Assigning Default Position for Digital Goods: Competition, Regulation, and Welfare

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Abstract. We analyze alternative ways to assign the default position for competing digital goods such as search engines. When two firms vie for the position through bidding, the higher-quality firm typically wins but delivers lower utility than the rival due to heightened monetization (e.g., unwanted ads), exploiting consumers' switching costs. Paradoxically, increasing via regulation the rival's default share tends to raise profit and harm consumers, at least in the short run. Delegating the default choice to consumers benefits them but harms the weaker firm. Our findings highlight the subtle welfare tradeoffs in default assignment, an important and controversial policy issue.

Keywords: Default Position, Digital Goods, Competition, Regulation

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1. INTRODUCTION

This paper addresses an important controversy regarding key digital products: how to choose the supplier whose product will be preset as the default for consumers? In many situations where competing products vie for the default position, the selection is made by a third party that supplies a different good to the consumers. For example, the manufacturer of a PC or a mobile device may choose the browser, search engine, or other software that will be pre-installed. Although consumers may switch to a non-default product, doing so can entail switching costs that create significant inertia; many consumers lack the technical savvy to switch or the willingness to incur the hassle. Thus, the firm whose product obtains default status can gain a substantial competitive edge over rivals.

A striking example comes from the landmark case brought by the U.S. Department of Justice (DOJ) against Microsoft in the late 1990s for exclusionary practices against Netscape's Navigator browser, the main competitor to Microsoft's Internet Explorer browser. A key piece of the DOJ's evidence was the much larger growth in Explorer's market share at Internet Service Providers (ISPs) that agreed to promote Explorer exclusively: from 20% to 90% vs. from 20% to 30% at other ISPs (Dunham, 2006). Even where switching appears easy—"just a click away" for some digital products—the default position can be valuable, as evidenced by the large payments that firms are willing to make for this position.[1] Google reportedly pays hundreds of millions of dollars annually to be the default search engine on Mozilla's Firefox browser; billions annually to be the default search engine on Apple's Safari browser; and considerable sums to other parties such as wireless carriers (DOJ, 2020; Ostrovsky, 2021). The European Commission's (2018) Android decision, finding that Google foreclosed distribution outlets to competing search engines, flagged such payments to third parties, as did the DOJ's (2020) lawsuit.[2]

The Google search controversy offers a useful springboard for addressing some questions

[1] Switching between browsers in the late 1990s was more difficult than switching between search engines today, notably due to the slowness of downloading a second browser via a narrowband Internet connection.

[2] Both agencies also stressed Google's requirement that device manufacturers preinstall its search app and Chrome browser (an important outlet for Google Search) in order to obtain access to the Google Play Store.

of broader interest. Often, as with Google in search, one of the firms vying for the default position enjoys a quality advantage, for example due to initially superior technology. Critics worry that the leading firm may prolong its dominance by paying for default positions at key distribution outlets to deprive rivals of the scale needed to compete effectively. Google, or a similarly situated leading firm, might plausibly counter that its willingness to outbid rivals for default status derives only from its product superiority (e.g., Walker, 2020), because a firm that expects to retain more consumers than would a lower-quality rival typically is willing to spend more to attract consumers. For example, in Milgrom and Roberts (1986) greater expenditure on advertising acts as a signal of quality. Athey and Ellison (2011) and Chen and He (2011) analyze auctions of ad positions to competing firms where a higher position will be searched earlier by consumers.[3] Although firms charge equal prices, a higher-quality firm outbids lower-quality rivals for a higher position because increased exposure yields it more product matches than to such rivals hence greater sales.

The greater-sales argument, however, may be less applicable to bidding for default position for search engines or some other digital products. If the default is won by an inferior firm, some of its customers may switch to the superior firm, which reduces the latter's willingness to pay for default status. Consider this simple example: a unit mass of consumers demand the product and are initially assigned to the default firm. Firm A provides higher quality than firm B, and both firms earn equal revenue per consumer, normalized to one. If firm A wins the default, it retains all consumers and firm B gets none. If firm B wins, a share q of consumers quit and move to firm A. Each firm's maximum bid is the difference between the number of its patronizing consumers with and without the default position: firm B's maximum bid is $(1-q)-0$ and firm A's is $1-q$, the same amount. Thus, it is not obvious that greater popularity alone would induce Google to outbid a rival for default.

Moreover, Google's view that it offers a superior product is disputed by some critics. They argue that while Google may deliver more relevant search results, it offers a worse overall consumer experience than some other search engines because it engages in excessive

[3]Similarly, in the literature on auctions of advertising positions by an online platform (e.g., Edelman et al. 2007), an ad placed at a higher position will be seen by more consumers.

monetization, e.g., through intrusive tracking or by prioritizing ads over natural search results. They contend that Google's enduring market share dominance is attributable to its ubiquitous default position, not to superior quality. This, in turn, raises the question: If Google offers an inferior product, how can it outbid rivals for the default position?

This paper addresses two broad issues using a parsimonious model that seeks to capture salient features of the search engine environment. First, what are the characteristics of the equilibrium when the default position is assigned through bidding? Second, compared to the bidding outcome, do alternative regulatory schemes for assigning the default position benefit consumers and total welfare?

Section 2 presents the model. Consumers choose between two competing suppliers of a given product that differ only in product qualities, with firm A offering higher quality than firm B.[4] Each firm sets a monetization level ('charge') which generates revenue but imposes disutility to the consumer. In traditional economic models, the charge is typically a firm's price to consumers. However, many digital products have zero price but firms can monetize them through other methods, such as selling consumer data to a third party, (unwanted) targeted advertising, using consumer information to engage in price discrimination for a related product, etc. Our formulation admits broad interpretations of the charge, monetary and/or non-monetary. Consumers are presented with one of the products as the default, and can switch to the other product by incurring a private switching cost randomly drawn from a known probability distribution. They decide whether to switch by considering the firms' (exogenous) qualities and their (simultaneously-chosen) charges. Their values for the products are assumed to be high enough so the market is fully covered.

In Section 3 we assume that a third party selects the default product for consumers and assigns the default position to the highest-bidding firm. We characterize the equilibrium in each of the alternative cases where the default position is assigned either to A or to B. The default firm will exploit its sticky consumers by raising its charge such that consumers'

[4] Our main model has fixed qualities, hence abstracts from concerns that Google maintains a quality advantage partly by denying rivals the scale needed to improve their quality. We will address this issue in a 'reduced form' manner in Section 4.3.

utility typically is *lower* from the default product than from the rival—even when the default product has higher quality (Proposition 1).[5] This pattern is consistent with claims made by some Google critics noted earlier. Since the default product yields lower utility in equilibrium, some consumers will switch to the non-default product, though fewer will switch when A is the default.

Firms bid for the default position anticipating the equilibrium outcomes under the two alternative default assignments. We first observe that a firm wins if and only if assigning the default to it rather than the rival results in higher industry profit (Remark 1). Hypothetically, if revenue per consumer were equal for both firms, then industry profit would be the same under either default assignment, implying equal bids despite the different qualities (Remark 2).[6] However, due to their different qualities the firms will choose different monetization levels (charges), and these anticipated endogenous choices lead to different bids. It is not obvious which default assignment yields higher industry profit in this asymmetric duopoly setting, hence which firm will win the default. Indeed, we provide an example where the lower-quality firm wins. Nevertheless, for broad classes of the revenue and switching cost functions, the higher-quality firm wins (Proposition 2 and Corollary 1). Consumer surplus in this case can be higher or lower than under the alternative assignment, i.e. if the default were awarded to the lower-quality firm (Corollary 2).

Instead of competitive bidding, in Section 4 we consider assigning the default position through regulation. One possibility is random assignment across consumers, so that A and B each become the default for half the market, and we characterize the resulting equilibrium (Proposition 3). Firm A now provides higher utility than B—unlike when firm A wins the default for all consumers—and switching will occur only from B to A: now that firm A competes for $B's$ default consumers, $A's$ equilibrium charge exceeds $B's$ by less than $A's$

[5] We assume there is no regulation that constrains firms' monetization charges. This allows us to focus on the assignment of default position and the resulting equilibrium characteristics. In practice, there could be regulations, especially for non-price charges, which would change the equilibrium outcomes. We briefly discuss such regulations in the concluding section.

[6] The logic was illustrated in the simple earlier example: firm A retains more consumers than firm B at the default position, but it also attracts more switching consumers at the non-default position.

quality advantage. Interestingly, this assignment yields higher industry profit and lower consumer surplus than when firm A wins the default position everywhere. Industry profit rises because when firm B holds the default for half the consumers instead of none, it raises its charge substantially, which induces firm A also to raise its charge (albeit by less). Thus, greater symmetry in firms' sticky-customer bases softens competition, unlike greater symmetry in costs or quality which intensifies competition.

Extending this analysis, we consider regulations that limit $A's$ default position to a share of consumers between one half and one, i.e. above the random assignment share but below that under competitive bidding when firm A wins. Consumer surplus again tends to be lower—while industry profit can be either higher or lower—than under competitive bidding; and both welfare measures can vary non-monotonically with firm $A's$ share (Proposition 4).

Section 4 further considers a scenario where firms can improve their product quality by serving more consumers. This scenario is at the heart of the DOJ's (2020) complaint against Google. DOJ argues that a search engine's algorithms improve with the number of users due to greater learning via experimentation. By obtaining default status at leading distribution outlets, Google deprives rival search engines of users and impairs their quality improvement, without necessarily raising its own quality as much. Evaluating this foreclosure argument in an equilibrium model is complex and beyond the scope of this paper.[7] Nevertheless, using our basic model we provide illustrative calculations of the effects of reducing the leading firm's share of default positions through regulatory intervention. Specifically, we report critical levels of the increase in the rival's quality relative to the decrease in the dominant firm's quality such that consumer surplus rises compared to no intervention.

A prominent alternative way of selecting the default product is to let individual consumers choose among the leading suppliers, as required by the European Union's (2022) Digital Markets Act. In our setting such a "choice screen" remedy, paradoxically, is worse for

[7] Gilbert and Newbery's (1982) result, that an incumbent monopolist would outbid a potential entrant for a single vital asset, does not immediately extend to the Google case because there are *multiple* distribution outlets for which firms can bid to attract users. The profitability of sustaining monopoly through bidding for assets is then an open question (Kamien and Zang, 1990; Malueg and Schwartz, 1991; Krishna, 1993).

the weaker firm than even the bidding outcome where the higher-quality rival obtains the default position everywhere. From the standpoint of consumer welfare, choice screen is likely to dominate regulatory assignment in the short run, but can be worse in the long run due to diminished competition. Our results underscore the intricacies in evaluating the welfare consequences of alternative assignment methods.

2. MODEL

The market contains two firms, A and B, that may provide a product to a unit mass of consumers through a third party. To reduce notation, we denote the firms' products also by $i = A, B$. Each consumer demands one unit of the product. By using $i = A$ or B, each consumer obtains utility $u_i = v_i - x_i$, where $x_i \geq 0$ is $i's$ action to monetize its product ('charge'), and $v_A > v_B$ so that A has higher quality. Firm i earns revenue $r(x_i)$ per consumer from its charge x_i, where $r'(x) > 0$, and production cost is normalized to zero. As noted in the Introduction, if x is the usual price, then $r(x) = x$; but our formulation allows general monetization activities, such as unwanted advertising, that are common for digital products (which often have zero price). Hence, $r(x)$ can have general forms including, for instance, concave or convex functions. We allow $r(0) \equiv b \geq 0$, where b can be interpreted as a firm's monetizing activities that have no disutility to consumers.

One of the two products is set as the default option for consumers by the third party. For instance, a PC manufacturer will preset the default search engine from among competing providers. If product i is the default, denoted by $D = i$, a consumer who wishes to use product $j \neq i$ will need to incur a switching cost s. Each consumer's switching cost is the realization of a random variable that has distribution $F(s)$ with density $f(s) > 0$ on support $[0, 1]$.[8] We assume that $v_A - v_B \equiv \Delta \in (0, 1)$ so that firm $A's$ quality lead is not

[8] We could alternatively assume that $s \in [0, \theta]$ for a given parameter $\theta > 0$. Then, as θ decreases, the average switching cost and its dispersion are both lower, which would intensify competition and reduce firms' charges. As $\theta \to 0$, the logic of Bertrant competition implies that $x_B \to 0$ and $x_A \to v_A - v_B$ regardless of whether $D = A$ or $D = B$. We normalize $\theta = 1$ to focus on analyzing the assignment of default position under a given support for $F(s)$.

too large relative to switching costs, and v_A and v_B are both high enough to ensure full market coverage in equilibrium.[9]

We maintain the following assumption:

$$\frac{d^2 (\ln f(s))}{ds^2} \leq 0, \qquad \text{and} \qquad \frac{d^2 (\ln r(x))}{dx^2} < 0. \tag{C1}$$

Assumption (C1) says that $f(s)$ is log-concave and $r(x)$ strictly log-concave; i.e., $f'(s)/f(s)$ is non-increasing and $r'(x)/r(x)$ is decreasing. Note that $d^2 (\ln f(s))/ds^2 \leq 0$ implies that $d\left(\frac{f(s)}{1-F(s)}\right)/ds < 0$, $d\left(\frac{f(s)}{F(s)}\right)/ds < 0$, and $f(s)$ can be either a convex or concave function; whereas a decreasing $r'(x)/r(x)$ also allows convex and concave $r(x)$. Assumption (C1) holds for broad classes of $r(x)$ and $F(s)$ functions. It also ensures that in our model x_A and x_B will be strategic complements, as in Bulow et al. (1985).

In our base model we assume that the default position is allocated through competitive bidding. In a variant of the model we shall examine regulated assignments of the default position. The game with competitive bidding proceeds as follows:

First, the firm that bids higher is assigned the default position and pays the lower bid. Next, either $D = A$ or $D = B$ starts a subgame in which A or B is the default for all consumers and the two firms simultaneously choose x_A and x_B. Finally, consumers choose which product to patronize, after observing D, x_A, and x_B.[10] If $D = i$ and a consumer chooses product $j \neq i$, she needs to incur her personal switching cost.

A strategy of firm i specifies its bid for the default position and its choice of x_i conditional on the assignment of the default position. A consumer's strategy specifies her decision on which product to use, based on D, x_A, x_B and her realized s. We study the subgame perfect Nash equilibrium of this market game, where the strategies of the firms and consumers

[9]For example, if $\frac{r'(1)}{r(1)} < \max\left\{\frac{f(\Delta)}{F(\Delta)}, \frac{f(\Delta)}{1-F(\Delta)}\right\}$, then the market coverage condition is satisfied if $v_A > v_B > 1$. The full-market coverage assumption seems reasonable for the type of goods we are interested in, such as the Internet search engine.

[10]We can allow the possibility that a certain part of a firm's monetizing activities is not observed by consumers before they decide to use its product. Both firms would then set the maximium level for such unobservable activities and consumers would rationally expect this. We may thus consider the x in our model as the observable monetizing charge beyond the unobservable level.

induce a Nash equilibrium in every subgame.

Before proceeding to our analysis, we discuss how our framework compares to some existing work. An extensive literature has studied competition in markets with consumer switching costs (e.g., Klemperer, 1987; Farrell and Klemperer, 2007). Switching costs may arise, for example, because of the time and effort needed to find a new supplier, learn about a new product, or set up a new product.[11] They are likely to vary across consumers, e.g., due to different values of time or technical savvy. The literature often considers firms with the same product quality, and has shown that even for firms that offer ex ante homogeneous products, switching costs can create market power and soften price competition. In our model, firms differ in product quality and can engage in general monetization activities ('charges').[12] Notably, consumers' switching pattern will be affected by which firm holds the default position—in addition to the firms' charges—and these foreseen switching patterns will themselves affect the firms' equilibrium bids for the default position.

As noted, our paper also is related to Athey and Ellison (2011) and Chen and He (2011). There, moving to a higher position in order to be visited by more consumers is worth more to a higher-quality firm because the probability that a product will meet a consumer's need and yield a sale rises with product quality.[13] In those papers, each consumer faces numerous products online that she may search for a match, hence it is fairly natural to consider product quality as uncertain and that total transactions volume may vary significantly with the order of the search process. For the type of ubiquitously-used products we have in mind, we adopt a model of known product quality, where total transactions are approximately fixed (the market is covered). Among other things, this implies that differences in willingness

[11] We abstract from behavioral factors that also may bias consumers to the status quo.

[12] Our formulation of heterogeneous consumer switching costs follows the approach in Chen (1997). This approach has been used to analyze a variety of competition issues, including exclusionary contracts, e.g., Bedre-Defolie and Biglaiser (2017).

[13] Armstrong et al. (2009) likewise find that the highest-quality firm gains most from becoming prominent, but its gain comes, additionally, from charging a higher price. In these models, consumer surplus is higher when the higher(est)-quality firm, rather than its rival(s), holds the default (prominent) position, whereas the reverse can occur in our model.

to bid for the default position are not driven by differences in quality directly, but only indirectly insofar as different assignments of the default position will yield different profiles of equilibrium charges.

3. MARKET EQUILIBRIUM

Denote the equilibrium profits of A and B by (i) $\hat{\pi}_A$ and $\hat{\pi}_B$ when $D = A$ and (ii) $\tilde{\pi}_A$ and $\tilde{\pi}_B$ when $D = B$. We first characterize the equilibrium choices of x_A and x_B under a given assignment of the default position, then analyze the firms' equilibrium bidding incentives and the equilibrium default assignment. We will also provide welfare results.

3.1 Equilibrium when A or B is the Default

First, consider the subgame where $D = A$. In this case, a consumer with switching cost (or 'type') s will remain with firm A if

$$v_A - x_A \geq v_B - x_B - s$$

and will switch to B otherwise. The profit functions of the two firms under $D = A$ are

$$\pi_A = r(x_A)\left[1 - F(x_A - x_B - \Delta)\right]; \qquad \pi_B = r(x_B)\, F(x_A - x_B - \Delta), \tag{1}$$

where $s = x_A - x_B - \Delta$ is the consumer who is indifferent between A and B.

Next, consider the subgame where $D = B$. In this case, a consumer with type s will remain with B if

$$v_B - x_B \geq v_A - x_A - s$$

and will switch to A otherwise. The profit functions of A and B under $D = B$ are

$$\pi_A = r(x_A)\, F(x_B - x_A + \Delta), \qquad \pi_B = r(x_B)\left[1 - F(x_B - x_A + \Delta)\right], \tag{2}$$

where $s = x_B - x_A + \Delta$ is the indifferent consumer.

Denote the equilibrium charges by $\hat{x}_A$, $\hat{x}_B$ when $D = A$, and $\tilde{x}_A$, $\tilde{x}_B$ when $D = B$. We have the following preliminary result:[14]

[14] Unless stated otherwise, proofs for formally-presented results are contained in the Appendix.

Lemma 1 *If* $\frac{r'(\Delta)}{r(\Delta)} > f(0)$ *and* $b \geq 0$ *is sufficiently small, then under either default assignment both firms set positive charges: for* $D = i = A, B$, $\hat{x}_i > 0$ *and* $\tilde{x}_i > 0$.

The conditions in Lemma 1 let us focus on situations where both firms attract consumers in equilibrium. Instead, if $\frac{r'(\Delta)}{r(\Delta)}$ were low or b were high, then firm A when it holds the default position ($D = A$) may prefer $\hat{x}_A = \Delta$ and $\hat{x}_B = 0$ to higher charges, in order to attract all the consumers. This is ruled out if $\frac{r'(\Delta)}{r(\Delta)} > f(0)$ and $b \geq 0$ is sufficiently small (recall that $b = r(0)$ is the firm's monetization benefit aside from its charge). The specific values of Δ and b under which $\hat{x}_i > 0$ and $\tilde{x}_i > 0$ depend on the functional forms of $r(x)$ and $F(s)$. In the rest of the paper, we shall assume the equilibrium is interior ($x_i > 0$ for each firm), as will be true in all our numerical examples.[15]

Proposition 1 below characterizes the unique interior equilibrium under each regime ($D = A$ or B). Denote the marginal switching consumer by $\hat{\sigma} \equiv \hat{x}_A - \hat{x}_B - \Delta$ when $D = A$, and $\tilde{\sigma} \equiv \tilde{x}_B - \tilde{x}_A + \Delta$ when $D = B$. The proposition references equations (3) and (4), which are based on the first-order conditions for the equilibrium charges (see proof of Proposition 1):

$$\frac{r'(\hat{x}_A)}{r(\hat{x}_A)} = \frac{f(\hat{\sigma})}{1 - F(\hat{\sigma})}, \qquad \frac{r'(\hat{x}_B)}{r(\hat{x}_B)} = \frac{f(\hat{\sigma})}{F(\hat{\sigma})}; \tag{3}$$

$$\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} = \frac{f(\tilde{\sigma})}{F(\tilde{\sigma})}, \qquad \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})}. \tag{4}$$

Proposition 1 *Under either default assignment, in the unique equlibrium the default product yields lower utility than the other product, some consumers switch to the other product, and there is less switching when the high-quality product is the default. Formally:*
(i) When $D = A$, $\hat{x}_A$ *and* $\hat{x}_B$ *uniquely solve (3),* $\hat{x}_A - \hat{x}_B > \Delta$, $\hat{\sigma} > 0$, *and* $0 < F(\hat{\sigma}) < \frac{1}{2}$.
(ii) When $D = B$, $\tilde{x}_A$ *and* $\tilde{x}_B$ *uniquely solve (4),* $\tilde{x}_A - \tilde{x}_B < \Delta$, $\tilde{\sigma} > 0$, *and either: if* $F(\Delta) \leq \frac{1}{2}$ *then* $\tilde{x}_A \leq \tilde{x}_B$ *and* $F(\tilde{\sigma}) \leq \frac{1}{2}$, *or if* $F(\Delta) > \frac{1}{2}$ *then* $\tilde{x}_A > \tilde{x}_B$ *and* $F(\tilde{\sigma}) > \frac{1}{2}$.
(iii) $\hat{\sigma} < \tilde{\sigma}$; $\hat{x}_A > \tilde{x}_B$ *and* $\tilde{x}_A > \hat{x}_B$.

[15] For some digital products, a firm may even pay consumers to use its product. We could extend our analysis to such situations by allowing $x < 0$, without the need to impose parameter restrictions that ensure $x_i > 0$ in equilibrium. However, it seems more relevant to focus on scenarios where there are monetization charges that are undesirable to consumers in equilibrium, and we do so to reduce the cases to consider.

For a given assignment of the default position, the equilibrium has several noteworthy features. First, the default product yields lower consumer surplus than the non-default product: (i) when $D = A$, $\hat{x}_A - \hat{x}_B > \Delta \Rightarrow v_A - \hat{x}_A < v_B - \hat{x}_B$; and (ii) when $D = B$, $\tilde{x}_A - \tilde{x}_B < \Delta \Rightarrow v_B - \tilde{x}_B < v_A - \tilde{x}_A$. The default firm clearly will not offer higher surplus than the rival because starting from such a case it could raise its charge while retaining all customers. At equal surplus, the default firm would still retain all consumers since switching requires some cost, but under the assumed conditions it prefers to raise its charge further, while ceding some consumers with low switching costs to the rival.[16] The property that product A offers lower utility in equilibrium when it holds the default, even though it has higher quality / product value, differs from many other settings. It is consistent with perceptions of some critics, discussed in the Introduction, that Google delivers a worse consumer experience due to excessive monetization.

Second, while switching costs prevent some consumers from moving to the non-default product, other consumers do switch in equilibrium and receive higher surplus—net of the switching cost—than the non-switchers. As might be expected, fewer consumers will switch when product A is the default, $\hat{\sigma} < \tilde{\sigma}$. The difference between the switching cost threshholds is $\hat{\sigma} - \tilde{\sigma} = (\hat{x}_A - \hat{x}_B - \Delta) - (\tilde{x}_B - \tilde{x}_A + \Delta)$, where the first term is product A's utility deficit relative to B when $D = A$ (product A is the default), while the second term is B's utility deficit when $D = B$. The difference can be expressed as $(\hat{x}_A - \hat{x}_B) - (\tilde{x}_B - \tilde{x}_A) - 2\Delta$. Although $\hat{x}_A - \hat{x}_B > \tilde{x}_B - \tilde{x}_A$, i.e., the difference between the charges of the default and non-default products is higher when $D = A$ than when $D = B$, this effect is outweighed by A's quality advantage, hence $\hat{\sigma} < \tilde{\sigma}$. Consequently, the deadweight loss from switching costs is lower when product A is the default.

The equilibrium charges under each assignment, together with the distribution of switching costs, determine the allocation of consumers (via $\hat{\sigma}$ and $\tilde{\sigma}$), hence firms' profits. These foreseen profits determine each firm's gain g_i from winning the default position and, hence,

[16] Since switching costs are heterogeneous, each firm faces a downward-sloping (not horizontal) demand, yielding the familiar outcome for Bertrand competition with imperfect substitutes, where both firms earn positive margins.

its maximum bid:

$$g_A = \hat{\pi}_A - \tilde{\pi}_A \qquad g_B = \tilde{\pi}_B - \hat{\pi}_B.$$

The maximum bids satisfy

$$g_A - g_B \gtreqless 0 \Leftrightarrow \hat{\pi}_A - \tilde{\pi}_A \gtreqless \tilde{\pi}_B - \hat{\pi}_B \Leftrightarrow \hat{\pi}_A + \hat{\pi}_B \gtreqless \tilde{\pi}_A + \tilde{\pi}_B.$$

We thus immediately have the following:

Remark 1 *Firm $i \in \{A, B\}$ is willing to bid more than the rival, and hence in equilibrium $D = i$, if and only if industry profit $\Pi \equiv \pi_A + \pi_B$ is higher when $D = i$.*

We will use Remark 1 to analyze which firm will win the bidding. The role of industry profit can be grasped as follows. Moving from $D = B$ to $D = A$ increases firm A's profit but decreases firm B's profit, and firm A outbids B if and only if its gain from this move exceeds B's loss, i.e., if industry profit is higher under $D = A$. The same logic underlies Gilbert and Newbery's (1982) classic result that an incumbent monopolist would outbid a potential entrant for a single innovation or, more generally, for any single asset needed to enter. (See also Tirole, 1988.) The market remains a monopoly if the incumbent wins the bidding but becomes a duopoly if the entrant wins, and the incumbent wins under the fairly weak condition that industry profit is higher under monopoly (over both technologies/products), because monopoly avoids rent dissipation from duopoly competition. Our comparison of industry profits is more complex, as it involves alternative asymmetric duopoly regimes.

Before addressing that comparison in Section 3.2 below, consider the hypothetical case where $x_A = x_B \equiv x_E$ so that $r(x_A) = r(x_B) = r(x_E) = \bar{r}$ is a constant under either $D = A$ or $D = B$. This could be the case, for example, if there is regulation that limits the firms' monetization actions to some common level. Then, from (1) and (2), $\widehat{\Pi} = \bar{r} = \widetilde{\Pi}$. The result below follows immediately from Remark 1.

Remark 2. *If x_A and x_B are constrained to take an equal value $x_E \geq 0$, so that each firm earns the same revenue per consumer $r(x_E)$, then their maximum bids are equal, $g_A = g_B$.*

Intuitively, if firms earn the same revenue per consumer then industry profit is unaffected when consumers are redistributed between the two firms, given that all consumers would purchase in all cases (the market always is covered). Hence, a higher consumer value for $A's$ product in itself does not imply that A will outbid the rival for the default position—generalizing the simple example from the Introduction.[17]

3.2 Equilibrium Assignment of the Default Position

With endogenous choices of x, equilibrium industry profits when $D = A$ and $D = B$ are

$$\widehat{\Pi} = r(\hat{x}_A)[1 - F(\hat{\sigma})] + r(\hat{x}_B)F(\hat{\sigma}), \qquad \widetilde{\Pi} = r(\tilde{x}_B)[1 - F(\tilde{\sigma})] + r(\tilde{x}_A)F(\tilde{\sigma}). \quad (5)$$

Industry profits generally will differ under the alternative default assignments, but it is not obvious which assignment generates the higher profit (hence which firm will win the bidding). Under $D = A$, the default firm earns higher revenue than under $D = B$, as its charge is higher ($\hat{x}_A > \tilde{x}_B$) and so is its share of consumers (since $\hat{\sigma} < \tilde{\sigma} \Longrightarrow [1 - F(\hat{\sigma})] > [1 - F(\tilde{\sigma})]$), but revenue to the non-default firm is lower than under $D = B$ (since $\hat{x}_B < \tilde{x}_A$ and $F(\hat{\sigma}) < F(\tilde{\sigma})$).

Nevertheless, we will provide a sufficient condition for $\widehat{\Pi} > \widetilde{\Pi}$ by comparing the arithmetic sum of per-consumer charges by the two firms ('total charge') under the two allocations: $\hat{x}_A + \hat{x}_B$ under $D = A$ versus $\tilde{x}_B + \tilde{x}_A$ under $D = B$. Since $\hat{x}_A > \tilde{x}_B$ but $\hat{x}_B < \tilde{x}_A$, the comparison of total charges is ambigous in general. The regularity condition below will allow us to make unambiguous comparisons:

$$f'(s) \geq 0, \qquad \text{and} \qquad \rho''(x) \leq 0, \quad \text{(C2)}$$

where $\rho(x) \equiv r(x)/r'(x)$, and $\rho'(x) > 0$ from assumption (C1). Notice that under $\rho''(x) \leq 0$, $r(x)$ can be either concave or convex.

Condition (C2) is consistent with (C1), but neither condition is necessarily implied by the other. Both (C1) and (C2) are satisfied for broad classes of revenue functions and switching cost distributions. In particular, they are satisfied for the following classes of $F(s)$ and

[17] Extending that example so the market is not always covered, suppose that of the share q that quit firm B if it holds the default, a fraction l would leave the market and $1 - l$ would switch to A. The bids become $g_A = 1 - q(1 - l)$, $g_B = 1 - q$, hence $\frac{g_A}{g_B} = 1 + l\frac{q}{1-q}$. For a given q, $\frac{g_A}{g_B} \longrightarrow 1$ as the dropout rate $l \longrightarrow 0$.

$r(x)$, that we shall denote as the *Extended-Power-Functions (EPF)* case:

$$F(s) = s^n \quad \text{and} \quad r(x) = ax^m + b, \tag{6}$$

where $n \geq 1$, and $a > 0$, $b \geq 0$, $b(1-m) \geq 0$, $m > \Delta > 0$.[18]

Lemma 2 *$\hat{x}_A + \hat{x}_B \geq \tilde{x}_B + \tilde{x}_A$ if (C2) holds, $\hat{x}_A + \hat{x}_B \leq \tilde{x}_B + \tilde{x}_A$ if the weak inequalities in (C2) are reversed, and $\hat{x}_A + \hat{x}_B = \tilde{x}_B + \tilde{x}_A$ if $\rho'' = 0$ and $f'(s) = 0$.*

The proof of Lemma 2 uses the result that $\rho(\hat{x}_A) + \rho(\hat{x}_B) \gtreqless \rho(\tilde{x}_A) + \rho(\tilde{x}_B)$ if $f'(s) \gtreqless 0$, following (3) and (4). The mean-value theorem is then used to show that $\hat{x}_A + \hat{x}_B \gtreqless \tilde{x}_A + \tilde{x}_B$ when $\rho''(x) \lesseqgtr 0$. Lemma 2 will play an important role in the comparisons of industry profit.

We now show that firm A will outbid firm B under fairly mild sufficient conditions:

Proposition 2 *Suppose (C2) holds. Then industry profit is greater when the higher quality product is the default if the revenue function is not too concave. Formally, there is some (small) $\varepsilon > 0$ such that $\widehat{\Pi} > \widetilde{\Pi}$ if $r''(x) \geq -\varepsilon$.*

When (C2) holds, $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$ (Lemma 2). Additionally, the ranking of charges satisfies

$$\hat{x}_B < \max\{\tilde{x}_B,\ \tilde{x}_A\} < \hat{x}_A \tag{7}$$

as shown in the proof of Proposition 2. Then, because $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$ and $F(\hat{\sigma}) < F(\tilde{\sigma})$, $\{\hat{x}_B, \hat{x}_A\}$ is a mean-increasing spread of $\{\tilde{x}_A, \tilde{x}_B\}$ so that

$$[1 - F(\hat{\sigma})]\,\hat{x}_A + F(\hat{\sigma})\,\hat{x}_B > [1 - F(\tilde{\sigma})]\,\tilde{x}_B + F(\tilde{\sigma})\,\tilde{x}_A.$$

Hence, $\widehat{\Pi} > \widetilde{\Pi}$ if $r(x)$ is not too concave (i.e., $r''(x) \geq -\varepsilon$).

We next illustrate Proposition 2 with $r(x)$ and $F(s)$ that belong to the extended-power-functions case given by (6), for which (C2) is satisfied.

Corollary 1 *Suppose (6) holds. Then $\widehat{\Pi} > \widetilde{\Pi}$ if $m \geq 1$, and possibly also if $1 > m > \Delta$.*

[18] On the other hand, a violation of (C2) may also violate (C1). For example, if $F(s) = s^n$ for $n < 1$, then (C2) is violated and so is (C1).

If $m \geq 1$, then $\widehat{\Pi} > \widetilde{\Pi}$ because $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$ from Lemma 2, and $r(x) = ax^m + b$ is convex when $m \geq 1$. But if $m < 1$, $r(x)$ is strictly concave, and it is perhaps surprising that $\widehat{\Pi} > \widetilde{\Pi}$ is possible when $\Delta < m < 1$, which allows for a wide range of concavity: $r''(x) = am(m-1)x^{m-2} < 0$. To understand this, notice that the comparison of industry profit depends both on the dispersion of $\{\hat{x}_B, \hat{x}_A\}$ relative to $\{\tilde{x}_B, \tilde{x}_A\}$ and on the degree of concavity. In the case here, it appears that when $r(x)$ becomes more concave, $\{\hat{x}_B, \hat{x}_A\}$ also become less dispersed relative to $\{\tilde{x}_B, \tilde{x}_A\}$, which offsets the effect of increasing concavity so that $\widehat{\Pi} > \widetilde{\Pi}$. For instance, when $F(s) = s$: $\hat{x}_A - \tilde{x}_B = \frac{2}{3}\Delta = \tilde{x}_A - \hat{x}_B$ if $m = 1$, while $\hat{x}_A - \tilde{x}_B = \frac{1}{2}\Delta = \tilde{x}_A - \hat{x}_B < \frac{2}{3}\Delta$ if $m = \frac{1}{2}$, which partly explains why $\widehat{\Pi} > \widetilde{\Pi}$ for all $m > \Delta$ in this case.

If (C2) is violated, for example, when $F(s) = s$ but $\rho''(x) > 0$, then $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$. However, since $F(\hat{\sigma}) < F(\tilde{\sigma})$, the highest charge ($\hat{x}_A$) is weighted most heavily when $D = A$, hence $\widehat{\Pi}$ can still be higher than $\widetilde{\Pi}$, even when $r(x)$ is concave:

Example 1 *Suppose* $r(x) = e^{-\frac{1}{x}}$ *and* $F(s) = s$. *Then* $r''(x) = -e^{\frac{1}{x}} \frac{2x+1}{x^4}$, $\rho'' = 2 > 0$. *Hence,* $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$. *Despite this, numerical analysis indicates that* $\widehat{\Pi} > \widetilde{\Pi}$ *for various values of* Δ. *For instance, if* $\Delta = 0.5$*, then* $\hat{x}_A = 0.943$, $\hat{x}_B = 0.333$, $\hat{\sigma} = 0.11$; $\tilde{x}_A = 0.707 = \tilde{x}_B$, $\tilde{\sigma} = 0.5$; *and* $\widehat{\Pi} = 0.314 > \widetilde{\Pi} = 0.243$.

In this example, even though $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, $\hat{x}_A$ is much higher than $\hat{x}_B$ and $F(\hat{\sigma})$ is much below $F(\tilde{\sigma})$, so that $r(\hat{x}_A)$ is weighted much more than $r(\hat{x}_B)$ under $D = A$, versus equal weights under $D = B$ for $r(\tilde{x}_A)$ and $r(\tilde{x}_B)$ (since $F(\tilde{\sigma}) = \frac{1}{2}$). As a result, $\widehat{\Pi}$ is higher than $\widetilde{\Pi}$ even though $r(x)$ is concave. For all other values of Δ that we checked in Example 1, $\widehat{\Pi} > \widetilde{\Pi}$; and the same is true, for instance, if $r(x) = e^{-\frac{1}{2x}}$.

Proposition 2, Corollary 1, and Example 1 suggest that under fairly mild conditions, industry profit is higher under $D = A$ than under $D = B$, so firm A will win the bidding for the default position. However, if (C2) is violated then $\widehat{\Pi} < \widetilde{\Pi}$ is possible, as in the next example:

Example 2 *Suppose* $r(x) = ax$ *for* $a > 0$ *and* $F(s) = s^{0.7}$. *Then* $f(s) = 0.7s^{-0.3}$, *violating the assumption* $f'(s) \geq 0$ *in (C2). If* $\Delta = 0.3$, *then* $\hat{x}_A = 0.5832$, $\hat{x}_B = 0.1666$, $\hat{\sigma} = 0.116\,6$;

$\tilde{x}_A = 0.4964$, $\tilde{x}_B = 0.5439$, $\tilde{\sigma} = 0.3475$; *and* $\widehat{\Pi} = 0.4906a < \widetilde{\Pi} = 0.5213a$. *Moreover, for other values of* Δ, *it is possible to have either* $\widetilde{\Pi} < \widehat{\Pi}$ *or* $\widetilde{\Pi} > \widehat{\Pi}$.

In Example 2, $\rho''(x) = 0$ but $f'(s) < 0$, which leads to a lower total charge under $D = A$ than under $D = B$: $\hat{x}_A$ is only slightly higher than $\tilde{x}_B$ while $\hat{x}_B$ is much lower than $\tilde{x}_A$. Industry profit could still be higher under $D = A$ because $\hat{x}_A > \tilde{x}_B > \tilde{x}_A > \hat{x}_B$ and $\hat{\sigma} < \tilde{\sigma}$ so that $r(\hat{x}_A)$ is weighted more heavily than $r(\tilde{x}_B)$. However, the mass of consumers with $s < \hat{s}$ is higher when $f(s)$ is decreasing than when it is increasing, and hence for a given $\hat{\sigma}$ more consumers will switch to B (under $D = A$) when $f(s)$ is decreasing—reducing the weight on $r(\hat{x}_A)$ sufficiently to yield a lower weighted average of $r(\hat{x}_A)$ and $r(\hat{x}_B)$ than under $D = B$. Hence, when $f'(s) < 0$, it is possible that

$$\widehat{\Pi} = r(\hat{x}_A)[1 - F(\hat{\sigma})] + r(\hat{x}_B) F(\hat{\sigma}) < r(\tilde{x}_B)[1 - F(\tilde{\sigma})] + r(\tilde{x}_A) F(\tilde{\sigma}) = \widetilde{\Pi}.$$

This also suggests that when (C2) holds so that $f'(s) \geq 0$, for a given $\hat{\sigma}$ fewer consumers will switch to B (under $D = A$), increasing the weight on $r(\hat{x}_A)$ and the weighted average of $r(\hat{x}_A)$ and $r(\hat{x}_B)$, and hence $\widehat{\Pi}$ is higher, which, together with $\rho''(x) \leq 0$, ensures a higher industry profit when A is the default. Therefore, while typically $\widehat{\Pi} > \widetilde{\Pi}$, the comparison of industry profits is subtle, and the three key elements of our model—$r(x)$, $F(s)$, and Δ—all matter for the profit ranking.

Consumer Surplus

Which default assignment will result in higher consumer surplus? When $D = A$, all consumers who stay with A obtain utility $v_A - \hat{x}_A$, while a consumer who switches to B at cost s obtains $v_B - \hat{x}_B - s$, and the switching consumers are those with $s \leq \hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta$. Similarly, when $D = B$, all consumers who stay with B obtain utility $v_B - \tilde{x}_B$, a consumer who switches to A obtains $v_A - \tilde{x}_A - s$, and the switchers have $s \leq \tilde{\sigma} = \tilde{x}_B - \tilde{x}_A + \Delta$. In the Appendix we show that consumer surpus takes the forms in (8):

$$\widehat{S} = v_A - \hat{x}_A + \int_0^{\hat{\sigma}} F(s)\, ds; \qquad \widetilde{S} = v_B - \tilde{x}_B + \int_0^{\tilde{\sigma}} F(s)\, ds. \tag{8}$$

In each case, consumer surplus equals the surplus all consumers would get if they stayed with the default product ($v_A - \hat{x}_A$ or $v_B - \tilde{x}_B$), plus the integral term which denotes the gain to those consumers who switch.[19] This gain arises because, from Proposition 1(i) and (ii), the default product offers lower utility in equilibrium than the rival product.

The difference in consumer surplus between the two default assignments is

$$\widehat{S} - \widetilde{S} = [\Delta - (\hat{x}_A - \tilde{x}_B)] - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds. \tag{9}$$

The square-bracketed term is the difference in utilities of all consumers had they stayed with the default product under $D = A$ compared to $D = B$: A's quality advantage, $\Delta \equiv v_A - v_B$, minus A's charge premium when A holds the default position compared to B's charge when B holds the default. The integral term is the extra gain to switchers under regime $D = B$ compared to $D = A$, where $\tilde{\sigma} > \hat{\sigma}$ from Proposition 1(iii).

Corollary 2 below provides sufficient conditions for consumer surplus to be higher under either $D = A$ or $D = B$.

Corollary 2 *Suppose $f(s)$ and $r(x)$ satisfy (6). Then (i) $\widehat{S} > \widetilde{S}$ if $n = 1$ and $b(1-m) = 0$, but (ii) if $n > 1$, it is possible that $\widehat{S} < \widetilde{S}$.*

Consumer surplus tends to be higher under $D = A$ than under $D = B$ when the total charge is not (much) higher under $D = A$. This holds, for example, if $F(s) = s$ and $b < \frac{a}{3}(1 - \Delta)$ in (6), for which the total charges under $D = A$ and $D = B$ are equal.[20] Notice that if the total charge is higher under $D = A$, then $v_A - \hat{x}_A - \hat{x}_B - (v_B - \tilde{x}_B - \tilde{x}_A)$ is reduced, which makes it more likely that $\widehat{S} < \widetilde{S}$; but

$$\tilde{\sigma} - \hat{\sigma} = v_A - \tilde{x}_A - (v_B - \tilde{x}_B) - [v_B - \hat{x}_B - (v_A - \hat{x}_A)]$$

[19] The integral term is the difference in consumer utility from the two products minus the expected switching cost. When, say, $D = A$, $\int_0^{\hat{\sigma}} F(s)\, ds = \hat{\sigma} F(\hat{\sigma}) - \int_0^{\hat{\sigma}} s f(s)\, ds$, where $\hat{\sigma} = x_A - x_B - \Delta$ is the gross gain to any consumer from switching to B (hence also denotes the consumer indifferent between between remaining at A or incurring $s = \hat{\sigma}$ to switch), while $\int_0^{\hat{\sigma}} s f(s)\, ds$ is the expected switching cost for the switchers.

[20] We also have $\widehat{S} > \widetilde{S}$ in Examples 1-2, where $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$.

may also be lower, which would reduce the difference in the mass of consumers who switch to benefit from the non-default product's higher utility; so, it is still possible that $\widehat{S} > \widetilde{S}$.

However, if the total charge is sufficiently higher when A is the default, then $\widehat{S} < \widetilde{S}$ is possible. For example, with $F(s) = s^2$ in (6) and a linear monetization function $r(x) = x$, $\hat{x}_A - \tilde{x}_B = \frac{10}{8}\Delta$ (see Proof of Corollary 2(ii)), so in (9) $[\Delta - (\hat{x}_A - \tilde{x}_B)] < 0$: conditional on holding the default position, firm $A's$ charge exceeds $B's$ by more than $A's$ quality advantage. Thus, consumers who stay with the default product are better off under $D = B$ than under $D = A$. (And the gain to switchers always is greater under $D = B$.) By contrast, retaining $r(x) = x$ but replacing $F(s) = s^2$ with the uniform distribution of switching costs $F(s) = s$ yields $\hat{x}_A - \tilde{x}_B = \frac{2}{3}\Delta \Longrightarrow [\Delta - (\hat{x}_A - \tilde{x}_B)] > 0$ (see Proof of Corollary 1), so consumers who stay with the default product are better off under $D = A$.

While Δ, $r(x)$, and $F(s)$ all matter for the equilibrium outcome, the switching cost distribution appears to play an especially pivotal role in the consumer surplus comparison. When $f'(s) > 0$, as opposed to $f'(s) \leq 0$, a larger mass of consumers will have higher switching costs, and hence, for given charges by the two firms, fewer consumers will switch to benefit from the higher utility offered by the non-default product. This emoldens firm A to set a much higher charge when it holds the default position, making it possible for $\widetilde{S} > \widehat{S}$ if $f'(s) > 0$. On the other hand, our results are fairly robust to the form of $r(x)$.

The Extended-Power-Functions case, (6), provides a useful benchmark and sufficient conditions where industry profit is higher under $D = A$, hence firm A wins the default position, but consumer surplus can be higher or lower under $D = B$.[21] In other models of competitive bidding for default or prominent positions, the winning by a higher-quality firm typically benefits consumers. It is perhaps surprising that in our context there are plausible situations where consumer surplus is higher when the lower-quality firm is assigned the default position. The discussion above suggests one key difference that may account for the different results: In our model, the higher-quality firm (when it wins the bidding) takes advantage

[21] For example, suppose that in (6) we have $a = m = 1$ and $b = 0$, so that $r(x) = x$ (a standard revenue technology), and $F(s) = s^n$ with $n \geqslant 1$. Then $\widehat{\Pi} > \widetilde{\Pi}$ from Corollary 1. If $n = 1$ (uniformly distributed search costs), we have $\widehat{S} > \widetilde{S}$, while if $n = 2, \widehat{S} < \widetilde{S}$.

of the default position and consumers' heterogeneous switching costs to set a charge that exceeds the rival's by more than its quality advantage.

Regarding total welfare, whereas total revenue is a pure transfer in standard environments, here the monetization activities x may be weighted differently by firms and consumers, hence can directly affect total welfare. Setting aside this complication, two factors push total welfare to be higher under $D = A$ vs. $D = B$. First, the deadweight loss from switching costs is lower when $D = A$ since less switching occurs (Proposition 1). Second, while total output is the same under both regimes given our assumption that the market is always covered, the share of consumers that use the higher-valued product A is likely to be higher when $D = A$.[22] In our numerical examples total welfare indeed tends to be higher under $D = A$ than under $D = B$.

4. WELFARE-IMPROVING REGULATION?

We now investigate how regulations governing default-position assignment may affect firms, consumers, and efficiency in this market.

4.1 Random Assignment of Default Position

If firms are not permitted to make payments for the default position, then one possibility is that A and B are randomly assigned as the default across consumers, so that $D = A$ or $D = B$ with equal probability $(\frac{1}{2})$. Thus, A and B each is the default for half of the consumers. We next examine the equilibrium in this case and compare it with that under competitive bidding.

If $v_A - x_A \geq v_B - x_B$, then the only consumers who may switch are those with $D = B$, from B to A, with the marginal switching consumer type being $s = \sigma = x_B - x_A + \Delta$. The

[22] The share of consumers that will use product A is $1 - F(\hat{\sigma})$ when $D = A$ and $F(\tilde{\sigma})$ when $D = B$. From Proposition 1, $1 - F(\hat{\sigma}) > \frac{1}{2}$ while $F(\tilde{\sigma}) \leq \frac{1}{2}$ if $F(\Delta) \leq \frac{1}{2}$. But if $F(\Delta) > \frac{1}{2}$, then $F(\tilde{\sigma}) > \frac{1}{2}$, rendering the comparison ambiguous.

profit functions of the two firms would then be

$$\pi_A = \frac{r(x_A)}{2}\left[1 + F(x_B - x_A + \Delta)\right], \qquad \pi_B = \frac{r(x_B)}{2}\left[1 - F(x_B - x_A + \Delta)\right]. \tag{10}$$

On the other hand, if $v_A - x_A < v_B - x_B$, then the only consumers who may switch are those with $D = A$, from A to B, with the marginal switching consumer being $s = \sigma = x_A - x_B - \Delta$.

In this case where the two firms have equal shares of default positions, denote $A's$ and $B's$ equilibrium choices by x_A^e and x_B^e, and similar notation is adopted for other outcome variables. The result below references the following equations, derived from (10) in the Appendix (Proof of Proposition 3):

$$\frac{r'(x_A^e)}{r(x_A^e)} = \frac{f(\sigma^e)}{1 + F(\sigma^e)}; \qquad \frac{r'(x_B^e)}{r(x_B^e)} = \frac{f(\sigma^e)}{1 - F(\sigma^e)}, \tag{11}$$

where $\sigma^e = \Delta - x_A^e + x_B^e \in (0,1)$, $0 \leq x_A^e - x_B^e < \Delta$, and $v_A - x_A^e > v_B - x_B^e$. The result also establishes that the equilibrium industry profit and consumer surplus are given by

$$\Pi^e = r(x_A^e)\frac{1 + F(\sigma^e)}{2} + r(x_B^e)\frac{1 - F(\sigma^e)}{2}; \quad S^e = \frac{1}{2}\left(v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)\,ds\right). \tag{12}$$

Proposition 3 *Under random assignment of the default position, the equilibrium x_A^e and x_B^e satisfy (11). Consumers with $D = B$ and $s < \sigma^e$ switch to A, and there is no equilibrium where consumers with $D = A$ switch to B. Equilibrium industry profit and consumer surplus are given by (12). Moreover, $\Pi^e > \widehat{\Pi}$ and $S^e < \widehat{S}$ if $\sigma^e \leq \hat{\sigma}$ or if (6) holds for various values of m and n; and $\widehat{W}$ can be either higher or lower than W^e.*

Notice that under random assignment, $x_A^e - x_B^e < \Delta$, in contrast to $\hat{x}_A - \hat{x}_B > \Delta$ when $D = A$ for all consumers under competitive bidding. That is, A's equilibrium charge now exceeds B's by less than A's quality advantage, resulting in $v_A - x_A^e > v_B - x_B^e$, so that consumers switch only from B to A, instead of only from A to B when $D = A$ for all consumers.

Since industry profit is typically higher under $D = A$ than under $D = B$, one might have expected that shifting half of the consumers from $D = A$ to $D = B$ (random assignment) would reduce industry profit. However, such a move tends to *raise* industry profit: competition is softened when each firm is assigned the default position for half of the consumers, resulting in higher charges. In fact, if $\sigma^e \leq \hat{\sigma}$, then $x_A^e > x_B^e \geq \hat{x}_A > \hat{x}_B$, so charges are uniformly higher when the two firms have equal shares of the default position than when firm B has none. Firm B now starts with $D = B$ for half of the consumers (veruss none in the bidding equilibrium), incentivzing it to raise x_B in order to exploit some of its default consumers (those with high switching costs). This 'installed base effect' causes x_B^e to rise substantially above $\hat{x}_B$. For firm A, its installed base is smaller than under competitive bidding (where its share is one), which motivates it to lower its charge; but the foreseen increase in x_B induces firm A to raise its charge because x_A and x_B are strategic complements, and this strategic effect tends to dominate. As a result, both firms choose higher charges than in the bidding equilibrium.

The condition $\sigma^e \leq \hat{\sigma}$, which is sufficient for $\Pi^e > \widehat{\Pi}$ and $S^e < \widehat{S}$ (Proposition 3), can hold in many situations. Moreover, it clearly is not necessary for these rankings, which hold also for $r(x)$ and $F(s)$ that belong to the class of extended-power-functions (6) under some parameter restrictions. For example, if $n = m = 1$, we have the *Linear-Uniform case*:

$$r(x) = ax + b, \qquad F(s) = s, \tag{13}$$

for which

$$x_A^e = 1 + \frac{\Delta}{3} - \frac{b}{a}, \quad x_B^e = 1 - \frac{\Delta}{3} - \frac{b}{a}; \qquad \hat{x}_A = \frac{2}{3} + \frac{\Delta}{3} - \frac{b}{a}, \quad \hat{x}_B = \frac{1}{3} - \frac{\Delta}{3} - \frac{b}{a},$$

so charges are uniformly higher under random assignment than under competitive bidding ($x_A^e - \hat{x}_A = \frac{1}{3}$, $x_B^e - \hat{x}_B = \frac{2}{3}$) for all $\Delta < 1$, even though $\sigma^e = \frac{\Delta}{3} \leq \hat{\sigma} = \frac{1-\Delta}{3}$ only if $\Delta \leq \frac{1}{2}$.

Consumer surplus under random assignment (S^e) can be lower than under competitive bidding ($\widehat{S}$) even when $\widehat{S} < \widetilde{S}$ (i.e., even when consumer surplus under the competitive bidding outcome, $D = A$, is lower than if firm B had obtained the default position for all consumers, $D = B$). For example, if $n = 2$ in (6) then $S^e < \widehat{S}$ is possible even though $\widehat{S} < \widetilde{S}$, because the total charge when default shares are equal (to $\frac{1}{2}$) is higher than under $D = A$, which in turn is higher than under $D = B$.

The comparison of total welfare (W) is generally ambiguous, due to the possible opposite changes in profit and consumer surplus. For example, in the linear-uniform case, total welfare is higher under competitive bidding if a is below $a^e \equiv \frac{\Delta^2+2\Delta+14}{4(4-2\Delta-\Delta^2)} \in \left(\frac{7}{8}, \frac{17}{4}\right)$, in which case the increase in industry profit is dominated by the decrease in consumer surplus under random assignment. But W is higher under random assignment when a is above a^e.

4.2 Other Shares of Default Position

Consider regulation that assigns $D = A$ for a portion $\lambda \in \left(\frac{1}{2}, 1\right)$ of consumers. The higher-valued product is then assigned as the default for more than half of the consumers but not all. Earlier we analyzed the cases $\lambda = \frac{1}{2}$ (random assignment) and $\lambda = 1$ (competitive bidding outcome when A wins), which are limiting cases of this more general setting. We next establish that for λ close to $\frac{1}{2}$ there exists a unique equilibrium similar to that when $\lambda = \frac{1}{2}$, whereas for λ close to 1 the unique equilibrium is similar to that when $\lambda = 1$. We will also discuss how profits and consumer surplus may change as λ varies in both ranges.

At the candidate equilibrium for λ close to $\frac{1}{2}$, where some consumers with $D = B$ will switch to A but none with $D = A$ will switch to B, the marginal switching consumer is

$$\sigma = x_B - x_A + \Delta \geq 0.$$

Then, the profits of the two firms are

$$\pi_A = r(x_A)\left[\lambda + (1-\lambda) F(x_B - x_A + \Delta)\right], \qquad \pi_B = r(x_B)(1-\lambda)\left[1 - F(x_B - x_A + \Delta)\right].$$

In this case, denote the equilibrium charges of the two firms by x_A^- and x_B^-, the marginal consumer by σ^-, and the other outcome variables by Π^-, S^-, W^-.

At the candidate equilibrium for λ close to 1, where some consumers with $D = A$ switch to B but none with $D = B$ switch to A, the marginal switching consumer is

$$\sigma = x_A - x_B - \Delta > 0.$$

Then, the profit functions of the two firms are

$$\pi_A = r(x_A)\lambda\left[1 - F(x_A - x_B - \Delta)\right], \qquad \pi_B = r(x_B)\left[1 - \lambda + \lambda F(x_A - x_B - \Delta)\right].$$

In this case, denote the equilibrium charges of the two firms by x_A^+ and x_B^+; and similarly for σ^+, Π^+, S^+, and W^+.

Proposition 4 *There exist* $\lambda^- \in \left(\frac{1}{2}, 1\right)$ *and* $\lambda^+ \in \left(\lambda^-, 1\right)$ *such that: (i) if* $\lambda \leq \lambda^-$, *then* $\Delta > x_A^- - x_B^- > 0$ *and there is consumer switching only from* B *to* A; *(ii) if* $\lambda \geq \lambda^+$, *then* $x_A^+ - x_B^+ > \Delta$ *and there is consumer switching only from* A *to* B. *(iii) In the Linear-Uniform case (13), as* λ *changes from* $\frac{1}{2}$ *to* 1, *industry profit and consumer surplus can vary non-monotonically, whereas total welfare can be higher or lower than under* $\lambda = 1$, *i.e.,* $\widehat{W}$.

Under (13), where $r(x) = ax + b$ and $F(s) = s$, equilibrium industry profit and consumer surplus can vary non-monotonically with λ as follows. For $\lambda < \lambda^-$, as λ rises industry profit also rises but consumer surplus falls. A higher λ raises A's customer base (consumers with A as the default), which induces a rise in x_A^- and a smaller rise in x_B^-. (The latter reflects the strategic response to the rise in x_A^-, which outweighs the effect on x_B of the reduction in $B's$ customer base.) Consequently, industry profit rises but consumer surplus falls. Thus, for all $\lambda \in \left(\frac{1}{2}, \lambda^-\right)$ we have $\Pi > \Pi^e > \widehat{\Pi}$ and $S < S^e < \widehat{S}$ (where the second inequalities follows from Proposition 3), so profit is higher but consumer surplus is lower than in the competitive bidding outcome when firm A wins the default.

For $\lambda > \lambda^+$, as λ rises consumer surplus now rises, but remains below $\widehat{S}$. The behavior of profit is more complex. If the quality difference $\Delta \leq 0.581$, profit decreases in λ but remains above $\widehat{\Pi}$ for all $\lambda \in \left(\lambda^+, 1\right)$. If $\Delta > 0.581$, profit may decrease or increase with λ, and can be lower or higher than $\widehat{\Pi}$. These patterns are roughly explained as follows. For λ near 1, $B's$ customer base is small, and as λ rises the decrease in B's customer base exerts a powerful downward effect on x_B, so that both x_B^+ and x_A^+ can fall, though the latter by less (since $A's$ customer base rose). However, with a higher λ, more consumers may patronize A, which has a higher charge. As a result, industry profit tends to—but not always—fall.

Recall from Proposition 3 that if the default position is distributed equally between the two firms (i.e., $\lambda = \frac{1}{2}$), then under plausible sufficient conditions industry profit is higher but consumer surplus is lower than in the competitive bidding outcome ($\lambda = 1$): $\Pi^e > \widehat{\Pi}$ and $S^e < \widehat{S}$. Compared to that outcome, a regulated assignment with $\lambda^- \in \left(\frac{1}{2}, 1\right)$ also tends to increase industry profit and decrease consumer surplus. Total welfare can rise or fall. For example, in the linear-uniform case, when a is above some threshold, the profit effect tends to dominate, resulting in higher total welfare than when $\lambda = 1$; and conversely if a is below the threshold.

4.3 Regulated Assignment with Endogenous Product Quality

Suppose there is learning by the firms, so that when more consumers use product B its quality v_B can increase. This scenario is at the heart of the DOJ's complaint against Google (DOJ, 2020). DOJ argues that search algorithms improve with experimentation and, hence, improve with a search engine's number of users. By obtaining default status at leading distribution outlets for search engines, Google deprives rival search engines of users and, hence, impairs their ability to improve their quality through learning. We take no position on the factual merits of the DOJ's argument, but will attempt to capture its essence and the potential welfare effects of reducing Google's share of default positions.

To formally model the quality improvement issues, one would need a dynamic model. For instance, one might consider a setting with two periods, where v_A and v_B are exogenously given in period 1, but may improve in period 2 due to learning in period 1, and greater improvement occurs when a firm serves more consumers in period 1. Then, if λ is reduced from $\lambda = 1$ to some lower value in period 1, potentially more consumers would use B in period 1, which could increase v_B and result in more consumers patronizing B in period 2.

There are significant complexities to analyze this scenario in an equilibrium model. In particular, the numbers of consumers that firms serve in period 1 are endogenous, depending on their choices of x_A and x_B in period 1. Therefore, the number of consumers who have $D = A$ and $D = B$ at the beginning of period 2 are also endogenous, assuming that a consumer who patronized firm i in period 1 will start with $D = i$ in period 2. Also, since the firms' choices of x_A and x_B in period 2 will depend on those numbers and the default assignment, the switching decisions of rational consumers in period 1 will hinge on their expectations about how the second-period equilibrium may depend on λ, x_A, and

x_B in period 1. Moreover, the incentives to win the default position may also change under competitive bidding. Since our purpose is mainly to gain insight on whether, with endogenous product quality, consumer surplus can be higher under regulation, we adopt a 'reduced-form' approach without analyzing a dynamic model that formally incorporates the intertemporal strategic interactions.

Specifically, consider the following: Under competitive bidding, $D = A$ for all consumers (i.e., $\lambda = 1$) with given v_A and v_B; while under regulated assignment, $D = A$ for some portion $\lambda \in \left(\frac{1}{2}, 1\right)$ of consumers and $A's$ and $B's$ product qualities with learning become $v_A^l \leq v_A$ and $v_B^l > v_B$. This can be viewed as the second period of a two-period model, assuming that (i) quality increases with scale but at a decreasing rate, so that if λ decreases from $\lambda = 1$ by a relatively small amount, then v_A will not be (much) lower in period 2 but v_B could be substantially higher; (ii) each period contains a separate unit mass of consumers, so consumers face no intertemporal choice; and (iii) the first period is much shorter than the second, so the welfare comparisons can be approximated by their comparisons for period 2. We investigate the quality improvement $v_B^l - v_B$ needed to achieve higher consumer surplus for some values of $\lambda < 1$ than under $\lambda = 1$. Our analysis incorporates the different equilibrium values of x_A and x_B under the bidding and regulated assignments.

For this analysis we assume the Linear-Uniform case (13) and let

$$v_A^l = v_A - \delta_A, \quad v_B^l = v_B + \delta_B, \quad \Delta^l = v_A^l - v_B^l = \Delta - \delta_A - \delta_B > 0,$$

for $\delta_A \geq 0$ and $\delta_B > 0$. We focus on situations where Δ^l is relatively small and λ relatively large, so that $\lambda \geq \lambda^+ = \frac{1}{2-\Delta^l}$ in Proposition 4. While δ_A and δ_B may well depend on λ, we treat them as parameters and inquire how the values of δ_A and δ_B, for a given change from $\lambda = 1$ to some $\lambda < 1$, may affect consumer surplus. Note that Δ^l is higher when δ_A and δ_B are lower (for given Δ), i.e., when the learning effect is weaker. Denote the consumer surplus in this case by S^l.

Corollary 3 *Assume the Linear-Uniform case and* $\lambda \geq \lambda^+ = \frac{1}{2-\Delta^l}$. *Then,* $S^l - \widehat{S}$ *decreases in* Δ^l *but increases in* λ *and* Δ. *Moreover,* $S^l - \widehat{S} > 0$ *if* $\Delta \geq 0.4$, $\lambda \geq 0.7$, *and* δ_A *and* Δ^l *are sufficiently small; but* $S^l - \widehat{S} < 0$ *if* $\delta_A = \delta_B$.

Thus, if regulation endows firm B with the default position for a portion $1-\lambda$ of consumers, which improves v_B possibly due to learning, then consumers can indeed benefit compared to the bidding outcome where firm A obtains the default position for all consumers. Consumers may benefit through several channels. An increase in v_B directly benefits consumers who use product B. But it also has strategic effects: a higher v_B, which reduces the quality asymmetry between the two products (i.e., Δ^l is smaller), may result in lower levels of x_A and x_B by the two firms due to the intensified competition when they are more symmetric. Also, x_A and x_B will be closer to each other if Δ^l is smaller, which would reduce the amount of switching, hence the total switching costs incurred by consumers.

The increase in v_B required for $S^l > \widehat{S}$ does not seem very high. For example, suppose $v_A = 5$ and $v_B = 4.5$ so $\Delta = 0.5$, and regulation lowers λ from 1 to 0.8. Then $S^l > \widehat{S}$ if (i) $\delta_B \geq 0.186$, or 4.13% of v_B, without lowering v_A; or if (ii) $\delta_B \geq 0.25$ and $\delta_A \leq 0.05$ (i.e., the reduction in v_A is less than 20% of the increase in v_B: $\delta_A/\delta_B \leq 20\%$). However, if v_A would decrease as much as v_B increases under the regulation, then consumer surplus is always higher when $\lambda = 1$.

4.4 Choice Screen

Instead of assigning a default product to consumers, an alternative policy known as "choice screen" allows consumers to choose their preferred default from a set of displayed options. This policy was first adopted by the European Commission in 2009: Microsoft was required to display alternative web browsers along with its own Internet Explorer instead of presetting Explorer as the default. Choice screen was also adopted in the Commission's Android (2018) case, where Google was required to display other search engines in addition to its own.[23] The Digital Markets Act adopted by the European Union (2022) requires large online platforms designated as "gatekeepers" to provide a choice screen for users to select

[23] In the Android case, unlike in Microsoft, the rival products displayed in the choice screen (for both search engines and web browsers) were determined through auctions conducted by Google starting in 2020. Ostrovsky (2021) shows that the identity of the winning bidders will depend on whether a bidder pays a flat fee for the right to be displayed in the choice screen, or a fee per user that installs its product ("per install").

their default apps for online search engines, virtual assistants, or web browsers.

In our setting, a choice screen policy would lead all consumers to choose product A. Consumers differ only in their switching costs and a choice screen allows each consumer to choose the preferred option at the outset before incurring a switching cost. The equilibrium then resembles Bertrand competition with asymmetric product qualities: the weaker firm B sets its charge x_B equal to marginal cost (that we normalized to zero), and firm A captures the entire market while charging a premium equal to its quality advantage: $x_A = x_B + \Delta$. Ironically, firm B would attract no customers in such a scenario, unlike the bidding-for-default outcome even when firm A wins.[24]

Because choice screen results in lower monetization charges by firms, if consumers face de minimis cost to set up the default themselves through the choices presented,[25] there is a strong presumption that choice screen would be the superior policy for consumer welfare in the short run. From a longer-run standpoint, however, a choice screen may be inferior to regulatory assignment if quality improves with learning (and, hence, with market share) as discussed earlier. Too few consumers would choose the lower-quality product B, because consumers individually ignore the positive competition externality they generate by enabling firm B to increase its quality. Hence, if the predominant concern is policy to enable improvement by the weaker firm, the choice screen approach is questionable.

5. CONCLUDING REMARKS

We analyzed several methods of assigning the default position for a product supplied by two competing firms with exogenously different qualities, when consumers face heterogeneous costs of switching from the default product to the rival. The default firm enjoys

[24] There, firm A exploits its customers' heterogeneous switching costs, setting $\hat{x}_A > \hat{x}_B + \Delta$, which in turn allows firm B to attract some (switching) customers in equilibrium. Essentially, firm A behaves like a "fat cat" (Tirole, 1988), exploiting its installed base by raising price, which allows the weaker firm to survive.

[25] For search engines, with a properly designed choice screen the cost may well be de minimis. However, in other situations it is possible that a consumer may need to incur costs to find the relevant alternatives or to install the default option. Then, a consumer's switching cost may reflect such 'search cost' or installation cost, which the consumer needs to incur even if she chooses the default.

market power over its inframarginal consumers, those with higher switching costs, which it exploits through greater monetization (e.g., unwanted advertising). Consequently, when the default position is assigned through competitive bidding for all consumers, the default winner provides lower utility than the rival, even when the winner is the higher-quality firm. That firm indeed tends to win (though we show a counter-example), not due to its quality advantage directly, but because industry monetization is greater when it rather than the rival holds the default.

Our analysis also yields some policy insights. Compared to the stronger firm winning the default position everywhere, assigning via regulation the default position to the rival for some minority share of consumers tends to *increase profit* and harm consumers. Profit rises because competition is softened when both firms have sticky (default) consumers. All consumers lose from the softened competition, and those who are assigned the lower-quality product suffer additional harm directly. We briefly considered another scenario where a firm's product quality is not fixed but instead improves at a decreasing rate with its share of users, possibly due to learning. Assigning the default position to the weaker firm for some share of consumers may then benefit consumers in the long run, but this must be weighed against the short run harm. Instead of assigning a default product, letting individual consumers select their preferred option from a choice screen will likely benefit consumers in the short run, but in our setting is worse for the weaker firm than the other assignment methods—regulation or bidding. Thus, a choice screen remedy is problematic if concern with longer term market structure is paramount.

Overall, attempts to promote consumer welfare by targeting default positions appear to suffer from serious limitations. An alternative policy approach is to reduce firms' monetization activities through regulations that, for example, restrict consumer tracking and unwanted ads, such as the European Union's GDPR. In our static setting, reducing monetization will clearly benefit consumers. It can also increase total welfare.[26] However, determining the 'adequate' revenue to cover fixed costs for digital products is problematic in practice. Additionally, restricting monetization is a form of behavioral regulation with well-known difficulties. Nonetheless, especially given the shortcomings of default oriented policies, regulating monetization deserves further analysis. Personal data protection and other regulations that limit firms' ability to monetize may be a more effective approach to benefit consumers and improve efficiency. Our analytical framework, where revenue is a general function of monetization activities, may aid such analysis. It also may be useful in other settings where firms earn revenue from a variety of monetization activities potentially harmful to consumers.

6. APPENDIX

The appendix contains proofs for Propositions 1-4, Lemmas 1-2, and Corollaries 1-3.

Proof of Lemma 1. (i) When $D = A$, the equilibrium $\hat{x}_A$ and $\hat{x}_B$, if they are strictly positive, satisfy the following first-order conditions obtained from (1):

$$\frac{\partial \pi_A}{\partial x_A} = r'(x_A)\left[1 - F(x_A - x_B - \Delta)\right] - r(x_A) f(x_A - x_B - \Delta) = 0, \quad (14)$$

$$\frac{\partial \pi_B}{\partial x_B} = r'(x_B) F(x_A - x_B - \Delta) - r(x_B) f(x_A - x_B - \Delta) = 0. \quad (15)$$

First, we show that $\hat{x}_B > 0$. If, to the contrary, $\hat{x}_B = 0$, then $\hat{x}_A > \Delta$ because $\left.\frac{\partial \pi_A}{\partial x_A}\right|_{x_A = \Delta} = r'(\Delta) - r(\Delta) f(0) > 0$ by assumption, and thus $\left.\frac{\partial \pi_B}{\partial x_B}\right|_{x_B = 0} = r'(0) F(\hat{x}_A - \Delta) - r(0) f(\hat{x}_A - \Delta) > 0$ if $r(0) = b$ is small enough, which contradicts $\hat{x}_B = 0$.

Next, $\hat{x}_A - \hat{x}_B \geq \Delta$, because if $\hat{x}_A - \hat{x}_B < \Delta$, A could increase its profit by raising x_A. It follows that $\hat{x}_A > \Delta > 0$.

(ii) When $D = B$, the equilibrium $\tilde{x}_A$ and $\tilde{x}_B$, if they are strictly positive, satisfy the

[26] For instance, since $r(0) = b$, total welfare is $W_0 = v_A + b$ when $x_A = x_B = 0$, where all consumers will patronize firm A. In the Linear-Uniform case where $r(x) = ax + b$ and $F(s) = s$, it can be verified that

$$\widehat{W} \gtreqless W_0 \Longleftrightarrow a \gtreqless \frac{8\Delta - \Delta^2 + 11}{2(2\Delta + 2\Delta^2 + 5)} \equiv a^* \quad \text{if } b = 0,$$

where $a^* > 1$. If $a < a^*$, welfare is maximized if monetizations that harm consumers are reduced to zero.

following first-order conditions obtained from (2):

$$\frac{\partial \pi_A}{\partial x_A} = r'(x_A) F(x_B - x_A + \Delta) - r(x_A) f(x_B - x_A + \Delta) = 0, \quad (16)$$

$$\frac{\partial \pi_B}{\partial x_B} = r'(x_B)[1 - F(x_B - x_A + \Delta)] - r(x_B) f(x_B - x_A + \Delta) = 0. \quad (17)$$

For any $\tilde{x}_B \geq 0$, firm A will choose $\tilde{x}_A > 0$ to profit from the switching consumers. It follows that $\tilde{x}_B > 0$ as well. ■

Proof of Proposition 1. (i) When $D = A$: Since $\hat{x}_A > 0$ and $\hat{x}_B > 0$, they satisfy the first-order conditions (14) and (15). Also, if $\hat{x}_A - \hat{x}_B = \Delta$, we would have $\left.\frac{\partial \pi_B}{\partial x_B}\right|_{\hat{x}_B} < 0$, contradicting $\hat{x}_B$ being optimal. Therefore $\hat{x}_A - \hat{x}_B > \Delta$ and hence $\hat{\sigma} > 0$. Notice that from (14), $\hat{x}_A \leq 1$ if $\frac{r'(1)}{r(1)} \leq \frac{f(\Delta)}{1-F(\Delta)}$, because $\frac{f(\Delta)}{1-F(\Delta)} < \frac{f(1-\hat{x}_B-\Delta)}{1-F(1-\hat{x}_B-\Delta)}$.

Observe that (14) and (15) can be rewritten as the two equations in (3). With $\sigma_A = x_A - x_B - \Delta$, letting

$$\mu(x) \equiv \frac{r'(x_A)}{r(x_A)}, \quad h(s) \equiv \frac{f(\sigma_A)}{1 - F(\sigma_A)}, \quad \text{and} \quad g(\sigma_A) \equiv \frac{f(\sigma_A)}{F(\sigma_A)},$$

where $\mu'(x) < 0$, $h'(\sigma_A) > 0$ and $g'(\sigma_A) < 0$ from (C1), we show that the $\hat{x}_A$ and $\hat{x}_B$ that satisfy (3) are unique. Each equation in (3) implicitly defines x_A as a function of x_B, and the curves in the (x_B, x_A)-space for the two functions, where x_A is on the vertical axis, respectively have the following slopes:

$$\frac{dx_A}{dx_B} = \frac{h'(\sigma_A)}{h'(\sigma_A) - \mu'(x_A)} \in (0,1), \qquad \frac{dx_A}{dx_B} = \frac{g'(\sigma_A) + \mu'(x_A)}{g'(\sigma_A)} > 1.$$

Thus the two curves intersect only once, implying that $\hat{x}_A$ and $\hat{x}_B$ exist uniquely. Notice that this also implies that the two firms' choices are strategic complements.

Finally, because $\hat{x}_A - \hat{x}_B > \Delta$, $\hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta > 0$, and $r(x)/r'(x)$ increases in x from (C1), we have

$$\frac{r(\hat{x}_A)}{r'(\hat{x}_A)} - \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} = \frac{1 - 2F(\hat{\sigma})}{f(\hat{\sigma})} > 0,$$

and hence $F(\hat{\sigma}) < \frac{1}{2}$.

(ii) When $D = B$: First, since both $\tilde{x}_A > 0$ and $\tilde{x}_B > 0$, we must have $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A > 0$, and hence $\tilde{x}_A - \tilde{x}_B < \Delta$. Next, equations (16) and (17) can be rewritten as the two equations in (4), each of which implicitly defines x_A as a function of x_B, and the curves in the (x_B, x_A)-space for the two functions, where x_A is on the vertical axis, respectively have the following slopes:

$$\frac{dx_A}{dx_B} = \frac{g'(\sigma)}{\mu'(x_A) + g'(\sigma)} \in (0,1), \qquad \frac{dx_A}{dx_B} = \frac{h'(\sigma) - \mu'(x_B)}{h'(\sigma)} > 1,$$

where $\sigma = \Delta + x_B - x_A$. Thus the two curves intersect only once, implying that $\tilde{x}_A$ and $\tilde{x}_B$ exist uniquely. Notice that this also implies that the two firms' choices are strategic complements.

Finally, if $F(\Delta) \leq \frac{1}{2}$, we show that $\tilde{x}_A \leq \tilde{x}_B$, and thus $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A \geq \Delta$, which further implies $F(\tilde{\sigma}) \leq \frac{1}{2}$. Suppose to the contrary that $\tilde{x}_A > \tilde{x}_B$, then $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A < \Delta$ and $\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} < \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)}$, which implies

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} < \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) < F(\tilde{\sigma}) \rightarrow \frac{1}{2} < F(\tilde{\sigma}) < F(\Delta),$$

a contradiction. Thus $\tilde{x}_A \leq \tilde{x}_B$. It follows that

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} = \frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} \geq \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) \geq F(\tilde{\sigma}) \rightarrow F(\tilde{\sigma}) \leq \frac{1}{2}.$$

On the other hand, if $F(\Delta) > \frac{1}{2}$, we show that $\tilde{x}_A > \tilde{x}_B$ and hence $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A < \Delta$. If , to the contrary, $\tilde{x}_A \leq \tilde{x}_B$, then $\tilde{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A \geq \Delta$ and $\frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} \geq \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)}$, which implies

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} \geq \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) \geq F(\tilde{\sigma}) \rightarrow \frac{1}{2} \geq F(\tilde{\sigma}) \geq F(\Delta),$$

a contradiction. Hence $\tilde{x}_A > \tilde{x}_B$. It follows that

$$\frac{f(\tilde{\sigma})}{F(\tilde{\sigma})} = \frac{r'(\tilde{x}_A)}{r(\tilde{x}_A)} < \frac{r'(\tilde{x}_B)}{r(\tilde{x}_B)} = \frac{f(\tilde{\sigma})}{1 - F(\tilde{\sigma})} \rightarrow 1 - F(\tilde{\sigma}) < F(\tilde{\sigma}) \rightarrow F(\tilde{\sigma}) > \frac{1}{2}.$$

(iii) Suppose, to the contrary, that $\tilde{\sigma} \leq \hat{\sigma}$. Then

$$\begin{aligned} \frac{r(\hat{x}_A)}{r'(\hat{x}_A)} &= \frac{1 - F(\hat{\sigma})}{f(\hat{\sigma})} \leq \frac{1 - F(\tilde{\sigma})}{f(\tilde{\sigma})} = \frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)} \Rightarrow \hat{x}_A \leq \tilde{x}_B, \\ \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} &= \frac{F(\hat{\sigma})}{f(\hat{\sigma})} \geq \frac{F(\tilde{\sigma})}{f(\tilde{\sigma})} = \frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} \Rightarrow \hat{x}_B \geq \tilde{x}_A. \end{aligned}$$

Hence

$$\tilde{\sigma} - \hat{\sigma} = \Delta + \tilde{x}_B - \tilde{x}_A - [\hat{x}_A - \hat{x}_B - \Delta] = 2\Delta + \tilde{x}_B - \hat{x}_A + \hat{x}_B - \tilde{x}_A > 0,$$

which produces a contradiction. Therefore $\tilde{\sigma} > \hat{\sigma}$. It follows that $\hat{x}_A > \tilde{x}_B$ and $\tilde{x}_A > \hat{x}_B$.

Moreover, for future reference, if $F(\Delta) \leq \frac{1}{2}$ so that $F(\tilde{\sigma}) \leq \frac{1}{2}$, then $\tilde{x}_A \leq \tilde{x}_B$ and hence $\hat{x}_A > \tilde{x}_B \geq \tilde{x}_A > \hat{x}_B$. If $F(\Delta) > \frac{1}{2}$ so that $F(\tilde{\sigma}) > \frac{1}{2}$, then $\tilde{x}_A > \tilde{x}_B$. ■

Proof of Lemma 2. First, from (3) and (4), because $\tilde{\sigma} > \hat{\sigma}$,

$$\begin{aligned} & \frac{r(\hat{x}_A)}{r'(\hat{x}_A)} + \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} - \left[\frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} + \frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)}\right] \\ = \; & \frac{1-F(\hat{\sigma})}{f(\hat{\sigma})} + \frac{F(\hat{\sigma})}{f(\hat{\sigma})} - \left[\frac{1-F(\tilde{\sigma})}{f(\sigma)} + \frac{F(\sigma)}{f(\sigma)}\right] = \frac{1}{f(\hat{\sigma})} - \frac{1}{f(\tilde{\sigma})} \gtreqless 0 \end{aligned}$$

if $f'(s) \gtreqless 0$.

Next, suppose $F(\Delta) \leq \frac{1}{2}$ so that $\tilde{x}_B \geq \tilde{x}_A$. Since $\rho(x) = \frac{r(x)}{r'(x)}$, $\rho'(x) = \frac{r'^2 - rr''}{r'^2} > 0$ from assumption (C1). If $f'(s) \geq 0$, using the mean-value theorem we have

$$\frac{r(\hat{x}_A)}{r'(\hat{x}_A)} - \frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)} + \frac{r(\hat{x}_B)}{r'(\hat{x}_B)} - \frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} = \rho'(\zeta_1)(\hat{x}_A - \tilde{x}_B) - \rho'(\zeta_2)(\tilde{x}_A - \hat{x}_B) \geq 0,$$

where $\zeta_1 \in (\tilde{x}_B, \hat{x}_A) > \zeta_2 \in (\hat{x}_B, \tilde{x}_A)$. If in addition $\rho'' \leq 0$ (so (C2) holds), which implies $\rho'(\zeta_1) \leq \rho'(\zeta_2)$, then $\hat{x}_A - \tilde{x}_B \geq \tilde{x}_A - \hat{x}_B$, or equivalently, $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$. On the other hand, if $f'(s) \leq 0$ and $\rho'' \geq 0$, then

$$\rho'(\zeta_1)(\hat{x}_A - \tilde{x}_B) - \rho'(\zeta_2)(\tilde{x}_A - \hat{x}_B) \leq 0$$

but $\rho'(\zeta_1) \geq \rho'(\zeta_2)$, and hence $\hat{x}_A - \tilde{x}_B \leq \tilde{x}_A - \hat{x}_B$. Furthermore, if $f'(s) = 0$ and $\rho'' = 0$, then $(\hat{x}_A - \tilde{x}_B) = (\tilde{x}_A - \hat{x}_B)$.

Suppose, instead, $F(\Delta) > \frac{1}{2}$ so that $\tilde{x}_A > \tilde{x}_B$. The mean-value theorem then implies

$$\frac{r(\hat{x}_A)}{r'(\hat{x}_A)} - \frac{r(\tilde{x}_A)}{r'(\tilde{x}_A)} - \left[\frac{r(\tilde{x}_B)}{r'(\tilde{x}_B)} - \frac{r(\hat{x}_B)}{r'(\hat{x}_B)}\right] = \rho'(\zeta_1)(\hat{x}_A - \tilde{x}_A) - \rho'(\zeta_2)(\tilde{x}_B - \hat{x}_B) \gtreqless 0,$$

where $\zeta_1 \in (\tilde{x}_A, \hat{x}_A) > \zeta_2 \in (\hat{x}_B, \tilde{x}_B)$. The rest of the proof is the same as that in the case of $F(\Delta) \leq \frac{1}{2}$.

Finally, if $F(s) = s^n$, then $f(s) = ns^{n-1}$, $f'(s) = n(n-1)s^{n-2} \geq 0$ if $n \geq 1$, and $f'(s)/f(s) = (n-1)/s$ decreases in s. Also, if $r(x) = ax^m + b$, then $\rho(x) = \frac{ax^m+b}{amx^{m-1}}$, and with $b(1-m) \geq 0$,

$$\rho'(x) = \frac{\partial\left(\frac{ax^m+b}{amx^{m-1}}\right)}{dx} = \frac{b(1-m) + ax^m}{amx^m} > 0, \quad \rho''(x) = \frac{\partial^2\left(\frac{ax^m+b}{amx^{m-1}}\right)}{dx^2} = \frac{b}{x^{m+1}}\frac{m-1}{a} \leq 0,$$

and $\rho'(x) > 0 \Leftrightarrow r'(x)/r(x)$ is decreasing. Therefore, both (C1) and (C2) hold if $r(x)$ and $F(s)$ satisfy (6). ■

Proof of Proposition 2. First, we show that when (C2) holds, the charges satisfy $\hat{x}_A > \max\{\tilde{x}_B,\ \tilde{x}_A\} > \hat{x}_B$. From Proposition 1: $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$, $\hat{\sigma} < \tilde{\sigma}$, $\hat{x}_A > \tilde{x}_B$ and $\tilde{x}_A > \hat{x}_B$. Thus, if $\tilde{x}_A \leq \tilde{x}_B$, then $\hat{x}_A > \tilde{x}_B \geq \tilde{x}_A > \hat{x}_B$. Suppose instead $\tilde{x}_A > \tilde{x}_B$. If $\tilde{x}_B \leq \hat{x}_B$, then $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$ implies $\hat{x}_A > \tilde{x}_A$; while if $\tilde{x}_B > \hat{x}_B$, then if $\hat{x}_A \leq \tilde{x}_A$, we would have $\hat{x}_A + \hat{x}_B < \tilde{x}_A + \tilde{x}_B$, contradicting the result from Lemma 1 that $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$ when (C2) holds. Hence, $\hat{x}_A > \tilde{x}_A$ if $\tilde{x}_A > \tilde{x}_B$. Thus $\hat{x}_A > \max\{\tilde{x}_B,\ \tilde{x}_A\} > \hat{x}_B$, This, together with $\hat{x}_A - \hat{x}_B > \Delta > \tilde{x}_A - \tilde{x}_B$, implies that $(\hat{x}_B, \hat{x}_A)$ is more dispersed than $(\tilde{x}_B,\ \tilde{x}_A)$.

Therefore, when (C2) holds so that $\hat{x}_A + \hat{x}_B \geq \tilde{x}_A + \tilde{x}_B$, the ranking $\hat{\sigma} < \tilde{\sigma}$ implies that $\{\hat{x}_B, \hat{x}_A\}$ is a mean-increasing spread of $\{\tilde{x}_B,\ \tilde{x}_A\}$, that is:

$$[1 - F(\hat{\sigma})]\,\hat{x}_A + F(\hat{\sigma})\,\hat{x}_B > [1 - F(\tilde{\sigma})]\,\tilde{x}_B + F(\tilde{\sigma})\,\tilde{x}_A.$$

Notice that for given Δ, $\widehat{\Pi} = [1 - F(\hat{\sigma})]\,r(\hat{x}_A) + F(\hat{\sigma})\,r(\hat{x}_B)$ exceeds $\widetilde{\Pi} = [1 - F(\tilde{\sigma})]\,r(\tilde{x}_B) + F(\tilde{\sigma})\,r(\tilde{x}_A)$ by a strictly positive number if $r''(x) = 0$. By continuity, there exists some $\varepsilon > 0$ such that $\widehat{\Pi} > \widetilde{\Pi}$ if $r''(x) \geq -\varepsilon$. ■

Proof of Corollary 1. Under (6), $r(x) = ax^m + b$ and $F(s) = s^n$ for $n \geq 1$.

First, if $m \geq 1$, then $r''(x) \geq 0$ and (C2) holds, implying $\widehat{\Pi} > \widetilde{\Pi}$ from Proposition 2. For example, if $r(x) = ax + b$ and $F(s) = s$, then

$$\hat{x}_A = \frac{2}{3} + \frac{\Delta}{3} - \frac{b}{a}, \quad \hat{x}_B = \frac{1}{3} - \frac{\Delta}{3} - \frac{b}{a} > 0, \quad \hat{\sigma} = \hat{x}_A - \hat{x}_B - \Delta = \frac{1}{3}(1 - \Delta);$$

$$\tilde{x}_A = \frac{1}{3} + \frac{\Delta}{3} - \frac{b}{a} > 0, \quad \tilde{x}_B = \frac{2}{3} - \frac{\Delta}{3} - \frac{b}{a} > 0, \quad \tilde{\sigma} = \tilde{x}_B - \tilde{x}_A + \Delta = \frac{1}{3}(1 + \Delta).$$

Thus, $\hat{x}_i > 0$ and $\tilde{x}_i > 0$ if $b < a\left(\frac{1-\Delta}{3}\right)$.

Next, suppose $1 > m > \Delta$. Then $\rho(x) = \frac{ax^m + b}{amx^{m-1}} = x + \frac{b}{am}x^{1-m}$ and $\rho''(x) \leq 0$ so that (C2) holds, but $r''(x) = am(m-1)\,x^{m-1} < 0$. If, for instance, $n = 1$, from (3) and (4):

$$\hat{x}_A = \frac{m + m\Delta + m^2}{2m + 1}, \quad \hat{x}_B = \frac{m(m - \Delta)}{2m + 1}, \quad \hat{\sigma} = \frac{m - \Delta}{2m + 1};$$

$$\tilde{x}_A = \frac{m\Delta + m^2}{2m+1}, \qquad \tilde{x}_B = \frac{m - m\Delta + m^2}{2m+1}, \qquad \tilde{\sigma} = \frac{m+\Delta}{2m+1}.$$

Numerical analysis indicates that $\widehat{\Pi} - \widetilde{\Pi} > 0$ for all $\Delta \in (0, m)$.

If, for instance, $n = 2$ and $\Delta = \frac{1}{4}$, then

$$\hat{x}_A = \frac{1}{16}\left(8\sqrt{\frac{1}{2}m + \frac{1}{2}m^2 + \frac{1}{64}} - 1\right)\frac{m+2}{m+1} + \frac{1}{4}, \quad \hat{x}_B = \frac{1}{16}m\frac{8\sqrt{\frac{1}{2}m + \frac{1}{2}m^2 + \frac{1}{64}} - 1}{m+1},$$

$$\tilde{x}_A = \frac{1}{16}m\frac{\sqrt{32m + 32m^2 + 1} + 1}{m+1}, \quad \tilde{x}_B = \frac{\left(-3m + m\sqrt{32m + 32m^2 + 1} + 2\sqrt{32m + 32m^2 + 1} - 2\right)}{16m + 16}.$$

Numerical analysis indicates that $\widehat{\Pi} - \widetilde{\Pi} > 0$ for all $m \geq \Delta = \frac{1}{4}$. ■

Proof of Corollary 2. First,

$$\begin{aligned} \widehat{S} &= (v_A - \hat{x}_A)[1 - F(\hat{\sigma})] + \int_0^{\hat{\sigma}} (v_B - \hat{x}_B - s) f(s)\, ds, \quad \text{and} \\ \widetilde{S} &= (v_B - \tilde{x}_B)[1 - F(\tilde{\sigma})] + \int_0^{\tilde{\sigma}} (v_A - \tilde{x}_A - s) f(s)\, ds. \end{aligned}$$

We can rewrite

$$\begin{aligned} \widehat{S} &= (v_A - \hat{x}_A) - (v_A - \hat{x}_A) F(\hat{\sigma}) + (v_B - \hat{x}_B) F(\hat{\sigma}) - \hat{\sigma} F(\hat{\sigma}) + \int_0^{\hat{\sigma}} F(s)\, ds \\ &= (v_A - \hat{x}_A) + [-\Delta + \hat{x}_A - \hat{x}_B - \hat{\sigma}] F(\hat{s}) + \int_0^{\hat{\sigma}} F(s)\, ds \\ &= (v_A - \hat{x}_A) + \int_0^{\hat{\sigma}} F(s)\, ds. \end{aligned}$$

Similarly,

$$\tilde{S} = v_B - \tilde{x}_B + \int_0^{\tilde{\sigma}} F(s)\, ds.$$

Thus,

$$\widehat{S} \gtreqless \widetilde{S} \Longleftrightarrow \Delta - \hat{x}_A + \tilde{x}_B \gtreqless \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds.$$

(i) If $n = 1$ and $m = 1$,

$$\widehat{S} = v_A - \left(\frac{2}{3} + \frac{\Delta}{3} - \frac{b}{a}\right) + \int_0^{\frac{1}{3}(1-\Delta)} s ds = v_A + \frac{b}{a} - \frac{1}{18}\left(11 + 8\Delta - \Delta^2\right). \qquad (18)$$

$$\widetilde{S} = v_B - \left(\frac{2}{3} - \frac{\Delta}{3} - \frac{b}{a}\right) + \int_0^{\frac{1}{3}(1+\Delta)} s ds = v_B + \frac{b}{a} - \frac{1}{18}\left(13 - 4\Delta + \Delta^2\right).$$

$$\widehat{S} - \widetilde{S} = \frac{1}{9}\left(3\Delta + \Delta^2 + 1\right) > 0.$$

If $n = 1$, $b = 0$, and $r(x) = ax^m$, then

$$\hat{x}_A = \frac{m + m\Delta + m^2}{2m+1}, \quad \hat{x}_B = \frac{m(m-\Delta)}{2m+1}, \quad \hat{\sigma} = \frac{m-\Delta}{2m+1}.$$

$$\tilde{x}_A = \frac{m\Delta + m^2}{2m+1}, \quad \tilde{x}_B = \frac{m - m\Delta + m^2}{2m+1}, \quad \tilde{\sigma} = \frac{m+\Delta}{2m+1};$$

$$\hat{S} - \tilde{S} = \Delta - (\hat{x}_A - \tilde{x}_B) - \int_{\hat{\sigma}}^{\tilde{\sigma}} F(s)\, ds > 0.$$

(ii) Next, if $n = 2$ (i.e., $F(s) = s^2$), $r(x) = x + b$, and $b < \frac{\sqrt{\Delta^2+4}-\Delta}{8}$, then

$$\hat{x}_A = \frac{5}{8}\Delta + \frac{3}{8}\sqrt{\Delta^2+4} - b, \quad \hat{x}_B = \frac{1}{8}\sqrt{\Delta^2+4} - \frac{1}{8}\Delta - b, \ \hat{\sigma} = \frac{1}{4}\left(\sqrt{\Delta^2+4} - \Delta\right)$$

$$\tilde{x}_A = \frac{1}{8}\Delta + \frac{1}{8}\sqrt{\Delta^2+4} - b, \quad \tilde{x}_B = \frac{3}{8}\sqrt{\Delta^2+4} - \frac{5}{8}\Delta - b, \ \tilde{\sigma} = \frac{1}{4}\left(\sqrt{\Delta^2+4} + \Delta\right)$$

$$\hat{S} - \tilde{S} = -\frac{1}{24}\Delta\left(\Delta^2 + 9\right) < 0.$$

If $n = 2$, $r(x) = ax^m$, and $\Delta = \frac{1}{4}$, then numerical analysis also indicates $\hat{S} - \tilde{S} < 0$. ■

Proof of Proposition 3. First, suppose

$$v_A - x_A > v_B - x_B$$

so that some of the consumers with $D = B$ will switch to A, but no consumer whose $D = A$ will switch to B. The marginal switching consumer with $D = B$ is

$$\sigma = \Delta - x_A + x_B.$$

From (10), the equilibrium x_A^e and x_B^e satisfy the first-order conditions

$$\partial\pi_A/\partial x_A = r'(x_A)\left[1 + F(\Delta - x_A + x_B)\right] - r(x_A) f(\Delta - x_A + x_B) = 0,$$

$$\partial\pi_B/\partial x_B = r'(x_B)\left[1 - F(\Delta - x_A + x_B)\right] - r(x_B) f(\Delta - x_A + x_B) = 0,$$

which can be rewritten as (11) if $x_A^e > 0$ and $x_B^e > 0$.

Next, $x_A^e - x_B^e \geq 0$, because otherwise $x_B^e > x_A^e > 0$, which implies

$$\frac{r'\left(x_A^e\right)}{r\left(x_A^e\right)} = \frac{f\left(\sigma^e\right)}{1+F\left(\sigma^{\diamond}\right)} > \frac{f\left(\sigma^e\right)}{1-F\left(\sigma^e\right)} = \frac{r'\left(x_B^e\right)}{r\left(x_B^e\right)} \to -F\left(\sigma^e\right) > F\left(\sigma^e\right),$$

a contradiction.

Next, $\sigma^e = \Delta - x_A^e + x_B^e > 0$, because if $\sigma^e < 0$, B can increase π_B by raising x_B; and if $\sigma^e = 0$, we would have $x_A^e = x_B^e$, which implies $\sigma^e = \Delta - x_A^e + x_B^e = \Delta > 0$, a contradiction. Hence $x_A^e - x_B^e < \Delta$. And $\sigma^e < 1$, because $\left.\frac{\partial \pi_B}{\partial x_B}\right|_{x_B = 1 + x_A^e - \Delta} < 0$.

The only other potential equilibrium may arise when $v_A - x_A < v_B - x_B$, in which case the marginal switching consumer whose $D = B$ is $\sigma = -\Delta + x_A - x_B > 0$, and the two firms' profit functions are

$$\pi_A = r\left(x_A\right)\frac{1}{2}\left[1 - F\left(-\Delta + x_A - x_B\right)\right], \quad \pi_B = r\left(x_B\right)\frac{1}{2}\left[1 + F\left(-\Delta + x_A - x_B\right)\right].$$

We next show that there can be no such an equilibrium. Suppose to the contrary that the equilibrium exists. Then at such an equilibrium, (x_A, x_B) satisfy the first-order conditions

$$\begin{aligned} r'\left(x_A\right)\left[1 - F\left(-\Delta + x_A - x_B\right)\right] - r\left(x_A\right) f\left(-\Delta + x_A - x_B\right) &= 0, \\ r'\left(x_B\right)\left[1 + F\left(-\Delta + x_A - x_B\right)\right] - r\left(x_B\right) f\left(-\Delta + x_A - x_B\right) &\leq 0. \end{aligned}$$

where

$$\sigma = -\Delta + x_A - x_B > 0 \Rightarrow x_A - x_B > \Delta \Rightarrow x_A > x_B \geq 0.$$

Hence, since $\frac{r'(x)}{r(x)}$ is decreasing,

$$\begin{aligned} \frac{r'\left(x_A\right)}{r\left(x_A\right)} &= \frac{f\left(-\Delta + x_A - x_B\right)}{1 - F\left(-\Delta + x_A - x_B\right)} < \frac{r'\left(x_B\right)}{r\left(x_B\right)} \leq \frac{f\left(-\Delta + x_A - x_B\right)}{1 + F\left(-\Delta + x_A - x_B\right)} \\ &\Rightarrow 1 + F\left(\sigma\right) < 1 - F\left(\sigma\right) \Rightarrow 2F\left(\sigma\right) < 0, \end{aligned}$$

a contradiction.

We next establish the expressions for S^e and Π^e: In equilibrium, consumers whose $D = B$ will switch to A if $s < \sigma^e$. Hence, consumer surplus is

$$\begin{aligned} S^e &= \frac{1}{2}\left(v_A - x_A^e\right) + \frac{1}{2}\left(v_B - x_B^e\right)\left[1 - F\left(\sigma^e\right)\right] + \frac{1}{2}\int_0^{\sigma^e}\left(v_A - x_A^e - s\right) f\left(s\right) ds \\ &= \frac{1}{2}\left[\left(v_A - x_A^e\right) + \left(v_B - x_B^e\right) + \int_0^{\sigma^e} F\left(s\right) ds\right]. \end{aligned}$$

The expression for Π^e follow directly from (10).

Moreover, suppose $\sigma^e \leq \hat{\sigma}$. From (4) and (11), together with the fact that $\frac{r'(x)}{r(x)}$ decreases from (C1),

$$\frac{r'\left(x_B^e\right)}{r\left(x_B^e\right)} = \frac{f\left(\sigma^e\right)}{1 - F\left(\sigma^e\right)} \leq \frac{f\left(\hat{\sigma}\right)}{1 - F\left(\hat{\sigma}\right)} = \frac{r'\left(\hat{x}_A\right)}{r\left(\hat{x}_A\right)},$$

and hence $x_B^e \geq \hat{x}_A$. It follows that $x_A^e > x_B^e \geq \hat{x}_A > \hat{x}_B$. Therefore,

$$\Pi^e = r\left(x_A^e\right) \frac{\left[1 + F\left(\sigma^e\right)\right]}{2} + r\left(x_B^e\right) \frac{\left[1 - F\left(\sigma^e\right)\right]}{2} > r\left(x_B^e\right),$$

while

$$\widehat{\Pi} = r\left(\hat{x}_A\right)\left[1 - F\left(\hat{\sigma}\right)\right] + r\left(\hat{x}_B\right) F\left(\hat{\sigma}\right) < r\left(\hat{x}_A\right).$$

Thus $\Pi^e > \widehat{\Pi}$. Also, because $\hat{x}_A \leq x_B^e < x_A^e$, $v_B < v_A$, and $\sigma^e \leq \hat{\sigma}$,

$$\begin{aligned} \widehat{S} &= v_A - \hat{x}_A + \int_0^{\hat{\sigma}} F(s)\, ds = \frac{2\left(v_A - \hat{x}_A\right) + 2\int_0^{\hat{\sigma}} F(s)\, ds}{2} \\ &> \frac{v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)\, ds}{2} = S^e. \end{aligned}$$

Next, suppose (6) holds. When $n = 1$ (i.e., $F(s) = s$), if $r = ax + b$, from (11) we have

$$x_A^e = 1 + \frac{1}{3}\Delta - \frac{b}{a}, \quad x_B^e = 1 - \frac{b}{a} - \frac{1}{3}\Delta > 0 \text{ when } b < a\left(1 - \frac{\Delta}{3}\right), \quad \sigma^e = \frac{1}{3}\Delta.$$

$$S^e = \frac{1}{2}\left(v_A + v_B - 2\right) + \frac{\Delta^2}{36} + \frac{b}{a}.$$

$$\Pi^e = \frac{1}{9}a\left(\Delta^2 + 9\right), \quad W^e = \frac{1}{9}a\left(\Delta^2 + 9\right) + \frac{1}{2}\left(v_A + v_B - 2\right) + \frac{\Delta^2}{36} + \frac{b}{a}.$$

$$\hat{S} - S^e = \frac{1}{36}\left(2\Delta + \Delta^2 + 14\right) > 0; \quad \widehat{\Pi} - \Pi^e = \frac{1}{9}a\left(2\Delta + \Delta^2 - 4\right) < 0.$$

$$\widehat{W} - W^e = \frac{1}{36}\left(14 - 16a + 2\Delta + 8a\Delta + \Delta^2 + 4a\Delta^2\right) \gtreqless 0 \Longleftrightarrow a \lesseqgtr \frac{\Delta^2 + 2\Delta + 14}{4\left(4 - 2\Delta - \Delta^2\right)} = a^e,$$

and $a^e \in \left(\frac{7}{8}, \frac{17}{4}\right)$ for $\Delta \in (0, 1)$.

If $r(x) = ax^m$, then

$$x_A^e = \frac{1}{2m+1}\left(m + m\Delta + 2m^2\right), \quad x_B^e = \frac{1}{2m+1}\left(m - m\Delta + 2m^2\right), \quad \sigma^e = \frac{\Delta}{2m+1}.$$

$$\Pi^e = a\left(\frac{1}{2m+1}\left(m + m\Delta + 2m^2\right)\right)^m \frac{1 + \frac{\Delta}{2m+1}}{2} + a\left(\frac{1}{2m+1}\left(m - m\Delta + 2m^2\right)\right)^m \frac{1 - \frac{\Delta}{2m+1}}{2}.$$

$$S^e = \frac{1}{2}\left(v_A - x_A^e + v_B - x_B^e + \int_0^{\sigma^e} F(s)\,ds\right)$$

$$\sigma^e - \hat{\sigma} = \frac{\Delta}{2m+1} - \frac{m-\Delta}{2m+1} = \frac{2\Delta - m}{2m+1} < 0 \Leftrightarrow m > 2\Delta.$$

Hence $\widehat{\Pi} < \Pi^e$, $\widehat{S} > S^e$ if $m > 2\Delta$.

When $n > 1$, if, for instance, $n = 2$, $r(x) = ax^m$, and $\Delta = \frac{1}{4}$,

$$x_A^e = \frac{17m + 32m^2 + 16m^3}{8m+8}, \qquad x_B^e = \frac{15m + 32m^2 + 16m^3}{8m+8}, \quad \sigma^e = \frac{1}{4(m+1)}.$$

$$\sigma^e - \hat{\sigma} = \frac{1}{4(m+1)} - \frac{1}{8}\frac{8\sqrt{\frac{1}{2}m + \frac{1}{2}m^2 + \frac{1}{64}} - 1}{m+1} < 0 \text{ if } m > 0.21 \Rightarrow \widehat{\Pi} < \Pi^e,\ \widehat{S} > S^e.$$

If, for instance, $n = 2$, $r(x) = x + b$, and $b < \frac{\sqrt{\Delta^2+4}-\Delta}{8}$,

$$x_A^e = \frac{1}{4\Delta}\left(-4b\Delta + \Delta^2 + 4\right), \qquad x_B^e = -\frac{1}{4\Delta}\left(4b\Delta + \Delta^2 - 4\right), \quad \sigma^e = \frac{1}{2}\Delta.$$

$$\sigma^e - \hat{\sigma} = \frac{1}{2}\Delta - \frac{1}{4}\left(\sqrt{\Delta^2+4} - \Delta\right) < 0 \Leftrightarrow 3\Delta < \sqrt{\Delta^2+4} \Leftrightarrow \Delta^2 < \frac{1}{2}.$$

But for any $\Delta < 1$,

$$\Pi^e = r(x_A^e)\frac{1 + F(\sigma^e)}{2} + r(x_B^e)\frac{1 - F(\sigma^e)}{2} = \frac{1}{16}\frac{\Delta^4 + 16}{\Delta},$$

$$S^e = \frac{1}{2}\left(v_A - \frac{1}{4\Delta}\left(-4b\Delta + \Delta^2 + 4\right) + v_B + \frac{1}{4\Delta}\left(4b\Delta + \Delta^2 - 4\right) + \int_0^{\frac{1}{2}\Delta} F(s)\,ds\right).$$

$$\widehat{\Pi} - \Pi^e = \frac{1}{64}\frac{-36\Delta^2 + 5\Delta^3\sqrt{\Delta^2+4} + 24\Delta\sqrt{\Delta^2+4} - \Delta\left(\Delta^2+4\right)^{\frac{3}{2}} - 64}{\Delta} < 0,$$

$$\widehat{S} - S^e = -\frac{132\Delta^2 + 8\Delta^4 - 3\Delta^3\sqrt{\Delta^2+4} - 96\Delta^2 + 72\Delta\sqrt{\Delta^2+4} - \Delta\left(\Delta^2+4\right)^{\frac{3}{2}} - 192}{192\Delta} > 0.$$

■

Proof of Proposition 4. (i) First, at the potential equilibrium where some consumers with $D = B$ switch to A, the marginal consumer is $\sigma = \Delta - (x_A - x_B) > 0$. The equilibrium x_A^- and x_B^- solve the first-order conditions

$$\begin{aligned}
\frac{\partial \pi_A}{\partial x_A} &= r'(x_A)\left[\lambda + (1-\lambda)F(\Delta - x_A + x_B)\right] - r(x_A)(1-\lambda)f(\Delta - x_A + x_B) = 0, \\
\frac{\partial \pi_B}{\partial x_B} &= r'(x_B)\left[1 - F(\Delta - x_A + x_B)\right] - r(x_B)f(\Delta - x_A + x_B) = 0;
\end{aligned}$$

which can be rewritten as

$$\frac{r'(x_A)}{r(x_A)} = \frac{(1-\lambda) f(\sigma)}{\lambda + (1-\lambda) F(\sigma)}, \qquad \frac{r'(x_B)}{r(x_B)} = \frac{f(\sigma)}{1-F(\sigma)}. \tag{19}$$

Since $\left(x_A^- - x_B^-\right) < \Delta$ if λ is sufficiently close to $\frac{1}{2}$, there is some $\lambda^- \in \left(\frac{1}{2}, 1\right)$ such that if $\lambda \leq \lambda^-$, in equilibrium $x_A^- > x_B^- > 0$, $\sigma^- = \Delta - \left(x_A^- - x_B^-\right) > 0$, and there is consumer switching only from B to A.

Next, compared to the bidding outcome when firm A wins (i.e., $\lambda = 1$), if $\sigma^- \leq \hat{\sigma}$, then

$$\frac{r'\left(x_B^-\right)}{r\left(x_B^-\right)} = \frac{f(\sigma^-)}{1-F(\sigma^-)} \leq \frac{f(\hat{\sigma})}{1-F(\hat{\sigma})} = \frac{r'(\hat{x}_A)}{r(\hat{x}_A)} \Rightarrow x_B^- \geq \hat{x}_A.$$

Thus $x_A^- > x_B^- \geq \hat{x}_A > \hat{x}_B$. Hence

$$\Pi^- = r\left(x_A^-\right)\left[\lambda + (1-\lambda) F\left(\sigma^-\right)\right] + r\left(x_B^-\right)(1-\lambda)\left[1-F\left(\sigma^-\right)\right] > r\left(x_B^-\right);$$

but $\widehat{\Pi} < r(\hat{x}_A)$. It follows that $\lambda^- > \widehat{\Pi}$. Also,

$$\begin{aligned} S^- &= \left[\lambda + (1-\lambda) F\left(\sigma^-\right)\right]\left(v_A - x_A^-\right) + (1-\lambda)\left[1-F\left(\sigma^-\right)\right]\left(v_B - x_B^-\right) - (1-\lambda)\int_0^{\sigma^-} s dF(s) \\ &= \lambda\left(v_A - x_A^-\right) + (1-\lambda)\left(v_B - x_B^-\right) + (1-\lambda)\int_0^{\sigma^-} F(s)\, ds. \end{aligned}$$

And since $x_A^- > x_B^- \geq \hat{x}_A > \hat{x}_B$,

$$\begin{aligned} \widehat{S} &= \lambda(v_A - \hat{x}_A) + (1-\lambda)(v_A - \hat{x}_A) + \int_0^{\hat{\sigma}} F(s)\, ds \\ &> \lambda\left(v_A - x_A^-\right) + (1-\lambda)\left(v_B - x_B^-\right) + (1-\lambda)\int_0^{\sigma^-} F(s)\, ds = S^-. \end{aligned}$$

(ii) Next, consider the potential equilibrium where some consumers with $D = A$ switch to B. In this case, the equilibrium x_A^+ and x_B^+ solve the first-order conditions

$$\begin{aligned} r'(x_A)\left[1 - F(-\Delta + x_A - x_B)\right] - r(x_A) f(-\Delta + x_A - x_B) &= 0, \\ r'(x_B)\left[1 - \lambda + \lambda F(-\Delta + x_A - x_B)\right] - r(x_B)\lambda f(-\Delta + x_A - x_B) &= 0; \end{aligned}$$

which can be written as

$$\frac{r'\left(x_A^+\right)}{r\left(x_A^+\right)} = \frac{f(\sigma^+)}{1-F(\sigma^+)}, \qquad \frac{r'\left(x_B^+\right)}{r\left(x_B^+\right)} = \frac{\lambda f(\sigma^+)}{1-\lambda+\lambda F(\sigma^+)}, \tag{20}$$

where $\sigma^+ = x_A^+ - x_B^+ - \Delta > 0$, or $x_A^+ - x_B^+ > \Delta$. As $\lambda \to 1$, this equilibrium exists as in Proposition 1. On the other hand, as $\lambda \to \frac{1}{2}$,

$$\frac{r'\left(x_A^+\right)}{r\left(x_A^+\right)} - \frac{r'\left(x_B^+\right)}{r\left(x_B^+\right)} = \frac{f\left(\sigma^+\right)}{1 - F\left(\sigma^+\right)} - \frac{\lambda f\left(\sigma^+\right)}{1 - \lambda + \lambda F\left(\sigma^+\right)}$$

$$\Rightarrow \frac{r'\left(x_A^+\right)}{r\left(x_A^+\right)} - \frac{r'\left(x_B^+\right)}{r\left(x_B^+\right)} = \frac{f\left(\sigma^+\right)}{1 - F\left(\sigma^+\right)} - \frac{f\left(\sigma^+\right)}{1 + F\left(\sigma^+\right)} > 0,$$

which cannot hold if $x_A^+ > \Delta + x_B^+$. Hence, there is some $\lambda^+ \in \left(\lambda^-, 1\right)$ such that (20) holds if and only if $\lambda > \lambda^+$.

(iii) Now suppose (13) holds; that is, $r\left(x\right) = ax + b$ and $F\left(s\right) = s$. Then

$$x_A^- = \frac{1}{3}\left(\frac{1 + \Delta + \lambda - \Delta\lambda}{\left(1 - \lambda\right)} - 3\frac{b}{a}\right), \qquad x_B^- = \frac{1}{3}\left(\frac{2 - \Delta - \lambda + \Delta\lambda}{\left(1 - \lambda\right)} - 3\frac{b}{a}\right).$$

$$\sigma^- = \Delta - \left(x_A^- - x_B^-\right) = \frac{1}{3}\frac{1 + \Delta - 2\lambda - \Delta\lambda}{1 - \lambda} > 0 \Leftrightarrow \lambda < \lambda^- \equiv \frac{\Delta + 1}{\Delta + 2}.$$

$$\Pi^- = \pi_A^- + \pi_B^- = \frac{1}{9}a\frac{5 - 2\Delta + 2\Delta^2 - 2\lambda\left(1 - \Delta\right)\left(1 - 2\Delta - \lambda + \Delta\lambda\right)}{1 - \lambda}. \tag{21}$$

$$\frac{d\Pi^-}{d\lambda} = \frac{1}{9}a\frac{3 + 4\Delta - 2\Delta^2 + 2\lambda\left(\Delta - 1\right)^2\left(2 - \lambda\right)}{\left(\lambda - 1\right)^2} > 0.$$

$$\Pi^- - \hat{\Pi} = \frac{1}{9}a\frac{4\Delta\left(2\lambda - 1\right) + \lambda\left(3 - 2\Delta^2\right) + 2\lambda^2\left(\Delta - 1\right)^2}{1 - \lambda} > 0.$$

$$S^- = \frac{-11a + 18\left(av_B + b\right)\left(1 - \lambda\right) + 8a\left(\Delta + \lambda - \lambda^2\right) + a\Delta^2 + a\Delta\lambda\left(-2\Delta - 2\lambda + \Delta\lambda - 6\right)}{18a\left(1 - \lambda\right)}.$$

$$\frac{dS^-}{d\lambda} = -\frac{1}{18}\frac{3 - 2\Delta + \Delta^2 + \lambda\left(\Delta + 2\right)\left(\Delta - 4\right)\left(\lambda - 2\right)}{\left(\lambda - 1\right)^2} < 0.$$

$$S^- - \hat{S} = \frac{1}{18}\frac{-2\Delta - 3\lambda - \lambda\left(4 - \Delta\right)\left(-\Delta + 2\lambda + \Delta\lambda\right)}{1 - \lambda} < 0.$$

On the other hand,

$$x_A^+ = \frac{1}{3a\lambda}\left(a + a\lambda + a\Delta\lambda - 3b\lambda\right), \qquad x_B^+ = \frac{1}{3a\lambda}\left(2a - a\lambda - a\Delta\lambda - 3b\lambda\right),$$

$$\sigma^+ = -\Delta + x_A^+ - x_B^+ = \frac{1}{3}\frac{2\lambda - \Delta\lambda - 1}{\lambda} > 0 \Leftrightarrow \lambda > \frac{1}{2 - \Delta} \equiv \lambda^+.$$

$$\lambda^+ - \lambda^- = \frac{1}{2 - \Delta} - \frac{\Delta + 1}{\Delta + 2} = \frac{\Delta^2}{\left(2 - \Delta\right)\left(\Delta + 2\right)} > 0.$$

$$\Pi^{+} = \frac{1}{9} a \frac{5 + 2\lambda (\Delta + 1)(\lambda + \Delta\lambda - 1)}{\lambda}, \tag{22}$$

$$\frac{d\Pi^{+}}{d\lambda} = \frac{1}{9} a \frac{2\lambda^2 (\Delta + 1)^2 - 5}{\lambda^2} \gtreqless 0 \text{ if } \lambda \gtreqless \frac{\sqrt{5/2}}{(\Delta + 1)}.$$

If $\Delta \leq 0.581$, $\frac{\sqrt{5/2}}{(\Delta+1)} \geq 1$, and hence $\frac{d\Pi^{+}}{d\lambda} < 0$ for all $\lambda > \lambda^{+}$; while if $\Delta > 0.581$, $\lambda^{+} = \frac{1}{2-\Delta} > \frac{1}{2-0.581} = 0.704\,72$, and $\frac{d\Pi^{+}}{d\lambda} < 0$ if $\lambda \in \left(\lambda^{+}, \frac{\sqrt{5/2}}{(\Delta+1)}\right)$ and $\frac{d\Pi^{+}}{d\lambda} > 0$ if $\lambda > \frac{\sqrt{5/2}}{(\Delta+1)}$.

$$\Pi^{+} - \widehat{\Pi} = \frac{1}{9} a (1 - \lambda) \frac{5 - 2\lambda (\Delta + 1)^2}{\lambda} \gtreqless 0 \text{ if } \lambda \lesseqgtr \frac{5/2}{(\Delta + 1)^2}.$$

If $\Delta \leq 0.581$, $\frac{5/2}{(\Delta+1)^2} \geq 1$, and hence $\Pi^{+} - \widehat{\Pi} > 0$ for all $\lambda > \lambda^{+}$; while if $\Delta > 0.581$, $\frac{5/2}{(\Delta+1)^2} < 1$, and hence $\Pi^{+} > \widehat{\Pi}$ if $\lambda \in \left(\lambda^{+}, \frac{5/2}{(\Delta+1)^2}\right)$ and $\Pi^{+} < \widehat{\Pi}$ if $\lambda > \frac{5/2}{(\Delta+1)^2}$.

Furthermore,

$$S^{+} = \lambda \left(v_A - x_A^{+}\right) + (1 - \lambda)\left(v_B - x_B^{+}\right) + \lambda \int_0^{\sigma^{+}} F(s)\, ds.$$

For the linear-uniform case:

$$S^{+} = \frac{1}{18} \frac{-11a + 8a\lambda + 18b\lambda + 18a\lambda v_B + 8a\Delta\lambda - a\lambda^2 (\Delta + 4)(2 - \Delta)}{a\lambda}. \tag{23}$$

$$\frac{dS^{+}}{d\lambda} = \frac{1}{18} \frac{-8\lambda^2 + \Delta^2\lambda^2 + 2\Delta\lambda^2 + 11}{\lambda^2} > 0,$$

$$S^{+} - \widehat{S} = \frac{1}{18} \frac{19\lambda + 2\Delta\lambda^2 - 2\lambda\Delta - 8\lambda^2 + \Delta^2\lambda^2 - \Delta^2\lambda - 11}{\lambda} < 0.$$

Finally,

$$\begin{aligned} & W^{-} - \widehat{W} \\ = \; & \frac{-2\Delta - \lambda (\Delta - 1)(\Delta - 3) - \lambda^2 (\Delta + 2)(4 - \Delta) + 2a\left(8\Delta\lambda - 4\Delta + 3\lambda - 2\Delta^2\lambda\right) + 4a\lambda^2 (\Delta - 1)^2}{18(1 - \lambda)} \\ \gtreqless \; & 0 \Leftrightarrow a \gtreqless \frac{2\Delta + \lambda (\Delta - 1)(\Delta - 3) + \lambda^2 (\Delta + 2)(4 - \Delta)}{2(8\Delta\lambda - 4\Delta + 3\lambda - 2\Delta^2\lambda) + 4\lambda^2 (\Delta - 1)^2} = a^{-}, \end{aligned}$$

where $a^{-} < 1$ if Δ is sufficiently small and λ is sufficiently close to $\frac{1}{2}$, while $a^{-} > 1$ if $\Delta \geq \frac{1}{4}$.

$$W^{+} - \widehat{W} = \frac{1}{18} (1 - \lambda) \frac{10a - 4a\lambda (\Delta + 1)^2 - 2\Delta\lambda - \Delta^2\lambda + 8\lambda - 11}{\lambda}.$$

If $10 \leq 4\lambda(\Delta+1)^2$, then $W^+ < \widehat{W}$. If $10 > 4\lambda(\Delta+1)^2$, which holds if $\Delta \leq 0.58$,

$$W^+ \gtreqless \widehat{W} \Leftrightarrow a \gtreqless \frac{2\Delta\lambda + \Delta^2\lambda - 8\lambda + 11}{10 - 4\lambda(\Delta+1)^2} = a^+,$$

where $a^+ > 1$ if $\Delta \in [0.27, 0.58]$ and $a^+ < 1$ if $\Delta \leq 0.2$. ■

Proof of Corollary 3. Since $\lambda \geq \lambda^+ = \frac{1}{2-\Delta^l}$, the analysis in Proposition 4 for $\lambda \geq \lambda^+$ applies. From (23),

$$S^l = v_A^l + \frac{b}{a} - \frac{1 + 6\lambda + 4\lambda^3 + \left(\Delta^l\right)^2 \lambda^2 (1-2\lambda) + 2\Delta^l \lambda (\lambda+1)^2}{18\lambda^2}. \tag{24}$$

Under competitive bidding, $\lambda = 1$, $\Delta = v_A - v_B$, and from (18)

$$\widehat{S} = v_A + \frac{b}{a} - \frac{1}{18}\left(11 + 8\Delta - \Delta^2\right).$$

Thus, since $v_A^l = v_A - \delta_A$ and $\Delta^l = \Delta - \delta_A - \delta_B > 0$,

$$S^l - \widehat{S} = -\delta_A + \frac{\lambda^2\left(11 + 8\Delta - \Delta^2\right) - 1 - 6\lambda - 4\lambda^3 - \Delta^l\lambda\left(4\lambda + \Delta^l\lambda + 2\lambda^2 - 2\Delta^l\lambda^2 + 2\right)}{18\lambda^2}.$$

Then, $S^l - \widehat{S}$ decreases in Δ^l and increases in λ, because

$$\frac{d\left(S^l - \widehat{S}\right)}{d\Delta^l} = \frac{1}{9}\frac{-2\lambda\left(1-\Delta^l\right) - \Delta^l\lambda - \lambda^2 - 1}{\lambda} < 0,$$

$$\frac{d\left(S^l - \widehat{S}\right)}{d\lambda} = \frac{1}{9}\frac{3\lambda - 2\lambda^3 + \left(\Delta^l\right)^2\lambda^3 + \Delta^l\lambda - \Delta^l\lambda^3 + 1}{\lambda^3} > 0.$$

Moreover,

$$-1 - 6\lambda - 4\lambda^3 + 11\lambda^2 + \lambda^2\left(8(0.4) - (0.4)^2\right) > 0$$

if $\lambda \geq 0.66$, implying $S^l - \widehat{S} > 0$ if $\lambda \geq 0.7$ while δ_A and Δ^l are sufficiently small.

On the other hand, suppose $\delta_A = \delta_B = \delta$. Then

$$\begin{aligned} & S^l - \widehat{S} \\ = \; & \frac{-1 - 6\lambda + 11\lambda^2 - 4\lambda^3 - 2\Delta\lambda(1-\lambda)(1-\lambda+\Delta\lambda) - 2\lambda\delta(2\lambda-1)(2-\lambda+2\Delta\lambda-2\lambda\delta)}{18\lambda^2} \\ < \; & 0. \end{aligned}$$

Finally, suppose $v_A = 5$ and $v_B = 4.5$, and $\lambda = 0.8$. Then $S^l > \widehat{S}$ if $\Delta^l \leq 0.314$ and $\delta_A = 0$; that is, if changing λ from 1 to 0.8 increases v_B by at least $0.5 - 0.314 = 0.186$ without lowering v_A. If $\delta_B = 0.25$, so that $v_B^l = 4.75$ and $v_A^l = 5 - \delta_A$. Then $v_A^l - v_A = -\delta_A$, $\Delta^l = 0.25 - \delta_A$, and

$$
\begin{aligned}
& S^l - \widehat{S} \\
= \quad & -\delta_A + \frac{\lambda^2\left(11 + 8\Delta - \Delta^2\right) - 1 - 6\lambda - 4\lambda^3 - \Delta^l\lambda\left(4\lambda + \Delta^l\lambda + 2\lambda^2 - 2\Delta^l\lambda^2 + 2\right)}{18\lambda^2} \\
= \quad & \frac{1}{180}\left(-102.0\delta_A + 6.0\delta_A^2 + 5.0\right) > 0
\end{aligned}
$$

if $\delta_A < 0.05$ (i.e., if $\delta_A < 20\%$ of δ_B). ■

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