with intuitive certainty. They were first published as "Tabulae pro calculandis eclipsibus satellitum Jovis" in *Acta Regiae societatis scientiarum Upsaliensis pro 1741* (1746).

Wargentin's values were far more accurate than those of his predecessors, but he nevertheless continued his observations and calculations, which he communicated to Lalande, with whom he corresponded regularly. Wargentin's revised tables of the satellites of Jupiter were published by Lalande in his enlarged edition of *Tables astronomiques de M. Halley* (1759); through new equations he had obtained improved calculations for the movements of the third and the fourth satellites. Wargentin continued to publish new contributions to this subject in Swedish and foreign scientific journals and until his last years he was engaged in improving the theory for the third satellite.

Among his contemporaries Wargentin was considered the outstanding expert in his field, and his tables of Jupiter's moons remained authoritative until the improvement of mathematical analysis made possible exact theoretical solutions of the problems. And his empiricism, even when compared with modern theory, must be considered surprisingly reliable.

Outward circumstances led Wargentin to his other lifelong scientific occupation, population statistics. In 1736 it was decreed that the pastors of Sweden should collect yearly reports on births and deaths, and within the Academy of Sciences the idea grew that the collected material should be submitted to statistical analysis. In 1754 the authorities ordered Wargentin to assume this task. The Royal Table Commission, established two years later with Wargentin as the guiding power, was officially assigned to work with the deposited population tables.

In 1754 Wargentin began publishing his results in a series of demographic articles in the *Transactions* of the Academy. He used both the older and the contemporary pioneers in the field of population statistics (Graunt, Petty, J. P. Süssmilch, Deparcieux); but in his later works he surpassed them and showed a sure, methodical touch. In his most important article, "Mortaliteten i Sverige" (1766), he calculated the mortality rate for different groups in the community: men, women, all inhabitants of Stockholm. He also dealt with birth and mortality rates in different months, the population increase of Stockholm, and the total population of the country. Wargentin may well have been the first to compile mortality tables based on exact figures. His results were of practical importance, especially for life insurance. Richard Price contacted Wargentin and then published the latter's mortality tables in his *Observations on Reversionary Payments* (1783).

Wargentin received many scientific distinctions and in 1783, shortly before his death, became one of the eight foreign members of the Paris Academy. Although not noted for brilliance or innovation, he had a clear and penetrating mind, even when dealing with mundane matters, almost unlimited energy, and a strong moral integrity.

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II. Secondary Literature. The basic biography is N. V. E. Nordenmark, *Pehr Wilhelm Wargentin* (Uppsala, 1939), in Swedish. See also Sten Lindroth, in *Kungliga Svenska vetenskapsakademiens historia 1739–1818*, I: pt. 1 (Stockholm, 1967), 48–59, 411–416; and his "Pehr Wilhelm Wargentin," in *Swedish Men of Science* (Stockholm, 1952), 105–112. On the moons of Jupiter, see Bertil Lindblad, "P. W. Wargentins arbeten över Jupitermånarna och modern teori," in *Populär astronomisk tidskrift*, **15** (1934), 9–19. Wargentin as a population statistician has been treated (apart from Nordenmark) by A. R. Cederberg, *Pehr Wargentin als Statistiker* (Helsinki, 1919); and O. Grönlund, *Pehr Wargentin och den svenska befolkningsstatistiken under 1700-talet* (Stockholm, 1946).

Sten Lindroth

WARING, EDWARD (*b.* Shrewsbury, England, *ca.* 1736; *d.* Plealey, near Shrewsbury, 15 August 1798), *mathematics*.

Little is known of Waring's early life. In 1753 he was admitted to Magdalene College, Cambridge, as a sizar, and his mathematical talent immediately attracted attention. He graduated B.A. as senior wrangler in 1757, was elected a fellow of the college, and in 1760 received the M.A. and resigned his fellowship to accept appointment, on the death of John Colson, as sixth Lucasian professor of mathematics. Although his Lucasian professorship was opposed in some quarters because of his age—he was still in his twenties—Waring soon effective-

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ly silenced his critics by publishing, in 1762, his *Miscellanea analytica de aequationibus algebraicis et curvarum proprietatibus*, which gave indisputable proof of his ability and at once established him as a mathematician of the first rank. He was elected a fellow of the Royal Society the following year.

The *Miscellanea* was described by Charles Hutton (in *Mathematical and Philosophical Dictionary*, II [1795], 584) as "one of the most abstruse books written in the abstrusest parts of Algebra." It deals largely with the theory of numbers (some of its chapters are "De fluxionibus fluentium inveniendis," "De methodo incrementorum," and "De infinitis seriebus"), a branch of mathematics for which Waring had a special gift. It contains, without proof, the theorem that every integer is the sum of four squares, nine cubes, nineteen biquadrates, "and so on." In 1770 Waring published *Meditationes algebraicae*, a work that was highly praised by Lagrange; in 1772 he brought out *Proprietates algebraicarum curvarum*; and 1776 saw the publication of *Meditationes analyticae*. In addition to these important treatises, he also, during this period, published a number of learned papers in the *Philosophical Transactions of the Royal Society*. His last major work, *Essay on the Principles of Human Knowledge*, published in 1794, is notable for his application of abstract science to philosophy.

As a mathematician, Waring was unfortunate in working at a time in which English mathematics were in a state of decline. This was in part due to the clumsy notation in which Newton had expounded his calculus and to the geometrical exposition that gave the *Principia* a somewhat archaic appearance and persuaded English readers that the great new mathematical tool forged by Newton and Leibniz (which was then being employed with great vigor and skill on the Continent, particularly by the Bernoullis) was, in fact, not really necessary. This melancholy state of affairs persisted for more than a century, despite the efforts of such distinguished mathematicians as Brook Taylor, Colin Maclaurin, and John Wallis, and led Lalande to observe in a "Notice sur la vie de Condorcet" (*Mercure de France*, 20 Jan. 1796, p. 143) that there was not a single first-rate analyst in all England. (Waring, however, stoutly maintained that his *Miscellanea Analytica* disproved Lalande's charge, and cited its commendation by d'Alembert, Lagrange, and Euler.)

Despite the spectacular improvements in notation by which fundamental mathematical operations were expressed on the Continent, Waring, in his own works, used both the *dei*sm of Leibniz and the *do*tage of Newton—the two great rival systems—indifferently, and made no notable contribution to the establishment of a permanent notation in any branch of mathematics. His method of writing exponents (as, for example, on page 8 of the 1785 edition of his *Meditationes analyticae*) was clumsy in the extreme, and in general his presentation is unattractive and his books difficult to follow. He suffered from an apparent lack of intellectual order that rendered his mathematical compositions so confused that they are almost impossible to follow in manuscript, while his published works, perhaps because of his extreme myopia, are riddled with typographical errors. His language, at best, was obscure.

Waring received the Copley Medal of the Royal Society in 1784. He was also elected a member of a number of European scientific societies, notably those of Göttingen and Bologna. He served as Lucasian professor until his death; he was also a commissioner of the important Board of Longitude. Nor were his activities exclusively mathematical; simultaneously with his composition of his books he turned to medicine, and received the M.D. from Cambridge in 1770. He does not appear ever to have practiced medicine, but it is believed that he carried out dissections in the privacy of his Cambridge rooms.

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WARMING, JOHANNES EUGENIUS BÜLOW (*b.* Mandø, Denmark, 3 November 1841; *d.* Copenhagen, Denmark, 2 April 1924), *botany*.

Warming, professor of botany at the University of Copenhagen from 1886–1911, laid the foundation of a new branch of botany, ecological plant geography, with the publication of his *Plantesamfund* (1895). During the preceding years he had published many papers on various botanical subjects, several of which rank high in the literature of that time. He also had published two excellent textbooks: *Haandbog i den systematiske botanik* (1879) and *Den almindelige botanik* (1880), both of which have since been enlarged, revised, and translated into several languages.

Warming's father was a Lutheran minister on Mandø, one of the north Frisian Islands. From his childhood he loved the west coast of Jutland, with its marshland and dunes, on which he wrote two volumes of *Dansk plantevaekst: Strandvegetationen* (1906) and *Klitterne* (1909); the third volume was *Skovene* (1919). The work is still important for research on the phytoecology of northwestern Europe.

While still a student, Warming became secretary to the Danish zoologist P. W. Lund, who was excavating fossil Bradypodidae at Lagoa Santa, Minas Gerais, Brazil. He spent 1863–1866 in the tropical savannah, carrying out the most detailed and thorough study of a tropical area undertaken at the time. It took twenty-five years for complete presentation of his large collections in "Symbolae ad floram Brasiliae centralis cognoscendandae," printed in *Videnskabelige Meddelelser fra Dansk naturhistorisk Forening i Kjøbenhavn* (1867–1893). Using the "Symbolae" as a basis, Warming published *Lagoa Santa, et bidrag til den biologiske plantegeografi* (1892) with a lengthy summary in French—perhaps his most outstanding work.

After returning from Brazil, Warming studied for a year under Martius, Naegeli, and Ludwig Radlkofer at Munich and, in 1871, under J. L. von Hanstein at Bonn. The morphological-organogenetic point of view was then the leading principle in botany, and within a few years Warming became one of the most prominent workers in this branch of botany. His main works during this period were *Er kjønnen hos vortemaelken (Euphorbia) en blomst eller en blomsterstand?* (1871); *De l'ovule* (1878); and his monograph on purple bacteria: *Om nogle ved Danmarks kyster levende bakterier* (1876).

In the 1870's, however, Warming adopted the theory of evolution. From then on, he became an ardent adherent of the Lamarckian view of the causes of evolution, and his research turned from ontogeny to phylogeny. In 1876 he published his first "Smaa biologiske og morphologiske bidrag" in *Botanisk Tidsskrift*, a series of papers that continued into 1878. They give a masterly account of the morphology and flower biology of numerous species, mostly Danish, pointing out their adaptation to the edaphic factors. Having assumed the difficult task of classifying the plants in a morphological-biological system, Warming published the first results in the monograph *Om skudbygning, overvintring og foryngelse* (1884), based on his examination of Scandinavian species. One of his main works, it illustrates both his comprehensive knowledge and his power to present a subject in an easily understood manner.

Warming was the founder of plant ecology. The term "ecology," first used by Haeckel in 1866, was introduced into botany by H. Reiter in 1885; but it was Warming who made ecology a preferred field of activity for many botanists. In *Plantesamfund* (1895) he formulated the program of his research: "To answer the question: Why each species has its own habit and habitat, why the species congregate to form definite communities and why these have a characteristic physiognomy."

The book created an enormous sensation as a new attempt at grouping and characterizing the plant communities—a new phytogeographical term by which Warming meant a group of species forming a physiognomically well-defined unity, such as a meadow. In all essentials the species of a community are subject to the same external conditions arising from the ecological factors. These factors are of fundamental importance to the ecology of the individual plant and the plant community. Considering water to be the most important factor, Warming divided plant communities into four types: hydrophytic, xerophytic, halophytic, and mesophytic.

Warming's ingenious way of elucidating the relation between the living plant and its surroundings