FREQUENCY, THEOREM AND FORMULA: REMEMBERING JOSEPH LARMOR IN ELECTROMAGNETIC THEORY

by

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1. INTRODUCTION

Joseph Larmor (1857–1942) was one of the most distinguished British mathematical physicists of the late-nineteenth century. He introduced both the electron and the so-called Lorentz transformations into physics. His book of 1900, *Aether and Matter*[1], helped to establish a research school that guided the development of mathematical electromagnetic theory in Cambridge until the end of World War I. Today, however, Larmor is widely remembered by scientists for just two formulae and one theorem which, although correctly attributed to him, have been seen by historians of science as tangential to his main research interests. Indeed, none of the recent scholarly studies of Larmor's scientific work even mention the now famous formulae and theorem.[2] In this essay I review Larmor's contribution to post-Maxwellian electromagnetic theory and explain the origin of the specific results upon which his reputation now rests.

2. LARMOR'S EARLY CAREER

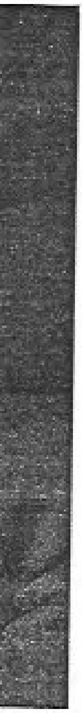
Born in 1857 at Magheragall in County Antrim, Larmor was educated at the Royal Belfast Academy.[3] At the Academy he distinguished himself in mathematics and classics before moving on to Queen's College (Belfast) to read mathematics. In 1876 Larmor went to St John's College, Cambridge, as a mathematics scholar. At Cambridge, he was coached for the Mathematical Tripos by the outstanding mathematics coach, E.J. Routh, and, in 1880, Larmor added to Routh's astonishing record by becoming senior wrangler and first Smith's prizeman.[4] The Mathematical Tripos of 1880 is especially noteworthy as the student beaten into second place by Larmor was J.J. Thomson. In 1884 Thomson succeeded Lord Rayleigh as Cavendish Professor of Experimental Physics, while Larmor followed George Stokes as Lucasian Professor of Mathematics in 1903. Both men were subsequently knighted for their services to science.[5] Larmor became a Fellow of the Royal Society in 1892

Sir Joseph Larmor, F.R.S.

and served as Secretary of the Society from 1901 until 1912.

Despite the fact that the two most distinguished theoreticians of mathematical electromagnetic theory in Britain, William Thomson and James Clerk Maxwell, were graduates of the Cambridge Mathematical Tripos, electromagnetic theory was completely excluded from the Tripos until the late 1860s.[6] However, following Maxwell's appointment as Professor of Experimental Physics in 1871 and the publication of his *Treatise on Electricity and Magnetism* in 1873, Maxwell's electrodynamics was taught to undergraduates.[7] By the early 1880s, Maxwell's work had become an appropriate topic of research for the most able graduates of the Mathematical Tripos. One of J.J. Thomson's first publications, for example, was a study of the electromagnetic effects produced by the steady motion of a charged conductor according to the principles set down by Maxwell in the *Treatise*.[8]

Following his success in the Mathematical Tripos, Larmor was elected to a Fellowship at St John's and appointed Professor of Natural Philosophy at Queen's



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College, Galway. In Galway he was cut off from the research community in Cambridge, but continued to study the *Treatise* and took Maxwell's account of electromagnetic induction as the starting point for his first major piece of research.[9] In 1885 Larmor returned to Cambridge to take up one of five newly-created University Lectureships in mathematics.[10] He continued to contribute occasional papers on the development of Maxwellian electrodynamics throughout the 1880s, but confined the bulk of his research during this period to more traditional wrangler problems in dynamics and analytical geometry.[11] Some time during the early 1890s, however, Larmor prepared a review article on magneto-optic rotation and recent theories of light propagation. The article was extremely comprehensive and was eventually published as a Report by the British Association.[12] While preparing this Report, Larmor's interest in electromagnetic theory, optical theory and dynamics, was piqued by a paper written by the Irish physicist George FitzGerald.

3. A Dynamical Theory of the Ether

FitzGerald was one of a small group of non-Cambridge trained men - including Oliver Lodge and Oliver Heaviside - who had discovered the *Treatise* for themselves and who were attempting to articulate Maxwell's electromagnetic theory more fully.[13] FitzGerald had noticed that a remarkable formal similarity existed between the expressions given by James MacCullagh in 1839 for the mechanical energy stored in his rotationally elastic ether, and those given by Maxwell for the energy stored in the electromagnetic field.[14] By replacing the mechanical symbols in MacCullagh's theory with appropriate electromagnetic symbols, and applying Hamilton's principle of least action to the resulting Lagrangian, FitzGerald was able to follow MacCullagh's analysis to obtain an electromagnetic theory of the propagation, refraction and reflection of light.

Larmor aligned himself closely with the distinguished school of mathematical physics associated with Trinity College Dublin - including James McCullagh, William Rowan Hamilton and George FitzGerald - and considered himself to be developing the tradition that they had begun. Furthermore, Larmor ascribed special importance to the *principle of least action*, believing it to represent the most fundamental formulation of mechanics and to be applicable in every branch of physics.[15] Through his work on the analytical dynamics of magneto-optic rotation and through reading FitzGerald's paper, Larmor became convinced that MacCullagh's ether could provide a common dynamical foundation for Maxwell's synthesis of electromagnetic and luminiferous phenomena. Larmor's goal at this point was to find the mechanical properties that had to be ascribed to the ether such that the application of the principle of least action to the resulting Lagrangian would generate Maxwell's equations.

4. The Electronic Theory of Matter

The fruits of Larmor's research were published by the Royal Society as 'A Dynamical Theory of the Electric and Luminiferous Medium' (referred to hereafter as Dynamical Theory), in three instalments (with various appendices) between 1894 and 1897; but during this period his theory changed considerably.[16] The first instalment came in for some powerful criticism from FitzGerald himself, who acted as a referee for the Royal Society. FitzGerald obtained permission to correspond directly with Larmor concerning his paper and, during the spring and summer of 1894, FitzGerald encouraged Larmor to introduce the concept of 'discrete electric nuclei', or 'electrons', into his theory.[17] But the introduction of the electron did far more than solve the immediate problems that troubled Larmor's theory; over the following three years it also had a profound effect upon his understanding of the relationship between the electromagnetic ether and gross matter.

According to Maxwellian electromagnetic theory, all electromagnetic effects were attributable to processes taking place in the ether. Consider, for example, the Maxwell interpretation of an electric current in a wire. The current was not thought of as a material flow of one or more electrical fluids, but rather as a spontaneous 'breaking down' of the electric tension, or 'displacement', in the ether in the vicinity of the wire. By some unexplained mechanism, the material presence of the conducting wire caused the electrostatic energy stored in the ether to be converted into heat. This conversion was accompanied – also via an unexplained mechanism – by the appearance of a magnetic field around the wire. The continuous nature of the electric current was accounted for by postulating that the discontinuous process of build-up and breakdown of displacement occurred many thousands of times every second. Prior to the introduction of the electron, the electromagnetic ether and real matter were thus distinct concepts whose mechanism of interaction was seldom discussed. With the introduction of the electron, however, the situation changed dramatically.

If electric conduction and associated electromagnetic effects were due solely to the motion of electrons, and if, as Larmor postulated, matter was itself composed exclusively of positive and negative electrons, then virtually every problem, both in electrodynamics and matter theory, became a problem in the electrodynamics of moving bodies. Indeed, these two previously distinct realms of physical theory – electrical theory and matter theory – became inseparable. Such well-known effects as the electric polarization and magnetization of matter (which previously had been ascribed to changes in the dynamical properties of the ether somehow brought about by the presence of matter) could now be explained in terms of the electronic micro-structure of matter. Polarization, for example, was now attributed to the micro-separation of the electrons of which matter was composed, while the magnetic properties of materials were attributed to the micro-circulations of their electrons. By 1897 Larmor had constructed a comprehensive Electronic Theory of Matter

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(hereafter ETM) that rendered redundant much of the Maxwellian physics of the 1880s and early 1890s.[18] According to the ETM, the universe consisted of a sea of ether populated solely by positive and negative electrons. These electrons could be thought of mechanically as point centres of radial strain in the ether. They were, moreover, the *sole* constituents of ponderable matter. This view of the universe diffused the problem of the relationship between ether and matter by reducing all matter to movable discontinuities in the ether.

The ETM also led Larmor to a new understanding of the null result obtained by Michelson and Morley in their ether-drift experiment of 1887. Maxwell's followers in Britain preferred to associate electromagnetic effects with the concept of a *non-convected* ether; that is, an ether that is not dragged along by the motion of the earth. According to non-convected ether theory, however, Michelson and Morley ought to have obtained a positive result.[18] In order to square the null result with the non-convected ether, FitzGerald suggested, in 1889, that moving matter must contract minutely in the direction of its motion through the ether.[20] FitzGerald's contraction hypothesis was not initially taken very seriously by the other Maxwellians because it lacked any theoretical foundation. In 1897, however, Larmor showed that the ETM *predicted* that moving matter would contract precisely in the way proposed by FitzGerald. This led Larmor to argue that, far from being problematic, the null result obtained in the famous Michelson-Morley ether drift experiment provided powerful evidence in support of the ETM.[21]

A further important aspect of Larmor's ETM was its incorporation of new space-time transformations to explain the electromagnetic measurements made in the rest frames of moving electrical systems. Larmor believed that Maxwell's field equations were only truly applicable in the stationary ether frame of reference. The fields measured in this frame, he claimed, represented real physical states of the ether. He knew perfectly well, however, that Maxwell's equations were also applicable on the surface of the Earth, which was, he believed, moving through the ether with a velocity of several miles a second. In order to explain this puzzling fact, he developed new electromagnetic and space-time transformations that correlated the fields measured by a moving observer with the *real* fields that would be measured by an observer who was stationary in the ether. By 1900, when he published his book *Aether and Matter*, these new space-time transformations had become precisely those that would be given by Lorentz in 1904 and Einstein in 1905.[22]

5. Larmor Precession and the Zeeman Effect

It was Larmor's interest in the electron and transformation theory that led him, in 1897, to derive the formulae and theorem that now bear his name. Soon after Larmor introduced electrons into his electromagnetic theory as the fundamental and unique carriers of electric charge, he postulated that molecules might be composed of groups of orbiting positive and negative electrons. This led him to speculate that orbiting electrons might be responsible for the characteristic line-spectra emitted by individual elements. Coupling this speculation with his interest in magneto-optic rotation, Larmor calculated what effect a powerful magnetic field would have on the periods of two electrons, one positive and one negative, orbiting under mutual electronic attraction.[23] He found that their periods would be altered in a well-defined way but, assuming the charge to mass ratio (*e/m*) of the electron to be approximately that of the hydrogen ion, concluded that the 'spectral effect would be inappreciable'.[24]

In 1896, however, the Dutch experimentalist, Pieter Zeeman, succeeded in producing a new magneto-optic effect very like the one postulated by Larmor. Zeeman showed that a very powerful magnetic field was capable of broadening each of the D-lines in the sodium spectrum. This effect became known as the Zeeman effect.[25] Zeeman's theoretically minded colleague, H.A. Lorentz, soon showed that the effect could be explained by attributing the sodium D-lines to the oscillations of charged ions within the sodium atom. Lorentz's analysis also made it possible to use Zeeman's results to calculate the charge-to-mass ratio (*e/m*) of the ions.[26] The value obtained by this method was 1000 times larger than for the then lightest known charged particle, the hydrogen ion.

Larmor learned of Zeeman's results early in 1897 and quickly modified his own electron theory.[27] He now assumed that the oscillating electrons responsible for the sodium D-lines carried the same charge as the hydrogen ion and had, therefore, to be 1000 times lighter than the hydrogen ion. This assumption meant that the electromagnetic mass of 'an ideal single molecule consisting of one positive and one negative electron revolving around each other' would be considerably less than the chemical masses of actual molecules.[28] In order to maintain consistency between the ETM and matter theory, Larmor suggested that molecules contained many electrons and that line spectra probably arose 'from one of the numerous epicycles superposed on the main orbits of the various electrons in the molecule [rather] than from a main orbit itself'.[29]

The announcement of the Zeeman effect thus led Larmor to conclude that atoms were built up from large numbers of orbiting electrons. This conclusion was further substantiated at the end of April 1897, when J.J. Thomson announced his *corpuscular* theory of cathode rays.[30] Thomson claimed that the corpuscles were small negatively-charged particles that were sub-atomic constituents of the atom. Furthermore, Thomson's experimental determination of the charge-to-mass ratio of the corpuscles accorded well with Zeeman's determination of the charge-to-mass ratio of Lorentz's ions. In a paper published in the December issue of the *Philosophical Magazine*, Larmor claimed that Lorentz's 'ions' and Thomson's 'corpuscles' were simply his own 'electrons'.[31]

This paper of Larmor was directed primarily at providing a clear explanation of the Zeeman effect in terms of orbiting atomic electrons. It was upon the work contained in this short paper that Larmor's reputation would come to rest. During the summer of 1897, Zeeman published the results of further experiments on atomic spectra. In these experiments he confirmed a number of predictions made by Lorentz's ion theory, including the claim that a powerful magnetic field would not merely broaden the spectral lines (when viewed perpendicular to the magnetic field lines), but split them into definite, polarized, triplets.[32] Larmor began his paper by acknowledging that Lorentz had given a 'theoretical analysis of a somewhat general character' of Zeeman's results, and then explained that he proposed to 'generalize' Lorentz's analysis in order to reveal the physical 'cause' of the effect.[33] Larmor considered the effect of a magnetic field of strength **B** on an electron describing an elliptic orbit around an attracting central charge. He drew upon his familiarity with transformation theory to show that for an observer moving with a frame of reference that *rotated* with angular velocity $\omega = -(e/2m)\mathbf{B}$, the effect of the magnetic field on the electron would, to a good approximation, be negated.[34]

This result enabled Larmor to give a simple physical explanation of the Zeeman effect. He argued that the magnetic field caused the orbits of the electrons to precess with angular velocity ω. The sense of the precession depended on the sense of the electron's orbit with respect to the applied magnetic field. A circular orbit described one way round the axis of the magnetic field would be accelerated; an orbit the other way round would be retarded; a linear component of acceleration along the axis of the magnetic field would remain unaffected. With this analysis Larmor gave a straightforward physical explanation of the triplication and polarization of the spectral lines in terms of his ETM. The frequency ω has since been known as the LARMOR FREQUENCY, while the phenomenon itself is known as LARMOR PRECESSION.[35]

Larmor concluded this section of his paper by noting that his analysis applied whatever the number of electrons revolving in the molecule and however they interacted with each other and the fixed central charge. In fact, for any charged particle – or group of charged particles, each with the same (e/m) ratio – subject to electric (**E**) and magnetic (**B**) fields, the effect of the magnetic field can be negated to a first-order approximation by transforming into a frame of reference that rotates with angular velocity $\omega = -(e/2m)\mathbf{B}$. This important result is now known as LARMOR'S THEOREM.

6. Larmor's Formula

Having shown that the Zeeman effect could be attributed to the orbital motions of sub-atomic electrons, Larmor continued his paper by deriving a general expression for the rate at which energy is emitted by an accelerating electron. By considering the path of an accelerating electron as composed of a series of infinitesimal virtual electric dipoles, he derived the following expression for the power (P) radiated by an accelerating electron in terms of its charge, the velocity of light (c) and the acceleration (a):

$$P = \frac{2}{3}\frac{e^2a^2}{c^3}$$

This expression, now qualified as non-relativistic, has since been known as LARMOR'S FORMULA.

Larmor's reason for deriving the above formula sprang directly from the ETM. In order to make plausible his claim that atoms were composed solely of orbiting electrons, Larmor had to counter the objection that the electrons would quickly lose their orbital energy through radiation. Larmor produced two arguments to counter this objection. First, it was well known that the electromagnetic energy of a moving electron was inversely proportional to the cube of the velocity of light.[36] Larmor's Formula above showed, however, that the energy *emitted* by an electron was inversely proportional to the *square* of the velocity of light. The energy radiated per second by an accelerating electron would therefore be smaller than its total energy by a factor of c^{-1}. Second, Larmor inferred from his calculation of the Larmor Formula that the amount of radiation actually lost from an atom would depend upon the vector sum of the electromagnetic waves emitted by *all* of the electrons in the atom. If the phases of a large number of orbiting electrons were arranged in an appropriate fashion, he argued, the amount of energy radiated could be made arbitrarily small. By these two arguments Larmor reckoned that the problem of the loss of internal energy from the atom by radiation could be surmounted.

Larmor wrote the above paper for the *Philosophical Magazine* during the few months that elapsed between the completion of his monumental paper, Dynamical Theory, and the commencement of his Adams-Prize winning essay 'On the Theory of the Aberration of Light'; the latter being published in 1900 as *Aether and Matter*. Thus all of the expressions that bear Larmor's name, and by which he is now remembered, were given in a single short paper that he published whilst working on much more ambitious projects. The issues raised in the paper were, nevertheless, highly relevant to his attempt to construct an electronic theory of matter.

7. THE LARMOR FREQUENCY AND QUANTUM MECHANICS

It is the Larmor Frequency for which Larmor is now most widely remembered, largely because of its fundamental importance to modern research in nuclear magnetic resonance.[37] Nowadays, however, the expression is derived from the quantum mechanical description of sub-atomic particles. Quantum mechanical analysis of the effects produced on atomic electrons by an external magnetic field follows a very different course from Larmor's analysis and relies upon physical of light (c) and the

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principles quite alien to the ETM. The modern theory nevertheless predicts that the expectation values of the components of the magnetic dipole moment (perpendicular to the applied magnetic field) of the electrons will change cyclically with precisely the Larmor Frequency.[38] It might seem surprising that an expression derived in 1897 has survived the fundamental changes in physical theory wrought by quantum mechanics, but this is not quite the extraordinary coincidence that it first appears.

As we have seen, Larmor did not derive his expression directly from his classical-electrodynamical model of the atom. Rather the expression emerged as a special case of Larmor's Theorem which states that the effects of a magnetic field on a moving charged particle are negated (to a good approximation) by transforming into a reference frame that rotates with the Larmor Frequency. In this sense Larmor's explanation of the Zeeman effect was based upon a straightforward mathematical property of the Lorentz force to which any theory that recognized macroscopic electric and magnetic fields would have to correspond. It was to be expected, therefore, that the quantum mechanical description of the Zeeman effect would be such that the effect of the external magnetic field on the atomic electrons would be negated by transforming into a frame of reference that rotated with the Larmor Frequency.

8. Conclusion

That Larmor's more fundamental contributions to electromagnetic theory – the electron and the Lorentz transformations – have now been forgotten is symptomatic of the way late-nineteenth-century British physics has, until recently, been portrayed by historians of science. Relativity theory and quantum theory have become definitive of theoretical physics in the twentieth century, and much of the work done by historians during the last thirty years has been directed towards explaining the origins of these theories. In the case of relativity, it is Lorentz's work on 'ion' physics, rather than Larmor's work on 'electron' physics, that is understood as the direct precursor of Einstein's relativistic electrodynamics. Indeed, many of Einstein's contemporaries conflated his work with Lorentz's by referring to the 'Lorentz-Einstein' principle of relativity.

British mathematical physics of this period has more typically been cast as the villain of the piece, with British physicists too obsessed by fanciful theories of the ether and *ad hoc* hypotheses to make any real contribution to electrodynamics. But as one of Larmor's students, J.W. Nicholson, reminded his readers in 1912, the principle of relativity could be regarded from two points of view: it could either be 'postulated, as by Einstein and others' or else 'derived, as originally by Larmor, from the result of an analytical transformation'.[39] Nicholson was pointing to an important difference between the interpretations of the principle of relativity adopted by Larmor and Einstein, but he might equally have contrasted Lorentz and Einstein.

Unlike Lorentz, however, Larmor and his students continued to work explicitly on the construction of a purely electronic theory of the world and flatly rejected Einstein's interpretation of the principle of relativity as empirically unfounded. By emphasizing the differences, rather than the similarities, between their work and what was to become a cornerstone of twentieth-century physics, Larmor's group became increasingly isolated. When their enterprise collapsed at the end of World War I, the foundational work undertaken by Larmor during the 1890s was quickly forgotten.

NOTES

1 J. Larmor, *Aether and Matter* (Cambridge, 1900).

2 See J.Z. Buchwald, *From Maxwell to Microphysics* (Chicago, 1985), 133–173; B.J. Hunt, *The Maxwellians* (Cornell, 1991), 209–228; A.C. Warwick, 'On the Role of the FitzGerald-Lorentz Contraction Hypothesis in the Development of Joseph Larmor's Electronic Theory of Matter', *Archive for History of Exact Sciences* 43 (1991), 29–91.

3 For further biographical details see A.S. Eddington, 'Joseph Larmor', *Obituary Notices of Fellows of the Royal Society*, 4 (1944), 197–207.

4 On Routh's coaching record and teaching techniques see W.W. Rouse Ball, 'The Cambridge School of Mathematics', *The Mathematical Gazette*, 6 (1912), 311–323, 320–321. The first and second Smith's prizes were awarded to the two top-scoring candidates in an examination in higher mathematics taken by the top wranglers shortly after the Tripos examinations. For the history of the prize and a list of the winners see J.R. Tanner (editor) *Historical Register of the University of Cambridge to the Year 1910* (Cambridge, 1910), 299–301.

5 Thomson was knighted in 1908, Larmor in 1909.

6 William Whewell had removed electricity and magnetism from the syllabus of the Mathematical Tripos in 1848 as part of a wider campaign of counter-reform against the wholesale introduction of French mathematical physics into Cambridge. See H. Becher, 'William Whewell and Cambridge Mathematics', *Historical Studies in Physical Sciences*, 11 (1980) 1–48.

7 J.C. Maxwell, *Treatise on Electricity and Magnetism* (Oxford, 1873), 2 Vols.

8 J.J. Thomson, 'On the Electric and Magnetic Effects produced by the Motion of Electrified Bodies', *Philosophical Magazine*, 11 (1881), 229–249.

9 J. Larmor, 'Electromagnetic Induction in Conducting Sheets and Solid Bodies', in *Mathematical and Physical Papers* (Cambridge, 1929), 2 Vols, Vol 2, 8–28.

10 On the new lectureships in mathematics,see W.W. Rouse Ball, 'The Cambridge School ...' (see note 4), 323.

11 The term 'Maxwellian electrodynamics' has been used to describe the work that was done by British physicists during the 1880s and 1890s to clarify and extend the work begun by Maxwell in the *Treatise*. See, for example, B.J. Hunt, *The Maxwellians* (see note 2).

12 J. Larmor, 'The Action of Magnetism on Light: with a Critical Correlation of the Various Theories of Light Propagation', *British Association Report* (1893); also *Mathematical and*

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Physical Papers (see note 9), 310–355.

13 On the non-Cambridge Maxwellians see B.J. Hunt, *The Maxwellians* (see note 2).

14 G.F. FitzGerald, 'On the Electromagnetic Theory of the Reflection and Refraction of Light', *Philosophical Transactions of the Royal Society*, **171** (1880), 691–711.

15 The *principle of least action* is often referred to as *Hamilton's principle* in recognition of the fundamental contributions made by William Rowan Hamilton to the study of mechanics and optics in terms of least action.

16 J. Larmor, 'A Dynamical Theory of the Electric and Luminiferous Medium', *Mathematical and Physical Papers* (see note 9), Part I, (Vol. I) 414–535; Part II, (Vol. 1) 543–597; Part III, (Vol. II) 11–132.

17 See J.Z. Buchwald, *From Maxwell to Microphysics* (see note 2), 162–167; and B.J. Hunt, *The Maxwellians* (see note 2), 216–226.

18 The development of Larmor's electronic theory of matter is discussed in detail in A.C. Warwick, 'On the Role of the FitzGerald-Lorentz Contraction ...' (see note 2).

19 The null result obtained by Michelson and Morley is only problematic for a *non-convected* ether theory. Theories in which the ether is *convected* (that is, dragged along at the surface of the earth) predict a null result as the apparatus is not moving with respect to the ether.

20 See B.J. Hunt, 'The Origins of the FitzGerald Contraction', *British Journal for the History of Science*, 21 (1988), 67–76.

21 For a discussion of Larmor's use of the Michelson-Morley experiment see A.C. Warwick, 'The Sturdy Protestants of Science: Larmor, Trouton and the Earth's Motion through the Ether', in J.Z. Buchwald (editor), *Table-Top Experiments* (Chicago, 1993), forthcoming.

22 For a discussion of the relationship between Larmor's work and that of Lorentz and Einstein see A.C. Warwick, 'On the Role of the FitzGerald-Lorentz Contraction ...' (see note 2), 60–69.

23 Dynamical Theory, Part I (see note 16), 524.

24 J. Larmor, 'On the Theory of the Magnetic Influence on Spectra; and on the Radiation from moving Ions', *Mathematical and Physical Papers* (see note 9), Vol II, 140–149; see footnote on 140.

25 For an overview of Zeeman's experimental investigations see *Dictionary of Scientific Biography*, Vol 14, 597–599.

26 The first complete account of Zeeman's work, including the claim that the calculated (e/m) ratio was approximately 1000 times smaller than for the hydrogen ion, appeared in *Nature* on 11 February 1897.

27 J. Larmor, 'On the Theory of the Magnetic Influence on Spectra ...' (see note 24), 149. Larmor explains in a footnote how he learned of Zeeman's experiments.

28 J. Larmor, 'The Influence of a Magnetic Field on Radiation Frequency', *Mathematical and Physical Papers* (see note 9), 138–139. Larmor generally used the term 'molecule' when speaking of molecules or atoms.

29 *ibid.*, 139.

30 Thomson's corpuscular theory of cathode rays is discussed in I. Falconer, 'Corpuscles, Electrons and Cathode Rays: J.J. Thomson and the "Discovery of the Electron"', *British Journal for the History of Science*, 20 (1987), 241–276.

31 J. Larmor, 'On the Theory of the Magnetic Influence on Spectra ...', (see note 24), 143.

32 P. Zeeman, 'Doublets and Triplets in the Spectrum produced by External Magnetic Forces', *Philosophical Magazine*, 44 (1897), 55–60, 255–259.

33 J. Larmor, 'On the Theory of the Magnetic Influence on Spectra ...' (see note 24), 140.

34 *ibid.*, 141.

35 The term LARMOR PRECESSION is rather misleading as Larmor did not derive the effect dynamically by analogy with a spinning top (as is sometimes implied in textbooks) but as the effect of a coordinate transformation.

36 This is discussed by Buchwald in *From Maxwell to Microphysics* (see note 2).

37 See, for example, C.P. Slichter, *Principles of Magnetic Resonance*, 2nd edition (Springer, 1980), 11–13.

38 *ibid.*, 14.

39 J.W. Nicholson, 'On Uniform Rotation, the Principle of Relativity, and the Michelson-Morley Experiment', *Philosophical Magazine*, 24 (1912), 820–827, 820.

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