Strategic Incentives When Supplying to Rivals
With an Application to Vertical Firm Structure

Serge Moresi and Marius Schwartz[*]

October 10, 2016

Abstract

We consider a vertically integrated input monopolist supplying to a differentiated downstream rival. With linear input pricing, at the margin the firm unambiguously wants the rival to *expand*—unlike standard oligopoly with no supply relationship—for either Cournot or Bertrand competition. With a two-part tariff for the input, the same result holds if downstream choices are strategic complements, but is reversed for Cournot with strategic substitutes. We analyze vertical delegation as one mechanism for inducing expansion or contraction by the rival/customer.

JEL Codes: L13, D43, L14, L22

Keywords: Strategic Competition against Customers, Vertical Delegation

[*] Moresi: Charles River Associates, Inc., Washington DC 2004 <smoresi@crai.com>.
Schwartz: Department of Economics, Georgetown University, Washington DC 20057 <mariusschwartz@mac.com>. We thank Axel Anderson, Yongmin Chen, Jay Ezrielev, Ian Gale, Justin Johnson, Nadav Levy, Patrick Rey, Bill Rogerson, Steve Salop, Yossi Spiegel, John Vickers, Aleks Yankelevich, and seminar and conference participants at the Dusseldorf Institute for Competition Economics, IIOC 2015 conference, CRESSE 2015 conference and Searle Center 2015 conference. Any errors are ours alone.

1. Introduction

Vertically integrated firms often supply inputs to other firms with whom they compete in a downstream market. To cite just a few examples, Qualcomm makes chips used in smartphones and licenses key patents to rival chip manufacturers (Benoit and Clark, 2015); Samsung supplies components for iPhones and produces competing devices; Comcast-NBCU supplies programming to video distributors and competes with them in video distribution (Rogerson, 2013); and the US Post Office supplies last mile distribution services of packages to private competitors such as FedEx and UPS (Panzar, 2015).

Tougher behavior or "expansion" by a rival/customer—an output increase or a price decrease—then has opposing effects on the integrated firm's profit: downstream profits fall, the *competition effect*, but input sales and upstream profits rise, the *supply effect*. At the margin, would the integrated firm gain or lose from expansion by its rival/customer? Specifically, consider the following thought experiment. Hold constant the integrated firm's downstream choice and the equilibrium input contract with the other firm, and suppose the integrated firm could change the other firm's downstream choice, anticipating how this affects input orders. Would it prefer to set a (marginally) higher quantity (or lower price) than the other firm chooses in equilibrium, i.e. for the other firm to become more aggressive than in the actual equilibrium or less aggressive?

The question is relevant because the integrated firm may have additional ways beyond the input contract to elicit the desired change in the rival/customer's choice. For example, post contract the integrated firm may discover a way to lower the other firm's marginal cost, such as by sharing an innovation, or to raise it by engaging in a new form of non-price discrimination. Its attitude towards the other firm's marginal cost will hinge on whether it prefers that firm to become more or less aggressive relative to the equilibrium choice. Additionally, the integrated firm might alter the other firm's choice—without necessarily changing the input contract—by making observable commitments that signal its own strategic posture in downstream competition, as discussed in the extensive literature on strategic commitments in oligopoly (surveyed by Shapiro, 1989). (We shall analyze one such mechanism, vertical delegation.) There, a firm may adopt a tough or soft

posture depending on whether the competitive choice variables are strategic substitutes or strategic complements (Fudenberg and Tirole, 1984; Bulow, Geanakoplos and Klemperer, 1985)—but the goal throughout is to induce softer behavior by a rival. In our setting, the downstream rival is also an input customer, which introduces an opposing incentive.

We consider an unregulated and vertically integrated input monopolist that chooses to supply the input also to a downstream firm selling a differentiated substitute product. With enough differentiation, or a sufficient cost advantage for the other firm, the input monopolist indeed will prefer not to foreclose entirely the other firm. Downstream competition may be Cournot or Bertrand, and we do not impose a functional form on demand. Despite the tradeoff between downstream profits and input profits, we are able to characterize under fairly general conditions the integrated firm's incentive regarding a marginal expansion by its rival/customer starting at the equilibrium contract.

When the input is sold under linear pricing, at the margin the integrated firm necessarily benefits from expansion by the rival/customer (Proposition 1). This sharp result holds whether downstream competition is in prices or quantities, and whether these variables are strategic substitutes or complements. It generalizes the finding for Cournot competition with linear (differentiated) demands by Arya, Mittendorf and Yoon (2008), and our proof reveals the underlying general forces. In hindsight, the logic is simple. Starting at the profit-maximizing input price, a contraction by the other firm must reduce the integrated firm's profit (holding constant its downstream choice and the input price) despite the gain downstream. If contraction were beneficial or neutral, the integrated firm would have increased its profit by charging a higher input price and gaining from the increased input margin. (This simple logic hinges on the assumption that a higher input price induces contraction by the other firm, as we will discuss further below.) Therefore, the only direction in which the integrated firm wants to change the other firm's choice relative to the equilibrium value is expansion, i.e. it wants to make the other firm more aggressive.

We also analyze the case where the input is sold under a two-part tariff and the integrated firm fully extracts the other firm's profit via the fixed fee. The equilibrium contract nevertheless fails to maximize industry profit—because the integrated firm cannot commit to the rival/customer regarding its own downstream choice—and the

integrated firm will still benefit at the margin from expansion or contraction by the other firm relative to the equilibrium choice. However, determining which of these it prefers is more intricate than under linear pricing, and the specifics of competition now matter. The previous result persists with Bertrand or Cournot competition if downstream choices are strategic complements; but with Cournot and the "normal" case of strategic substitutes, the result is reversed: expansion by the rival/customer would harm the integrated firm (Proposition 2; the effect is ambiguous with Bertrand and strategic substitutes).

The logic underlying these patterns is subtle, and the following is only a sketch. Raising the input price generally will signal a change in the integrated firm's competitive toughness downstream, which affects the other firm's expected profit and, hence, the maximal fixed fee that the integrated firm can charge. If raising the input price signals increased toughness, the fixed fee must be lowered (by more than the increase in revenue from inframarginal input sales). Thus, holding the input contract and the integrated firm's choice constant at their equilibrium values, the integrated firm now must gain from a marginal contraction by the rival/customer—since inducing contraction by raising the input price would have been costly—the opposite of Proposition 1. The reverse holds if raising the input price signals softer competition by the integrated firm: it then gains from a marginal expansion by the rival/customer. Whether a higher input price will signal toughness or softness by the integrated firm depends on whether downstream variables are strategic substitutes or complements, and whether competition is Cournot or Bertrand. The latter distinction matters independently because in our setting the input price affects the integrated firm's shadow marginal cost of its downstream output—hence its expected downstream aggressiveness—only under Bertrand.

We consider one potential mechanism to elicit expansion or contraction by the rival/customer: *vertical delegation*. The input supplier grants its downstream unit autonomy to set price or quantity to maximize its objective, rather than integrated profit as in the standard *centralization* case (or spins off control of the downstream unit to outside investors but retains majority passive ownership); and charges that unit an input price

observable to the rival/customer.[1] Because the input price is not treated as a purely internal transfer, it can be used to signal the downstream unit's competitive posture and (indirectly) alter the rival/customer's choice. Focusing on linear input pricing (for reasons explained later), we apply Proposition 1 to show how the supplier can gain under delegation by adjusting the input price to its downstream unit relative to its shadow marginal cost under centralization: raise the input price if competition involves strategic substitutes—to signal contraction by its downstream unit and induce expansion by the rival/customer—or lower the input price with strategic complements (Proposition 3). With Bertrand competition and strategic complements, delegation also creates an incentive to raise the input price to the other firm, but not for standard foreclosure reasons.

The paper is organized as follows. Section 2 presents the model and addresses linear input pricing. Section 3 analyzes two-part tariffs. Section 4 applies the analysis to vertical delegation, and Section 5 concludes. The Appendix provides technical details and a numerical example comparing the equilibrium values and welfare under various regimes (no integration, integration with centralization, and vertical delegation), for Cournot or Bertrand competition downstream, and either linear input pricing or a two-part tariff. The example also is used to check the robustness of our results to changes in the assumptions.

2. The Setting and Linear Input Pricing

An input monopolist, firm 1, supplies to its downstream unit and to an independent downstream rival, firm 2, a setting sometimes described as partial forward integration or dual distribution. The firms produce imperfect substitutes, and with enough differentiation firm 1 indeed will prefer to supply also to firm 2 rather than foreclose it entirely. Downstream choice variables x_1 and x_2 are either quantities (q_1 and q_2) or per-unit prices (p_1 and p_2), thereby allowing Cournot or Bertrand competition downstream. To streamline the exposition, we define $x_i \equiv q_i$ for Cournot and $x_i \equiv -p_i$ for Bertrand; thus, an increase in x_i represents greater "aggressiveness" or "toughness" because it reduces downstream

[1] We therefore abstract from issues of unobservability and renegotiation (e.g. Katz, 1991; Caillaud and Rey, 1995). We will discuss these issues briefly in Section 4, along with the plausibility of our vertical delegation model and the related literature.

profit of the other firm. The timing is as follows. First, firm 1 sets a per-unit input price w_2 to firm 2. Then, firms 1 and 2 simultaneously set downstream variables $\boldsymbol{x} = (x_1, x_2)$, consumers purchase, and firm 2 pays for the required quantity of firm 1's input.

Each firm requires one unit of input per unit of output, and we use $Q_k(\boldsymbol{x})$ for both k's output and input amounts conditional on the downstream variables.[2] Firm 2 chooses x_2 to maximize its profit $\Pi_2(\boldsymbol{x}; w_2)$, and firm 1 chooses x_1 to maximize its total profits:

$$V(\boldsymbol{x}; w_2, c) = \Pi_1(\boldsymbol{x}; c) + (w_2 - c)Q_2(\boldsymbol{x}). \tag{1}$$

Here, c is firm 1's marginal cost of producing the input, assumed constant over the relevant range; $\Pi_1(\boldsymbol{x}; c)$ is firm 1's profit from its output sales; and $(w_2 - c)Q_2(\boldsymbol{x})$ is its profit from input sales to firm 2.[3] This is a standard representation of behavior by an integrated firm that also supplies to a rival.[4] All the relevant functions are assumed differentiable.

Note that if firm 1 did not supply inputs to firm 2, or was a regulated monopolist that must supply to firm 2 at cost (i.e. $w_2 = c$), then firm 1's profit would come solely from its own sales, i.e. $V(\boldsymbol{x}; c) = \Pi_1(\boldsymbol{x}; c)$. In that case, firm 1 would prefer that firm 2 be less aggressive, $\partial\Pi_1/\partial x_2 < 0$, i.e. reduce output when downstream competition is Cournot, or raise price if competition is Bertrand, so as to increase the demand for 1's product.

In our setting, firm 1 sells inputs to firm 2 at a markup (i.e. $w_2 > c$), so firm 2's downstream choice, x_2, has two opposing effects on firm 1's profits:

$$\frac{\partial V}{\partial x_2} = \frac{\partial \Pi_1}{\partial x_2} + (w_2 - c)\frac{\partial Q_2}{\partial x_2}, \tag{2}$$

where $\partial\Pi_1/\partial x_2$, is the "downstream competition" effect discussed previously and $(w_2 - c)\,\partial Q_2/\partial x_2$ is the "input supply" effect which typically runs in the opposite direction. For

[2] Under Bertrand, $Q_k(\boldsymbol{x})$ is the demand for firm k's product as a function of the prices set by the two firms. Under Cournot, $Q_k(\boldsymbol{x}) = x_k$ is the quantity set by firm k.

[3] For convenience, we will often suppress the third argument of V, the exogenous marginal cost c.

[4] Under this representation, Chen (2001) compares partial integration to no integration. Arya, Mittendorf, and Sappington (2008) compare the outcomes under partial integration when downstream competition is Bertrand or Cournot.

example, under downstream Bertrand competition, a price rise by firm 2 increases firm 1's profit in the output market, but lowers its profit in the input market because firm 2 will produce less, and hence purchase fewer inputs, when it raises its output price. Proposition 1 below will show that, in equilibrium, the input sales effect *always* dominates, and thus the integrated firm wants the rival/customer to become *more aggressive*.

We make the following assumptions:

Assumption 1. For any input price w_2 (within the relevant range), there exists a unique Nash equilibrium in the downstream competition stage, with the choice of firm k (k = 1, 2) denoted $X_k^*(w_2)$ and its corresponding output level and input level denoted $Q_k^*(w_2) \equiv Q_k\big(X_1^*(w_2), X_2^*(w_2)\big)$.

Assumption 2. Let $R_1(x_2; w_2)$ denote firm 1's "reaction function" for its downstream choice, i.e. the value of x_1 that maximizes $V(\boldsymbol{x}; w_2)$, and denote firm 2's reaction function by $R_2(x_1; w_2)$. Over the relevant range, a firm's reaction function is either strictly decreasing in the rival's choice or strictly increasing: $\partial R_i/\partial x_j < 0$ (strategic substitute) or $\partial R_i/\partial x_j > 0$ (strategic complement), with slope smaller than unity in absolute value, $|\,\partial R_i/\partial x_j\,| < 1$.

Assumption 3. An increase in the input price makes firm 2 less aggressive—that is, firm 2 reduces output under Cournot competition or increases price under Bertrand competition: $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$ (recalling that $x_i \equiv q_i$ for Cournot and $x_i \equiv -p_i$ for Bertrand).

Although Assumption 3 is intuitive, it deserves some discussion. When an integrated firm sells an input to a downstream rival, its *shadow marginal cost* of supplying to its own downstream unit includes both the resource marginal cost c and an *opportunity cost* from reduced input sales to the rival (e.g. Chen, 2001; Sappington, 2005).[5] This shadow marginal cost can be expressed as:

[5] Moresi and Salop (2013) analyze unilateral pricing incentives in vertical mergers (with no regulation) and show that this opportunity cost creates upward pressure on the price charged to consumers by the downstream unit of the merged firm. Similarly, in the literature on efficient regulation of access pricing, the opportunity cost is the reduction in profit to the integrated firm per unit of access provided to the downstream *competitor* (e.g. Armstrong, Doyle, and Vickers, 1996).

$$C_1(w_2, \boldsymbol{x}) \equiv c + (w_2 - c)D_{21}(\boldsymbol{x}), \tag{3}$$

where $D_{21} \equiv -\frac{\partial Q_2/\partial x_1}{\partial Q_1/\partial x_1}$ is the input diversion ratio—i.e. decreased input sales to firm 2 per extra unit of input to firm 1—and the opportunity cost is D_{21} multiplied by the input margin.[6] With inputs purchased after downstream sales occur, $D_{21} > 0$ under Bertrand competition, since firm 1's downstream expansion induced by cutting price will divert some sales from firm 2; but $D_{21} = 0$ under Cournot, since firm 1 takes firm 2's output—and thus its input purchases—as given when setting its own quantity.[7] Thus, raising w_2 increases firm 1's shadow marginal cost under Bertrand competition but does not affect it under Cournot. It follows that firm 1's downstream "reaction function" $R_1(x_2; w_2)$, defined in Assumption 2, exhibits $\partial R_1/\partial w_2 = 0$ under Cournot competition, but $\partial R_1/\partial w_2 < 0$ under Bertrand competition.[8] This distinction is relevant for Assumption 3 as follows.

Appendix A shows that $\mathrm{d}X_2^*/\mathrm{d}w_2$ takes the sign of $\partial R_2/\partial w_2 + (\partial R_2/\partial x_1)(\partial R_1/\partial w_2)$. The term $\partial R_2/\partial w_2$ is the "direct cost effect" which is negative under general conditions—a higher input price makes firm 2 less aggressive, holding x_1 constant. The second term is the "indirect strategic effect." It is zero under Cournot competition since $\partial R_1/\partial w_2 = 0$.

---

[6] Define $G(\boldsymbol{x}) \equiv \Pi_1(\boldsymbol{x}; c) + cQ_1(\boldsymbol{x})$ as firm 1's profit from its downstream sales *gross* of the cost of the internally provided input (so G is unaffected by c). Using (1), firm 1's overall profit can be re-written as $V(\boldsymbol{x}; w_2, c) = G(\boldsymbol{x}) - cQ_1(\boldsymbol{x}) + (w_2 - c)Q_2(\boldsymbol{x})$, hence:

$$\frac{\partial V}{\partial x_1} = \frac{\partial G}{\partial x_1} - \left[c + (w_2 - c)\left(-\frac{\partial Q_2 / \partial x_1}{\partial Q_1 / \partial x_1} \right) \right] \frac{\partial Q_1}{\partial x_1}.$$

The term in square brackets is the integrated firm's shadow marginal cost as defined in (3).

[7] See Rey and Tirole (2007) p. 2213: "There is an extra twist under Bertrand competition, though: Because orders lag price setting, a change in the wholesale price w_i charged to a downstream competitor i affects its final price p_i and thus the profit $(w_j - c)D(p_j, p_i)$ made on downstream competitor j." Arya, Mittendorf, and Sappington (2008) invoke this distinction to show that, unlike in standard duopoly, Bertrand competition downstream can yield higher prices than Cournot, when a partially integrated input monopolist sells also to a downstream rival, because the monopolist internalizes an opportunity cost only under Bertrand competition.

[8] Differentiating the first-order condition $\partial V/\partial x_1 = 0$ and using the second-order condition $\partial^2 V/\partial x_1^2 < 0$ shows: sign $(\partial R_1/\partial w_2)$ = sign $(\partial^2 V/\partial x_1 \partial w_2)$. From (1), $\partial^2 V/\partial x_1 \partial w_2 = \partial Q_2/\partial x_1$. Under Bertrand $\partial Q_2/\partial x_1 < 0$ (recalling $x_i \equiv -p_i$), while under Cournot $\partial Q_2/\partial x_1 = 0$.

With Bertrand competition, $\partial R_1/\partial w_2 < 0$. Thus, when prices are strategic complements ($\partial R_2/\partial x_1 > 0$), the indirect strategic effect is negative and reinforces the direct cost effect. If instead prices are strategic substitutes, we assume the direct cost effect dominates. In sum, Assumption 3 generally holds under Cournot competition, and also under Bertrand competition if (but *not* only if) the demand system makes prices strategic complements.

In the first stage, firm 1 sets w_2 to maximize $V^*(w_2) \equiv V(X_1^*(w_2), X_2^*(w_2); w_2)$. Let w_2^* denote the profit-maximizing choice, and (x_1^*, x_2^*) the resulting equilibrium downstream choices. We now state a sharp result:

Proposition 1. Suppose firm 1 sells the input to firm 2 using linear pricing. Under Assumptions 1-3, at the equilibrium outcome $(w_2, x_1, x_2) = (w_2^*, x_1^*, x_2^*)$, firm 1 wants firm 2 to behave more aggressively than firm 2's actual equilibrium choice, i.e. $\partial V/\partial x_2 > 0$.

Proof. The first-order condition (FOC) with respect to w_2 is

$$\frac{\mathrm{d}V^*}{\mathrm{d}w_2} \equiv \frac{\partial V}{\partial x_1}\frac{\mathrm{d}X_1^*}{\mathrm{d}w_2} + \frac{\partial V}{\partial x_2}\frac{\mathrm{d}X_2^*}{\mathrm{d}w_2} + \frac{\partial V}{\partial w_2} = 0. \tag{4}$$

The FOC for x_1 in the downstream competition implies $\partial V/\partial x_1 = 0$, and (1) implies $\partial V/\partial w_2 = Q_2^*$, so that (4) can be rewritten as

$$\frac{\partial V}{\partial x_2}\frac{\mathrm{d}X_2^*}{\mathrm{d}w_2} = -Q_2^*. \tag{5}$$

The assumptions $Q_2^* > 0$ and $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$ imply $\partial V/\partial x_2 > 0$ at the equilibrium point.

Proposition 1 identifies opposite incentives to those in a standard duopoly setting where firm 1 does not supply firm 2. There, holding x_1 constant, firm 1 would want firm 2 to behave less aggressively (raise price under Bertrand competition or reduce output under Cournot). This "softening downstream competition" effect is present here as well, but is necessarily dominated by the opposing "input supply" effect.[9]

[9] At the end of Section 3, we will discuss how this result, and its counterpart for the case where the input is sold using a two-part tariff, would change if the input supplier were not integrated.

The logic is shown in (4). An increase in the input price w_2 affects firm 1's profit via three channels. (a) It alters firm 1's downstream equilibrium choice X_1^*, but this has a zero first-order effect on profit by the envelope theorem, since firm 1 chooses x_1 optimally given w_2. (b) It increases firm 1's profit from inframarginal input sales, in proportion to firm 2's equilibrium output ($\partial V/\partial w_2 = Q_2^*$). (c) It changes firm 2's downstream equilibrium choice ($\mathrm{d}X_2^*/\mathrm{d}w_2$) which affects firm 1's profit both downstream and from input sales ($\partial V/\partial x_2$). Under Assumption 3, a higher w_2 makes firm 2 less aggressive ($\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$), which benefits firm 1 downstream but reduces its profitable input sales.[10] The net effect of such contraction by firm 2 must be harmful to firm 1 starting at the equilibrium input price w_2^*: otherwise firm 1 could have increased the input price and profited from the higher input margin (effect (b) above). Thus, holding w_2 at w_2^* and x_1 at x_1^*, if the integrated firm could induce its rival/customer to expand input purchases and downstream sales somewhat, it would gain on balance, despite the loss in downstream profits. (We show in Section 4 how vertical delegation can be used to induce the desired change in x_2.)

Observe that the argument relies on starting at firm 1's profit-maximizing input price. If the input price were constrained—by regulation or by a fringe of competitive input suppliers—to be sufficiently low, Proposition 1 would be reversed: firm 1 then would prefer that firm 2 contract rather than expand.

3. Two-Part Tariff for the Input

When the integrated firm sells the input to firm 2 using a two-part tariff—i.e. a pair (w_2, f_2), where f_2 is a fixed upfront fee—its total profits are $V^T(\boldsymbol{x}; w_2, f_2) = V(\boldsymbol{x}; w_2) + f_2$, where $V(\boldsymbol{x}; w_2)$ is given in (1). If firm 2 accepts the two-part tariff offer (w_2, f_2) (as it will in equilibrium), the downstream outcome is given by the functions $X_1^*(w_2)$ and $X_2^*(w_2)$ as with linear pricing. Let $\Pi_2^*(w_2) = \Pi_2(X_1^*(w_2), X_2^*(w_2); w_2)$ denote firm 2's profit gross of the fixed fee. In equilibrium, firm 1 will extract firm 2's profit by setting $f_2 = \Pi_2^*(w_2)$. Therefore, it sets w_2 to maximize $V^{*T}(w_2) = V^*(w_2) + \Pi_2^*(w_2)$, where $V^*(w_2)$ is the same as

[10] If instead a higher input price were to make firm 2 *more aggressive*, Proposition 1 would be reversed: firm 1 would then want firm 2 to contract or raise price relative to its equilibrium choice. However, as explained, Assumption 3 is likely to hold quite broadly.

with linear pricing. Let w_2^T and $f_2^T \equiv \Pi_2^*(w_2^T)$ denote the profit-maximizing two-part tariff, and (x_1^T, x_2^T) denote the resulting equilibrium downstream choices.

The next result addresses the same experiment as Proposition 1—the effect on firm 1' s profit V, defined in (1), of a small change in firm 2's choice x_2—but this time holding other variables constant at their *two-part tariff* equilibrium values. Before proceeding, it is worth clarifying why the integrated firm would benefit from a suitable change in firm 2's choice even though it fully extracts firm 2's profit in the two-part tariff equilibrium. The key point is that in our contracting environment the integrated firm cannot maximize overall industry profit, downstream plus upstream, even when charging a two-part tariff because it is unable to commit to firm 2 regarding the level of its own downstream choice.[11]

Denote the choices that maximize industry profit as the "monopoly solution," x_1^m and x_2^m. Firm 1 could induce x_2^m by setting $w_2 = X_2^{*-1}(x_2^m)$. However, since firm 2 pays its fixed fee f_2 before x_1 is determined, firm 1's best response in the downstream competition would not be x_1^m but $R_1(x_2^m; X_2^{*-1}(x_2^m))$, the value of x_1 that maximizes firm 1's profit ignoring firm 2's profit (since f_2 is "sunk" when firm 1 sets x_1). At $x_1 = R_1(x_2^m; X_2^{*-1}(x_2^m))$, we have $\partial V/\partial x_1 = 0$ hence $\partial(V + \Pi_2)/\partial x_1 = \partial \Pi_2/\partial x_1 < 0$: the integrated firm's best response to x_2^m entails an excessive downstream output (or too low a price) relative to the monopoly solution. Because firm 2 foresees at the contracting stage that firm 1 will subsequently act to maximize only its profit, the two-part tariff equilibrium choices (x_1^T, x_2^T) will generally differ from the monopoly solution (x_1^m, x_2^m).

Proposition 2 below characterizes how firm 1's profit would be affected by a marginal change in firm 2's choice from the two-part tariff equilibrium, holding constant the equilibrium input contract and firm 1's downstream choice.

---

[11] Hart and Tirole (1990) highlighted the problem caused by lack of commitment. In one of their scenarios (*ex post* monopolization), an input monopolist supplies under nonlinear contracts to downstream duopolists that produce homogeneous products at equal and constant marginal costs, and the commitment inability causes equilibrium output to exceed the monopoly level. Vertical integration with one firm and cutting off supply to the other would achieve the monopoly outcome. In our setting, the integrated supplier prefers (by assumption) to supply also to the *differentiated* downstream rival, and this structure fails to maximize overall profits, as discussed in the text.

Proposition 2. Suppose firm 1 sells the input to firm 2 using a two-part tariff. Under Assumptions 1-3, starting at the equilibrium outcome $(w_2, f_2, x_1, x_2) = (w_2^T, f_2^T, x_1^T, x_2^T)$, a small change in firm 2's choice x_2 affects firm 1's profit V as follows:
(i) With Cournot competition, $\partial V/\partial x_2 < 0$ if $\partial R_1/\partial x_2 < 0$ (strategic substitute) and $\partial V/\partial x_2 > 0$ if $\partial R_1/\partial x_2 > 0$ (strategic complement).
(ii) With Bertrand competition, $\partial V/\partial x_2 > 0$ if $\partial R_1/\partial x_2 > 0$, and the sign of $\partial V/\partial x_2$ is ambiguous if $\partial R_1/\partial x_2 < 0$.

Proof. Firm 1 sets w_2 to maximize $V^{*T}(w_2) = V^*(w_2) + \Pi_2^*(w_2)$. The FOC is

$$\frac{\mathrm{d}V^{*T}}{\mathrm{d}w_2} \equiv \frac{\mathrm{d}V^*}{\mathrm{d}w_2} + \frac{\mathrm{d}\Pi_2^*}{\mathrm{d}w_2} = \frac{\partial V}{\partial x_2}\frac{\mathrm{d}X_2^*}{\mathrm{d}w_2} + Q_2^* - Q_2^* + \frac{\partial \Pi_2}{\partial x_1}\frac{\mathrm{d}X_1^*}{\mathrm{d}w_2} = 0, \tag{6}$$

where the first equality makes use of $\partial V/\partial x_1 = 0$ and $\partial \Pi_2/\partial x_2 = 0$ at $(x_1, x_2) = (x_1^T, x_2^T)$. Decomposing $\mathrm{d}X_1^*/\mathrm{d}w_2$ and rearranging gives

$$\frac{\partial V}{\partial x_2}\frac{dX_2^*}{dw_2} = -\frac{\partial \Pi_2}{\partial x_1}\left\{\underbrace{\frac{\partial R_1}{\partial x_2}\frac{dX_2^*}{dw_2}}_{\text{strategic effect}} + \underbrace{\frac{\partial R_1}{\partial w_2}}_{\text{opportunity cost effect}}\right\} \tag{7}$$

In (7), $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$ by Assumption 3, and $\partial \Pi_2/\partial x_1 < 0$ by definition of x_1. Thus, $\partial V/\partial x_2$ takes the opposite sign of the term in brackets. Given $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$, (7) then implies:
(i) With Cournot competition, sign $\partial V/\partial x_2$ = sign $\partial R_1/\partial x_2$ (since $\partial R_1/\partial w_2 = 0$).
(ii) With Bertrand competition, $\partial V/\partial x_2 > 0$ if $\partial R_1/\partial x_2 > 0$ (since $\partial R_1/\partial w_2 < 0$). If instead $\partial R_1/\partial x_2 < 0$, the term in parentheses in (7) has an ambiguous sign, hence so does $\partial V/\partial x_2$.

Recapping, a marginal increase in x_2 (firm 2's aggressiveness) would benefit firm 1 under Cournot or Bertrand competition if firm 1's downstream choice is a strategic complement for firm 2's choice; would harm firm 1 under Cournot competition if firm 1's quantity is a strategic substitute for firm 2's quantity; and would have an ambiguous effect under Bertrand competition if firm 1's price is a strategic substitute for firm 2's price. Thus, for Cournot competition and the "normal" case of strategic substitutes, the pattern is reversed from the case with no fixed fee (i.e. $f_2 \equiv 0$) addressed in Proposition 1: starting at

the equilibrium outcome, the integrated firm now benefits from an exogenous *decrease* in q_2. For Bertrand competition and the "normal" case of strategic complements, the pattern from Proposition 1 persists: the integrated firm benefits from a reduction in p_2, which constitutes more aggressive behavior by firm 2.

These results, and their relationship to linear input pricing, can be understood as follows.[12] With linear input pricing, an increase in w_2 would raise firm 1's revenue from inframarginal input sales; thus, to sustain w_2^* as firm 1's optimum it must be true that contraction by firm 2—induced by increasing w_2—would by itself raise firm 1's profit V. With a two-part tariff for the input, however, w_2 no longer affects firm 1's profit from inframarginal input sales, due to the compensating change in firm 2's fixed fee (term Q_2^* cancels in (6)). Instead, raising w_2 now affects firm 1's profit via the induced contraction by firm 2 (the term $(\partial V/\partial x_2)(\mathrm{d}X_2^*/\mathrm{d}w_2)$ in (6) as before) *and by signaling a change in firm 1's downstream choice*, which alters firm 2's profit and therefore the *fixed fee* that firm 1 can extract (collectively, the last term in (6), $(\partial \Pi_2/\partial x_1)(\mathrm{d}X_1^*/\mathrm{d}w_2)$). Signaling softer behavior lets firm 1 extract a higher fixed fee.

Whether signaling softness entails raising or lowering w_2 depends on the sign of $\mathrm{d}X_1^*/\mathrm{d}w_2 = (\partial R_1/\partial x_2)(\mathrm{d}X_2^*/\mathrm{d}w_2) + \partial R_1/\partial w_2$. The last term is the *opportunity cost effect* of w_2 on firm 1's (downstream) choice, which is zero under Cournot competition (recall the discussion after equation (3)). The first term is the *strategic effect*—how the foreseen change in firm 2's equilibrium choice induced by altering w_2 alters firm 1's choice. With Cournot competition, if firm 1's output is a strategic substitute for firm 2's output ($\partial R_1/\partial x_2 < 0$), then increasing firm 2's output by cutting w_2 will signal a lower output by firm 1 and boost the fixed fee. This effect "distorts" the equilibrium w_2 downwards to increase firm 2's output for the strategic goal of reducing firm 1's output.[13] Thus, starting at

[12] The results *cannot* be understood based on intuitions on how the centralization equilibrium compares to the monopoly solution (x_1^m, x_2^m). Proposition 2 describes local changes in x_2 that benefit firm 1 holding x_1 (and the input contract) constant, whereas moving to (x_1^m, x_2^m) entails changing both x_1 and x_2. We elaborate on this in Section 4 (see the two-part tariffs discussion).

[13] A similar strategic effect operates in Reisinger and Tarantino (2015). With homogeneous Cournot competition downstream, an integrated firm that supplies the input to a lower-cost downstream

the two-part tariff equilibrium contract and *holding firm 1's output constant*, firm 1 would gain from an exogenous *contraction* by firm 2. If, instead, firm 1's output is a strategic complement ($\partial R_1/\partial x_2 > 0$), then firm 1 would gain from exogenous expansion by firm 2.

Under Bertrand competition and the "normal" strategic complement case, firm 1 signals softness by raising w_2, which increases firm 1's price for two reasons: the foreseen rise in firm 2's price, and the increased opportunity cost of firm 1's output.[14] Because the equilibrium w_2 is distorted upwards, the previous logic implies that firm 1 would now gain from a marginal price reduction by firm 2, as with linear input pricing. If firm 1's price is a strategic substitute for firm 2's price, the strategic and opportunity cost effects work in opposite directions, rendering the direction of distortion in w_2 ambiguous.

Finally, compare our Propositions 1 and 2 with the alternative case where the input supplier (S) is not vertically integrated and sells to two downstream firms (firms 1 and 2). Under Cournot competition, S always gains if firm 2 (or firm 1) expands output, holding the output of the other firm and the input contracts constant. This tracks Proposition 1, where the integrated firm sells to firm 2 using linear pricing. But with a two-part tariff, and when quantities are strategic substitutes, it wants firm 2 to contract (Proposition 2(i))—opposite to S's incentive. For Bertrand competition, the integrated firm gains if firm 2 reduces price when the input contract is a linear price (Proposition 1) or a two-part tariff and prices are strategic complements (Proposition 2(ii)). The same is likely to be true for S, but one cannot invoke the same logic.[15]

---

rival under a two-part tariff may reduce the per-unit input price below marginal cost, to signal a reduction its own equilibrium downstream output and extract a higher fixed fee.

[14] In (7), the increase in the opportunity cost makes firm 1 less aggressive, i.e. $\partial R_1/\partial w_2 < 0$, regardless of whether prices are strategic complements or substitutes. See footnote 9.

[15] With Bertrand competition and no integration, the supplier wants firm 2 to reduce price (holding the price of firm 1 and the input contracts constant) if the marginal price of the input paid by firm 2 is higher than the *effective marginal cost* of supplying the input to firm 2, i.e.

$$w_2 > c + (w_1 - c)DR_{12}$$

This condition is satisfied if the downstream firms are symmetric and the input diversion ratio DR_{12} is less than 1. We have not found any asymmetric example where the above condition would be violated and, at the same time, an integrated supplier would still want to supply firm 2.

4. Application: Vertical Delegation to Signal Downstream Behavior

The literature on strategic commitments in oligopoly mentioned in the Introduction identifies various mechanisms a firm may use to signal a change in its behavior in the subsequent competition (shift its best-response curve) so as to alter a rival's behavior.[16] An integrated firm potentially could also employ several such mechanisms to induce the desired change by its rival/customer beyond varying its input price.

We analyze one possible mechanism, *vertical delegation*. The firm establishes an autonomous downstream unit, division 1, and charges it a publicly observable input price; and the division treats an increase in the input price as raising its perceived marginal cost, not as an irrelevant internal transfer. For expositional simplicity, we assume that division 1 maximizes solely its own profit. At the end of this section we will explain why the logic extends to other objective functions, discuss the plausibility of vertical delegation, and compare our results to related literature. We begin with linear pricing of the input.

Under delegation the game is as follows. First, firm 1 publicly commits to a pair of input prices $\boldsymbol{w} = (w_1, w_2)$ where w_1 is the price to its division 1. Then division 1 and firm 2 make downstream choices $\boldsymbol{x}$ simultaneously, consumers purchase, and firm 1 receives input payments. Firm 2's best response function, $R_2(x_1; w_2)$, is unchanged from the original game; but now division 1 chooses x_1 to maximize only its profit, $\Pi_1(\boldsymbol{x}; w_1)$. Firm 1 sets both w_2 and the new instrument w_1 to maximize its integrated profit V, defined in (1).[17]

We make similar assumptions as before:

Assumption 4. For any input prices $\boldsymbol{w}$ (within the relevant range), there exists a unique Nash equilibrium in the competition stage, with the choice of downstream rival k (k = 1, 2)

[16] These mechanisms include: investments to shift downstream marginal cost (Spence, 1977; Dixit, 1980); advertising (Schmalensee, 1983); managerial incentives schemes (Vickers, 1985); capital structure (Brander and Lewis, 1986); vertical contracts (Bonnano and Vickers, 1988; Rey and Stiglitz, 1995). For additional mechanisms and references, see Shapiro (1989).

[17] Although V does not depend directly on w_1, since division 1's input payments accrue as revenue upstream, w_1 will affect V indirectly by changing the equilibrium values of $\boldsymbol{x}$.

denoted $X_k^D(\boldsymbol{w})$, its output and input level denoted $Q_k^D(\boldsymbol{w}) \equiv Q_k\big(X_1^D(\boldsymbol{w}), X_2^D(\boldsymbol{w})\big)$, and $\partial X_k^D/\partial w_k < 0$ (where $x_k \equiv -p_k$ for Bertrand competition).[18]

Firm 1 sets w_1 and w_2 to maximize the continuation equilibrium profit function under delegation, $V^D(\boldsymbol{w}) \equiv V(X_1^D(\boldsymbol{w}), X_2^D(\boldsymbol{w}); w_2)$. Let (w_1^D, w_2^D) denote the optimal choice. We will compare this regime to the one from Section 2, that we now label *centralization*. Let $C_1^*(w_2) \equiv C_1^*(w_2, X_1^*(w_2), X_2^*(w_2))$ denote firm 1's shadow marginal cost, defined in (3), evaluated at the continuation equilibrium under centralization.

Under delegation, if the integrated firm sets $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$, i.e. an input price to division 1 equal to the shadow marginal cost in the centralization equilibrium, it will induce the same downstream choices as with centralization and earn the same profit. But it generally can do better, by suitably choosing $w_1 \neq C_1^*(w_2^*)$ and $w_2 \neq w_2^*$. Specifically, under delegation firm 1 achieves the profit it would earn under centralization if it were the Stackelberg leader in downstream competition, rather than making its downstream choice simultaneously with firm 2.[19] Intuitively, the Stackelberg equivalence arises because choosing w_1 serves as a commitment regarding the level of x_1.[20]

The next result characterizes how the integrated firm under delegation can profitably change w_1 locally relative to $C_1^*(w_2^*)$. Proposition 1 showed that starting at the centralization equilibrium with linear input pricing, firm 1 benefits if firm 2 becomes

---

[18] For linear demand, Assumption 4 is satisfied for all $\boldsymbol{w}$ that lead to marginal costs that satisfy the assumptions in Singh and Vives (1984). See also Appendix B.

[19] See Moresi and Schwartz (2015, Propositions 3 and 4). Lu, Moresi, and Salop (2007, Appendix 4) show the Stackelberg equivalence result for Bertrand competition. Arya, Mittendorf, and Yoon (2008) establish these results, and a result analogous to our Proposition 3 below, for the restricted case of linear (differentiated) demands and Cournot competition. They further show that decentralization (our delegation) increases the integrated firm's profit if the input price to the downstream division is determined by bargaining.

[20] On the general connection between strategic delegation and Stackelberg leadership in the competition game, see Vickers (1985, Sections I and II). Adapted to our setting, Vickers's agent appointment game—that precedes downstream competition—corresponds to whether the integrated firm initially adopts the centralization structure and sets only an input price w_2 or the delegation structure and sets an additional input price w_1. The agent appointment game maximizes the integrated firm's profit if and only if it implements the same downstream outcome as Stackelberg leadership by the integrated firm. See also Heifetz, Shannon, and Spiegel (2007).

marginally more aggressive. Delegation enables firm 1 to induce the desired change by using w_1 to signal a suitable change in division 1's downstream choice as follows:

Proposition 3. Suppose firm 1 sells the input to firm 2 using linear pricing. Under Assumption 4 and holding w_2 constant at the centralization equilibrium level w_2^*, in the delegation regime:
(i) If firm 2's choice is a strategic substitute for division 1's choice ($\partial R_2/\partial x_1 < 0$), firm 1 gains from a small increase in w_1 above $C_1^*(w_2^*)$, to signal softer behavior by division 1.
(ii) If firm 2's choice is a strategic complement for division 1's choice ($\partial R_2/\partial x_1 > 0$), firm 1 gains from a small decrease in w_1 below $C_1^*(w_2^*)$, to signal tougher behavior by division 1.

Proof. Starting at $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$, under delegation the integrated firm's desired change in w_1 is determined by the sign of (from $V^D(\boldsymbol{w}) \equiv V(X_1^D(\boldsymbol{w}), X_2^D(\boldsymbol{w}); w_2)$):

$$\frac{\partial V^D}{\partial w_1} = \frac{\partial V}{\partial x_2}\frac{\partial X_2^D}{\partial w_1} = \frac{\partial V}{\partial x_2}\frac{\partial R_2}{\partial x_1}\frac{\partial X_1^D}{\partial w_1}, \tag{8}$$

where the first equality follows from $\partial V/\partial x_1 = 0$. From Proposition 1, $\partial V/\partial x_2 > 0$ and, by assumption, $\partial X_1^D/\partial w_1 < 0$. Thus, the sign of $\partial V^D/\partial w_1$ is the opposite of that of $\partial R_2/\partial x_1$.

Proposition 3 shows that with Cournot competition and strategic substitutes, firm 1 under vertical delegation can gain by raising w_1 (i.e. setting w_1 above $C_1^*(w_2^*)$) to signal a reduction in division 1's output and thereby induce an increase in firm 2's output.[21] With Bertrand competition and strategic complements, firm 1 can gain by lowering w_1 to signal a decrease in division 1's price and induce a decrease in firm 2's price.

Interestingly, with Bertrand and strategic complements, firm 2's equilibrium *output* is likely to fall under delegation. Seemingly this contradict Proposition 1, which states that firm 1 gains from a marginal expansion by firm 2, but in fact there is no contradiction. Starting at the centralization equilibrium, a small decrease in p_1 under delegation (signaled

[21] We purposely say "can gain by raising w_1" instead of "gains by raising w_1" because we have not characterized the equilibrium input prices under delegation, and are instead describing a profitable local deviation from centralization. We elaborate on this shortly, when discussing an example.

by setting w_1 below $C_1^*(w_2^*)$) becomes profitable only because it induces a decrease in p_2. This reduction in p_2 in turn benefits firm 1 solely by increasing firm 2's sales and input purchases, which shows that unilateral expansion by firm 2—the experiment of Proposition 1—indeed benefits firm 1. In other words, a small reduction in p_1 from p_1^* under delegation is profitable only because the reduction in firm 2's price now *mitigates the decrease in firm 2's output and input purchases* caused by the decrease in p_1.

Another noteworthy feature under Bertrand and strategic complements is that delegation incentivizes firm 1 not only to set w_1 below $C_1^*(w_2^*)$) but likely also to *raise* w_2 above w_2^*. This incentive is superficially reminiscent of the familiar raising-rivals' costs effect (e.g. Salop and Scheffman, 1983). However, it is not driven by a desire to foreclose firm 2—by assumption, firm 1 prefers to supply also to firm 2—but by an entirely different force: firm 2's pass-through rate is lower when firm 1 chooses delegation instead of centralization, as explained next.[22]

Starting at $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$, and recalling that $\partial V/\partial x_1 = 0$, under delegation firm 1's desired change in w_2 is given by the sign of

$$\frac{\partial V^D}{\partial w_2} = \frac{\partial V}{\partial x_2}\frac{\partial X_2^D}{\partial w_2} + \frac{\partial V}{\partial w_2}. \tag{9}$$

Because $\partial V/\partial w_2 = Q_2^*(w_2^*)$, (9) would be zero if firm 2's "pass-through rate" under delegation, $\partial X_2^D/\partial w_2$, were equal to that under centralization, $\mathrm{d}X_2^*/\mathrm{d}w_2$ (see (4)). However, with Bertrand competition and strategic complements, $\partial X_2^D/\partial w_2$ likely is smaller in absolute value than $\mathrm{d}X_2^*/\mathrm{d}w_2$ (see Appendix A), in which case (9) is positive and firm 1 gains from raising w_2 when holding w_1 at $C_1^*(w_2^*)$. Intuitively, under centralization, raising w_2 increases firm 1's opportunity cost of its own output,[23] leading it to increase p_1; this effect of w_2 on p_1 is absent under delegation, because division 1 ignores upstream profit. Recognizing this differential response of p_1, firm 2 raises its price by less—hence reduces its input purchases by less—under delegation following a given increase in w_2. This lower

[22] On the role of downstream pass-through in affecting input prices, see Gaudin (2015).

[23] Recall that this opportunity cost effect is absent with Cournot competition, since firm 1's sales do not affect firm 2's sales (hence input purchases).

pass-through rate emboldens firm 1 to raise w_2 under delegation. (These results are illustrated by an example in Appendix B: see discussion of Tables 1 and 2.)

Example: Equilibrium in Various Regimes. Proposition 3 conducts a local experiment for w_1, while holding constant the input price w_2 to the rival/customer, firm 2. Appendix B reports a linear-demand example showing the equilibrium values of all the variables under centralization and delegation, for Cournot or Bertrand competition. We also report the equilibrium outcome if the input supplier were not integrated and supplied to independent downstream firms 1 and 2.

In the example, the firms' quantities are strategic substitutes while prices are strategic complements. Moving to delegation, the equilibrium values of w_1 and firm 1's downstream choice x_1 change in the direction suggested by Proposition 3. With Cournot competition (Appendix B, Table 1), the integrated firm raises w_1 (above $C_1^*(w_2^*)$)) to signal a reduction in its own output in order to induce an output expansion by firm 2. With Bertrand competition (Appendix B, Table 2), the integrated firm reduces w_1 to signal a reduction in its downstream price in order to induce a price reduction by firm 2; however, firm 2's output falls (and we explained why this is consistent with Proposition 1).

The change in the input price to firm 2 for Bertrand competition also is consistent with the earlier discussion: w_2 rises under delegation despite the decrease in firm 1's final price p_1, which by itself calls for reducing w_2.[24] The increase in w_2 is motivated by the lower pass-through rate from w_2 to p_2 under delegation than under centralization, which occurs only with Bertrand competition and strategic complements.[25]

Turning to welfare, while delegation necessarily raises firm 1's profit relative to centralization, the effects on firm 2 and consumers vary. Under Cournot competition, firm

[24] Decreasing p_1 makes it more attractive to reduce w_2 for two reasons: the decreased margin on firm 1's sales renders output diversion away from firm 1 less costly ("margin effect"); and the reduction in firm 2's input demand due to the decrease in p_1 ("demand effect").

[25] The equilibrium w_2 increases with delegation also under Cournot competition, but that is explained by the margin effect and demand effect caused by raising w_1. Holding w_1 constant at the shadow marginal cost under centralization ($C_1^*(w_2^*)$), firm 1's constrained optimal w_2 under delegation would remain at w_2^* with Cournot competition but rise with Bertrand.

2's profit rises with delegation but consumer surplus falls—because both downstream prices increase. Under Bertrand competition, these patterns are reversed: delegation benefits consumers, though the rival is harmed. The effect on total surplus is generally complex because downstream outputs change in opposite directions and by different amounts. Under Cournot competition, delegation improves the output mix by raising firm 2's output while lowering firm 1's, and the total price-cost margin is higher on 2's output. But firm 2's output rises by much less than the decrease in firm 1's output, and total surplus falls. Under Bertrand competition, delegation worsens the output mix, but firm 2's output falls by much less than the increase in firm 1's output, and total surplus rises. In this example, therefore, delegation benefits the integrated firm, but does not systematically benefit or harm other parties or total surplus.

The focus of our paper is on the pricing and vertical structure of an integrated firm that supplies also to a downstream rival, not on comparing integration to no integration. Nevertheless, for completeness we report the equilibrium outcome under no integration as well. In the example, for linear pricing, the following patterns hold when comparing no integration to integration (under either the centralization or delegation regime), regardless of whether downstream competition is Cournot or Bertrand (Appendix B, Tables 1 and 2). With integration, the input price to firm 2 does *not* increase but firm 2's profit falls, because the elimination of double marginalization between the input supplier and firm 1 increases the competition faced by firm 2 and leads to a downstream output mix that is more heavily skewed in favor of product 1 than under no integration. Consumer surplus rises with integration since both prices fall, and total surplus also rises, due to the large expansion in firm 1's output resulting from elimination of double marginalization. We caution, however, that these results are specific to this example, and in general the effects of vertical integration on consumer and total welfare are ambiguous.[26]

Two-Part Tariff for the Input. Suppose that in stage one, firm 1 publicly offers a two-part tariff (w_2, f_2) to firm 2, and a per-unit input price w_1 to division 1. Downstream competition then occurs as above. Firm 1 can now earn the maximal industry profit, by

[26] See, for example, the surveys by Church (2008) and Salinger (2014).

setting input prices at the levels w_1^m and w_2^m that induce division 1 and firm 2 to choose (unilaterally) the monopoly solution (x_1^m, x_2^m) and setting f_2 to extract firm 2's profits. However, with observable two-part tariffs, firm 1 could achieve the same maximal profit by vertically separating entirely, setting the input prices (w_1^m, w_2^m), and charging fixed fees that fully extract each downstream firm's profits. Thus, delegation for pricing purposes is arguably of less interest when observable two-part tariffs are feasible.[27]

Moreover, relative to firm 1's shadow marginal cost under centralization, $C_1^*(w_2^T)$, the "change" in w_1 needed to induce the monopoly solution generally will be in a different direction from that needed to induce the change in firm 2's choice described in Proposition 2. The logic is as follows. Holding firm 2's choice constant at the centralization level, firm 2's profit *and overall industry profit* can be increased by making *firm 1 less aggressive* downstream. (Firm 1 is too aggressive under centralization because it sets x_1 to maximize only its own profit, taking firm 2's fixed fee as given). With delegation and a *two-part tariff*, firm 1 gains from committing to less aggression by setting a "high" w_1, because it can capture the increase in firm 2's profit via a higher fixed fee. As the example in Appendix B suggests, under centralization there is a bias towards favoring firm 1's output, and moving to the monopoly solution (via delegation and a two-part tariff) reduces firm 1's output and increases firm 2's, for either Cournot competition (Table 3) or Bertrand (Table 4). In both cases, moving to the monopoly solution entails setting w_1 above $C_1^*(w_2^T)$; the prime role of w_1 becomes to "rein in" firm 1's aggressiveness.[28] For these reasons, we forgo a result analogous to Proposition 3 for the case of delegation with a two-part tariff.[29]

---

[27] By contrast, delegation and complete separation are not equivalent when firm 1 charges linear input prices. We showed that delegation then dominates centralization, and under reasonable conditions centralization dominates complete separation (as it eliminates double marginalization between the upstream and downstream units of firm 1).

[28] Whereas w_1 would have to be *lowered* in order to induce the changes in x_2 identified in Proposition 2 for the case of Cournot competition with strategic substitutes or Bertrand competition with strategic complements. See the discussion of Tables 3 and 4 in Appendix B.

[29] Also, one can no longer use a simple envelope argument to show that firm 1 benefits from a small change in w_1 to division 1 from $C_1^*(w_2^T)$. Although $\partial V/\partial x_1 = 0$ at $\boldsymbol{w} = (C_1^*(w_2^T), w_2^T)$, the change in x_1 induced by changing w_1 would alter firm 2's profit and, hence, the attainable fixed fee f_2.

Although vertical delegation may be of less interest when two-part tariffs are feasible, Proposition 2 can still have practical relevance. For example, Milliou and Petrakis (2015) consider a vertically integrated firm that can share cost-reducing knowledge with a downstream rival to whom it supplies a key input. Our Proposition 2 suggests that even if the scope for knowledge sharing arises unexpectedly *after* the two-part tariff for the input has been set, some sharing would be profitable in identified cases (e.g. Bertrand with strategic complements), because on the margin the integrated firm's gain from expanded input sales outweighs its loss in the downstream market (and this is *always* true under linear input pricing, by Proposition 1).

Discussion and Related Work. Our vertical delegation mechanism has two requirements: the supplier can commit to charge its division an input price observable to downstream rivals; and the division does not "undo" the strategic effects of that price by fully internalizing how its choice affects the affiliated supplier's upstream profits. To streamline the exposition, we assumed the division maximizes solely its own profit, but all our results extend to any objective function where the input price affects the division's downstream choice.[30]

In practice, integrated firms often treat divisions as profit centers. Eccles and White (1988) examine in detail the main transfer pricing policies observed in the intensive field study by Eccles (1985), and find that one such policy, exchange autonomy, grants divisions great latitude in transacting with one other. Consistent with (some) divisional autonomy, Crawford, Lee, Whinston, and Yurukoglu (2015) present empirical evidence that video distributors who are vertically integrated into programming do not fully internalize the effects of their pricing and program carriage decisions on their profits from programming.

Instead of relying solely on internal protocols, another way to prevent the downstream unit from acting as a passive arm of its upstream supplier is by having

[30] For example, if the division maximizes a weighted average of its profit and the integrated firm's profit, the supplier can implement its preferred downstream outcome by suitably raising the input price to its division to compensate for the division treating a fraction of that price as a pure transfer (Arya, Mittendorf, and Yoon, 2008, Proposition 3).

minority outside shareholders (e.g. O'Brien and Salop, 2000). In principle, an initially integrated firm could retain an arbitrarily high share of the downstream unit's profits while still inducing it to maximize solely downstream profit by *spinning off the control rights* (voting stock) to outside shareholders and becoming a passive majority owner.[31] Alternatively, the initially integrated firm could spin off to the same outside shareholders the upstream unit and non-voting stock of the downstream unit, thereby retaining the control rights over the downstream unit. Our analysis illustrates a strategic benefit of such spin-offs when the supplier also sells to downstream rivals.

Besides downstream autonomy, our signaling through vertical delegation mechanism requires that the input price charged to the division (or spun-off unit) be observed by the downstream rival and cannot be secretly renegotiated. This assumption is more questionable for unregulated firms,[32] and its validity will be context specific. One potential justification is the supplier's incentive to maintain a reputation for not acting opportunistically against customers by offering secret discounts to their rivals (e.g. O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994). To preserve such a reputation, it may adopt a policy of committing to public and transparent pricing.

In addition, Katz (1991) and Caillaud and Rey (1994) point out that scope for renegotiation will be limited when the supplier sells the input using linear pricing. Consider delegation and linear pricing. With Bertrand competition downstream and strategic complements, firm 1 specifies a relatively low input price to division 1 (w_1) in order to induce firm 2 to set a lower price p_2 than under centralization (Proposition 3). Firm 1 would then like to renegotiate with division 1 and secretly increase w_1, but that would reduce division 1's profit and, therefore, division 1 would refuse (since with only linear pricing, firm 1 cannot compensate division 1).[33] With Cournot competition and

[31] Family run firms sometimes retain control rights but spin off majority ownership for purposes of raising capital. Here, the initially integrated firm spins off control rights solely to signal that the downstream unit will act independently, not to raise capital.

[32] Regulated firms may be subject to rules governing input pricing to subsidiaries as well as the subsidiaries' behavior through imputation requirements (e.g. Laffont and Tirole, 2000).

[33] Allowing unconstrained transfers to division 1 would make delegation irrelevant in our model.

strategic substitutes, in order to induce firm 2 to expand, firm 1 now specifies a relatively *high* w_1, and this time both it and division 1 could benefit by secretly renegotiating to *reduce* this input price. Such renegotiation may be deterred, however, by inserting an MFN clause in the contract between firm 1 and firm 2, that caps w_2 at some percentage of w_1, and provides firm 2 with audit rights and a suitably structured penalty for breach by firm 1.

Bonanno and Vickers (1988) noted early on the strategic advantage of commitment to observable input prices. They consider two supply chains in differentiated Bertrand competition with prices as strategic complements. By vertically separating and charging its single retailer an input price somewhat above marginal cost, each supplier can profitably coax an increase in the rival's downstream price. Our setting differs by having a single supplier, who is vertically integrated and supplies to a downstream rival. Consequently, under linear input pricing the supplier benefits from inducing tougher rather than softer behavior from the rival/customer. In the same vein, our analysis differs from the literature on strategic advantages from creating autonomous *competing divisions* (e.g. Schwartz and Thompson, 1986; Baye, Crocker, and Ju, 1996). There, the role of divisional autonomy is to signal toughness against rivals. Here, the division is at a different vertical stage and the strategic gain may involve presenting a tough *or soft* posture.

In Appendix C we briefly explore the alternative case where firm 2 observes division 1's input contract only after accepting its own two-part tariff contract but before competing downstream, what Rey and Vergé (2004) call *interim observability*. It is no longer obvious whether delegation will now benefit or harm firm 1 compared to centralization. Delegation allows firm 1 to commit to an input price for division 1, which firm 2 will observe before competing downstream. As noted, this enables firm 1 to implement the monopoly solution if the commitment is made before firm 2 pays its fixed fee. But with interim observability, firm 2 bases the fixed fee it is willing to pay at the contracting stage on its belief about the input price that will be charged to division 1 *after* firm 2 has paid its fee. In the example of Appendix C, delegation reduces firm 1's profit relative to centralization (under Cournot or Bertrand competition downstream, and under either passive or wary beliefs by firm 2).[34]

[34] A more systematic exploration of interim observability is beyond the scope of this paper.

5. Concluding Remarks

We considered a vertically integrated input monopolist that both sells output in the downstream market and supplies inputs to a differentiated downstream rival. We showed that under linear pricing of the input, the integrated firm would unambiguously gain from exogenous expansion by its rival/customer. Under a two-part tariff for the input, the integrated firm would still benefit from exogenous expansion by the rival/customer if downstream competition is Bertrand with strategic complements, but would lose under Cournot competition with strategic substitutes. We analyzed one of potentially many mechanisms to induce expansion or contraction by the rival/customer, beyond relying solely on the input price to that firm—vertical delegation.

This research potentially could be extended in several directions. For instance, the results may change under non-linear contracts different from two-part tariffs.[35] Here we discuss a different extension, to bargaining. Propositions 1 and 2 characterize the supplier's incentives when it can make a take-it-or-leave it contract offer, under linear input pricing or a two-part tariff. Suppose, instead, that the per-unit input price is determined through bargaining with no fixed fee.[36] Under standard conditions, w_2 will be lower than when firm 1 unilaterally sets it under centralization (w_2^*). Consider the other polar case where firm 2 has all the bargaining power and denote the equilibrium input price w_2^L ($< w_2^*$). Note that w_2^L cannot be too low (specifically, it exceeds firm 1's marginal cost c) because it must yield firm 1 no less profit than by serving the downstream market solely with its own product. If w_2^L is sufficiently close to w_2^*, by continuity Proposition 1 will still hold.[37] If instead w_2^L is sufficiently below w_2^*, however, Proposition 1 might

[35] For example, consider a "quantity forcing contract" that specifies a *fixed quantity* of input to be purchased by firm 2 and a fixed fee to be paid by firm 2. Under this type of contract, firm 1 *always* benefits if firm 2 exogenously reduces output or increases price in the competition stage. With a two-part tariff, Proposition 2 shows that firm 1 benefits from less aggressive behavior by firm 2 *only if* choice variables are strategic substitutes.

[36] One reason to focus on linear pricing is that our most general result (Proposition 1) addresses this case.

[37] Under regularity conditions, Proposition 1 would hold also for intermediate bargaining power that yields w_2 between w_2^L and w_2^*.

reverse, in which case firm 1 would want firm 2 to be less aggressive. For the example in Appendix C, under either Bertrand or Cournot downstream, we find that w_2^L is about half-way between c and w_2^*, and firm 1 still wants firm 2 to be more aggressive.

Another extension involves alternative market structures. The case where the input supplier sells to multiple downstream rivals would not raise conceptual difficulties if rivals can observe each other's input prices, but with secret offers, beliefs about off-equilibrium offers will introduce familiar complexities. Introducing upstream competition from a competitive fringe also would be fairly straightforward. Our Propositions 1 and 2 would extend if the fringe's marginal cost caps the supplier's price close enough to its unconstrained optimum, but would reverse if the constraint is sufficiently tight. Allowing for strategic competition upstream, however, would significantly complicate the analysis.

A different but complementary direction involves applications of the analysis: mechanisms the supplier might employ to alter the rival/customer's choice. Regarding vertical delegation, an important empirical issue is how the supplier may set an observable input price. Alternatives to vertical delegation could also be explored.

Appendix A: Direct Cost Effect and Indirect Strategic Effect

Assumption 3 says that an increase in the input price makes firm 2 less aggressive, i.e. $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$ (recalling that $x_i \equiv q_i$ for Cournot and $x_i \equiv -p_i$ for Bertrand). This derivative can be analyzed by differentiating

$$X_1^* = R_1(X_2^*; w_2) \quad \text{and} \quad X_2^* = R_2(X_1^*; w_2) \tag{A1}$$

where R_k denotes the reaction function of downstream rival k (k=1,2). Thus:

$$\frac{\mathrm{d}X_2^*}{\mathrm{d}w_2} = \frac{\frac{\partial R_2}{\partial w_2} + \left(\frac{\partial R_2}{\partial x_1}\frac{\partial R_1}{\partial w_2}\right)}{1 - \frac{\partial R_2}{\partial x_1}\frac{\partial R_1}{\partial x_2}} \tag{A2}$$

Assumption 2 implies that the denominator in (A2) is positive. In the numerator, $\partial R_2/\partial w_2$ is the "direct cost effect" and we assume it is negative—i.e. a higher input price makes firm 2 less aggressive, holding x_1 constant.[38] The term in parentheses is the "indirect strategic effect" and it is zero with Cournot competition.[39] With Bertrand competition, we assumed that inputs are purchased after downstream sales occur, hence output expansion by firm 1 reduces input purchases by firm 2.[40] Firm 1's shadow marginal cost therefore increases with w_2, and we assume that a higher shadow marginal cost makes firm 1 less aggressive, i.e. $\partial R_1/\partial w_2 < 0$. Thus, the indirect strategic effect is negative, like the direct cost effect, if prices are strategic complements, i.e. if $\partial R_2/\partial x_1 > 0$. If instead prices are strategic substitutes, we assume that the direct cost effect dominates. It follows that $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$.

[38] This is a standard assumption. See, for example, Rey and Tirole (2007) at p. 2212.

[39] Since firm 1 takes firm 2's output and input purchases as given, firm 1's shadow marginal cost in (3) is unaffected by w_2, implying $\partial R_1/\partial w_2 = 0$.

[40] With Bertrand competition but inputs purchased before downstream sales occur, there would be no input diversion, hence no indirect strategic effect, as in Rey and Tirole (2007). They note (at p. 2159): "In Appendix C, we discuss the [other] case in which … the downstream firms produce to order. Technically, the difference between these two modes of production resembles the distinction between Cournot and Bertrand competition."

Assumption 4 says that division 1 is less aggressive if w_1 increases (holding w_2 constant), i.e. $\partial X_1^D/\partial w_1 < 0$. Similarly, Assumption 3 now says $\partial X_2^D/\partial w_2 < 0$. These derivatives can be analyzed by differentiating

$$X_1^D = R_1^D(X_2^D; w_1) \quad \text{and} \quad X_2^D = R_2(X_1^D; w_2). \tag{A3}$$

Thus:

$$\frac{\partial X_1^D}{\partial w_1} = \frac{\frac{\partial R_1^D}{\partial w_1}}{1 - \frac{\partial R_1^D}{\partial x_2}\frac{\partial R_2}{\partial x_1}} \quad \text{and} \quad \frac{\partial X_2^D}{\partial w_2} = \frac{\frac{\partial R_2}{\partial w_2}}{1 - \frac{\partial R_2}{\partial x_1}\frac{\partial R_1^D}{\partial x_2}} \tag{A4}$$

Holding x_1 constant, we assume as before that firm 2 is less aggressive if w_2 increases, i.e. $\partial R_2/\partial w_2 < 0$. It follows that $\partial X_2^D/\partial w_2 < 0$. Similarly, $\partial X_1^D/\partial w_1 < 0$.

Finally, observe that under Bertrand competition, strategic complements and linear demand, an increase in the input price makes firm 2 less aggressive *and more so* under centralization than under delegation, i.e. $\mathrm{d}X_2^*/\mathrm{d}w_2 < \partial X_2^D/\partial w_2 < 0$. This follows from (A2), (A4), $(\partial R_2/\partial x_1)\,(\partial R_1/\partial w_2) < 0$ and $\partial R_1/\partial x_2 = \partial R_1^D/\partial x_2$.[41] Intuitively, an increase in w_2 raises firm 1's shadow marginal cost of the input under centralization (which is $C_1(w_2, \boldsymbol{x})$ from (3)) but will not affect division 1's perceived marginal cost under delegation (which is w_1). Thus, following an increase in w_2, firm 2 expects a larger price increase by firm 1 under centralization than under delegation, leading to a greater equilibrium pass-through rate, i.e. $\mathrm{d}P_2^*/\mathrm{d}w_2 > \partial P_2^D/\partial w_2 > 0$. In turn, firm 2's lower pass-through rate under delegation encourages firm 1 to raise w_2.

Appendix B: Example

Proposition 3 shows how firm 1 can raise its profit under delegation with a local deviation: maintain the input price to firm 2 at the centralization level w_2^* and offer division 1 an input

[41] With linear demand, the derivatives of the reaction functions are scalars and hence do not depend on price, quantity or input price levels. Under Cournot competition and linear demand, we have $\partial X_2^D/\partial w_2 = \mathrm{d}X_2^*/\mathrm{d}w_2 < 0$ because $\partial R_1/\partial w_2 = 0$ and $\partial R_1/\partial x_2 = \partial R_1^D/\partial x_2$.

price different from the shadow marginal cost under centralization, $w_1 \neq C_1^*(w_2^*)$. In the example below, we find the actual equilibrium values under both regimes. We also report the equilibrium outcome if the input supplier were not integrated and supplied to independent downstream firms 1 and 2. In addition, we will use this example in Appendix C to investigate the robustness of our results to two changes in the assumptions.

Assume that the marginal cost of the input is constant, $c = 1$, and there are no other downstream costs. Consumer demand is given by $q_i = 500 - 200p_i + 100p_j$, where q_i denotes the output of firm i, and p_i and p_j denote the prices of firms i and j, respectively $(i, j \in \{1,2\}, i \neq j)$.[42] One can check that Assumptions 1, 2, 3, and 4 are satisfied.[43]

In the ensuing Tables, under delegation, $W_1(w_2^*)$ is firm 1's optimal choice for w_1 given $w_2 = w_2^*$, and $W_2\big(C_1^*(w_2^*)\big)$ is the optimal choice for w_2 given $w_1 = C_1^*(w_2^*)$, where the superscript * denotes equilibrium values under centralization. (NA means Not Applicable.)

1. Linear Input Price

The "Centralization" and "Delegation" columns in Table 1 below illustrate Proposition 3(i): with Cournot competition downstream, starting from the centralization outcome and holding w_2 constant, firm 1 wants to *increase* firm 2's quantity q_2. It thus signals a reduction in q_1 (since quantities here are strategic substitutes) by setting w_1 to division 1

---

[42] Inverse demand is given by $p_i = 5 - (2q_i + q_j)/300$. If firm 1 does not supply the input to firm 2, then $q_2 = 0$ and firm 1 maximizes $(4 - q_1/150)q_1$, yielding $q_1 = 300$, $p_1 = 3$ and a profit of 600 for firm 1. As we will see shortly, in this example firm 1 obtains a higher profit by supplying the input to firm 2. See also Arya, Mittendorf, and Sappington (2008).

[43] With centralization, for any given $w_2 \in [0,4)$ there exists a unique Nash equilibrium in the downward competition stage; under Cournot, quantities are strategic substitutes, $X_1^* = 220 + 20w_2$ and $X_2^* = 320 - 80w_2$; under Bertrand, prices are strategic complements, $X_1^* = -(29 + 6w_2)/15$ and $X_2^* = -(26 + 9w_2)/15$; and an increase in w_2 makes firm 2 less aggressive, $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$. With delegation, for any given $\boldsymbol{w} \in \mathbb{R}_+^2$ such that $2w_j - 5 < w_i < 5$ $(i, j \in \{1,2\}, i \neq j)$ there exists a unique Nash equilibrium in the downstream competition stage; under Cournot, quantities are strategic substitutes, $X_1^D = 300 - 80w_1 + 20w_2$ and $X_2^D = 300 - 80w_2 + 20w_1$; under Bertrand, prices are strategic complements, $X_1^D = -(25 + 8w_1 + 2w_2)/15$ and $X_2^D = -(25 + 8w_2 + 2w_1)/15$; and an increase in w_k makes firm k less aggressive, $\mathrm{d}X_k^D/\mathrm{d}w_k < 0$. See Singh and Vives (1984).

well above the shadow marginal cost under centralization (which here equals $C_1^* = c = 1$): holding $w_2 = w_2^*$, firm 1 would charge division 1 $W_1(w_2^*) = 1.28 > 1 = C_1^*$.

Table 1: Cournot Competition Downstream (Linear Input Price)

	Centralization	Delegation	No Integration
Input Price Charged to Downstream Rival, w_2	2.97	3	3
Shadow Cost to Supply Input Internally, $C_1^*(w_2^*)$	1	NA	NA
Input Price Charged to Division 1, w_1	NA	1.29	3
$W_1(w_2^*)$	NA	1.28	NA
$W_2(C_1^*(w_2^*))$	NA	2.97	NA
Output of Integrated Firm, q_1	279.31	257.14	120
Output of Downstream Rival, q_2	82.76	85.71	120
Output Price of Integrated Firm, p_1	2.86	3	3.8
Output Price of Downstream Rival, p_2	3.52	3.57	3.8
Profit of Integrated Firm	682.76	685.71	NA
Profit of Downstream Rival	45.66	48.98	96
Profit of Upstream Firm	NA	NA	480
Producer Surplus	728.42	734.69	672
Consumer Surplus	359.93	318.37	144
Total Surplus	1088.35	1053.06	816

In the actual delegation equilibrium, firm 1 raises w_2 slightly, from 2.97 to 3, and sets w_1 at 1.29. The integrated firm's output falls and the rival's output rises, consistent with the local incentives shown in Proposition 3(i). (The rise in firm 2's output and input demand in response to the foreseen large decrease in q_1 explains the increase in w_2.) Profits of both firms increase, while consumer surplus decreases as both downstream prices are higher with delegation than with centralization. Total surplus also is lower with delegation. Note that the downstream rival prefers no integration (the last column) to either integration regime, while the opposite is true for consumers and society as a whole.

Table 2 below illustrates Proposition 3(ii): with Bertrand competition downstream, starting from the centralization outcome and holding w_2 constant, firm 1 wants to induce a *decrease* in firm 2's price p_2. It thus signals a reduction in p_1 (since prices here are strategic complements) by setting w_1 well below the shadow marginal cost under centralization: $W_1(w_2^*) = 1.74 < 1.98 = C_1^*(w_2^*)$.

Table 2: Bertrand Competition Downstream (Linear Input Price)

	Centralization	Delegation	No Integration
Input Price Charged to Downstream Rival, w_2	2.97	3	3
Shadow Cost to Supply Input Internally, $C_1^*(w_2^*)$	1.98	NA	NA
Input Price Charged to Division 1, w_1	NA	1.75	3
$W_1(w_2^*)$	NA	1.74	NA
$W_2(C_1^*(w_2^*))$	NA	3.04	NA
Output of Integrated Firm, q_1	227.27	250	133.33
Output of Downstream Rival, q_2	109.09	100	133.33
Output Price of Integrated Firm, p_1	3.12	3	3.67
Output Price of Downstream Rival, p_2	3.52	3.5	3.67
Profit of Integrated Firm	696.97	700	NA
Profit of Downstream Rival	59.50	50	88.89
Profit of Upstream Firm	NA	NA	533.33
Producer Surplus	756.47	750	711.11
Consumer Surplus	294.49	325	177.78
Total Surplus	1050.96	1075	888.89

Table 2 also illustrates the discussion after the proof of Proposition 3(ii). Under delegation, if w_1 were set at the centralization shadow marginal cost, $C_1^*(w_2^*)$, firm 1 would gain by raising w_2: $W_2(C_1^*(w_2^*)) = 3.04 > 2.97$. This incentive emerges because an increase in w_2 induces a smaller price increase by firm 2 under delegation than under centralization: firm 2 recognizes that under centralization firm 1 will raise p_1 when w_2 increases, due to

the increased opportunity cost, while under delegation division 1 does not internalize this effect. Firm 2's lower pass-through makes its input demand less elastic with respect to w_2.

In the actual delegation equilibrium, w_2 rises by less, from 2.97 to 3.00, because firm 2's input demand falls due to the reduction in p_1 induced by setting w_1 at 1.75, significantly below $C_1^*(w_2^*)$. The decrease in w_1 and increase in w_2 cause firm 1's output to rise and firm 2's to fall. (But the fall in q_2 is *mitigated* by the reduction in p_2 prompted by the observed reduction in w_1, which is why firm 1 benefits from reducing p_1 under delegation.) Unlike the Cournot case, delegation now reduces firm 2's profit, increases consumer surplus as both downstream prices are lower than with centralization, and increases total surplus. Like the Cournot case, the downstream rival prefers no integration to either integration regime, while the opposite is true for consumers and society as a whole.

2. Two-Part Tariff Input Price

Tables 3 and 4 below show the equilibrium outcomes under the same three regimes (centralization, delegation, and no integration) for Cournot and Bertrand competition, respectively, when firm 1 sells the input using a *two-part tariff*. Under delegation, firm 1 implements the fully integrated monopoly solution, i.e. generates and collects the maximal industry total profits (here, 800). Under centralization, firm 1 cannot achieve this outcome, despite fully extracting firm 2's profit through the two-part tariff, because it cannot commit not to behave opportunistically against firm 2 in the competition stage by selling "too much" output. Centralization with a two-part tariff thus yields firm 1 only a second-best outcome.

Starting at the centralization outcome with a two-part tariff (w_2^T, f_2^T), Proposition 2 characterizes the (local) change in firm 2's choice that would benefit firm 1, holding constant the two-part tariff contract and firm 1's downstream choice. In the move from centralization to delegation, however, firm 1 has an incentive to use delegation as a means to commit to behave less aggressively, *even if that did not change firm 2's choice*, in order to extract a larger fee f_2 from firm 2. Thus, firm 1 has an incentive to charge its division 1 a relatively high input price w_1 to signal less aggressive behavior in the competition stage. (This contrasts with the case of linear pricing, where firm 1 had an incentive to use

delegation *only* for the purpose of altering firm 2's choice.) Thus, the patterns in Tables 3 and 4 differ from those identified in Proposition 2, as discussed next.

Table 3: Cournot Competition Downstream (Two-Part Tariff Input Price)

	Centralization	Delegation	No Integration
Input Price Charged to Downstream Rival, w_2	1.692	1.667	1.667
Shadow Cost to Supply Input Internally, $C_1^*(w_2^T)$	1	NA	NA
Input Price Charged to Division 1, w_1	NA	1.667	1.667
$W_1(w_2^T)$	NA	1.666	NA
$W_2(C_1^*(w_2^T))$	NA	1.692	NA
Output of Integrated Firm, q_1	253.85	200	200
Output of Downstream Rival, q_2	184.62	200	200
Output Price of Integrated Firm, p_1	2.69	3	3
Output Price of Downstream Rival, p_2	2.92	3	3
Profit of Integrated Firm	784.62	800	NA
Fixed Fee Paid by Downstream Rival	227.22	266.67	266.67
Profit of Upstream Firm	NA	NA	800
Producer Surplus	784.62	800	800
Consumer Surplus	484.62	400	400
Total Surplus	1269.23	1200	1200

Table 3 contrasts with Proposition 2(i), which states that for Cournot competition and strategic substitutes (as here) starting from the centralization with two-part tariff outcome, firm 1 would benefit if firm 2 exogenously reduced its output q_2. This might suggest that, under delegation, firm 1 should signal an increase in q_1 by setting w_1 below 1 (the shadow marginal cost under Cournot). However, Table 3 shows the opposite is true: with delegation, firm 1 raises the input price to its division above the shadow marginal cost ($1.667 > 1$), q_1 decreases, and q_2 *increases*. Although firm 1 would benefit if firm 2 exogenously reduced output *ceteris paribus* (Proposition 2), firm 1 benefits *even more* by committing to behave less aggressively in the competition stage. That is exactly what

delegation and a two-part tariff allow firm 1 to do: implement the fully integrated monopoly solution. The main role of w_1 becomes to make division 1 less aggressive so as to maximize industry profits, rather than induce firm 2 to reduce output.

Similarly, Table 4 contrasts with Proposition 2(ii): for Bertrand competition and strategic complements (as here), starting from the centralization outcome, firm 1 would benefit if firm 2 exogenously reduced its price p_2. This might suggest that under delegation, firm 1 should signal a reduction in p_1 by setting w_1 below the shadow marginal cost under centralization (1.60). However, the opposite is true since firm 1 raises the input price to its division ($2 > 1.60$). The reason is similar to that given for Table 3. In both cases, Cournot or Bertrand competition, the move to delegation lowers firm 1's output and raises firm 2's output, with the input price rising substantially to division 1 and falling (by less) to firm 2.

Table 4: Bertrand Competition Downstream (Two-Part Tariff Input Price)

	Centralization	Delegation	No Integration
Input Price Charged to Downstream Rival, w_2	2.19	2	2
Shadow Cost to Supply Input Internally, $C_1^*(w_2^T)$	1.60	NA	NA
Input Price Charged to Division 1, w_1	NA	2	2
$W_1(w_2^T)$	NA	2.01	NA
$W_2(C_1^*(w_2^T))$	NA	1.98	NA
Output of Integrated Firm, q_1	242.86	200	200
Output of Downstream Rival, q_2	171.43	200	200
Output Price of Integrated Firm, p_1	2.81	3	3
Output Price of Downstream Rival, p_2	3.05	3	3
Profit of Integrated Firm	790.48	800	NA
Fixed Fee Paid by Downstream Rival	146.94	200	200
Profit of Upstream Firm	NA	NA	800
Producer Surplus	790.48	800	800
Consumer Surplus	433.33	400	400
Total Surplus	1223.81	1200	1200

Finally, with two-part tariffs, the upstream input supplier also can implement the fully integrated monopoly solution under *no integration* (compare the second and third columns in Tables 3 and 4) assuming that the downstream firms can observe each other's per-unit input price before accepting the input contracts—as we assumed under delegation. Thus, firm 1 has no incentive to be vertically integrated in this case. (Vertical integration with *centralization* would reduce downstream prices and industry profits, while increasing both consumer surplus and total surplus.)

Appendix C: Robustness

We now use the above example to investigate the robustness of our results to two changes in the assumptions. First, for the case with centralization and linear pricing, we assume that firm 2 has bargaining power and sets the input price w_2 that it pays to firm 1. We find that firm 2 sets w_2 significantly below w_2^*, which reduces firm 1's margin on input sales to firm 2 by about half (relative to our assumption that firm 1 sets w_2). Interestingly, starting from this outcome, we still find that firm 1 would benefit if firm 2 increased quantity or reduced price *ceteris paribus*, and hence Proposition 1 still holds.

Second, for the case with delegation and a two-part tariff, we assume "interim observability" (Rey and Vergé, 2004), i.e. firm 2 observes the input price w_1 that firm 1 charges to its division only after firm 2 has accepted its own contract (w_2, f_2) but before the competition stage begins. Relative to centralization, we find that delegation *now* reduces firm 1's profits, in both the Cournot and Bertrand cases, and for either passive or wary beliefs.

1. Bargaining Power and Proposition 1

Firm 1's profit function is given by $V = (p_1 - c)q_1 + (w_2 - c)q_2$. In the example of Appendix B, if firm 1 does not supply firm 2, firm 1's profit equals 600 (see footnote 39).

Given w_2 and Cournot competition (see footnote 40), firm 1's profit is given by

$$V^*(w_2) = (8/3)(1 + 172w_2 - 29{w_2}^2) \qquad \text{(C1)}$$

Firm 2 thus sets w_2 such that $V^*(w_2) = 600$, i.e. $w_2^L \cong 1.93$, and the Cournot equilibrium quantities are $(q_1^L, q_2^L) \cong (258.6\ , 165.5)$. We have

$$\partial V/\partial q_2 = w_2 - c + q_1\, \partial p_1/\partial q_2 = 0.069 > 0 \qquad \text{(C2)}$$

where the second equality follows from $w_2 = w_2^L$, $c = 1$, $q_1 = q_1^L$, and $\partial p_1/\partial q_2 = -1/300$. Thus, like in Proposition 1, firm 1 would benefit if firm 2 were to increase q_2 *ceteris paribus*.

Given w_2 and Bertrand competition (see footnote 40), firm 1's profit is given by

$$V^*(w_2) = (8/9)(-89 + 588w_2 - 99{w_2}^2) \qquad \text{(C3)}$$

Firm 2 thus sets w_2 such that $V^*(w_2) = 600$, i.e. $w_2^L \cong 1.92$, and the Bertrand equilibrium prices are $(p_1^L, p_2^L) \cong (2.70\ , 2.89)$. We have

$$\partial V/\partial p_2 = (p_1 - c)(\partial q_1/\partial p_2) + (w_2 - c)(\partial q_2/\partial p_2) = -13.9 \qquad \text{(C4)}$$

where the second equality follows from $p_1 = p_1^L$, $c = 1$, $\partial q_1/\partial p_2 = 100$, $w_2 = w_2^L$, and $\partial q_2/\partial p_2 = -200$. Thus, like in Proposition 1, firm 1 would benefit if firm 2 were to reduce p_2 *ceteris paribus*.

2. Interim Observability and Delegation

Rey and Vergé (2004) analyze *interim* observability when the upstream firm is not vertically integrated and sells the input to two independent downstream competitors using two-part tariffs. In contrast, we assume that the upstream firm (firm 1) is vertically integrated with one of the downstream competitors (division 1), and we focus on firm 1's incentive to use either centralization or delegation.[44] As discussed in Section 4, when contract offers are *ex ante* observable, the integrated firm always prefers delegation to centralization because delegation allows it to solve the commitment problem that exists with firm 2 under centralization. Note that, under centralization, the issue of observability

[44] Delegation is different from no integration. With delegation, firm 1 does not use a fixed fee to extract division 1's profit (since it owns division 1) and therefore firm 1 faces an opportunism problem only when contracting with firm 2.

is mute, since the input contract between firm 1 and firm 2 is observable by both firms, and there is no other contract. Thus, *interim* observability affects our results only for the delegation regime.[45]

Under delegation and *interim* observability, the (continuation) equilibrium choices are $X_k^D(\boldsymbol{w})$ as in our model with *ex ante* observability. Similarly, firm 1's profit function is still given by $V^D(\boldsymbol{w}) + f_2$, but now $f_2 = \Pi_2^D(w_1^b, w_2) \equiv \Pi_2(X_1^D(w_1^b, w_2), X_2^D(w_1^b, w_2); w_2)$, where w_1^b is firm 2's belief (conjecture) about w_1 at the contracting stage. It follows that, under passive beliefs, the equilibrium contract terms (w_1^T, w_2^T, f_2^T) are the solution of the following equations:

$$(w_1^T, w_2^T) = \arg\max_{(w_1, w_2)} V^D(\boldsymbol{w}) + \Pi_2^D(w_1^b, w_2) \tag{C5}$$

$$w_1^b = w_1^T \tag{C6}$$

$$f_2^T = \Pi_2^D(w_1^T, w_2^T) \tag{C7}$$

If instead one assumes wary beliefs,[46] then the equilibrium contract terms are the solution of the following equations (with some abuse of notation):

$$w_1^b(w_2) = \arg\max_{w_1} V^D(\boldsymbol{w}) \tag{C8}$$

$$(w_1^T, w_2^T) = \arg\max_{(w_1, w_2)} V^D(\boldsymbol{w}) + \Pi_2^D(w_1^b(w_2), w_2) \tag{C9}$$

$$f_2^T = \Pi_2^D(w_1^T, w_2^T) \tag{C10}$$

For the example described in Appendix B and footnote 40, the above equations lead to the results reported below in Tables 5 and 6 for Cournot and Bertrand competition,

[45] We ignore the no integration regime analyzed in Rey and Vergé (2004), and focus on delegation *versus* centralization. Also, we confine attention to two-part tariffs because, under linear pricing, delegation yields the same outcome under interim observability as under public commitment (shown in Tables 1 and 2). See Gaudin (2016).

[46] As explained in Rey and Vergé (2004), wary beliefs are more plausible than passive beliefs whenever the contract actually offered to one downstream firm affects the upstream monopolist's incentives when dealing with the other downstream firm.

respectively. Interestingly, firm 1 prefers centralization to delegation, both for Cournot and Bertrand competition, and regardless of whether beliefs are passive or wary.

Table 5: Cournot and *Interim* Observability (Two-Part Tariff Input Price)

	Centralization	Delegation Passive	Delegation Wary
Input Price Charged to Downstream Rival, w_2	1.692	1.694	1.857
Shadow Cost to Supply Input Internally, $C_1^*(w_2^T)$	1	NA	NA
Input Price Charged to Division 1, w_1	NA	0.959	1
Output of Integrated Firm, q_1	253.85	257.14	257.14
Output of Downstream Rival, q_2	184.62	183.67	171.43
Output Price of Integrated Firm, p_1	2.69	2.67	2.71
Output Price of Downstream Rival, p_2	2.923	2.918	3
Profit of Integrated Firm	784.62	782.67	783.67
Fixed Fee Paid by Downstream Rival	227.22	224.91	195.92

Table 6: Bertrand and *Interim* Observability (Two-Part Tariff Input Price)

	Centralization	Delegation Passive	Delegation Wary
Input Price Charged to Downstream Rival, w_2	2.19	1.979	2.125
Shadow Cost to Supply Input Internally, $C_1^*(w_2^T)$	1.60	NA	NA
Input Price Charged to Division 1, w_1	NA	1.458	1.5
Output of Integrated Firm, q_1	242.86	250	250
Output of Downstream Rival, q_2	171.43	187.5	175
Output Price of Integrated Firm, p_1	2.81	2.71	2.75
Output Price of Downstream Rival, p_2	3.05	2.92	3
Profit of Integrated Firm	790.48	786.46	787.5
Fixed Fee Paid by Downstream Rival	146.94	175.78	153.13

References

Armstrong, M., Doyle, C., and Vickers, J. 1996. "The Access Pricing Problem: A Synthesis." *Journal of Industrial Economics* 44 (2): 131-50.

Arya, A., Mittendorf, B., and Sappington, D.E.M. 2008. "Outsourcing, Vertical Integration, and Price vs. Quantity Competition." *International Journal of Industrial Organization* 26 (1): 1-16.

Arya, A., Mittendorf, B., and Yoon, D-H. 2008. "Friction in Related Party Trade When a Rival is Also a Customer." *Management Science* 54 (11): 1850-60.

Baye, M.R., Crocker, K.J., and Ju, J. 1996. "Divisionalization, Franchising, and Divestiture Incentives in Oligopoly." *American Economic Review* 86 (1): 223-36.

Benoit, D. and Clark, D. "Activist Puts Pressure on Qualcomm." *Wall Street Journal*, April 13, 2015.

Bonanno, G. and Vickers, J. 1988. "Vertical Separation." *Journal of Industrial Economics* 46 (3): 257-65.

Brander, J.A.. and Lewis, T.R. 1986. "Oligopoly and Financial Structure: The Limited Liability Effect." *American Economic Review* 76 (5): 956-70.

Bulow, J.I., Geanakoplos, J.D., and Klemperer, P.D. 1985. "Multiproduct Oligopoly: Strategic Substitutes and Complements." *Journal of Political Economy* 93 (3): 488-511.

Caillaud, B. and Rey, P. 1995. "Strategic Aspects of Vertical Separation." *European Economic Review* 39 (3-4): 421-31.

Chen, Y. 2001. "On Vertical Mergers and Competitive Effects." *RAND Journal of Economics* 32 (4): 667-85.

Church, J.R. 2008. "Vertical Mergers." In W.D. Collins ed., *Issues in Competition Law and Policy*. Vol. 2. Chicago: American Bar Association.

Crawford, G.S., Lee, R.S., Whinston, M.D., and Yurukoglu, A. 2015. "The Welfare Effects of Vertical Integration in Multichannel Television Markets." Mimeo, September 4.

Dixit, A.K. 1980. "The Role of Investment in Entry Deterrence." *Economic Journal* 90 (357): 95-106.

Eccles, R.G. 1985. *The Transfer Pricing Problem: A Theory for Practice*. Lexington, Mass.: Lexington.

Eccles, R.G. and White, H.C. 1988. "Price and Authority in Inter-Firm Profit Center Transactions." *American Journal of Sociology* 94, Supplement: Organizations and Institutions: Sociological and Economic Approaches to the Analysis of Social Structure, S17-S51.

Fudenberg, D. and Tirole, J. 1984. "The Fat-Cat Effect, The Puppy-Dog Ploy, and the Lean and Hungry Look." *American Economic Review* 74 (2): 361-6.

Gaudin, G. 2015. "Vertical Bargaining and Retail Competition: What Drives Countervailing Power?" Mimeo, September. Available at http://ssrn.com/abstract=2655726.

Gaudin, G. 2016. "Vertical Relations, Opportunism, and Welfare. Mimeo, August. Available at http://ssrn.com/abstract=2819489.

Hart, O. and Tirole, J. "Vertical Integration and Market Foreclosure." *Brookings Papers on Economic Activity: Microeconomics* 1990: 205-285.

Heifetz, A., Shannon, C., and Spiegel, Y. 2007. "What To Maximize If You Must." *Journal of Economic Theory* 133 (1): 31-57.

Katz, M.L. 1991. "Game-Playing Agents: Unobservable Contracts as Precommitments." *RAND Journal of Economics* 22 (3): 307-28.

Laffont, J.J. and Tirole, J. 2000. *Competition in Telecommunications*. Cambridge: MIT Press.

Lu, S., Moresi, S., and Salop, S.. 2007. "A Note on Vertical Mergers with an Upstream Monopolist: Foreclosure and Consumer Welfare Effects." Mimeo. Available at http://crai.com/sites/default/files/publications/Merging-with-an-upstream-monopolist.pdf.

McAfee, R.P. and Schwartz, M. 1994. "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity." *American Economic Review* 84 (1): 210-30.

Milliou, C. and Petrakis, E. 2015. "Vertical Integration, Knowledge Disclosure and Decreasing Rival's Cost." University of Crete, Working Paper 15-07, November 17.

Moresi, S. and Salop, S.C. 2013. "vGUPPI: Scoring Unilateral Pricing Incentives in Vertical Mergers." *Antitrust Law Journal* 79 (1): 185-214.

Moresi, S. and Schwartz, M. 2015. "Strategic Incentives when Supplying to Rivals." Georgetown University, Department of Economics Working Paper 15-15-05.

O'Brien, D.P. and Salop, S.C. 2000. "Competitive Effects of Partial Ownership: Financial Interest and Corporate Control." *Antitrust Law Journal* 67 (3): 559-614.

O'Brien, D.P. and Shaffer, G. 1992. "Vertical Control with Bilateral Contracts." *RAND Journal of Economics* 23 (3): 299-308.

Panzar, J.C , 2015. "Co-opetition in Parcel Delivery: An Exploratory Analysis." RARC-WP-12-002, Office of Inspector General, United States Postal Service, November 2.

Rey, P. and Stiglitz, J. 1995. "The Role of Exclusive Territories in Producers' Competition." *RAND Journal of Economics* 26 (3): 431-51.

Rey, P. and Tirole, J. 2007. "A Primer on Foreclosure." In M. Armstrong and R. Porter eds., *The Handbook of Industrial Organization.* Vol. 3. Amsterdam: Elsevier.

Rey, P. and Vergé, T. 2004. "Bilateral Control with Vertical Contracts." *RAND Journal of Economics* 35 (4): 728-746.

Reisinger, M. and Tarantino, E. 2015. "Vertical Integration, Foreclosure, and Productive Efficiency." *RAND Journal of Economics* 46 (3): 461-79.

Rogerson, W.P. , 2013. "Vertical Mergers in the Video Programming and Distribution Industry: The Case of Comcast-NBCU." In J. Kwoka and L. White eds., *The Antitrust Revolution*, 6th Ed., New York: Oxford U Press.

Salinger, M. 2014. "Vertical Mergers." In R.D. Blair and D.D. Sokol eds., *The Oxford Handbook of International Antitrust Economics*. Vol. 1. Oxford: Oxford University Press.

Salop, S. and Scheffman, D. 1983. "Raising Rivals' Costs." *American Economic Review* 73 (2): 267-271.

Sappington, D.E.M. 2005. "On the Irrelevance of Input Pricing for Make-or-Buy Decisions." *American Economic Review* 95 (5), pp. 1631-8.

Schmalensee, R. 1983. "Advertising and Entry Deterrence: An Exploratory Model." *Journal of Political Economy* 91 (4): 636-53.

Schwartz, M. and Thompson, E. A. 1986. "Divisionalization and Entry Deterrence." *Quarterly Journal of Economics* 101 (2): 307-21.

Singh, N. and Vives, X. 1984. "Price and Quantity Competition in a Differentiated Duopoly." *RAND Journal of Economics* 15 (4): 546-554.

Spence, A.M. 1977. "Entry, Capacity, Investment, and Oligopolistic Pricing." *Bell Journal of Economics* 8 (2): 534-44.

Shapiro, C. 1989. "Theories of Oligopoly Behavior." In R. Schmalensee and R.D. Willig eds., *The Handbook of Industrial Organization*. Vol. 1. Amsterdam: Elsevier.

Vickers, J. 1985. "Delegation and the Theory of the Firm." *Economic Journal* 95 (380a): 138-47.