Product Innovation with Vertical Differentiation: Is a Monopolist's Incentive Weaker?

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Abstract

Extant literature shows that Arrow's famous result—a secure monopolist gains less from a nondrastic process innovation than would a competitive firm—does not always extend to nondrastic *product innovations*. If the new product is horizontally differentiated, the monopolist can have a greater incentive to add the new product than a firm that would face competition from the old product; but the monopolist's incentive to add the new product cannot be greater if the new product is vertically differentiated with higher quality than the old. This paper compares the incentives when the new product is vertically differentiated but of *lower quality*, a common case empirically. We show that, as with horizontal differentiation, the monopolist can have the greatest incentive to add the new product. However, in all the cases analyzed, consumer welfare (though not total welfare) is lower under monopoly, even when only the monopolist would add the new product. Our analysis also helps clarify why the ranking of incentives depends on the type of product differentiation and on whether the market is covered or not.

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1. Introduction

The effect of market structure on incentives for innovation is of longstanding interest to economists and policymakers. In a seminal early analysis, Arrow (1962) considered a perfectly patentable *process innovation* that reduces the constant marginal cost for an existing homogeneous product. He compared the innovator's gross gain (before subtracting the fixed cost of obtaining the innovation) under two alternative market structures. One is a secure monopolist.[1] In the other market structure, homogeneous Bertrand competitors initially set price at their common marginal cost. The innovator obtains a lower marginal cost but faces potential competition from the higher-cost firms. If the innovation is drastic—the old technology does not constrain the innovator's profit—the incentive to innovate clearly is lower under monopoly than under competition. In both cases, the innovator earns the unconstrained monopoly profit with the innovation, whereas only the monopolist earns positive profit without the innovation.

If instead the innovation is *nondrastic*, then there is an opposing effect: post-innovation profit will be greater to a monopolist that also controls the old technology than to an innovator that is constrained by competition from the old technology. Arrow (1962) nevertheless showed that the monopolist's gross gain from a process innovation again is lower because the reduction in marginal cost applies to a smaller industry output than under perfect competition (see also Tirole, 1988).

However, for a *nondrastic product innovation*—yielding a differentiated substitute product that coexists with the old product—total output is no longer sufficient to rank the innovation incentives across alternative market structures. [2] The innovator's gain will depend also on the output mix and prices. Compared to a firm that initially earns no profit (an existing competitive firm or a new entrant), a monopolist's gain from adding a new product is subject to opposing forces. The monopolist diverts profit from its old product (the "replacement effect"), but can coordinate its product prices to increase overall profit. In particular, it might raise the price of the old product to boost profit from the new product. A priori, either effect might dominate—diversion or coordination—depending on the specifics of product differentiation.

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[1] Its gain comes solely from using the new technology, unlike a threatened monopolist that has an added "defensive" incentive to preempt and deny the innovation to a potential entrant (Gilbert and Newbery, 1982). We shall return to this distinction.

[2] A product innovation may be drastic under one market structure but nondrastic under another. For example, perfectly competitive firms are willing to sell the old good at any price above marginal cost, but a monopolist over the old and new goods may prefer to eliminate the old good in order to increase profit from its new good. (Hereafter, we shall use "product" and "good" interchangeably.)

Two papers close to ours that compare product innovation incentives in a setting similar to Arrow's are Greenstein and Ramey (1998, hereafter G&R) and Chen and Schwartz (2013, hereafter C&S). G&R consider vertically differentiated products, with the innovation being a higher-quality product.[3] In C&S, the innovation yields a horizontally differentiated product à la Hotelling. G&R find that, when the innovator is a monopolist also over the old product, its gain from innovating (G^m) is equal to the gain when the innovator instead faces perfect competition from the old product (G^c), whereas C&S find $G^m > G^c$, reversing Arrow's ranking for nondrastic process innovations.

Our paper considers an innovation that yields a vertically-differentiated product of *lower quality*.[4] In practice, a product innovation certainly can take this form: stripping out costly features or using lower-grade materials to create a more affordable product offers consumers beneficial variety. For example, the cost and quality of a disposable camera are lower than for a regular camera,[5] and the same is true of cubic zirconia compared to diamonds.[6] Other potential examples include frozen vs. fresh vegetables, and clothing made from polyester vs. natural fibers such as cotton. Also, it is common for manufacturers of smartphones or consumer electronics to introduce lower-quality models at lower prices.[7]

The relative incentives to add the new good under monopoly vs. competition may well differ when the quality of the new good is lower rather than higher. Under constant marginal cost and perfect competition for the old good, its price always remains constant post-innovation. The same holds under monopoly but only when the monopolist adds a higher-quality good (as in G&R, who found $G^m = G^c$): it keeps the price of the old, low-quality good constant since raising it would cause low-valuation consumers to drop out. However, when the new good is of lower quality, low-

[3] The analysis of vertical differentiation was developed by Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983). See also Tirole (1988).

[4] We thank Shane Greenstein for suggesting this scenario in correspondence with one of the authors.

[5] Cameras were expensive, and would often be left at home for safety. In the late 1940s, H. M. Stiles invented a way to enclose 35mm film in an inexpensive enclosure without the expensive precision film transport mechanism. A company called Photo-Pac produced such a cardboard camera in 1949, and a French company FEX introduced a disposable bakelite plastic camera in 1966, but neither became popular. The currently familiar and popular disposable camera was released by Fujiilm in 1986. See Disposable camera, Wikipedia.

[6] The synthesized material cubic zirconia (CZ) is hard and usually colorless, but may be made in a variety of different colors. Because of its low cost, durability, and close visual likeness to diamond, synthetic cubic zirconia has remained the most gemologically and economically important competitor for diamonds since commercial production began in 1976. See Cubic zirconia, Wikipedia.

[7] Lower-quality products sometimes do not reduce costs and are used only as a form of second-degree price discrimination (Deneckere and McAfee, 1996). However, there are also numerous situations where reducing quality does yield savings in variable costs, as in some of the above examples.

valuation consumers will purchase the new good rather than the old. Thus, a monopolist that adds the new good could raise the price of its old good without losing total sales, potentially increasing its gain from the innovation.

We compare innovation incentives across the same three market structures as G&R and C&S.

> *Monopoly*: Initially a high-quality product H is sold by a monopolist. The monopolist is the only firm that can develop and sell a lower-quality product L (secure monopolist). The innovator's gross gain G^m is the difference between monopoly profit with both products vs. with only the old product H.
>
> *Competition*: Initially product H is sold by a perfectly competitive industry. A potential entrant can develop and sell product L.[8] The innovator's gross gain G^c is its post-innovation profit from good L when competing against good H priced at marginal cost.
>
> *Duopoly (post-innovation)*: Initially product H is sold by a monopolist. A potential entrant is the only firm that can develop and sell product L. The innovator's gross gain, G^d, is its profit from good L when competing against the monopolist seller of good H.[9]

The Competition and Duopoly market structures ("regimes") span the polar extremes of rivalry that an innovator might face from the old good: the latter is sold by perfectly competitive firms or by a monopolist. Thus, G^c and G^d are, respectively, the lowest and highest incentive to add the new good for a firm that is not the initial monopolist.

Note that our Monopoly regime assumes a secure monopolist, not a preempting monopolist as in Gilbert and Newbery (1982). There, the monopolist expects that if it does not acquire the innovation an entrant will, leading to duopoly profits (π_H^d, π_L^d). A preempting monopolist thus has an added incentive to acquire the innovation: to prevent a fall in its profit from the monopoly level (π_H^m) to the duopoly level (π_H^d). Let π_{HL}^m denote the monopoly profit with both goods. A preempting monopolist's incentive is $G^{mp} = \pi_{HL}^m - \pi_H^d$ while for a secure monopolist $G^m = \pi_{HL}^m - \pi_H^m$, hence $G^{mp} - G^m = \pi_H^m - \pi_H^d > 0$. Gilbert and Newbery showed that the incentive of a preempting monopolist to acquire the innovation exceeds an entrant's incentive $(G^d = \pi_L^d)$, whenever industry profit is greater under monopoly than under duopoly, $G^{mp} - G^d = \pi_{HL}^m - (\pi_H^d + \pi_L^d) > 0$. However, since $G^{mp} > G^m$, Gilbert and Newbery's result is not sufficient to rank G^m versus G^d.

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[8] The "entrant" may be a new firm or one of $n \geq 3$ homogeneous Bertrand competitors in product H. In either case, the entrant earns zero profit initially from H and would face a price $P_H = c_H$ after bringing product L.

[9] The Competition and Monopoly regimes were considered by Arrow (1962), but for a process innovation.

While our main goal is to advance the understanding of product-innovation incentives under alternative market structures, comparing G^m to G^c or G^d also can have policy relevance in certain scenarios. For example, suppose that a secure monopolist over the old good holds a patent vital for developing the new good. A policy intervention requiring the monopolist to divest the blocking patent to a single firm, so as to promote competition to the old product, would change the incentive to innovate from G^m to G^d. Instead, suppose that the same blocking patent is also responsible for sustaining the monopoly over the old good. An intervention to void the patent could induce perfect competition in the old good and change the incentive to add the new product from G^m to G^c.

Comparing G^m to G^c or G^d also can be relevant in a (perhaps far-fetched) merger scenario. Suppose that all the Bertrand competitors in the old product propose to merge, and only those firms are capable of adding the new product. If there are at least three such firms, approving their merger would change the incentive to innovate from G^c to G^m. If instead there are only two such firms, then if one of them adds the new good it will abandon the old good (Judd, 1985), hence the incentive to innovate is G^d without the merger and G^m with the merger.[10] Understanding how incentives for product innovation compare under various regimes can help assess the merits of some policy interventions against monopoly.

Returning to prior work, G&R found $G^m = G^c < G^d$, hence a (secure) monopolist's incentive to add a *higher-quality* product is no higher and is sometimes lower than for a firm that earns zero profit initially and would face rivalry from the old, lower-quality product. Their result holds for general distributions of consumer types (i.e., willingness to pay for increased quality) regardless of whether the market is *not covered* (i.e., a small reduction in the price of the low-quality product would expand total sales), as they assume, or *covered* (as we show in Proposition 2(i)). When the new good instead is of *lower quality* and the market is not covered, we also find $G^m = G^c$ if the distribution of consumer types is uniform (our Proposition 1), but otherwise $G^m < G^c$ or $G^m > G^c$ are possible (Proposition 2(ii)). However, in our examples it remains true that $G^m < G^d$, as in G&R.

In contrast, C&S found that $G^m > G^d$ can occur in the standard Hotelling model of horizontal differentiation. We thus also analyze the case where the market is *covered* (i.e., a small reduction in the price of the low-quality product would not expand total sales). One purpose is to explore whether $G^m > G^d$ becomes possible also in our setting. If so, the lower-quality product innovation might occur only under Monopoly (recall that G^d is the highest profit that could be earned by an

[10] Recent analyses of the effects of mergers on incentives for product innovation under uncertainty include Giulio, Langus and Valletti (2018) and Motta and Peitz (2020).

entrant when competing against the old good). Retaining the C&S assumption that consumer types are uniformly distributed allows us to focus on how innovation incentives might depend on the nature of product differentiation. When the new good is of lower quality and the market is covered, we find $G^m > G^c$ (Proposition 3), as in C&S. Moreover, $G^m > G^d$ (Proposition 4), which also can occur in C&S.

Turning to welfare, consumers are always worse off under Monopoly, even when the product innovation occurs only under Monopoly (Proposition 5). However, in the latter cases total welfare can be lower or higher under Monopoly (Proposition 6).

Section 2 of the paper presents the model. We focus on the case where the innovation is a lower-quality product. In Section 3, we compare innovation incentives across the three alternative market structures when the market is always not covered, and in Section 4 when it is always covered. Section 5 discusses welfare, and Section 6 offers concluding remarks, including potential policy implications of our findings.

2. Vertical differentiation model and alternative market structures

Initially only product H is sold to consumers. A differentiated substitute, product L, potentially can be developed. We assume that the potential new product L has lower quality and lower unit cost than product H , i.e., $v_L < v_H$ and $c_L < c_H$.[11] In addition, there is a fixed cost f to add product L.

Each consumer demands at most one unit. Consumers are heterogeneous in their willingness to pay for increased quality. Formally, let θv_i denote the (maximum) willingness to pay for product i of a consumer of type θ. Except in Section 3.3, the parameter θ is uniformly distributed across the population of consumers between $\underline{\theta}$ and $\overline{\theta} = \underline{\theta} + 1$. The main purpose of the uniform-distribution assumption is to let us characterize conditions under which the innovation is non-drastic and the market is covered versus not covered. We normalize the mass of consumers to one.

Let P_i denote the price of product i. A consumer of type θ chooses among three options (initially only between the first two) that yield the following levels of surplus S:

Buy none: $S = 0$

Buy product H: $S = \theta v_H - P_H$

Buy product L: $S = \theta v_L - P_L$

[11] G&R instead assume that the new product has higher quality and higher cost than the old product.

It follows that, if in equilibrium both products are sold, then the consumer type who is indifferent between buying product H and buying product L is:

$$\theta_{HL} = (P_H - P_L)/(v_H - v_L)$$

and the consumer type who is indifferent between buying product L and not buying any of the two products is:

$$\theta_{L0} = P_L/v_L$$

Consumers with type $\theta \in [\underline{\theta}, \theta_{L0})$ do not buy any products, those with $\theta \in [\theta_{L0}, \theta_{HL})$ buy good L, and those with $\theta \in \left[\theta_{HL}, \overline{\theta}\right]$ buy good H.

If instead only good H is sold, the consumer type who is indifferent between buying good H and not buying it is:

$$\theta_{H0} = P_H/v_H$$

Consumers with type $\theta \in [\underline{\theta}, \theta_{H0})$ do not buy the product, and those with $\theta \in \left[\theta_{HO}, \overline{\theta}\right]$ buy it.

The innovator obtains a monopoly over product L but may face competition from product H. We consider the three alternative market structures (Monopoly, Competition, and Duopoly post-innovation) that were analyzed in G&R and C&S, and that we described in Section 1. We rank the incentive to innovate across these three market structures, where "incentive to innovate" in each market structure means the gross profitability of adding the new product before subtracting the fixed cost f. The innovator's gross gain is denoted G^c under Competition, G^m under Monopoly and G^d under Duopoly.

Throughout, we consider parameter values such that the product innovation is *nondrastic*: both products enjoy positive sales after product L is added. Remarks 1-3 also will apply throughout.

Remark 1. $G^d > G^c$.

The above ranking follows because the innovator's initial profit is zero in both regimes, and its post-innovation profit will be larger if good H is sold by a monopolist that will set $P_H > c_H$ than if good H is sold by competitive firms that would set $P_H = c_H$.

Remark 2. If parameter values are such that the market is *not covered post-innovation under Competition*, then it also will not be covered post-innovation under Monopoly or Duopoly, as well as pre-innovation.

The logic for Remark 2 runs as follows: Under Competition $P_H = c_H$, and post-innovation P_L will be lower than in the alternative market structures. Under Duopoly, the innovator competes against good H sold by a monopolist at $P_H > c_H$, hence the innovator sets P_L higher than under Competition. Under Monopoly, both goods are sold by the same firm, which has a further incentive to raise the price of good L because it internalizes the diversion of sales to good H. Thus, if some (low type) consumers choose not to buy under Competition post-innovation, the same will be true when they face higher prices post-innovation. A fortiori, the market also will not be covered pre-innovation, because consumers would lack the option of good L, and the price of good H would be no lower than c_H (under Competition, $P_H = c_H$, while under Monopoly $P_H > c_H$).

By similar logic, consumers' *worst* option is pre-innovation (when only good H is available) and under Monopoly, implying:

Remark 3. If parameter values are such that the market is *covered pre-innovation under Monopoly*, it also will be covered pre-innovation under Competition, and post-innovation under all regimes.

We will compare the innovator's incentives G^c, G^m and G^d when (a) the market is always not covered or (b) always covered.[12] Remarks 2 and 3 will enable us to provide a sufficient condition for each case. In either case, if $G^m = G^c$ then $G^m < G^d$ by Remark 1.

3. Innovation incentives when the market is not covered

Here and in Section 4, we make the following assumptions:

Assumption 1 $0 < c_L < c_H$, $0 < v_L < v_H$ and $\underline{\theta} \geq 0$

Assumption 2 $c_L/v_L < c_H/v_H < \overline{\theta}$

Assumption 3 $\underline{\theta}(v_H - v_L) < c_H - c_L < \overline{\theta}(v_H - v_L)$

Assumption 1 says that product L has lower cost and lower quality than product H, and that utility increases in quality. Assumptions 2 and 3 ensure that product H has positive sales initially, that product L will have positive sales, and that the innovation is nondrastic, i.e., there will be sales of product H after product L is introduced.

In this section, we assume $\underline{\theta} < (c_H/v_H + c_L/v_L)/2$ so the market is not covered post-innovation under Competition. Thus, from Remark 2, the market is not covered in all cases, as in G&R. (In Section 4, the market will be covered in all cases, as in C&S.)

[12] An extension could consider hybrid cases. Our goal here is to illustrate the possibilities while avoiding a tedious taxonomy.

3.1. Competition regime

Both before and after the innovation, product H is sold at the competitive price $P_H = c_H$. It follows that pre-innovation $\theta_{H0} = c_H/v_H$ and consumers with $\theta < \theta_{H0}$ do not buy product H while consumers with $\theta \geq \theta_{H0}$ buy product H.[13]

Post-innovation, $\theta_{HL} = (c_H - P_L)/(v_H - v_L)$ and $\theta_{L0} = P_L/v_L$. Consumers with $\theta < \theta_{L0}$ do not buy any of the two products, consumers with $\theta_{L0} \leq \theta < \theta_{HL}$ buy product L, and consumers with $\theta \geq \theta_{HL}$ buy product H. The entrant sets P_L to maximize $(P_L - c_L)Q_L$ where $Q_L = \theta_{HL} - \theta_{L0}$. Thus, the monopoly price of product L set by the entrant when it faces a competitive supply of product H is $P_L = c_L + (c_H/v_H - c_L/v_L)v_L/2 \ (= P_L^{cm})$, and the entrant earns a gross profit equal to:[14]

$$G^c = \frac{(c_H v_L - c_L v_H)^2}{4 v_H v_L (v_H - v_L)} \tag{1}$$

3.2. Monopoly regime

The monopolist initially sets P_H to maximize $(P_H - c_H)Q_H$ where $Q_H = \overline{\theta} - \theta_{H0}$ and $\theta_{H0} = P_H/v_H$. The monopoly price is $P_H^m = [\overline{\theta} v_H + c_H]/2$ and the monopoly profit is $\pi_H^m = [\overline{\theta} v_H - c_H]^2/(4v_H)$.[15]

Post-innovation, the monopolist sets P_H and P_L to maximize $(P_H - c_H)Q_H + (P_L - c_L)Q_L$ where $Q_H = \overline{\theta} - \theta_{HL}$ and $Q_L = \theta_{HL} - \theta_{L0}$. The monopoly prices are $P_i^{mm} = [\overline{\theta} v_i + c_i]/2$ post-innovation.[16] Note that the monopolist does not change the price of product H post-innovation (i.e., $P_H^{mm} = P_H^m$).

The difference between the monopolist's profit post-innovation and pre-innovation is equal to:

$$G^m = \frac{(c_H v_L - c_L v_H)^2}{4 v_H v_L (v_H - v_L)} \tag{2}$$

The following result follows immediately from comparing (1) and (2).

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[13] Assumption 2 and $\underline{\theta} < (c_H/v_H + c_L/v_L)/2$ imply $\underline{\theta} < \theta_{H0} < \overline{\theta}$.

[14] Assumptions 1-3 and $\underline{\theta} < (c_H/v_H + c_L/v_L)/2$ imply $P_L^{cm} > c_L$ and $\underline{\theta} < \theta_{L0} < \theta_{HL} < \overline{\theta}$. We will use the superscripts "cm" and "mm", to distinguish the post-innovation monopoly price of product L when the old product H is supplied by competitive firms *versus* by the monopolist.

[15] Our assumptions imply $P_H^m > c_H$ and $\underline{\theta} < \theta_{H0} < \overline{\theta}$.

[16] Our assumptions imply $P_i^{mm} > c_i$ and $\underline{\theta} < \theta_{L0} < \theta_{HL} < \overline{\theta}$. We will use the superscripts, "mm" and "m", to distinguish the post-innovation monopoly price of product H when the monopolist sells both goods *versus* the pre-innovation monopoly price of product H when the monopolist sells only one good.

Proposition 1. Assume $\underline{\theta} < (c_H/v_H + c_L/v_L)/2$ so that the market is not covered in all regimes. The incentive to innovate and introduce a lower-quality product L is the same for an innovator under the Monopoly or Competition regimes, i.e., $G^m = G^c$, and hence $G^m < G^d$.

The above result assumes that consumer types θ are uniformly distributed. As we will show in Section 3.3., the result $G^m = G^c$ when the market is not covered holds for general distribution functions if the new good, instead, is of *higher* quality than the old good (as in G&R). However, if the new good is of lower quality as in our setting (and the market is not covered), the result $G^m = G^c$ holds for the uniform distribution but not for general distribution functions.

For the uniform distribution case, Figure 1 provides some intuition for why the monopolist does not change the price of good H after adding good L (i.e., $P_H^{mm} = P_H^m$) when the market is not covered. Pre-innovation, the marginal consumer type is θ_{H0}. The monopolist does not want to raise P_H any higher because it would lose some consumers to the outside good, and that loss would exactly offset the gain on the remaining consumers of good H. Suppose the monopolist maintains P_H constant after adding good L (at the conditionally-optimal price, $P_L(P_H^m) < P_H^m$). Some new consumers buy good L (the types between θ_{L0} and θ_{H0}) while others switch to good L from good H (the types between θ_{H0} and θ_{HL}). Unlike when the old good had the lower quality, the monopolist now *could* raise the price of the old good without losing total sales: raising P_H would only divert consumers to good L instead of losing them to the outside good. Thus, raising P_H would cause a smaller loss of profit from reduced sales of good H than it would pre-innovation.[17] However, the monopolist *also would gain less* from raising P_H because, with the introduction of good *L,* fewer consumers are buying good H. With the uniform distribution of consumer types, it turns out that the (smaller) loss is still equal to the (smaller) gain, maintaining the optimality of the initial, pre-innovation price P_H.

[17] The monopolist still would incur a loss from the reduced sales of good H, because the margin on good L is smaller than on good H and, furthermore, for a given increase in P_H, the diversion to good L would be larger than the diversion to the outside good pre-innovation.

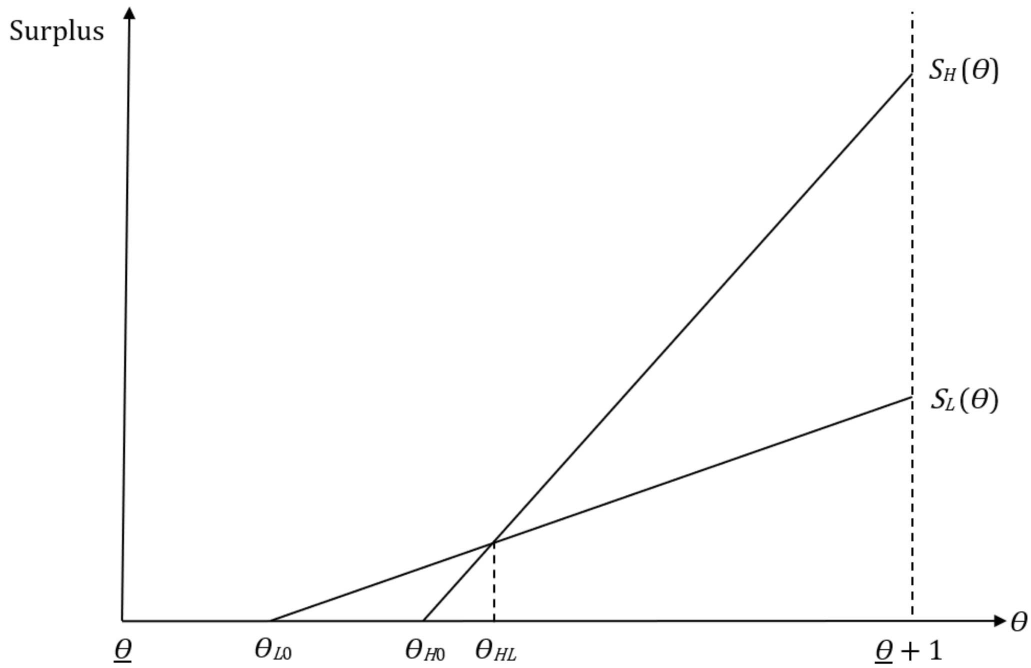

Figure 1. New Good *L* has Lower Quality and Market is Not Covered

3.3. Non-uniform distributions of consumer types

Suppose θ is distributed on the interval $[\underline{\theta}, \overline{\theta}]$, where $\overline{\theta} > \underline{\theta} \geq 0$, with cumulative distribution function $F(x) \equiv prob\{\theta \leq x\}$. (The uniform distribution case has $\overline{\theta} = \underline{\theta} + 1$ and $F(x) = x - \underline{\theta}$.)

Consider a monopolist selling both products and assume that the market is not covered. The monopolist's profit function is given by:

$$\tilde{\pi}(P_L, P_H) = (P_L - c_L)\left[F\left(\frac{P_H - P_L}{v_H - v_L}\right) - F\left(\frac{P_L}{v_L}\right)\right] + (P_H - c_H)\left[1 - F\left(\frac{P_H - P_L}{v_H - v_L}\right)\right]$$

Let (P_L^{mm}, P_H^{mm}) denote the monopoly prices that maximize $\tilde{\pi}(P_L, P_H)$.

3.3.1. New good has higher quality than old good

Define $\Delta z \equiv z_H - z_L$ for $z = P, v, c$. We then can rewrite the profit function as:

$$\pi(P_L, \Delta P) = (P_L - c_L)\left[1 - F\left(\frac{P_L}{v_L}\right)\right] + (\Delta P - \Delta c)\left[1 - F\left(\frac{\Delta P}{\Delta v}\right)\right]$$

Note that $\pi(P_L, \Delta P)$ is additively separable in the price of the low-quality good, P_L, and the price premium for the high-quality good, $\Delta P = P_H - P_L$, that is:

$$\pi(P_L, \Delta P) = \pi_L(P_L) + \pi_\Delta(\Delta P)$$

where

$$\pi_L(P_L) = (P_L - c_L)\left[1 - F\left(\frac{P_L}{v_L}\right)\right]$$

$$\pi_\Delta(\Delta P) = (\Delta P - \Delta c)\left[1 - F\left(\frac{\Delta P}{\Delta v}\right)\right]$$

In words, $\pi_L(P_L)$ is the profit from selling the low-quality good when the high-quality good is not available, and $\pi_\Delta(\Delta P)$ is the incremental profit from selling also the high-quality good. Let $(P_L^{mm}, \Delta P^{mm})$ denote the monopoly prices that maximize $\pi(P_L, \Delta P)$. Clearly, $P_H^{mm} = P_L^{mm} + \Delta P^{mm}$.

Several key results follow directly from this separability property of $\pi(P_L, \Delta P)$.[18]

Result 1. When the monopolist sells both the low-quality and high-quality goods, the monopoly price of the low-quality good is the same as when the monopolist sells only the low-quality good.
Proof. For any ΔP, $P_L = P_L^{mm}$ maximizes $\pi(P_L, \Delta P)$ and also $\pi_L(P_L)$.

Result 1 shows that, when a monopolist initially sells a low-quality good and then introduces a new, high-quality good, the monopolist does not change the price of the old, low-quality good.

Result 2. When the monopolist sells both the low-quality and high-quality goods, the optimal price premium ΔP^{mm} for the high-quality good and the maximized incremental profit $\pi_\Delta(\Delta P^{mm})$ from selling also the high-quality good do not depend on the price P_L of the low-quality good.
Proof. For any P_L, $\Delta P = \Delta P^{mm}$ maximizes $\pi(P_L, \Delta P)$ and also $\pi_\Delta(\Delta P)$.

Result 2 implies that the monopolist's incentive to introduce the new, high-quality good would not change if the price of the old, low-quality good was constrained to be equal to, say, marginal cost (i.e., $P_L = c_L$). The next result then is straightforward.

Result 3. When the monopolist sells both the low-quality and high-quality goods, the incremental profit G^m that the monopolist earns from selling also the high-quality good is equal to the profit G^c that an entrant would earn from selling the high-quality good while facing a perfectly competitive supply of the low-quality good.
Proof. $G^m = \pi_\Delta(\Delta P^{mm})$ when $P_L = P_L^{mm}$, and $G^c = \pi_\Delta(\Delta P^{mm})$ when $P_L = c_L$. Result 2 then implies $G^m = G^c$.

[18] The ensuing results are present in G&R, but they frame their argument in terms of quantities rather than prices. Our reformulation helps to contrast our case where the new good is of lower quality, Section 3.3.2.

Result 3 shows that, when a monopolist initially sells a low-quality good, its incentive to introduce a new, high-quality good is the same as the incentive of an entrant that would be selling the high-quality good while facing a perfectly competitive supply of the low-quality good.

This result, $G^m = G^c$ when the new product is of higher quality, also holds if one assumes that the market is *covered*. Intuitively, the monopolist then sets $P_L = \underline{\theta} v_L$ and its maximized incremental profit G^m from selling also good H would not change if instead it were to set $P_L = c_L$ (by Result 2) and hence is equal to the profit G^c that an entrant would earn in the Competition regime.

3.3.2. New good has lower quality than old good

Let us now rewrite $\tilde{\pi}(P_L, P_H)$ as follows:

$$\tilde{\pi}(P_L, P_H) = (P_H - c_H)\left[1 - F\left(\frac{P_H}{v_H}\right)\right]$$

$$- \big(P_H - c_H - (P_L - c_L)\big)\left[F\left(\frac{P_H - P_L}{v_H - v_L}\right) - F\left(\frac{P_H}{v_H}\right)\right]$$

$$+ (P_L - c_L)\left[F\left(\frac{P_H}{v_H}\right) - F\left(\frac{P_L}{v_L}\right)\right]$$

The first term is the profit from selling the high-quality good when the low-quality good is not available, the second term is the (negative) incremental profit due to some consumers switching from the high-quality good to the low-quality good, and the third term is the (positive) incremental profit due to some consumers switching from the outside good to the low-quality good.

Let $\pi_H(P_H) = (P_H - c_H)\left[1 - F\left(\frac{P_H}{v_H}\right)\right]$ denote the profit from selling the high-quality good when the low-quality good is not available, so that $\tilde{\pi}(P_L, P_H) - \pi_H(P_H)$ is the incremental profit from selling also the low-quality good. From above, we have:

$$\tilde{\pi}(P_L, P_H) - \pi_H(P_H) = -(\Delta P - \Delta c)F\left(\frac{\Delta P}{\Delta v}\right)$$

$$+(P_H - c_H)F\left(\frac{P_H}{v_H}\right) - (P_H - \Delta P - c_L)F\left(\frac{P_H - \Delta P}{v_L}\right)$$

where $\Delta P = P_H - P_L$ is the price discount for the low-quality good.

This shows that the incremental profit from adding the low-quality good, $\tilde{\pi}(P_L, P_H) - \pi_H(P_H)$, depends on both ΔP and P_H, while we showed in the previous section that $\tilde{\pi}(P_L, P_H) - \pi_L(P_L)$ depends only on ΔP. Therefore, if the new good is of lower quality than the old good, one cannot use the same approach as in the previous section to show that $G^m = G^c$ for general distribution functions. We will thus test the robustness of the result $G^m = G^c$ (when the market is not covered) by considering a family of distribution functions with linear densities that contains the uniform distribution as a special case.[19]

Assume θ is distributed on $[0,1]$ with cumulative distribution function:

$$F(x) = x[1 - b\,(1 - x)/2]$$

where $b \in [-2,2]$. The density function is $F'(x) = 1 - b(1 - 2x)/2$ for $x \in [0,1]$, and is positive if and only if $b \in [-2,2]$. These density functions are linear and intersect at $x = 1/2$ and $F'(1/2) = 1$. When $b < 0$, low consumer types are more frequent than high consumer types, and the reverse holds when $b > 0$. The uniform distribution corresponds to $b = 0$. Assume further the following parameter values:

$$c_L = 1, \;\; v_L = c_H = 3, \quad v_H = 6$$

These parameter values satisfy the assumptions of Proposition 1 and, therefore, we know that $G^m = G^c$ if $b = 0$. Table 1 summarizes the results for five different values of the parameter b, i.e., $b \in$ {-2, -1, 0, 1, 2}.[20] It shows that $G^m < G^c$ if $b \in$ {-2, -1} (density is greater for lower than higher consumer types θ) and conversely that $G^m > G^c$ if $b \in$ {1, 2}. In all cases, $G^d > \max\{G^m, G^c\}$.

	$\boldsymbol{b = -2}$	$\boldsymbol{b = -1}$	$\boldsymbol{b = 0}$	$\boldsymbol{b = 1}$	$\boldsymbol{b = 2}$
G^m	0.037	0.038	0.042	0.048	0.055
G^c	0.042	0.042	0.042	0.042	0.042
G^d	0.075	0.100	0.122	0.144	0.164

Table 1. Innovation incentives under linear density functions

The above results are summarized as follows:

[19] Future work could consider alternative distributions.

[20] The calculations were implemented in *Mathematica* and are available upon request. Those calculations also show how the monopolist changes the price of the old good, P_H, after adding good L: it raises P_H if $b = 1$ or $b = 2$, lowers P_H if $b = -1$, and does not change P_H if $b = 0$ or $b = -2$. The fact that G^c takes the same value (0.042) for all values of b hinges on our particular parameter values.

Proposition 2. Under general distributions of consumer types:

(i) When the new product has higher quality, the incentive to innovate is the same under the Monopoly or Competition regimes, i.e., $G^m = G^c$, regardless of whether the market is covered or not covered.

(ii) When the new product has lower quality and the market is not covered, each of the following rankings is possible: $G^m < (=) > G^c$.

4. Innovation incentives when the market is covered

In this section, we assume $\underline{\theta} > 1 + c_H/v_H$ so that the market is covered pre-innovation under Monopoly. By Remark 3, it is covered both pre-innovation and post-innovation for all market structures (as in C&S). Consumer types θ are assumed uniformly distributed on $[\underline{\theta}, \underline{\theta} + 1]$.

4.1. Competition regime

As in Section 3, under Competition $P_H = c_H$ both before and after the innovation. Pre-innovation, since $\theta v_H > c_H$ for all θ, all consumers buy good H.

Post-innovation, consumers with $\theta < \theta_{HL} = (c_H - P_L)/(v_H - v_L)$ switch to good L, the entrant's profit is $(P_L - c_L)Q_L$ where $Q_L = \theta_{HL} - \underline{\theta}$, so the entrant sets $P_L = c_L + (c_H - c_L - \underline{\theta}(v_H - v_L))/2$ $(= P_L^{cm})$.[21] The entrant's maximized profit, and hence its gross gain from the innovation, is:

$$G^c = \frac{(c_H - c_L - \underline{\theta}(v_H - v_L))^2}{4(v_H - v_L)} \tag{3}$$

4.2. Monopoly regime

When $\underline{\theta} > 1 + c_H/v_H$, the monopolist initially sells product H at price $P_H^m = \underline{\theta} v_H$, all consumers buy product H, and the monopolist earns a profit $\pi_H^m = \underline{\theta} v_H - c_H$.

Post-innovation, the market remains covered: the monopolist sells good L at price $P_L^{mm} = \underline{\theta} v_L$ and consumers with $\theta < \theta_{HL} = (P_H - \underline{\theta} v_L)/(v_H - v_L)$ will switch to good L. The monopolist thus sets P_H to maximize $(P_H - c_H)Q_H + (\underline{\theta} v_L - c_L)Q_L$ where $Q_H = \overline{\theta} - \theta_{HL}$ and $Q_L = \theta_{HL} - \underline{\theta}$, hence the price of good H increases to $P_H^{mm} = (c_H - c_L + v_H - v_L + \underline{\theta}(v_H + v_L))/2$ post-innovation.[22]

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[21] Our assumptions imply $c_L < P_L^{cm} < \underline{\theta} v_L$ and $\underline{\theta} < \theta_{HL} < \overline{\theta}$.

[22] Our assumptions imply $\underline{\theta} < \theta_{HL} < \overline{\theta}$, $P_H^{mm} > P_H^m > c_H$, $P_L^{mm} > c_L$, and that the monopolist has no incentive to raise the price of product L.

The difference between the monopolist's post-innovation profit and its pre-innovation profit is

$$G^m = \frac{(c_H - c_L - (\underline{\theta} - 1)(v_H - v_L))^2}{4(v_H - v_L)} \tag{4}$$

The following result follows immediately from comparing (3) and (4).

Proposition 3. Assume $\underline{\theta} > 1 + c_H/v_H$ so that the market is covered in all regimes. The incentive to innovate and introduce a lower-quality product L is greater for a monopolist of product H than for an entrant that faces perfect competition from product H, i.e., $G^m > G^c$.

Recapping, if the new good is of higher quality and the market is not covered, G&R found that $G^m = G^c$ for general distributions of consumer types. We showed that their result also holds if the market is *covered* (Proposition 2(i)). In contrast, if the new good is of *lower* quality and the market is not covered, we found $G^m = G^c$ for uniform distributions of consumer types (Proposition 1) but $G^m > G^c$ and $G^m < G^c$ are possible under general distributions (Proposition 2(ii)). Proposition 3 assumes a uniform distribution of consumer types and shows that $G^m > G^c$ if the new good is of *lower* quality and the market is covered.

The above results are linked to pricing. When the old good is supplied competitively, its price is always equal to its (constant) marginal cost, hence it does not change after the new good is added. Interestingly, in all cases where we found $G^m = G^c$, the monopolist also leaves the price of the old good unchanged after it adds the new good. By contrast, when $G^m > G^c$ (which requires the new good to be of lower quality), the monopolist raises the price of the old good H after adding good L. This raises the profitability from adding good L compared to the profitability to an entrant that adds good L but faces competition from good H priced at (constant) marginal cost.

Figure 2 illustrates the monopolist's optimal pricing when adding good L. Initially, P_H was set to yield zero surplus to the lowest-type consumer $\underline{\theta}$. Holding P_H constant, once the lower-quality good L is added (at its conditionally-optimal price P_L) low-type consumers would switch to it and all types would earn strictly positive surplus. By raising P_H and P_L equally the monopolist maintains the allocation of consumers between the two goods (the same indifferent type θ_{HL}) and increases profits. It raises both prices until the lowest type $\underline{\theta}$ earns zero surplus (dashed lines in Figure 2).

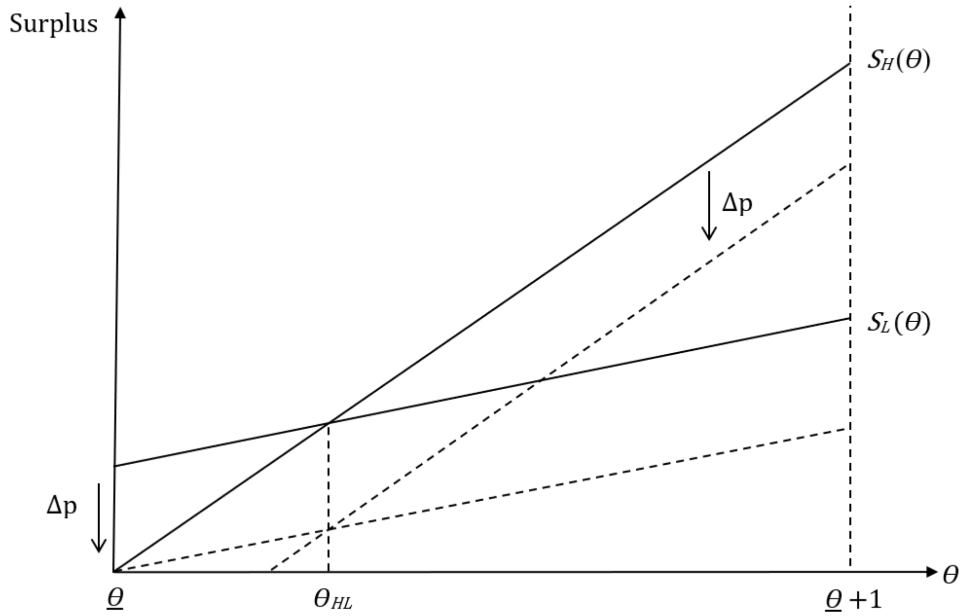

Figure 2. New Good *L* has Lower Quality and Market is Covered

Similar logic applies in the Hotelling framework of C&S. When covering the market with only good A, the monopolist sets P_A to yield zero surplus for the most distant consumer (located at the other end of the line, $x = 1$). Suppose the monopolist adds good B (at $x = 1$) and sets its price at the optimal level conditional on the unchanged P_A. The consumers who are distant from good A and close to good B switch to good B. Thus, the monopolist can raise P_A without losing customers, because the marginal types would not drop out but instead switch to good B. In fact, it will raise P_A as well as P_B from the prior conditionally optimal level, until the consumer who is indifferent between the two goods obtains zero surplus. Figure 3 illustrates the case of symmetric goods: the monopolist sets equal prices that yield zero surplus to the consumer located in the middle of the line.

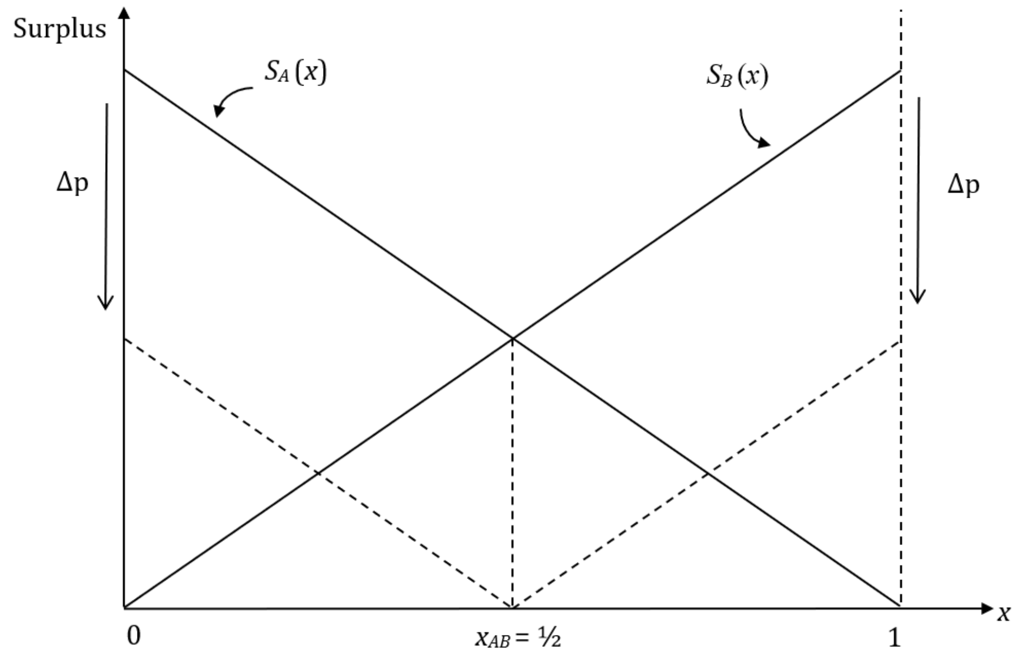

Figure 3. New Good *B* is Horizontally Differentiated and Market is Covered

4.3. Duopoly regime

We now address the case where the innovator is not the monopolist supplier of product H and, instead, is an entrant. Thus, post-innovation there is duopolistic competition between the incumbent supplier of product H and the new supplier of product L. Since $G^d > G^c$ (Remark 1), the finding in Proposition 3 that $G^m > G^c$ does not tell us how G^m compares to G^d.

The market remains covered post-innovation: the entrant sets P_L below $\underline{\theta}v_L$ and all consumers buy either product L or product H. The incumbent supplier sets P_H to maximize $(P_H - c_H)Q_H$ where $Q_H = \overline{\theta} - \theta_{HL}$, and the entrant sets P_L to maximize $(P_L - c_L)Q_L$ where $Q_L = \theta_{HL} - \underline{\theta}$. Therefore, $P_H^d = (2c_H + c_L + (\underline{\theta} + 2)(v_H - v_L))/3$ and $P_L^d = (2c_L + c_H - (\underline{\theta} - 1)(v_H - v_L))/3$.[23] The entrant's profit is equal to:

$$G^d = \frac{(c_H - c_L - (\underline{\theta} - 1)(v_H - v_L))^2}{9(v_H - v_L)} \tag{5}$$

[23] Our assumptions imply $\underline{\theta} < \theta_{HL} < \overline{\theta}$, $P_i^d > c_i$, and $P_L^d < \underline{\theta}v_L$.

The following result follows immediately from comparing (4) and (5).

Proposition 4. Assume $\underline{\theta} > 1 + c_H/v_H$ so that the market is covered in all regimes. The incentive to innovate and introduce a lower-quality product L is stronger for a monopolist of product H than for an entrant that faces competition from the monopolist of product H, i.e., $G^m > G^d$.

Recall that Duopoly is the most favorable regime for an entrant (it competes against a single seller of the old good). Proposition 4 therefore shows that a monopolist's incentive to add a lower-quality product can be larger than an entrant's incentive for any intensity of competition the entrant would face from the old product.

When the new good is of *higher* quality, we showed that $G^m = G^c$ always holds (Proposition 2(i)), which implies $G^m < G^d$ (by Remark 1). Therefore, under vertical differentiation with the market covered, the incentive to add a new good can be *smaller or larger* for the initial monopolist than for an entrant that would compete against the monopolist (Duopoly post-innovation), depending on whether the new good is of higher or lower quality. C&S similarly showed that either ranking is possible under horizontal differentiation with product asymmetries.[24] Table 2 summarizes our results and those of G&R and C&S.

[24] In their Hotelling setting (with consumers uniformly distributed on the line), C&S found $G^m > G^d$ if the new product is stronger than the old, and $G^m < G^d$ if the new product is weaker. "Stronger" means that before transport costs, all consumers value that good more than the other by an equal amount (or the marginal cost of that good is lower). This highlights a difference between vertical and horizontal differentiation when the market is covered in all cases. Under vertical differentiation, $G^m > G^d$ when the new good is of *lower* quality (our Proposition 4); whereas if the new good is of higher quality, then $G^m < G^d$. In C&S's Hotelling setting, $G^m > G^d$ when the new product is stronger (leading to a larger market share) and $G^m < G^d$ when the new product is weaker.

	Vertical Differentiation *Higher-quality new good*	*Vertical Differentiation* *Lower-quality new good*	*Horizontal Differentiation*
Market is Not Covered	$G^m = G^c$ (G&R)	$G^m = G^c$ (Proposition 1)* $G^m < (=) > G^c$ (Proposition 2(ii))	
Market is Covered	$G^m = G^c$ (Proposition 2(i))	$G^m > G^c$ (Proposition 3)* $G^m > G^d$ (Proposition 4)*	$G^m > G^c$ (C&S)* $G^m > (<)\, G^d$ (C&S)*

Table 2. Summary of Results

- The row "Market is Not Covered" assumes parameter values for which the market is not covered under Competition post-innovation, hence not covered under all other regimes (Remark 2).
  The row "Market is Covered" assumes parameter values for which the market is covered under Monopoly pre-innovation, hence covered under all other regimes (Remark 3).
- Abbreviations: G&R – Greenstein and Ramey (1998); C&S – Chen and Schwartz (2013).
- Propositions are from current paper.
- Asterisk (*) means the result holds under a uniform distribution of consumer types. All other results hold for general distribution functions.
- Throughout, $G^c < G^d$ (Remark 1). Thus, whenever $G^m = G^c$, it follows that $G^m < G^d$.

5. Welfare analysis

As noted in the introduction, policy interventions sometimes could alter the market structure from secure monopoly (only the monopolist over the old good may add the new good) to one where the innovator faces rivalry from the old good — either perfect competition or monopoly in the supply of that old good. The ranking of product innovation incentives under these alternative regimes, G^m, G^c and G^d, can affect whether innovation will occur and, hence, might affect the ranking of consumer surplus (S) and total welfare (W) across regimes.

We first compare (total) consumer surplus under Monopoly vs. the other regimes, Competition and Duopoly (post-innovation), for *vertical differentiation* in the four cases shown in Table 2: the new good is of higher or lower quality than the old good and the market is always not covered or always covered.[25] Whenever $G^m \leq G^d$, the innovation never occurs more often (i.e., for more values of the fixed cost f) under Monopoly. As expected, consumer surplus then is never higher under Monopoly and sometimes is lower. The logic is as follows.

Monopoly vs. Competition regimes. When $G^m = G^c$ ($< G^d$), the new good is added if the associated fixed cost f is below the same threshold under Competition or Monopoly. If the new good is not added (f is above the threshold), then consumer surplus under Competition is higher than under Monopoly, because the old good is priced lower under Competition. If instead the new good is added (f is below the threshold), then its price also will be lower under Competition since the innovator would compete against the old good priced at marginal cost, whereas a single monopolist over both goods would price both above marginal cost. Thus, consumer surplus is always higher under Competition than under Monopoly.

Monopoly vs. Duopoly (post-innovation) regimes. In both cases, the old good is priced at the monopoly level pre-innovation. Thus, if the new good is not added in either case ($f > G^d$) both regimes yield the same consumer surplus. However, if the new good is added in either case ($f < G^m \leq G^d$), both goods will be priced lower under Duopoly. And if the new good is added only under Duopoly ($G^m < f < G^d$), then consumer surplus under duopoly is greater than under monopoly with only the old good. Thus, consumer surplus is never lower under Duopoly than Monopoly, and is higher whenever the new good is added under Duopoly ($f < G^d$).

Suppose the innovation incentive is highest under Monopoly, $G^m > G^d > G^c$. This can occur when the new good is of lower quality and the market is covered in all regimes (Proposition 4). Consider

[25] Welfare comparisons under horizontal differentiation when the market is always covered, the remaining case in Table 1, are discussed by C&S.

values for f such that good L is added only under Monopoly, $G^m > f > G^d$. Denote by $\Delta S^m \equiv S_{HL}^m - S_H^m$ the change in consumer surplus under Monopoly from adding good L. If $\Delta S^m < 0$ then $S_{HL}^m < S_H^m = S_H^d < S_H^c$, so post-innovation Monopoly yields lower consumer surplus than either the Duopoly or Competition regimes. Figure 1 shows that indeed $\Delta S^m < 0$: S_H^m is the area under the steeper bold line, while S_{HL}^m is the area under the upper envelope of the two dashed lines, both of which lie below the bold line. Adding good L enables the monopolist to extract more consumer surplus, essentially through a form of second degree (indirect) price discrimination. (Intuitively, introducing a lower-quality good allows the monopolist to relax the individual rationality constraint and thus extract more informational rents from all types of consumers.) Thus, all consumers are harmed when the monopolist adds the lower-quality product and market coverage does not expand.[26]

The above findings are summarized as follows:

Proposition 5. Under Monopoly, consumer surplus is always lower than under Competition and is never higher than under Duopoly (post-innovation), regardless of whether the incentive to innovate and add the lower-quality product L is lower or higher under Monopoly.

Within our model, therefore, consumers are worse off under monopoly even when product innovation would only occur under monopoly.

Next, consider total welfare, consumer surplus plus profits. We focus again on the case where the incentive to add the new product L is greatest under Monopoly ($G^m > G^d > G^c$), which can occur when the market is covered. We showed that adding good L under Monopoly reduces consumer surplus, $\Delta S^m < 0$. As explained next, however, before subtracting the fixed cost f of adding product L, the monopolist's profit gain from the innovation, G^m, exceeds consumers' loss: $G^m + \Delta S^m > 0$. Therefore, the change in total welfare will depend on the size of the fixed cost f.

The logic for $G^m + \Delta S^m > 0$ runs as follows. Hold P_H fixed at the pre-innovation monopoly level $P_H^m = \underline{\theta} v_H$, with the market covered and the lowest type earning zero surplus. Now add good L at the monopolist's conditionally-optimal price P_L. Good L would yield positive surplus to the lowest consumer type $\underline{\theta}$ (and all higher types). The types between $\underline{\theta}$ and θ_{HL} would switch to good L and gain surplus (by revealed preference), while types above θ_{HL} would stay with good H. So, total

[26] By contrast, maintaining the assumption that θ is distributed uniformly, when the monopolist adds the lower-quality good and the market is *not covered*, its price for the old good remains unchanged (Section 3.2). Thus, no consumers lose, and those who buy the new good gain – some who switched from the old good, and others that constitute market expansion.

consumer surplus would increase from the innovation. And the monopolist would gain profit on the consumers who switched (otherwise it would not offer good L).[27] Therefore, total welfare would increase. Now raise P_H and P_L equally, thereby maintaining the same indifferent consumer θ_{HL}, and do so while maintaining the market covered. Production costs are unchanged, and the loss to consumers is exactly offset by a revenue gain to the monopolist. So, these price increases have no effect on total welfare. It follows that, at the equilibrium prices, $G^m + \Delta S^m > 0$.

Net of the fixed cost, the change in total welfare from adding product L under Monopoly is $\Delta W^m = G^m - f + \Delta S^m$. In order for product L to be *added only under Monopoly*, f must lie in the range $G^d < f < G^m$. (For $f < G^d$ product L is added also under Duopoly.) We thus consider the limiting case with $f = G^d$ and $\Delta W^m = G^m - G^d + \Delta S^m$. The terms G^m, G^d and ΔS^m depend on the values of the model parameters. Consider the set of parameter values that satisfy Assumptions 1-3 and $\underline{\theta} > 1 + c_H/v_H$ so the market is always covered. Within that set, there are parameter values such that $G^m - G^d + \Delta S^m < 0$ and, therefore, $G^m - f + \Delta S^m < 0$ for all $f \in \left(G^d, G^m\right)$: whenever the product innovation would occur only under Monopoly, it would reduce not only consumer welfare but also total welfare. There are also parameter values such that $G^m - G^d + \Delta S^m > 0$ and thus the product innovation would increase total welfare for sufficiently low f, but decrease it for sufficiently high f. (As $f \to G^m$ from below, the monopolist still adds good L and $\Delta W^m \to \Delta S^m < 0$.) This partition of the set of parameter values is characterized by whether $(c_H - c_L)/(v_H - v_L)$, which measures the extra cost of the high-quality product relative to its extra value, is below or above a certain threshold. The following result is proved using Mathematica (the code is available on request):

Proposition 6. Assume $\underline{\theta} > 1 + c_H/v_H$ so the market is covered in all regimes.

(i) If $(c_H - c_L)/(v_H - v_L) < \left(\underline{\theta} + \frac{17}{19}\right)$ then $G^m - G^d + \Delta S^m < 0$ and thus $\Delta W^m < 0$ for all f such that good L would be added only under Monopoly.

(ii) If instead $(c_H - c_L)/(v_H - v_L) > \left(\underline{\theta} + \frac{17}{19}\right)$ then $G^m - G^d + \Delta S^m > 0$ and thus $\Delta W^m > 0$ for f in a neighborhood above G^d, but $\Delta W^m < 0$ for f near G^m.

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[27] To see that adding good L is profitable, suppose the monopolist maintained $P_H = P_H^m = \underline{\theta} v_H$ and offered $\tilde{P}_L = P_H^m - (c_H - c_L)$. The lowest type would earn positive surplus by switching to good L since $\underline{\theta} v_L - \tilde{P}_L = c_H - c_L - \underline{\theta}(v_H - v_L) > 0$ from the first part of Assumption 3. Thus, some low types would switch to good L, with no effect on profit since $\tilde{P}_L - c_L = P_H^m - c_H$. By continuity, the monopolist could set a price P_L somewhat higher than $\tilde{P}_L$ that would still induce switching and raise profit.

An implication of Proposition 6(ii) is the following. Suppose the fixed cost f is such that the innovation would occur and increase total welfare under Monopoly but would not occur under Duopoly (post-innovation) or Competition. Since the market is always covered, moving from Monopoly to any other market structure would not expand market coverage, hence would yield no welfare gain, but would forgo a welfare-increasing innovation.

6. Concluding remarks

The main goal of this paper is to provide a fuller understanding of the relative incentives for nondrastic product innovation under secure monopoly compared to more competitive market structures, within the classic framework of vertical product differentiation. We showed, inter alia, that the incentive to add a product that offers lower quality and lower marginal cost can be greatest under Monopoly if the market is covered. Here, we will offer an industry example that may approximate that setting, and then discuss potential policy implications of our broader analysis.

Consider smartphones, e.g., Apple's iPhone. While Apple is hardly a monopolist, it does command significant customer loyalty. Suppose the current product is a high-quality phone and the new product has lower quality, e.g., less memory, shorter battery life, lower resolution screen, slower processor. Adopting these inferior features reduces variable cost. Consumers vary in their willingness to pay for higher quality, depending on which features they use and how intensively; their type θ lies in a range $[\underline{\theta}, \overline{\theta}]$. Extending our model, suppose there are also lower types, $\theta \epsilon [0, \alpha]$ with $\alpha \ll \underline{\theta}$, who do not value many smartphone features, such as web surfing and various apps. They may value some features not offered by "regular" phones, e.g., email (so we can allow $\alpha > 0$). But there is a gap between their valuations and the lowest type $\underline{\theta}$ in the higher group. If that gap $\underline{\theta} - \alpha$ is large enough, the monopolist will prefer to serve only types $\theta \geq \underline{\theta}$ even after introducing the lower-quality good L, instead of cutting P_L further to attract low types below α. Thus, adding good L does not expand market coverage. (This may be a reasonable approximation for smartphones in their mature phase.) For purposes of our analysis, therefore, "market covered" does not require literally that all potential consumers have been served.

Our setting is too rudimentary for strong policy prescriptions, as it abstracts from various important factors such as innovation spillovers and internal slack under Monopoly. Nevertheless, the analysis has some policy relevance.

We identified in the Introduction several scenarios where policy interventions could alter the market structure from (secure) monopoly to more competitive regimes. A common claim in policy circles is that monopoly discourages innovation, often invoking Arrow's (1962) seminal work on incentives to invest in a cost-reducing innovation fors a homogeneous product.[28] However, as noted by Greenstein and Ramey (1998), Arrow's analysis does not extend to nondrastic product innovations, where the level of total output is no longer sufficient to rank innovation incentives across market structures. Indeed, Chen and Schwartz (2013) found that a monopolist over an initial product can have a greater incentive to add a horizontally differentiated product than would a firm facing competition from the first product. We showed that this ranking can occur also with vertical product differentiation if the new product has lower rather than higher quality. Our findings thus reinforce the caveat against claims that monopoly unequivocally discourages product innovation.

From a welfare standpoint, do the above findings weigh against the common policy stance that disfavors monopoly? We found that even when the incentive to innovate is greatest under Monopoly and only a monopolist would add the new (lower-quality) product, consumer welfare is still lower under Monopoly than under more competitive market structures. However, in those cases, if the marginal cost advantage of the lower-cost product is sufficiently large (relative to its quality disadvantage) then total welfare can be higher under Monopoly if the fixed cost of adding the new product is relatively low. The desirability of intervention that shifts a market structure away from monopoly then could depend on whether the policymaker's criterion is consumer welfare or total welfare.

We caution, however, that—via product innovation incentives—several factors could make the welfare effects of monopoly more favorable for consumers than in our setting. For example, in cases where only a monopolist would introduce the new product, consumers could benefit if the monopolist would *also* add the new (lower-quality) product in a different geographic market where it does not sell the old (higher-quality) product.[29] The gains to consumers from the new product in that second market may then exceed the loss to consumers in the first market. As a second example, the fixed cost of adding a second product may be lower when done by a monopolist that already offers the initial product than by a new firm. Our model abstracts from such economies of scope.

[28] An online search conducted on January 29, 2023 found that Arrow's article had 16,981 RePEc citations.

[29] That extension of the model would preserve the incentive rankings, since the innovator sets the monopoly price for good L in that other market in all regimes (Monopoly, Duopoly, or Competition). See also Chen and Schwartz (2010, Appendix A).

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