Strategic Incentives When Supplying to Rivals

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Abstract

We consider an unregulated, vertically integrated input monopolist that supplies to a differentiated downstream rival. With linear input pricing, the integrated firm unambiguously wants to induce *expansion* by the rival—the opposite incentive from that in standard oligopoly settings with no supply relationship, even though the downstream competition effect is still present here. This result holds whether downstream competition involves prices or quantities and strategic substitutes or complements. If the firm charges a two-part tariff for the input, the result continues to hold under Bertrand competition in the "normal" case of prices as strategic complements, but is reversed for Cournot and strategic substitutes. We analyze one mechanism for influencing the independent downstream firm, vertical delegation, whereby the integrated firm charges its downstream unit an observable input price, and the downstream unit does not treat that price as a pure internal transfer. Vertical delegation is shown to dominate centralized behavior by the integrated firm, and we characterize how the input price should be set in order to alter the independent firm's choice depending on the specifics of downstream competition.

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1. Introduction

A vast literature studies a firm's incentives to elicit softer competitive behavior from oligopolistic rivals—contraction, a higher price or lower output—by taking observable and irreversible actions that alter the firm's own strategic posture in the ensuing competition (Shapiro, 1989). Mechanisms to achieve such strategic commitment include investment that lowers the firm's marginal cost (Spence, 1977; Dixit, 1980), advertising (Schmalensee, 1983), subsidies to domestic firms for exports or R&D (Spencer and Brander, 1983), capital structure (Brander and Lewis, 1986), managerial incentive schemes (Vickers, 1985; Fershtman and Judd, 1987), and vertical contractual arrangements (Bonanno and Vickers, 1988; Rey and Stiglitz, 1995). To influence its rivals, the firm may adopt a tough or soft strategic posture, depending on whether it wishes to deter entry or accommodate rivals and, in the latter case, whether the competitive choice variables are strategic substitutes or complements (Fudenberg and Tirole, 1984; Bulow, Geanakoplos and Klemperer, 1985). But throughout, the goal is to induce softer competition from rivals.

How do these incentives change when a firm not only competes with rivals downstream but also supplies them with inputs? Such situations are fairly common. To cite just a few examples, Qualcomm makes chips used in smartphones and licenses key patents to rival chip manufacturers (Benoit and Clark, 2015); Samsung supplies components for the iPhone and competes against the iPhone; and Comcast-NBCU supplies programming to video distributors and competes with them in video distribution (Rogerson, 2013).

We consider an unregulated, vertically integrated input monopolist that faces differentiated-products competition downstream. Unsurprisingly, the integrated firm now perceives a tradeoff: softer behavior by a rival/customer increases the firm's downstream profit but reduces its profitable input sales. More surprisingly, we show that with linear input pricing, the effect on input sales necessarily dominates. The vertically integrated supplier unambiguously wants to induce *expansion* by a rival/customer—the opposite incentive to that in standard two-stage games with no supply relationship. This sharp result holds whether downstream competition is in prices or quantities and whether these variables are strategic substitutes or complements.

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The analysis becomes more intricate when the input supplier charges the rival a two-part tariff instead of linear pricing. The previous result continues to hold under Bertrand competition in the "normal" case where prices are strategic complements, but the incentive is reversed for Cournot and strategic substitutes: the integrated firm then wants to induce contraction by its rival/customer. We explain why the incentives depend on the specifics of downstream competition only when the input is sold under a two-part tariff.

As noted, the literature identifies numerous practices a firm may use to alter its strategic posture—shift its best-response function in the ensuing competition to signal a change in its downstream choice—so as to influence rivals' choices. We apply our analysis to one such mechanism, *vertical delegation*: the firm establishes a downstream unit whose objective function treats a portion (no matter how small) of the input price charged to it by the upstream unit as a cost rather than a purely internal transfer; and commits to charge the downstream unit a publicly observable input price.[1] Echoing familiar ideas (e.g. Vickers, 1985; Katz, 2006; Heifetz, Shannon and Spiegel, 2007), we show that with delegation the integrated firm can replicate the downstream outcome arising when the firm acts as a centralized decision maker (e.g. as in Chen, 2001) and generally does better, even if it can charge a two-part tariff for the input under both regimes. Specifically, it attains the outcome it would achieve under centralization if it were the Stackelberg leader in downstream interaction. We also characterize the firm's incentives regarding the input price to its downstream unit, depending on the downstream competition—Bertrand or Cournot and strategic substitutes or complements. For example, with prices as strategic complements, the integrated firm wants to induce a reduction in the downstream price of its rival/customer, which requires lowering the input price to its downstream unit.[2]

The next section of the paper presents the model and main results. Section 3 applies the analysis to vertical delegation, and Section 4 offers brief concluding remarks.

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[1] We therefore abstract from issues of unobservability and renegotiation (e.g. Katz, 1991; Caillaud and Rey, 1994), but will discuss these issues briefly in Section 3.

[2] For differentiated Cournot competition with linear demands and linear pricing of the input, Arya, Mittendorf and Yoon (2008) show that under delegation the integrated firm would price the input to its downstream division so as to induce expansion by the rival/customer, consistent with our general result discussed earlier.

2.   The Setting and Main Results

2.1   Linear Pricing of the Input

An input monopolist, firm 1, supplies to its downstream unit and to an independent downstream rival, firm 2, a setting often described as partial integration. The downstream choice variables x_1 and x_2 are either per-unit prices (p_1 and p_2) or quantities (q_1 and q_2), thereby allowing Bertrand or Cournot competition downstream.[3] The timing is as follows. First, firm 1 sets a per-unit input price w_2 to firm 2. Then, firms 1 and 2 simultaneously set downstream variables $\boldsymbol{x} = (x_1, x_2)$, consumers purchase, and firm 2 pays for firm 1's input. To simplify, we assume each firm requires one unit of input per unit of output, and use $Q_k(\boldsymbol{x})$ for both k's output and input amounts conditional on the downstream variables.

Firm 2 chooses x_2 to maximize its profit $\Pi_2(\boldsymbol{x}; w_2)$, and firm 1 chooses x_1 to maximize its total profits:

$$V(\boldsymbol{x}; w_2) = \Pi_1(\boldsymbol{x}; c) + (w_2 - c)Q_2(\boldsymbol{x}). \tag{1}$$

Here, c denotes firm 1's marginal cost of producing the input, assumed constant over the relevant range, $\Pi_1(\boldsymbol{x}; c)$ is firm 1's profit from its output sales, and $(w_2 - c)Q_2(\boldsymbol{x})$ is its profit from input sales to firm 2. This is a standard representation of behavior by an integrated firm that also supplies to a rival.[4]

Observe that if firm 1 did not supply inputs to firm 2, or was a regulated monopolist that must supply to firm 2 at cost (i.e. $w_2 = c$), then firm 1's profit would come solely from its sales, i.e. $V(\boldsymbol{x}; c) = \Pi_1(\boldsymbol{x}; c)$. In that case, firm 1 would want firm 2 to *reduce* output—i.e. $\partial \Pi_1 / \partial x_2 < 0$ when downstream competition is Cournot, and $\partial \Pi_1 / \partial x_2 > 0$ if it is Bertrand.[5]

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[3] The integrated input monopolist prefers to maintain firm 2 as an active buyer of the input rather than foreclose firm 2 (under both Bertrand and Cournot downstream competition) if firm 2 is a sufficiently efficient competitor. *See* Arya, Mittendorf, and Sappington (2008).

[4] Under this representation, Chen (2001) compares partial integration to no integration. Arya, Mittendorf, and Sappington (2008) compare the outcomes under partial integration when downstream competition is Bertrand or Cournot.

[5] We assume that all the relevant functions are differentiable. Implicitly, there are no binding capacity constraints and the downstream products are differentiated (i.e. imperfect substitutes).

This is because a contraction by firm 2 would increase the demand faced by firm 1.

In our setting, firm 1 sells inputs to firm 2 at a markup (i.e. $w_2 > c$), so firm 2's downstream choice, x_2, has two opposing effects on firm 1's profits:

$$\frac{\partial V}{\partial x_2} = \frac{\partial \Pi_1}{\partial x_2} + (w_2 - c)\frac{\partial Q_2}{\partial x_2}. \tag{2}$$

where $\partial \Pi_1 / \partial x_2$, is the "downstream competition" effect discussed previously and $(w_2 - c) \partial Q_2 / \partial x_2$ is the "input supply" effect which typically runs in the opposite direction. For example, under Bertrand competition downstream, a price increase by firm 2 raises firm 1's profit in the output market, but lowers its profit in the input market because firm 2 will produce less, and hence purchase fewer inputs, when it raises its output price. (Under Cournot, an output reduction by firm 2 increases firm 1's profit in the output market, but decreases its profit in the input market.) Proposition 1 below will show that, in equilibrium, the input sales effect *always* dominates, and thus the integrated firm wants the rival/customer to *expand* output.

For any input price w_2, we assume there exists a unique downstream Nash equilibrium in pure strategies, with the choice of downstream rival k ($k = 1, 2$) denoted $X_k^*(w_2)$ and its corresponding output level denoted $Q_k^*(w_2)$.[6,7] Finally, we assume that an increase in the input price to firm 2 leads it to reduce output under Cournot competition (i.e. $\mathrm{d}X_2^*/\mathrm{d}w_2 < 0$) and to increase price under Bertrand competition (i.e. $\mathrm{d}X_2^*/\mathrm{d}w_2 > 0$).[8]

Firm 1 sets w_2 to maximize $V^*(w_2) \equiv V(X_1^*(w_2), X_2^*(w_2); w_2)$. Let w_2^* denote the profit-maximizing choice, and (x_1^*, x_2^*) denote the resulting equilibrium downstream choices. Recalling that $x_k = p_k$ under Bertrand competition and $x_k = q_k$ under Cournot ($k = 1,2$), we now state our main result:

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[6] We assume the firms' reaction functions downstream are either strictly increasing (strategic complements) or strictly decreasing (strategic substitutes), and have slopes smaller than unity in absolute value. These conditions are assumed to hold over the relevant range of the variables.

[7] We thus have $Q_k^*(w_2) \equiv Q_k(X_1^*(w_2), X_2^*(w_2))$. Under Cournot, $Q_k(x) = x_k$ and $Q_k^*(w_2) = X_k^*(w_2)$.

[8] We assume that the direct cost effect dominates the indirect strategic effect. See Appendix A.

Proposition 1. Assume firm 1 sells the input to firm 2 under linear pricing. At the equilibrium outcome $(w_2, x_1, x_2) = (w_2^*, x_1^*, x_2^*)$, firm 1 wants firm 2 to expand output, i.e. $\partial V / \partial x_2 > 0$ if downstream competition is Cournot, and $\partial V / \partial x_2 < 0$ if it is Bertrand.

Proof. The first-order condition (FOC) with respect to w_2 is

$$\frac{dV^*}{dw_2} \equiv \frac{\partial V}{\partial x_1} \frac{dX_1^*}{dw_2} + \frac{\partial V}{\partial x_2} \frac{dX_2^*}{dw_2} + \frac{\partial V}{\partial w_2} = 0. \tag{3}$$

The FOC with respect to x_1 implies $\partial V / \partial x_1 = 0$, and (1) implies $\partial V / \partial w_2 = Q_2^*$, so that (3) can be rewritten as

$$\frac{\partial V}{\partial x_2} \frac{dX_2^*}{dw_2} = -Q_2^*. \tag{4}$$

It then follows from (4) and $Q_2^* > 0$ that

$$\frac{\partial V}{\partial x_2} \frac{dX_2^*}{dw_2} < 0. \tag{5}$$

If downstream competition is Bertrand, then $dX_2^*/dw_2 > 0$ and (5) implies $\partial V / \partial x_2 < 0$. If downstream competition is Cournot, then $dX_2^*/dw_2 < 0$ and (5) implies $\partial V / \partial x_2 > 0$.

Proposition 1 identifies opposite incentives to those in a standard duopoly setting where firm 1 does not supply firm 2. There, holding x_1 constant, firm 1 would gain from a rise in 2's price under Bertrand competition or a fall in 2's output under Cournot. This is the usual "softening downstream competition" effect. It is present also here, but dominated by an opposing "input supply" effect. The logic is shown in (4). Given that firm 1 sets w_2 at the profit-maximizing level, a reduction in w_2 would have the following equal but opposite effects: profit from inframarginal input sales would fall, implying that profit must rise from increased input sales—after incorporating the loss in firm 1's downstream profit caused by firm 2's output expansion (see (2)). Thus, holding w_2 constant at w_2^* and x_1 constant at x_1^*, the integrated firm would gain on balance, despite the loss in its downstream profit, if firm 2 were exogenously to increase its input purchases and downstream sales. (We show in Section 3 that vertical delegation can induce a change in x_2 by signaling a change in x_1.)

2.2 Two-Part Tariff for the Input

When the integrated firm sells the input to firm 2 using a two-part tariff—i.e. a pair (w_2, f_2), where f_2 is a fixed upfront fee—its total profits are $V^T(\boldsymbol{x}; w_2, f_2) = V(\boldsymbol{x}; w_2) + f_2$, where $V(\boldsymbol{x}; w_2)$ is given in (1). As we will explain, the integrated firm cannot achieve the vertically integrated monopoly profit despite the use of a two-part tariff and, therefore, at the equilibrium, it still wants firm 2 to change its output. The analysis is more intricate than with linear pricing and the results will depend on the specifics of downstream competition. In that regard, let $R_1(x_2; w_2)$ denote firm 1's "reaction function" for its downstream choice, i.e. the value of x_1 that maximizes $V(\boldsymbol{x}; w_2)$. Note that $\partial R_1/\partial w_2 > 0$ if downstream competition is Bertrand, and $\partial R_1/\partial w_2 = 0$ if it is Cournot.[9] By definition, $\partial R_1/\partial x_2 > 0$ if downstream choices are strategic complements, and $\partial R_1/\partial x_2 < 0$ for strategic substitutes.

If firm 2 accepts the two-part tariff offer (w_2, f_2) (as it will in equilibrium), the downstream outcome is given by the functions $X_1^*(w_2)$ and $X_2^*(w_2)$ as with linear pricing. Denote firm 2's profit gross of the fixed fee as $\Pi_2^*(w_2) = \Pi_2(X_1^*(w_2), X_2^*(w_2); w_2)$. In equilibrium, firm 1 will extract firm 2's profit by setting $f_2 = \Pi_2^*(w_2)$. Therefore, it sets w_2 to maximize $V^{*T}(w_2) = V^*(w_2) + \Pi_2^*(w_2)$, where $V^*(w_2)$ is the same as with linear pricing. Let w_2^T and $f_2^T \equiv \Pi_2^*(w_2^T)$ denote the profit-maximizing two-part tariff, and (x_1^T, x_2^T) denote the resulting equilibrium downstream choices.

Proposition 2. Assume firm 1 sells the input to firm 2 under a two-part tariff. At the equilibrium outcome $(w_2, f_2, x_1, x_2) = (w_2^T, f_2^T, x_1^T, x_2^T)$, firm 1 wants firm 2 to expand output if downstream choices are strategic complements, and wants firm 2 to contract output if downstream competition is Cournot and quantities are strategic substitutes:
(i) $\partial V^T/\partial x_2 < 0$ if downstream competition is Cournot and quantities are strategic substitutes, and $\partial V^T/\partial x_2 > 0$ if quantities are strategic complements.
(ii) $\partial V^T/\partial x_2 < 0$ if downstream competition is Bertrand and prices are strategic complements, and $\partial V^T/\partial x_2$ is ambiguous if prices are strategic substitutes.

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[9] Differentiating the FOC $\partial V/\partial x_1 = 0$ and using the SOC $\partial^2 V/\partial x_1^2 < 0$, one finds: $\text{sign}(\partial R_1/\partial w_2) = \text{sign}(\partial^2 V/\partial x_1 \partial w_2)$. From (1), we have $\partial^2 V/\partial x_1 \partial w_2 = \partial Q_2/\partial x_1$. Under Bertrand $\partial Q_2/\partial x_1 > 0$ while, under Cournot, $\partial Q_2/\partial x_1 = 0$. We explain the intuition in Section 3, after (9).

Proof. The integrated firm sets w_2 to maximize $V^{*T}(w_2) = V^*(w_2) + \Pi_2^*(w_2)$. The FOC is

$$\frac{dV^{*T}}{dw_2} \equiv \frac{dV^*}{dw_2} + \frac{d\Pi_2^*}{dw_2} = \frac{\partial V}{\partial x_2}\frac{dX_2^*}{dw_2} + Q_2^* - Q_2^* + \frac{\partial \Pi_2}{\partial x_1}\frac{dX_1^*}{dw_2} = 0, \qquad (6)$$

where $\partial V/\partial x_1 = 0$ by the Envelope Theorem. Decomposing dX_1^*/dw_2 and rearranging,

$$\frac{\partial V}{\partial x_2}\frac{dX_2^*}{dw_2} = -\frac{\partial \Pi_2}{\partial x_1}\left(\frac{\partial R_1}{\partial x_2}\frac{dX_2^*}{dw_2} + \frac{\partial R_1}{\partial w_2}\right). \qquad (7)$$

(i) For Cournot competition: $dX_2^*/dw_2 < 0$, $\partial \Pi_2/\partial x_1 < 0$, and $\partial R_1/\partial w_2 = 0$; (7) implies $\partial V/\partial x_2 < 0$ and hence $\partial V^T/\partial x_2 < 0$ if quantities are strategic substitutes ($\partial R_1/\partial x_2 < 0$), and $\partial V^T/\partial x_2 > 0$ if instead $\partial R_1/\partial x_2 > 0$.

(ii) For Bertrand competition: $dX_2^*/dw_2 > 0$, $\partial \Pi_2/\partial x_1 > 0$, and $\partial R_1/\partial w_2 > 0$; (7) implies $\partial V^T/\partial x_2 < 0$ if prices are strategic complements ($\partial R_1/\partial x_2 > 0$). If instead $\partial R_1/\partial x_2 < 0$, the term in parentheses in (7) has an ambiguous sign, hence so does $\partial V^T/\partial x_2$.

Interestingly, for Cournot competition and the "normal" case of strategic substitutes, the pattern is reversed relative to Proposition 1 with no fixed fee (i.e. $f_2 \equiv 0$): starting at the equilibrium outcome, the integrated firm wants to induce a *decrease* in q_2. For Bertrand competition and the "normal" case of strategic complements, we have the same pattern as in Proposition 1 since the integrated firm wants to induce a decrease in p_2.

These patterns can be understood by comparing the FOCs with linear input pricing versus a two-part tariff. With linear input pricing, a decrease in w_2 is costly to firm 1 since revenue falls from inframarginal input sales, term $-Q_2^*$ in (4); thus, at the optimal w_2, the other effect of a decrease in w_2—expansion by firm 2—must increase the integrated firm's profit, V. Therefore, the integrated firm wants firm 2 to expand (Proposition 1). With a two-part tariff for the input, a decrease in w_2 does not reduce firm 1's profit from inframarginal input sales, due to the compensating increase in firm 2's fixed fee (term Q_2^* cancels in (6)). Instead, a decrease in w_2—besides inducing expansion by firm 2—affects firm 1's profit by altering its downstream choice, which changes firm 2's profit and, hence, the fixed fee (terms $(\partial \Pi_2/\partial x_1)(dX_1^*/dw_2)$ in (6)).

In turn, dX_1^*/dw_2 is the sum of two effects shown in parentheses in (7). With Cournot competition, an increase in w_2 will reduce firm 2's output ($dX_2^*/dw_2 < 0$) and increase firm 1's output in the "normal" case of strategic substitutes ($\partial R_1/\partial x_2 < 0$).[10] Firm 1's increased output reduces firm 2's gross profit and thus the fixed fee, which by itself reduces firm 1's total profit. From the optimality of w_2^T, the other effects of firm 2's contraction (induced by an increase in w_2) necessarily increase firm 1's profit. Thus, starting at the optimal two-part tariff, firm 1 wants firm 2 to *contract* if quantities are strategic substitutes.[11]

With Bertrand competition, a decrease in w_2 will reduce firm 1's downstream price in the "normal" case of strategic complements due to the fall in firm 2's price, *and* for a second reason: by lowering the profit from input sales to firm 2 and, hence, the opportunity cost of expanding firm 1's output.[12] Since this reduction in firm 1's price reduces firm 2's gross profit and thus the fixed fee, the previous logic implies that at the optimal two-part tariff, firm 1 would benefit from *expansion* by firm 2, as with linear input pricing.[13]

Why would the integrated firm benefit from an exogenous change in firm 2's choice, x_2, notwithstanding the assumed ability to extract firm 2's profit through the fixed fee? The (perhaps obvious) answer is that in our contracting environment the integrated firm lacks sufficient instruments to maximize overall industry profit, downstream plus upstream. Denote the required downstream choices as the "monopoly solution," x_1^m and x_2^m. The integrated firm can induce x_2^m by appropriately setting $w_2 = X_2^{*-1}(x_2^m)$. However, since we assumed that firm 2's fixed fee f_2 cannot be contingent on x_1, the integrated firm's best response will not be x_1^m but $R_1(x_2^m; X_2^{*-1}(x_2^m))$, the value of x_1 that maximizes firm 1's profit while ignoring the effect on firm 2 (since the fixed fee is "sunk" when firm 1 sets x_1). At $x_1 = R_1(x_2^m; X_2^{*-1}(x_2^m))$, we have $\partial V/\partial x_1 = 0$, implying $\partial(V + \Pi_2)/\partial x_1 = \partial\Pi_2/\partial x_1$

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[10] In (7), $\partial R_1/\partial w_2 = 0$ since $\partial Q_2/\partial x_1 = 0$ under Cournot competition. See footnote 9.

[11] If quantities are strategic complements, a decrease in w_2 leads firm 1 to expand and thus reduce the fixed fee (ceteris paribus), which implies that firm 1 wants firm 2 to expand.

[12] In (7), $\partial R_1/\partial w_2 > 0$ since $\partial Q_2/\partial x_1 > 0$ under Bertrand competition. *See* footnote 9.

[13] If prices are strategic substitutes, a decrease in w_2 has opposing effects on firm 1's downstream price (the terms in parentheses in (7) run in opposite directions), and the net effect is ambiguous.

which is positive under Bertrand competition and negative under Cournot. Thus, industry profit would increase if the integrated firm acted less aggressively downstream (raised price or reduced output). Intuitively, given the inability to condition f_2 on x_1, firm 2 expects firm 1 to favor its own downstream unit at firm 2's expense, and reduces accordingly the fixed fee it is willing to pay. Thus, the two-part tariff solution under our informational assumptions fails to maximize industry profit, leaving room for the integrated firm to do better with additional instruments even though it already extracts firm 2's profit.

Indeed, (7) shows that under a two-part tariff, the optimal input price w_2^T does "double duty" by balancing two effects: how w_2 affects the integrated firm's variable profit by altering firm 2's downstream choice $(\partial V/\partial x_2)(dX_2^*/dw_2)$; and how w_2 affects firm 2's profit (and, hence, f_2) by altering firm 1's downstream choice $(\partial \Pi_2/\partial x_1)(dX_1^*/dw_2)$. Thus, w_2 is used partly to alter firm 2's choice and partly as a signal to firm 2 about firm 1's downstream choice. This logic suggests that the integrated firm would benefit if it found a way to signal credibly about the level of x_1 beyond relying solely on the input price w_2 to firm 2. The next section considers this issue.

3.   Application: Altering Rival's Choice through Vertical Delegation

As noted in the Introduction, the literature on strategic competition in two-stage games identifies various commitment mechanisms a firm may use to visibly alter its incentives in the subsequent competition (shift its best-response function) and signal a change in its downstream choice, so as to alter a rival's choice. In our context, the integrated input supplier potentially could use any of those mechanisms—depending on their cost—to induce the desired change by its rival/customer. For example, firm 1 could make an investment that changes its marginal cost of selling output in the downstream market.

Here, we focus on a mechanism that does not require an investment and does not have any direct cost (at least not explicitly modeled), vertical *Delegation*: the integrated firm establishes a downstream unit, division 1, and commits to charge it a publicly observable input price; and the division treats a portion (no matter how small) of the input price charged to it by the upstream unit as a cost rather than a purely internal transfer. To simplify the exposition, we assume that division 1 maximizes solely its own profit, and will

explain later how the results extend to other objective functions. We begin with linear pricing of the input, and then discuss a two-part tariff.

Under Delegation, the game is as follows. First, firm 1 publicly commits to a pair of input prices $\boldsymbol{w} = (w_1, w_2)$ where w_1 denotes the price to its division 1. Given these observed prices, division 1 and firm 2 make downstream choices $\boldsymbol{x}$ simultaneously, consumers purchase, and firm 1 receives input payments. Division 1 chooses x_1 to maximize only its profit $\Pi_1^d(\boldsymbol{x}; w_1) \equiv \Pi_1(\boldsymbol{x}; c) - (w_1 - c)Q_1(\boldsymbol{x})$, but in the prior stage firm 1 now sets both w_2 and the new instrument w_1 to maximize integrated profit, $V(\boldsymbol{x}; w_2)$.[14] For any given $\boldsymbol{w}$, assume there exists a unique downstream equilibrium in pure strategies, with the choice of downstream rival k denoted $X_k^D(\boldsymbol{w})$ and its output level denoted $Q_k^D(\boldsymbol{w})$. We assume $\partial X_k^D / \partial w_k > (<) \, 0$ if downstream competition is Bertrand (Cournot).[15] Under this Delegation regime, firm 1 sets w_1 and w_2 to maximize the continuation equilibrium profit function $V^D(\boldsymbol{w}) \equiv V(X_1^D(\boldsymbol{w}), X_2^D(\boldsymbol{w}); w_2)$. Let (w_1^D, w_2^D) denote the optimal choice.

We will compare this Delegation structure to the structure from Section 2, that we now label *Centralization*.[16] For later use, observe that the best response function of firm 2, $R_2(x_1; w_2)$, is the same under both regimes. Also, let $W_1(w_2)$ denote the integrated firm's optimal choice for w_1 given w_2, i.e. $w_1 = W_1(w_2)$ maximizes $V^D(\boldsymbol{w})$.

It is useful to introduce the concept of *shadow marginal cost* of the input. When an integrated firm sells inputs to rivals, its relevant marginal cost for expanding its downstream output includes both the resource marginal cost c and an opportunity cost of lost profits from reduced input sales to its downstream rival(s) (e.g. Sappington, 2005). In (1), we decompose firm 1's profit from its downstream output sales as $\Pi_1(\boldsymbol{x}; c) \equiv G(\boldsymbol{x}) - cQ_1(\boldsymbol{x})$, where $G(\boldsymbol{x})$ is firm 1's profit on its downstream sales gross of the cost of the internally provided input, and write:

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[14] Firm 1's total profit, V, does not depend directly on w_1 since input payments from division 1 are a pure transfer, but w_1 will affect V indirectly by changing the equilibrium values of $\boldsymbol{x}$.

[15] See Appendix A.

[16] Arya, Mittendorf, and Yoon (2008) establish results similar to Propositions 3 through 5 below for the case of linear (differentiated) demands and Cournot competition. They further show that decentralization (our Delegation) increases the integrated firm's profit if the input price to the downstream division is determined by bargaining.

$$\frac{\partial V}{\partial x_1} = \frac{\partial G}{\partial x_1} - \left[c + (w_2 - c) \left(-\frac{\partial Q_2 / \partial x_1}{\partial Q_1 / \partial x_1} \right) \right] \frac{\partial Q_1}{\partial x_1}. \tag{8}$$

The term in square brackets can be interpreted as the integrated firm's shadow marginal cost for expanding its downstream output. We write it more compactly as

$$C_1(w_2, \boldsymbol{x}) \equiv c + (w_2 - c) D_{21}(\boldsymbol{x}), \tag{9}$$

where $D_{21} \equiv -\frac{\partial Q_2 / \partial x_1}{\partial Q_1 / \partial x_1}$ is the input diversion ratio—i.e. decreased input sales to firm 2 per extra unit of input to division 1. Note that $D_{21} > 0$ if downstream competition is Bertrand, since division 1's expansion (induced by cutting price) displaces some sales of firm 2; but $D_{21} = 0$ if competition is Cournot, since division 1 then takes firm 2's output, and thus also firm 2's input purchases, as given.[17] Let $C_1^*(w_2) \equiv C_1^*(w_2, X_1^*(w_2), X_2^*(w_2))$ denote the shadow marginal cost at the continuation equilibrium under Centralization.

Proposition 3. (i) Under Delegation, the integrated firm can achieve the same profit as under Centralization by setting $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$, i.e. $V^*(w_2^*) = V^D(C_1^*(w_2^*), w_2^*)$. (ii) Therefore, the integrated firm's equilibrium profit is (weakly) higher with Delegation: $V^*(w_2^*) = V^D(C_1^*(w_2^*), w_2^*) \leq V^D(W_1(w_2^*), w_2^*) \leq V^D(w_1^D, w_2^D)$.

Proof. (i) Under Delegation, suppose the integrated firm set $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$, i.e. an input price to division 1 equal to the shadow marginal cost under Centralization. This would induce the same downstream equilibrium choices as with Centralization:

$$X_1^D(C_1^*(w_2^*), w_2^*) = x_1^* \quad \text{and} \quad X_2^D(C_1^*(w_2^*), w_2^*) = x_2^*. \tag{10}$$

To see this, observe first that under Centralization, firm 1's best response function maximizes $V(\boldsymbol{x}; w_2)$ and, recalling (1), is implicitly determined by:

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[17] Arya, Mittendorf, and Sappington (2008) invoke this distinction to show that, unlike in standard duopoly, Bertrand competition downstream can yield higher prices than Cournot when a partially integrated input monopolist sells also to a downstream rival, because the monopolist internalizes a higher opportunity cost under Bertrand than under Cournot.

$$\frac{\partial V}{\partial x_1} \equiv \frac{\partial \Pi_1}{\partial x_1} + (w_2 - c) \frac{\partial Q_2}{\partial x_1} = 0, \tag{11}$$

whereas with Delegation, division 1's best response function maximizes $\Pi_1^d(\boldsymbol{x}; w_1)$ and, hence, using $\Pi_1(\boldsymbol{x}; c) \equiv \Pi_1^d(\boldsymbol{x}; w_1) + (w_1 - c)Q_1(\boldsymbol{x})$, is implicitly determined by

$$\frac{\partial \Pi_1^d}{\partial x_1} \equiv \frac{\partial \Pi_1}{\partial x_1} - (w_1 - c) \frac{\partial Q_1}{\partial x_1} = 0. \tag{12}$$

Setting $w_1 = C_1(w_2, \boldsymbol{x})$ from (9) makes (12) identical to (11), so the best response for x_1 will be the same function of w_2 and x_2 in both regimes. Since firm 2's best response to w_2 and the expected x_1 is also the same function in both regimes, setting $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$ under Delegation would replicate the Centralization outcome.

(ii) Under Delegation, the integrated firm's optimal choice for w_1 conditional on $w_2 = w_2^*$ is typically not $C_1^*(w_2^*)$. Profit could be (weakly) increased by setting w_1 at the optimal level $W_1(w_2^*)$. The actual optimum will generally involve changing both w_1 and w_2, thereby further increasing profit compared to Centralization.

Observe that the ability under Delegation to charge division 1 an observable input price $w_1 \neq C_1^*(w_2^*)$ benefits the integrated firm solely because it alters firm 2's choice by signaling a change in division 1's downstream choice.[18] Instead of Delegation, the same outcome could be achieved under Centralization if the integrated firm could act as a Stackelberg leader in the downstream competition, as in the following game.[19]

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[18] The direct effect of changing w_1 (holding $\boldsymbol{x}$ constant) on firm 1's profit is zero; and the change in x_1 induced by a small change in w_1 also has no first-order effect because, given $w_2 = w_2^*$ and $x_2 = x_2^*$, setting $w_1 = C_1^*(w_2^*)$ induced the level of x_1 that maximizes firm 1's profit.

[19] Lu, Moresi, and Salop (2007) show the same result as the ensuing Proposition 4 assuming Bertrand competition. On the general connection between strategic delegation and Stackelberg leadership in the competition game, *see* Vickers (1985, sections I and II). Adapted to our setting, his agent appointment game—that precedes downstream competition—corresponds to whether the integrated firm initially adopts the Centralization structure and sets only an input price w_2 or the Delegation structure and sets an additional input price w_1. The agent appointment game maximizes the integrated firm's profit if and only if it implements the same downstream outcome as Stackelberg leadership by the integrated firm. See also Heifetz, Shannon, and Spiegel (2007).

Leadership: As with Centralization, firm 1 publicly commits only to w_2. But now downstream choices occur sequentially, with first firm 1 choosing x_1 to maximize $V(x_1, R_2(x_1; w_2), w_2)$ and firm 2 then choosing its best response, $x_2 = R_2(x_1; w_2)$.

Proposition 4. The integrated firm's profit is the same under Delegation or Leadership.

Proof. Under Leadership, the integrated firm sets w_2 and x_1 to maximize:

$$V(x_1, R_2(x_1; w_2); w_2). \tag{13}$$

Under Delegation, let $w_1 = H_1(x_1; w_2)$ be the inverse function of $x_1 = X_1^D(w_1, w_2)$. Setting w_1 and w_2 to maximize $V(X_1^D(\boldsymbol{w}), X_2^D(\boldsymbol{w}); w_2)$ is equivalent to setting w_2 and x_1 to maximize:

$$V(x_1, X_2^D(H_1(x_1; w_2), w_2); w_2). \tag{14}$$

Since $X_2^D(H_1(x_1; w_2), w_2) \equiv R_2(x_1; w_2)$, the two maximization problems are identical.

The next result identifies the integrated firm's incentive to change w_1 under Delegation, relative to the shadow marginal cost under Centralization, $C_1^*(w_2^*)$.

Proposition 5. Holding w_2 constant at w_2^*, under Delegation:
(i) With Bertrand competition, the integrated firm wants to induce a fall in 2's price, implying $W_1(w_2^*) < (>) C_1^*(w_2^*)$ if prices are strategic complements (substitutes).
(ii) With Cournot competition, the integrated firm wants to induce a rise in 2's quantity, implying $W_1(w_2^C) > (<) C_1^*(w_2^*)$ if quantities are strategic substitutes (complements).

Proof. Starting at $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$, under Delegation the integrated firm's desired change in w_1 is determined by the sign of (from $V^D(\boldsymbol{w}) \equiv V(X_1^D(\boldsymbol{w}), X_2^D(\boldsymbol{w}); w_2)$):

$$\frac{\partial V^D}{\partial w_1} = \frac{\partial V}{\partial x_1}\frac{\partial X_1^D}{\partial w_1} + \frac{\partial V}{\partial x_2}\frac{\partial X_2^D}{\partial w_1} = \frac{\partial V}{\partial x_2}\frac{\partial X_2^D}{\partial w_1}. \tag{15}$$

Observe that $\partial X_2^D/\partial w_1 = (\partial R_2/\partial x_1)(\partial X_1^D/\partial w_1)$, and we assumed $\partial X_1^D/\partial w_1 > (<) 0$ if downstream competition is Bertrand (Cournot), while by definition $\partial R_2/\partial x_1 > (<) 0$ if downstream variables are strategic complements (substitutes).

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Setting w_1 here is akin to an "investment" that affects division 1's marginal cost in a standard two-stage game and thereby shifts its best-response function in downstream competition. The change in w_1 needed to induce the desired change in x_2 will therefore depend on familiar issues — whether downstream competition is in prices or quantities and whether these choice variables are strategic complements or substitutes. For brevity, we will therefore explain only part (i) of Proposition 5; the logic for (ii) is similar.

With Bertrand competition, Proposition 1 shows that firm 1's profit would increase if firm 2 were to *lower* its price p_2. Under Delegation, firm 1 can induce firm 2 to cut p_2 by lowering w_1 to signal a reduction in division 1's price p_1, in the "normal" case where prices are strategic complements (or raising w_1 if prices are strategic substitutes). Lowering p_1 is an imperfect way to shrink firm 2's margin and *reduce double marginalization* in input sales to firm 2. However, because the reduction in p_2 is induced by lowering p_1, firm 2's output is likely to decline on balance relative to Centralization.[20]

Interestingly, when moving from Centralization to Delegation, firm 1 likely will raise w_2 above w_2^* under Bertrand competition. Formally, starting at $w_2 = w_2^*$ and $w_1 = C_1^*(w_2^*)$ under Delegation, and using $\partial V/\partial x_1 = 0$, firm 1's desired change in w_2 is determined by the sign of

$$\frac{\partial V^D}{\partial w_2} = \frac{\partial V}{\partial x_2}\frac{\partial X_2^D}{\partial w_2} + \frac{\partial V}{\partial w_2}. \tag{16}$$

Since $\partial V/\partial w_2 = Q_2^*(w_2^*) > 0$, (16) would be zero if firm 2's "pass-through rate" under Delegation, $\partial X_2^D/\partial w_2$, was equal to that under Centralization, dX_2^*/dw_2 (see (4)). However, with Bertrand competition and strategic complements, $\partial X_2^D/\partial w_2$ likely is smaller than dX_2^*/dw_2, as discussed in Appendix A, in which case (16) is positive and firm 1 has an incentive to raise w_2 when holding w_1 constant at the shadow marginal cost $C_1^*(w_2^*)$. Intuitively, raising w_2 increases firm 1's marginal profitability of selling to firm 2, which induces a downstream price increase by firm 1 under Centralization but not under

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[20] The linear demand example in Appendix B supports this intuition. In equilibrium, the integrated firm typically will adjust also w_2, as discussed momentarily. However, the equilibrium change in w_1 and x_2 can be expected to track the direction from Propositions 5 and 1, respectively, as illustrated by the example in Appendix B.

Delegation; this expected differential response leads firm 2 to raise its price by less, hence reduce its input purchases by less, under Delegation following a given increase in w_2.

Now suppose firm 1 charges firm 2 a two-part tariff for the input. Proposition 3 continues to hold—firm 1 earns (weakly) higher profit under Delegation. With Delegation, firm 1 can both maximize industry profit and collect it, by setting the marginal input prices at the levels w_1^m and w_2^m that induce division 1 and firm 2 to choose (unilaterally) the "monopoly solution" x_1^m and x_2^m, and setting f_2 to extract firm 2's profits.[21] With Centralization, firm 1 generally cannot implement this solution even with two-part tariffs (see Section 2.2).

Proposition 4 also continues to hold. Under Leadership, the integrated firm can achieve the monopoly outcome by setting $w_2 = w_2^m$ and $x_1 = x_1^m$, which will induce firm 2 to set $x_2 = x_2^m$. And by definition, it cannot earn more than the monopoly profit. Thus, Delegation and Leadership yield the same payoff, the monopoly profit. Proposition 5, characterizing the incentives regarding w_1 under Delegation, must be modified for the case of a two-part tariff, but for brevity we forgo that exercise.

We end this section with a brief discussion of additional related work, and the plausibility of Delegation by a vertically integrated firm. Bonanno and Vickers (1988) noted the strategic advantage of commitment to observable input prices. They consider differentiated Bertrand competition between two suppliers with prices as strategic complements. If both suppliers are vertically integrated, each provides the input to its downstream retail unit at marginal cost. If both are vertically separated, each sells through a different single retailer and captures the latter's profit with a two-part tariff. For any input price set by one supplier (including marginal cost if integrated), the other supplier prefers to vertically separate and raise the input price somewhat above marginal cost, to coax an increase in the rival's downstream price. Our setting features an input monopolist

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[21] When division 1 and firm 2 are competitors, to implement x_1^m and x_2^m the integrated firm typically must set the marginal input prices above the marginal cost c. The qualitative results do not change if firm 2 has an alternative source of supply and, as a result, the integrated firm cannot extract all the profits of firm 2.

engaged in downstream competition, and we showed that because the rival is also an input customer, the incentive is to induce a *reduction* in its downstream price.

Our assumption that under Delegation the downstream division acts (at least partly) in its own interest is reminiscent of the literature on strategic advantages of creating autonomous *competing divisions* (e.g. Schwartz and Thompson, 1986; Baye, Crocker, and Ju, 1996). There, divisional autonomy makes the firm a tougher competitor, to deter entry or induce output contraction by oligopoly rivals when outputs are strategic substitutes. Here, autonomy involves a vertical division and the strategic gain does not hinge on presenting a tough posture, e.g. under Cournot competition and linear input pricing, the integrated firm uses Delegation to make itself a softer competitor downstream.

Regarding the feasibility of vertical Delegation, there are two requirements: the vertically integrated supplier can commit to charge its downstream division an input price observable to downstream rivals; and the division does not "undo" the strategic effects of that input price by fully internalizing how its choice affects the upstream profits of the affiliated supplier. For expositional convenience we assumed the division maximizes solely its own downstream profit, but all our results extend to any objective function for which the input price affects the division's downstream choice. For example, if the division maximizes a weighted average of its profit and the integrated firm's profit, the supplier can implement its preferred downstream outcome by suitably raising the input price to its division to compensate for the division treating a fraction of that price as a pure transfer (Arya, Mittendorf, and Yoon, 2008, Proposition 3). In practice, integrated firms may find it feasible and beneficial to establish divisions as profit centers for internal incentive reasons. Another mechanism that may prevent the downstream unit from acting as a passive arm of an integrated firm is to allow minority shareholders in the downstream unit, which may limit the ability to treat input payments to the upstream affiliate as a pure transfer.[22]

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[22] For example, O'Brien and Salop (2000) state: "in making decisions that affect the acquired firm [the analogue of our downstream division], the Board of Directors of the acquired firm is constrained to ignore the impact of its actions on the acquiring firm, even if the acquiring firm has a large financial interest in the acquired firm. Instead, the Board must manage the acquired firm to act like an independent, stand-alone entity." (p. 580)

Our analysis suggests that mechanisms that yield divisional autonomy can have strategic benefits in partial integration settings, provided a commitment can be made to charge the division an observable input price.[23] The latter assumption is more questionable for unregulated firms.[24] One consideration that may aid such commitment is the supplier's interest in maintaining a reputation for not acting opportunistically to disadvantage independent customers by secretly favoring its division (e.g. McAfee and Schwartz, 1994). To preserve such a reputation, it may adopt a policy of public and transparent pricing.

4.   Concluding Remarks

We considered a vertically integrated input monopolist that both sells output in the downstream market and supplies inputs to a differentiated downstream rival. Expansion by the rival harms the integrated firm downstream but benefits it upstream from increased input sales. Our main result shows that under linear pricing of the input, the integrated firm would unambiguously gain from exogenous expansion by its rival/customer. The optimal input price balances the revenue loss on inframarginal input sales from a small cut in the input price against the induced expansion by the rival/customer, implying that such expansion must increase the integrated firm's profit, despite the downstream loss. If the input is sold, instead, under a two-part tariff, the inframarginal sales effect is absent, because of the compensating change in the fixed fee, and is replaced by a different effect: how changing the input price alters the supplier's own optimal downstream choice and how that, in turn, affects the attainable fixed fee. The profit effect of expansion by the rival/customer then depends on the specifics of downstream competition. With Bertrand competition and prices as strategic complements, the integrated firm would still benefit from exogenous expansion by the rival/customer, but would lose under Cournot competition with strategic substitutes.

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[23] For a general discussion on firms' ability to make commitments, *see* Shapiro (1989, p. 382).

[24] Regulated firms may be subject to rules governing input pricing to subsidiaries as well as the subsidiaries' behavior through imputation requirements (e.g. Laffont and Tirole, 2000).

We analyzed one of potentially many mechanisms to induce expansion or contraction by the rival/customer, beyond relying solely on the input price to that firm. That mechanism is vertical delegation: establishing a downstream division that treats its input price from the upstream affiliate partly as a cost (rather than a purely internal transfer), and charging that division an input price observable by the rival/customer. Vertical delegation was shown to dominate centralized behavior by the integrated input supplier, due to the increased strategic ability to alter the behavior of the rival/customer.

To focus on the new incentives created when the downstream rival is also an input customer of the vertically integrated supplier, we abstracted throughout from upstream competition. An important extension would be to explore how the results are affected by the presence of upstream competition.

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Appendix

A. Own cost effect and the indirect strategic effect

In Proposition 1, we assume that an increase in the input price leads firm 2 to increase price under Bertrand competition, i.e. $dX_2^*/dw_2 > 0$, and reduce output under Cournot competition, i.e. $dX_2^*/dw_2 < 0$. These derivatives can be analyzed by differentiating the downstream equilibrium conditions

$$X_1^* = R_1(X_2^*; w_2) \quad \text{and} \quad X_2^* = R_2(X_1^*; w_2) \tag{A1}$$

where R_k denotes the reaction function of downstream rival k (k=1,2). Thus:

$$\frac{dX_2^*}{dw_2} = \frac{\dfrac{\partial R_2}{\partial w_2} + \dfrac{\partial R_2}{\partial x_1}\dfrac{\partial R_1}{\partial w_2}}{1 - \dfrac{\partial R_1}{\partial x_2}\dfrac{\partial R_2}{\partial x_1}}. \tag{A2}$$

In (A2), the denominator is positive since we assume the reaction functions have slopes smaller than unity in absolute value. The first term in the numerator, $\partial R_2/\partial w_2$, is the "direct cost effect" and we assume it is strictly positive under Bertrand competition—i.e. holding p_1 constant, an increase in w_2 leads firm 2 to raise p_2—and strictly negative under Cournot competition—i.e. holding q_1 constant, an increase in w_2 leads firm 2 to decrease q_2. The second term is the "indirect strategic effect" and it is zero with Cournot competition (since there is no input diversion, firm 1's shadow marginal cost in (9) is unaffected by w_2, implying $\partial R_1/\partial w_2 = 0$). With Bertrand competition, the indirect strategic effect is positive (like the direct cost effect) if prices are strategic complements. If instead prices are strategic substitutes, we assume that the direct cost effect dominates the indirect strategic effect.

In Proposition 5, we assume division 1's output falls if w_1 increases (holding constant w_2), i.e. $\partial X_1^D/\partial w_1 > (<) 0$ if downstream competition is Bertrand (Cournot). These derivatives can be analyzed by differentiating

$$X_1^D = R_1^D(X_2^D; w_1) \quad \text{and} \quad X_2^D = R_2(X_1^D; w_2). \tag{A3}$$

Note that division 1's reaction function under Delegation, R_1^D, is different from firm 1's reaction function under Centralization, R_1. In particular, under Delegation there is no indirect strategic effect since $\partial R_1^D / \partial w_2 = 0$ (and $\partial R_2 / \partial w_1 = 0$). Thus:

$$\frac{\partial X_1^D}{\partial w_1} = \frac{\dfrac{\partial R_1^D}{\partial w_1}}{1 - \dfrac{\partial R_1^D}{\partial x_2}\dfrac{\partial R_2}{\partial x_1}} \tag{A4}$$

(and a similar expression for $\partial X_2^D / \partial w_2$).

Under Bertrand competition, strategic complements and linear demand, we have $dX_2^* / dw_2 > \partial X_2^D / \partial w_2 > 0$ because $(\partial R_2 / \partial x_1)(\partial R_1 / \partial w_2) > 0$, $\partial R_1^D / \partial w_2 = 0$ and $\partial R_1 / \partial x_2 = \partial R_1^D / \partial x_2$.[25] Intuitively, an increase in w_2 will raise firm 1's shadow marginal cost under Centralization (which is $C_1(w_2, \boldsymbol{x})$ from (9)) but will not affect division 1's perceived marginal cost under Delegation (which is simply w_1). Thus, following an increase in w_2, firm 2 expects a larger price increase by firm 1 under Centralization than under Delegation, leading to a greater equilibrium pass-through rate, $dp_2^* / dw_2 > \partial p_2^D / \partial w_2 > 0$. In turn, firm 2's lower pass-through rate under Delegation encourages firm 1 to raise w_2 above w_2^*.[26]

This incentive helps explain a seeming puzzle in Appendix B, Table 1, where the equilibrium input price charged to firm 2 under Delegation (3.00) is higher than under Centralization (2.97), despite two forces that push in the opposite direction: firm 1's output price under Delegation (3.00) is lower than under Centralization (3.12), which by itself increases the profitability to the integrated firm of diverting sales to firm 2 by reducing w_2; and firm 2's equilibrium output under Delegation (100) is lower than under Centralization (109), which also by itself calls for reducing w_2.

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[25] With linear demand, the derivatives of the reaction functions are scalars and hence do not depend on price, quantity or input price levels.

[26] This incentive does not arise under Cournot competition and linear demand. That is, $dq_2^* / dw_2 = \partial q_2^D / \partial w_2$ since $\partial R_1 / \partial w_2 = \partial R_1^D / \partial w_2 = 0$ and $\partial R_1 / \partial x_2 = \partial R_1^D / \partial x_2$.

B. Example

Assume that the marginal cost of the input is constant, $c = 1$, and that there are no other downstream costs. Consumer demand for the products of firms 1 and 2 is given by $q_i = 500 - 200p_i + 100p_j$, where q_i denotes the output of firm i, and p_i and p_j denote the prices of firms i and j, respectively $(i, j \in \{1,2\}, i \neq j)$.[27]

We begin with the case of Bertrand competition. Table 1 below shows the equilibrium profits, outputs and prices under Centralization and Delegation.

Table 1: Bertrand Competition Downstream

	Centralization	Delegation
Profit of Integrated Firm	696.97	700.00
Profit of Downstream Rival	59.50	50.00
Output of Integrated Firm, q_1	227.27	250.00
Output of Downstream Rival, q_2	109.09	100.00
Output Price of Integrated Firm, p_1	3.12	3.00
Output Price of Downstream Rival, p_2	3.52	3.50
Input Price Charged to Downstream Rival, w_2	2.97	3.00
Shadow Cost of Supplying Input to Division 1, $C_1^*(w_2^*)$	1.98	NA
Input Price Charged to Division 1, w_1	NA	1.75
$W_1(w_2^*)$	NA	1.74

Table 1 illustrates Proposition 3: Delegation allows the integrated firm to increase its profit relative to Centralization. It also illustrates Proposition 5(i): starting from the Centralization solution, the integrated firm wants to induce a *reduction* in firm 2's downstream price p_2; and since linear demands imply that prices are strategic

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[27] In this example where firms 1 and 2 are symmetric, vertical integration of firm 1 and the input monopolist does not lead to foreclosure of firm 2 under Centralization. *See* Arya, Mittendorf, and Sappington (2008). Our results imply that this also is true under Delegation.

complements, inducing a reduction in p_2 requires signaling a reduction in p_1 by lowering w_1: $W_1(w_2^*) < C_1^*(w_2^*)$. That is, at the input price charged to firm 2 under Centralization (2.97), the profit-maximizing input price to division 1 under Delegation is lower than the shadow marginal cost under Centralization, 1.74 < 1.98. In the actual Delegation equilibrium, the integrated firm raises w_2 slightly, from 2.97 to 3.00, and reduces w_1 to 1.75 (instead of 1.74) from the shadow marginal cost of 1.98. The reduction in w_1 and increase in w_2 lead to a (substantial) decrease in the profit of the downstream rival.

   We now turn to the case of Cournot competition.

Table 2: Cournot Competition Downstream

	Centralization	Delegation
Profit of Integrated Firm	682.76	685.71
Profit of Downstream Rival	45.66	48.98
Output of Integrated Firm, q_1	279.31	257.14
Output of Downstream Rival, q_2	82.76	85.71
Output Price of Integrated Firm, p_1	2.86	3.00
Output Price of Downstream Rival, p_2	3.52	3.57
Input Price Charged to Downstream Rival, w_2	2.97	3.00
Shadow Cost of Supplying Input to Division 1, $C_1^*(w_2^*)$	1.00	NM
Input Price Charged to Division 1, w_1	NM	1.29
$W_1(w_2^*)$	NM	1.28

Table 2 also illustrates Proposition 3—the integrated firm's profit again is higher under Delegation. It also illustrates Proposition 5(ii): starting from the Centralization solution, the integrated firm wants to induce an *increase* in firm 2's quantity q_2; and since linear demands imply that quantities are strategic substitutes, this requires signaling a reduction in q_1 by raising w_1. Thus, moving to Delegation the integrated firm wants to raise the input price to division 1 above the shadow marginal cost under Centralization (which, under Cournot competition, simply equals the resource marginal cost, $C_1 = c = 1$, independent of w_2): $W_1(w_2^*) = 1.28 > 1.00 = C_1$. In the actual Delegation equilibrium, w_1 rises

substantially, to 1.29 (from $C_1 = 1$), while w_2 rises slightly, from 2.97 to 3.00, causing the integrated firm's output q_1 to fall and the rival's output q_2 to rise, consistent with the incentives described above. This time, the rival's profit increases, unlike in the Bertrand case.