Third-Degree Price Discrimination and Output: Generalizing a Welfare Result

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One of the best-known conjectures in the economics of price discrimination is that a move by a monopolist from uniform pricing to third-degree price discrimination—charging different prices in different exogenously identifiable markets—reduces the sum of consumer surplus and profit (hereinafter "welfare") if total output decreases. This conjecture can be found, at least implicitly, as far back as A. C. Pigou (1920). It is of some interest, since it suggests a welfare test that only requires knowledge of observable magnitudes. Richard Schmalensee (1981) proves the conjecture assuming that the monopolist can perfectly separate markets and that marginal cost is constant. Hal Varian (1985) extends the result by allowing imperfect arbitrage, so that demand in any market can depend on prices in other markets, and by allowing marginal cost to be constant or increasing. (Schmalensee and Varian establish additional useful results on the welfare effects of third-degree price discrimination.) Using a revealed-preference argument, this note generalizes the result to the case in which marginal cost is decreasing, a serious possibility in the context of monopoly.

In order to motivate the revealed-preference approach, it is helpful to review the intuition for the result when marginal cost is constant or increasing and show why that intuition can break down when marginal cost is decreasing. Suppose that the monopoly output under uniform pricing is q^{u} and that moving to discrimination yields a total output q^{d} below q^{u}. Welfare under discrimination will be no higher than if the same output q^{d} is allocated through uniform pricing: uniform pricing allocates a given total output optimally (it leaves no unexploited gains from reshuffling output between markets), while discriminatory pricing in general will induce misallocations by distorting consumers' choices. Also, welfare achieved if q^{d} is allocated through uniform pricing will be lower than if the higher output q^{u} is allocated through uniform pricing. This follows because q^{u} is the monopolist's choice under uniform pricing, so the demand curve lies above the marginal cost curve at q^{u}. If marginal cost is nondecreasing, demand will lie above marginal cost also at lower outputs; hence, reducing output below q^{u} will reduce welfare.

If marginal cost is decreasing, this type of argument is inconclusive. At some outputs below q^{u} the demand curve might now lie below the marginal cost curve, as illustrated in Figure 1. Thus, welfare under uniform pricing, $W^{\mathrm{u}}(q)$, can increase over some range as output falls below q^{u}. I therefore proceed along a different tack, using a revealed-preference argument that relies on q^{u} being a profit-maximizing output under uniform pricing.

Consider a monopolist selling to n exogenously identifiable markets. Let p_i and q_i respectively denote the price and output sold in market i, $i = 1, \ldots, n$. The monopolist's total cost function is $C(\Sigma q_i)$; that is, total cost depends only on total output and not on its distribution among markets. The markets can be viewed, for example, as different types of customers (e.g., students, senior citizens), different times of purchase (e.g., lunch vs. dinner), or different locations to which the monopolist ships its output. (In the last case, cost can be independent of the output's distribution among markets if, for example, markets are

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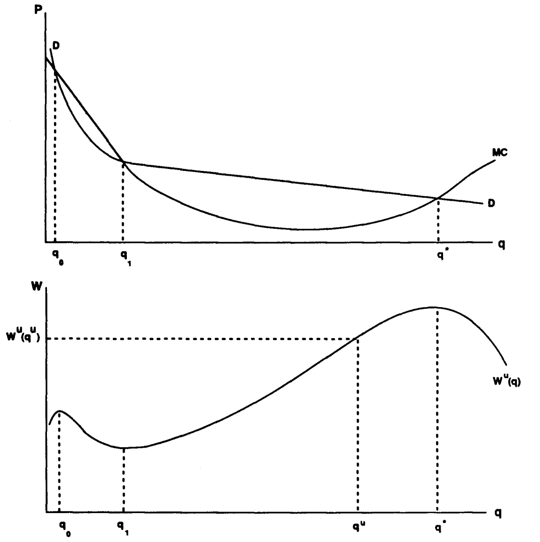

FIGURE 1. WELFARE AND OUTPUT UNDER UNIFORM PRICING: EXAMPLE IN WHICH WELFARE FUNCTION IS NOT SINGLE-PEAKED

equidistant to the monopolist's plant and transport cost is constant.) Following Varian (1985), I allow imperfect arbitrage among markets (with perfect arbitrage, of course, price discrimination would be impossible). That is, if price differentials are sufficiently high, then goods or customers might move between locations, nonstudents might obtain fake student ID's, and dinner patrons might switch to lunch.

It is not necessary to get into details of the arbitrage technology. One simply thinks of the n markets as representing different goods to consumers and allows each individual's indirect utility function to depend on the prices of all n goods. In order to use the classical welfare measure of total consumer surplus plus profit, each individual's indirect utility function is assumed to be quasi-linear in the vector of n prices and in all other goods, which are treated as a composite commodity y and used as the numeraire. Under the quasi-linear preferences, one can also aggregate across consumers and think of the indirect utility function of a representative individual whose endowment in the numeraire is y_0: $f(p_1,\dots,p_n,y_0) = v(p_1,\dots,p_n)+y_0$. The function v embodies whatever substitutability exists among the n goods or, equivalently, whatever arbitrage is possible among the n markets. (For discussions of consumer surplus, aggregation, quasi-linear utility, and the composite commodity theorem see Angus Deaton and John Muellbauer [1980] or Varian [1984].)

If the monopolist's n goods are sold under uniform pricing ($p_i = p$ for all i), then one can simplify further and think also of these n goods as a composite commodity whose price is p, and write the indirect utility function as

$$F(p,y_0)=V(p)+y_0.$$

Note that $V(p)$ gives consumer surplus from purchasing the monopolist's composite good at price p (if one normalizes V by setting $V(p)\to 0$ as $p\to\infty$). $V(p)$ is always strictly decreasing and weakly convex. Since F is linear in y_0, the negative of the derivative of V, where it exists, gives the demand function for the composite good: $q = D(p) = -V'(p)$. The only substantive assumption is that $V(p)$ is strictly convex, that is, that the demand for the monopolist's composite good is a strictly decreasing function of price.

Let $W^u(q)$ denote welfare when the monopolist maximizes profit subject to being constrained to charge uniform prices and to sell a given total quantity q:

$$(1) \qquad W^u(q)=V(h(q))+\Pi(q)$$

where $h(q)$ is the inverse demand function and $\Pi(q)=h(q)q-C(q)$ is profit and where, for simplicity, we omit from welfare the endowment term y_0, which is constant. Observe that V is strictly increasing in q, since it is strictly decreasing in p and since the inverse demand function is strictly decreasing. That is, given a downward-sloping demand curve, consumer surplus is higher if a higher output is sold. Whether pricing is uniform or not, welfare (again ignoring y_0) can also be expressed as utility minus cost:

$$(2) \quad W(q_1,\dots,q_n)=U(q_1,\dots,q_n)-C(\Sigma q_i).$$

It is now possible to establish the welfare result.

PROPOSITION 1: *Suppose that* p^{u} *and* $q^{\mathrm{u}} = D(p^{\mathrm{u}})$ *are a profit-maximizing price and output pair when the monopolist is constrained to charge uniform prices. Consider any discriminatory price vector* $\mathbf{p}^{\mathrm{d}} = (p_1, \dots, p_n)$, $p_i \neq p_j$ *for at least some* $i \neq j$, *which yields an associated output vector* $\mathbf{q}^{\mathrm{d}} = (q_1, \dots, q_n)$, *and denote the total output by* $q^{\mathrm{d}} = \Sigma q_i$. *If total output is lower under discrimination, then welfare also is lower. That is, if* $q^{\mathrm{d}} < q^{\mathrm{u}}$, *then* $W(\mathbf{q}^{\mathrm{d}}) < W^{\mathrm{u}}(q^{\mathrm{u}})$.

PROOF:

I show that $W(\mathbf{q}^{\mathrm{d}}) \leq W^{\mathrm{u}}(q^{\mathrm{d}}) < W^{\mathrm{u}}(q^{\mathrm{u}})$. Consider the first inequality. For any total output q, let $W^*(q)$ denote the solution to the planner's problem: $\max U(q_1, \dots, q_n) - C(\Sigma q_i)$ subject to $\Sigma q_i = q$. Since cost is fixed, the planner's problem is equivalent to $\max U(q_1, \dots, q_n)$ subject to $\Sigma q_i = q$. Now consider $W^{\mathrm{u}}(q)$. Since q is the quantity of the monopolist's composite good, $q = D(p) = \Sigma q_i(p)$, where the outputs $[q_1(p), \dots, q_n(p)]$ maximize utility given $p_i = p$. This means that $D(p)$ solves $\max U(q_1, \dots, q_n)$ subject to $p \Sigma q_i = pq$, which coincides with the planner's problem. Thus, $W^{\mathrm{u}}(q) = W^*(q)$. Since $W^*(q)$ is the maximum feasible welfare given the constraint $\Sigma q_i = q$, the first inequality is established.

Consider the second, more novel inequality. Given $q^{\mathrm{d}} < q^{\mathrm{u}}$, it is known that $V(h(q^{\mathrm{d}})) < V(h(q^{\mathrm{u}}))$. Since q^{u} is a profit-maximizing output (not necessarily unique) under uniform pricing, $\Pi(q^{\mathrm{d}}) \leq \Pi(q^{\mathrm{u}})$. Thus, by expression (1), $q^{\mathrm{d}} < q^{\mathrm{u}}$ implies $W^{\mathrm{u}}(q^{\mathrm{d}}) < W^{\mathrm{u}}(q^{\mathrm{u}})$.

Intuitively, the first inequality reflects the fact that, if the cost function depends only on total output and not on its distribution among goods or markets, then the constraint $\Sigma q_i = q$ can be interpreted as a particular transformation function, one with marginal transformation rates of unity. Uniform pricing reflects these marginal rates of transformation. Thus, a uniform-price equilibrium will maximize welfare for the given level of total output, while discriminatory prices generally will not. This is just the same logic that underlies the first welfare theorem.

The second inequality is where the revealed preference argument comes in. It shows that—regardless of the shape of the cost function—welfare under uniform pricing is higher at a profit-maximizing output q^{u} than at any lower output q^{d}. For more intuition, express welfare under uniform pricing as total valuation minus total cost: $W^{\mathrm{u}}(q) = B(q) - C(q)$, where B is the integral under the demand curve from 0 to q. Since q^{u} maximizes profit, moving from a lower output q^{d} to q^{u} must increase revenue by at least as much as cost: $\Delta R \geq \Delta C$. Since increasing quantity demanded from q^{d} to q^{u} would require lowering price, total valuation would increase by more than revenue: $\Delta B > p^{\mathrm{u}}(q^{\mathrm{u}} - q^{\mathrm{d}}) > \Delta R$. Therefore, $\Delta W = \Delta B - \Delta C > \Delta R - \Delta C \geq 0$, so welfare must increase if, under uniform pricing, output is raised to a profit-maximizing level. Correspondingly, Figure 1 shows welfare at q^{u} to be higher than at any lower output.

Note that if marginal cost is decreasing and the comparison is of two *arbitrary* outputs, both below the efficient level, then one cannot be sure that welfare will be higher at the higher output. When the cost function is concave, welfare—value minus cost—need not be concave everywhere (even though value is concave) and therefore need not be single-peaked. It is because the higher output represents a profit maximum that one can be sure that welfare there is higher.

I conclude with two remarks about the policy relevance of the analysis. First, the welfare result rests on the assumption that demand curves faced by the monopolist generate adequate measures of welfare. This condition can fail, for example, when the monopolist is selling to distorted intermediate-good markets rather than to final consumers. Consider an input monopolist selling at a uniform price to several unrelated intermediate-good industries. Suppose that in equilibrium the proportional price-cost markups are different in the various industries due to different degrees of competition (rather than different demand elasticities). Then, allocating a given quantity of the input through uniform pricing does not maximize welfare for that input quantity; lower input prices should be charged to the industries with the higher markups. If price discrimination by the input monopolist results in such a pattern, then welfare can be higher under discrimination even if the total quantity of the input is lower. (Such desirable discrimination might be profit-maximizing for the monopolist if, for instance, those industries with the higher markups also have greater ability to substitute in production away from the monopolist's input.) That is, price discrimination by the input monopolist could help counteract the downstream distortions. This is a standard second-best ambiguity.

The second remark concerns the information needed for my result and for those of Schmalensee (1981) and Varian (1985) to provide useful welfare tests in practice (assuming that areas under demand curves do accurately reflect welfare). What must the policymaker know in order to infer that welfare is lower under discrimination if output is observed to be lower? My proposition requires the policymaker to be confident that the monopolist knows demand and cost and that the output observed under uniform pricing, q^{u}, is profit-maximizing. Schmalensee (1981) and Varian (1985) require only that marginal cost at q^{u} be less than price (q^{u} need not be profit-maximizing, because of the monopolist's imperfect knowledge about cost and demand), provided the policymaker knows also that marginal cost is nondecreasing at lower outputs. Thus, more information is required for the monopolist but less for the policymaker: the policymaker must know only that the monopolist possesses the requisite information needed to maximize profit under uniform pricing.

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