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Parallel Imports, Demand Dispersion, and International Price Discrimination

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Abstract

Parallel imports, or gray-market imports, are genuine goods imported through unauthorized channels. Opponents see them as free riding on authorized distributors. Proponents see them as arbitraging international price discrimination. Abstracting from free riding, this paper asks whether the price discrimination explanation justifies the prevailing support for parallel imports from the standpoint of global welfare. The paper compares uniform and discriminatory pricing for a monopolist serving multiple markets. It finds that price discrimination is superior with high demand dispersion across markets, because uniform pricing would cause too many markets to go unserved. The paper also considers "mixed systems" in which manufacturers are required to charge a single price in a designated group of markets but can discriminate across groups. Mixed systems can yield significantly greater welfare than either polar regime (and may provide a Pareto improvement over uniform pricing). The paper identifies the optimal mixed system in the class of "no holes," i.e., if two markets are in the same group, so are all markets with demand elasticities between the two. Surprisingly perhaps, global welfare is even greater in mixed systems with holes.

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1. Introduction

Parallel imports, or "gray-market" imports, are genuine products—not counterfeits—imported by unauthorized resellers. A common example is where one firm holds the national trademarks in different countries, each trademark conferring the exclusive distribution right in that country, and a third party obtains the product in one country (typically from wholesalers rather than the trademark holder) and diverts it to another without the authorization of the local trademark holder. There are numerous variations on this scenario, and the legal treatment of parallel imports can differ across the various settings.[1] The basic economic question, however, is the same: Should a producer be able to enforce exclusive distribution territories internationally?

Although accurate data on parallel imports are limited, because the business is inherently rather secretive, the phenomenon appears important. Parallel imports into the U.S. increased dramatically with the dollar's rapid appreciation in the early 1980s, and by the mid-1980s were estimated at $7–10 billion, or 2–3% of the U.S. import bill. Moreover, they were disproportionately concentrated in particular products—typically name-brand consumer goods such as cosmetics and fragrances, luxury automobiles, and cameras—products in which they accounted for 15–20% of all sales (Chard and Mellor, 1989; Business Week, 1985 and 1988). Although parallel imports into the U.S. appear to have peaked in the mid 1980s, they remain significant both in the U.S. and worldwide. For example, parallel imports of pharmaceuticals alone within the EC were estimated at more than $500 million in 1990 and projected to grow rapidly (REMIT, 1992); a surge of parallel imports of pharmaceuticals from Mexico to the U.S. also is predicted if the North American Free Trade Agreement materializes (Drug Store News, 1993).[2]

The attention parallel imports command further attests to their importance. In the U.S., extensive litigation continues, and both opponents and proponents have repeatedly tried to enact federal legislation (see Hearings on S.

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[1] The products may be covered by other intellectual property rights (IPRs)—in the parallel imports context trademarks are the most common IPRs, then copyrights, and occasionally patents—or by no IPRs. The authorized national distributors may be commonly owned as above, linked by licensing, or entirely independent. And the imported goods are usually produced abroad, but sometimes are produced domestically, exported, then reimported. For details on the legal treatment of parallel imports in the U.S. and the EC see Hawk (1991).

[2] More broadly, articles found in a LEXIS-NEXIS search of the trade press, January 1989–May 1993, reveal the following. Parallel imports continue in many of the same products as in the mid-1980s: consumer electronics, automobiles, spirits, watches, cosmetics and fragrances (where their estimated worldwide market share is 20–30%). They have extended to additional consumer goods including haircare products, athletic sneakers, camcorders, and personal computers. They also have been reported in industrial products such as semiconductor chips and construction equipment. A survey of U.S. exporters to Asia (Palia and Keown, 1991) offers a clue to the pervasiveness of parallel imports: of 141 respondents that used sole import distributors (55% of all respondents), 41% reported problems with parallel imports in the past five years.

626, 1990). In the EC, a recent controversy centers on the role of national exclusive-distribution territories in sustaining different car prices between member states (Financial Times, 1992). And Japan's Fair Trade Commission recently issued enforcement guidelines focusing prominently on parallel imports (JFTC, 1991).

Generally, policies worldwide firmly support parallel imports.[3] This support stems largely from a belief that parallel imports are driven not so much by free riding on promotional efforts of authorized distributors, but by international price discrimination. Why has the price discrimination view generated such strong support for parallel imports, whereas use of exclusive territories within a country typically elicits much less hostility? We see two likely explanations. First, the scope for price discrimination is probably greater internationally: disparities in demand elasticities across countries are likely to be greater than across regions within a country, most obviously because of the greater differences in per capita incomes between countries than regionally within countries.[4] A priori, therefore, international price discrimination is a plausible explanation for parallel imports.

Second, relative to uniform pricing, geographic price discrimination within a country typically harms consumers in some regions but benefits others. In contrast, countries facing relatively high prices under international price discrimination tend to ignore the gains from discrimination to consumers in low-price countries. Thus, high-price countries generally perceive that their welfare will be higher under uniform pricing.[5] Such perceptions account for the support for parallel imports in many countries, including the U.S., Japan, and Australia.[6]

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[3] The U.S. Supreme Court's decision in *K mart Corp. v. Cartier, Inc.*, 486 U.S. 281 (1988), upheld the Customs Service's position of not excluding parallel imports of trademarked goods whenever the U.S. trademark holder and the trademarkholder in the country where the parallel imports originated are commonly owned or controlled. The EC, in a stance that remains controversial, insists that unimpeded parallel imports among member states are central to promoting a common market (Hawk, 1991). It treats as unlawful, under the competition provisions and free movement of goods provisions of the Treaty of Rome, the use of exclusive territories that overlap with national borders. The JFTC's (1991) guidelines arguably go furthest, prohibiting actions against parallel imports not only on Japanese soil but also abroad.

[4] The following figures illustrate these differences in 1990. The quintile of the U.S. population represented by the states with the greatest per capita incomes had a per capita income approximately 1.5 times as great as did the quintile of the population in the poorest states (see U.S. Bureau of the Census, 1992, Tables 25 and 687). In contrast, for the EC countries, instead of the states in the U.S., the comparable ratio is about 2.3 (see U.S. Bureau of the Census, 1992, Table 25 1370). Worldwide these income differences were significantly greater. Among World Bank member countries, the richest quintile (based on world population) had a per capita GNP approximately 75-85 times as great as in the poorest quintile (World Bank, 1992, Table 1). Even after adjusting for purchasing power parity, as done in the United Nations' International Comparison Program, this ratio lay in the range 15-20 (World Bank, 1992, Table 30).

[5] We will explain in Section 5 why these perceptions need not be correct, however.

[6] The EC is a special case; as noted earlier, price discrimination within the EC is condemned for somehow delaying economic integration, not for its distributional impact.

This paper departs from the traditional debate by analyzing parallel imports from the perspective of *global* welfare (the sum of profit and consumers' surplus in all markets) rather than national welfare. National welfare is an inadequate criterion for designing international trading rules, since quid pro quos among countries to compensate losers are generally possible in multilateral negotiations (perhaps via concessions in other areas). Our change of focus, therefore, is relevant for evaluating multilateral approaches to the question of parallel imports (see also the discussion in Section 5).

In our analysis, we shall abstract from free-rider explanations for parallel imports—even though parallel imports *are* partly explained by free riding (see Section 2). We choose to focus solely on the price-discrimination explanation purely as a modeling strategy: the price discrimination scenario is probably the "best case" for allowing parallel imports, and our theme is that the case for parallel imports is tenuous even then.

We consider a monopolist producer of a final good, facing markets with different known demands.[7] If legally permitted to curb parallel imports, the monopolist can charge a different monopoly price in each market. We call this *complete discrimination*, to distinguish it from mixed systems (discussed shortly), which permit discrimination between but not within designated groups of markets. (Note that complete discrimination is not "perfect discrimination," since demand in each market is downward sloping.) If prohibited from curbing parallel imports, the monopolist is constrained by the threat of arbitrage to set a uniform price.

This formulation embodies several simplifying assumptions. One is that arbitrage by end users is not feasible, otherwise markets could not be fully segmented even if parallel imports (which typically are brought in by resellers) were legally prohibited.[8] The assumption that, if legally permitted, parallel imports would mandate a uniform price across all markets embodies two ingredients. First, consumers view parallel imports as perfect

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[7] Most parallel imports involve differentiated products, characterized by high fixed costs (e.g., R&D, advertising, and marketing expenses). A perfectly competitive model therefore is inappropriate. To focus on price discrimination free of strategic complications, we opt to study monopoly rather than imperfect competition. In practice the ability to price discriminate requires only some market power, which is likely to exist for many of the branded products featured in the parallel import trade. Indeed, price discrimination apparently can be found even in fairly competitive industries, such as gasoline retailing (Borenstein, 1991; Shepard, 1991). Regarding our assumption of a final good, as noted earlier parallel imports indeed are concentrated in consumer goods.

[8] DeGraba (1991) and Malueg (1992) show that if demand in a market depends also on prices in other markets, then conventional comparisons between price discrimination and uniform pricing can break down. For example, even with "normal" demand functions, the uniform price need not be bounded by the discriminatory prices.

substitutes for authorized goods.[9] Second, the supply of parallel imports would be perfectly elastic (at some constant price differential between markets, which for simplicity we set at zero).[10] These assumptions abstract from some interesting issues; our goal, however, is to develop a tractable and transparent model focused on the price discrimination aspect.

It is well known that, in general, overall welfare can be higher under uniform-price monopoly or under discrimination. Demand dispersion, however, is likely to be greater internationally than between regions within a country, and we conjecture that high dispersion makes welfare higher under discrimination. This conjecture holds in familiar two-market examples with non increasing marginal cost (see, e.g., Tirole, 1988, or Hausman and MacKie-Mason, 1988): for large enough dispersion, a uniform-price monopolist would serve only the high-demand market, but under discrimination would add the second market and not raise price in the first. The two-market case, however, conceals a potential tradeoff that can arise with more than two markets. In those markets that would continue to be served under uniform pricing, increased dispersion could increase the loss from discrimination, because of the greater scope for misallocating output; this effect must be weighed against the greater number of markets served under discrimination. The conjecture that dispersion tends to favor discrimination from the standpoint of global welfare therefore remains to be verified in a model with more than two markets.

The remainder of the paper is organized as follows. Section 2 reviews the debate over the causes of parallel imports, concluding that discrimination has played a part. Section 3 presents a model with a continuum of markets and confirms the conjecture that for "high" dispersion, discrimination yields higher welfare—because of the powerful effect of serving more markets. Section 4 considers "mixed systems." Markets are placed into designated groups (e.g., by international agreement), with different prices allowed between but not within groups; such groupings may preserve the benefits of discrimination from serving more markets, but limit the misallocation effects. We present one such mixed system that yields higher welfare than "complete discrimination" and Pareto dominates uniform

[9] In practice, parallel imports can be somewhat differentiated from authorized goods (in packaging, warranty coverage, and other dimensions) and issues of consumer confusion and damage to product reputation have been raised in the debate (see Hearings on S. 626). Such issues, however, are conceptually distinct from the basic question of whether a supplier should be allowed to price discriminate.

[10] We believe that a more general formulation, in which the supply of parallel imports increased continuously with the price differential between markets, would not change our qualitative results. In a different context Hunter et al. (1991) find, through simulations in a general equilibrium model, that *quantitative* estimates of gains from liberalizing U.S.-Mexico auto trade are quite sensitive to the strength of arbitrage assumed. Closer to our context, Baker (1987) models parallel imports as a competitive fringe with rising marginal cost, but focuses on national rather than global welfare.

pricing. This system is the optimal mixed system with no "holes"—if two markets are in a group, so are all markets that have marginal valuations (at any quantity) lying between the two; but mixed systems that contain holes can yield even greater welfare. Section 5 concludes that some price discrimination is probably desirable for world welfare, and discusses countries' incentives to curb parallel imports to sustain discrimination given the distributional issues involved.

2. The Debate over Parallel Imports

Simplifying somewhat, opponents argue that parallel imports are profitable mainly because parallel traders free ride on investments of authorized distributors at various levels of the distribution chain, e.g., on national advertising by the authorized importer or on local advertising, display, or other services provided by wholesalers or retailers (Lexecon, 1985; COPIAT, 1986; DeMuth, 1990). Parallel importers can obtain goods at prices that do not reflect many of these costs, by purchasing abroad sufficiently high up the distribution chain (e.g., from wholesalers) for sale to a different market. They are then able to undercut the domestic authorized distributors, which do incur the expenses needed to cultivate and maintain local demand and reputation for the product. Opponents argue that such free riding disrupts a supplier's overall marketing plan, discourages various investments, and is generally inefficient.

Under the free-rider hypothesis, parallel trade does not rely on price differentials or other international asymmetries. Indeed, it can profitably occur in both directions, by the same logic as in the reciprocal "dumping" model of Brander and Krugman (1983).[11] In contrast, proponents argue that parallel imports are primarily an arbitrage response to international price discrimination that a supplier tries to sustain via exclusive distribution territories over different national markets. Proponents laud parallel imports as undermining such discrimination.

It is difficult to determine empirically whether the free-riding or discrimination hypothesis predominates, even in a single industry, as the hypotheses are not mutually exclusive. The difficulties stem from both the paucity of appropriate data and the subtlety of the testable implications. For instance, retail price differentials between countries could be due to differences in distribution costs and not price discrimination by a manufacturer; potentially more informative are the manufacturer's export prices to different markets, but these are not generally available.

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[11] Note, however, that the Brander-Krugman model implies price discrimination in favor of the foreign market—the reverse of what is alleged by supporters of parallel imports.

Observing parallel imports flowing only in one direction also does not prove discrimination; it could merely reflect differences in demand attributable to different levels of "free-ridable" distributor investments in the two countries.[12] These caveats notwithstanding, there is some suggestive evidence regarding the causes of parallel imports.

Generally, the type of goods in which parallel imports are concentrated—name-brand consumer goods, entailing heavy promotional investments—suggests free riding. But the timing of parallel imports points to an arbitrage explanation. Parallel imports have generally surged as a country's exchange rate appreciated, suggesting "incomplete pass-through"—that import prices in the destination currency were not reduced in the same proportion as the appreciation of that currency, thereby creating scope for arbitrage. An extensive literature documents that, indeed, incomplete pass-through is quite common [see Knetter (1989), Marston (1990) and Kasa (1992), and the references they cite]. As noted by several authors (Tarr, 1985; Dornbusch, 1987; Krugman, 1987; Feenstra, 1989), for plausible demand conditions in the foreign market, incomplete pass-through can be an equilibrium response by an imperfectly competitive firm or by a monopolist to the new demand conditions caused by the appreciation of the destination currency when the exporting firm's costs are sticky in its own currency.[13]

Neither piece of evidence is conclusive. The concentration of parallel imports in upscale consumer goods might alternatively arise because such goods are highly differentiated, and the associated market power encourages considerable price discrimination. Also, the co-movement of parallel imports with the importing country's exchange rate can alternatively be due to increased scope for free riding. If a manufacturer's export price to the U.S. reflects marketing costs incurred in dollars, a dollar appreciation can expand the gap between the manufacturer's prices to the U.S. and to other markets purely on cost grounds; this increased gap can further attract free-rider–based parallel imports (Tarr, 1985; DeMuth, 1990). Finally, even if incomplete pass-through is not cost-based, it need not imply price discrimination. It could instead reflect adjustment costs and various dynamic considerations, especially when exchange rate movements might be transitory or unexpected (Hilke, 1988; Froot and Klemperer, 1989; Kasa, 1992).

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[12] Such differential expenses can, in turn, reflect differences in costs of promotion across markets due to factor prices, or different levels of promotion reflecting different brand-positioning strategies.

[13] Consider a monopolist exporter with constant marginal cost c denominated in its own currency, facing domestic and foreign inverse demand functions, $P(q)$ and $eP^*(q^*)$, respectively, where P^* is expressed in foreign currency and e is the exchange rate (domestic currency per unit of foreign). Appreciation of the foreign exchange rate, an exogenous increase in e, increases the foreign inverse demand function when expressed in terms of the exporter's (i.e., home) currency. For many demand functions, including all concave ones, the optimal export price will increase in terms of the exporter's currency.

For the U.S. in the early 1980s, Hilke (1988) concludes that while no single explanation of parallel imports was completely adequate, incomplete pass-through (due to either conscious discrimination or adjustment lags) was more persuasive than was the free-rider hypothesis. Hilke's inference rests largely on the co-movement of parallel imports with the dollar, which we noted is also consistent with a free-rider explanation. However, for some products for which data were available, Tarr (1985) finds that the differentials in the manufacturers' prices to the U.S. versus their domestic markets exceed plausible estimates of the differential marketing costs.[14] He concludes that free riding was an important factor in some industries, such as perfumes, but that discrimination was a substantial factor in other industries, including German automobiles, Japanese cameras, ski equipment, and champagne.

More generally, there is widespread evidence that manufacturers engage in international price discrimination. Knetter (1989) interprets his findings of incomplete pass-through by German exporters from 1977 to 1985 in these terms. Marston (1990) detects incomplete pass-through by Japanese exporters and, significantly, finds that this behavior is not primarily explained by adjustment lags. Cross-sectional comparisons also find price discrimination internationally, e.g., in luxury automobiles (Ginsburg and Mertens, 1985), pharmaceuticals (Schut and Van Bergeijk, 1986), and books (Prices Surveillance Authority, 1989).

Our reading of the above findings is that price discrimination does play a role (though not necessarily the major role) in explaining parallel imports. The support for parallel imports on these grounds therefore merits closer scrutiny.

3. Uniform Pricing vs. Complete Discrimination

A. The Basic Model

We consider a monopolist with zero marginal cost. The monopolist faces a continuum of markets, the continuum having total mass of 1. Each market can be viewed as a country. The inverse demand function in market "a" is $p(q) = a(1 - q)$. Thus, demand functions are linear with the same horizontal intercept but different vertical intercepts (or choke prices), a; note that at a given price, higher demands have lower elasticities. This demand structure emerges, e.g., if demanders have identical utility functions separable in the monopolist's product

[14] For example, Tarr notes that despite a 40% appreciation of the dollar against the mark from 1980 to 1984, Mercedes-Benz's U.S. prices remained constant in dollars, implying a 40% increase in terms of marks. Assuming that Mercedes-Benz's expenses on U.S. marketing were 20% of its total costs (production accounting for 80%), and that in the U.S. these costs must be paid in dollars, the differential in marketing costs changed by only 8%.

and income but differ in their incomes, hence also in their marginal utilities of income.[15] The demand intercept a is uniformly distributed over $[1 - x, 1 + x]$, where the parameter $x \in [0, 1]$ measures demand dispersion for the continuum.

The assumption that demands are linear aids tractability. Its main, purpose, however, is to ensure a welfare tradeoff between discrimination and uniform pricing. Discrimination has the advantage of opening up new markets. We want to allow uniform pricing to be *potentially* superior. The simplest way of ensuring this is with linear demands. If a set of linear-demand markets is served under both pricing schemes (each market purchases positive output), then total output from those markets would be the same under the two regimes;[16] whenever total output is no higher under discrimination than under uniform pricing, welfare will be lower under discrimination (if demands are continuous and strictly decreasing)—because discrimination fails to equate marginal valuations.[17] The assumption of linear demands therefore is relatively favorable for uniform pricing: if under uniform pricing the monopolist drops no markets, uniform pricing yields higher welfare than does discrimination.

Consider first the equilibrium under complete discrimination. Given our zero cost and "rotating demands" assumptions, for all values of the dispersion parameter x the monopolist serves all markets, sets the same output in each market, and charges a price that increases with type: $q(a) = 1/2$, $p(a) = a(1 - q(a)) = a/2$. Total output thus remains unchanged as x varies. Because the demand intercept a is distributed uniformly over $[1-x, 1+x]$, the mean price is 1. The monopolist's profit is therefore $\Pi^d = 1/4$. For linear demands, consumer surplus in each market is one-half of profit. Thus, for all $x \in [0, 1]$, complete discrimination (hereinafter "discrimination" for brevity) yields profit, consumer surplus, and welfare of $\Pi^d = 1/4$, $S^d = 1/8$, and $W^d = 3/8$, respectively.

Under uniform pricing, the aggregate demand function facing the monopolist consists of two segments, depending on whether, at the given price, all markets are served. In Case 1, price falls in the range $0 \leq p \leq 1 - x$, and

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[15] This discussion is based on Tirole (1988, ch. 2). Let y denote consumption of the numeraire good and q consumption of the monopolist's good, and suppose the consumer has utility function $V(y, q) = u(y) + q - (1/2)q^2$. If a consumer has income I and the monopolist's price per unit of the good is p, then the inverse demand function, in implicit form, is $p(q) = (1 - q)/(u'(I - pq))$, which is 0 at $q = 1$ for all I, and is increasing in I, assuming u is concave. For pq "small," $u'(I - pq)$ approximately equals $u'(I)$, so the inverse demand function is approximately linear. This is simply the inverse demand function of the model, with the vertical intercept *a* corresponding to $1/u'(I)$; thus, higher values of *a* correspond to higher incomes. For tractability, we assume that *a* is uniformly distributed.

[16] See, e.g., Schmalensee (1981), and the discussion in Section 4 following Lemma 1.

[17] This welfare result holds for any marginal cost function (see Schwartz, 1990).

all markets are served.[18] Case 2 has price in the range $1-x < p \leq 1+x$; in this case, only types $a \in (p, 1+x]$ are served. Letting $b = \max\{p, 1-x\}$, aggregate demand is therefore

$$Q(p) = \frac{1}{2x}\int_b^{1+x}\left(1 - \frac{p}{a}\right)da$$

$$= \begin{cases} 1 - \dfrac{p}{2x}\log\left(\dfrac{1+x}{1-x}\right) & \text{if } 0 \leq p \leq 1-x \\ \dfrac{1+x}{2x} - \dfrac{p}{2x}\left(1 + \log\left(\dfrac{1+x}{p}\right)\right) & \text{if } 1-x < p \leq 1+x. \end{cases}$$

Figure 1 depicts a typical aggregate demand function. It is straightforward to verify that the demand function Q(p) is continuous and strictly decreasing everywhere. For prices below $1-x$, the function is linear, since all the (linear-demand) markets are served; for prices above $1-x$, it is strictly convex, because lowering price increases output at an increasing rate due to the newly served markets. Also, marginal revenue (MR) is continuous (despite the change in curvature of Q(p) at $p = 1-x$). At outputs to the right of that at which the "kink" occurs, $Q(1-x)$, marginal revenue is decreasing and linear, corresponding to the linear segment of Q(p). To the left of $Q(1-x)$, corresponding to the convex segment of Q(p), marginal revenue is either positive everywhere or crosses the horizontal axis only once (hence the profit function is single-peaked also over this range of prices).

Figure 2 illustrates how the dispersion parameter x affects aggregate demand. As x increases, the linear segment of Q(p) pivots counterclockwise around the point $q = 1$, $p = 0$, and the "kink" output, $Q(1-x)$, increases. We can visualize how the profit-maximizing output and price vary with x. There are two cases to consider, according to whether $x \leq x^*$ or $x > x^*$, with x^* denoting the value at which the "kink" output $Q(1-x)$ equals 1/2 (as shown later, $x^* \approx 0.5568$). For $x \leq x^*$, the monopolist's relevant marginal revenue segment is linear, and this linear segment equals the monopolist's cost of 0 at $q = 1/2$. (This is so because, for all x, the linear segment of Q(p) passes through (1, 0); hence, the associated marginal revenue curve equals 0 at $q = 1/2$.) Starting from $x = 0$, as x increases, the monopolist initially keeps total output at 1/2 and reduces price. That is, for low demand dispersion, the monopolist serves all markets. For $x \geq x^*$, the monopolist drops some markets. Geometrically, for high x the relevant segment of MR is that corresponding to the convex segment of Q(p) (recall that the kink output

[18] In the case where $p = 1-x$, the market with $a = 1-x$ purchases $q = 0$, but this market is itself of measure zero. In this case we continue to say that (almost) all markets are served.

$Q(1-x)$ shifts right as x increases). The monopolist's output is determined where this segment of MR equals zero. As x increases in this high range, the monopolist's output falls and price rises.

The relevant equilibrium expressions under uniform pricing are derived in Appendix 1 and reported in Table 1. In Table 1, y is the unique number greater than 1 satisfying $y = 1 + 2\log(y)$: $y \approx 3.5128$. The monopolist chooses to serve all markets for "low dispersion," $x \leq x^*$, and drop some for $x > x^*$, where $x^* \equiv (y-1)/(y+1) \approx 0.5568$.

B. Welfare Comparisons

We are interested in comparing the two pricing regimes for different values of x. Recall that output, profit, and consumer surplus—and therefore also welfare—all are constant under discrimination. (This is a special feature of zero cost and our "rotating demands," as discussed in subsection C below.)

Now turn to uniform pricing. Figure 3 shows that the uniform price decreases with x until x^* and increases thereafter. The price decreases initially because at $p < 1 - x$, increases in x raise the elasticity of aggregate demand. (Increasing x is a mean-preserving spread for the inverse but not for the direct demand functions, and the increase of elasticity in low-demand markets outweighs the decrease in high-demand markets.) For $x > x^*$, however, the monopolist sets $p > 1 - x$. In this range, increases in x lead to the dropping of high-elasticity (low a) markets, thereby decreasing the elasticity of aggregate demand and inducing the monopolist to raise price.

Figure 4 shows that total output, $Q(p)$, remains 1/2 for all $x \in [0, x^*]$ but decreases with x for $x \in (x^*, 1]$.[19] As x increases beyond x^*, output drops for two reasons: the pure effect of dispersion and the induced change in price. Holding price constant, an increase in dispersion would reduce output, because some markets are dropped (their demands become zero at this price). The fact that some markets are dropped in turn leads the monopolist to re-optimize and choose a higher price, further reducing output.

Figure 5 graphs consumer surplus, profit, and welfare under the two regimes. Clearly profit is always lower under uniform pricing, but the rankings of consumer surplus and welfare depend on x. Consider first the range over which the monopolist serves all markets, $x \in [0, x^*)$. As x increases in this range, output remains at 1/2. Had the

[19] Output remains at 1/2 initially because for $x < x^*$ the monopolist serves all markets and, hence, operates in the linear segment of aggregate demand. As x increases, the linear segment pivots counterclockwise about the horizontal intercept of 1, hence the corresponding marginal revenue curve always cuts the monopolist's zero marginal cost at the output 1/2. For $x > x^*$, the monopolist operates in the strictly convex portion of aggregate demand, and the output at which the corresponding marginal revenue equals zero decreases with x.

allocation of this output across markets also remained constant (as it does under discrimination), welfare would have remained constant, since the mean inverse demand curve remains unchanged. But under uniform pricing, increased x causes reshuffling of output from low-demand markets to high-demand ones, which equates marginal valuations and thus increases welfare. In contrast, profit falls because output is constant but price is cut. The welfare gain as x increases thus reflects increased consumer surplus; hence, consumer surplus is higher than under discrimination.

Consider next the range over which the monopolist drops some markets, $x \in (x^*, 1]$. As x increases beyond x^*, profit continues to fall (albeit more slowly than before), but now the reduction in output and increase in price cause consumer surplus also to fall (in contrast to the range $x \leq x^*$). Nevertheless, consumer surplus remains higher than under discrimination. Welfare is subject to two opposing effects as x increases: improved output allocation across markets (as in the range $x \leq x^*$), but a decrease in total output. The latter effect dominates for all $x > x^*$. Thus, once the dispersion in demands is large enough that the monopolist drops some markets under uniform pricing, any further increase in dispersion reduces welfare.

Figure 6 shows that the ratio of welfare under discrimination to that under uniform pricing, W^d/W^u, starts at 1 (for $x = 0$), falls to a minimum of about 0.965 (at $x^* \approx 0.56$), and rises to a maximum of about 1.048. Since both pricing regimes yield the same total output when all markets are served ($x < x^*$), greater demand dispersion initially creates an increasing disadvantage for discrimination, due to the greater misallocation of output. When some markets are dropped ($x^* < x \leq 1$), welfare under uniform pricing decreases with x due to the fall in output, but is constant under discrimination; eventually discrimination dominates.[20] Proposition 1 summarizes these findings:

Proposition 1. For zero demand dispersion, $W^d/W^u = 1$. For "small" demand dispersion ($x < x^*$), all markets are served under both discrimination and uniform pricing; in this range, W^d/W^u decreases monotonically with dispersion. For "large" dispersion ($x > x^*$), some markets are dropped under uniform pricing; in this range, W^d/W^u increases monotonically with dispersion and exceeds 1 when dispersion is sufficiently large.

Interestingly for policy, discrimination yields the largest welfare gain when demand dispersion is largest; it is also when demand dispersion is largest that the monopolist has the greatest incentive to engage in discrimination.

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[20] To see analytically that discrimination eventually dominates, observe that $W^u(1) = (y-1)/2y \approx 0.3577$ whereas $W^d = 0.375$. From Table 1, $W^u = W^d$ for $x = x_e$, which is the solution to $[(y-1)(1+x)^2]/[xy] = 3$. Therefore the two regimes yield equal welfare for dispersion level $x_e = \{(y+1) - [3y(4-y)]^{1/2}\}/2(y-1) \approx 0.646$.

Regarding magnitudes, in our model discrimination can yield anywhere from about a 4% loss in welfare to a 5% gain, relative to uniform pricing. These magnitudes are modest by the standards of the rent-seeking literature. Compared with standard estimates of losses from allocative inefficiency, however, such as Harberger's for monopoly or others' for trade barriers, these numbers are significant.[21] Moreover, the magnitudes can be significantly larger when the distribution of markets is not uniform but skewed (see subsection C below).

It is instructive to explore further why dropping markets eventually makes uniform pricing inferior to discrimination. As noted earlier, there are two effects. First, there is the direct welfare loss from dropping markets. Second, the monopolist increases price to those markets still served. To assess the effect of the latter, we computed $W^u(x)$ for $x \in [x^*, 1]$ holding price fixed at $1 - x^*$ (≈ 0.4432); $1 - x^*$ is the monopolist's price when dispersion is at the maximum level consistent with the monopolist choosing to serve all markets. Let $W^u(x \mid p = 1 - x^*)$ denote this hypothetical welfare level. We find that $W^d/W^u(x \mid p = 1 - x^*) < 1$ for all $x \in [x^*, 1]$:

Remark 1. At demand dispersion x^* the monopolist would charge a uniform price $p = 1 - x^*$ and just serve all markets. Holding the uniform price fixed at $1 - x^*$, for all $x > x^*$ welfare at this price would be higher than under discrimination, despite the dropping of markets under uniform pricing.[22]

Remark 1 and Proposition 1 show that increased dispersion eventually makes uniform pricing inferior to discrimination not because of the dropping of markets itself, but because dropping markets leads the monopolist to re optimize with respect to the markets still served.

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[21] Note also that the empirical literature often reports welfare gains as a percentage of GNP, which excludes consumer surplus; doing so in our model, by computing welfare gain as a percentage only of revenue—which in the model coincides with profit—would increase the above maximum percentage gain from discrimination substantially, from about 4.8% to about 8% (since for $x = 1$, profit and consumer surplus under uniform pricing are respectively about 0.204 and 0.155, while welfare under discrimination is 0.375). Moreover, as shown by Malueg (forthcoming), for different demand functions than those assumed here, the welfare difference between discrimination and uniform pricing can be much larger, although we have not investigated the effects of dispersion for those different demand functions.

[22] For $x \in [x^*, 1]$ the ratio $W^d/W^u(x \mid p = 1 - x^*)$ lies in the interval [0.934, 0.970] and is not monotonic in x; it first rises, then falls. In contrast, for this range of x the actual solution under uniform pricing yields W^d/W^u monotonically increasing in x, rising from 0.965 (at x^*) to 1.048 (at $x = 1$).

C. Robustness

Here we discuss the robustness of the welfare patterns described in Proposition 1. We consider three variations of the basic model: positive marginal cost, a family of "parallel" rather than "rotating" demands, and rotating demands with a skewed rather than uniform distribution of markets.

Positive Cost. It is difficult to solve explicitly for the uniform price as a function of constant marginal cost, c, and dispersion when only some markets are served. However, for maximum dispersion in this model, $x = 1$, we find that $W^d/W^u > 1$ for all values of cost c that we checked between 0 and 1. Thus, W^d/W^u follows the pattern of Proposition 1 also for any positive cost $c \in [0, 1)$.[23]

Parallel Demands. We also consider an alternative family of inverse demands, $p = a - q$, where the intercept a is uniformly distributed over $[1-x, 1+x]$ and $x \in [0, 1]$ (derivations of this case are available on request). Thus, demands would be parallel shifts of each other rather than rotations as earlier.[24] Under discrimination, output still is constant (at 1/2) for all x, but this time profit, consumer surplus, and welfare all increase with x: $\Pi^d = (3+x^2)/12$, $S^d = \Pi^d/2$, $W^d = 3\Pi^d/2$. The reason is that, in contrast to the rotating-demands case, higher-demand markets always receive relatively greater outputs ($q(a) = a/2$), and a symmetric increase in dispersion induces such efficient reallocation. Consumer surplus and profit therefore both increase with dispersion.

Under uniform pricing, all markets are served if $x \in [0,1/2]$. In this range, price and output remain constant at 1/2 and the other relevant values are: $\Pi^u = 1/4$, $S^u = (3+4x^2)/24$, $W^u = (9+4x^2)/24$. Thus, profit is constant, but consumer surplus and therefore welfare increase with x. The reason again is the increased allocation of output to the higher-demand markets. If $x \in (1/2, 1]$, $p = (1+x)/3$. Output decreases with x over this range in which markets are dropped. (Again there are two reasons: dropping markets at the initial price and concomitantly raising price to

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[23] We know that $W^d/W^u = 1$ at $x = 0$ and decreases initially (since all markets are then served for small x, hence output is the same under the two regimes but discrimination misallocates it). After some intermediate level of x, output becomes lower under uniform pricing, as markets are dropped, and W^d/W^u increases, since we verified that it eventually exceeds 1 (for the case of maximum dispersion, $x = 1$). Note that, unlike the zero-cost case, total output under both regimes now decreases with x over the entire range [0,1], but this difference does not affect the relative welfare patterns (though W^d/W^u, while it remains above 1, does decrease with cost at the maximum dispersion level of $x = 1$).

[24] Such parallel demands can emerge, for example, by assuming the same conditions that gave rise to our rotating demands (see subsection A above) but adding the assumption that the markets with higher per capita incomes also have larger populations of consumers. We focus on the rotating demands in the paper because it is higher per capita income and not different populations of identical consumers that generates the difference in demand elasticities.

the remaining markets.) In contrast to the rotating-demand case, the beneficial reshuffling of output toward the higher demands is now strong enough to outweigh the adverse drop in output, thereby causing profit, consumer surplus and thus welfare to continue increasing in x: $\Pi^u = (1+x)^3/27x$, $S^u = \Pi^u/2$, $W^u = 3\Pi^u/2$.

Overall, therefore, dispersion affects the variables of interest quite differently than with the "rotating demands." Interestingly, however, the ratio of welfare under discrimination to that under uniform pricing, W^d/W^u, behaves in the same way as described in Proposition 1.[25]

Skewed Distributions of Demands. A more important extension is to allow for the distribution of demands to be skewed rather than uniform, since actual distributions of per capita income are skewed towards low incomes. We address skewness by considering the basic model of zero cost and rotating demands. As before, inverse demand in market *a* is given by $p(q) = a(1-q)$. But now we assume that the intercepts, a, are distributed according to the density function $f(a|t,\underline{a}) = k(t,\underline{a})\,(a-\underline{a})^t$, where $a \in (\underline{a}, 1)$, $t > -1$, $1 > \underline{a} \geq 0$, and where, given t and $\underline{a}$, $k(t,\underline{a})$ is a parameter that makes the density integrate to 1.[26] Figure 7 shows, for the case of $\underline{a} = 0$, how skewness of this distribution depends on t. The uniform distribution case discussed above corresponds to $t = 0$. For $t > 0$ the distribution of intercepts is skewed toward the right (higher intercepts), and for $t < 0$ the distribution is skewed toward the left. As t approaches -1 (from above) the distribution of intercepts becomes more concentrated at the lower end (near $\underline{a}$).

For $p \leq 1$, the monopolist's aggregate demand function is given by

$$Q(p) = k(t,\underline{a})\int_b^1 (a-\underline{a})^t(1-(p/a))\,da,$$

where $b = \max\{\underline{a}, p\}$. Therefore, the monopolist's profit if it charges a uniform price p is simply $\Pi(p) = pQ(p)$. Using numerical methods, we found the price, p^u, that maximizes the monopolist's profit under uniform pricing. Thus, consumer surplus under uniform pricing is

$$S(p^u) = k(t,\underline{a})\int_b^1 (a-\underline{a})^t\,\frac{(a-p^u)^2}{2a}\,da,$$

[25] The range of W^d/W^u is now [0.975, 1.0125], less than for rotating demands.

[26] This description of demands represents a normalization of the basic model. If the vertical axis in the basic model is rescaled by dividing by 1+x, then the top demand intercept is 1, and the lowest of the demand intercepts is (1−x)/(1+x). Therefore, the relationship $\underline{a} = (1-x)/(1+x)$ links our current presentation to the dispersion parameter x introduced earlier.

where $b = \max\{\underline{a}, p^u\}$. Total welfare is then given by $W^u = \Pi(p^u) + S(p^u)$. Observe that each of these surplus functions depends on the level of skewness, t, and the amount of "gap" at the bottom, $\underline{a}$.

Under complete discrimination, the monopolist treats each market individually, setting price a/2 in market a. Thus, the average price under discrimination, $\overline{p}$, is

$$\overline{p} \equiv k(t,\underline{a})\int_{\underline{a}}^{1}(a/2)(a-\underline{a})^t\,da.$$

Consequently, under discrimination, total output equals 1/2, monopoly profit equals $\overline{p}/2$, consumer surplus equals $\overline{p}/4$, and total welfare equals $W^d = 3\overline{p}/4$. As with $\overline{p}$, these surplus functions depend on t and $\underline{a}$.

Table 2 presents results found using numerical methods to solve this model. The columns refer to different values of skewness, where moving right (towards $t = -1$) means skewing the distribution more towards low demands. The rows consider four levels of demand dispersion: $\underline{a}$ = 0, 0.05, 0.1, and 0.2. In each case, Table 2 reports three magnitudes for each level of skewness: relative welfare, W^d/W^u; the monopolist's uniform price, p^u; and the proportion of all markets served at this price, propserved. For example, if t = 0 (uniform distribution), then for $\underline{a}$ = 0 (maximum dispersion) we have relative welfare of 1.04846, a uniform price of .284668, and only 0.715332 of all markets served under uniform pricing.

Consider first the case of $\underline{a}$ = 0 (maximum dispersion). As skewness increases toward low demands (t falls toward −1), the monopolist reduces its uniform price. However, the price reduction is "not fast enough": increased skewness over this range leads to a drop in the proportion of markets served under uniform pricing. Correspondingly, W^d/W^u increases with skewness (recall that all markets are served under discrimination), until discrimination can offer a welfare gain of over 17.6% relative to uniform pricing. In contrast, for the same level of dispersion, the gain with a uniform distribution of markets (t = 0) is only about 4.8%.

Now consider $\underline{a}$ = 0.05. For modest levels of skewness (t > −0.775), the above pattern recurs, with W^d/W^u eventually exceeding 1.2. As t nears −1, however, the distribution becomes concentrated around $\underline{a}$, and the monopolist's profit function becomes double-peaked, achieving one local maximum at a price above $\underline{a}$ and the second at a price below $\underline{a}$. At some level of skewness t* (for $\underline{a}$ = 0.05, t* is approximately −0.775), these two local maxima yield equal profit. As t falls below t*, the monopolist discontinuously drops its uniform price below $\underline{a}$ and serves all markets. At that point, welfare under discrimination is less than under uniform pricing, and further increases in skewness magnify this difference, with W^d/W^u eventually falling below 0.78.

The cases $\underline{a}$ = 0.05, 0.1 and 0.2 suggest that the switch level of skewness, t^*, decreases with demand dispersion. For instance, when $\underline{a}$ = 0.2 the monopolist drops its uniform price and serves all markets once $t = -.25$. For sufficiently low levels of dispersion (not shown), all markets would be served under uniform pricing for all values of skewness t; hence uniform pricing would then yields higher welfare than discrimination for all skewness.

Summarizing, skewness increases the potential difference in welfare under price discrimination and uniform pricing, but need not erode the advantage of discrimination. For low demand dispersion, skewness favors uniform pricing. For high dispersion, however, moderate levels of skewness favor price discrimination—since in such cases uniform pricing results in more markets going unserved than with a uniform distribution of markets.

4. Mixed Systems

So far we have considered two polar regimes: uniform pricing, whereby all markets are charged the same price, and complete discrimination, whereby each market receives its own price. For policy purposes, one can envisage intermediate regimes in which the monopolist is permitted to engage in some but not complete multimarket discrimination. The idea behind such regimes would be to allow enough discrimination to ensure that all markets are served but only that much discrimination, so as to limit the harmful misallocation of output.

Specifically, we consider *mixed systems*: markets are placed into groups, with all markets in a group required to receive the same price but prices are allowed to differ across groups. Mixed systems might be attainable through multilateral agreements that allow parallel imports within but not between groups of countries (see also Section 5). Arbitrage within groups would then push toward uniform pricing without the need for cumbersome hands-on price regulation. (Section 5 discusses some practical problems with implementing such systems.) We analyze mixed systems for the basic model of Section 3, though some of the results generalize.

Definition. A *countable partition* of $[1-x, 1+x]$ is a countable collection $\wp = \{P_i\}_i$ of (Lebesgue measurable) sets such that (i) $\bigcup_i P_i = [1-x, 1+x]$ and (ii) for any i, j with $i \neq j$, $P_i \cap P_j = \emptyset$.

Definition. A *mixed system* is a countable partition of $[1-x, 1+x]$, $\wp = \{P_i\}_i$, together with the requirement that, for each i, all markets in P_i are charged the same price. In mixed system $\wp$, group P_i is said to be *fully served* if $\mu\{a \in P_i \mid q(p_i \mid a) > 0\} = \mu(P_i)$, where p_i denotes the monopolist's profit-maximizing price charged to group P_i, $q(p_i \mid a)$ denotes the demand of market *a* at price p_i, and μ denotes Lebesgue measure.

Alternatively, group P_i is fully served at price p_i if almost all group members have choke prices above p_i.

Lemma 1 provides intuition for several ensuing results and extensions. This lemma essentially covers all collections of linear demand functions (including the "parallel" demands and "rotating" demands with non-uniform distributions of markets discussed earlier).

Lemma 1. Suppose the monopolist has constant marginal cost $c \geq 0$. *Consider the family of demand functions* $Q(p/a) = \alpha(a) - \beta(a)p$, *where* $a \in [0, 1]$, *and* α *and* β *are (measurable) functions from* $[0, 1]$ *to* R_+. *Suppose index a is distributed according to the cumulative distribution function F. Let* $\wp = \{P_i\}_i$ *be a mixed system for this family of demands. If group* P_i *is fully served under* $\wp$, *then the total output supplied to group* P_i *under mixed system* $\wp$ *equals the sum to the outputs that group* P_i*'s member markets would have received under complete discrimination.*

Appendix 2 contains the proofs of Lemma 1 and all subsequent results. Lemma 1 can be understood as follows. For uniform prices at which all markets in group i are served, group i's aggregate demand is simply the horizontal sum of the demands of all group i's markets. Because this aggregate demand segment is linear, so is its corresponding marginal revenue curve, $MR_i(Q_i)$. Therefore, in a mixed system that fully serves group i, total output to group i is determined where $MR_i(Q_i)$ cuts the monopolist's marginal cost (here zero). Under discrimination, total output to these same markets is determined where the horizontal sum of all individual marginal revenue curves cuts marginal cost. But for linear demands, when all markets are served, the horizontal sum of marginal revenue curves coincides with $MR_i(Q_i)$; total output therefore is the same if the mixed system fully serves group i.[27]

A. No Holes

We say that a mixed system has *no holes* if each member of the associated partition is an interval. We say that a mixed system contains *holes* if at least one member of the associated partition is not an interval. For example, some relatively high-demand markets might be grouped together with relatively low demands, with the medium-demand markets placed in a different group.

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[27] The above logic shows that Lemma 1 would hold for any marginal cost function if the mixed system consisted of only one group. This is the well-known result mentioned in Section 2: that if uniform pricing fully serves a set of linear-demand markets, then total output is the same as under discrimination. In a mixed system with multiple groups, however, our added assumption of constant marginal cost is needed to allow the monopolist to optimize separately for each group.

Consider the following mixed system with no holes, involving "recursive" divisions into groups:

(4.1) $$I_0 = \{0\}, \quad I_1 = \left[\frac{t}{y}, t\right], \quad I_n = \left[\frac{t}{y^n}, \frac{t}{y^{n-1}}\right), \quad \text{where } t = 1 + x \text{ and } n = 2, 3, \ldots.$$

Recall from Table 1 that for dispersion levels that induce the monopolist to drop some markets ($x > x^*$), the monopolist's profit-maximizing uniform price is t/y. Group I_1 thus consists of those markets that the monopolist would have served if constrained to charge a uniform price; faced with system (4.1), the monopolist clearly would charge this group t/y. For the other markets there are two possibilities, depending on the value of x: either the monopolist would set a second price that serves all these remaining markets, or it would set t/y^2 (t/y^2 because once types I_1 are excluded the new "top" demand has vertical intercept t/y, and, by a simple rescaling argument, the profit-maximizing price to any *uniform* distribution of markets—if some markets would still be dropped at the lower price—is $1/y$ times the top intercept). In the latter case, the remaining markets not served at price t/y^2 would ultimately be served at the price(s) chosen for the lower group(s). The system (4.1) thus ensures that all markets are served. (Some intervals would contain no consumers, except in the case of full spread, $x = 1$.)

Proposition 2. (a) If demand dispersion is small enough that the monopolist would serve all markets under uniform pricing ($x \leq x^*$), then uniform pricing and the mixed system (4.1) are identical and yield higher welfare than discrimination. (b) If demand dispersion is large enough that the monopolist would drop some markets under uniform pricing ($x > x^*$), then the mixed system (4.1) yields higher welfare than discrimination and is a Pareto improvement over uniform pricing.

Part (a) of Proposition 2 is straightforward. The mixed system (4.1) boils down to uniform pricing for dispersion in the range $x \leq x^*$, because all markets then fall in the top group. Uniform pricing then dominates discrimination because with all markets served under uniform pricing, total output is the same under the two regimes (Lemma 1). And discrimination misallocates this output, whereas uniform pricing allocates it optimally.

Turn to part (b). Clearly the mixed system is a Pareto improvement over uniform pricing: those markets that would be served under uniform pricing are still served and at the same price, but the mixed system serves additional markets. Now compare the mixed system with discrimination. Consider the markets in any group i (that has strictly positive measure). Because all markets are served under the mixed system, total output to the markets in group i is the same as they would collectively receive under discrimination (Lemma 1). Because all markets in group i are charged a uniform price under the mixed system, misallocation of output among these markets is avoided; thus, the welfare to group i is strictly higher than under discrimination (Schwartz, 1990).

Observe that, if marginal cost is constant, then for any distribution (not just uniform) of markets with linear demands, one can recursively construct a mixed system similar to (4.1) that will satisfy Proposition 2. (That welfare would be higher than under complete discrimination follows by application of Lemma 1.) Moreover, for any distribution of (possibly nonlinear) demands, as long as uniform pricing would not serve all markets, there exists a mixed system that Pareto dominates uniform pricing; e.g., form two groups, one consisting of those markets served under uniform pricing and the other consisting of all remaining markets. This observation is interesting for policy purposes because, as noted earlier, prevailing sentiment worldwide favors uniform pricing.

For our basic model with a uniform distribution of "rotating-demand" markets, we can show the following.

Proposition 3. The recursive system (4.1) maximizes welfare in the class of mixed systems with no holes.

The intuition behind Proposition 3 is roughly that discrimination can increase welfare only if it leads the monopolist to increase total output. The proof shows that in designing groups, the top group should then be made as large as possible consistent with it being fully served. The lower boundary of the top group is therefore t/y. The same argument is repeated for the remaining markets (recall that the distribution of markets is uniform).

Example. To indicate the potential welfare gain from mixed systems, consider the case of maximum dispersion, $x = 1$ (the distribution of demand intercepts is uniform over $[0, 2]$). For this case, the recursive system (4.1) would require an infinite number of groups for the monopolist to choose to serve all markets. Let $W(n)$ denote welfare under this recursive system for groups I_1 through I_n and observe that $W(1)$ is just welfare under uniform pricing (the monopolist would charge $p = 2/y$ and drop all lower markets). It can be shown, for all $n \geq 1$, that

$$W(n) = \frac{y}{2(y+1)}\left(1 - \frac{1}{y^{2n}}\right) \quad \text{and} \quad \frac{W(n)}{W(1)} = \left(\frac{y^2}{y^2 - 1}\right)\left(1 - \frac{1}{y^{2n}}\right).$$

Because $y \approx 3.5128$, $W(\infty)/W(1) \approx 1.0882$, that is, the recursive system (4.1) increases welfare by approximately 8.82% relative to uniform pricing. (The gain relative to discrimination is about 3.84%.) This gain is significant.

Even marginal-cost pricing would yield welfare of only 0.5, or a gain relative to uniform pricing of under 40%; and marginal-cost pricing is infeasible for many of the products experiencing parallel imports, given that these products often entail substantial fixed costs. Interestingly, the bulk of the gain from the mixed system is achieved with a small number of groups; with $n = 2$ the gain is already 8.10%, and with $n = 3$ it is 8.76%.

B. Holes

Proposition 3 considers only mixed systems without holes. Surprisingly, perhaps, welfare can be further increased by mixed systems that contain holes.

Proposition 4. If demand dispersion is large enough that the monopolist would drop markets under uniform pricing ($x > x^*$), then the recursive system (4.1) is not optimal among the class of mixed systems that allow for holes.

Starting with the mixed system (4.1), the optimal mixed system with no holes, we show that moving a small set of markets into one group from the group above it will increase welfare, due to the improved allocation of total output. Referring to Figure 8, we define the following sets of markets:

$$s_1 = (t-\varepsilon, t], \quad s_2 = \left[\frac{t}{y}, t-\varepsilon\right], \quad s_3 = \left[z, \frac{t}{y}\right),$$

where $\varepsilon > 0$ is "small," $t = 1 + x$, and $z = \max\{1 - x, t/y^2\}$. Consider the partition of $[z, 1+x]$ without holes, $\{A, B\}$, and the partition with holes, $\{A', B'\}$, where

$$A = s_1 \cup s_2, \quad B = s_3, \quad \text{and} \quad A' = s_2, \quad B' = s_1 \cup s_3 .$$

The sets A and B correspond to the top two groups of markets in system (4.1). There are two cases to consider, depending on the level of demand dispersion x. If $1 - x > t/y^2$, the monopolist charges group A the price $p_A = t/y$ and group B a price $p_B \in (t/y^2, 1 - x)$. Group B is fully served with *slack*—even if the price to that group rose slightly, group B would still be fully served. If $1 - x \le t/y^2$, the monopolist charges the prices $p_A = t/y$ and $p_B = t/y^2$; group B is fully served, but with *no slack*.

In Appendix 2 we show that

$$(4.2) \qquad \left.\frac{d(W_A + W_B)}{de}\right|_{e=0} > 0 \text{ for both the case } 1-x > \frac{t}{y^2} \text{ (slack) and the case } 1-x \le \frac{t}{y^2} \text{ (no slack),}$$

where W_i denotes the welfare—consumer surplus plus profit—attained from serving group i (i = A, B) at the profit-maximizing price chosen by the monopolist for that group. (The derivative in (4.2) is the right-hand derivative found as ε approaches zero through positive numbers.) The cases require different proofs: with slack, shifting a small set of the top markets $(t - \varepsilon, t]$ into group B would result in group B's continuing to be fully served (even though the monopolist would raise price after the addition of these high markets); with no slack, such a shift would result in some of the bottom markets being dropped (because of the induced price increase).

Here we present the intuition for why holes can increase welfare in the case of slack. Suppose the monopolist is faced instead with the partition with holes, {A', B'}; this simply involves moving the top markets s_1. The monopolist's prices would satisfy the following inequalities:

$$p_B < p_{B'} < p_{A'} < p_A ,$$

where the first and third inequalities hold because the highest demand set s_1 is included in B' and A but not in B and A', and the second because this transferred set s_1 is sufficiently "small." The key to understanding the change in welfare is to determine what happens to total output and to its allocation among various markets.

Consider first the allocation of output. Price to s_1 falls; hence, its consumption increases: $p_{B'} < p_A \Rightarrow Q_1' > Q_1$. Similarly for s_2: $p_{A'} < p_A \Rightarrow Q_2' > Q_2$. For s_3, price rises; hence, consumption falls: $p_{B'} > p_B \Rightarrow Q_3' < Q_3$. Thus, output is reallocated from markets in s_3 to markets in s_1 and s_2. Denote by MV_1, MV_2, and MV_3 the initial marginal valuations in markets 1, 2, and 3, respectively, and by MV_1', MV_2', and MV_3' the equilibrium marginal valuations after the move of s_1. The marginal valuations in markets s_1 and s_3 differed initially but are now equated: $MV_3 = p_B < p_A = MV_1$ but $MV_3' = p_{B'} = MV_1'$. The marginal valuation in s_2 still exceeds that in s_3 but the gap has narrowed: $MV_2' - MV_3' = p_{A'} - p_{B'} < p_A - p_B = MV_2 - MV_3$. Therefore, the allocation of total output following the move of s_1 is more efficient.

Now consider the level of total output. All markets are served before *and* after the move of s_1. This follows because originally all markets are served with slack ($p_B < 1-x$), so all markets also will be served after the move ($p_{B'} < 1-x$) provided the transferred set s_1 is sufficiently "small." By Lemma 1, we therefore know that total output remains unchanged. Given the improved allocation, the move of s_1 (introducing a "hole") must increase welfare.

Note that if initially group B were served with *no slack* ($1 - x \leq t/y^2$ and $p_B = t/y^2$), then total output would fall after the move of s_1, because $p_{B'} > p_B = t/y^2$. The proof that such a small move nevertheless would increase welfare therefore relies on the gain from reallocation of output outweighing the loss from the output reduction. Observe, however, that for any x, $x^* < x < 1$, only a finite number of the groups in (4.1) contain a positive measure of markets, and generically the bottom group will be served with slack. Shifting a small group of consumers from the second-from-the-bottom group to the bottom group will then increase welfare, by the same argument as in the example above involving the top groups I_1 and I_2. Moreover, that reasoning holds for any family of linear demands (not just "rotating") and for any mixed system with no holes that fully serves with slack the lower of the two adjacent groups (because the reasoning relies only on Lemma 1, which holds for all linear demands).

While we have not characterized an optimal mixed system, Proposition 4 shows that the welfare gains from such systems relative to uniform pricing exceed the significant gains, illustrated in the example, that are achieved by the no-holes system (4.1).

5. Conclusion

This paper examined whether the strong policy support that parallel imports enjoy internationally is justified from the standpoint of global welfare, even assuming that parallel imports are caused entirely by international price discrimination, not free riding. An advantage of allowing at least some international price discrimination is that additional countries would likely be served. If parallel imports are prevented, at least between certain groups of countries, firms could offer lower prices to lower-demand (more elastic) countries without fear of the products resurfacing in high-price markets. Absent such (partial) segmentation, firms may well choose relatively high uniform prices, at which many low-demand countries are likely to go unserved.[28]

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[28] A famous example of such behavior is the decision by Distillers Company Limited to drop its premium Red Label brand from the U.K. market in response to 1978 decisions by the European Commission and Court of Justice that Distillers could not discourage parallel exports from the lower-priced U.K. market to the Continent (Hawk, 1991). Also in response to the EC's prohibitions of curbs on parallel trade, manufacturers are reportedly curtailing supplies of pharmaceuticals to low-price countries (REMIT, 1992). Harder to measure, but potentially more important, are decisions by manufacturers not to enter certain low-price markets in the first place, for fear that parallel exports from these markets would undercut high prices in other countries.

In a stylized model we compared world welfare under uniform pricing to that under complete discrimination (a different price in each country) and under mixed systems, which allow discrimination between groups of countries but not within a group. Whether mixed systems are feasible will depend on the possibility of tracing the original source of the goods. This problem of re-exportation, or "trade deflection" (Corden, 1984), is well known in the literature on free-trade areas. Its severity will vary across countries and industries. Where mixed systems are not feasible, complete discrimination may still be possible—because it requires curbing unauthorized imports regardless of origin. Our welfare comparison between complete discrimination and uniform pricing is relevant for such cases.

Comparing complete discrimination with uniform pricing, we found that when demand dispersion is large enough, welfare is higher under discrimination. The beneficial effect of higher output under discrimination from continuing to serve low-demand markets outweighs the misallocation effect of discrimination.

The welfare gains relative to uniform pricing can be higher still if suppliers are granted some but not complete discretion to price discriminate. We constructed a mixed system, (4.1), that yields higher welfare than complete discrimination and is a Pareto improvement over uniform pricing (whereas complete discrimination can yield higher welfare than uniform pricing but harms the high-demand markets). Interestingly, a small number of groups suffices to achieve the bulk of the welfare gain. Thus, one could imagine grouping countries into a few blocks based on per capita income—e.g., low, middle, and high—and allowing discrimination only between blocks. System (4.1) was shown to yield highest welfare in the class of systems with no holes. Surprisingly, perhaps, introducing holes—by grouping together some markets with low and high demands while grouping separately some intermediate ones—can increase welfare still further.[29]

In evaluating price discrimination it is important to recognize, however, that discrimination not only affects global welfare but also raises distributional questions, both within countries (producers vs. consumers) and between countries. Since any system that involves some international discrimination will invite arbitrage, it is important to

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[29] As a practical matter, there may be political difficulties in implementing systems with "holes," due to the distributional issues raised. Such systems group together high-demand countries with low-demand ones, and the welfare gain arises from reshuffling output from the latter to the former—for instance, putting the U.S. in a group with the developing countries in order to "pull down" the price that the monopolist would charge the U.S. In the absence of adequate compensation mechanisms (discussed further shortly), such groupings may be politically awkward; grouping together countries with similar demands, as in our recursive system, may be more palatable. On the other hand, the finding that holes can increase welfare suggests that if holes are introduced inadvertently, because a no-holes system such as (4.1) is not feasible, say due to geographical considerations, the welfare advantage of limited discrimination over uniform pricing need not be eroded.

examine more closely countries' incentives to uphold such a system by curbing parallel imports. Moving from uniform pricing to complete discrimination, even if it increases global welfare, is likely to harm consumers in high-price countries (it need not if the supplier has decreasing marginal cost, cf. Hausman and MacKie-Mason, 1988). But manufacturers of products prone to parallel imports also are predominantly from richer (more industrialized) countries, and those manufacturers would gain from discrimination. Thus, allowing complete international price discrimination need not systematically reduce the national welfare of industrialized countries.

Nevertheless, the above "cancellation" tendency is unlikely to leave all rich countries better off under complete discrimination. For instance, some high-price countries may be relatively under-represented in the manufacturing of goods involved in parallel trade and therefore may capture little of the increased profit from discrimination. Potentially more promising are mixed systems such as (4.1) that, by permitting lower prices only to markets that otherwise would not be served, mitigate the distributional impact of discrimination. System (4.1) in fact is a Pareto improvement over uniform pricing in our model. In practice, of course, things will be messier. For example, the appropriate sorting of countries into groups will vary by industry (as national demands vary across products), but a country's policies and laws toward parallel imports may have to be relatively uniform across industries. Given such inflexibility, some rich countries might still lose from a regime that permits limited discrimination between groups. If so, such countries could perhaps be compensated by poorer countries that stand to gain from discrimination in their favor. Such compensation need not be direct, but could instead take the form of offsetting concessions in multilateral negotiations over a range of trade issues; e.g., in exchange for rich countries curbing parallel imports, poor countries might offer stronger protection for intellectual property rights.[30]

The basic idea that some international price discrimination might be beneficial has implications for policy toward parallel imports both within the EC and between less developed countries (LDCs) and industrialized ones (North-South trade). Within the EC demand conditions still vary considerably, because of differences in incomes and in national policies such as price controls and the strength of IPRs (hence the availability of counterfeit substitutes). Insisting as the EC does on unencumbered parallel imports that arbitrage national price differences may well lead manufacturers to sharply curtail sales to certain countries. This prospect suggests that *some* price discrimination

[30] Indeed, a recent U.S. proposal (May 1990) to the talks on Trade Related Aspects of Intellectual Property Rights (TRIPs) of the Uruguay Round of the GATT addressed, inter alia, both traditional issues concerning the strength of IPRs (forced licensing, penalties for infringement, etc.) and the rights of holders of copyrights and patents to prevent parallel imports.

should be permitted. As a rough cut, low-income countries such as Greece, Ireland, and Portugal might be grouped in a block separate from the richer countries, with parallel imports allowed within but not between blocks. The EC's unwavering support for parallel imports is therefore questionable.[31]

Potentially larger gains from international price discrimination could arise by permitting suppliers to offer lower prices to LDCs, whose demand elasticities are likely to be much higher than in industrialized countries due to vastly lower per capita incomes. There are signs that suppliers indeed would price favorably to LDCs. Schut and Van Bergeijk (1986) found that pharmaceuticals prices internationally varied widely and were strongly positively correlated with per capita incomes. Their finding is corroborated by anecdotal evidence (see Drug Diversion, 1985) that some U.S. pharmaceutical companies sold their products for as little as one-quarter the domestic price when destined for export to LDCs (although the "exports" were frequently diverted back to the U.S. market). Discounts of the same order are offered by some economics journals (e.g., Econometrica) to subscribers in LDCs.[32]

Offering lower prices to LDCs, in addition to its inherent advantage of "opening markets," may also soften LDCs' reluctance to grant stronger protection of IPRs. Many LDCs fear that most of their consumers could not afford the high prices that would be charged for products embodying intellectual property if they did grant stronger

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[31] Simulation studies (e.g., Smith and Venables, 1988; Mercenier, 1992) often find that *all* EC member countries would benefit from moving to uniform pricing (by completing the market integration). These authors assume imperfectly competitive firms rather than monopoly, and attribute their results to a decrease in the average degree of market power when moving to uniform pricing. The rationale given by Smith and Venables (1988, p. 1522) is that a firm's market share is typically larger at home than in foreign markets (say due to preferences for home products); integration (uniform pricing) reduces market power by reducing concentration, since "the relevant measure of concentration is for the EC as a whole."

This argument, however, is incomplete. The aggregate EC shares mask a persisting preference for home products and therefore need not accurately proxy the change in market power. Consider a symmetric model with two countries, each with a single firm. Given preference for home products, with segmented markets each firm has a larger share of its home market and charges a higher price (at least under Smith and Venables' CES preferences). We conjecture that requiring a uniform price would lead each firm to lower its domestic price but raise its foreign price; moreover, the uniform price may well exceed the average discriminatory price, because the domestic market is relatively more important. Thus, requiring uniform pricing in oligopoly need not create a systematic tendency toward lower prices. Given Holmes' (1989) results that the comparison is generally ambiguous, we believe that the simulation findings above result from special assumptions, e.g., about the particular nature of product differentiation. For more on this point, see Haaland and Wooton (1992).

[32] To be sure, poor countries sometimes pay higher prices than rich ones. Such "reversals" are partly attributable, however, to various government policies in LDCs that reduce competition in their markets, e.g., by discouraging foreign entry. Pricing reversals may also arise from choices by suppliers to target their products to the rich rather than the mass market in LDCs; but such choices may themselves be dictated by the inability to offer lower prices selectively to LDCs if parallel exports from LDCs cannot be controlled. Thus, pricing reversals do not refute the argument that price discrimination, if feasible, likely would favor poorer countries.

IPRs (Diwan and Rodrik, 1991). Preventing parallel trade between LDCs and richer countries, however, would help make it possible to offer selectively lower prices in LDCs.

In conclusion, our analysis casts doubt on the view that world welfare would be enhanced by encouraging unrestricted parallel imports in order to undermine price discrimination. Importantly, we have focused on price discrimination as the sole cause of parallel imports, abstracting entirely from other factors that exclusive territories are designed to combat, such as free riding and consumer confusion (when parallel imports differ from the authorized products targeted to the local market). Those other roles of exclusive territories are often efficient. Moreover, these efficiencies are likely to be at least as great in the international context as within countries, given that substantial country-specific investments are often required to introduce new products and that such investments are often best elicited by awarding sole-import distributorships. The case against parallel imports is correspondingly strengthened.

Appendix 1: Equilibrium Under Uniform Pricing

In this Appendix we derive the values presented in Table 1 for the case of uniform pricing by the monopolist. For convenience, we recall the aggregate demand function facing the monopolist under uniform pricing:

$$Q(p) = \begin{cases} 1-\frac{p}{2x}\log\left(\frac{1+x}{1-x}\right) & \text{if } 0 \le p \le 1-x \\ \frac{1+x}{2x}-\frac{p}{2x}\left(1+\log\left(\frac{1+x}{p}\right)\right) & \text{if } 1-x < p \le 1+x. \end{cases}$$

We consider two cases, corresponding to whether or not all markets are served at a particular uniform price.

Case 1 (all markets are served): $0 \le p \le 1-x$.

We show below that this case pertains if $x \le x^*$. Using the formula for Q(p) over this price range gives

$$\Pi(p) = pQ(p) = p\left[1-\frac{p}{2x}\log\left(\frac{1+x}{1-x}\right)\right],$$

$$\Pi'(p) = 1-\frac{p}{x}\log\left(\frac{1+x}{1-x}\right),$$

$$\Pi''(p) = -\frac{1}{x}\log\left(\frac{1+x}{1-x}\right) < 0.$$

Let p_a solve $\Pi'(p) = 0$. (The subscript a means that *all* markets are served.) Thus,

(A.1) $$p_a = \frac{x}{\log\left(\frac{1+x}{1-x}\right)},$$

and p_a provides a local maximum if $p_a < 1-x$. The resulting profit is

$$\Pi(p_a) = p_a\left[1-\frac{1}{2x}\log\left(\frac{1+x}{1-x}\right)\left(\frac{x}{\log\left(\frac{1+x}{1-x}\right)}\right)\right] = \frac{1}{2}p_a.$$

Straightforward calculations show that, with all markets served, consumer surplus is

$$S(p_a) = \frac{1}{2x}\int_{1-x}^{1+x}\left(\frac{1}{2a}(a-p_a)^2\right)da = \frac{1}{2}-\frac{3}{4}p_a.$$

Therefore, welfare is

$$W(p_a) = \Pi(p_a) + S(p_a) = \frac{1}{2}-\frac{1}{4}p_a.$$

Case 2 (some markets are dropped): $1 - x < p \le 1+x$.

For this price range, the expression for Q(p) implies that

$$\Pi(p) = \frac{1}{2x}\left[p\,(1+x-p) - p^2 \log\left(\frac{1+x}{p}\right)\right],$$

$$\Pi'(p) = \frac{1}{2x}\left[1 + x - p - 2p \log\left(\frac{1+x}{p}\right)\right],$$

$$\Pi''(p) = \frac{1}{2x}\left[1 - 2 \log\left(\frac{1+x}{p}\right)\right].$$

By the first-order condition (FOC), we have

(A.2) $$\Pi'(p) = 0 \;\Rightarrow\; \frac{1+x}{p} = 1 + 2\log\left(\frac{1+x}{p}\right).$$

There are two solutions to the FOC. One solution is $p = 1+x$, which yields zero sales. Let p_s be the (other) solution to the FOC with $p_s < 1+x$ (the subscript s indicates that only *some* markets are served). To find p_s, we express the FOC in the form $z = 1 + 2\log(z)$, where $z = (1+x)/p$. There exists a unique number y, $y > 1$, with the property that

$$y = 1 + 2\log(y);$$

moreover, $y \approx 3.5128$. Thus, the FOC implies

(A.3) $$p_s = \frac{1+x}{y} \approx \frac{1+x}{3.5128}.$$

The second-order condition for a maximum is satisfied at p_s:

$$\begin{aligned}\Pi''(p_s) &= \frac{1}{2x}\left[1 - 2\log\left(\frac{1+x}{p_s}\right)\right]\\ &= \frac{1}{2x}\left[1 - \left(\frac{1+x}{p_s} - 1\right)\right]\\ &= \frac{1}{2x}[2 - y]\\ &< 0 \quad \text{because } y > 2,\end{aligned}$$

where the second line follows by using (A.2) and the third by (A.3). Hence, the FOC identifies the monopolist's optimum.

Substituting from (A.3) into the expression for $\Pi(p_s)$ and performing some routine algebra gives profit as

$$\Pi(p_s) = \frac{1}{4x} p_s\,(1+x-p_s) = \frac{y-1}{4x} p_s^2.$$

Consumer surplus is

$$S(p_s) = \frac{1}{2x}\int_{p_s}^{1+x} \frac{(a-p_s)^2}{2a}\, da$$

$$= \frac{1}{4x}\left[\frac{1}{2}\left((1+x)^2 - p_s^2\right) - 2(1+x-p_s)p_s + p_s^2 \log\left(\frac{1+x}{p_s}\right)\right].$$

Using condition (A.3), recalling that y = 1 + 2 log y, and regrouping terms gives

$$S(p_s) = \frac{(y-1)(y-2)}{8x} p_s^2 = \left(\frac{y}{2} - 1\right)\Pi(p_s).$$

Therefore, welfare is

$$W(p_s) = \frac{y(y-1)}{8x} p_s^2 = \left(\frac{y}{2}\right)\Pi(p_s).$$

Now consider when each of the two cases applies. Expression (A.1) for p_a presumes $p_a \leq 1-x$, which is equivalent to

(A.4) $$\frac{x}{1-x} \leq \log\left(\frac{1+x}{1-x}\right).$$

Expression (A.3) for p_s presumes $(1+x)/y = p_s > 1-x$, which is equivalent to

(A.5) $$x > x^* \equiv \frac{y-1}{y+1}.$$

recalling that y > 1 satisfies y = 1 + 2 log(y). Therefore, the two cases are mutually exclusive and exhaustive.† Since y ≈ 3.5128, x* ≈ 0.5568. All the relevant expressions are reported in Table 1 in the text.

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†Recall that Case 1 pertains if and only if x/(1–x) ≤ log((1+x)/(1–x)). But we see

$$\frac{x}{1-x} \leq \log\left(\frac{1+x}{1-x}\right) \Leftrightarrow 1 + \frac{2x}{1-x} \leq 1 + 2\log\left(\frac{1+x}{1-x}\right)$$

$$\Leftrightarrow \frac{1+x}{1-x} \leq 1 + 2\log\left(\frac{1+x}{1-x}\right)$$

$$\Leftrightarrow \frac{1+x}{1-x} \leq y$$

$$\Leftrightarrow x \leq \frac{y-1}{y+1},$$

where the third line follows from the property of the function 1 + 2log(x). Thus, the cases described by (A.4) and (A.5) are complementary.

Appendix 2: Proofs

Proof of Lemma 1. Because marginal cost is constant, the monopolist can optimize with respect to each element of the mixed system independently of the prices charged to the other groups. Let P_i be an element of the mixed system $\wp$, and assume that this group is fully served when the monopolist is required to charge the same price to all members of the group. For prices at which this group is fully served, demand by group P_i equals

$$Q(p) = \int_{P_i} 1 - (p/a)\, dF(a) = A - Bp,$$

where $A = \int_{P_i} dF(a)$ and $B = \int_{P_i} (1/a)\, dF(a)$. Given that the monopolist's optimum involves fully serving group P_i, it follows that the price to this group maximizes $(p-c)Q(p)$, which implies that total sales to group P_i equal $Q^u = (A-Bc)/2$ (recall that marginal cost is identically equal to c).

If all markets are served under uniform pricing, then it must be that $c \le 1-x$. Consequently, all markets will also be served under complete discrimination. If the monopolist is allowed to discriminate, then it sets price equal to $(a+c)/2$ in market a, yielding total sales under discrimination equal to

$$Q^d = \int_{P_i} 1 - \frac{a+c}{2} \times \frac{1}{a}\, dF(a) = \frac{1}{2}\left(\int_{P_i} dF(a) - c\int_{P_i} \frac{1}{a}\, dF(a) \right) = \frac{A-Bc}{2}.$$

Thus, assuming group P_i is fully served under mixed system $\wp$, then total sales to P_i under discrimination equals the total sales to group P_i when the monopolist must charge the same price to all markets in P_i must face the same price. Q.E.D.

Remark: In the text, variations in demand dispersion were considered by maintaining the mass of markets to equal 1, with the intercepts of demand uniformly distributed over $[1 - x, 1 + x]$, and then considering variations in x. In this formulation the associated density of market intercepts equaled 1/2x over the relevant interval. In the remainder of this Appendix, we do not consider changes in dispersion. Therefore, for notational simplicity, we assume that the density of market demand intercepts is equal to 1 over the interval $[\underline{a}, 1]$, where $0 \le \underline{a} < 1$. This simplifies the formulas to follow and does not affect the relative welfare comparisons.

The following lemma is used in the proof of Proposition 3.

Lemma A.1: Let $b \in (0, 1)$ and define $h: (b, 1) \to R$ as follows:

$$h(s) = \frac{(1-s)^2}{\log(1/s)} + \frac{(s-b)^2}{\log(s/b)}.$$

The function h is quasiconcave.

Proof. Observe that

$$h'(s) = \frac{-2(1-s)\log(1/s) + \frac{1}{s}(1-s)^2}{[\log(1/s)]^2} + \frac{2(s-b)\log(s/b) - \frac{1}{s}(s-b)^2}{[\log(s/b)]^2}$$

$$= \frac{1}{[\log(1/s)]^2[\log(s/b)]^2} \times \left\{\left(-2(1-s)\log(1/s) + \frac{1}{s}(1-s)^2\right)[\log(s/b)]^2 + \left(2(s-b)\log(s/b) - \frac{1}{s}(s-b)^2\right)[\log(1/s)]^2\right\}.$$

Let N(s) denote the term in braces. Then the sign of h' is the same as the sign of N. We see

$$N(s) = \left(-2(1-s)\log(1/s) + \frac{1}{s}(1-s)^2\right)[\log(s/b)]^2 + \left(2(s-b)\log(s/b) - \frac{1}{s}(s-b)^2\right)[\log(1/s)]^2$$

$$= 2(1-s)\log(s)[\log(s/b)]^2 + \frac{(1-s)^2}{s}[\log(s/b)]^2$$

$$+2(s-b)\log(s/b)[\log(s)]^2 - \frac{(s-b)^2}{s}[\log(s)]^2$$

$$= 2\log(s)\log(s/b)\{(1-s)\log(s/b) + (s-b)\log(s)\}$$

$$+\frac{1}{s}\left\{[(1-s)\log(s/b)]^2 - [(s-b)\log(s)]^2\right\}$$

$$= 2\log(s)\log(s/b)\{(1-s)\log(s/b) + (s-b)\log(s)\}$$

$$+\frac{1}{s}\{(1-s)\log(s/b) + (s-b)\log(s)\} \times \{(1-s)\log(s/b) - (s-b)\log(s)\}$$

$$= \{(1-s)\log(s/b) + (s-b)\log(s)\} \times \left\{\frac{1}{s}(1-s)\log(s/b) - \frac{1}{s}(s-b)\log(s) + 2\log(s)\log(s/b)\right\}$$

$$= [(1-s)\log(s/b) - (s-b)\log(1/s)]$$

(A.6)
$$\times\left\{\frac{1}{s}(1-s)\log(s/b) + \frac{1}{s}(s-b)\log(1/s) - 2\log(1/s)\log(s/b)\right\}.$$

In order to understand N(s), it is necessary to understand the two factors in the product of the previous equation.

First we show that $\left[(1-s)\log(s/b)-(s-b)\log(1/s)\right]$ is strictly positive. To see this, observe that

$$(1-s)\log(s/b)-(s-b)\log(1/s)>0$$

$$\Leftrightarrow (1-b)\log(s)-(1-s)\log(b)>0$$

$$\Leftrightarrow \frac{\log(s)}{1-s}>\frac{\log(b)}{1-b}. \tag{A.7}$$

Now consider the function $F(s) = \log(s)/(1-s)$ for $s \in (b, 1)$. We have

$$F'(s)=\frac{\frac{1}{s}-1-\log\left(\frac{1}{s}\right)}{(1-s)^2}.$$

It is readily checked that if $z > 0$ and $z \neq 1$, then $z > 1 + \log(z)$. Therefore, it follows that for all s in the relevant range, $F'(s) > 0$, and, because $s > b$, it follows that inequality (A.7) is valid. Consequently, the sign of N(s) is the same as the sign of the term in braces in (A.6). Let f(s) denote the term in braces, that is,

$$f(s)=\frac{1}{s}(1-s)\log(s/b)+\frac{1}{s}(s-b)\log(1/s)-2\log(1/s)\log(s/b).$$

The remainder of the proof consists of showing that f has the shape depicted in Figure A.1:

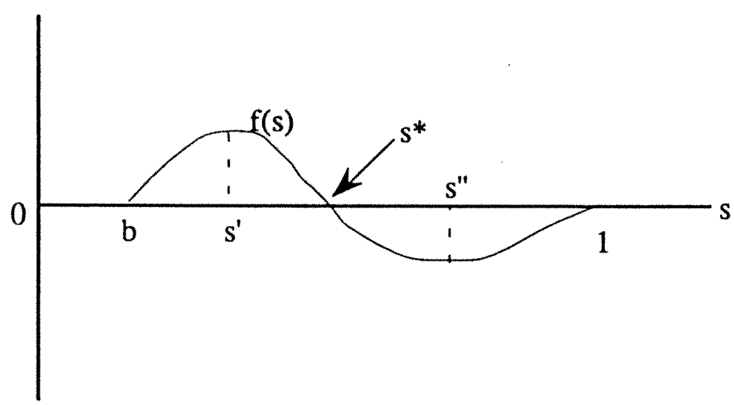

Figure A.1

In order to see why this suffices, observe that if f(s) has this shape, then h' is initially positive, up to some point s*, after which it is negative. This simply says that h(s) is increasing up to some point s* between b and 1 and it is decreasing for all larger s, implying that h(s) is single-peaked. First observe that $f(b) = f(1) = 0$. Furthermore,

$$f'(s)=\frac{1}{s^2}\left[2s\log(s/b)+2s\log(s)-\log(s/b)+1-2s+b-b\log(s)\right].$$

Therefore,

$$f'(b) = \frac{1}{b^2}\left[b\log(b) + 1 - b\right] = \frac{1}{b}\left[\frac{1}{b} - \log\left(\frac{1}{b}\right) - 1\right] > 0,$$

where the inequality follows from the fact, noted above, that for all $z > 1$, $z > 1 + \log(z)$. Similarly,

$$f'(1) = \left[\log(1/b) - 1 + b\right] = \left[-\log(b) - 1 + b\right] > 0.$$

Because $f'(b) > 0$ and $f'(1) > 0$, and because f is continuous, it follows that, between b and 1, f(s) must "turn" at least twice; that is, $f'(s) = 0$ for at least two distinct points, say s' and s", with b < s' < s" < 1. It remains to show that f(s) turns at most twice, from which we will conclude that f(s) has the shape depicted above. Define g(s) on [b, 1] as follows:

$$g(s) = 2s\log(s/b) + 2s\log(s) - \log(s/b) + 1 - 2s + b - b\log(s).$$

In order to show $f'(s) = 0$ at at most 2 points, it suffices to show that g(s) = 0 for at most 2 values of s. Observe that

$$g''(s) = \frac{4}{s} + \frac{1+b}{s^2}.$$

Therefore, g is strictly convex, implying that g(s) can indeed equal 0 for at most 2 values of s. Hence, in light of the earlier discussion, there are *exactly* 2 values of s, say s' and s", at which $f'(s) = 0$. We can conclude that there is s* strictly between b and 1 such that $f'(s) > 0$ for $s \in (b, s^*)$ and $f'(s) < 0$ for $s \in (s^*, 1)$. Therefore, it follows that over (b, 1) the function h(s) is quasiconcave. Q.E.D.

Proof of Proposition 3. By the assumption of the proposition, there is a positive mass of consumers with type less than 1/y (otherwise no markets would be dropped under uniform pricing), that is, $\underline{a} < 1/y$. The proof establishes that any mixed system without holes that is not essentially equivalent to (4.1) can be reorganized into a new mixed system without holes that yields higher welfare.† The proof shows that the "top" group of an optimal no-holes mixed system is (essentially) [1/y, 1]. This result is then applied to the remaining set of markets $[\underline{a}, 1/y)$, for which the optimal "top" group is $[1/y^2, 1/y)$. This argument is repeated until all markets are included in some group (it is simplest to allow one element of the partition to be the singleton set {0}, the group I_0 in (4.1)), implying that the mixed system (4.1) is optimal among mixed systems without holes.

Let $\wp = \{P_i\}_i$ denote any mixed scheme with no holes. Define set A by

$$A = \bigcup\{P_i : P_i \subset [1/y, 1]\}.$$

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† Two mixed systems without holes are *essentially equivalent* if they differ only on a set of measure zero. For mixed systems without holes, this means two mixed systems are equivalent if they differ only with respect to the endpoints of the intervals in their partitions of [1–x, 1+x]. It is worth noting that the proof actually establishes that (4.1) is the essentially unique welfare maximizing no-holes partition of [1–x, 1+x].

Because interval A $\subset$ [1/y, 1], it follows that group A is fully served (recall that under a uniform price the monopolist drops markets only if the lowest-demand intercept is below 1/y). By Lemma 1, total consumption by markets in group A equals total consumption in those markets under the mixed system $\wp$. Therefore, if two elements in the definition of A have strictly positive measure, then this merging of groups has already created a "no-holes" mixed system that yields greater welfare than the mixed system $\wp$, namely the mixed system $\wp'$ described as follows (cf. Schwartz, 1990):

$$\wp' = \{A\} \cup \{ P_i : P_i \not\subset [1/y, 1] \}.$$

Suppose instead that there is only one set in the construction of A that has strictly positive measure. Without loss of generality, we may assume A is closed, and therefore it can be written as $[\hat{s}, 1]$. Clearly, $\hat{s} \geq 1/y$. If $\hat{s} = 1/y$, then we are done, because then A = [1/y, 1], and the top interval in partition $\wp$ is essentially equivalent to the top interval in (4.1). Next, by repeatedly applying the above logic to the remaining markets it will be found that either $\wp$ is essentially equivalent to (4.1) or that, at some stage, regrouping markets as described in the remainder of the proof will yield a no-holes partition that generates strictly greater welfare that system $\wp$.

Next, suppose $1/y < \hat{s} < 1$. Consider the element of $\wp$ that contains 1/y. This is an interval, which we may take to be of the form $[b, \hat{s})$, where $b < 1/y$. If $[b, \hat{s})$ is not fully served, then the partition can be improved upon by replacing it in the partition with the elements $[\hat{s}/y, \hat{s})$ and $[b, \hat{s}/y)$. Therefore, assume $[b, \hat{s})$ is fully served, which implies $b \geq \hat{s}/y$. In the remainder of this proof we need consider only the case in which $1/y^2 < b < 1/y$ and $1/y < \hat{s} \leq by$. We show that welfare can be increased by partitioning the markets [b, 1] into [b, 1/y) and [1/y, 1], rather than $[b, \hat{s})$ and $[\hat{s}, 1]$.

For $s \in [1/y, by]$, define A = A(s) = [s, 1] and B = B(s) = [b, s). Under the requirement of uniform pricing to each group, the monopolist fully serves each group (group A because $s \geq 1/y$, and group B because $b \geq s/y$). The monopolist's pricing decision ($p_A = (1-s)/(2\log(1/s))$ to group A and $p_B = (s-b)/(2\log(s/b))$ to group B) yields the following welfare to groups A and B:

$$W_A(s) = \int_s^1 \frac{a}{2} - \frac{(p_A)^2}{2a}\,da = \frac{1}{4}\left(1 - s^2\right) - \frac{1}{4} \times \frac{(1-s)^2}{\log(1/s)}$$

and

$$W_B(s) = \int_b^s \frac{a}{2} - \frac{(p_B)^2}{2a}\,da = \frac{1}{4}\left(s^2 - b^2\right) - \frac{1}{4} \times \frac{(s-b)^2}{\log(s/b)}.$$

Therefore, total welfare to markets [b, 1], as a function of the "dividing point" s, is

$$W^*(s) = W_A(s) + W_B(s) = \frac{1}{4}\left(1 - b^2\right) - \frac{1}{4}\left\{\frac{(1-s)^2}{\log(1/s)} + \frac{(s-b)^2}{\log(s/b)}\right\}.$$

Recall that for groups A and B to be fully served, s must lie in the interval [1/y, by]. Next we show that over the interval [1/y, by], W* is maximized at s = 1/y.

From Lemma A.1, it follows that W* is quasiconvex over the interval [1/y, by]. Therefore, in order to show that over the interval [1/y, by], W* is maximized at s = 1/y, it suffices to show that h(1/y) < h(by), where h(s) is as defined in Lemma A.1. We have

$$h(1/y) = \frac{1}{\log(y)\times\log(1/by)}\left\{\log(1/by)\left(1-\frac{1}{y}\right)^2 + \log(y)\left(\frac{1}{y}-b\right)^2\right\}$$

and

$$h(by) = \frac{1}{\log(y)\times\log(1/by)}\left\{\log(y)(1-by)^2 + \log(1/by)(by-b)^2\right\}.$$

Therefore, recalling that y = 1 + 2log(y),

$$h(by) > h(1/y)$$

$$\Leftrightarrow \log(y)(1-by)^2 + \log(1/by)(by-b)^2 > \log(1/by)(1-(1/y))^2 + \log(y)\left(\frac{1}{y}-b\right)^2$$

$$\Leftrightarrow \log(y)\left\{(1-by)^2 - \left(\frac{1}{y}-b\right)^2\right\} > \log(1/by)\left\{(1-(1/y))^2 - (by-b)^2\right\}$$

$$\Leftrightarrow \left(\frac{y-1}{2y^2}\right)(1-by)^2\left(y^2-1\right) > \frac{\log(1/by)}{y^2}(y-1)^2\left(1-b^2y^2\right)$$

(A.8) $$\Leftrightarrow \left(\frac{y+1}{2}\right)(1-by) > (1+by)\log(1/by).$$

Recall that b lies between $1/y^2$ and 1/y. Define f on $[1/y^2, 1/y]$ as follows:

$$f(b) = \left(\frac{y+1}{2}\right)(1-by) - (1+by)\log(1/by).$$

Then,

(A.9) $$f''(b) = \frac{1}{b^2}(by-1) < 0.$$

In order to show that inequality (A.8) holds for all relevant b, it suffices to show that f(s) is strictly positive for all b in the interval $(1/y^2, 1/y)$. Because f is strictly concave (see (A.9)), it suffices to show that $f(1/y) = f(1/y^2) = 0$. It is immediate that f(1/y) = 0. Furthermore,

$$f(1/y^2) = \frac{1-(1/y)}{2}(y+1) + \left(1+\frac{1}{y}\right)\log(1/y)$$

$$= \left(\frac{y+1}{y}\right)\left(\frac{y-1}{2} - \log(y)\right)$$

$$= 0,$$

where the third equality follows from y = 1 + 2log(y). Thus, inequality (A.8) is satisfied for all relevant b, from which it follows that, over the interval [1/y, by], h(s) is minimized at s = 1/y, which implies that, over the interval [1/y, by], W* is maximized at s = 1/y. Q.E.D.

Proof of Proposition 4. By the assumption of the proposition, there is a positive mass of markets with intercept less than 1/y (otherwise no markets would be dropped under uniform pricing), so $\underline{a} < 1/y$. From Proposition 3 it follows that for the optimal mixed system with no holes, "top" group is [1/y, 1], and the "second" group of markets is [z, 1/y), where z = max {$\underline{a}$, $1/y^2$}. It will be shown that markets in these first two groups can be regrouped to yield a mixed system generating higher welfare, showing that the mixed system of Proposition 3 is not optimal among the broader class of mixed system that allows "holes."

Let $\varepsilon \geq 0$ be small and consider the sets A = [1/y, 1 − ε] and B = [z, 1/y) ∪ (1 − ε, 1]. It is shown, considering only the markets [z, 1], welfare can be increased by replacing the initial partition of this interval by the sets A and B for some small ε > 0. Now, either z > $1/y^2$ or z = $1/y^2$. We consider these two cases separately.

Case 1: $\underline{a} > 1/y^2$ (the case of "slack")

Group A is fully served. For prices p < 1/y, the demand by group A is

$$Q_a(p) = \int_{1/y}^{1-\varepsilon} 1 - \frac{p}{a}\,da = 1 - \varepsilon - \frac{1}{y} - p\log((1-\varepsilon)y).$$

The monopolist required to charge a uniform price to group A will sets its price equal to maximize $pQ_a(p)$, yielding

$$p_a = \frac{1-\varepsilon-(1/y)}{2\log(y(1-\varepsilon))}.$$

Therefore,

$$\frac{dp_a}{d\varepsilon} = \frac{-\log(y(1-\varepsilon)) + 1 - \dfrac{1}{y(1-\varepsilon)}}{2\{\log(y(1-\varepsilon)\}^2}.$$

Hence, recalling that y satisfies y = 1 + 2log(y),

$$\left.\frac{dp_a}{d\varepsilon}\right|_{\varepsilon=0} = \frac{-\log(y)+1-\frac{1}{y}}{2\{\log(y)\}^2} = \frac{2-y}{y(y-1)},$$

where this is interpreted as a right-hand derivative. In the remainder of this proof, every derivative evaluated at $\varepsilon = 0$ is to be interpreted as a right-hand derivative. The welfare to group A, given the monopolist's pricing decision, is

$$W_a = \int_{1/y}^{1-\varepsilon} \frac{a}{2} - \frac{(p_a)^2}{2a}\, da = \frac{1}{4}\left[(1-\varepsilon)^2 - \frac{1}{y^2}\right] - \frac{1}{2}(p_a)^2 \log(y(1-\varepsilon)).$$

Therefore,

$$\frac{dW_a}{d\varepsilon} = -\frac{1}{2}(1-\varepsilon) - p_a \log(y(1-\varepsilon))\frac{dp_a}{d\varepsilon} + \frac{1}{2(1-\varepsilon)}(p_a)^2.$$

Again noting that $y = 1 + 2\log(y)$ and $p_a|_{\varepsilon=0} = 1/y$, we have

$$\left.\frac{dW_a}{d\varepsilon}\right|_{\varepsilon=0} = -\frac{1}{2} - p_a \log(y)\left.\frac{dp_a}{d\varepsilon}\right|_{\varepsilon=0} + \frac{1}{2}\left(p_a|_{\varepsilon=0}\right)^2$$

$$= -\frac{1}{2} - \frac{1}{y}\left(\frac{y-1}{2}\right)\left(\frac{2-y}{y(y-1)}\right) + \frac{1}{2y^2}$$

$$= -\frac{1}{2} - \frac{2-y}{2y^2} + \frac{1}{2y^2}$$

(A.10) $$= -\frac{1}{2y^2}\left(1-y+y^2\right).$$

For $p \le z$, the aggregate demand for group B is given by

$$Q_b(p) = \int_{1-\varepsilon}^{1} 1-\frac{p}{a}\, da + \int_{z}^{1/y} 1-\frac{p}{a}\, da = \left(\varepsilon + \frac{1}{y} - z\right) - p\log\left(\frac{1}{(1-\varepsilon)yz}\right).$$

When there is "slack" and ε is small, the monopolist's optimal uniform price to group B is less than z. Therefore, the monopolist's price to group B is p_b, where

$$p_b = \frac{\varepsilon + (1/y) - z}{2\log[1/zy(1-\varepsilon)]}.$$

Differentiating with respect to ε, we obtain

$$\frac{dp_b}{d\varepsilon} = \frac{-\log(zy(1-\varepsilon)) - \left(\varepsilon + \frac{1}{y} - z\right)\left(\frac{1}{1-\varepsilon}\right)}{2\{\log((1-\varepsilon)zy)\}^2}.$$

Therefore,

$$\left.\frac{dp_b}{d\varepsilon}\right|_{\varepsilon=0} = \frac{-\log(zy) - \left(\frac{1}{y} - z\right)}{2\{\log(zy)\}^2}$$

$$= \frac{\log\left(\frac{1}{zy}\right) - \left(\frac{1-zy}{y}\right)}{2\left\{\log\left(\frac{1}{zy}\right)\right\}^2}$$

$$= \frac{1}{2\log\left(\frac{1}{zy}\right)}\left\{1 - \frac{1-zy}{y\log\left(\frac{1}{zy}\right)}\right\} = \frac{1}{2\log\left(\frac{1}{zy}\right)}\left\{1 - 2p_b\big|_{\varepsilon=0}\right\}$$

Also, for group B, welfare is given by

$$W_b = \int_{1-\varepsilon}^{1} \frac{a}{2} - \frac{(p_b)^2}{2a}\, da + \int_{z}^{1/y} \frac{a}{2} - \frac{(p_b)^2}{2a}\, da$$

$$= \frac{1}{4}\left[1 - (1-\varepsilon)^2 + \left(\frac{1}{y^2} - z^2\right)\right] + \frac{1}{2}(p_b)^2 \log(zy(1-\varepsilon)).$$

Therefore,

$$\frac{dW_b}{d\varepsilon} = \frac{1-\varepsilon}{2} + p_b \log(zy(1-\varepsilon))\frac{dp_b}{d\varepsilon} - \frac{(p_b)^2}{2(1-\varepsilon)}.$$

Hence,

$$\left.\frac{dW_b}{d\varepsilon}\right|_{\varepsilon=0} = \left.\left\{\frac{1}{2} + p_b \log(zy)\frac{dp_b}{d\varepsilon} - \frac{1}{2}(p_b)^2\right\}\right|_{\varepsilon=0} = \left.\frac{1}{2}\left(1 - p_b + p_b{}^2\right)\right|_{\varepsilon=0}.$$

The expression $1 - p_b + (p_b)^2$ is decreasing in p_b as long as $p_b < 1/2$, which is certainly the case under consideration because $p_b < 1/y < 1/3$. Therefore, over the relevant range, $\left.\frac{dW_b}{d\varepsilon}\right|_{\varepsilon=0}$ is smallest when $p_b\big|_{\varepsilon=0}$ is largest, which occurs as z approaches 1/y. Applying L'Hopital's Rule, we find

$$\lim_{z\uparrow 1/y} p_b\big|_{\varepsilon=0} = 1/2y.$$

Hence, for all $z \in (1/y^2, 1/y)$,

$$\left.\frac{dW_b}{d\varepsilon}\right|_{\varepsilon=0} \geq \frac{1}{2}\left(1 - \frac{1}{2y} + \frac{1}{4y^2}\right).$$

Therefore, for all $z \in (1/y^2, 1/y)$,

$$\left.\frac{d(W_a + W_b)}{d\varepsilon}\right|_{\varepsilon=0} \geq -\frac{1}{2y^2}(1-y+y^2)+\frac{1}{2}\left(1-\frac{1}{2y}+\frac{1}{4y^2}\right)$$

$$= \frac{-4+4y-4y^2+4y^2-2y+1}{8y^2} = \frac{2y-3}{8y^2} > 0,$$

showing that the initial "no-holes" partition is not optimal.

Case 2: $1/y^2 \geq a$ (the case of "no-slack")

Formula (A.10) for $\left.\frac{dW_a}{d\varepsilon}\right|_{\varepsilon=0}$ is valid whether or not there is slack. Hence, it will be used in this case, too.

When there is no slack and ε is "small," we know the monopolist's optimal price to group B will exceed $1/y^2$ but be less than $1/y$. Consider such a price. The aggregate demand by the types $[1-\varepsilon, 1]$ at this price is

$$\int_{1-\varepsilon}^{1} 1-\frac{p}{a}\,da = \varepsilon - p\log(1/(1-\varepsilon)).$$

The aggregate demand by the types $[1/y^2, 1/y)$ (note that some markets are dropped) is

$$\int_{p}^{1/y} 1-\frac{p}{a}\,da = 1/y - p[1+\log(1/yp)].$$

Therefore, aggregate demand by group B at prices between $1/y^2$ and $1/y$ is equal to

$$Q_b(p) = \varepsilon + \frac{1}{y} - p\left(1+\log\left(\frac{1}{(1-\varepsilon)yp}\right)\right).$$

The monopolist's optimal price, p_b, to group B maximizes $pQ_b(p)$. The solution satisfies the following first-order condition:

(A.11) $$0 = \varepsilon + \frac{1}{y} - p\left(1+2\log\left(\frac{1}{(1-\varepsilon)yp}\right)\right).$$

(One can verify that, for small ε, the second-order condition for a maximum is satisfied at the solution to (A.7).) Treating p_b as a function of ε and differentiating (A.11) with respect to ε yields

(A.12) $$0 = 1 - \frac{dp_b}{d\varepsilon} - 2\log\left(\frac{1}{(1-\varepsilon)y}\right)\frac{dp_b}{d\varepsilon} - 2p\left(\frac{1}{1-\varepsilon}\right) + 2\log(p)\frac{dp_b}{d\varepsilon} + 2\frac{dp_b}{d\varepsilon}.$$

Note that $p_b\big|_{\varepsilon=0} = 1/y^2$. Therefore, upon rearranging terms of (A.12) and taking limits as $\varepsilon \to 0$, we obtain

(A.13) $$\left.\frac{dp_b}{d\varepsilon}\right|_{\varepsilon=0}=\left(\frac{1}{y^2}\right)\left(\frac{y^2-1}{y-2}\right).$$

Because $p_b \geq 1/y^2$, the welfare to group B is equal to

$$W_b=\int_{1-\varepsilon}^{1}\frac{a}{2}-\frac{(p_b)^2}{2a}\,da \;+\; \int_{p_b}^{1/y}\frac{a}{2}-\frac{(p_b)^2}{2a}\,da$$

$$=\frac{1}{4}\left((1-(1-\varepsilon)^2\right)+\frac{(p_b)^2}{2}\log(1-\varepsilon)+\frac{1}{4}\left(\frac{1}{y^2}-(p_b)^2\right)+\frac{(p_b)^2}{2}\log(yp_b).$$

Therefore,

$$\frac{dW_b}{d\varepsilon}=\frac{1-\varepsilon}{2}-\frac{(p_b)^2}{2(1-\varepsilon)}+p_b\log((1-\varepsilon)yp_b)\frac{dp_b}{d\varepsilon}.$$

Hence, recalling (A.13) and the fact that $\left.p_b\right|_{\varepsilon=0}=1/y^2$, we find, after some routine manipulations, that

$$\left.\frac{dW_b}{d\varepsilon}\right|_{\varepsilon=0}=\frac{1}{2}-\frac{1}{2y^4}+\frac{1}{y^2}\log(1/y)\left.\frac{dp_b}{d\varepsilon}\right|_{\varepsilon=0}$$

$$=\frac{1}{2}-\frac{1}{2y^4}-\frac{1}{y^2}\left(\frac{y-1}{2}\right)\left(\frac{y^2-2}{y^2(y-2)}\right)$$

$$=\frac{1}{2}-\frac{1}{2y^4}-\frac{(y-1)(y^2-2)}{2y^4(y-2)}.$$

Therefore, when $z=1/y^2$,

$$\left.\frac{d(W_a+W_b)}{d\varepsilon}\right|_{\varepsilon=0}=-\left\{\frac{1}{2y^2}\left(1-y+y^2\right)\right\}+\left\{\frac{1}{2}-\frac{1}{2y^4}-\frac{(y-1)(y^2-2)}{2y^4(y-2)}\right\}$$

$$=-\frac{1}{2y^4}\left[y^2(1-y)+1+\frac{(y-1)(y^2-2)}{(y-2)}\right]$$

$$=\frac{1}{2y^4}\left[y^3-y^2-1-\frac{(y-1)(y^2-2)}{(y-2)}\right]$$

$$=\frac{1}{2(y-2)y^4}\left(y^4-4y^3+3y^2+y\right)$$

(A.14) $$=\frac{1}{2y^3(y-2)}\left[y(y-3)(y-1)+1\right]\;>\;0,$$

where the inequality follows because $y > 3$. From (A.14) it follows that the initial "no-holes" partition is not optimal. Q.E.D.

Table 1. Equilibrium under Uniform Pricing and Discrimination

	Case 1 $0 \le x \le x^*$	Case 2 $x^* < x \le 1$	Discrimination
p	$p_a = \dfrac{x}{\log\left(\dfrac{1+x}{1-x}\right)}$	$p_s = \dfrac{1+x}{y}$	$p(a) = \dfrac{a}{2}$
Q(p)	$\dfrac{1}{2}$	$\left(\dfrac{y-1}{y}\right)\left(\dfrac{1+x}{4x}\right)$	$\dfrac{1}{2}$
Π	$\dfrac{p_a}{2}$	$\left(\dfrac{y-1}{4x}\right)p_s^2$	$\dfrac{1}{4}$
S	$\dfrac{1}{2} - \dfrac{3}{4}p_a$	$\dfrac{(y-1)(y-2)}{8x}p_s^2$	$\dfrac{1}{8}$
W	$\dfrac{1}{2} - \dfrac{1}{4}p_a$	$\dfrac{y(y-1)}{8x}p_s^2$	$\dfrac{3}{8}$

Table 2. Welfare and Price Results with Skewness

t —>	2.0	1.0	0	–.25	–.4	–.5	–.6	–.7	–.725	–.75	–.775	–.8	–.9	–.95	–.999
a=0: w^d/w^u	1	1.01254	1.04846	1.06613	1.07998	1.09087	1.10356	1.11834	1.12241	1.12676	1.13118	1.13579	1.15665	1.16819	1.17673
p^u	.366025	.333355	.284668	.268477	.257717	.249985	.241809	.233103	.230837	.228569	.226226	.223839	.213961	.208612	.201853
propserved	.950962	.888874	.715332	.627025	.556712	.500015	.433253	.353953	.331796	.30856	.284233	.258712	.142896	.0753722	.0015989
a=.05: w^d/w^u	.997639	1.00828	1.04591	1.06902	1.09059	1.11108	1.13996	1.18499	1.20029	1.21808	.833189	.833346	.831258	.817671	.772283
p^u	.370473	.336824	.284701	.266553	.254161	.245151	.235319	.224669	.221805	.2189	.0463993	.0439039	.0354825	.0323166	.0299136
propserved	.961611	.908844	.752946	.670101	.602489	.546765	.47991	.398345	.375172	.350653	1	1	1	1	1
a=.1: w^d/w^u	.995428	1.00359	1.03798	1.06186	1.08552	1.10923	1.1443	1.20098	.87694	.877019	.877132	.877126	.86673	.837622	.76444
p^u	.375354	.340857	.284668	.263946	.249144	.237982	.225302	.210523	.0865275	.0830844	.0797421	.0765177	.0651849	.0607209	.0572512
propserved	.971362	.92838	.794814	.721167	.659891	.608448	.54555	.466956	1	1	1	1	1	1	1
a=.2: w^d/w^u	.992003	.993896	1.00652	1.00922	.932394	.929908	.928439	.928016	.927872	.927526	.926772	.925281	.89707	.847253	.74789
p^u	.386604	.350994	.284668	.253556	.195638	.18066	.165143	.149343	.145408	.141505	.137653	.133873	.120043	.114334	.109784
propserved	.987309	.964376	.894165	.86839	1	1	1	1	1	1	1	1	1	1	1

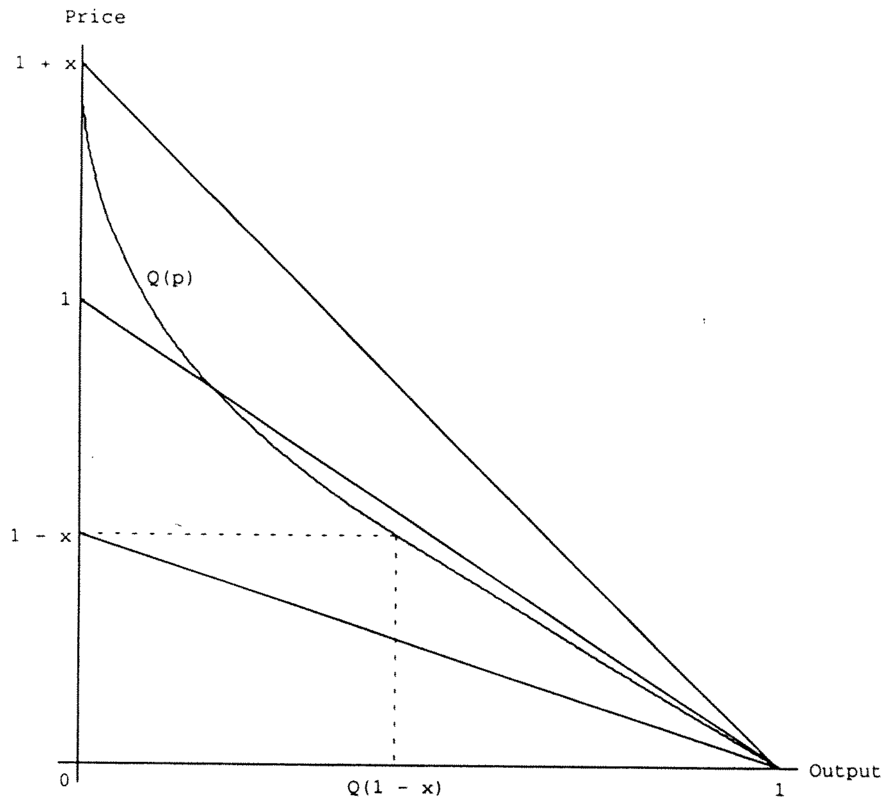

Figure 1. Aggregate Demand Under Uniform Pricing

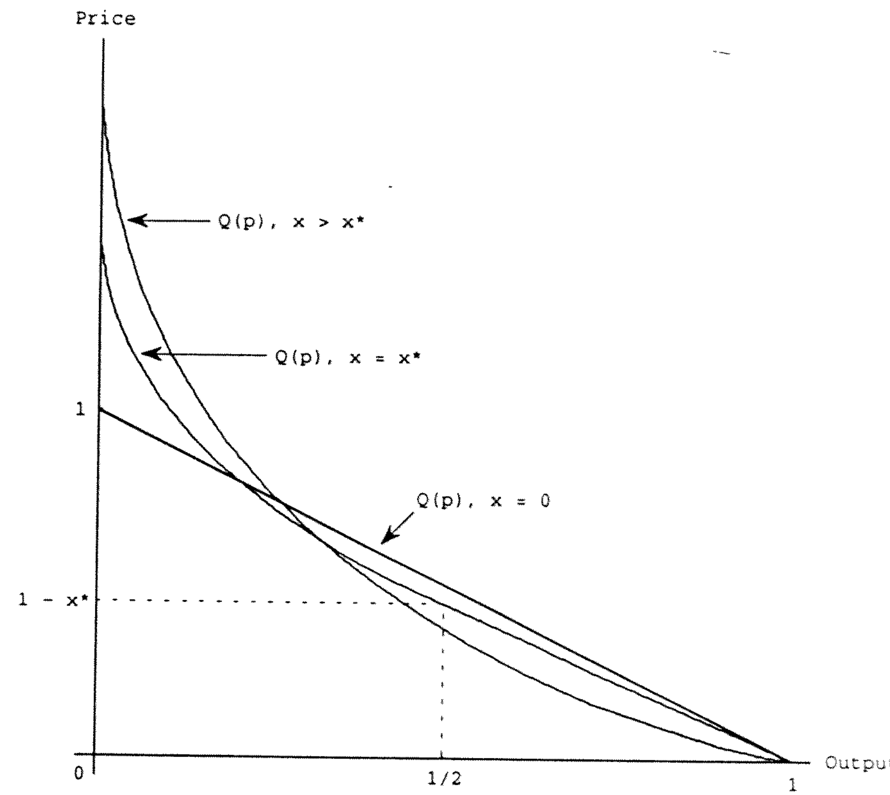

Figure 2. Aggregate Demand for Different Levels of Dispersion

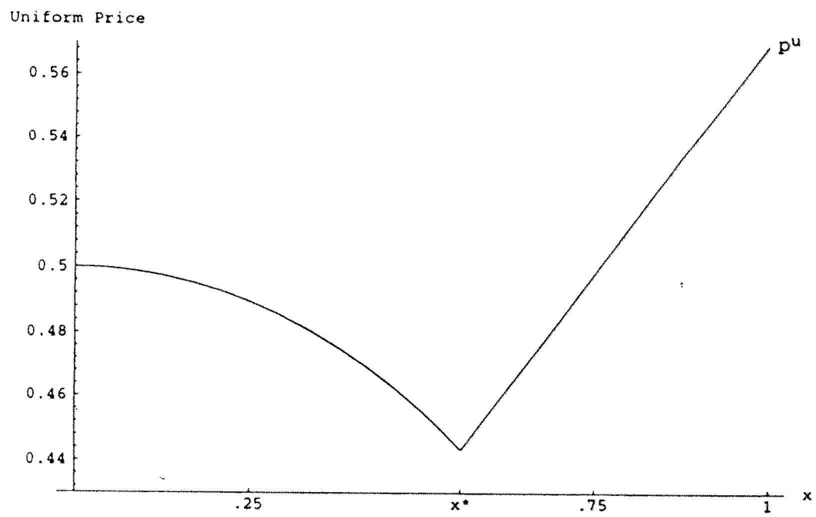

Figure 3. The Monopolist's Uniform Price

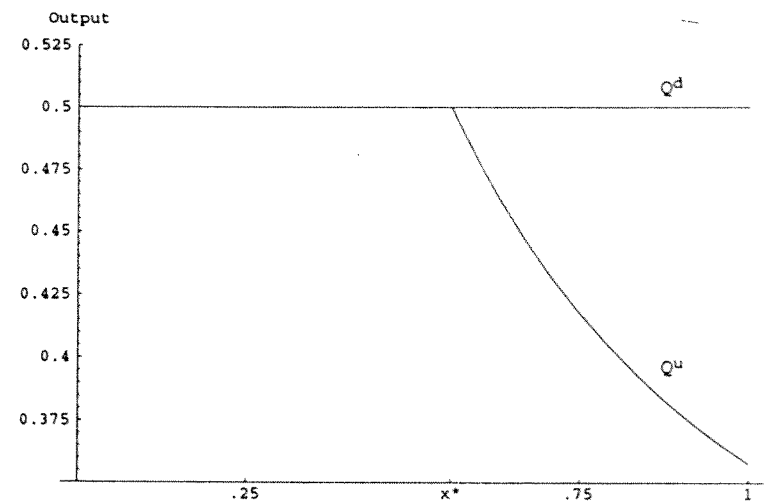

Figure 4. Total Output Under Discrimination and Uniform Pricing

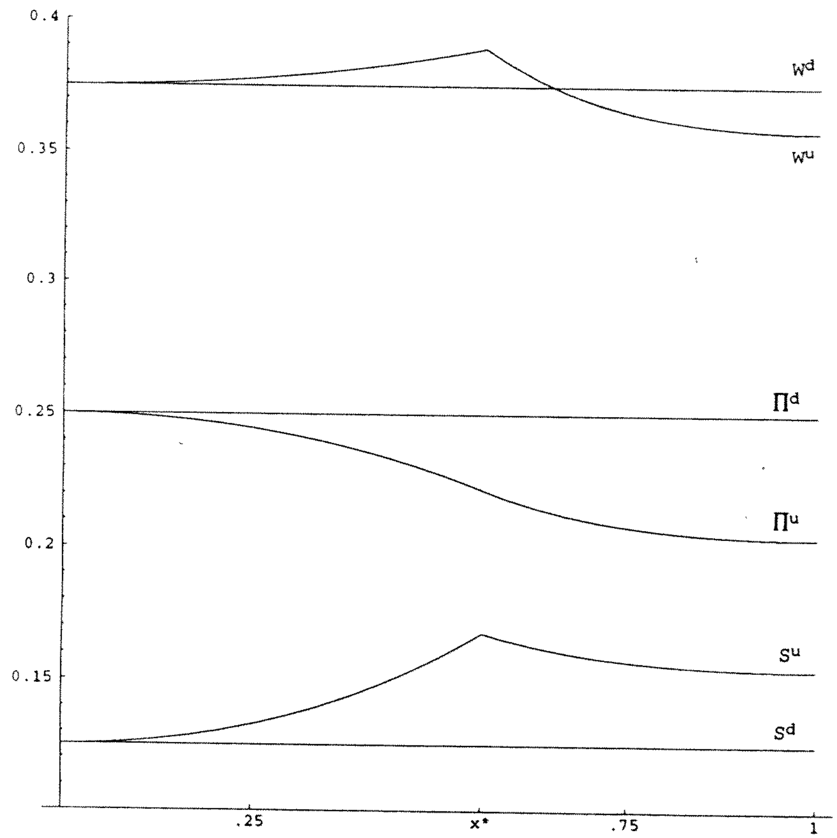

Figure 5. Consumer Surplus, Profit, and Welfare Under Discrimination and Uniform Pricing

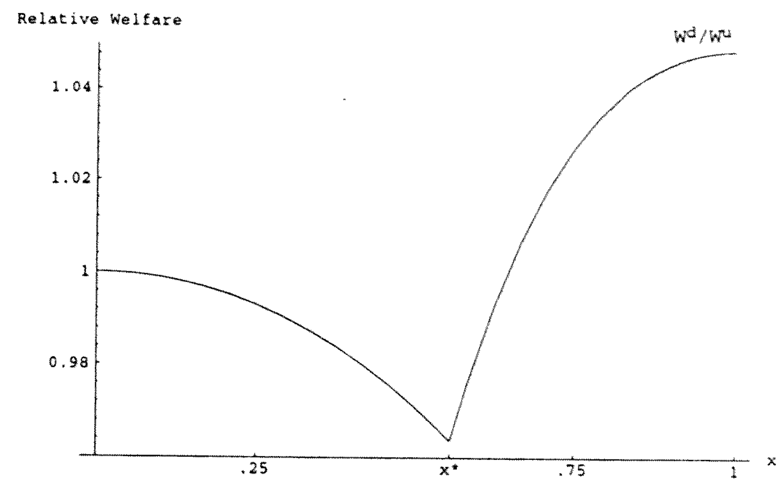

Figure 6. Relative Welfare: W^d/W^u

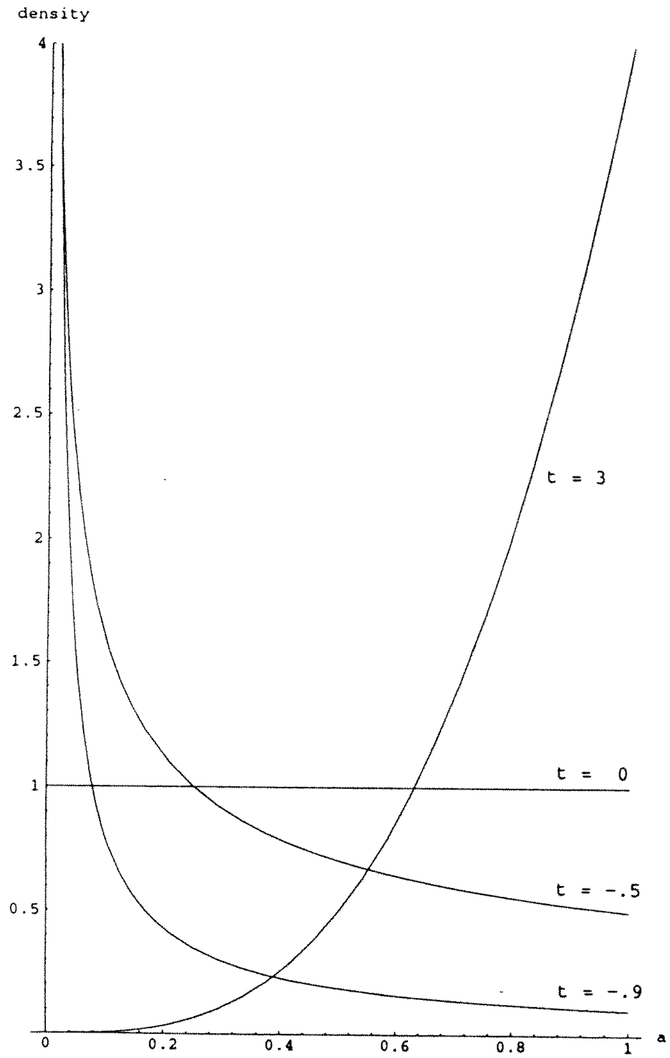

Figure 7. Skewed Distributions of Markets for $\underline{a} = 0$:

$f(a|t) = (t+1)a^t$ for $0 < a < 1$

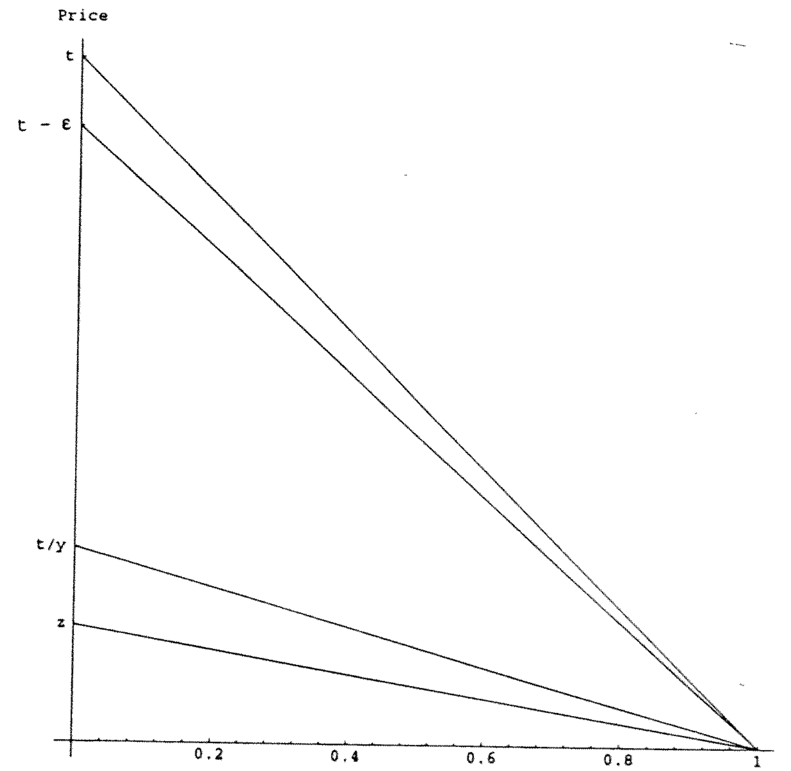

Figure 8. Adding Holes Can Raise Welfare

REFERENCES

Baker, Jonathan B., "The Gray Market in Video, Consumer Welfare, and Public Policy: An Economic Analysis," forthcoming in a volume from the Conference on "The International Market in Film and Television Programs" at Columbia University, October 23, 1987.

Borenstein, Severin, "Selling Costs and Switching Costs: Explaining Retail Gasoline Margins," *RAND Journal of Economics*, Autumn 1991, 22, 354–69.

Brander, James and Krugman, Paul, "A Reciprocal Dumping Model of International Trade," *Journal of International Economics*, November 1983, 15, 313–21.

Business Week, "The $7 Billion Gray Market: Where It Stops, Nobody Knows," April 15, 1985.

Business Week, "A Red-Letter Day for Gray Marketeers," June 13, 1988.

Chard, J.S. and Mellor, C.J., "Intellectual Property Rights and Parallel Imports," *World Economy*, March 1989, 12, 69–83.

Corden, W. Max, "The Normative Theory of International Trade," in Ronald W. Jones and Peter B. Kenen eds., *Handbook of International Economics*, Volume 1, Amsterdam: North Holland, 1984.

COPIAT, Comments of the Coalition to Preserve the Integrity of American Trademarks (COPIAT), submitted to U.S. Customs Service, October 20, 1986, in the Matter of Importations Bearing Recorded U.S. Trademarks; Solicitation of Public Comments on Gray Market Policy Options, 51 Fed. Reg. 22005, June 17, 1986.

DeGraba, Patrick, "The Relationship Between Optimal Third-Degree Discriminatory Prices and the Optimal Uniform Price," Cornell University, mimeograph, 1991.

DeMuth, Christopher, Statement Before the Subcommittee on Patents, Copyrights and Trademarks Committee on the Judiciary, Hearing on S. 626, Senate Committee on the Judiciary, 101st Cong., 2nd Sess., April 4, 1990.

Diwan, Ishac and Rodrik, Dani, "Patents, Appropriate Technology, and North-South Trade," *Journal of International Economics*, February 1991, 30, 27–47.

Dornbusch, Rudiger, "Exchange Rates and Prices," *American Economic Review*, March 1987, 77, 93–106.

Drug Diversion, A Staff Report by the Subcommittee on Oversight and Investigations of the Committee on Energy and Commerce, U.S. House of Representatives, June 1985, Washington DC: USGPO, 1985.

Drug Store News, "One If By Land, Two If By sea; Offshore Mail Order Dispensaries Dumping Worries Local Drugstores," April 26, 1993.

Feenstra, Robert C., "Symmetrical Pass-Through of Tariffs and Exchange Rates Under Imperfect Competition: An Empirical Test," *Journal of International Economics*, 1989, 27, 25–45.

Financial Times, "Sir Leon Cracks the Whip," June 25, 1992.

Froot, Kenneth A. and Klemperer, Paul A., "Exchange Rate Pass-Through When Market Share Matters," *American Economic Review*, September 1989, 79, 637–54.

Haaland, Jan I. and Wooton, Ian, "Market Integration, Competition, and Welfare," in Alan Winters Ed., *Trade Flows and Trade Policy after '1992'*, Glasgow: Cambridge University Press, 1992.

Hausman, Jerry A. and MacKie-Mason, Jeffrey, "Price Discrimination and Patent Policy," *RAND Journal of Economics*, Summer 1988, 19, 253-265.*Economics*, Summer 1988, 19, 253–65.

Hawk, Barry E., *United States, Common Market and International Antitrust: A Comparative Guide*, Second Edition, Englewood Cliffs, NJ: Prentice Hall Law & Business, 1991.

Hearing On S. 626, Legislation to Amend the Lanham Trademark Act Regarding Gray Market Goods: Hearing on S. 626 Before the Subcommittee on Patents, Copyrights and Trademarks, Senate Committee on the Judiciary, 101st Cong., 2nd Sess., April 4, 1990.

Hilke, John C., "Free-Trading or Free-Riding: An Examination of the Theories and Available Empirical Evidence on Gray Market Imports," *World Competition*, February 1988, Number 32, 75–91.

Holmes, Thomas J., "The Effects of Third-Degree Price Discrimination in Oligopoly," *American Economic Review*, March 1989, 79, 244–50.

Hunter, Linda, Markusen, James R. and Rutherford, Thomas F., "Trade Liberalization in a Multinational-Dominated Industry: A Theoretical and Applied General Equilibrium Analysis," mimeo, University of Colorado, December, 1991.

JFTC, The Antimonopoly Act Guidelines Concerning Distribution Systems and Business Practices, Executive Office, Japanese Fair Trade Commission, Tokyo, July 11, 1991.

Kasa, Kenneth, "Adjustment Costs and Pricing to Market: Theory and Evidence," *Journal of International Economics*, February 1992, 32, 1–30.

Knetter, Michael M., "Price Discrimination by U.S. and German Exporters," *American Economic Review*, March 1989, 79, 198–210.

Krugman, Paul, "Pricing to Market When the Exchange Rate Changes," in S. W. Arndt and J. D. Richardson, eds., *Real-Financial Linkages Among Open Economies*, Cambridge: MIT Press, 1987.

Lexecon Inc., *The Economics of Gray-Market Imports*, Washington DC, May 1985.

Malueg, David A., "Direction of Price Changes in Third-Degree Price Discrimination: Comment," Tulane University, A.B. Freeman School of Business working paper # 92–ECAN–05.

Malueg, David A., "Bounding the Welfare Effects of Third-Degree Price Discrimination," forthcoming in *American Economic Review*.

Marston, Richard C., "Pricing to Market in Japanese Manufacturing," *Journal of International Economics*, November 1990, 29, 217–36.

Mercenier, Jean, "Completing the European Internal Market: A General Equilibrium Evaluation Under Alternative Market Structure Assumptions," University of Montreal, Department of Economics, Discussion Paper 9208, March 1992.

Mertens, Yves and Ginsburgh, Victor, "Product Differentiation and Price Discrimination in the European Community: The Case of Automobiles," *Journal of Industrial Economics*, December 1985, 34, 151–66.

Palia, Aspy P. and Keown, Charles F., "Combating Parallel Importing: Views of U.S. Exporters to the Asia-Pacific Region," *International Marketing Review*, 1991, 8, 47–56.

Prices Surveillance Authority, Inquiry into Book Prices: Final Report, Report No. 25, Sydney, Australia: 19 December, 1989.

REMIT Consultants, London, *Impediments to Parallel Trade in Pharmaceuticals Within the European Community*, Final Report Prepared for DGIV of the European Commission, Luxembourg: Office for Official Publications of the European Communities, 1992.

Schmalensee, Richard, "Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination," *American Economic Review*, March 1981, 71, 242–7.

Schut, Frederick T. and Van Bergeijk, Peter A. G. "International Price Discrimination: The Pharmaceutical Industry," *World Development*, September 1986, 14, 1141–50.

Schwartz, Marius, "Third-Degree Price Discrimination and Output: Generalizing a Welfare Result," *American Economic Review*, December 1990, 80, 1259–62.

Shepard, Andrea, "Price Discrimination and Retail Configuration," *Journal of Political Economy*, February 1991, 99, 30–53.

Smith, Alasdair and Venables, Anthony J., "Completing the Internal Market in the European Community: Some Industry Simulations," *European Economic Review*, 1988, 32, 1501–25.

Tarr, David G., "An Economic Analysis of Gray Market Imports," mimeo, Federal Trade Commission, September 25, 1985.

Tirole, Jean, *The Theory of Industrial Organization*, Cambridge, MA: MIT Press, 1988.

U.S. Bureau of the Census, *Statistical Abstract of the United States: 1992*, 112th edition, Washington, DC, 1992.

World Bank, *World Development Report 1992*, New York, NY: Oxford University Press, 1992.