - \delta_B > 0,$$

for $\delta_A \geq 0$ and $\delta_B > 0$. We focus on situations where Δ^l is relatively small and λ relatively large, so that $\lambda \geq \lambda^+ = \frac{1}{2-\Delta^l}$ in Proposition 4. While δ_A and δ_B may well depend on λ, we treat them as parameters and inquire how the values of δ_A and δ_B, for a given change from $\lambda = 1$ to some $\lambda < 1$, may affect consumer surplus, denoted in this case by S^l. Note that, for a given Δ, the quality gap Δ^l will be higher when δ_A and δ_B are lower i.e., when the learning effect is weaker.

Corollary 3 *Assume $m = n = 1$ under (C1) and $\lambda \geq \lambda^+ = \frac{1}{2-\Delta^l}$. Then, $S^l - \widehat{S}$ decreases in Δ^l but increases in λ and Δ. Moreover, $S^l - \widehat{S} > 0$ if $\Delta \geq 0.4$, $\lambda \geq 0.7$, and δ_A and Δ^l are sufficiently small; but $S^l - \widehat{S} < 0$ if $\delta_A = \delta_B$.*

Thus, if regulation endows firm B with the default position for a portion $1 - \lambda$ of consumers, which improves v_B possibly due to learning, then consumers can indeed benefit compared to the bidding outcome where firm A obtains the default position for all consumers. Consumers may gain through several channels. An increase in v_B *directly benefits* consumers who use product B. But it also has strategic effects: a higher v_B, which reduces the quality asymmetry between the two products (i.e., Δ^l is smaller), may result in lower levels of x_A and x_B by the two firms due to the *intensified competition* when they are more symmetric. Also, x_A and x_B will be closer to each other if Δ^l is smaller, which would reduce the amount of switching, hence *reduce the switching costs* incurred by consumers.

To illustrate the increase in v_B required for $S^l > \widehat{S}$, suppose $v_A = 5$ and $v_B = 4.5$ so $\Delta = 0.5$, and regulation lowers λ from 1 to 0.8. Then $S^l > \widehat{S}$ if (i) $\delta_B \geq 0.186$, or 4.13% of v_B, without lowering v_A; or if (ii) $\delta_B \geq 0.25$ and $\delta_A \leq 0.05$ (i.e., the reduction in v_A is less than 20% of the increase in v_B: $\delta_A/\delta_B \leq 20\%$). However, if v_A would decrease as much as v_B increases under the regulation, then consumer surplus is always higher when $\lambda = 1$.

4.4 Choice Screen

Instead of assigning a default product to consumers, an alternative policy known as "choice screen" allows consumers to choose their preferred default from a set of displayed options. This policy was first adopted by the European Commission in 2009: Microsoft was required to display alternative web browsers along with its own Internet Explorer instead of presetting Explorer as the default. A choice screen was also adopted in the Commission's Android (2018) case, where Google was required to display other search engines in addition to its own.[24] The Digital Markets Act adopted by the European Union (2022) requires large

[24] In the Android case, unlike in Microsoft, the rival products displayed in the choice screen (for both search engines and web browsers) were determined through auctions conducted by Google starting in 2020.

online platforms designated as “gatekeepers” to provide a choice screen for users to select their default apps for online search engines, virtual assistants, or web browsers.

In our setting, a choice screen policy would lead all consumers to choose product A. Consumers differ only in their switching costs and a choice screen allows each consumer to choose the preferred option at the outset before incurring a switching cost. The equilibrium then resembles Bertrand competition with asymmetric product qualities: the weaker firm B sets its charge x_B equal to marginal cost (that we normalized to zero), and firm A captures the entire market while charging a premium equal to its quality advantage: $x_A = x_B + \Delta$. Ironically, firm B would attract no customers in such a scenario, unlike the bidding-for-default outcome even when firm A wins.[25]

This stark pattern emerges in our setting because consumers are heterogeneous only in switching costs, not in their preferences between the competing firms. In fact, Decarolis et al. (2023) found that Google incurred modest decreases in its search market share after the introduction of a choice screen.[26] Such a pattern could be explained by factors outside our model, notably, richer product differentiation between the firms, that would allow both firms to attract consumers under Bertrand competition with no preassigned defaults.[27] For example, consumers may differ in their valuation of quality (as with standard vertical differentiation) or their ‘location’ (horizontal differentiation a la Hotelling, e.g. the weight placed on accuracy of search results versus invasion of privacy). Therefore, we are not

Ostrovsky (2023) shows that the identity of the winning bidders will depend on whether a bidder pays a flat fee for the right to be displayed in the choice screen, or a fee per user that installs its product (“per install”).

[25] There, firm A exploits its customers' heterogeneous switching costs, setting $\hat{x}_A > \hat{x}_B + \Delta$, which in turn allows firm B to attract some (switching) customers in equilibrium. Essentially, firm A behaves like a “fat cat” (Tirole, 1988), exploiting its installed base by raising price, which allows the weaker firm to survive.

[26] Over roughly a two-year period, Google's share in mobile search fell about 2 percentage points in the European Economic Area and between 5-7 percentage points in Russia.

[27] Another possibility is that consumers selected from the choice screen based on firms' past ‘charges’ (e.g. advertising levels), either because charges had not been adjusted to reflect the new competitive environment, or had not yet been observed by consumers. Since the default platform yields lower utility in our model due to its charge premium, eliminating the default while holding charges fixed would induce some switching to the rival. These scenarios, however, involve non-equilibrium behavior.

suggesting that a choice screen would necessarily reduce the weaker firm's market share. Nevertheless, our analysis offers the following robust insights.

There is a strong presumption that a choice screen would be the superior policy for consumer welfare in the short run if consumers face de minimis cost to set up the default themselves through the choices presented.[28] Consumers would then obtain their preferred choice. Additionally, because competition is intensified when firms must compete for a larger share of the market instead of having a base of default consumers, consumers would further benefit via lower monetization charges. From a longer-run standpoint, however, a choice screen may be inferior to some regulatory default assignments. If product quality improves with a firm's share of consumers at a diminishing rate, then shifting some consumers to the weaker firm will increase the latter's quality more than it reduces the leader's quality and ultimately consumers can benefit, directly and from stronger competition. Under a choice screen, too few consumers would choose the lower-quality product because consumers individually ignore the positive competition externality they generate by enabling the weaker firm to improve its quality. Thus, if the predominant policy concern is to enable improvement by the weaker firm, a choice screen approach can be problematic.

5. CONCLUDING REMARKS

We analyzed several methods of assigning the default position for a product supplied by two competing firms with exogenously different qualities, when consumers face heterogeneous costs of switching from the default product to the rival. The default firm enjoys market power over its inframarginal consumers, those with higher switching costs, which it exploits through greater monetization such as unwanted advertising. Consequently, when the default position is assigned through competitive bidding for all consumers, the default winner provides lower utility than the rival, even when the winner is the higher-quality firm.

[28] For search engines, with a properly designed choice screen the cost may well be de minimis (and we are setting aside any psychic costs of making a choice). However, in other situations a consumer may need to incur costs if (s)he chooses the default. Whereas switching costs are avoided, the consumer may need to incur costs to find the relevant alternatives or to install the default option.

That firm indeed tends to win (though we show a counter-example), not due to its quality advantage directly, but because industry monetization is greater when it rather than the rival holds the default. Interestingly, the shape of the switching cost distribution plays important roles in determining whether the higher-quality firm wins the bidding and whether consumer surplus is higher or lower under this default assignment.

Our analysis also yields some policy insights. Compared to the stronger firm winning the default everywhere, assigning via regulation the default to the rival for some minority share of consumers tends to increase profit and harm consumers. Profit rises because competition is softened when both firms have sticky (default) consumers. All consumers lose from the softened competition, and those who are assigned the lower-quality product suffer additional harm directly. We briefly considered another scenario where product quality is not fixed but instead improves at a decreasing rate with the firm's share of users, possibly due to learning. Assigning the default position to the weaker firm for some share of consumers may then benefit consumers in the long run, but this must be weighed against the short run harm. An alternative approach is to let consumers select their preferred option from a choice screen. This approach will likely benefit consumers in the short run, but can be problematic for longer term competition and consumer welfare if learning effects are paramount. Too few consumers will choose the weaker product because they ignore the beneficial externality they would generate by helping the weaker firm improve its quality.

Finally, we note that our model omits some features that could yield a better alignment between consumer welfare and the default assignment under competitive bidding. The leading firm may have an advantage not only in quality but also in monetization efficiency (e.g. better targeting of ads), that would yield it greater revenue than to the rival per dollar harm to consumers. Alternatively, or in addition, it may enjoy greater utilization of its product by consumers than would the rival, instead of our assumption of fixed aggregate consumption. Lastly, the third-party may assign the default position not solely based on the highest bid, but also weighing its customers' utility from the competing products products (e.g. Apple claims it selects the default search engine that is best for iPhone buyers). Future work could explore such extensions.

6. APPENDIX

Proof that (C) holds under (C1): Under (C1),

$$f(s) = ns^{n-1}, \quad f'(s) = n(n-1)s^{n-2} \geq 0, \quad \frac{f'(s)}{f(s)} = \frac{(n-1)}{s} \text{ decreases in } s;$$

$$\rho(x) = \frac{ax^m}{amx^{m-1}} = x/m, \quad \rho'(x) = 1/m > 0, \quad \rho''(x) = 0,$$

$\rho(\Delta) f(0) = \frac{\Delta}{m}\Delta^n < 1$, and $r(0) = 0$. Thus, condition (C) holds.

The rest of this appendix contains proofs for Propositions 1-4 and Corollaries 1-3.

Proof of Proposition 1. (i) When $D = A$, the equilibrium $\hat{x}_A$ and $\hat{x}_B$, if they are strictly positive, satisfy the following first-order conditions obtained from (1):

$$\frac{\partial \pi_A}{\partial x_A} = r'(x_A)[1 - F(x_A - x_B - \Delta)] - r(x_A) f(x_A - x_B - \Delta) = 0, \tag{12}$$

$$\frac{\partial \pi_B}{\partial x_B} = r'(x_B) F(x_A - x_B - \Delta) - r(x_B) f(x_A - x_B - \Delta) = 0. \tag{13}$$

First, we show that $\hat{x}_B > 0$. If, to the contrary, $\hat{x}_B = 0$, then $\hat{x}_A > \Delta$ because $\left.\frac{\partial \pi_A}{\partial x_A}\right|_{x_A=\Delta}$ $= r'(\Delta) - r(\Delta) f(0) > 0$ by Assumption (C), and thus $\left.\frac{\partial \pi_B}{\partial x_B}\right|_{x_B=0} = r'(0) F(\hat{x}_A - \Delta) - r(0) f(\hat{x}_A - \Delta) > 0$ if $r(0)$ is small enough, which contradicts $\hat{x}_B = 0$.

Next, $\hat{x}_A - \hat{x}_B \geq \Delta$, because if $\hat{x}_A - \hat{x}_B < \Delta$, A could increase its profit by raising x_A. Also, if $\hat{x}_A - \hat{x}_B = \Delta$, we would have $\left.\frac{\partial \pi_B}{\partial x_B}\right|_{\hat{x}_B} < 0$, contradicting $\hat{x}_B$ being optimal. Therefore $\hat{x}_A - \hat{x}_B > \Delta$.

Observe that (12) and (13) can be rewritten as the two equations in (3). With $\sigma_A = x_A - x_B - \Delta$, let

$$\mu(x) \equiv \frac{r'(x_A)}{r(x_A)}, \quad h(s) \equiv \frac{f(\sigma_A)}{1 - F(\sigma_A)}, \quad \text{and} \quad g(\sigma_A) \equiv \frac{f(\sigma_A)}{F(\sigma_A)},$$

where $\mu'(x) < 0$, $h'(\sigma_A) > 0$ and $g'(\sigma_A) < 0$, because $f(s)$ is logconcave from (C). We show that the $\hat{x}_A$ and $\hat{x}_B$ that satisfy (3) are unique. Each equation in (3) implicitly defines x_A as a function of x_B, and the curves in the (x_B, x_A)-space for the two functions, where x_A is on the vertical axis, respectively have the following slopes:

$$\frac{dx_A}{dx_B} = \frac{h'(\sigma_A)}{h'(\sigma_A) - \mu'(x_A)} \in (0,1), \qquad \frac{dx_A}{dx_B} = \frac{g'(\sigma_A) + \mu'(x_A)}{g'(\sigma_A)} > 1.$$

Thus the two curves intersect only once, implying that $\hat{x}_A$ and $\hat{x}_B$ exist uniquely. Notice that the positive slopes imply that the two firms' choices are strategic complements.

Finally, because $\hat{x}_A - \hat{x}_B > \Delta$, $\hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta > 0$, and $r(x)/r'(x)$ increases in x from (C), we have

$$\frac{r(\hat{x}_A)}{r'(\hat{x}_A)} - \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} = \frac{1 - 2F(\hat{\sigma})}{f(\hat{\sigma})} > 0,$$

and hence $F(\hat{\sigma}) < \frac{1}{2}$.

(ii) When $D = B$, the equilibrium $\tilde{x}_A$ and $\tilde{x}_B$, if they are strictly positive, satisfy the following first-order conditions obtained from (2):

$$\frac{\partial \pi_A}{\partial x_A} = r'(x_A) F(x_B - x_A + \Delta) - r(x_A) f(x_B - x_A + \Delta) = 0, \quad (14)$$

$$\frac{\partial \pi_B}{\partial x_B} = r'(x_B)[1 - F(x_B - x_A + \Delta)] - r(x_B) f(x_B - x_A + \Delta) = 0. \quad (15)$$

For any $\tilde{x}_B \geq 0$, firm A will choose $\tilde{x}_A > 0$ to profit from the switching consumers. It follows that $\tilde{x}_B > 0$ as well.

Next, since both $\tilde{x}_A > 0$ and $\tilde{x}_B > 0$, we must have $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A > 0$, and hence $\tilde{x}_A - \tilde{x}_B < \Delta$. Equations (14) and (15) can be rewritten as the two equations in (4), each of which implicitly defines x_A as a function of x_B, and the curves in the (x_B, x_A)-space for the two functions, where x_A is on the vertical axis, respectively have the following slopes:

$$\frac{dx_A}{dx_B} = \frac{g'(\sigma_B)}{\mu'(x_A) + g'(\sigma_B)} \in (0,1), \qquad \frac{dx_A}{dx_B} = \frac{h'(\sigma_B) - \mu'(x_B)}{h'(\sigma_B)} > 1,$$

where $\sigma_B = \Delta + x_B - x_A$. Thus the two curves intersect only once, implying that $\tilde{x}_A$ and $\tilde{x}_B$ exist uniquely. Notice that this also implies that the two firms' choices are strategic complements.

If $F(\Delta) \leq \frac{1}{2}$, we show that $\tilde{x}_A \leq \tilde{x}_B$, and thus $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A \geq \Delta$, which further implies $F(\tilde{\sigma}) \leq \frac{1}{2}$. Suppose to the contrary that $\tilde{x}_A > \tilde{x}_B$, then $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A < \Delta$ and $\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} < \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)}$, which implies

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} < \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) < F(\tilde{\sigma}) \rightarrow \frac{1}{2} < F(\tilde{\sigma}) < F(\Delta),$$

a contradiction. Thus $\tilde{x}_A \leq \tilde{x}_B$. It follows that

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} = \frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} \geq \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) \geq F(\tilde{\sigma}) \rightarrow F(\tilde{\sigma}) \leq \frac{1}{2}.$$

On the other hand, if $F(\Delta) > \frac{1}{2}$, we show that $\tilde{x}_A > \tilde{x}_B$ and hence $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A < \Delta$. If, to the contrary, $\tilde{x}_A \leq \tilde{x}_B$, then $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A \geq \Delta$ and $\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} \geq \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)}$, which implies

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} \geq \frac{f(\tilde{\sigma})}{1-F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) \geq F(\tilde{\sigma}) \rightarrow \frac{1}{2} \geq F(\tilde{\sigma}) \geq F(\Delta),$$

a contradiction. Hence $\tilde{x}_A > \tilde{x}_B$. It follows that

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} = \frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} < \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)} = \frac{f(\tilde{\sigma})}{1-F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) < F(\tilde{\sigma}) \rightarrow F(\tilde{\sigma}) > \frac{1}{2}.$$

(iii) Suppose, to the contrary, that $\tilde{\sigma} \leq \hat{\sigma}$. Then

$$\begin{aligned} \frac{r(\hat{x}_A)}{r'(\hat{x}_A)} &= \frac{1-F(\hat{\sigma})}{f(\hat{\sigma})} \leq \frac{1-F(\tilde{\sigma})}{f(\tilde{\sigma})} = \frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)} \Rightarrow \hat{x}_A \leq \tilde{x}_B, \\ \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} &= \frac{F(\hat{\sigma})}{f(\hat{\sigma})} \geq \frac{F(\tilde{\sigma})}{f(\tilde{\sigma})} = \frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} \Rightarrow \hat{x}_B \geq \tilde{x}_A. \end{aligned}$$

Hence

$$\tilde{\sigma} - \hat{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A - [\hat{x}_A - \hat{x}_B - \Delta] = 2\Delta + \tilde{x}_B - \hat{x}_A + \hat{x}_B - \tilde{x}_A > 0,$$

which produces a contradiction. Therefore $\tilde{\sigma} > \hat{\sigma}$. It follows that $\hat{x}_A > \tilde{x}_B$ and $\tilde{x}_A > \hat{x}_B$.

Moreover, for future reference, if $F(\Delta) \leq \frac{1}{2}$ so that $F(\tilde{\sigma}) \leq \frac{1}{2}$, then $\tilde{x}_A \leq \tilde{x}_B$ and hence $\hat{x}_A > \tilde{x}_B \geq \tilde{x}_A > \hat{x}_B$. If $F(\Delta) > \frac{1}{2}$ so that $F(\tilde{\sigma}) > \frac{1}{2}$, then $\tilde{x}_A > \tilde{x}_B$. ■

Proof of Proposition 2. We prove the proposition in two steps.

Step 1: We show that $\hat{x}_A + \hat{x}_B \geq \tilde{x}_B + \tilde{x}_A$. First, from (3) and (4), because $\tilde{\sigma} > \hat{\sigma}$,

$$\begin{aligned} & \frac{r(\hat{x}_A)}{r'(\hat{x}_A)} + \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} - \left[\frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} + \frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)}\right] \\ = & \frac{1-F(\hat{\sigma})}{f(\hat{\sigma})} + \frac{F(\hat{\sigma})}{f(\hat{\sigma})} - \left[\frac{1-F(\tilde{\sigma})}{f(\sigma)} + \frac{F(\sigma)}{f(\sigma)}\right] = \frac{1}{f(\hat{\sigma})} - \frac{1}{f(\tilde{\sigma})} \gtreqless 0 \end{aligned}$$

if $f'(s) \gtreqless 0$.

Next, suppose $F(\Delta) \leq \frac{1}{2}$ so that $\tilde{x}_B \geq \tilde{x}_A$. Since $\rho(x) = \frac{r(x)}{r'(x)}$ and $\rho'(x) > 0$ from assumption (C), if $f'(s) \geq 0$, using the mean-value theorem we have

$$\frac{r(\hat{x}_A)}{r'(\hat{x}_A)} - \frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)} + \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} - \frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} = \rho'(\zeta_1)(\hat{x}_A - \tilde{x}_B) - \rho'(\zeta_2)(\tilde{x}_A - \hat{x}_B) \geq 0,$$

where $\zeta_1 \in (\tilde{x}_B, \hat{x}_A) > \zeta_2 \in (\hat{x}_B, \tilde{x}_A)$. Moreover, since $\rho'' \leq 0$ from (C), which implies $\rho'(\zeta_1) \leq \rho'(\zeta_2)$, we have $\hat{x}_A - \tilde{x}_B \geq \tilde{x}_A - \hat{x}_B$, or equivalently, $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$.

Step 2: We show that there exists some $\varepsilon > 0$ such that $\widehat{\Pi} > \widetilde{\Pi}$ if $r''(x) \geq -\varepsilon$.

From Proposition 1: $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$, $\hat{\sigma} < \tilde{\sigma}$, $\hat{x}_A > \tilde{x}_B$ and $\tilde{x}_A > \hat{x}_B$. Thus, if $\tilde{x}_A \leq \tilde{x}_B$, then $\hat{x}_A > \tilde{x}_B \geq \tilde{x}_A > \hat{x}_B$. Suppose instead $\tilde{x}_A > \tilde{x}_B$. If $\tilde{x}_B \leq \hat{x}_B$, then $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$ implies $\hat{x}_A > \tilde{x}_A$; while if $\tilde{x}_B > \hat{x}_B$, then if $\hat{x}_A \leq \tilde{x}_A$, we would have $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, contradicting the result that $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$. Hence, $\hat{x}_A > \tilde{x}_A$ if $\tilde{x}_A > \tilde{x}_B$. Thus $\hat{x}_A > \max\{\tilde{x}_B,\ \tilde{x}_A\} > \hat{x}_B$, This, together with $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$, implies that $(\hat{x}_B, \hat{x}_A)$ is more dispersed than $(\tilde{x}_B,\ \tilde{x}_A)$.

Therefore, since $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$ and $\hat{\sigma} < \tilde{\sigma}$, the pair $\{\hat{x}_B, \hat{x}_A\}$ is a mean-increasing spread of $\{\tilde{x}_B,\ \tilde{x}_A\}$, that is:

$$[1 - F(\hat{\sigma})]\,\hat{x}_A + F(\hat{\sigma})\,\hat{x}_B > [1 - F(\tilde{\sigma})]\,\tilde{x}_B + F(\tilde{\sigma})\,\tilde{x}_A.$$

Notice that for given Δ, $\widehat{\Pi} = [1 - F(\hat{\sigma})]\, r(\hat{x}_A) + F(\hat{\sigma})\, r(\hat{x}_B)$ exceeds $\widetilde{\Pi} = [1 - F(\tilde{\sigma})]\, r(\tilde{x}_B) + F(\tilde{\sigma})\, r(\tilde{x}_A)$ by a strictly positive number if $r''(x) = 0$. By continuity, there exists some $\varepsilon > 0$ such that $\widehat{\Pi} > \widetilde{\Pi}$ if $r''(x) \geq -\varepsilon$. ■

Proof of Corollary 1. Under (C1), since

$$f(s) = ns^{n-1}, \quad f'(s) = n(n-1)\,s^{n-2} \geq 0, \quad \frac{f'(s)}{f(s)} = \frac{(n-1)}{s} \text{ decreases in } s;$$

$$\rho(x) = \frac{ax^m}{amx^{m-1}} = x/m, \quad \rho'(x) = 1/m > 0, \quad \rho''(x) = 0,$$

$\rho(\Delta)\, f(0) = \frac{\Delta}{m}\Delta^n < 1$, and $r(0) = 0$, Assumption (C) holds. Thus, if $m \geq 1$, then $r''(x) \geq 0$, and $\widehat{\Pi} > \widetilde{\Pi}$ from Proposition 2.

On the other hand, if $m \in (\Delta, 1)$, then $r''(x) = am(m-1)\,x^{m-1} < 0$, but since $n = 1$, from (3) and (4) we obtain:

$$\hat{x}_A = \frac{m + m\Delta + m^2}{2m+1}, \quad \hat{x}_B = \frac{m(m-\Delta)}{2m+1}, \quad \hat{\sigma} = \frac{m-\Delta}{2m+1};$$

$$\tilde{x}_A = \frac{m\Delta + m^2}{2m+1}, \quad \tilde{x}_B = \frac{m - m\Delta + m^2}{2m+1}, \quad \tilde{\sigma} = \frac{m+\Delta}{2m+1},$$

where $\hat{x}_i > 0$ and $\tilde{x}_i > 0$. Numerical analysis indicates that $\widehat{\Pi} - \widetilde{\Pi} > 0$ for all $\Delta \in (0, m)$. $\blacksquare$

Proof of Corollary 2. First,

$$\begin{aligned}
\widehat{S} &= (v_A - \hat{x}_A)\left[1 - F(\hat{\sigma})\right] + \int_0^{\hat{\sigma}} (v_B - \hat{x}_B - s)\, f(s)\, ds, \quad \text{and} \\
\widetilde{S} &= (v_B - \tilde{x}_B)\left[1 - F(\tilde{\sigma})\right] + \int_0^{\tilde{\sigma}} (v_A - \tilde{x}_A - s)\, f(s)\, ds.
\end{aligned}$$

We can rewrite

$$\begin{aligned}
\widehat{S} &= (v_A - \hat{x}_A) - (v_A - \hat{x}_A) F(\hat{\sigma}) + (v_B - \hat{x}_B) F(\hat{\sigma}) - \hat{\sigma} F(\hat{\sigma}) + \int_0^{\hat{\sigma}} F(s)\, ds \\
&= (v_A - \hat{x}_A) + \left[-\Delta + \hat{x}_A - \hat{x}_B - \hat{\sigma}\right] F(\hat{s}) + \int_0^{\hat{\sigma}} F(s)\, ds \\
&= (v_A - \hat{x}_A) + \int_0^{\hat{\sigma}} F(s)\, ds.
\end{aligned}$$

Similarly,

$$\tilde{S} = v_B - \tilde{x}_B + \int_0^{\tilde{\sigma}} F(s)\, ds.$$

Thus,

$$\widehat{S} \gtreqless \widetilde{S} \Longleftrightarrow \Delta - \hat{x}_A + \tilde{x}_B \gtreqless \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds.$$

Next, we have:.

(i) If $r(x) = ax$ and $F(s) = s$, then,

$$\hat{x}_A = \frac{2}{3} + \frac{\Delta}{3} > 0, \quad \hat{x}_B = \frac{1}{3} - \frac{\Delta}{3} > 0, \quad \hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta = \frac{1}{3}(1 - \Delta);$$

$$\tilde{x}_A = \frac{1}{3} + \frac{\Delta}{3} > 0, \quad \tilde{x}_B = \frac{2}{3} - \frac{\Delta}{3} > 0, \quad \tilde{\sigma} = \tilde{x}_B - \tilde{x}_A + \Delta = \frac{1}{3}(1 + \Delta).$$

$$\widehat{S} = v_A - \left(\frac{2}{3} + \frac{\Delta}{3}\right) + \int_0^{\frac{1}{3}(1-\Delta)} s ds = v_A - \frac{1}{18}\left(11 + 8\Delta - \Delta^2\right). \tag{16}$$

$$\widetilde{S} = v_B - \left(\frac{2}{3} - \frac{\Delta}{3}\right) + \int_0^{\frac{1}{3}(1+\Delta)} s ds = v_B - \frac{1}{18}\left(13 - 4\Delta + \Delta^2\right).$$

$$\widehat{S} - \widetilde{S} = \frac{1}{9}\left(3\Delta + \Delta^2 + 1\right) > 0.$$

Moreover, if $r(x) = ax^m$ and $F(s) = s$, then

$$\hat{x}_A = \frac{m + m\Delta + m^2}{2m+1} > 0, \quad \hat{x}_B = \frac{m(m - \Delta)}{2m+1} > 0, \quad \hat{\sigma} = \frac{m - \Delta}{2m+1}.$$

$$\tilde{x}_A = \frac{m\Delta + m^2}{2m+1} > 0, \qquad \tilde{x}_B = \frac{m - m\Delta + m^2}{2m+1} > 0, \qquad \tilde{\sigma} = \frac{m+\Delta}{2m+1};$$

$$\hat{S} - \tilde{S} = \Delta - (\hat{x}_A - \tilde{x}_B) - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds > 0.$$

(ii) Suppose $r(x) = ax$ and $F(s) = s^2$. Then

$$\hat{x}_A = \frac{5}{8}\Delta + \frac{3}{8}\sqrt{\Delta^2+4}, \quad \hat{x}_B = \frac{1}{8}\sqrt{\Delta^2+4} - \frac{1}{8}\Delta, \ \hat{\sigma} = \frac{1}{4}\left(\sqrt{\Delta^2+4} - \Delta\right);$$

$$\tilde{x}_A = \frac{1}{8}\Delta + \frac{1}{8}\sqrt{\Delta^2+4}, \quad \tilde{x}_B = \frac{3}{8}\sqrt{\Delta^2+4} - \frac{5}{8}\Delta, \ \tilde{\sigma} = \frac{1}{4}\left(\sqrt{\Delta^2+4} + \Delta\right),$$

$$\begin{aligned}
\widehat{S} - \widetilde{S} &= [\Delta - (\hat{x}_A - \tilde{x}_B)] - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds \\
&= \Delta - \left(\frac{5}{8}\Delta + \frac{3}{8}\sqrt{\Delta^2+4} - \left(\frac{3}{8}\sqrt{\Delta^2+4} - \frac{5}{8}\Delta\right)\right) - \int_{\frac{1}{4}\left(\sqrt{\Delta^2+4}-\Delta\right)}^{\frac{1}{4}\left(\sqrt{\Delta^2+4}+\Delta\right)} s^2 ds \\
&= -\frac{1}{24}\Delta\left(\Delta^2+9\right) < 0
\end{aligned}$$

The comparison of profits (and for later reference also of total welfare), is as follows:

$$\begin{aligned}
\widehat{\Pi} &= a\hat{x}_A\left(1-\hat{\sigma}^2\right) + a\hat{x}_B\hat{\sigma}^2 \\
&= a\begin{pmatrix} \left(\frac{5}{8}\Delta + \frac{3}{8}\sqrt{\Delta^2+4}\right)\left(1 - \left(\frac{1}{4}\left(\sqrt{\Delta^2+4}-\Delta\right)\right)^2\right) \\ +\left(\frac{1}{8}\sqrt{\Delta^2+4} - \frac{1}{8}\Delta\right)\left(\frac{1}{4}\left(\sqrt{\Delta^2+4}-\Delta\right)\right)^2 \end{pmatrix} \\
&= \frac{1}{64}a\left(40\Delta + 24\sqrt{\Delta^2+4} - \left(\Delta^2+4\right)^{\frac{3}{2}} - 3\Delta^3 + 5\Delta^2\sqrt{\Delta^2+4} - \Delta\left(\Delta^2+4\right)\right),
\end{aligned}$$

$$\begin{aligned}
\widetilde{\Pi} &= a\tilde{x}_A\tilde{\sigma}^2 + a\tilde{x}_B\left(1-\tilde{\sigma}^2\right) \\
&= a\begin{pmatrix} \left(\frac{1}{8}\Delta + \frac{1}{8}\sqrt{\Delta^2+4}\right)\left(\frac{1}{4}\left(\sqrt{\Delta^2+4}+\Delta\right)\right)^2 \\ +\left(\frac{1}{8}\sqrt{\Delta^2+4} - \frac{1}{8}\Delta\right)\left(1 - \left(\frac{1}{4}\left(\sqrt{\Delta^2+4}-\Delta\right)\right)^2\right) \end{pmatrix} \\
&= \frac{1}{64}a\left(-8\Delta + 8\sqrt{\Delta^2+4} + \Delta^3 + 3\Delta\left(\Delta^2+4\right)\right),
\end{aligned}$$

$$\widehat{\Pi} - \widetilde{\Pi} = \frac{1}{64}a\left(32\Delta + 16\sqrt{\Delta^2+4} - \left(\Delta^2+4\right)^{\frac{3}{2}} - 8\Delta^3 + 5\Delta^2\sqrt{\Delta^2+4}\right) > 0$$

$$
\begin{aligned}
\widehat{W}-\widetilde{W} &= \frac{1}{64}a\left(32\Delta+16\sqrt{\Delta^2+4}-\left(\Delta^2+4\right)^{\frac{3}{2}}-8\Delta^3+5\Delta^2\sqrt{\Delta^2+4}\right)-\frac{1}{24}\Delta\left(\Delta^2+9\right) \\
&\gtreqless 0 \Longleftrightarrow a \gtreqless \frac{8}{3}\frac{\Delta\left(\Delta^2+9\right)}{32\Delta+16\sqrt{\Delta^2+4}-(\Delta^2+4)^{\frac{3}{2}}-8\Delta^3+5\Delta^2\sqrt{\Delta^2+4}} \in (0, 0.45)
\end{aligned}
$$

Thus $\widehat{W} > \widetilde{W}$ if $a \geq 0.45$. ■

Proof of Proposition 3. First, suppose

$$v_A - x_A > v_B - x_B$$

so that some of the consumers with $D = B$ will switch to A, but no consumer whose $D = A$ will switch to B. The marginal switching consumer with $D = B$ is

$$\sigma = \Delta - x_A + x_B.$$

From (9), the equilibrium x_A^e and x_B^e satisfy the first-order conditions

$$\partial\pi_A/\partial x_A = r'(x_A)\left[1 + F(\Delta - x_A + x_B)\right] - r(x_A) f(\Delta - x_A + x_B) = 0,$$

$$\partial\pi_B/\partial x_B = r'(x_B)\left[1 - F(\Delta - x_A + x_B)\right] - r(x_B) f(\Delta - x_A + x_B) = 0,$$

which can be rewritten as (10) if $x_A^e > 0$ and $x_B^e > 0$.

Note that $x_A^e - x_B^e \geq 0$, because otherwise $x_B^e > x_A^e > 0$, which implies

$$\frac{r'\left(x_A^e\right)}{r\left(x_A^e\right)} = \frac{f(\sigma^e)}{1 + F(\sigma^\diamond)} > \frac{f(\sigma^e)}{1 - F(\sigma^e)} = \frac{r'\left(x_B^e\right)}{r\left(x_B^e\right)} \to -F(\sigma^e) > F(\sigma^e),$$

a contradiction.

Next, $\sigma^e = \Delta - x_A^e + x_B^e > 0$, because if $\sigma^e < 0$, B can increase π_B by raising x_B; and if $\sigma^e = 0$, we would have $x_A^e = x_B^e$ (from (10) since F would be 0), which implies $\sigma^e = \Delta - x_A^e + x_B^e = \Delta > 0$, a contradiction. Hence $x_A^e - x_B^e < \Delta$. And $\sigma^e < 1$, because $\left.\frac{\partial\pi_B}{\partial x_B}\right|_{x_B = 1 + x_A^e - \Delta} < 0$.

The only other potential equilibrium may arise when $v_A - x_A < v_B - x_B$, in which case the marginal switching consumer whose $D = B$ is $\sigma = -\Delta + x_A - x_B > 0$, and the two firms' profit functions are

$$\pi_A = r(x_A)\frac{1}{2}\left[1 - F(-\Delta + x_A - x_B)\right], \quad \pi_B = r(x_B)\frac{1}{2}\left[1 + F(-\Delta + x_A - x_B)\right].$$

We now show there can be no such an equilibrium. Suppose to the contrary that the equilibrium exists. Then at such an equilibrium, (x_A, x_B) satisfy the first-order conditions

$$\begin{aligned}
r'(x_A)\left[1 - F(-\Delta + x_A - x_B)\right] - r(x_A) f(-\Delta + x_A - x_B) &= 0, \\
r'(x_B)\left[1 + F(-\Delta + x_A - x_B)\right] - r(x_B) f(-\Delta + x_A - x_B) &\leq 0.
\end{aligned}$$

where

$$\sigma = -\Delta + x_A - x_B > 0 \Rightarrow x_A - x_B > \Delta \Rightarrow x_A > x_B \geq 0.$$

Hence, since $\rho'(x) > 0 \Longleftrightarrow \frac{r'(x)}{r(x)}$ is decreasing,

$$\begin{aligned}
\frac{r'(x_A)}{r(x_A)} &= \frac{f(-\Delta + x_A - x_B)}{1 - F(-\Delta + x_A - x_B)} < \frac{r'(x_B)}{r(x_B)} \leq \frac{f(-\Delta + x_A - x_B)}{1 + F(-\Delta + x_A - x_B)} \\
&\Rightarrow 1 + F(\sigma) < 1 - F(\sigma) \Rightarrow 2F(\sigma) < 0,
\end{aligned}$$

a contradiction.

We next establish the expressions for S^e and Π^e: In equilibrium, consumers whose $D = B$ will switch to A if $s < \sigma^e$. Hence, consumer surplus is

$$\begin{aligned}
S^e &= \frac{1}{2}(v_A - x_A^e) + \frac{1}{2}(v_B - x_B^e)\left[1 - F(\sigma^e)\right] + \frac{1}{2}\int_0^{\sigma^e} (v_A - x_A^e - s) f(s)\, ds \\
&= \frac{1}{2}\left[(v_A - x_A^e) + (v_B - x_B^e) + \int_0^{\sigma^e} F(s)\, ds\right].
\end{aligned}$$

The expression for Π^e follow directly from (9).

Moreover, suppose $\sigma^e \leq \hat{\sigma}$. From (4) and (10), together with the fact that $\rho'(x) > 0$,

$$\frac{r'(x_B^e)}{r\left(x_B^e\right)} = \frac{f(\sigma^e)}{1 - F(\sigma^e)} \leq \frac{f(\hat{\sigma})}{1 - F(\hat{\sigma})} = \frac{r'(\hat{x}_A)}{r(\hat{x}_A)},$$

and hence $x_B^e \geq \hat{x}_A$. It follows that $x_A^e > x_B^e \geq \hat{x}_A > \hat{x}_B$. Therefore,

$$\Pi^e = r(x_A^e)\frac{\left[1 + F(\sigma^e)\right]}{2} + r(x_B^e)\frac{\left[1 - F(\sigma^e)\right]}{2} > r(x_B^e),$$

while

$$\widehat{\Pi} = r(\hat{x}_A)\left[1 - F(\hat{\sigma})\right] + r(\hat{x}_B) F(\hat{\sigma}) < r(\hat{x}_A).$$

Thus $\Pi^e > \widehat{\Pi}$. Also, because $\hat{x}_A \leq x_B^e < x_A^e$, $v_B < v_A$, and $\sigma^e \leq \hat{\sigma}$,

$$\begin{aligned}\widehat{S} &= v_A - \hat{x}_A + \int_0^{\hat{\sigma}} F(s)\,ds = \frac{2(v_A - \hat{x}_A) + 2\int_0^{\hat{\sigma}} F(s)\,ds}{2} \\ &> \frac{v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)\,ds}{2} = S^e.\end{aligned}$$

Next, suppose (C1) is satisfied: $r(x) = ax^m$ and $F(s) = s^n$.

(i) If $n = 1$, then

$$x_A^e = \frac{1}{2m+1}\left(m + m\Delta + 2m^2\right), \quad x_B^e = \frac{1}{2m+1}\left(m - m\Delta + 2m^2\right), \quad \sigma^e = \frac{\Delta}{2m+1}.$$

$$\Pi^e = a\left(\frac{1}{2m+1}\left(m + m\Delta + 2m^2\right)\right)^m \frac{1 + \frac{\Delta}{2m+1}}{2} + a\left(\frac{1}{2m+1}\left(m - m\Delta + 2m^2\right)\right)^m \frac{1 - \frac{\Delta}{2m+1}}{2}.$$

$$S^e = \frac{1}{2}\left(v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)\,ds\right)$$

$$\sigma^e - \hat{\sigma} = \frac{\Delta}{2m+1} - \frac{m - \Delta}{2m+1} = \frac{2\Delta - m}{2m+1} < 0 \Leftrightarrow m > 2\Delta.$$

Thus, $m > 2\Delta \Longrightarrow \sigma^e < \hat{\sigma}$, which we showed was sufficient for $\widehat{\Pi} < \Pi^e$ and $\widehat{S} > S^e$.

(ii) If $m = n = 1$, then from (10) we have

$$x_A^e = 1 + \frac{1}{3}\Delta, \quad x_B^e = 1 - \frac{1}{3}\Delta > 0\ , \quad \sigma^e = \frac{1}{3}\Delta.$$

$$S^e = \frac{1}{2}(v_A + v_B - 2) + \frac{\Delta^2}{36}.$$

$$\Pi^e = \frac{1}{9}a\left(\Delta^2 + 9\right), \quad W^e = \frac{1}{9}a\left(\Delta^2 + 9\right) + \frac{1}{2}(v_A + v_B - 2) + \frac{\Delta^2}{36}.$$

$$\hat{S} - S^e = \frac{1}{36}\left(2\Delta + \Delta^2 + 14\right) > 0; \quad \widehat{\Pi} - \Pi^e = \frac{1}{9}a\left(2\Delta + \Delta^2 - 4\right) < 0.$$

$\widehat{W} - W^e = \frac{1}{36}\left(14 - 16a + 2\Delta + 8a\Delta + \Delta^2 + 4a\Delta^2\right) \gtreqless 0 \Longleftrightarrow a \lesseqgtr a_1^e(\Delta) = \frac{\Delta^2 + 2\Delta + 14}{4(4 - 2\Delta - \Delta^2)}$, with $a_1^e(\Delta) \in \left(\frac{7}{8}, \frac{17}{4}\right)$ for $\Delta \in (0,1)$.

(iii) If $m = 1$ and $n = 2$, then

$$x_A^e = \frac{1}{4\Delta}\left(-4b\Delta + \Delta^2 + 4\right), \qquad x_B^e = -\frac{1}{4\Delta}\left(4b\Delta + \Delta^2 - 4\right), \quad \sigma^e = \frac{1}{2}\Delta.$$

$$\sigma^e - \hat{\sigma} = \frac{1}{2}\Delta - \frac{1}{4}\left(\sqrt{\Delta^2 + 4} - \Delta\right) < 0 \Leftrightarrow 3\Delta < \sqrt{\Delta^2 + 4} \Leftrightarrow \Delta^2 < \frac{1}{2}.$$

But for any $\Delta < 1$,

$$\Pi^e = r\left(x_A^e\right)\frac{1+F\left(\sigma^e\right)}{2} + r\left(x_B^e\right)\frac{1-F\left(\sigma^e\right)}{2} = \frac{a}{16}\frac{\Delta^4+16}{\Delta},$$

$$S^e = \frac{1}{2}\left(v_A - \frac{1}{4\Delta}\left(-4b\Delta+\Delta^2+4\right) + v_B + \frac{1}{4\Delta}\left(4b\Delta+\Delta^2-4\right) + \int_0^{\frac{1}{2}\Delta} F\left(s\right)ds\right).$$

$$\widehat{\Pi} - \Pi^e = \frac{a}{64}\frac{-36\Delta^2+5\Delta^3\sqrt{\Delta^2+4}+24\Delta\sqrt{\Delta^2+4}-\Delta\left(\Delta^2+4\right)^{\frac{3}{2}}-64}{\Delta} < 0,$$

$$\widehat{S} - S^e = -\frac{132\Delta^2+8\Delta^4-3\Delta^3\sqrt{\Delta^2+4}-96\Delta^2+72\Delta\sqrt{\Delta^2+4}-\Delta\left(\Delta^2+4\right)^{\frac{3}{2}}-192}{192\Delta} > 0.$$

$$\begin{aligned}\widehat{W} &\gtreqless W^e \\ \iff a &\lesseqgtr a_2^e\left(\Delta\right) = \frac{1}{3}\frac{36\Delta^2+8\Delta^4-3\Delta^3\sqrt{\Delta^2+4}+72\Delta\sqrt{\Delta^2+4}-\Delta\left(\Delta^2+4\right)^{\frac{3}{2}}-192}{-36\Delta^2+5\Delta^3\sqrt{\Delta^2+4}+24\Delta\sqrt{\Delta^2+4}-\Delta\left(\Delta^2+4\right)^{\frac{3}{2}}-64} \in \left(0.035, 1\right).\end{aligned}$$

■

Proof of Proposition 4. First, at the potential equilibrium where some consumers with $D = B$ switch to A, the marginal consumer is $\sigma = \Delta - (x_A - x_B) > 0$. The equilibrium x_A^- and x_B^- solve the first-order conditions

$$\begin{aligned}\frac{\partial \pi_A}{\partial x_A} &= r'\left(x_A\right)\left[\lambda + (1-\lambda)F\left(\Delta - x_A + x_B\right)\right] - r\left(x_A\right)(1-\lambda)f\left(\Delta - x_A + x_B\right) = 0, \\ \frac{\partial \pi_B}{\partial x_B} &= r'\left(x_B\right)\left[1 - F\left(\Delta - x_A + x_B\right)\right] - r\left(x_B\right)f\left(\Delta - x_A + x_B\right) = 0;\end{aligned}$$

which can be rewritten as

$$\frac{r'\left(x_A\right)}{r\left(x_A\right)} = \frac{(1-\lambda)f\left(\sigma\right)}{\lambda+(1-\lambda)F\left(\sigma\right)}, \qquad \frac{r'\left(x_B\right)}{r\left(x_B\right)} = \frac{f\left(\sigma\right)}{1-F\left(\sigma\right)}. \tag{17}$$

Since $\left(x_A^- - x_B^-\right) < \Delta$ if λ is sufficiently close to $\frac{1}{2}$, there is some $\lambda^- \in \left(\frac{1}{2}, 1\right)$ such that if $\lambda \leq \lambda^-$, in equilibrium $x_A^- > x_B^- > 0$, $\sigma^- = \Delta - \left(x_A^- - x_B^-\right) > 0$, and there is consumer switching only from B to A.

Next, consider the potential equilibrium where some consumers with $D = A$ switch to B. In this case, the equilibrium x_A^+ and x_B^+ solve the first-order conditions

$$\begin{aligned} r'(x_A)\left[1 - F(-\Delta + x_A - x_B)\right] - r(x_A) f(-\Delta + x_A - x_B) &= 0, \\ r'(x_B)\left[1 - \lambda + \lambda F(-\Delta + x_A - x_B)\right] - r(x_B)\lambda f(-\Delta + x_A - x_B) &= 0; \end{aligned}$$

which can be written as

$$\frac{r'\left(x_A^+\right)}{r\left(x_A^+\right)} = \frac{f(\sigma^+)}{1 - F(\sigma^+)}, \qquad \frac{r'\left(x_B^+\right)}{r\left(x_B^+\right)} = \frac{\lambda f(\sigma^+)}{1 - \lambda + \lambda F(\sigma^+)}, \tag{18}$$

where $\sigma^+ = x_A^+ - x_B^+ - \Delta > 0$, or $x_A^+ - x_B^+ > \Delta$. As $\lambda \to 1$, this equilibrium exists as in Proposition 1. On the other hand, as $\lambda \to \frac{1}{2}$,

$$\begin{aligned} \frac{r'\left(x_A^+\right)}{r\left(x_A^+\right)} - \frac{r'\left(x_B^+\right)}{r\left(x_B^+\right)} &= \frac{f(\sigma^+)}{1 - F(\sigma^+)} - \frac{\lambda f(\sigma^+)}{1 - \lambda + \lambda F(\sigma^+)} \\ &\Rightarrow \frac{r'\left(x_A^+\right)}{r\left(x_A^+\right)} - \frac{r'\left(x_B^+\right)}{r\left(x_B^+\right)} = \frac{f(\sigma^+)}{1 - F(\sigma^+)} - \frac{f(\sigma^+)}{1 + F(\sigma^+)} > 0, \end{aligned}$$

which cannot hold if $x_A^+ > \Delta + x_B^+$. Hence, there is some $\lambda^+ \in \left(\lambda^-, 1\right)$ such that (18) holds if and only if $\lambda > \lambda^+$.

Now suppose $r(x) = ax$ and $F(s) = s$. Then

$$x_A^- = \frac{1}{3}\left(\frac{1 + \Delta + \lambda - \Delta\lambda}{(1 - \lambda)}\right), \qquad x_B^- = \frac{1}{3}\left(\frac{2 - \Delta - \lambda + \Delta\lambda}{(1 - \lambda)}\right).$$

$$\sigma^- = \Delta - \left(x_A^- - x_B^-\right) = \frac{1}{3}\frac{1 + \Delta - 2\lambda - \Delta\lambda}{1 - \lambda} > 0 \Leftrightarrow \lambda < \lambda^- \equiv \frac{\Delta + 1}{\Delta + 2}.$$

$$\Pi^- = \pi_A^- + \pi_B^- = \frac{1}{9}a\frac{5 - 2\Delta + 2\Delta^2 - 2\lambda(1 - \Delta)(1 - 2\Delta - \lambda + \Delta\lambda)}{1 - \lambda}. \tag{19}$$

$$\frac{d\Pi^-}{d\lambda} = \frac{1}{9}a\frac{3 + 4\Delta - 2\Delta^2 + 2\lambda(\Delta - 1)^2(2 - \lambda)}{(\lambda - 1)^2} > 0.$$

$$\Pi^- - \hat{\Pi} = \frac{1}{9}a\frac{4\Delta(2\lambda - 1) + \lambda\left(3 - 2\Delta^2\right) + 2\lambda^2(\Delta - 1)^2}{1 - \lambda} > 0.$$

$$S^- = \frac{-11a + 18(av_B + b)(1 - \lambda) + 8a\left(\Delta + \lambda - \lambda^2\right) + a\Delta^2 + a\Delta\lambda(-2\Delta - 2\lambda + \Delta\lambda - 6)}{18a(1 - \lambda)}.$$

$$\frac{dS^-}{d\lambda} = -\frac{1}{18}\frac{3 - 2\Delta + \Delta^2 + \lambda(\Delta + 2)(\Delta - 4)(\lambda - 2)}{(\lambda - 1)^2} < 0.$$

$$S^{-}-\hat{S}=\frac{1}{18}\frac{-2\Delta-3\lambda-\lambda(4-\Delta)(-\Delta+2\lambda+\Delta\lambda)}{1-\lambda}<0.$$

On the other hand,

$$x_A^{+}=\frac{1}{3a\lambda}(a+a\lambda+a\Delta\lambda), \quad x_B^{+}=\frac{1}{3a\lambda}(2a-a\lambda-a\Delta\lambda),$$

$$\sigma^{+}=-\Delta+x_A^{+}-x_B^{+}=\frac{1}{3}\frac{2\lambda-\Delta\lambda-1}{\lambda}>0\Leftrightarrow\lambda>\frac{1}{2-\Delta}\equiv\lambda^{+}.$$

$$\lambda^{+}-\lambda^{-}=\frac{1}{2-\Delta}-\frac{\Delta+1}{\Delta+2}=\frac{\Delta^2}{(2-\Delta)(\Delta+2)}>0.$$

$$\Pi^{+}=\frac{1}{9}a\frac{5+2\lambda(\Delta+1)(\lambda+\Delta\lambda-1)}{\lambda}, \tag{20}$$

$$\frac{d\Pi^{+}}{d\lambda}=\frac{1}{9}a\frac{2\lambda^2(\Delta+1)^2-5}{\lambda^2}\gtreqless 0\text{ if }\lambda\gtreqless\frac{\sqrt{5/2}}{(\Delta+1)}.$$

If $\Delta\leq 0.581$, $\frac{\sqrt{5/2}}{(\Delta+1)}\geq 1$, and hence $\frac{d\Pi^{+}}{d\lambda}<0$ for all $\lambda>\lambda^{+}$; while if $\Delta>0.581$, $\lambda^{+}=\frac{1}{2-\Delta}>\frac{1}{2-0.581}=0.704\,72$, and $\frac{d\Pi^{+}}{d\lambda}<0$ if $\lambda\in\left(\lambda^{+},\frac{\sqrt{5/2}}{(\Delta+1)}\right)$ and $\frac{d\Pi^{+}}{d\lambda}>0$ if $\lambda>\frac{\sqrt{5/2}}{(\Delta+1)}$.

$$\Pi^{+}-\widehat{\Pi}=\frac{1}{9}a(1-\lambda)\frac{5-2\lambda(\Delta+1)^2}{\lambda}\gtreqless 0\text{ if }\lambda\lesseqgtr\frac{5/2}{(\Delta+1)^2}.$$

If $\Delta\leq 0.581$, $\frac{5/2}{(\Delta+1)^2}\geq 1$, and hence $\Pi^{+}-\widehat{\Pi}>0$ for all $\lambda>\lambda^{+}$; while if $\Delta>0.581$, $\frac{5/2}{(\Delta+1)^2}<1$, and hence $\Pi^{+}>\widehat{\Pi}$ if $\lambda\in\left(\lambda^{+},\frac{5/2}{(\Delta+1)^2}\right)$ and $\Pi^{+}<\widehat{\Pi}$ if $\lambda>\frac{5/2}{(\Delta+1)^2}$.

Furthermore,

$$S^{+}=\lambda\left(v_A-x_A^{+}\right)+(1-\lambda)\left(v_B-x_B^{+}\right)+\lambda\int_0^{\sigma^{+}}F(s)\,ds.$$

Hence:

$$S^{+}=\frac{1}{18}\frac{-11a+8a\lambda+18a\lambda v_B+8a\Delta\lambda-a\lambda^2(\Delta+4)(2-\Delta)}{a\lambda}. \tag{21}$$

$$\frac{dS^{+}}{d\lambda}=\frac{1}{18}\frac{-8\lambda^2+\Delta^2\lambda^2+2\Delta\lambda^2+11}{\lambda^2}>0,$$

$$S^{+}-\widehat{S}=\frac{1}{18}\frac{19\lambda+2\Delta\lambda^2-2\lambda\Delta-8\lambda^2+\Delta^2\lambda^2-\Delta^2\lambda-11}{\lambda}<0.$$

Finally,

$$
\begin{aligned}
& W^{-}-\widehat{W} \\
= \; & \frac{-2\Delta-\lambda(\Delta-1)(\Delta-3)-\lambda^{2}(\Delta+2)(4-\Delta)+2a\left(8\Delta\lambda-4\Delta+3\lambda-2\Delta^{2}\lambda\right)+4a\lambda^{2}(\Delta-1)^{2}}{18(1-\lambda)} \\
\gtreqless \; & 0 \Leftrightarrow a \gtreqless \frac{2\Delta+\lambda(\Delta-1)(\Delta-3)+\lambda^{2}(\Delta+2)(4-\Delta)}{2\left(8\Delta\lambda-4\Delta+3\lambda-2\Delta^{2}\lambda\right)+4\lambda^{2}(\Delta-1)^{2}}=a^{-},
\end{aligned}
$$

where $a^{-}<1$ if Δ is sufficiently small and λ is sufficiently close to $\frac{1}{2}$, while $a^{-}>1$ if $\Delta \geq \frac{1}{4}$.

$$
W^{+}-\widehat{W}=\frac{1}{18}(1-\lambda)\frac{10a-4a\lambda(\Delta+1)^{2}-2\Delta\lambda-\Delta^{2}\lambda+8\lambda-11}{\lambda}.
$$

If $10 \leq 4\lambda(\Delta+1)^{2}$, then $W^{+}<\widehat{W}$. If $10>4\lambda(\Delta+1)^{2}$, which holds if $\Delta \leq 0.58$,

$$
W^{+} \gtreqless \widehat{W} \Leftrightarrow a \gtreqless \frac{2\Delta\lambda+\Delta^{2}\lambda-8\lambda+11}{10-4\lambda(\Delta+1)^{2}}=a^{+},
$$

where $a^{+}>1$ if $\Delta \in[0.27,0.58]$ and $a^{+}<1$ if $\Delta \leq 0.2$. ■

Proof of Corollary 3. Since $\lambda \geq \lambda^{+}=\frac{1}{2-\Delta^{l}}$, the analysis in Proposition 4 for $\lambda \geq \lambda^{+}$ applies. From (21),

$$
S^{l}=v_{A}^{l}-\frac{1+6\lambda+4\lambda^{3}+\left(\Delta^{l}\right)^{2}\lambda^{2}(1-2\lambda)+2\Delta^{l}\lambda(\lambda+1)^{2}}{18\lambda^{2}}. \tag{22}
$$

Under competitive bidding, $\lambda=1$, $\Delta=v_{A}-v_{B}$, and from (16)

$$
\widehat{S}=v_{A}-\frac{1}{18}\left(11+8\Delta-\Delta^{2}\right).
$$

Thus, since $v_{A}^{l}=v_{A}-\delta_{A}$ and $\Delta^{l}=\Delta-\delta_{A}-\delta_{B}>0$,

$$
S^{l}-\widehat{S}=-\delta_{A}+\frac{\lambda^{2}\left(11+8\Delta-\Delta^{2}\right)-1-6\lambda-4\lambda^{3}-\Delta^{l}\lambda\left(4\lambda+\Delta^{l}\lambda+2\lambda^{2}-2\Delta^{l}\lambda^{2}+2\right)}{18\lambda^{2}}.
$$

Then, $S^{l}-\widehat{S}$ decreases in Δ^{l} and increases in λ, because

$$
\frac{d\left(S^{l}-\widehat{S}\right)}{d\Delta^{l}}=\frac{1}{9}\frac{-2\lambda\left(1-\Delta^{l}\right)-\Delta^{l}\lambda-\lambda^{2}-1}{\lambda}<0,
$$

$$
\frac{d\left(S^{l}-\widehat{S}\right)}{d\lambda}=\frac{1}{9}\frac{3\lambda-2\lambda^{3}+\left(\Delta^{l}\right)^{2}\lambda^{3}+\Delta^{l}\lambda-\Delta^{l}\lambda^{3}+1}{\lambda^{3}}>0.
$$

Moreover,

$$-1-6\lambda-4\lambda^3+11\lambda^2+\lambda^2\left(8\,(0.4)-(0.4)^2\right)>0$$

if $\lambda \geq 0.66$, implying $S^l-\widehat{S}>0$ if $\lambda \geq 0.7$ while δ_A and Δ^l are sufficiently small.

On the other hand, suppose $\delta_A=\delta_B=\delta$. Then

$$\begin{aligned} & S^l-\widehat{S} \\ = \; & \frac{-1-6\lambda+11\lambda^2-4\lambda^3-2\Delta\lambda\,(1-\lambda)\,(1-\lambda+\Delta\lambda)-2\lambda\delta\,(2\lambda-1)\,(2-\lambda+2\Delta\lambda-2\lambda\delta)}{18\lambda^2} \\ < \; & 0. \end{aligned}$$

Finally, suppose $v_A=5$ and $v_B=4.5$, and $\lambda=0.8$. Then $S^l>\widehat{S}$ if $\Delta^l\leq 0.314$ and $\delta_A=0$; that is, if changing λ from 1 to 0.8 increases v_B by at least $0.5-0.314=0.186$ without lowering v_A. If $\delta_B=0.25$, so that $v_B^l=4.75$ and $v_A^l=5-\delta_A$. Then $v_A^l-v_A=-\delta_A$, $\Delta^l=0.25-\delta_A$, and

$$\begin{aligned} & S^l-\widehat{S} \\ = \; & -\delta_A+\frac{\lambda^2\left(11+8\Delta-\Delta^2\right)-1-6\lambda-4\lambda^3-\Delta^l\lambda\left(4\lambda+\Delta^l\lambda+2\lambda^2-2\Delta^l\lambda^2+2\right)}{18\lambda^2} \\ = \; & \frac{1}{180}\left(-102.0\delta_A+6.0\delta_A^2+5.0\right)>0 \end{aligned}$$

if $\delta_A<0.05$ (i.e., if $\delta_A<20\%$ of δ_B). ■

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