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# PHILOSOPHICAL TRANSACTIONS,

OF THE

# ROYAL SOCIETY

OF

# LONDON.

VOL. LXIX. For the Year 1779.

PART I.

LONDON,

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MDCCLXXIX.

have been cured with the uſe of vinegar, really never had the hydrophobia, although he had been aſſured, that Dr. Bertoſſi ſaw him in the hydrophobous ſtate. That man, it was true, did receive a very ſlight and ſuperficial ſcratch upon his cheek from the ſame dog, who bit the other two perſons, who became hydrophobous, and afterwards died; but the perſon, of whom the account was publiſhed, about the uſeful diſcovery of a cure by vinegar, was in reality never arrived to the ſtate of the hydrophobia; that is to ſay, to ſuch a degree of the malady, as moſt frequently follows the bite of a mad dog, and which, after ſome weeks, diſcovers itſelf by an uneaſineſs in attempting to drink; and after drinking, by a fever, delirium, convulſions, vomiting, ſweating, and death, within the fifth, and ſometimes within the fourth day.

Dr. Reghellini, having thus found, that the account firſt given him, and the confirmation of it from his friend at Padua, were doubtful, or rather a miſapprehenſion, wrote again to Florence and Piſa, retracting his former account, and relating the fact, as upon a more ſtrict examination he had found it truly to be, and which is exactly agreeable to the account here incloſed.

XXII. *Two*



## XXII. *Two Theorems, by* Edward Waring, *M. A.* Lucaſian *Profeſſor of Mathematics in the Univerſity of* Cambridge, *and F. R. S. In a Letter to* Charles Morton, *M. D. Sec. R. S.*

### THEOREMA I.

FIGURA I.

Read April 25, 1765.

IN datâ Ellipſi inſcribantur duo ($n$) Laterum Polygona $abcde$, &c. et $pqrst$, &c. ad Puncta reſpectiva $a, b, c, d, e$, &c. $p, q, r, s, t$, &c. ducantur Tangentes A B, B C, C D, D E, &c. et P Q, Q R, R S, S T, &c. et ſint

$\angle abB = \angle cbC$, $\angle bcC = \angle dcD$, $\angle cdD = \angle edE$, &c. et $\angle pqQ = \angle rqR$, $\angle qrR = \angle srS$, $\angle rsS = \angle tsT$, et ſic deinceps.

Et erit Summa Laterum

$ab + bc + cd + de +$ &c. $= pq + qr + rs + st +$ &c.

FIGURA 2.

Cor. Ducatur in Ellipſi Polygonum $abcde$ &c. ($n$) Laterum Methodo ſupra traditâ; inſcribatur etiam aliud Polygonum $abklm$ &c. ($n$) Laterum quovis alio

alio Modo, cujus unus Angulus ponitur ad Punctum ($a$), et Summa $ab + bc + cd + de +$ &c. major est quam Summa $ab + bk + kl + lm +$ &c.

## THEOREMA II.

### TAB. IV. FIGURA I.

Describantur circa datam Ellipsim duo ($n$) Laterum Polygona ABCDE &c. et PQRST &c. quorum Puncta Contactuum respective sunt $a, b, c, d, e,$ &c. et $p, q, r, s, t,$ &c.

Et sint

$\overline{\text{Tang.} + \text{Seca. Comp.}} \angle aBb : \overline{\text{Tan.} + \text{Seca. Comp.}} \angle cCb :: bC : bB$, et

$\overline{\text{Tang.} + \text{Seca. Comp.}} \angle cCb : \overline{\text{Tan.} + \text{Seca. Comp.}} \angle cDd :: cD : cC$, et

$\overline{\text{Tang.} + \text{Seca. Comp.}} \angle cDd : \overline{\text{Tan.} + \text{Seca. Comp.}} \angle eEd :: Ed : aD$ &c.

Et sic

$\overline{\text{Tang.} + \text{Seca. Comp.}} \angle pQq : \overline{\text{Tan.} + \text{Seca. Comp.}} \angle qRr :: qR : qQ$, et

$\overline{\text{Tang.} + \text{Seca. Comp.}} \angle qRr : \overline{\text{Tan.} + \text{Seca. Comp.}} \angle sSr :: Sr : rR$, et

$\overline{\text{Tang.} + \text{Seca. Comp.}} \angle sSr : \overline{\text{Tan.} + \text{Seca. Comp.}} \angle tTs :: Ts : sS$, et sic deinceps.

Et erit Summa Laterum

$AB + BC + CD + DE +$ &c. $= PQ + QR + RS + ST +$ &c.

FIGURA

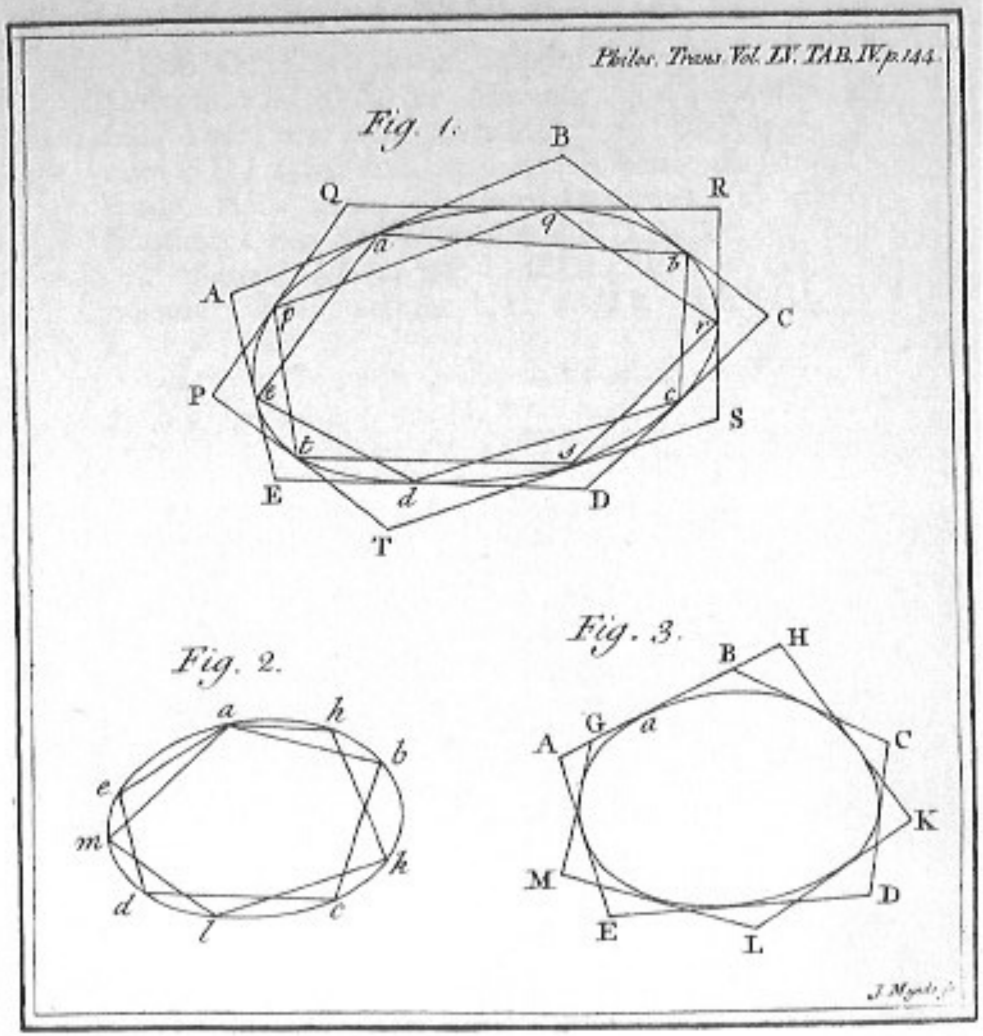

FIGURA 3.

Cor. Defcribatur circa Ellipfim Polygonum ($n$) Laterum A B C D E, &c. Methodo, quæ prius data fuit; Defcribatur etiam circa Ellipfim aliud Polygonum G H $K$ L M, &c. ($n$) Laterum quavis aliâ Methodo, cujus unum Punctum Contactus ($a$) eft Punctum Contactus Polygoni A B C D E, &c.

Et Summa A B + B C + C D + D E + &c. minor erit quam Summa G H + H K + K L + L M + &c.

Confimiles Proprietates affirmari poffunt de Polygonis Hyperbolas defcriptis, &c.

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Preſent Undertakings, Studies, *and* Labours,

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IN MANY

Conſiderable Parts of the WORLD.

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VOL. LV. For the Year 1765.

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*LONDON:*
Printed for L. Davis and C. Reymers,
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againſt *Gray's-Inn Gate*, in *Holbourn*.
M.DCC.LXVI.

The teſtimony of Richard King, which he is ready to make oath to, ſaith, that coming from Kitty-Vitty to St. John's on Sunday the 4th of May, between the King's Bridge and the Garriſon, he ſaw towards the garriſon as if there was a ſtar ſhooting, falling greatly, but only it made too large *. It was as large as a man's head, and juſt before it came to the ground it broke all to pieces, which made like large ſparks of fire flying from it; and in that time it was as light as ever he ſaw all day: And in leſs than two or three minutes there was a rumbling noiſe in the air, ſomething like thunder.

Several other perſons in St. John's were prodigiouſly ſurpriſed at the ſame light.

* His meaning ſeems to be, it was too large for a ſtar.

XXXV. *An*



XXXV. *Some New Properties in Conic Sections, diſcovered by* Edward Waring, *M. A.* Lucaſian *Profeſſor of the Mathematics in the Univerſity of* Cambridge, *and F R. S. to* Charles Morton, *M. D. Sec. R. S.*

## THEOR. I.

Read June 21, 1764.

SIT ellipſis APBQCRDSET, &c. deſcribantur circa eam duo polygona [TAB. XIII. Fig. 1.] ($abcdef$, &c. $pqrstv$, &c.) eundem laterum numerum habentia, & quorum latera ad reſpectiva contactuum puncta (APBQCRDS, &c.) in duas æquales partes dividuntur, i. e. $aA = Ab$, $bB = Bc$, $cC = Cd$, &c. $pP = Pq$, $qQ = Qr$, $rR = Rs$, &c. & erit ſumma quadratorum ex ſingulis unius polygoni lateribus æqualis ſummæ quadratorum ex ſingulis alterius polygoni lateribus, i. e. $ab^2 + bc^2 + cd^2 + de^2 + ef^2 +$, &c. $= pq^2 + qr^2 + rs^2 + st^2 + tv^2 +$, &c.

Cor. Ducantur lineæ AB, BC, CD, DE, EF, &c. PQ, QR, RS, ST, TV, &c. & erit $AB^2 + BC^2 + CD^2 + DE^2 + EF^2 +$ &c. $= PQ^2 + QR^2 + RS^2 + ST^2 + TV^2 +$ &c.

## THEOR. II.

Iisdem positis sit O centrum ellipseos, & ducantur lineæ OA, OP, OB, OQ, OC, OR, OD, OS, &c. erit

$OA^2 + OB^2 + OC^2 + OD^2 + \&c. = OP^2 + OQ^2 + OR^2 + OS^2 + \&c.$

Cor. Ducantur etiam lineæ $Oa$, $Op$, $Ob$, $Oq$, $Oc$, $Or$, $Od$, $Os$, &c. & erit

$Oa^2 + Ob^2 + Oc^2 + Od^2 + \&c. = Op^2 + Oq^2 + Or^2 + Os^2 + \&c.$

Hæc etiam vera sunt de polygonis inter conjugatas hyperbolas eodem modo descriptis.

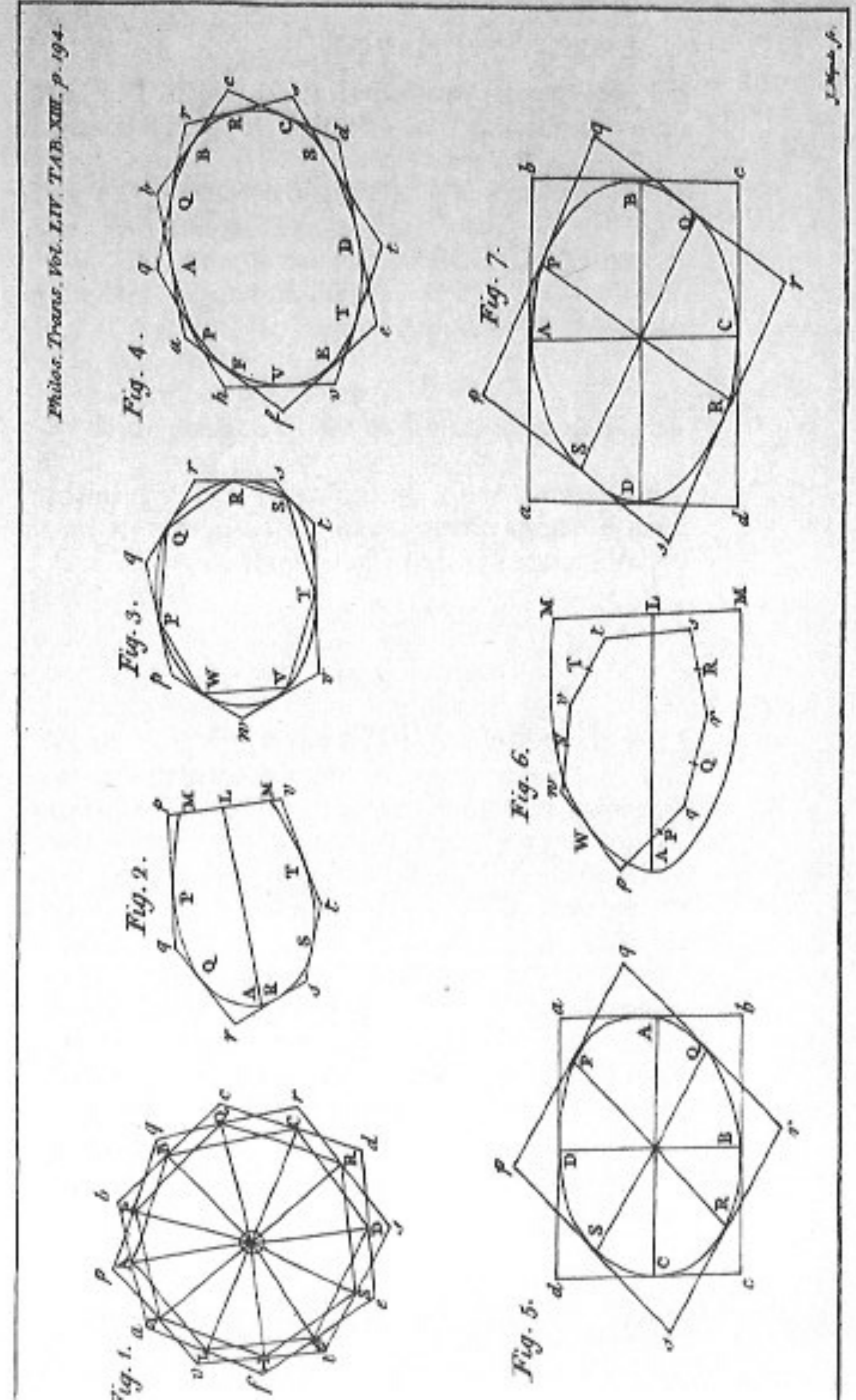
## THEOR. III.

Sit conica sectio MPQRSTM′ &c. [Fig. 2.] cujus diameter sit AL, et ejus ordinata ML; sit $Mp = Mv$, & consequenter $Lp = Lv$.

Ducantur lineæ $pq$, $qr$, $rs$, $st$, $tv$, &c. quæ respective tangant conicam sectionem in punctis P, Q, R, S, T, &c. & erit contentum

$pP \times qQ \times rR \times Ss \times \&c. = Pq \times Qr \times Rs \times St \times Tv \times \&c.$ vel, quod idem est, summa omnium hujus generis rationum ($Pp : Pq$, $Qq : Qr$, $Rr : Rs$, $Ss : St$, &c.) erit nihilo æqualis.

Cor. 1. Sit ellipsis PQRSTV &c. circa eam describatur quodcunque polygonum ($p\,q\,r\,s\,t\,u\,w$, &c.), [Fig.



[Fig. 3.] cujus latera reſpective tangant ellipſim in punctis P, Q, R, S, T, V, &c. & erit contentum

$pP \times qQ \times rR \times sS \times tT \times vV$ &c. $= Pq \times Qr \times Rs \times St \times Tv \times Vw \times$ &c.

Cor. Ducantur lineæ PQ, QR, RS, ST, &c. & pro ſinibus angulorum $WPp$, $QPq$, $RQr$, $QRr$, $SRs$, $TSt$, &c. ſcribantur reſpective $a$, $p$, $b$, $q$, $c$, $r$, $d$, $s$, &c. & erit

$abcd$ &c. $= pqrs$ &c.

Et ſic de polygonis inter conjugatis hyperbolas inſcriptis.

Idem verum eſt de polygono, cujus laterum ſumma vel area minima ſit, circa quamcunque ovalem in ſeſe ſemper concavam deſcripto, ut conſtat e noſtra Miſcell. Anal.

## THEOR. IV.

Sit ellipſis PAQBRCSDTEVF, &c. [Fig. 4.] circa eam deſcribantur duo polygona $abcdef$, &c. $pqrstu$, &c. eundem laterum numerum habentia; eorum latera $ab$, $bc$, $cd$, $de$, $ef$, &c. $pq$, $qr$, $rs$, $st$, $tv$, &c. reſpective tangant ellipſim in punctis A, B, C, D, E, F, &c. & P, Q, R, S, T, U, &c. & ſit $aA : Ab :: pP : Pq$, & $bB : Bc :: qQ : Qr$ & $cC : Cd :: rR : Rs$ & $dD : De :: sS : St$, & ſic deinceps. Et area polygoni $abcdef$, &c. æqualis erit areæ polygoni $pqrstv$, &c.

Cor. Duo parallelogramma ($abcd$ & $pqrs$) circa datæ ellipſeos conjugatas diametros (AC & BD; PR, QS) [Fig. 5.] deſcripta, erunt inter ſe æqualia.

 In

In hoc caſu enim $aA = Ab$, $bB = Bc$, $cC = Cd$, $dD = Da$, & $pP = Pq$, $qQ = Qr$, $rR = Rs$, $sS = Sp$; & conſequenter $aA : Ab :: pP : Pq$ & $bB : Bc :: qQ : Qr$, & ſic deinceps: ergo per theorema hæc duo parallelogramma erunt inter ſe æqualia, quæ eſt notiſſima ellipſeos proprietas.

Idem dici poteſt de polygonis inter conjugatas hyperbolas eodem modo deſcriptis.

## THEOR. V.

Rotetur conica ſectio circa diametrum ejus (AL), & ſit $MAM'$, &c. ſolidum exinde generatum; ſint $pq$, $qr$, $rs$, $st$, $tv$, $vw$, $wp$, &c. [Fig. 6.] lineæ, quæ tangant ſolidum in reſpectivis punctis P, Q, R, S, T, V, W, &c. & erit contentum

$$pP \times qQ \times rR \times sS \times tT \times vV \times wW \times \&c. = Pq \times Qr \times Rs \times St \times Tv \times Vw \times \&c.$$

## THEOR. VI.

Sit ellipſis APBQCR, &c. rotetur circa diametrum ejus BD; & circa conjugatas diametros (AC & BD, PR & QS) deſcribantur elliptici cylindri ($pqrs$ & $acbd$) [Fig. 7.] ſolidum generatum circumſcribentes, & erunt hi duo cylindri inter ſe æquales.

Sint duo ſolida e truncatis conis compoſita, ſolidum generatum circumſcribentibus, & quorum latera continuo



tinuo eâdem ratione ad puncta contactuum dividuntur; erunt hæc duo ſolida inter ſe æqualia.

Et ſic de ſolidis inter conjugatas hyperboloides eodem modo deſcriptis.

Facile conſtant plures conſimiles conicarum ſectionum proprietates.

Hujus generis proprietates affirmari poſſunt de infinitis aliis curvis, ut facile deduci poteſt e noſtrâ Miſcell. Anal.

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M.DCC.LXV.

# XLVI. *Problems by* Edward Waring, *M. A. and* Lucaſian *Profeſſor of Mathematics in the Univerſity of* Cambridge, *F. R. S.*

## P R O.

Read April 21, 1763.

1. INvenire, quot radices impoſſibiles habet data biquadratica æquatio $x^4 + qx^2 - rx + s = 0$.

1mo Sit $256 s^3 - 128 q^2 s^2 + \overline{144 r^2 q + 16 q^4} \times s - 27 r^4 - 4 r^2 q^3$ negativa quantitas, & duas & non plures impoſſibiles radices habet data æquatio.

2do Sit affirmativa quantitas, & vel $-q$ vel $q^2 - 4s$ negativa quantitas, & datæ æquationis quatuor radices erunt impoſſibiles.

3tio. Sit nihilo æqualis, & vel $-q$ vel $q^2 - 4s$ negativa quantitas, & datæ æquationis duæ inæquales radicis erunt impoſſibiles.

2. Invenire, quot radices impoſſibiles habet data æquatio $x^5 + qx^3 - rx^2 + sx - t = 0$.

1mo Si ſigna terminorum æquationis $w^{10} + 10 q w^9 + \overline{39 q^2 + 10 s} \times w^8 + \overline{80 q^3 + 50 qs + 25 r^2} \times w^7 + \overline{95 q^4 + 124 q^2 s - 95 s^2 + 92 q r^2 + 200 rt} \times w^6 + \overline{66 q^5 - 360 q s^2 + 196 q^3 s + 118 q^2 r^2 + 260 r^2 s + 625 t^2 + 400 qrt} \times w^5 + \overline{25 q^6 + 40 s^3 - 53 r^4 + 52 q^3 r^2 - 522 q^2 s^2 + 194 q^4 s + 708 q r^2 s + 240 q^2 rt + 1750 q t^2 - 950 srt} \times w^4 + \overline{4 q^7 + 106 q^5 s - 80 q s^3 - 308 q^3 s^2 - 102 q r^4 - 7 q^4 r^2 + 570 r^2 s^2 + 612 q^2 r^2 s + 700 r^2 t - 3750 t^2 s + 2500 t^2 q^2 + 80 rt q^3 - 2150 qrst} \times w^3 + 400 s^4 - 360 q^2 s^3 - 15 q^4 s^2 + 24 q^6 s - 8 q^5 r^2$

$-45$



$-45 q^2 r^4 - 270 r^4 s + 140 r^2 s q^3 + 960 r^2 s^2 q$ …
$t^2 r^2 + 1000 trs^2 - 5000 t^2 qs + 1750 t^2 q^3 +$ …
$+ 1000 tr^3 q - 1650 trsq^2 \times w^2 + 36 q^3 s^3 - 2$ …
$+ 320 qs^4 + 4 q^3 r^4 + 27 r^6 - 40 r^2 s^3 + 434 r$ …
$24 r^3 sq^4 - 198 r^4 qs + 5000 t^2 s^2 - 450 tr^3 s$ …
$t^3 r + 675 t^2 q^4 - 3750 t^2 q^2 s + 3000 t^2 r^2 q +$ …
$+ 200 trs^2 q - 330 trq^3 s \times w + 3125 t^4 - 37$ …
$+ 2000 s^2 q + 2250 r^2 s - 900 sq^3 + 825 r^2 q^2 +$ …
$\times t^2 - 1600 s^3 r - 560 rq^2 s^2 - 16 r^3 q^3 + 630$ …
$72 rsq^4 - 108 r^3 \times t + 256 s^5 - 128 q^2 s^4 + 14$ …
$+ 16 q^4 s^3 - 27 r^4 s^2 - 4 r^2 q^3 s^2 = 0$. continuo … tur de + in −, & − in +; nullas impoſſib… dices habet data æquatio.

2do. Si ſigna terminorum æquationis haud … nuo mutentur de + in − & − in +; d… quatuor datæ æquationis radices erunt impo… prout ultimus ejus terminus ſit negativa vel aff… va quantitas.

3tio. Si ultimus ejus terminus nihilo ſit æqu… ſigna terminorum æquationis haud continuo mu… de + in − & − in +; tum vel quatuor vel d… dices datæ æquationis erunt impoſſibiles, prou… & non plures ultimi datæ æquationis termini … ſint æquales, necne.

## P R O.

Sint $x$, $y$, $v$, abſciſſa, ordinata & area datæ … & ſit $y^n + \overline{a + bx} \times y^{n-1} + \overline{c + dx + ex^2} \times y^{n-2} + \overline{f \ldots + hx^2 + kx^3} \times y^{n-3} +$ &c. $= 0$: invenire, utrum (v) quadrari poteſt, necne.

Supponamus æquationem ad aream eſſe $\overline{A + Bx + Cx^2}\, v^{n-1} + \overline{D + Ex + Fx^2 + Gx^3 + \ldots}$

*[P]roblems by* Edward Waring, *M. A.*
[L]u[c]asian *Professor of Mathematics in* [the Un]i[v]ersity *of* Cambridge, *F. R. S.*

P R O.

1. INvenire, quot radices impossibiles habet data biquadratica æquatio
$x$ … $-s = 0$.
5 … $s^3 - 128 q^2 s^2 + \overline{144 r^2 q + 16 q^4} \times s -$
… $q^3$ negativa quantitas, & duas & non
… [impossi]biles radices habet data æquatio.
… [affir]mativa quantitas, & vel $-q$ vel $q^2 - 4s$
… [quan]titas, & datæ æquationis quatuor radices
… [impossib]iles.
… [ni]hilo æqualis, & vel $-q$ vel $q^2 - 4s$
… [quan]titas, & datæ æquationis duæ inæquales
… [i]mpossibiles.
… quot radices impossibiles habet data
… $x^3 - r x^2 + s x - t = 0$.
… terminorum æquationis $w^{10} + 10 q w^9$
… $s \times w^8 + \overline{80 q^3 + 50 q s + 25 r^2} \times w^7 +$
… $s - 95 s^2 + 92 q r^2 + 200 r t \times w^6 +$
… $+ 196 q^3 s + 118 q^2 r^2 + 260 r^2 s + 625$
… $w^5 + 25 q^6 + 40 s^3 - 53 r^4 + 52 q^3 r^2 -$
… $q^4 s + 708 q r^2 s + 240 q^2 r t + 1750$
… $\times w^4 + 4 q^7 + 106 q^5 s - 80 q s^3 - 308$
… $- 7 q^4 r^2 + 570 r^2 s^2 + 612 q^2 r^2 s + 700$
… $+ 2500 t^2 q + 80 r t q^3 - 2150 q r s t$
… $- 360 q^2 s^3 - 15 q^3 s^2 + 24 q^5 s - 8 q^3 r^2$
$- 45$



$- 45 q^2 r^4 - 270 r^4 s + 140 r^2 s q^2 + 960 r^2 s^2 q + 1875$
$t^2 r^2 + 1000 t r s^2 - 5000 t^2 q s + 1750 t^2 q^2 + 40 t r q^4$
$+ 600 t r^3 q - 1650 t r s q^2 \times w^2 + 36 q^3 s^3 - 224 q^2 s^4$
$+ 320 q s^4 + 4 q^3 r^4 + 27 r^6 - 40 r^2 s^3 + 434 r^2 q^2 s^2 -$
$24 r^2 s q^4 - 198 r^4 q s + 5000 t^2 s^2 - 450 t r^3 s - 6250$
$t^3 r + 675 t^2 q^4 - 3750 t^2 q^2 s + 3000 t^2 r^2 q + 60 t r^3 q^2$
$+ 200 t r s^2 q - 330 t r q^3 s \times w + 3125 t^4 - 3750 q r t^3$
$+ 2000 s^2 q + 2250 r^2 s - 900 s q^3 + 825 r^2 q^2 + 108 q^5$
$\times t^2 - 1600 s^3 r - 560 r q^2 s^2 - 16 r^3 q^3 + 630 r^3 q s +$
$72 r s q^4 - 108 r^5 \times t + 256 s^5 - 128 q^2 s^4 + 144 r^2 q s^3$
$+ 16 q^4 s^3 - 27 r^4 s^2 - 4 r^2 q^3 s^2 = 0$. continuo mutentur de + in —; & — in +; nullas impossibiles radices habet data æquatio.

2$^{do}$. Si signa terminorum æquationis haud continuo mutentur de + in — & — in +; duæ vel quatuor datæ æquationis radices erunt impossibiles, prout ultimus ejus terminus sit negativa vel affirmativa quantitas.

3$^{tio}$. Si ultimus ejus terminus nihilo sit æqualis, & signa terminorum æquationis haud continuo mutentur de + in — & — in +; tum vel quatuor vel duæ radices datæ æquationis erunt impossibiles, prout duo & non plures ultimi datæ æquationis termini nihilo sint æquales, necne.

P R O.

Sint $x, y, v$, abscissa, ordinata & area datæ curvæ, & sit $y^n + \overline{a + b x} \times y^{n-1} + \overline{c + d x + e x^2} \times y^{n-2} + \overline{f + g x + h x^2 + k x^3} \times y^{n-3} +$ &c. $= 0$: invenire, utrum area ($v$) quadrari potest, necne.

Supponamus æquationem ad aream esse $v^n + \overline{A + B x + C x^2}\, v^{n-1} + \overline{D + E x + F x^2 + G x^3 + H x^4} \times$

$v^{n-2} + \overline{I + Kx + Lx^2 + Mx^3 + Nx^4 + Ox^5 + Px^6} \times v^{n-3} + \&c. = 0$. & confequenter erit

$$\left.\begin{matrix} nyv^{n-1} + \overline{n-1}\,\overline{A+Bx+Cx^2}\,yv^{n-2} + \overline{n-2} \times \overline{D+Ex+Fx^2+Gx^3+Hx^4} \times yv^{n-3} + \&c. \\ \overline{B+2Cx}\; v^{n-1} + \overline{E+2Fx+3Gx^2+4Hx^3} \times v^{n-2} + \&c. \end{matrix}\right\} = 0.$$

Ex quibus æquationibus, fi methodis notis extermi- netur $(v)$, habebimus æquationem, quæ exprimit relationem inter $(x)$ & $(y)$. Hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2} + y^{n-2} + \&c. = 0$; & fi quantitates A, B, C, &c. exinde determinari poffunt, curva quadratur, eft enim $v^n + \overline{A+Bx+Cx^2} \times v^{n-1} + \overline{D+Ex+Fx^2+Gx^3+Hx^4} \times v^{n-2} + \&c. = 0$; aliter autem quadrari non poteft.

Ex. Sit data æquatio $y^2 + x^2 - 1 = 0$, & fupponamus æquationem ad aream $v^2 + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; & erit $2vy + E + 2Fx + 3Gx^2 + 4Hx^3 = 0$, ita reducantur hæ duæ æquationes in unam, ut exterminatur $(v)$, & refultat æquatio $y^2 + \frac{16H^2x^6 + 24HGx^5 + \overline{16HF+9G^2}\,x^4 + \overline{8EH+12FG}\,x^3 + \overline{6GE+4F^2}\,x^2 + 4FEx + E^2}{4 \times \overline{Hx^4 + Gx^3 + Fx^2 + Ex + D}} = 0$; debet autem fractio $\frac{16H^2x^6 + 24HGx^5 + \overline{16HF+9G^2}\,x^4 + \overline{8EH+12FG}\,x^3 + \overline{6GE+4F^2}\,x^2 + 4FEx + E^2}{4 \times \overline{Hx^4 + Gx^3 + Fx^2 + Ex + D}}$ effe $x^2 - 1$; & confequenter





$$\begin{aligned} 4H &= 16H^2 \\ 4G &= 24HG \\ 4F - 4H &= 16HF + 9G^2 \\ 4E - 4G &= 8HE + 12FG \\ 4D - 4F &= 6GE + 4F^2 \\ -4E &= 4FE \\ -4D &= E^2 \end{aligned}$$

fed e methodo communes divifores inveniendi conftat has æquationes inter fe contradictorias effe, & confequenter curvam haud generaliter effe quadrabilem.

## THEO.

Sint $x$, $y$, $v$, abfciffa & ordinatæ curvarum ABCD EFGHI &c. & A$\beta\gamma\delta\varepsilon$ &c. & fit $y = px^n$, & $v = \frac{n}{2.3}pa^{n-1}x - \frac{n \times \overline{n-1} \times \overline{n-2}}{30 \times 2 \times 3}pa^{n-3}x^3 + \frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4}}{42 \times 2 \times 3 \times 4 \times 5}pa^{n-5}x^5 - \frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6}}{30 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}pa^{n-7}x^7 + \frac{5n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \overline{n-7} \times \overline{n-8}}{66 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9}pa^{n-9}x^9 - \frac{691 \times n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \overline{n-7} \times \overline{n-8} \times \overline{n-9} \times \overline{n-10}}{2730 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11}pa^{n-11}x^{11} + \&c.$ cujus ultimus terminus debet effe $x^{n-1}$ vel $x^{n-2}$, prout $(n)$ eft par vel impar numerus.

Sit $x = AP = a$, bifecetur AP in T in duas æquales partes, & ducatur linea ET$\delta$, & fi AE, EM, AM, jungantur; erit triangulum AEM = TP$\varepsilon\delta$T areæ.

**Deinde,** biſecentur T P, A T in R and V, & ducantur R G, C V $\gamma$; & jungantur A C, C E, E G, G M; & erunt duo triangula A C E + E G M = V T $\delta\gamma$ V areæ.

Eodem modo, ſi partes A V, V T, T R, R P iterum biſecentur in W, U, S, Q, & ducantur lineæ B W $\beta$, U D, S F, Q H; & jungantur A B, B C, C D, D E, E F, F G, G H, H M; erunt quatuor triangula A B C + C D E + E F G + G H M = W V $\gamma\beta$ W areæ; & ſic deinceps.

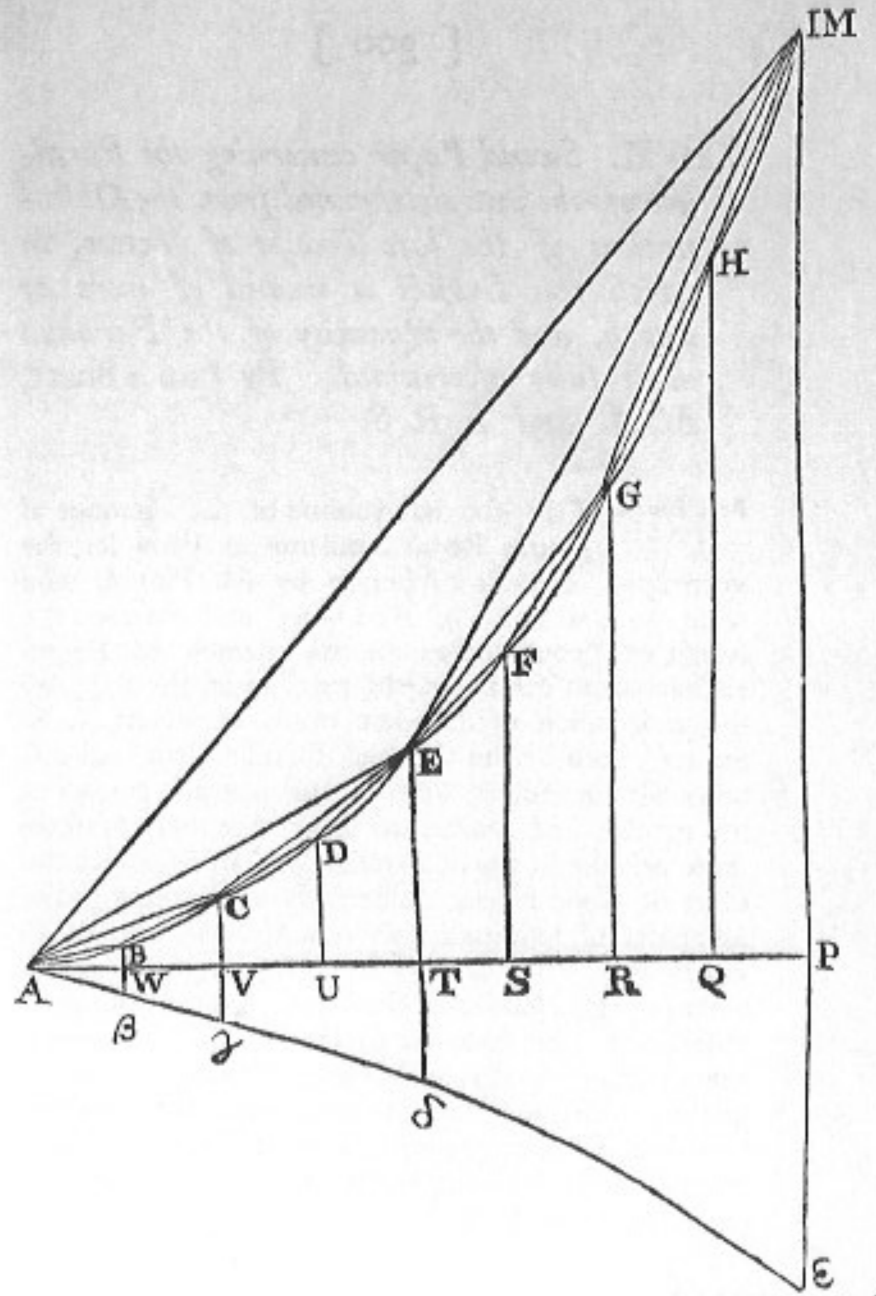
Cor. 1. Si curva A B C & M ſit conica parabola, $(c, e)\ y = p i x^2$, erit $v = \frac{1}{3} p a x$; & A $\beta\gamma\delta$ &c. erit recta linea; & propoſitio eadem eſt cum notiſſimâ propoſitione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = p x^3$, erit $v = \frac{1}{4} p a^2 x$, & A $\beta\gamma\delta$ &c. iterum recta linea.

Cor. 3. Datâ curvâ, cujus æquatio eſt $y = p x^{2n}$, inveniri poteſt altera curva, cujus dimenſiones ſunt $(2n-1)$, in quâ ſummæ triangulorum ad ſingulas biſectiones erunt reſpectivè æquales ſummis triangulorum datæ curvæ.

His adjici poteſt, quod ſi loco biſectionis abſciſſa A P aliâ quâvis ratione in æquales partes dividatur, ſummæ triangulorum curvæ A B C D &c. ad ſingulas diviſiones æquales erunt ſegmentis curvæ A $\beta\gamma\delta$ &c.

IM




XLVII. *Second*