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The electron theories of Larmor and Lorentz: A comparative study

DURING THE LAST quarter of the 19th century physicists became aware of the necessity of combining Maxwell's theory with the atomic hypothesis. Originally, Maxwell dealt with a single continuous medium, and absorbed in a few parameters of this medium the electromagnetic effects of matter. But at least four kinds of phenomena seemed to require a more detailed consideration of the structure of matter in relation to ether: optical dispersion, which obviously contradicted Maxwell's relation between optical index and dielectric permittivity; the magneto-optical effects discovered by Faraday and Kerr; the unexpectedly high transparency of metal sheets to light; and electrolysis.[1]

In his own limited discussions of such anomalies, Maxwell tended to avoid detailed microscopic analysis, and when he did not, in the case of electrolysis, he regarded the proposed mechanism as fictitious and out of harmony with the rest of his work. Maxwell's closest heirs did not proceed differently. They either ignored the anomalies or tried to explain them away by macroscopic modifications of Maxwell's theory. The main initiators of atomistic electrodynamics (Helmholtz, Lorentz, Larmor, and Wiechert) were more distant admirers of Maxwell.[2]

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The following abbreviations are used: *AHES*, *Archive for the history of exact sciences*; *AN*, *Archives néerlandaises*; *AP*, *Annalen der Physik*; *HWA*, H. von Helmholtz, *Wissenschaftliche Abhandlungen*, 3 vols. (Leipzig, 1882, 1883, 1895); *LaMPP*, J. Larmor, *Mathematical and physical papers* (2 vols., Cambridge, 1929); *LCP*, H.A. Lorentz, *Collected papers* (9 vols., The Hague, 1939); *PLMS*, London Mathematical Society, *Proceedings*; *PM*, *Philosophical magazine*; *PRA*, Royal Academy of Amsterdam, *Proceedings*; *PRS*, Royal Society of London, *Proceedings*; *PT*, Royal Society of London, *Philosophical transactions*; *VKA*, Koninklijke Akademie van Wetenschappen, Amsterdam, *Verslagen*.

1. Cf. H.A. Lorentz, "Over de theorie der terugkaatsing en breking van het licht," *Academisch Proefschrift* (Leiden, 1875), translated as, "Sur la théorie de la réflexion et de la réfraction de la lumière," in *LCP*, *1*, 193–383, on 382–383; J. Buchwald, *From Maxwell to microphysics: Aspects of electromagnetic theory in the last quarter of the nineteenth century* (Chicago, 1985); B. Hunt, *The Maxwellians* (Ithaca, 1991), 209–210.

2. J.C. Maxwell, *A treatise on electricity and magnetism* (Oxford, 1873), chapt. 4. Cf. Buchwald (ref. 1).

At the end of the century, two extensive theories by Lorentz and Larmor emerged as the most complete and successful combinations of electromagnetism and atomism. Both theories involved a stationary ether and new subatomic particles, the electrons, that were held entirely responsible of the electromagnetic properties of matter. Both theories removed the earlier anomalies and solved another outstanding problem of the 1890s, the electrodynamic reduction of the optics of moving bodies.

The present study is devoted to a comparison of Lorentz' and Larmor's electron theories. They will first be presented separately in their own contexts, then compared. Some overlap is to be expected with previous studies by Tetu Hirosige, Jed Buchwald, Bruce Hunt, and Andy Warwick.[3] But I hope to provide more complete answers to the following questions: How did Larmor and Lorentz depart from Maxwell's original system? What kind of unification of physics did they seek? To what extent did Larmor's theory depend on Lorentz'? How did the two theorists arrive at the Lorentz transformation? Did they interpret it similarly?

1. LORENTZ

Unlike most of his British and German colleagues, Hendrik Lorentz had no mentor and belonged to no school. As a Dutchman open to his neighbors' cultures he read indiscrimately from German, English, and French sources. His heroes, Helmholtz, Maxwell, and Fresnel, belonged to quite different, sometimes conflicting traditions. While in an average mind the eclecticism could have created confusion, Lorentz profited from it. He selected elements from each system and made his own syntheses.[4]

In his inaugural lecture for the Leyden chair in theoretical physics, the twenty-five year old Lorentz developed the epistemological correlate of his eclecticism. The aim of physical theories, he argued, was to unify and organize knowledge under a few simple principles; but the principles had no absolute truth and could not receive *a priori* justification. After listing

3. T. Hirosige, "Origins of Lorentz' theory of electrons and the concept of the electromagnetic field," *HSPS, 1* (1969), 151–209; Buchwald (ref. 1); Hunt (ref. 1); A. Warwick, "On the role of the FitzGerald-Lorentz contraction hypothesis in the development of Joseph Larmor's electronic theory of matter," *AHES, 43* (1991), 29–91. See also E. Whittaker, *A history of the theories of aether and electricity*, 2nd ed., vol.1: *The classical theories* (London, 1951); R. McCormmach, "H.A. Lorentz and the electromagnetic view of nature," *Isis, 61* (1970), 459–497; K.F. Schaffner, *Nineteenth-century aether theories* (Oxford, 1972).

4. Cf. McCormmach, "Lorentz, Hendrik Antoon," *DSB, 8*, 487–500.

foundational principles tried by previous physicists—Newton's invisible forces, Faraday's contiguous actions, and William Thomson's vortex rings—Lorentz concluded:[5]

> I have now given you an idea of a few directions in which one has tried to find an explanation to natural phenomena. All of these led to a principle which is not capable of further explanation. This being the case, one can in my opinion not be considered exempt of thoughtlessness when, as sometimes happens, one considers one of these directions as the only true one. On the contrary I believe it to be highly profitable, that various investigators take each his own way in this matter, for only in this manner will one be able, in due course, to decide, not which way entirely discloses the secrets of nature, but which one leads to the simplest fundamental principle.

The pluralistic stand is typical of physicists who have proposed new systems and fear or experience intolerance. For instance, a plea for developing different theories in parallel is found in the foreword of Maxwell's *Treatise* and in Boltzmann's popular lectures. In 1878 Lorentz had no new principle of his own to defend, but, as an unpartial witness of the multiplicity of competing systems in his own time, he was wise before being old.

Openness and pluralism do not exclude personal preferences. Lorentz' Leiden address, on "molecular theories in physics," forcefully asserted the superiority of the principle of atomism. The assertion was natural in van der Waals' country, and it had, at that time, no strong opponent anywhere. Much of Lorentz' later research was concerned with the kinetic theory of matter. In his most influential works he brought atomic structure to bear in domains in which it had previously been neglected, like optics and electrodynamics.

A Helmholtzian thesis

Lorentz' first major work, his dissertation of 1875, was not concerned with the molecular hypothesis, but with a footnote found in Helmholtz' fundamental memoir of 1870 on the equations of motion of electricity. After noting Maxwell's analogy between electric motions in a dielectric and motions in the optical ether, Helmholtz had written:[6]

5. Lorentz, "De moleculaire theoriën in de Natuurkunde" (Leiden lecture, 1878), English translation, "Molecular theories in physics," *LCP*, 9, 26–49, on 34.

6. Lorentz (ref. 1); H. von Helmholtz, "Über die Theorie der Elektrodynamik. I. Über die Bewegungsgleichungen der Elektricität für ruhende leitende Körper" (1870), *HWA, 1*, 545–628. One may wonder why Maxwell did not himself provide a derivation of Fresnel's relations. A plausible answer (Whittaker, ref. 3, on 266) is that Maxwell's picture of displacement suggested wrong continuity conditions.

> This analogy is also relevant in another, very important respect, which Maxwell has not touched. So far the mechanical state of the luminiferous ether in transparent media has been identified with that of solid elastic bodies. Yet, at the limit between two transparent media this assumption gives boundary conditions which are not the ones needed to explain the reflection and refraction of light at this limit, so that there remains an unsolved contradiction in theoretical optics. However, the theory of electric oscillations [in dielectrics] gives the laws of wave propagation, reflection and refraction which apply to light.

Lorentz' dissertation was a full explication of this concise footnote. He first verified Helmholtz' claim that previous optical theories did not yield the correct boundary conditions for the reflection and refraction of light. In general the conditions are continuity of displacement and continuity of strain on the surface between the two media, and they cannot be satisfied simultaneously by the reflected and refracted rays if, as usually assumed, there are only two such rays. As Lorentz knew, Neumann's and Fresnel's theory gave correct formulas for the intensities of the reflected and refracted rays, but only at the price of (unconsciously) ignoring part of the boundary conditions.

The origin of the difficulty is that in reality the reflection and refraction of an incident ray gives rise to four rays. Two of the emerging rays are unwanted, because they correspond to unobserved longitudinal vibrations. Yet they are necessary if all boundary conditions are to be satisfied. Cauchy's theory, Lorentz noted, properly took into account the longitudinal vibrations. But it gave no satisfactory account of the invisibility of the additional rays. Moreover, Lorentz could show that the theory led to unacceptable predictions for refraction in anisotropic media. None of the optical theories known to Lorentz could properly explain the laws for the reflection and refraction of light.[7]

In the rest of his dissertation, Lorentz applied "Maxwell's theory"—he meant Helmholtz' interpretation of it—to the problem. He relied on the action at a distance of charges and currents, including those produced by the local, invisible displacement of charges pertaining to dielectrics. If vacuum was itself a highly polarizable dielectric, the physical predictions of Maxwell's theory were retrieved. Lorentz performed all calculations with the full set of Helmholtz' equations and took the limit of infinite polarizability only at the end.

Lorentz justified his preference for Helmholtz' system: "I shall start with instantaneous action at a distance: thus we will be able to found the

7. MacCullagh's theory could, but Lorentz was not aware of it. Cf. Buchwald (ref. 1), appendix 2.

theory on the most direct interpretation of observed facts." Here Lorentz probably meant that previously observed electric actions were instantaneous. "However," he went on, "I do not regard *actio in distans* as a rigid dogma; the true starting point of the theory is the initial differential equations, not action at a distance." The position was in harmony with Lorentz' general disbelief in ultimate foundations. But, we may ask, if the initial differential equations are the true starting point, why not start from Maxwell's equations, which are much simpler than Helmholtz'?[8]

Lorentz' unstated rationale is not difficult to guess. Maxwell's system was difficult to penetrate. The very original field-based notions of charge and current were obscured by ambiguous wording. As a consequence, Lorentz did not find them compelling: "Whatever be our understanding of the nature of electricity," he wrote, "we will always find it difficult not to conceive a current as the motion of a certain substance which is contained by all good conductors of electricity."[9]

After exposing Helmholtz' theory in great detail, Lorentz gave the continuity conditions at the surface separating two different media. As in modern reasoning based on Maxwell's equation, he required that no observable quantity become infinite on the surface. He found formulas for the orientation and intensity of reflected and refracted rays in agreement with experiment—Fresnel's laws in the isotropic case, Neumann's laws in the anisotropic case—if he took the dielectric permittivity of vacuum to be extremely large. The agreement should not be surprising, since in this limit Helmholtz' theory is phenomenologically equivalent to Maxwell's. However, Lorentz' calculations were far more complex than the modern ones, because, as mentioned, he maintained a finite value of vacuum polarization to the end of the calculations.

For Lorentz, complexity did not imply confusion. His discussion of optical theories, as well as his account and application of Helmholtz' theory, were unusually clear and precise. In his command of mathematical methods, his virtuosity in calculation, and his style of exposition he already equalled the best theorists of his time. Although he was one the first few physicists who almost never performed experiments, he explored every possible contact of his theory with experiment. His qualities were promptly recognized by Dutch authorities, who offered him a chair for theoretical physics at Leyden three years after he completed his doctorate. Unfortunately, Lorentz lacked personal connections abroad and few noticed his dissertation. It contained several seeds of his later, popular electron theory.[10]

8. Lorentz (ref. 1), 224.
9. Ibid., 235. On Maxwell's concepts of charge and current, cf. Buchwald (ref. 1), chapt. 3, and Darrigol, "The electrodynamic revolution in Germany as documented by early German expositions of 'Maxwell's theory'," *AHES*, *45* (1993), 189–280.
10. Cf. McCormmach (ref. 4).

Through the work on his dissertation, Lorentz became convinced of the superiority of "Maxwell's theory," to "the old wave theory." At the same time, he emphasized the limitations of his and Maxwell's considerations. For instance, neither could account for dispersion phenomena or obtain a value anywhere near the empirical one for the absorption of light by metals and electrolytes. In each case, Lorentz expected that a future combination of Maxwell's theory with the molecular hypothesis would remove the anomalies. He formulated a precise program:

> Let us think about the dispersion phenomenon, the rotation of the plane of polarization, and the manner in which these phenomena are related to the molecular structure; then about the mechanical forces which perhaps play a role in certain light phenomena. Then let us think how external forces and the motion of the medium influence light; and let us think about the emission and absorption phenomena and the radiating heat. . . .Finally the theory of light should reveal the link between [molecular] electric motions and the physical and chemical state of matter, a link that lies at the basis of spectral analysis, with its wealth of surprising results.

Very strikingly, this program was precisely the one that Lorentz developed in subsequent years, except for the last subject, which other physicists would take on.[11]

Dispersion

Most of Lorentz' dissertation treated dielectrics in Helmholtz' economical manner. Polarizability was assumed without microscopic mechanism; and vacuum differed from other dielectrics only by the value of the constant χ in the relation $\mathbf{P} = \chi\mathbf{E}$ between polarization and electromotive force. However, Lorentz sketched how molecular structure could be taken into account. Assuming that all bodies were made of ether and molecules, he suggested:[12]

> If one whishes to give an absolutely complete description of electric motions in such bodies one will have to take into account ether first, then the inbedded molecules. The distance, the magnitude, and the shape of the latter then come into play, which very probably entails the possibility of explaining dispersion and the rotation of the polarization plane. Here I will leave these questions aside. I shall only remark that in gases, for which the influence of molecules is very small [as verified from the fact that the optical index is almost the same as in vacuo], this inluence can very simply be taken into account in a first approximation. For this purpose, *we shall suppose that ether has absolutely the same properties in gases as in a vacuum.*

11. Lorentz (ref. 1), 382; ibid., 382–383. Cf. Hirosige (ref. 3), 173.
12. Lorentz (ref. 1), 279 (my emphasis).

The latter assumption, the simplest Lorentz could make, had the essential advantage of leading to unambiguous calculations in Helmholtz' framework, if only a definite polarizability κ was attributed to each molecule. He had only to superpose the polarization of the molecules on that of vacuum, according to

$$\mathbf{P} = \chi_0\mathbf{E} + N\kappa\mathbf{E}, \tag{1}$$

where $\mathbf{E}$ is the electromotive force, $\mathbf{P}$ the total polarization, N the number of molecules per unit volume, and χ_0 is the polarizability of vacuum. Accordingly, the effective polarizability χ is

$$\chi = \chi_0 + N\kappa, \tag{2}$$

and the propagation velocity of transverse vibrations is

$$c_\perp \propto \chi^{-1/2} = (\chi_0 + N\kappa)^{-1/2}. \tag{3}$$

The corresponding optical index,

$$n = \sqrt{1 + N\kappa/\chi_0}, \tag{4}$$

implies that, for a given gas, $n^2 - 1$ is proportional to the density, in conformity with the law established by Arago and Biot.[13]

The possibility of such simple reasoning depended on Lorentz' reinterpretation of Maxwell in Helmholtz' terms. In Maxwell's genuine theory, the assumption that the properties of ether remained unchanged in the neighborhood of a molecule would have made no sense, since the electric properties of the molecule implied, by definition, discontinuities in the properties of the medium. Moreover, the polarizability of vacuum and the polarizability of a molecule, which were of the same essence in Helmholtz' view, could only be very different things in Maxwell's theory. For a Maxwellian, ether polarization was a primitive notion, and molecular polarization a third-level construct: it was the moment of the charge distribution of the molecule, charge being itself defined as an heterogeneity in the polarization of ether.

In 1878 Lorentz further explored the contribution of molecules to dielectric polarization. Again he used Helmholtz' version of Maxwell's theory and assumed a nearly incorruptible ether: "Whether and in what way the properties of ether are changed by the presence of molecules is entirely

13. Ibid., 280.

unknown to us. We shall here make the very simple supposition that—except perhaps in the immediate neighborhood of the particles—the properties of ether are the same as in a vacuum." In his calculations, Lorentz no longer reasoned in terms of an effective, large-scale polarizability, presumably because he realized that such reasoning would only apply to gases. Instead he discussed the electromagnetic actions of individual molecules on the basis of the retarded potentials satisfying Helmholtz' equations. The highly complex calculations led to the "Lorenz- Lorentz" law:[14]

$$\frac{n^2-1}{n^2+2} \propto N\kappa. \tag{5}$$

To direct his reasoning, Lorentz first represented a molecule by a point-like electric and magnetic moment within a spherical cavity in ether. He then verified that other representations led to the same results. Among those was the picture of a mobile, submolecular charged particle in an unmodified ether. Here the electric moment of the charged particle was identified with a polarization in Helmholtz' sense.[15]

Lorentz tried to explain optical dispersion in this framework. As was known since Fresnel and Cauchy, a microscopic heterogeneity in the constitution of the propagating medium implied a dependence of propagation on frequency. But whereas in molecular theories of the elastic ether the effect of heterogeneity was considerable, Lorentz found it to be quite negligible in his theory. He therefore concluded: "If we accept the electromagnetic theory of light, there is nothing left, in my opinion, but to look for the cause of dispersion in the molecules of the medium themselves." More exactly, the frequency dependence had to be sought in the polarizability κ of the molecule.[16]

Lorentz identified the molecular polarization with the moment $e\mathbf{r}$ of an elastically bound charged particle, and, for this particle, assumed the equation of motion

$$m\ddot{\mathbf{r}} + g\mathbf{r} = e\mathbf{E}, \tag{6}$$

where m is the mass of the particle, e its charge, g the elastic constant, and $\mathbf{E}$ the local electromotive force. If the latter is a periodic function of time with the pulsation ω, the forced oscillations satisfy

14. Lorentz, "Over het verband tusschen de voortplantings snelheid en samestelling der middenstofen," *VKA*, *18* (1878). English transl., "Concerning the relation between the velocity of propagation of light and the density and composition of media," *LÇP2*, 3–119, on 24.

15. Ibid., 24–27. The latter assumption departs from Helmholtz' potential theory, for which the polarizing electric force differs from the electric force acting on a charged body. But the difference disappears in the limit of infinite vacuum polarizability.

16. Ibid., 79–80.

$$e\mathbf{r} = \kappa\mathbf{E}, \quad \text{with} \quad \kappa = \frac{e^2}{g - m\omega^2}. \tag{7}$$

This implies, through the relation (5) between optical index and molecular polarizability, a dispersion formula that Lorentz found to agree well with experiments. A footnote in Lorentz' paper indicated that a force proportional to the velocity could be inserted in equation (6) to account for the absorption of light.[17]

This was the first electromagnetic theory of dispersion, but not the first to imply a molecular resonance phenomenon. In the context of the theory of the elastic ether, such a resonance had already been imagined by Sellmeier in 1872 and perfected by Helmholtz in 1875. Their principal aim was to account for anomalous dispersion, an occasional decrease of index with increasing frequency. Unlike these theories and Helmholtz' later electromagnetic theory (1893), Lorentz' paper remained practically unknown, if only because it appeared in Dutch. But it already contained essential features of the later electron theory, namely, the divorce of ether from matter, the idea of an electromagnetic coupling between the two, and some calculation techniques.[18]

Fresnel versus Stokes

Lorentz did not return to electromagnetic molecular theory during the 1880's. Most of his work dealt with thermodynamics and kinetic theory, and when he did deal with optics and electrodynamics he adopted a macroscopic outlook. However, in a memoir of 1886 on the optics of moving bodies, Lorentz drew conclusions that pointed to his later work in electron theory.[19]

First of all, Lorentz convinced himself that Stokes' theory of aberration could no longer be held, despite Michelson's then recent support. According to Stokes, ether had to be completely dragged by the earth to explain that optical instruments on earth behaved normally, and the ether flow had to be irrotational to give light rays emitted from stars the same apparent direction as if ether was at rest. But Lorentz found the two assumptions to be incompatible with the idea of an incompressible ether. Indeed, the velocity potential of an incompressible fluid must be a harmonic function, which is, according to a well-known theorem of Green's, completely determined by its value at infinity and the value of its normal derivative at the

17. Ibid., 80.

18. On early theories of dispersion, cf. Buchwald (ref. 1), 233–237.

19. Lorentz, "De l'influence du mouvement de la terre sur les phénomènes lumineux," *AN* (1887), also in *LCP*, *4*, 153–214, first published in Dutch in *VKA* (1886).

surface of the earth. The corresponding tangential derivative, which is the tangential velocity, must in general differ from zero. From this consideration, Lorentz concluded that Stokes' original theory was impossible.[20]

With his usual cautiousness, Lorentz did not infer, however, that Fresnel's theory, with its unperturbed ether, was the only possible one. Instead he provided a modification of Stokes' theory that accounted for stellar aberration and apparent optical laws. He maintained the condition of irrotational flow, but dropped the boundary condition at the earth's surface. Then the light rays from stars followed the same straight trajectory as in Fresnel's theory, but the refraction in optical instruments no longer obeyed Descartes' law. Just as in Fresnel's theory, the latter defect could be remedied by superposing on the motion of ether in vacuo a partial dragging by transparent matter.

The modified Stokes theory allowed for a partial dragging of ether by the earth and therefore could account for the negative result of Michelson's double-arm interference experiment. But Lorentz did not believe the experiment of 1881 to be decisive. He found that Michelson's theoretical estimate of the fringe shift (by assumption of a stationary ether) was wrong by a factor two, and that the true theoretical shift was within the error brackets. Hence Lorentz had two reasons to reject Michelson's claim that his experiment excluded Fresnel's theory and confirmed Stokes'. On the one hand, measurements were not sufficently accurate to exclude Fresnel's theory; on the other, Stokes' theory could not be retained without modification.[21]

In absence of a crucial experiment, Lorentz favored Fresnel's theory over the modified Stokes' theory, because the former led to a simpler picture of the permeability of matter to ether. For a supporter of Stokes' theory, matter in small quantity had to be transparent to ether, since ether could not be pumped from a container (ether was just as dense in the barometric "vacuum" as anywhere else); but the earth's matter had to be completely impermeable to ether. Instead, Fresnel's theory assumed complete permeability. To which Lorentz commented:[22] "It seems to me that

20. Ibid. Stokes could hardly have overlooked Lorentz' point. In fact, in his first paper on the subject, he did not assume incompressibility. He did so in later papers; but the structure of ether, and its condition at the surface of the earth, became highly complex, so that it is not clear whether Lorentz' criticism applies. Cf. Larmor, *Aether and matter* (Cambridge, 1900). On the general history of the optics of moving bodies, cf. Larmor, ibid. chapt. 13; E. Ketteler, *Astronomische Undulations-Theorie, oder die Lehre von der Aberration des Lichtes* (Bonn, 1873); T. Hirosige, "The ether problem, the mechanistic worldview, and the origins of the theory of relativity," *HSPS, 7* (1976), 3–82; A. Mayrargue, "L'aberration des étoiles et l'éther de Fresnel," thèse de doctorat (Paris 7, 1991); M. Pietrocola Pinto de Oliveira, "Elie Mascart et l'optique des corps en mouvement," thèse de doctorat (Paris 7, 1992); Whittaker (ref. 3).

21. Lorentz (ref. 19), 212.

22. Ibid., 203.

the latter view is at least as simple as the former, if not simpler. It may be that what we call an atom can perfectly well occupy the same place as a portion of ether, that for example an atom is nothing but a modification of the state of this medium; then one could understand that an atom could move without dragging ether around it." Lorentz had another reason to find the stationary ether more attractive: he knew from his previous work that the electromagnetic version of this assumption was most useful in linking the optical properties of matter to molecular structure.

Perusing Maxwell

As mentioned, Lorentz was not dogmatically committed to action at a distance, and his original reason for prefering Helmholtz' theory over Maxwell's was that it was "founded on the most direct interpretation of observed facts." After Hertz' experiments, the same criterion induced him to adopt Maxwell's theory, and to judge Helmholtz' theory "artificial." However, he quickly conceived a possibility of "bringing together old and new theory, at least with regard to form." One just had to imagine the existence of small "charged particles," the accumulation of which would represent an electric charge, and the flow of which would constitute a conduction current, as was already recognized in the case of electrolysis. This resembled Weber's theory, except for two things. For Weber's immaterial particles of electricity, Lorentz had to substitute material, electrified particles; and the interaction between the particles was no longer an action at a distance, but a derived interaction propagated through ether.[23]

In a long memoir published in 1892, Lorentz developed what he meant by Maxwell's theory and started working out his corpuscularist program. I will first examine the extent of his conversion to Maxwell's views.[24]

For Lorentz, the first characteristic of Maxwell's electrodynamics, as exposed in the *Treatise*, was that an electric current in a conductor implied a motion not only in the conductor but also in the surrounding magnetic field. Like Poincaré, Lorentz praised Maxwell's use of the Lagrangian method, which allowed deriving the fundamental dynamical equations of the field without detailed knowledge of the mechanism in the field. He also recognized the advantages of Hertz' conception, which instead took the field equations as axioms. Judging, however, that "one is always tempted to return to mechanical explanations," he proceeded to improve the dynamical foundation of Maxwell's theory.[25]

23. Lorentz, "Electriciteit en ether" (1891), *LCP*, *9*, 89–101, on 99. For a more detailed analysis of this text, cf. Hirosige (ref. 3), 183–186.

24. Lorentz, "La théorie électromagnétique de Maxwell et son application aux corps mouvants," *AN* (1892), also in *LCP*, *2*, 164–321.

25. Ibid., 165.

Maxwell's dynamical considerations, Lorentz noted, had three defects. They were limited to the case of linear conductors; they were based on quantities, the potentials, which Heaviside and Hertz had advantageously eliminated from the theory; and they did not completely cover the case of moving bodies. To correct these defects, Lorentz first had to generalize an essential assumption of Maxwell's dynamics of linear currents, that the quantity of electricity that had crossed a given section of each conductor since the origin of time controlled the configuration of an unspecified mechanical system representing the magnetic field. For the electrodynamics of bodies at rest, this was readily done by taking, for the controlling variables, a vector field λ of which the current $\mathbf{J}$ is the time derivative.

Following Maxwell, Lorentz expressed the kinetic energy of the system as

$$T = \frac{1}{2}\int \mathbf{B}\cdot\mathbf{H}\,d\tau, \tag{8}$$

$$\text{with } \mathbf{B} = \mu\mathbf{H} \text{ and } \nabla\times\mathbf{H} = \mathbf{J}. \tag{9}$$

The latter equation implies that every current is closed, a central assumption of Maxwell's theory. At that point, it would seem natural to write the Lagrange equations of the system as Maxwell had done for linear currents. Lorentz used instead d'Alembert's principle of virtual work. He had several good reasons to do so. First, the Lagrange equations for a continuum cannot be written without the notion of functional derivative, which was not available at Lorentz' time. Second, the variables λ are not independent, since they are constrained by the divergence-less character of the current. Third and last, the Lagrange equations boost the vector-potential to the forefront of the theory, as the generalized momentum conjugated with λ; whereas Lorentz, like Hertz and Heaviside, wished to eliminate the potentials.[26]

However, for the relevant holonomous constrained system, d'Alembert's principle is strictly equivalent to the Lagrange equations for the modified Lagrangian

$$L = T + \int \xi\nabla\cdot\mathbf{J}\,d\tau, \tag{10}$$

where ξ is a Lagrange parameter. For the convenience of the modern reader, I will here substitute the Lagrangian method for d'Alembert's principle. Introducing, as Lorentz does, the "accessory variable" $\mathbf{A}$, such that

26. Another possibility would have been to use the principle of least action. But this would have excluded impressed electromotive forces and dissipative counter-electromotive forces.

$$\mathbf{B} = \nabla\times\mathbf{A}, \tag{11}$$

the effective Lagrangian reads

$$L = \int\left(\frac{1}{2}\mathbf{J}\cdot\mathbf{A} + \xi\nabla\cdot\mathbf{J}\right)d\tau, \tag{12}$$

The corresponding Lagrange equations yield the generalized inertial force

$$\mathbf{E} = -\frac{\partial}{\partial t}\frac{\delta L}{\delta\mathbf{J}} = -\frac{\partial\mathbf{A}}{\partial t} + \nabla\frac{\partial\xi}{\partial t}, \tag{13}$$

where the δ symbols indicate a functional derivative. Eliminating the Lagrange parameter yields the equation

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \tag{14}$$

which Lorentz regarded, following Heaviside and Hertz in this respect, as a fundamental equation of the theory.

The extension of the reasoning to moving bodies requires an assumption about the connection between the seat of the current $\mathbf{J}$ and the seat of the magnetic field motion. Lorentz first examined the consequences of Hertz' and Heaviside's view that the two seats were identical or moving together, with a velocity $\mathbf{v}(\mathbf{r})$. In this case the effective Lagrangian is still given by (12), but there are additional configuration variables that determine the position of the medium, and the fluxes λ of electricity must be defined with respect to fixed particles of the medium. The Lagrange equations for the λ variables then imply

$$\mathbf{E} = -\frac{D}{Dt}(\mathbf{A} - \nabla\xi), \tag{15}$$

where D/Dt is the convective derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - \mathbf{v}\times(\nabla\times\ ) + \nabla(\mathbf{v}\cdot\ ). \tag{16}$$

After eliminating the Lagrange parameter, we reach one of the fundamental equations of Hertz' electrodynamics of moving bodies:[27]

27. The other fundamental equation,

$$\nabla\times(\mathbf{H} + \mathbf{v}\times\mathbf{D}) = \mathbf{j} + \partial\mathbf{D}/\partial t + \mathbf{v}(\nabla\cdot\mathbf{D}),$$

results from $\nabla\times\mathbf{H} = \mathbf{J}$, where $\mathbf{J}$ is $\mathbf{j}$ (the conduction current) plus the convective derivative of the *flux* $\mathbf{D}$.

$$\nabla \times (\mathbf{E} - \mathbf{v} \times \mathbf{B}) = -\frac{\partial \mathbf{B}}{\partial t}. \tag{17}$$

Regarding the dynamical foundation of the theory, Lorentz appears to have been more faithful to Maxwell than other Maxwellians. It remains to be seen how much he retained of the other central component of Maxwell's theory, the field-based concept of electric charge and current. Like Maxwell, Lorentz called "electricity" the thing of which **J** is a flow. In a conductor, he identified **J** with the ordinary current **j** entering Ohm's law. In a dielectric he assumed the flow of "electricity" to be elastically resisted, and, like Maxwell, called **D** the choice of λ that vanishes in the relaxed state. Then Lorentz defined the electric charge of an isolated conductor as the quantity of "electricity" that would leave it when connected to earth through a conducting wire, and proved that this charge was given by

$$q = \oint_{\sigma} \mathbf{D} \cdot d\mathbf{S}, \tag{18}$$

if σ is any closed surface surrounding the conductor.[28]

The proof is straighforward. Owing to the divergence-less character of **J**, during the discharging process we have

$$\int_{w} \mathbf{j} \cdot d\mathbf{S} + \int_{d} \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S} = 0, \tag{19}$$

where the first integral is taken over the portion of σ which is crossed by the wire, and the second integral over the rest of σ. This may be rewritten as

$$-\frac{dq}{dt} + \frac{d}{dt}\int_{d} \mathbf{D} \cdot d\mathbf{S} = 0, \tag{20}$$

which yields the desired result after integration over the duration of the process.

From this consideration Lorentz could deduce the relation

$$\rho = \nabla \cdot \mathbf{D} \tag{21}$$

between charge density and displacement *in an insulator*. He recognized that "it was impossible to produce such a charge in a medium which is entirely devoid of conductivity." But he imagined, as a Maxwellian would have done, that a volume element of a dielectric could be charged by a process similar to that given for a conductor, something like a local injection of "electricity" by means of a conducting needle of higher infinitesimal

28. Lorentz (ref. 24), on 189.

order. Then the relation (18) applies to a surface σ surrounding the volum element, which implies the relation (21).[29]

These arguments were in harmony with Maxwell's ideas. They mad conduction a precondition of electric charge and they did not confuse a con duction current with a flow of electric charge. What flowed was an ima ginary fluid, which, despite being called "electricity," could not accumu late anywhere since it was incompressible. However, Lorentz said nothin more precise about charge and current in Maxwells' theory. He gave n rule for determining the precise distribution of charge in the space occupie by a conductor. Also, he could not decide how media displaying both con ductivity and dielectric capacity should behave. He believed, like Poincaré that Maxwell's prediction, which simply added the conduction current an the displacement current was not the only possible one.[30]

In order to be more specific, Lorentz would have needed Maxwell's more precise definition of electric charge as a discontinuity of displacement and also some definition of displacement within a conductor. But he gave none of this, thereby omitting essential elements of Maxwell's views. There is a plausible explanation of this silence. Lorentz must have had difficulties with Maxwell's concept of polarization, which could not be made to agree with Helmholtz'. Also, he could hardly have been receptive to the Maxwellian picture of electric conduction as a continual decay of displacement, which seemed so remote from the intuitive notion of a flow. On such questions he is likely to have sought more light in Poincaré's and Hertz' clearer writings. But this could only have led him further astray, since, as I have shown elsewhere, Poincaré crudely misrepresented, and Hertz completely rejected, Maxwell's pictures.[31]

In brief, Lorentz' presentation of Maxwell's theory did not contradict Maxwell's views, but it was essentially incomplete. Fortunately, the incompleteness did not impede Lorentz' progress with the corpuscular approach, for a simple reason: conductors in Maxwell's sense did not occur in the new microscopic theory; every current was reduced to convection and displacement.

29. Ibid., 200.

30. Ibid., 201–202. On Poincaré, cf. Darrigol (ref. 9), on 216.

31. Ibid., 215–220, 251–254. See also P. Heimann, "Maxwell, Hertz, and the nature of electricity," *Isis, 62* (1971), 149–157, and Buchwald (ref. 1), 192–193. In a footnote, Lorentz (ref. 24, 190) referred to Poincaré's two fluids ("électricité" and "fluide inducteur").

Charged particles

In the rest of his memoir, Lorentz developed "the theory of a system of charged particles which move across ether without dragging this medium." As in Weber's theory, an electric charge had to be conceived as an accumulation of particles, a conduction current as a flow of particles, and a material dielectric polarization as local shifts of elastically bound particles. "From the atoms of electric fluids to charged corpuscles," Lorentz wrote, "the distance is not great."[32]

However, the particles now had to interact via ether. The particles' motion being the source of all electric phenomena, including the stationary ones, Lorentz needed a precise assumption about the effect of this motion on ether. As in his dissertation, he assumed a perfect transparency of matter to ether. The assumption was the simplest, it was compatible with Fresnel's theory of aberration, and Lorentz had already obtained some results with it within the framework of Helmholtz' theory.

A perfectly stationary ether is absurd from a Maxwellian point of view. The assumption implies that in absence of true conduction the total current is always given by $\partial\mathbf{D}/\partial t$; then the charge density $\nabla\cdot\mathbf{D}$ cannot vary in time, and moving charged particles cannot exist. The only truly Maxwellian assumption is that ether is fully dragged by matter, because for Maxwell matter modifies not only the state of ether, but also its *constitution*, by altering constitutive parameters of permittivity, permeability, and conductivity.

Lorentz circumvented the difficulty by avoiding any further reference to the picture of displacement as the shift of an incompressible fluid within ether and simply "borrowed from Hertz" the following expression for the total current:

$$\mathbf{J} = \rho\mathbf{v} + \frac{\partial\mathbf{D}}{\partial t}, \tag{22}$$

where $\mathbf{v}$ is the velocity of *matter*. The borrowing was narrowly selective, since, as Lorentz himself showed in a previous section of his memoir, Hertz' justification was based on a fully dragged ether. Moreover, for Hertz the total current involved a third term, the curl of $\mathbf{D}\times\mathbf{v}$.[33]

More convincingly, Lorentz refered to Rowland's experiment and to current views on electrolysis as supporting the existence of the convection current.[34] He also verified that his total current was divergence-less:

32. Lorentz (ref. 24), 229.

33. Lorentz (ref. 24), 230. For Hertz' total current, see note 27.

34. Lorentz (ref. 24), 231: "A l'appui de cette hypothèse, que j'ai empruntée à M. Hertz, on peut rappeler l'expérience bien connue de M. Rowland, dans laquelle la rotation rapide d'un disque chargé a produit les mêmes effets électromagnétiques qu'un système de courants

$$\nabla\cdot\left(\rho\mathbf{v} + \frac{\partial\mathbf{D}}{\partial t}\right) = \frac{\partial\rho}{\partial t} + \mathbf{v}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{v} = \frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{v}, \tag{23}$$

and the last member represents the variation of the charge of a material volume element, which is zero by assumption.[35]

In order to establish the dynamical equations of his system, Lorentz appealed to d'Alembert's principle of virtual work, now assuming that the values of $\mathbf{D}$ and the positions of all charged particles controlled the configuration of the system. In rationalized electrostatic units,[36] the electrokinetic energy of the system is

$$T = \frac{1}{2}\int B^2 d\tau, \tag{24}$$

$$\text{with}\quad \nabla\times\mathbf{B} = \frac{1}{c}\left(\rho\mathbf{v} + \frac{\partial\mathbf{D}}{\partial t}\right). \tag{25}$$

Introducing the vector potential $\mathbf{A}$, this is the same as

$$T = \frac{1}{2c}\int \mathbf{A}\cdot\left(\rho\mathbf{v} + \frac{\partial\mathbf{D}}{\partial t}\right) d\tau. \tag{26}$$

The potential energy is

$$U = \frac{1}{2}\int D^2 d\tau. \tag{27}$$

As before, I use Lagrange's method, with two Lagrange parameters ξ and η corresponding to the constraints $\nabla\cdot\mathbf{J} = 0$ and $\rho = \nabla\cdot\mathbf{D}$. The electromagnetic contribution to the effective Lagrangian is

$$L = T - U + \int \xi\nabla\cdot\left(\rho\mathbf{v} + \frac{\partial\mathbf{D}}{\partial t}\right) d\tau + \int \eta(\rho - \nabla\cdot\mathbf{D})\, d\tau. \tag{28}$$

The Lagrange equations with respect to $\mathbf{D}$ are

$$\frac{\partial}{\partial t}\frac{\delta L}{\delta\dot{\mathbf{D}}} - \frac{\delta L}{\delta\mathbf{D}} = \mathbf{0}, \tag{29}$$

circulaires. Elle a démontré que le déplacement d'un corps chargé constitue un vrai courant électrique, ce qui d'ailleurs est conforme à la théorie généralement acceptée de l'électrolyse." On electrolysis, cf. Whittaker (ref. 3), chap. 11.

35. Lorentz limited his consideration to solid distributions. The more general reasoning is found in his *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern* (Leiden, 1895), also in *LCP5*, 1–139, on 15–16.

36. Until 1904 Lorentz used electromagnetic units.

$$\text{or } \mathbf{D} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \nabla(\eta + \dot{\xi}). \tag{30}$$

Eliminating the Lagrange parameters, we reach the fundamental equation

$$\nabla\times\mathbf{D} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}. \tag{31}$$

We now write the Lagrange equation with respect to the position $\mathbf{r}(t)$ of the charge q contained in the material volume element $d\tau$. The relevant part of the electromagnetic Lagrangian is

$$L_q = \frac{q}{c}\mathbf{v}\cdot\mathbf{A}(\mathbf{r}) - q\mathbf{v}\cdot\nabla\xi(\mathbf{r}) + q\eta(\mathbf{r}). \tag{32}$$

The electromagnetic force acting on q is

$$\mathbf{f}_q = -\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}} + \frac{\partial L}{\partial \mathbf{r}}, \tag{33}$$

or

$$\mathbf{f}_q = q[-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \nabla(\eta + \dot{\xi})] + \frac{q}{c}\mathbf{v}\times\mathbf{B}. \tag{34}$$

Therefore the electromagnetic force density is given by "Lorentz' formula":[37]

$$\mathbf{f} = \rho(\mathbf{D} + \frac{1}{c}\mathbf{v}\times\mathbf{B}). \tag{35}$$

At the end of his reasoning, Lorentz recapitulated his fundamental equations as

$$\nabla\cdot\mathbf{D} = \rho, \quad \nabla\cdot\mathbf{B} = 0,$$
$$\nabla\times\mathbf{B} = \frac{1}{c}(\rho\mathbf{v} + \frac{\partial \mathbf{D}}{\partial t}), \quad \nabla\times\mathbf{D} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{36}$$
$$\mathbf{f} = \rho(\mathbf{D} + \frac{1}{c}\mathbf{v}\times\mathbf{B}).$$

He summed up his attitude toward these equations as follows:[38]

> On the way to these equations we have encountered more than one serious difficulty, and little satisfaction is likely to be drawn from a theory which, far from unveiling the mechanism of phenomena, leaves us, at the best, the hope of discovering it someday. Physicists who would feel this way, can nevertheless admit the fundamental idea which was the basis of Faraday's and Maxwell's researches, and they can regard the above equations as fairly simple hypothetical equations that might be used in the description of phenomena.

37. In his calculations, Lorentz assumed solid charged particles. In general, the force acting on the particle—which is obtained by integration of the Lorentz force over the volume of the particle—depends on the rotation of the particle. In order to get rid of this effect, Lorentz assumed a sufficiently large mechanical inertia of the particles.

38. Lorentz (ref. 24), on 246–247.

Lorentz thus recognized that his equations were simpler than their dynamical foundation. In subsequent writings, he omitted the complex dynamical arguments, and presented his equations as a consistent modification of Hertz' equations compatible with the energy principle. The changes were dictated by empirical considerations: Rowland's phenomenon suggested the $\rho\mathbf{v}$ term; Ampère's law for the magnetic force acting on a current element, once interpreted in microscopic terms, suggested the $\rho(\mathbf{v}/c)\times\mathbf{B}$ term in the force formula. In this view the dynamical method was not even heuristically important.[39]

We may now measure the full distance that separated Lorentz from Maxwell. Lorentz omitted the aspects of Maxwell's picture of electricity that were not necessary to him; he felt free to contradict this picture whenever he had practical reasons to do so; despite honorable and largely successful efforts at a dynamical foundation of the theory, he ended up preferring Hertz' axiomatic presentation. Yet some important features of Maxwell's theory survived: the field concept, the field equations in vacuo, and the requirement that all currents should be closed.

Lorentz could have remained closer to Maxwell without impeding the progress of the microphysical approach. Without conflict with his optics of moving bodies, Lorentz could have assumed that ether was fully dragged by electric particles, the dragging being confined to the interior and the immediate vicinity of the particles. In this case the field between and the forces acting on the particles, as calculated by Heaviside and other Maxwellians, are the same as in Lorentz' theory, as long as the particles' velocity remains small compared to c.[40]

Lorentz did not proceed in this way, presumably because his equations were simpler. Had he done so, his theory would still have differed from Maxwell's in important respects: by the systematic appeal to a molecular description of matter, by the complete elimination of true conduction, by making material dielectrics qualitatively different from ether in vacuo, and, more generally, by dropping the implicit assumption of a complete similarity between microscopic and macroscopic electrodynamics.

39. For example in the *Versuch* (ref. 35).

40. Cf. Buchwald (ref. 1), appendix 1.

Electromagnetic optics

Once he had established his fundamental equations, Lorentz quickly showed that they implied, by averaging over the electric particles, the usual electrodynamic forces, Faraday's induction law, and the screening property of dielectrics. But his main endeavor was to solve the optical problems that he had tackled within the framework of Helmholtz' theory. As before, he first solved the problem of a single elastically bound electric particle interacting with the electromagnetic field, and then superposed the solutions corresponding to all the particles of a portion of a dielectric medium.

This cumbersome method had some interesting subproducts: the electromagnetic mass of an electric particle, and the radiative damping force $(e^2/4\pi c)\dddot{\mathbf{v}}$, although Lorentz here missed a factor 2/3. The method also gave a new proof of the Lorenz-Lorentz formula, and, most spectacularly, a derivation of Fresnel's dragging coefficient.[41]

Some of the tricks used in the latter calculation are worth mention. Lorentz first introduced axes for which the transparent body is at rest. Calling **u** the velocity of this body with respect to ether, the equations of the field with respect to the moving axes result from the substitution

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \mathbf{u}\cdot\nabla. \tag{37}$$

Accordingly, the equations for the potentials involve the operator

$$\Box_u = \Delta - \frac{1}{c^2}\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial x}\right)^2, \tag{38}$$

if the x axis lies along **u**. Using a standard technique for solving differential equations, Lorentz tried to find new variables x', y', z', t' that would bring him back to a more familiar equation. He found that the substitution

$$x' = \gamma x,\ \ y' = y,\ \ z' = z\ ,\ \ t' = t - \gamma^2 ux/c^2\ , \tag{39}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{40}$$

changed the operator (38) into[42]

$$\Delta' - \frac{\gamma^2}{c^2}\frac{\partial^2}{\partial t'^2}. \tag{41}$$

41. Lorentz (ref. 24), on 268ff., 281, 319.
42. Lorentz (ref. 24), on 297.

There is an immediate corollary, which, however, Lorentz did not stat
The d'Alembertian

$$\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{42}$$

is invariant under the substitution

$$x' = \gamma(x - ut),\ \ t' = \gamma(t - ux/c^2), \tag{43}$$

which is obtained by combining, in this order, the change of axes, th transformation (39), and the division of the time variable by γ.

Lorentz' first derivation of the Fresnel coefficient linked a mathematic invariance property, the Lorentz invariance of the d'Alembertian, to th physical invariance property that makes the laws of refraction independe of the earth's motion. But the connection was highly indirect: the Loren invariance helped in calculating the retarded potential for a moving sourc the potentials corresponding to the various vibrating electrical particle were superposed; the velocity of a wave in the moving medium wa derived, leading to Fresnel's formula; finally Fresnel's formula could b used to prove the invariance of the laws of refraction. Much of Lorentz later efforts aimed at simplifying this involved procedure.

Macroscopic field equations

Very soon Lorentz introduced a first improvement in his optics of mov ing bodies. Through a proper averaging procedure, he derived equations fo the fields that were accessible to macroscopic measurement. The equation were published for the first time in 1892, but the proof appeared later, i the *Versuch* of 1895.[43]

As Lorentz started to do in this period, I will use small letters fo microscopic fields and capital letters for macroscopic fields. Calling ρ_m th microscopic charge density, and **v** the microscopic charge velocity, Lorentz microscopic field equations read:

$$\nabla\cdot\mathbf{d} = \rho_m,\ \ \nabla\cdot\mathbf{h} = 0\,,$$

$$\nabla\times\mathbf{h} = \frac{1}{c}(\rho_m\mathbf{v} + \dot{\mathbf{d}}),\ \ \nabla\times\mathbf{d} = -\frac{1}{c}\dot{\mathbf{h}}\,, \tag{44}$$

43. Lorentz, "On the reflection of light by moving bodies," *VKA* (1892), also in *LCP*, 4 215–218; *Versuch* (ref. 35).

$$\mathbf{f} = \mathbf{d} + \frac{1}{c}\mathbf{v}\times\mathbf{h}\,.$$

In the last equation, **f** is defined as the force acting on a unit charge moving with the velocity **v**.[44]

Lorentz first rewrote these equations with respect to axes moving with the velocity **u**. The necessary modifications are obtained by the substitutions

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \mathbf{u}\cdot\nabla\,,\quad \mathbf{v} \to \mathbf{v}+\mathbf{u}, \tag{45}$$

which lead to[45]

$$\begin{gathered} \nabla\cdot\mathbf{d} = \rho_m\,,\quad \nabla\cdot\mathbf{h} = 0\,, \\ \nabla\times(\mathbf{d} + \frac{1}{c}\mathbf{u}\times\mathbf{h}) = -\frac{1}{c}\dot{\mathbf{h}}\,,\quad \nabla\times(\mathbf{h} - \frac{1}{c}\mathbf{u}\times\mathbf{d}) = \frac{1}{c}(\rho_m\mathbf{v} + \dot{\mathbf{d}})\,, \\ \mathbf{f} = \mathbf{d} + \frac{1}{c}\mathbf{v}\times\mathbf{h} + \frac{1}{c}\mathbf{u}\times\mathbf{h}\,. \end{gathered} \tag{46}$$

Lorentz' next step was an averaging, denoted by an horizontal bar, over domains that contain a large number of molecules but could still be regarded as infinitesimal with respect to macroscopic dimensions. As in Poisson's and Helmholtz' theory of polarization, Lorentz introduced the polarization density

$$\mathbf{P} = \overline{\rho_m\mathbf{r}}\,. \tag{47}$$

He also introduced the vector

$$\mathbf{D} = \overline{\mathbf{d}} + \mathbf{P}, \tag{48}$$

which formally corresponds to Maxwell's displacement, because of the relations

$$\nabla\cdot\mathbf{D} = \overline{\rho_m} + \nabla\cdot\mathbf{P} = \overline{\rho_m} - \rho_P = \rho, \tag{49}$$

where ρ_P is the part of the macroscopic charge that corresponds to a gradient or discontinuity of material polarization, and ρ is the remainder of this charge. The latter charge density is easily seen to play the same role as Maxwell's ρ in macroscopic electrostatics.[46]

44. Lorentz (ref. 35), 14–21.
45. Ibid., 35.
46. Ibid., 63–64.

Consider now an isotropic transparent body at rest with respect to the moving axes of coordinates. After a careful inspection of the microscopic process of polarization, Lorentz found that for monochromatic waves propagating in the body, one could reasonably assume the relation[47]

$$\mathbf{P} = \chi\mathbf{E}, \tag{50}$$

where χ is a function of the frequency of the waves, and

$$\mathbf{E} = \overline{\mathbf{d} + \frac{1}{c}\mathbf{u}\times\mathbf{h}}\,. \tag{51}$$

Averaging the field equations in (44) and neglecting second order terms in u/c, Lorentz arrived at the system

$$\begin{gathered} \nabla\cdot\mathbf{D} = 0\,,\quad \nabla\cdot\mathbf{H} = 0\,, \\ \nabla\times\mathbf{E} = -\frac{1}{c}\dot{\mathbf{H}}\,,\quad \nabla\times(\mathbf{H} - \frac{1}{c}\mathbf{u}\times\mathbf{E}) = \frac{1}{c}\dot{\mathbf{D}}\,, \\ \mathbf{D} = \varepsilon\mathbf{E} - \frac{1}{c}\mathbf{u}\times\mathbf{H}\,, \end{gathered} \tag{52}$$

where $\varepsilon = 1 + \chi$. For a plane-wave solution with the velocity V, the equations lead to the relation

$$V = \frac{c}{\sqrt{\varepsilon}} - \frac{u_\perp}{\varepsilon}, \tag{53}$$

where $u_\perp$ is the component of **u** normal to the wave. The case $\mathbf{u} = \mathbf{0}$ leads to $\varepsilon = n^2$, where n is the refractive index. Consequently, the velocity $V + u$ with which the wave travels with respect to the stationary ether is the same as if the moving dielectric partially dragged ether with a coefficient $1 - 1/n^2$, in conformity with Fresnel's hypothesis.[48]

Corresponding states

The Fresnel coefficient, once combined with Huygens' and Doppler's principles, was sufficient to explain most known experiments in the optics of moving bodies. In particular, the coefficient explained why an optical experiment performed on earth should give, to first order in u/c, the same result as if earth did not move. But Lorentz wanted to show this directly from the system (52), which governs the macroscopic fields in reference to moving axes.

47. Ibid., 75. Lorentz also treated the anisotropic case.
48. Lorentz (ref. 43), 216.

For this purpose he thought of using the change of variables (39), which had the virtue of turning the wave operator for moving dielectrics into that for dielectrics at rest, the d'Alembertian. To first order in u/c the new variables are

$$x' = x,\ \ y' = y,\ \ z' = z, \qquad t' = t - ux/c^2. \tag{54}$$

The corresponding differentials are

$$\nabla' = \nabla + \frac{\mathbf{u}}{c^2}\frac{\partial}{\partial t}, \qquad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t}. \tag{55}$$

Consequently, in the same approximation the system (52) may be rewritten as

$$\begin{gathered} \nabla'\cdot\mathbf{D}' = 0,\ \ \nabla'\cdot\mathbf{H}' = 0, \\ \nabla'\times\mathbf{E} = -\partial\mathbf{H}'/c\partial t',\ \ \nabla'\times\mathbf{H}' = \partial\mathbf{D}'/c\partial t', \\ \mathbf{D}' = \varepsilon\mathbf{E}, \end{gathered} \tag{56}$$

$$\text{with}\ \ \mathbf{D}' = \mathbf{D} + \frac{1}{c}\mathbf{u}\times\mathbf{H},\ \ \text{and}\ \ \mathbf{H}' = \mathbf{H} - \frac{1}{c}\mathbf{u}\times\mathbf{E}. \tag{57}$$

Noting that the new system had exactly the same form as the system for bodies at rest, Lorentz stated the following theorem: "*If, for a given system of bodies at rest, a state of motion is known for which* **D**, **E**, *and* **H** *are certain functions of x, y, z, and t, then in the same system drifting with the velocity* **u**, *there exists a state of motion for which* **D′**, **E′**, *and* **H′** *are the same functions of x, y, z, and t′.*" Lorentz called t' the "local time" (*Ortszeit*) because it could be regarded as "the time reckoned from an instant that depends on the position of the relevant point [of space]," and he called states connected by the theorem "corresponding states."[49]

Since, according to the relations (57), **D** and **H** vanish together if and only if **D′** and **H′** do so, the surface delimiting a light beam is the same for two corresponding states. Consequently, the laws of reflection and refraction, which control the shape of a light beam, are the same in the drifting system. A similar invariance holds for interference experiments, since the position of dark fringes is the same for corresponding states.[50]

49. Lorentz (ref. 35), 85; ibid., 50.
50. Ibid., 85–87.

Finally, Lorentz used the theorem to simplify further the derivation of the Fresnel coefficient. To a plane wave in the system at rest with the phase

$$\phi = \omega t - \mathbf{k}\cdot\mathbf{r}, \tag{58}$$

the theorem connects a plane wave in the moving system with the phase

$$\phi = \omega t' - \mathbf{k}\cdot\mathbf{r} = \omega t - (\mathbf{k} + \omega\mathbf{u}/c^2)\cdot\mathbf{r}. \tag{59}$$

The corresponding phase velocity is

$$V = \frac{\omega}{\left|\mathbf{k} + \omega\mathbf{u}/c^2\right|}. \tag{60}$$

Since the phase velocity in a transparent body at rest is $\omega/k = c/n$, to first order in u/c we have

$$V = \frac{c}{n} - \frac{u_\parallel}{n^2}, \tag{61}$$

which agrees with Fresnel's hypothesis.[51]

The Lorentz contraction

Lorentz applied a similar technique to the electrostatics of drifting bodies.[52] There he started from the static case of the microscopic field equations (46) with respect to moving axes:

$$\begin{gathered} \nabla\cdot\mathbf{d} = \rho_m,\ \ \nabla\cdot\mathbf{h} = 0, \\ \nabla\times\mathbf{h} = \frac{1}{c}\rho_m\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{d},\ \ \nabla\times\mathbf{d} = \frac{1}{c}(\mathbf{u}\cdot\nabla)\mathbf{h}, \\ \mathbf{e} = \mathbf{d} + \mathbf{u}\times\mathbf{h}, \end{gathered} \tag{62}$$

where **e** is the electric force acting on a unit charge at rest with respect to the moving axes. Then he deduced the quadratic equations

$$\Delta_u\mathbf{d} = [\nabla - \frac{1}{c^2}\mathbf{u}(\mathbf{u}\cdot\nabla)]\rho_m,\ \ \Delta_u\mathbf{h} = \frac{1}{c}\mathbf{u}\times\nabla\rho_m, \tag{63}$$

51. Ibid., 95–97.
52. Lorentz, "De relative beweging van de aarde en den aether," *VKA* (1892). English translation, "The relative motion of the earth and the ether," in *LCP*, *4*, 220–223. More detailed considerations are in ref. 35, 35–39, 119–124.

$$\text{where } \Delta_u = \Delta - \frac{1}{c^2}(\mathbf{u}\cdot\nabla)^2. \tag{64}$$

Selecting the x-axis so that it lies along $\mathbf{u}$, and setting

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}, \tag{65}$$

we also have

$$\Delta_u = \gamma^{-2}\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \tag{66}$$

To any solution ξ of the equation

$$\Delta_u\xi + \rho_m = 0, \tag{67}$$

corresponds a solution of the system (63), as given by

$$\mathbf{d} = -[\nabla - \frac{1}{c^2}\mathbf{u}(\mathbf{u}\cdot\nabla)]\xi, \ \mathbf{h} = -\frac{1}{c}\mathbf{u}\times\nabla\xi. \tag{68}$$

According to the last equation of system (62), the corresponding value of the electric force field is

$$\mathbf{e} = -\gamma^{-2}\nabla\xi. \tag{69}$$

"In order to clarify the meaning of the previous formulas," Lorentz imagined a second, ficticious system S′. The latter system is at rest, it has the same constitution as the orginal system (S) and carries the same charges, but its dimensions are dilated by a factor γ in the direction of the x-axis. In other words, to a point of S with the coordinates x, y, z, corresponds a point of S′ with the coordinates

$$x' = \gamma x, \ y' = y, \ z' = z. \tag{70}$$

In the second system, the charge density is $\rho' = \gamma^{-1}\rho_m$; the corresponding potential ξ' is given by the Poisson equation

$$\Delta'\xi' + \rho' = 0, \tag{71}$$

and the electric force field is

$$\mathbf{e}' = -\nabla'\xi'. \tag{72}$$

Comparing equations (67) and (71), we have

$$\xi = \gamma\xi'. \tag{73}$$

Therefore, the force field e, which is given by (69), is related to the force field $\mathbf{e}'$ by

$$e_x = e_x', \ e_y = \gamma^{-1}e_y', \ e_z = \gamma^{-1}e_z'. \tag{7}$$

The simplicity of the relation between S and S', especially the fact th
$\mathbf{e}$ vanishes if and only if $\mathbf{e}'$ vanishes, warrants that the qualitative appea
ance of electrostatic phenomena is not modified by the earth's motion: the
is no electric force within conductors, and there are only surface charges f
a system of conductors in equilibrium. Lorentz also noted that the predicte
influence of motion on electrostatic forces and charge distribution was
the second order in u/c, which left little hope for an experiment
confirmation.[53]

However, Lorentz imagined a connection between the electrostatics
moving bodies and the Michelson-Morley experiment. In 1887 the tw
American experimenters had repeated the experiment of 1881 with a muc
larger interferometer floating on a mercury bath, and had found no fring
shift of the order of magnitude implied by a stationary ether. Despite thi
fact, Lorentz maintained his faith in the stationary ether. In 1992, a littl
after his memoir on Maxwell's theory and the optics of moving bodies, h
suggested a way out of the contradiction.[54]

There would be no shift of the interference fringes, he noted, if the arr
of the interferometeter parallel to the earth's velocity $\mathbf{u}$ was contracted b
an amount

$$\gamma^{-1} = \sqrt{1-u^2/c^2}. \tag{75}$$

The contraction was small enough to have remained unnoticed in previous
terrestrial experiments, but was theoretically conceivable. Argued Lorentz:[55]

> What determines the size and shape of a solid body? Evidently the intensity
> of the molecular forces....Nowadays we may safely assume that the electric
> and magnetic forces act by means of the intervention of the ether. It is no
> far-fetched to suppose the same to be true of molecular forces. But then i
> may make all the difference whether the line joining two material particles
> shifting together through the ether, lies parallel or crosswise to the directio
> of that shift.

If, Lorentz went on, molecular forces in moving bodies behaved exacly as electrostatic forces, the theory *implied* the contraction. He reasoned as follows.

If the correspondence theorem applies to molecular forces, the relation (74) between the forces in "corresponding states" implies that a state of equilibrium in S′ is also a state of equilibrium of S. Since, by assumption,

53. Lorentz (ref. 35), 38.
54. Lorentz (ref. 52).
55. Ibid., 221.

molecular forces completely determine the dimensions of a solid for a given molecular arrangement, the dimensions of S′ are those which the solid S would take if it were brought to rest. By definition of S′, these dimensions are obtained from those of the moving S by a dilatation along **u** in the proportion γ. Therefore S must contract along the direction of its motion.[56]

Lorentz was well aware of some defects of the argument. Nothing required that molecular forces behaved like electrostatic forces with respect to the "correspondence." Moreover, thermodynamical evidence implied that the equilibrium of a solid was not of a static nature, but involved microscopic motions. Still the argument was likely to be improved in the future. The Lorentz contraction was not, in Lorentz' mind, a completely *ad hoc* hypothesis.[57]

Improvements

Lorentz obtained the preceding results on macroscopic field equations, corresponding states, and the contraction of lengths in 1892. Three years later a full, systematic account of his theory appeared in German, under the title: *Attempt at a theory of electrical and optical phenomena in moving bodies*. Here Lorentz postulated again the existence of electrical particles, which he now called ions in allusion to electrolysis. He took his microscopic field equations and the "Lorentz force" as plausible axioms, and focussed on the consequences of these equations for macroscopic moving bodies.[58]

By the end of the century, this contribution to the increasingly popular ionic theory was regarded as most outstanding, for at least two reasons: the clarity and simplicity of the basic assumptions, and the extent of the empirical ground covered. From the same general assumptions, nearly all known electromagnetic and optical phenomena were interpreted, including standard electrodynamic effects, dispersion, crystal optics, magneto-optic effects, and the optics of moving bodies. However, the structure of the developments connecting the basic assumptions to experiments appeared to be unduly complex, even in Lorentz' own eyes.[59]

56. In his justification of the Lorentz contraction, Lorentz implicitly assumed that for a given arrangement of the molecules of a solid at rest there is only one state of the solid that is compatible with the equilibrium of molecular forces. Another implicit assumption, which Lorentz later found to be problematic, was that the definition of the corresponding states restablishing the Poisson equation was unique.

57. FitzGerald's contraction hypothesis (of which Lorentz was not yet aware in 1892) was not purely ad hoc either. Cf. Hunt (ref. 1), 189–197.

58. Lorentz (ref. 35).

59. Although, when it was published, the *Versuch* was just one ionic theory among many others (cf. Buchwald, ref. 1, 198–199), after the Düsseldorf meeting of 1898 (cf. Hirosige, ref.

The technique of "corresponding states," as exposed in the *Versuch* led to very different kinds of considerations, according to the problem t which it was applied. In the optical case, it led to the cancellation of firs order effects of motion, whereas in the electrostatic case, it led to the pred iction of observable second order effects, like the Lorentz contraction Even at the purely calculational level, Lorentz' procedure was incon veniently heterogeneous. In some cases (Fresnel coefficient, reflexion an refraction, interference), the technique of corresponding states was applie at the macrosopic level, while in some others (Doppler effect, aberration electrostatics) it was applied at the microscopic level. Moreover, the tech nique was not well adapted to some simple electrodynamic systems, like a electric charge and a current-carrier travelling together, which Lorentz ha to discuss separately.[60]

The complexity was the more embarrassing because it occurred in the proof of a very simple result: that the motion of the earth could not be detected by the terrestrial devices used so far by physicists. Lorentz wrote in 1898: "Although this very simple fact can be derived from the Fresnel theory [or from the corresponding ionic theory], it appears—one could almost say—as a fortuitous consequence of rather complicated considerations. Wouldn't it be much simpler to have ether follow the motion of the earth, which would immediately explain the negative results of the mentioned experiments?" Lorentz rejected the alternative, because, in his opinion, it gave no satisfactory account of aberration. But he was certainly aware of the necessity of simplifying his own scheme.[61]

Another defect of Lorentz' theory was incompleteness. The basic assumptions concerned only ether and the electromagnetic properties of ions. But some experiments on the optics of moving bodies could not be explained without further assumptions. For example, the Michelson-Morley experiment required that molecular forces behaved like electrostatic forces. For Poincaré, who insisted that the relativity principle held generally, this was a major defect of Lorentz' theory. Would a new kind of explanation be needed for every order in u/c?[62]

Lorentz could not have been much disturbed by the objection initially, because he believed that future refined terrestrial experiments would detect

22, on 33) and the Lorentz jubilee of 1900 Lorentz' theory became the center of all experts' discussions.

60. Lorentz (ref. 35), 39–43.

61. Lorentz, "Die Fragen welche die translatorische Bewegung des Lichtäthers betreffen" (Düsseldorf paper), Versammelung Deutscher Naturforscher und Aertze, *Verhandlungen* (1898), also in *LCP*, *7*, 101–115, 102.

62. H. Poincaré, "Relations entre la physique expérimentale et la physique mathématique," Congrès international de physique, *Rapports* (4 vols., Paris, 1900), *1*, 22–23.

the earth's motion. However, in subsequent years he found more and more reasons to accept Poincaré's criticism. It was not out of concern for third-order effects, but because he found that he had to be ever more subtle in explaining all available second-order experiments. Mascart's old experiment concerning the rotation of polarization of light was the subject of a highly technical controversy between Larmor and Lorentz. New experiments, like the ones by Rayleigh and Brace and by Trouton and Noble, failed to detect effects that seemed to result from Lorentz' theory. Their explanation challenged every expert including Lorentz.[63]

Fortunately, Lorentz managed to improve the technique of corresponding states in a way that would meet this challenge as well as Poincaré's criticism. In 1899 he took a first step in this direction by applying directly to the microscopic field equations (refered to moving axes) the change of coordinate (39). Since 1892 Lorentz knew the pleasant effect of this change on the quadratic differential operators of the theory. Now he could see that the change turned the field equations into equations similar to the equations refered to ether, but in terms of the new fields

$$\mathbf{d}' = (1,\gamma)(\mathbf{d} + \frac{1}{c}\mathbf{u}\times\mathbf{h}),\quad \mathbf{h}' = (1,\gamma)(\mathbf{h} - \frac{1}{c}\mathbf{u}\times\mathbf{d}), \tag{76}$$

if the symbol (α, β) means a multiplication by α of the component parallel to $\mathbf{u}$ and a multiplication by β of the component perpendicular to $\mathbf{u}$. Lorentz also noticed that the similarity was improved by introducing a third time coordinate, $t'' = \gamma^{-1}t'$ and that a common dilatation of space and time coordinates by a factor ε, together with a proper rescaling of the fields and forces, maintained the form of the field equations. Of course ε must be a function of u that takes the value *unity* when u reaches zero. The final coordinate and field transformations read:

$$x' = \gamma\varepsilon x,\quad y' = \varepsilon y,\quad z' = \varepsilon z,\quad t' = \varepsilon(\gamma^{-1}t - \gamma ux/c^2), \tag{77}$$

$$\mathbf{d}' = \varepsilon^{-2}(1,\gamma)(\mathbf{d} + \frac{1}{c}\mathbf{u}\times\mathbf{h}),\quad \mathbf{h}' = \varepsilon^{-2}(1,\gamma)(\mathbf{h} - \frac{1}{c}\mathbf{u}\times\mathbf{d}). \tag{78}$$

The transformation obtained by combining, in order, the Galilean transformation of velocity $\mathbf{u}$, and the given change of coordinates for $\varepsilon = 1$, is the so-called Lorentz transformation. Together with the field transformation, the Lorentz transformation restablishes the original form of the field equations *in vacuo*.[64]

63. Cf. Hirosige (ref. 20), 33–41.

64. Lorentz, "Théorie simplifiée des phénomènes électriques et optiques dans des corps en mouvement," Dutch in *VKA* (1899), French in *AN* (1902) and *LCP5*, 139–155. As we will see, in 1897 Larmor had already given the same field transformation, except for a global scale factor (γ). However, at that time Larmor was only aware of the *second-order* invariance of the field equations in vacuo. He asserted the exact invariance only in *Aether and matter* (1900).

However, Lorentz' exploitation of this invariance was impeded by two difficulties. First, he did not introduce transformed expressions for the charge density ρ and the current $\rho\mathbf{v}$, so that the inhomogeneous terms in the field equations and the expression for the Lorentz force $\mathbf{f}$ remained complicated. For this reason, the benefit of the new transformation was meager and the technique of corresponding states worked only for the cases that Lorentz had already been able to treat, electrostatics and first order optics.

The other difficulty had to do with the interpretation of corresponding states. Were they only states of an imaginary system, or were they the states that the moving system would really take if brought to rest? Lorentz' treatments of electrostatics and first-order optics required only the first conception. But his derivation of the Lorentz contraction appealed to the second, more realistic conception. Now a difficulty with the latter conception is that the correspondence defining the "corresponding states" is ambiguous owing to the factor ε in the transformation formulae. Although this embarrassing factor had to have a definite value in the realistic conception of corresponding states, the theory in its present stage was unable to determine it.[65]

Fortunately, the ambiguity did not impede the explanation of the Michelson-Morley experiment, because what determines the absence of fringe shift is the *relative* length of the two arms of the interferometer. The realistic conception of corresponding states was not endangered. In his paper of 1899 Lorentz generalized it to all second-order optical phenomena: "We shall admit that, when a translatory motion is imparted to a system S′ orginally at rest, this system goes *of itself* to the state S." This implied, Lorentz showed, the invariance of a large class of second-order experiments, expecially a variant of the Michelson-Morley experiment in which, as suggested by Liénard, the air was replaced by a solid or liquid dielectric.[66]

The ultimate improvement of Lorentz' electromagnetics of moving bodies, published in 1904, rested on nearly the same mathematical transformation as had been used in 1899. The only difference was a new transformation for velocities, that read

$$\mathbf{v}' = (\gamma^2, \gamma)\mathbf{v}. \tag{79}$$

This transformation gave $\nabla'\times\mathbf{h}'$ the same form as for a system at rest, but the expressions for $\nabla'\cdot\mathbf{d}'$ and for the velocity-dependent part of the Lorentz force remained complicated. The transformation is not consistent with the

65. Ibid., 154.

66. Ibid., 153.

coordinate transformation (77), a point that Lorentz seems to have overlooked. Nevertheless, Lorentz managed to counter Poincaré's criticism by extending the technique of corresponding states to any order in u/c and to nearly all conceivable experiments in the electrodynamics and optics of moving bodies.[67]

Lorentz first circumvented the complexity of the source terms in his transformed equations by showing that, for a variable dipole, the form of the emitted fields was invariant. Then he made sure that every object in S′ would look the same as in S brought to rest by introducing two additional assumptions: that the longitudinal dimension of an electron was altered by translation in the proportion $1/\varepsilon\gamma$, and the transversal dimensions in the proportion $1/\varepsilon$; and that the forces between uncharged particles, as well as those between such particles and electrons, were influenced by translation in quite the same way as the electric forces in an electrostatic system. The former assumption warrants that an electron in S′ would have the same shape and size as an electron at rest. According to the second assumption, and by the reasoning Lorentz had used for the Michelson-Morley experiment, the dimensions of a solid, macroscopic body in S′ are the same as if S′ were S brought to rest.[68]

What remained to be checked was whether a possible evolution in S′ corresponded to a possible evolution in S. For this, not only the expression of the fields in terms of their sources, but also the equations of motion of microscopic charges had to be invariant. Lorentz assumed that all forces behaved like electromagnetic forces and that all inertia behaved as if it were of purely electromagnetic origin. Then he had only to examine the transformation of the equation of motion of a purely electromagnetic electron. He first established this equation in the following manner.[69]

The electromagnetic momentum of an electron moving at the constant velocity **u** is, by definition,

$$\mathbf{p} = \frac{1}{c}\int \mathbf{d}\times\mathbf{h}\, d\tau, \tag{80}$$

where **d** and **h** represent the electromagnetic field of the electron. Using the field (78) and coordinate (77) transformations, and noting that **h**′ is zero, we have

67. Lorentz, "Electromagnetic phenomena in a system moving with any velocity smaller than that of light," *FRA* (1904), also in *LCP5*, 172–197. Lorentz refers to Poincaré's criticism, ibid., 173.

68. Ibid., 182–183.

69. Ibid., 184–185.

$$\mathbf{p} = \frac{1}{c^2}\varepsilon\gamma\mathbf{u}\int \mathbf{d}'^2_{\perp}\, d\tau'. \tag{81}$$

Taking with Lorentz the electron (at rest) to be a uniformly charged spherical shell with the total charge e and the radius R, and noting that **d**′ is the electrostatic field created by this charge distribution, we get

$$\int \mathbf{d}'^2_{\perp}\, d\tau' = \frac{2}{3}\int \mathbf{d}'^2\, d\tau' = \frac{e^2}{6\pi}\int_R^\infty \frac{dr}{r^2} = \frac{e^2}{6\pi R}, \tag{82}$$

$$\text{and}\quad \mathbf{p} = m_0\gamma\varepsilon\mathbf{u} \quad\text{with}\quad m_0 = \frac{e^2}{6\pi c^2 R}. \tag{83}$$

Following Abraham, Lorentz wrote for the equation of motion of a moderately accelerated electron

$$\mathbf{f} = \frac{d\mathbf{p}}{dt} = m_{||}\mathbf{a}_{||} + m_{\perp}\mathbf{a}_{\perp}, \tag{84}$$

where **f** represents the force acting on the electron, and the **a**'s denote the components of the accelerations parallel and perpendicular to the trajectory. The given expression for **p** yields

$$m_{||} = m_0\frac{d\varepsilon\gamma u}{du}, \quad m_{\perp} = m_0\gamma\varepsilon. \tag{85}$$

The result was in itself interesting, because it challenged the predictions of Abraham's previous theory of the rigid electron (1902), which Kaufmann claimed to have confirmed by electromagnetic deflection experiments. Lorentz performed long calculations to verify that Kaufmann's data were no less compatible with his own theory. Yet his main purpose remained the determination of the scale factor ε.[70]

For this purpose, Lorentz assumed that the velocity of all electrons in the moving system S remained close to the global velocity **u** of the system. Consequently, the equation of motion of an electron of S has the form (84) just given, with constant longitudinal and transverse masses given by (85). Lorentz then reexpressed this equation in terms of transformed variables. On the one hand, a double differentiation of the coordinate transformation (77) connects the transformed and the true accelerations according to

$$a'_{||} = \varepsilon^{-1}\gamma^3 a_{||}, \quad a'_{\perp} = \varepsilon^{-1}\gamma^2 a_{\perp}. \tag{86}$$

70. On Kaufmann's experiments and Abraham's theory, see A. Miller, *Albert Einstein's special theory of relativity. Emergence (1905) and early interpretation (1905–1911)* (Reading, 1981); J.T. Cushing, "Electromagnetic mass, relativity, and the Kaufmann experiments," *American journal of physics*, *49* (1981), 1133–1149.

On the other hand, the force **f** acting on the electron transforms, by assumption, like the electromotive force $e[\mathbf{d}+(\mathbf{u}+\mathbf{v})\times\mathbf{h}]$ (in which **d** and **h** are external fields). This force can be expressed in terms of the transformed fields through the relations (78), which gives, for a negligible relative velocity,

$$\mathbf{f}_{\parallel} = \varepsilon^2 e\mathbf{d}'_{\parallel},\ \ \mathbf{f}_{\perp} = \gamma^{-1}\varepsilon^2 e\mathbf{d}'_{\perp},\ \text{or} \tag{87}$$

$$\mathbf{f}_{\parallel} = \varepsilon^2\mathbf{f}'_{\parallel},\ \ \mathbf{f}_{\perp} = \gamma^{-1}\varepsilon^2\mathbf{f}'_{\perp}, \tag{88}$$

since $e\mathbf{d}'$ is identical to the force $\mathbf{f}'$ that acts on the corresponding electron of S′. Then, combining the equations (86), (88), and (85), with the normal equation of motion for electrons in S′, $\mathbf{f}' = m_0\mathbf{a}'$, we will obtain the equivalent of the equation of motion (84) for the corresponding electrons in S if and only if

$$\frac{d\varepsilon\gamma u}{du} = \varepsilon\gamma^3 . \tag{89}$$

As results from the expression (40) of γ, this condition is equivalent to ε = constant. The constant must be *unity* since for $u=0$ no contraction or dilatation of lengths is expected to happen.

The scope of Lorentz' method of corresponding states was no doubt widely increased by these considerations. Even the negative result of Trouton and Noble, which contradicted other electron theories, received an explanation. Yet, a critical reader of Lorentz could hardly have been satisfied. The invariance proof of 1904 still required approximations: the dipolar approximation for the microscopic sources of radiation, the approximation of small relative velocities, and the neglect of spinning motion for electrons and other ions. Even if this inconvenience could be elimated, the difficulty of the very notion of corresponding states would remain. The states were in a sense fictitious, since they were obtained from the real states of the moving system S by a mathematical transformation. But they were discussed as real states of the system that would be obtained by bringing S to rest.

Lorentz was prudent in his own judgment: "It need hardly be said," he wrote, "that the present theory is put forward with all due reserve." He did not exclude that future terrestrial experiments would detect the motion of the earth. In his theory, the invariance of electromagnetic phenomena depended on the behavior of the ultimate entities of matter, the electrons, of which very little was known: "Our assumption about the contraction of the electrons," he commented, "cannot in itself be pronounced to be either plausible or inadmissible." He was certainly pleased that his values for the longitudinal and transversal masses agreed with Kaufmann's data, but he did not prejudge the outcome of future, improved experiments.[71]

2. LARMOR

In the period 1894–1897 Larmor brewed a theory of electrodynamics that, in its final stage, was in several respects similar to Lorentz'. It involved electrons, a stationary ether, and derivations of the optical properties of bodies at rest and in motion. Before analyzing the contents, something must be said about appearance and style. Whereas Lorentz was known for his clarity and directness, Larmor's writings were notoriously difficult to read. The complexity and opacity of his memoirs on "a dynamical theory of the electric and luminiferous medium," which led to the electron theory, have several sources. These texts represent a complex of evolving ideas, not a definitive, synthetic system. A more fundamental difficulty is that essential elements of Larmor's theory, even in its final stage, were only expressed in words and pictures. Precise mathematization only occurred at the more phenomenological level. Larmor seems to have been unable to express himself concisely, even on straightforward matters. This verbal generosity reflected in good part his way of thinking, which involved much historical and philosophical digression. In brief, Larmor was neither a practical man nor a rigorous thinker. He was a natural philosopher.[72]

Background

As a Cambridge graduate, Larmor was taught to revere the principle of least action "as the fundamental formulation in dynamics and physics." Early in his career, in 1884, he praised the method of minima for conveying "a clearer and more compact mode of representation...and an easier grasp of mathematical relations as a whole, than any other." He did not think, however, that the action principle freed one from the duty of illustrating physical theories. Instead, he emphasized that the principle could ease the finding of dynamical analogies. He referred to Clausius' and Boltzmann's dynamical analogies for the second principle of thermodynamics and to Helmholtz' analogy between vortices in an ideal fluid and electric currents. In each case the analogy appears most directly by examining the action of the compared systems.[73]

72. On Larmor's style, see Buchwald (ref. 1), 141–142.

73. Larmor, "On least action as the fundamental formulation in dynamics and physics," *PLMS*, *15* (1884), 158–194, also in *LaMPP1*, 31, 55–56. Cf. Buchwald (ref. 1), 135–136, and Hunt (ref. 1), 212. Larmor was not aware that forces between currents and forces between vortices have opposite signs.

Later, while in search of a universal medium for electric and optical phenomena, Larmor acknowledged the more abstract virtue of least action: "its allowing us to ignore or leave out of account altogether the details of the mechanism, whatever it is, that is in operation in the phenomena under discussion." Maxwell had brilliantly demonstrated this power of Hamilton's principle in his dynamical theory of the electromagnetic field, to the applause of Poincaré and Helmholtz. But Larmor did not believe that a physical theory could do without pictures. He explained:[74]

> The problem of the correlation of the physical forces is...divisible into two parts, (i) the determination of the analytical function which represents the distribution of energy [more exactly, the Lagrangian] in the primordial medium which is assumed to be the ultimate seat of all phenomena, and (ii) the discussion of what properties may be most conveniently and simply assigned to that medium, in order to describe the play of energy in it most vividly, in terms of the stock of notions which we have derived from the observation of that part of the interaction of natural forces which presents itself directly to our senses, and is formulated under the name of natural law.

Larmor admitted that the first part involved in itself the solution of the whole problem. But he still demanded the second part, for the purpose of "illustration and explanation."

Before 1893, Larmor wrote electromagnetic and optical theory without the principle of least action. Instead, he developed models of ether in William Thomson's style. For example, in 1890 he generalized Thomson's model of a gyrostatically loaded ether which illustrated the rotation of the plane of polarization of light in a magnetized medium, that is, the Faraday effect. The model involved an elastic solid representing pure ether and small spherical cavities in this solid supporting permanently rotating flywheels. The resistance of the axis of the flywheels to rotation implied a modification of the elastic properties of the medium, which was to explain its rotary power.[75]

Concerning this baroque but expedient picture, Larmor commented:

> The hypothesis of gyrostatic cells interspersed throughout the medium, though at first sight artificial, is a correct realization of the current views of the influence of ponderable matter on the undulations of the aether. Any exhaustive optical investigation must take cognizance of the mutual influence of the two interpenetrating media, the aether and the ordinary matter.

74. Larmor, "A dynamical theory of the electric and luminiferous medium" (abstract), *PRS*, *54* (1893), 438–461, also in *LaMPPI*, 389–413, on 389–390.

75. Larmor, "Rotary polarization, illustrated by the vibrations of a gyrostatically loaded chain," *PLMS*, *21* (1890), 423–432, also in *LaMPPI*; "The equations of propagation of disturbances in gyrostatically loaded media, and the circular polarization of light," *PLMS*, *23* (1891), 127–135, also in *LaMPPI*. On Thomson's model, cf. C. Smith and N. Wise, *Energy and empire: A biographical study of Lord Kelvin* (Cambridge, 1989), 439, 473–474 .

More exactly, the two media had to be regarded as two linked dynamical systems. This view was not self-evident, because in most of Maxwell's *Treatise* and also in old optical theories, there was only one medium, with variable charateristics (permittivity, permeability; density, elasticity) depending on the presence of matter. But Thomson's old analysis of the Faraday effect, as interpreted by Maxwell and Larmor, left no other possibility: in this case the circular motion induced by the external magnetic field had to pertain to matter, while the motion implied in the light waves belonged to ether.[76]

Gyrostatic loading could not be a general representation of matter. For this, Thomson had something better to offer, his theory of vortex atoms, for which Larmor had much sympathy. Since Helmholtz' influential memoir on fluid motion, vortices in an ideal, incompressible fluid were known to be permanent structures that could start or end only at the surface of the liquid. In his theory of matter Thomson assumed a primitive medium similar to Helmholtz' ideal fluid and took atoms to be small vortex rings in this fluid. The picture did not explain much more than the permanence of atoms and their ability to combine. But Thomson, Larmor, and many others hoped for future improvements.[77]

To sum up, by 1890 Larmor's picture of the physical world involved an optical ether with molecular loading, the molecules being themselves permanent structures in a primitive, continuous medium. The molecules could eventually carry electric charge in discrete amounts, as Helmholtz had argued in his discussion of electrolysis. Larmor was fully aware of the strength of Helmholtz' arguments, since he had derived an estimate of molecular sizes from Helmholtz' interpretation of electrode polarization.[78] What the picture lacked was an account of electrodynamic phenomena.

As appears from his work on electromagnetic induction in rotating bodies, published in 1884, Larmor studied Maxwell's *Treatise* early in his career. After a period of confusion, he came to grasp the essentials of Maxwell's conception of charge and current. For instance, in 1893 he wrote: "The electric current is in a dielectric the rate of change of the

76. Larmor, "The equations" (ref. 75), *LaMPPI*, 248. On Thomson's analysis of the Faraday effect, see Larmor, "The action of magnetism on light: with a critical correlation of the various theories of light-propagation," British Association, *Report* (1893), *LaMPPI*, 310–355, on 314; Maxwell, *A treatise on electricity and magnetism* (2 vols., Oxford, 1873), sec. 831; See also O. Knudsen, "The Faraday effect and physical theory," *AHES*, *15* (1976), 235–281.

77. See Larmor (ref. 74), *LaMPPI*, 390; Smith and Wise (ref. 75), 417–425.

78. Larmor, "On the molecular theory of galvanic polarization," *PM* (1885), 422–435, also in *LaMPPI*.

electric displacement, which is of an elastic character; in a conducting medium part of the current is due to the continual damping of electric displacement in frictional modes.'' However, Larmor was never completely satisfied with Maxwell's theory. In 1891, well after Hertz' experiments, he provided a very detailed study of Helmholtz' polarization theory, which, he wrote, offered ''a more general view of the nature of dielectric polarization'' and represented, in the limit of infinite polarizability, ''a concrete illustration of the general statements of Maxwell with respect to electric displacement.'' Larmor had no taste for Maxwell's picture of the imaginary incompressible fluid and had difficulties with the notion of a primitive, irreducible polarization. Nor did Larmor uncritically espouse Helmholtz' theory. Within a few months Larmor found that the theory gave a wrong value for the electrostatic pressure at the boundary between two dielectrics. This result, incorrect as we know it was, prompted him to reject Helmholtz' theory.[79]

After this brief episode, Larmor remained dissatisfied with Maxwell's theory, as appears in an extract of 1893: ''Maxwell's views involve difficulties, not to say contradictions, and in place present obstacles which are to be surmounted, not by logical argument or any clear representation, but by the physical intuition of a mind saturated with this aspect of the phenomena.'' Most fundamentally, Larmor reproached Maxwell with not giving a complete model of the electromagnetice field, so that ''the nature of electric displacement, of electric and magnetic forces on matter, of what Maxwell calls the electrostatic and magnetic stress in the medium, of electrochemical phenomena, are all left obscure.'' Indeed, for electric dispacement Maxwell did not pretend to give more than a suggestive illustration; for stresses he offered no mechanism at all, and on electrolysis he could only offer ''a method by which we may remember a good many facts.'' Moreover, Maxwell's theory was not an ''ultimate dynamical theory'' in Larmor's sense, for it renounced any picture of the hidden motion in magnetic fields.[80]

By contamination Maxwell's electromagnetic theory of light suffered similar defects. In Larmor's eyes, the old, nonelectromagnetic theories of light were more attractive, because they usually rested on a clear, definite model. Consequently, a proper unifying foundation of physics could

79. Larmor, ''Electromagnetic induction in conducting sheets and solid bodies,'' *PM* (1884), also in *LaMPP1*, 8–28; ref. 76, in *LaMPP1*, 339; ''On a generalized theory of electrodynamics,'' *PRS*, *49* (1891), 521–536, in *LaMPP1*, 233; ''On the theory of electrodynamics, as affected by the nature of the mechanical stresses in excited dielectrics,'' *PRS*, *52* (1892), 55–66, also in *LaMPP1*; on Larmor's error, see Buchwald (ref. 1), 139–140, 320.

80. Larmor (ref. 74), *LaMPP1*, 397; Maxwell (ref. 76), sec. 260; Larmor, ''The equations'' (ref. 75), *LaMPP1*, 253.

perhaps be found among these theories. By 1894 Larmor had no doubt in this regard: ''Many of these obstacles [encountered in Maxwell's theory] may be removed by explaining electric actions on the basis of a mechanical theory of radiation, instead of radiation on the basis of electric actions.''[81]

MacCullagh's ether

What triggered this reverse reduction, from optical ether to electromagnetism, was Larmor's discovery of MacCullagh's optical theory of 1839. The theory was the only one that had given correct continuity conditions at the surface between different media. But because of criticism by Stokes, to which I will return, MacCullagh's theory was little cultivated. In 1880 FitzGerald resurrected it and showed its close connection with Maxwell's electromagnetic theory of light. He applied it to a study of the reflection and refraction of light with and without a magnetic field. In an extensive review of magneto-optical effects written in 1893, Larmor became aware of MacCullagh's theory through FitzGerald's electromagnetic interpretation.[82]

MacCullagh's ether had the peculiar property of elastic resistance to the rotation, *not* the translation, of its particles. To Larmor's satisfaction, MacCullagh had cast his theory in Lagrangian form, with the Lagrangian

$$L = \frac{1}{2}\int \mu\left(\frac{\partial \xi}{\partial t}\right)^2 d\tau - \int \frac{1}{2\varepsilon}(\nabla\times\xi)^2 d\tau, \tag{90}$$

where ξ is the position of a particle of ether, μ is the density of ether, and ε is the elastic constant. Varying the corresponding action gives

$$\mu\frac{\partial^2\xi}{\partial t^2} = -\nabla\times\frac{\nabla\times\xi}{\varepsilon}, \tag{91}$$

which, according to FitzGerald, can be interpreted electromagnetically as

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}, \tag{92}$$

$$\text{if } \mathbf{B} = \mu\frac{\partial \xi}{\partial t}, \text{ and } \mathbf{D} = \varepsilon\mathbf{E} = \nabla\times\xi. \tag{93}$$

81. Larmor (ref. 74), *LaMPP1*, 397.

82. J. MacCullagh, ''An essay toward a dynamical theory of crystalline reflexion and refraction,'' Royal Irish Academy, *Transactions*, *21* (1839), 17–50; G.F. FitzGerald, ''On the electromagnetic theory of the reflexion and refraction of light,'' *PT*, *171* (1880), 691–711; Larmor, ''The action of magnetism'' (ref. 76), *LaMPP1*, 340–343. On MacCullagh's theory, see Buchwald (ref. 1), 283–284.

Conversely, the magnetic induction **B** becomes a momentum, and Maxwell's displacement **D** becomes twice the local rotation of the medium.[83]

At the interface between two portions of this medium with different elastic constants, Hamilton's principle implies that ξ and the tangential component of $(\nabla\times\xi)/\varepsilon$ are continuous. These conditions lead to Fresnel's formula for the intensity and direction of reflected and refracted rays; they are equivalent to those deduced by Helmholtz and Lorentz from "Maxwell's theory."[84]

A serious defect of MacCullagh's ether is that it cannot be illustrated by any realistic elastic solid. Larmor supplied the defect by showing that Kelvin's gyrostatically loaded medium provided a model of this ether if the elasticity originated entirely in the gyrostatic effect. Such a medium had no resistance to local translation whatsoever, but resisted elastically local rotations. The first property explained the perfect fluidity of ether with respect to irrotational motion, and the second its optical properties.[85]

One of Stokes's objections remained: the net torque acting on a volume element of MacCullagh's ether is not zero, so that the medium cannot be in internal equilibrium. From his gyrostatic model, Larmor argued that Newton's notion of absolute space saved the situation. When a spinning gyrostat resists rotation, it does so with respect to absolute space, not with respect to nearby matter. As Larmor put it, "the gyrostat may be considered as a kind of connection binding that system to absolute immovable space by means of the forcive which it opposes to rotation; and this is the reason why the element of mass in a gyrostatic medium remains in equilibrium with its translational kinetic reactions, although the tractions in the surrounding parts on its surface are unbalanced and result in a couple."[86]

Another difficulty is that a gyrostat, when moved, loses part of its kinetic energy. Larmor assumed that the initial spinning energy was very

83. The e.m. interpretation of MacCullagh's equation implies that γ can take arbitrarily large values, in constant magnetic fields. In this case, equation (91) is no longer exact and the partial derivative in (92) must be replaced by a convective derivative. However, for the highly dense ether of Larmor (after Lodge's experiment), the correction is negligible.

84. The latter conditions are the continuity of the tangential components of **E** and **H**, and the continuity of the normal components of **D** and **B**; for $\mu=1$, this will hold if and only if **B**, the tangential component of **E**, and the normal component of **D** are continuous; thanks to the relations (93), the first condition corresponds to the continuity of ξ, the second to the continuity of the tangential component of $(\nabla\times\xi)/\varepsilon$, and the last to the continuity of the normal component of $\nabla\times\xi$, which is an immediate consequence of the continuity of ξ.

85. W. Thomson, "On a gyrostatic adynamic constitution for 'ether'," Royal Society of Edinburgh, *Proceedings*, *17* (1890), 127–132; Larmor (ref. 76), *LaMPP1*, 354; ref. 74, *LaMPP1*, 390–391.

86. Larmor (ref. 74), 408.

high so that its damping on a human time scale would be negligible. Altogether the model had no resemblance to any known macroscopic state of matter and its properties could be pictured only for limited stretches of time. According to Larmor, this difficulty merely signaled the ideal character of the ultimate medium. For the proper unifying basis of physics he demanded a clear picture, but not necessarily a familiar, palpable one.[87]

A last difficulty pertained to the dynamics of MacCullagh's ether, in which there could be no dissipation, since its motion derived from the principle of least action, which excluded dissipative forces.[88] This want of dissipation created a problem even in optics: propagation in or reflection by metals required empirical viscous terms in the equations. Refering to Helmholtz' then recent study of least action in electrodynamics, Larmor judged the absence of dissipation to be normal at the foundational level: "It is, in fact, clear that the scientific method, in forming a dynamical theory, is to restrict it in the first instance to systems in which the interaction of stress and motion has free play, without the interference with its results that is produced by frictional agencies."[89] In electrodynamics, it was also standard to specify first actions in pure dielectrics, then to discuss conductivity. As Larmor knew, Maxwell's theory did not proceed differently, since it first introduced the elastic properties of a dielectic, then defined conduction as a continual breakdown of these properties.

Vortices in ether

By the fall of 1893, Larmor believed that MacCullagh's ether was a legitimate departure for a new, unified theory of optics and electromagnetism. But this was not all. Larmor soon realized that permanent vortices were possible in this medium, so that Kelvin's vortex atoms could be used to explore the connection between ether and matter. In Larmor's own words:[90]

> The considerations...amount to an attempt to extend the regions of contact between three ultimate theories which have all been widely developed, but in such a way as not to have much connection to one another. These theories are Maxwell's theory of electric phenomena..., Lord Kelvin's vortex atom theory of matter, and the purely dynamical theories of light and radiation that have been proposed by Green, MacCullagh, and other authors.

87. Larmor, "A dynamical theory of the electric and luminiferous medium. Part III: Relations with material media," *PT* (1897), also in *LaMPP2*, 11–132, on 17.

88. Moreover, Heaviside had shown in 1891 that no additional dissipative terms in the equations of the rotational ether could represent conductivity. See Buchwald (ref. 1), 68–70, and Hunt (ref. 1), on 213–214.

89. Larmor (ref. 76), *LaMPP1*, 323–324.

90. Larmor (ref. 74), 411. See Hunt (ref. 1), 212–213, and Buchwald (ref. 1), chap. 16.

By the end of the year Larmor had a first sketch of the new theory, which he published as a summary of a forthcoming "dynamical theory of the electric and luminiferous medium."

A precondition for developing the theory was that permanent vortices should be able to exist in the rotational ether, as they do in Helmholtz' ideal fluid. Around a vortex, the circulation of the velocity $\mathbf{B}/\mu$ of the medium is not zero. This implies, in electrical terms, a displacement current in every section of the vortex, so that the elastic state of the medium cannot be stationary, and the vortex cannot be permanent. In order to avoid this difficulty, Larmor assumed that permanent vortices formed "faults" in ether. The vortices had to be hollow, or, if they were not, the medium had to lose its rotational elasticity along their axes. In brief, permanent vortices could exist if there were a proper linear discontinuity in the medium. Then atoms could be represented by vortex rings as in Kelvin's model.[91]

Larmor used the vortex atoms to discuss the relation between ether and matter in optics. For optical refraction he offered the following explanation: "The presence of vortex atoms, forming faults, so to speak in the aether will clearly diminish its effective rotational elasticity; thus it is to be expected that the specific inductive capacity of material dielectrics should be greater than the inductive capacity of a vacuum." Beyond this qualitative picture Larmor had no detailed microscopic theory. The same limitation is found in his theory of dispersion. In this case he accounted for the relevant effect of molecular vortices by simply modifying the macroscopic expression of the potential energy of the medium, with terms including higher derivatives of ξ. Unlike Sellmeir's or Lorentz' theories of dispersion, Larmor's did not treat the atoms as separate dynamical entities, and it could not explain resonant absorption nor anomalous dispersion. Larmor recognized the superiority of theories "of the Young-Sellmeir type" in this regard, but made no attempt to emulate them.[92]

Larmor also discussed the optics of moving bodies. In his opinion, the vortex-atom theory implied that ether should be stationary, as assumed by Fresnel. Indeed, it was known that a vortex ring in an ideal incompressible fluid, if not too small, could move freely in the surrounding fluid. If the ether also has rotational elasticity, the fluid motion outside the vortices remains the same since it is irrotational. By itself this reasoning suggested that ether would freely flow between the atoms of matter. Larmor also argued that a global dragging of ether would imply a yet undetected

91. Larmor (ref. 74), *LaMPP1*, 400, 406. It seems likely that Larmor first introduced the hollow vortices in trying to represent an electric current and then thought of using them to build vortex atoms.

92. Larmor (ref. 74), *LaMPP1*, 406–407; "A dynamical theory of the electric and luminiferous medium. Part I," *PT*, *185* (1894), 719–822, also in *LaMPP1*, 414–535, on 438–443.

magnetic field, since in his theory magnetic induction was the momentum of the medium. The only escape from absurdity was to assume a very low ether density. But in that case, the velocities of ether implied by ordinary magnetic fields would be very high, which raised difficulty: the magnetic field of magnets would deviate the path of light rays; also the velocity of ether in a magnetized piece of matter would be high, which would seem to imply a strong dependence of the magnetic permeablility μ on chemical composition, at variance with the fact that the permeability of most substances is nearly the same as for vacuum.[93]

In the following months, Oliver Lodge verified that a strong magnetic field had no effect on the propagation of light, which implied, in Larmor's theory, that material bodies could not drag ether. As a more direct consequence, the density of ether had to be comparable to that of ordinary matter, which did not bother Larmor more than other strange properties of ether, because, he repeated, "ether is an intangible medium and is not apparently amenable in any way to direct perception."[94]

Having proved the stationarity of ether, Larmor addressed the justification of Fresnel's coefficient. Here again the vortex ring was too vague or too complex to provide guidance. "The nature of the further slight alteration of...elasticity produced by a motion of the matter as a whole," Larmor deplored, "there appears to be no means of exactly determining." The best he could do was to show by questionable reasoning that a violation of Fresnel's formula, and the corresponding violation of the ordinary laws of reflexion and refraction in moving bodies, would imply a violation of the second law of thermodynamics. Larmor offered a similar thermodynamical argument about Michelson's experiment and also suggested a special alteration of wave-length, other than the normal Doppler effect, in the reflection of light by a moving mirror. He did not mention the contraction hypothesis, although he was in contact with FitzGerald, who reviewed his manuscript. In short, the best Larmor could do for the optics of moving bodies was to show that his microscopic theory, as far as it was definite, did not contradict standard experiments, and that general macroscopic principles, especially the second law of thermodynamics, perhaps implied the results of these experiments.[95]

93. Larmor (ref. 74), *LaMPP1*, 391; Larmor (ref. 92), *LaMPP1*, 476–478. Larmor also mentioned Lorentz' argument against Stokes' theory (ibid., 478–479), but did not regard it as definitive, because Stokes's concept of the fully dragged ether was more complex than Lorentz assumed.

94. On Lodge's experiments see Hunt (ref. 1), 214–215; Larmor's reactions are in ref. 74, *LaMPP1*, 413, and ref. 92, *LaMPP1*, 483–484.

95. Larmor (ref. 92), *LaMPP1*, 476; for the thermodynamic arguments see ibid., 479–480, 482. Cf. Warwick (ref. 3), 37–40.

In order to deduce electromagnetic phenomena from the rotational ether, Larmor had to face the difficulty that, according to Maxwell, the electric charge of a conductor is equal to the flux of **D** across a surface surrounding the conductor. But if **D** is the curl of the linear displacement of the medium, its flux accross any closed surface must be zero, even if the linear displacement is not everywhere defined within the surface. Larmor did not let such mathematical paradoxes discourage him. He argued: "The legitimate inference is that the electric displacement [**D**] in the medium which corresponds to an actual charge cannot be set without some kind of discontinuity or slip in the linear displacement [ξ] of the medium; nor can it lose a charge once received without a similar rupture." This way of thinking was in harmony with Maxwell's view, according to which a conductor could not sustain electric displacement, and the charging or discharging of the conductor required a conducting path, that is, a breakdown of the elastic property of the surrounding dielectric.[96]

Some obscurity remained regarding the definitions of ξ and **D** after the charging and the corresponding rupture of ether have ended. To clarify this point, I will define $\xi(\mathbf{r},t)$ globally, as the position occupied at time t by the particle of the medium that, in a previous relaxed reference state of the medium, was in the position **r**. This implies that around a charged conductor ξ is only defined through the charging process and is therefore ambiguous. Suppose that the charging occurs through a filamentary current along the line l. Then the function ξ will have a singularity on l. When the filament is withdrawn, this singularity remains, but becomes a mathematical artifact: it results from our reckoning linear displacement from a relaxed state that the system can no longer take. From a more physical point of view, the repaired medium must have homogeneous properties, and its physical state should not depend on the choice of the line l. In fact, what characterizes the physical state is **D**, the curl of ξ, which is continuous despite the singularity of ξ. Consequently, the elastic energy D^2/ε is unambiguous.

The more mathematically inclined reader might prefer to see the meaning of ξ independent of any ether surgery. In the space around the charge, the displacement **D** is divergenceless. Therefore, it can *locally* be written as the curl of a certain quantity ξ. This property, by itself, allows the interpretation of **D** as a rotational strain. The definition of the linear elastic distorsion ξ needs only to be local, and the flux of **D** across a surface surrounding the charge does not have to vanish.

96. Larmor (ref. 74), *LaMPP1*, 398–399 (insertion dated 7 Dec 1893). See Buchwald (ref. 1), 143–150.

Having thus defined an electric charge, Larmor proceeded to electric currents. A vortex in the rotational ether, he observed, would be surrounded by closed lines of magnetic induction, since the induction is proportional to the velocity of the fluid. This connection suggested that an electric current was nothing but a macroscopic vortex. In absence of any fault in the medium, the vortex and the corresponding current could only be transitory, but a channel of elastic breakdown along the axis of the vortex could secure a permanent current, as earlier discussed.[97]

As Larmor knew from Helmholtz, the kinetic energy of a system of vortices has the same form as the energy of a system of linear currents as given by Neumann's formula. This equivalence is easily seen by introducing for each vortex a tubular surface S to which **B** is tangent, and the potential **A** such that $\mathbf{B} = \nabla\times\mathbf{A}$. The kinetic energy of the medium is

$$T = \int\left(\frac{\partial\xi}{\partial t}\right)^2 d\tau = \int B^2 d\tau = \int \mathbf{B}\cdot(\nabla\times\mathbf{A})d\tau \qquad (94)$$

or, after integration by parts,

$$T = \int_V \mathbf{A}\cdot(\nabla\times\mathbf{H})d\tau + \Sigma\int_S (\mathbf{A}\times\mathbf{H})\cdot d\mathbf{S}, \qquad (95)$$

where V is the volume of the medium outside the tubes S, and the sum is taken over the various linear currents. For stationary or quasi-stationary currents the volume integral vanishes since the displacement current is negligible.

The remaining surface integrals may be rewritten as

$$\int_S (\mathbf{A}\times\mathbf{H})\cdot d\mathbf{S} = \int_S (\mathbf{H}\times d\mathbf{S})\cdot\mathbf{A} = \int I\mathbf{A}\cdot d\mathbf{l}, \qquad (96)$$

if I represents the circulation of **H** around a section of the tube S. According to the latter definition of I, **A** is just the vector potential of a linear current with the intensity I, so that the kinetic energy T is identical with Maxwell's electrokinetic energy for a system of linear currents. On this basis, and by dynamical reasoning similar to Maxwell's, Larmor deduced the current expressions for the electromotive and ponderomotive forces acting in and on electric circuits.[98]

For the ponderomotive forces between two electrically charged bodies, Larmor applied the energy principle. When one of the two charges is moved slowly, the elastic energy of the medium varies without any additional kinetic energy. If no frictional dissipation is involved, a Coulomb force must act on the conductor in order to compensate this variation.

97. Larmor (ref. 74), *LaMPP1*, 399; ref. 92, *LaMPP1*, 454–455.
98. Larmor (ref. 74), *LaMPP1*, 400; ref. 92, *LaMPP1*, 457–465.

Larmor made a considerable effort to give an intuitive understanding of the origin of electrostatic forces. The stress in the rotational ether could not be used to this end, for it is a linear function of **D** and is responsible only for the linear displacement ξ of the medium. There cannot be any direct connection between this stress system and the motion of matter, which is represented by an agglomerate of atomic faults in the medium. Hence Larmor's theory had no room for Faraday's stresses. The origin of electrostatic forces had to be dynamical. Larmor imagined that the motion of a conductor implied an encroachement of the surrounding medium, that is, that the superficial molecules of the conductor dissolved the strain of the portions of ether they crossed. During this dissolution, wavelets of elastic rearrangement had to be emitted; the mechanical force was nothing but the dynamical reaction to this emission process. In short, Larmor's theory rested neither on action at a distance, nor on contact action. It was a theory of dynamically propagated action.[99]

Troubles and electrons

Very soon Larmor encountered serious difficulties with his theory. His worries, which in good measure reflected FitzGerald's criticism, appeared between added parentheses, first in the summary of December 1893, then in the memoir of June 1894. The common source of all troubles was the concept of a vortex in the rotational ether.[100]

First, Larmor's theory inherited the problems of Kelvin's theory of vortex atoms. Thermal agitation within the medium implied a continuous spectrum of sizes for the molecular vortices, which made it difficult to understand why the emission spectrum of a given element was strictly independent of temperature. Larmor proposed that molecules could be designed in such a way that the rotation frequency of atomic charges would be independent of the energy of the molecule. A proper combination of charged vortices would explain the known properties of molecules, including chemical binding. This was a loose speculation even for Larmor, the more so because, as he feared by December 1893, charged vortices could well be unstable.[101]

A more general anomaly appeared to Larmor. According to a well-known theorem by Helmholtz, the strength of a vortex ring in an ideal incompressible fluid is invariable. Consequently, all closed currents should

99. Larmor (ref. 92), *LaMPP1*, 451–453. Other Maxwellians could not make sense of Larmor's dissolution picture; Hunt (ref. 1), 217–218.

100. On FitzGerald's role, see Hunt (ref. 1), 216, and Buchwald (ref. 1), part 3.

101. Larmor (ref. 74), *LaMPP1*, 406–407.

be invariable in Larmor's theory. The property nicely represented Amperean molecular currents in magnets, but made a problem for macroscopic currents, as Larmor realized by December 1893. He proposed the following escape: "Ordinary currents must. . .be held to flow in incomplete conducting circuits, and to be completed either by convection across an electrolyte or by electric displacement or discharge across the interval between the molecules." A generator, sliding contacts, or a voltameter could make the necessary discontinuity in a macroscopic circuit. The medium's strain could gain a purchase on an incomplete circuit and, perhaps, produce the expected induction phenomena.[102]

For microscopic currents, strict permanence was also problematic because it contradicted a popular explanation of diamagnetism. According to Weber, negative magnetic susceptibility could be explained as a microscopic induction phenomenon affecting molecular currents. There were, however, alternative explanations of diamagnetism so Larmor was not yet constrained to give up the molecular vortices.[103]

Kelvin struck the most damaging blow to the idea of vortex currents when he reminded Larmor that the analogy between vortex filaments and linear current was imperfect: two parallel vortices with the same sign repel each other, while the corresponding currents attract each other. By June 1894 Larmor realized that general dynamical considerations made the sign problem inescapable. Being invariable, the strength of a vortex had to be treated as a generalized momentum, not as a velocity. Consequently, Maxwell's dynamical method for linear currents did not apply. In the proper modified method, the kinetic energy T, not $-T$, acted as the potential for ponderomotive forces.[104]

In the case of macroscopic currents, Larmor's previous introduction of breaches, which the elasticity of the medium could grab hold of, might solve the difficulty. But there remained a crude contradiction for the forces between magnets, to which the hypothesis of microscopic vortex currents gave the wrong sign. Here Larmor noted that Kelvin's objection only applied to vortices containing a solid core, on which the fluid pressure could act. If the vortex rings representing Amperean currents were hollow vortices, Kelvin's expression of the force between vortices no longer applied.[105]

102. Larmor (ref. 74), *LaMPP1*, 400–401.

103. Larmor (ref. 92), *LaMPP1*, 468. Larmor referred to Thomson's explanation of diamagnetism in analogy with the percolation of a fluid through a porous medium; ibid., 477–478.

104. Larmor (ref. 92), *LaMPP1*, 504.

105. Ibid., 506.

Larmor went on to imagine a complex statistical mechanism involving thermal rotary fluctuations of the Amperean vortices, by which the average, measurable force between magnets would take the right sign. This mechanism, according to Larmor, had another virtue: it provided a microscopic picture for the induction in a conductor moving in the field of a permanent magnet. It would overcome the difficulty that in a strictly constant magnetic field no electric force existed to produce the induction.[106]

The complexity and opacity of these arguments turned Larmor's theory into a baroque monster. As Buchwald puts it, in the elaboration of his vortex theory Larmor "replaced contradictions with mysteries." Under FitzGerald's pressure, Larmor grew more and more hesitant, and formulated the latest "improvements" of his theory in the conditional mode.[107]

In two months Larmor solved all difficulties by giving up the vortices and introducing a new subatomic particle, the electron. He had already made a first step in this direction in the discussion of electric charging and discharging in an addendum of June 1894 to the memoir published that year. There Larmor judged his previous introduction of a disruption in the elasticity of ether as "the most unnatural feature of the present scheme" and proposed to regard electric flow, whenever possible, as a convection of charged atoms.[108] In other words, he supposed that nearly all macroscopic currents might be similar to the ionic currents within an electrolyte. The exception was the current occurring at the electrodes of the voltameter, which involved a charge or discharge of molecular ions. Earlier, Larmor had tried to explain the electrode current, and the discreteness of the involved charges, by imagining special paths of disruptive charge between two atoms.[109] He now proposed "a more fundamental view." All atoms and molecules had to be made of an elementary "monad." A monad was a point singularity in ether that carried a unit charge, with positive or negative sign. The idea of a primordial atom or protyle was not original, and had often been proposed as an explanation for the limited number of elements existing in the universe. In Larmor's eyes it had another virtue: it made the charge or discharge of an ion a transfer of monads.[110]

In another addendum of June 1894, Larmor replaced atomic vortices with circular, rotating chains of "small charged bodies." The resulting convection currents were no longer invariable and could be influenced by induction, as necessary in Weber's explanation of diamagnetism. A more

106. Ibid., 506–508. Cf. Buchwald (ref. 1), 155–159.

107. Buchwald (ref. 1), 160; Larmor (ref. 92), *LaMPP1* on 508: "This explanation, *if* valid, would carry with it. . . ."

108. Ibid., 475.

109. Larmor, *LaMPP1* (ref. 74), 406.

110. Ibid., 475.

important remark appeared at the beginning of a later addendum, of 13 August 1894: microscopic convection currents interacted like ordinary currents, so that they could play the role of Amperean currents in magnets.[111]

By that date, Larmor had given up the idea of vortex currents and vortex atoms. He now tried to build matter out of monads, which he renamed "electrons" after Stoney. The electrons were centers of intrinsic radial twist in the medium. They could be mentally constructed by the kind of ether surgery earlier practiced for the charge of a conductor: remove a filament of the medium ending at the position of the future electron; rotate the walls of the resulting cylindrical cavity; refill the cavity homogeneously. As in J.J. Thomson's and Heaviside's treatments of electric convection, the energetic properties of the field around a moving electron implied an inertial mass. Larmor took this mass to be the whole mass of an electron, in conformity with the primacy of ether.[112]

The electron theory: first sketch

In the new picture, all electromotive and electrodynamic actions had to be understood as interactions between electrons mediated via ether. Accordingly, Larmor first determined the forces acting between two moving electrons. He worked in the quasi-stationary approximation, for which the elastic strain around an electron travels together with the electron. The velocity of the medium around an electron carrying the charge e and moving with the velocity $\mathbf{v}$ is

$$\frac{\partial \xi}{\partial t} = \mathbf{B} = \frac{1}{4\pi} e\mathbf{v}\times\nabla\frac{1}{r}, \tag{97}$$

and the kinetic energy of the medium around two electrons has the form

$$T = \frac{1}{2}mv^2 + \frac{1}{2}m'v'^2 + Meve'v', \tag{98}$$

$$\text{where}\quad m = e^2/6\pi a\,,\quad m' = e'^2/6\pi a'\,, \tag{99}$$

$$\text{and}\quad M = \frac{1}{4\pi r}\cos(\mathbf{v},\mathbf{v}') + \frac{1}{8\pi}\frac{\partial^2 r}{\partial s \partial s'}. \tag{100}$$

In these formulae a represents the radius of the first electron, and s the curvilinear abscissa along the trajectory of the first electron; the accented variables refer to the second electron; and r denotes the distance between the

111. Ibid., 468, 515.

112. Ibid., 516, 520.

two electrons. To obtain the forces between two electrons, Larmor applied Lagrange's equations to the Lagrangian T.[113]

Larmor's electrodynamics of macroscopic currents rested on the assumption that a conduction current was nothing but a dense circulation of electrons (positive or negative). Larmor did not sum the forces acting on individual electrons, which would have been natural; instead he derived Maxwell's Lagrangian for a system of currents by summing the contributions of all relevant electrons to the kinetic energy of the medium. The contribution of the kinetic energy to the coupling between a current $\mathbf{j}$ and an external closed current $\mathbf{j}_0$ turns out to be

$$L = \int \mathbf{j}\cdot\mathbf{A}_0 d\tau, \text{ with } \mathbf{A}_0 = \frac{1}{4\pi}\mathbf{j}_0 * \frac{1}{r}, \tag{101}$$

since the second term of (100) does not contribute.[114]

According to Maxwell, this Lagrangian gives the force density

$$\mathbf{f} = \mathbf{j}\times(\nabla\times\mathbf{A}_0), \tag{102}$$

and the electromotive force

$$\mathbf{E} = -\frac{\partial \mathbf{A}_0}{\partial t} - \nabla\psi. \tag{103}$$

Larmor retained the latter formula, but contested Maxwell's expression for electrodynamic forces. In his opinion $\mathbf{j}d\tau$ was a "physical current element" that followed the current carrier during deformations and controlled the energy $\mathbf{j}\cdot\mathbf{A}_0 d\tau$. Consequently, for a displacement $\delta\mathbf{r}$ of this element, he wrote the relation

$$\delta(\mathbf{j}\cdot\mathbf{A}_0 d\tau) = \mathbf{j}d\tau\cdot(\delta\mathbf{r}\cdot\nabla)\mathbf{A}_0 = \mathbf{f}d\tau\cdot\delta\mathbf{r}, \tag{104}$$

which implies

$$\mathbf{f} = \mathbf{j}\times(\nabla\times\mathbf{A}_0) + (\mathbf{j}\cdot\nabla)\mathbf{A}_0. \tag{105}$$

The second term, a tension along the electric circuit, was unknown to Maxwell's theory, and prompted reactions from more faithful Maxwellians, as we will see.[115]

Except for this new tension, the new electronic conception seemed to be able to reproduce standard electrodynamics without any rupture of the medium other than the original creation of electrons. It also furnished a microscopic explanation of diamagnetism, ferromagnetism, the Hall effect

113. Ibid., 521–523.
114. Ibid., 528–529.
115. Ibid., 529.

and, possibly, for the constitution of atoms. Larmor hoped to find in J.J. Thomson's researches on cathode rays a confirmation of the existence of electrons. However, the characteristics of the electrons remained vague. To play the role of "monads" they could not exist with many different masses and their mass had to be a small fraction of a typical atomic mass. But Larmor said nothing precise in this regard.[116]

More urgently, Larmor had to show that the new conception accounted for optical phenomena at least as well as MacCullagh's original theory did. His discussion of refraction and dispersion was purely verbal and obscure. Electrons, being centers of pure strain, altered the effective elasticity of the medium without changing its inertia, as the vortex atoms had done before. MacCullagh's theory of reflection and refraction was thus retrieved at the macroscopic level. Also Larmor's previous formal theory of dispersion could be maintained, since it did not depend on the precise entity, vortex or electron, that altered the elasticity of the medium. Again, Larmor pointed to the necessity of a theory of Sellmeir's type for explaining absorption and anomalous dispersion, of which he vaguely foresaw an electric formulation.[117]

Larmor was slightly more specific about the optics of moving bodies. He distinguished between the rotational strain $\mathbf{D}_1$, "which belongs to the waves and provides the stress by which they are propagated," and the rotational strain $\mathbf{D}_2$ "due to orientation of the molecules." Between the two strains he assumed the relation

$$\mathbf{D}_1 + \mathbf{D}_2 = \varepsilon\mathbf{D}_1, \tag{106}$$

which retrieves Maxwell's electrostatics if $\mathbf{D}_1$ plays the role of Maxwell's $\mathbf{E}$ and $\mathbf{D}_1 + \mathbf{D}_2$ plays the role of Maxwell's $\mathbf{D}$. For a transparent body moving with the velocity u along the x axis, and for a wave propagating along the same axis, he wrote

$$\frac{\partial^2\mathbf{D}_1}{\partial t^2} + \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)^2\mathbf{D}_2 = c^2\frac{\partial^2\mathbf{D}_1}{\partial x^2}. \tag{107}$$

Clearly, he supposed that the convective derivative in front of $\mathbf{D}_2$ would take into account the fact that the corresponding strain was bound to the molecules of the transparent body. To first order in u/c the resulting propagation velocity is

116. Ibid., 256 (Hall effect), 516 (atoms), 523, 524n (cathode rays).
117. Ibid., 530–533, 533n.

$$V = \frac{c}{n} + u\left(1 - \frac{1}{n^2}\right), \text{ with } n = \sqrt{\varepsilon}, \qquad (108)$$

in conformity with Fresnel's dragging formula.[118]

Larmor's reasoning was almost certainly independent of Lorentz' and Reiff's previous derivations of Fresnel's coefficient.[119] In conformity with his reductionist endeavor, Larmor reasoned entirely in terms of ether strains. Formally, Lorentz' polarization (P) played the same role as Larmor's $\mathbf{D}_2$, but in terms of electronic shifts, not of ethereal displacements. There are other differences. Larmor's proof, unlike Lorentz' of 1892, was entirely macroscopic and therefore much simpler. But it did not provide a precise justification of the convective derivative in front of $\mathbf{D}_2$. For this he would have needed a physical interpretation of the time derivatives entering the wave equation. Most probably, Larmor reasoned in analogy with Glazebrook's earlier derivation of Fresnel's coefficient in a loaded mechanical ether, which he presented in parallel with his own derivation. Larmor's derivation of the Fresnel coefficient was the shortest, but also the loosest, available.

Drawing on Lorentz

So far Larmor's theory had evolved independently of other ionic theories. In the winter of 1894/95, he became aware of Helmholtz' second dispersion theory and of Reiff's subsequent derivation of Fresnel's formula; most important, he read Lorentz' *Versuch*.[120] In a second part of "A dynamical theory of the electric and luminiferous medium" entitled "Theory of electrons," Larmor drew heavily on these works, although he maintained the basic idea of a rotational ether from which everything, including electrons, had to be built. Larmor's claim that his work remained largely independent should not be accepted. As we will see, the dramatic improvement of his theory, from a rough and partially misconceived scheme to a precise deductive theory, owed much to Lorentz' insights.[121]

In one change, Larmor gave up the idea of a macroscopic Lagrangian and adopted Lorentz' procedure, according to which macroscopic forces and macroscopic field equations had to be derived by averaging microscopic relations over a macroscopic volume element. Schematically, the pre-Lorentz Larmor used the scheme

Micro-Lagrangian → Macro-Lagrangian → Macroscopic actions.

The new Larmor preferred

Micro-Lagrangian → Microscopic actions → Macroscopic actions.

118. Ibid., 534.

119. Lorentz (ref. 24); R. Reiff, "Die Fortpflanzung des Lichtes in bewegten Medien nach der elektrischen Lichttheorie," *AP*, *1* (1893), 361–367.

120. Cf. Warwick (ref. 3), 54.

121. Larmor, "A dynamical theory of the electric and luminiferous medium. Part II: Theory of electrons," *PT*, *186* (1895), 695–743; also in *LaMPP1*, 543–597.

A first aspect of this change concerned "physical current elements." Having learned form Lodge and Fitzgerald that the predicted tension in current-carrying conductors did not exist,[122] Larmor condemned macroscopic dynamical considerations and derived all forces by averaging elementary electronic actions. Like Lorentz, he took for the force acting on the carrier of a conduction current $\mathbf{j}$, the sum of the magnetic forces $e\mathbf{v}\times\mathbf{B}$ acting on the electrons of a volume element. That gave $\mathbf{j}\times\mathbf{B}$, since $\mathbf{j}$ is the algebraic average of all $e\mathbf{v}$'s.[123]

Another of Lorentz' ideas was to write macroscopic field equations. Previously Larmor had written field equations only for the ether between electrons, because electrons were faults in ether; his calculation of the magnetic field or ether velocity around a moving electron was based on the picture of a traveling center of radial rotational strain. Larmor now found that the magnetic field around a moving electron was the field generated by the "total current" obtained by adding to the displacement current outside the electron the "convection current," that is, the product of the charge of the electron by its velocity. For Larmor, the convection current was only "a kinematic fiction" because it occurred at a fault in the medium. This view differed from Lorentz', according to which the convection current was neither more nor less real than the displacement current, and also from Maxwell's, which dealt only with continuous media and made the convection current part of the displacement current. Nevertheless, the formal purpose was the same: to complete the first circuital equation in a consistent manner.[124]

Thanks to the convection current, Larmor could average over volume elements containing a large number of electrons and obtain the macroscopic circuital equation,

$$\nabla\times\mathbf{H} = \mathbf{J}, \qquad (109)$$

wherein $\mathbf{H}$ is the average magnetic force and $\mathbf{J}$ is the average total current. In general, the average convection current is made of three parts: the first represents the convection of the total charge $\rho - \nabla\cdot\mathbf{P}$ at the global velocity

122. Cf. Hunt (ref. 1), 223–228.

123. Larmor (ref. 121), *LaMPP1*, 577–578.

124. Ibid., 556.

$\mathbf{u}$; the second, $\mathbf{j}$, represents the conduction current; and the third, $(\partial/\partial t + \mathbf{u}\cdot\nabla)\mathbf{P}$, represents the polarization current. Altogether, Larmor had

$$\mathbf{J} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + (\rho - \nabla\cdot\mathbf{P})\mathbf{u} + \mathbf{j} + (\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla)\mathbf{P}, \tag{110}$$

where $\mathbf{E}/c^2$ is the average of the microscopic electric displacement. Between $\mathbf{E}$, $\mathbf{P}$, and ρ, the relation

$$\nabla\cdot\frac{\mathbf{E}}{c^2} = \rho - \nabla\cdot\mathbf{P} \tag{111}$$

obtains so that at the macroscopic level the vector

$$\mathbf{D} = \frac{\mathbf{E}}{c^2} + \mathrm{P} \tag{112}$$

plays the role of Maxwell's displacement.[125]

When there is no magnetic polarization, the macroscopic formula for the electric field strength directly reflects its microscopic counterpart since the averaging procedure is trivial in this case. Larmor wrote

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\psi, \tag{113}$$

$$\text{with } \mathbf{B} = \mu\mathrm{H} = \nabla\times\mathbf{A}. \tag{114}$$

The electromotive force $\mathbf{F}$ in a moving conductor with the velocity $\mathbf{v}$ is not equal to the average field strength. Its value results from averaging the force acting on the electrons of the conductor, which yields:[126]

$$\mathbf{F} = \mathbf{E} + \mathbf{v}\times\mathbf{B}. \tag{115}$$

In order to complete the system of equations, Larmor needed a relation controling the evolution of the polarization $\mathbf{P}$. As a remnant of his older approach, he first wrote a macroscopic form, in Helmholtz' and Reiff's manner:

125. Ibid., 573. The general expression for the polarization current is

$$\mathbf{j}_P = \partial\mathbf{P}/\partial t + \mathbf{u}(\nabla\cdot\mathbf{P}) - \nabla\times(\mathbf{u}\times\mathbf{P}),$$

as Larmor noted in ref. 87, 31n, and in *Aether and matter: A development of the dynamical relations of the aether to material systems on the basis of the atomic constitution of matter* (Cambridge, 1900), 102–104.

126. Larmor (ref. 122), 572–573.

$$\mathbf{D} - \varepsilon\mathbf{E} = \alpha\frac{\partial^2\mathbf{P}}{\partial t^2}. \tag{116}$$

In the static case, this yields Maxwell's linear relation between displacement and electromotive force. In the dynamic case and for non-conducting bodies at rest, Larmor's system of equations is nearly the same as that in Helmholtz' theory of dispersion; Larmor had no difficulty recovering the Sellmeir-Helmholtz dispersion formula apart from the damping term.[127]

With the same kind of macroscopic reasoning, Larmor tried to derive the Fresnel formula. For the first circuital equation in a moving transparent body, he wrote:

$$\nabla\times\mathbf{H} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + (\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla)\mathbf{P}, \tag{117}$$

in conformity with the expression (110) of the total current. Then:[128]

> From the mode in which the other circuital relation appears in the dynamical theory of the medium, $d\mathbf{B}/dt$ must mean the total acceleration of the velocity of the ether, due in part to change of time and in part to movement of the material dielectric; thus this d/dt also, when it operates on $\mathbf{P}$, must be replaced by $\partial/\partial t + u\partial/\partial x$.

Based on this dubious argument, he wrote the wave equation:

$$c^2\Delta\mathbf{B} = \frac{\partial^2\mathbf{B}}{\partial t^2} + (\varepsilon - 1)(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x})^2\mathbf{B}, \tag{118}$$

which has the same form as equation (107), and therefore leads to Fresnel's coefficient.

Larmor also made a more rigorous derivation similar to Lorentz', except for the choice of the coordinate system. He now considered the microscopic mechanism by which polarization appeared in a dielectric. In the quasistatic approximation (dispersion neglected), he reached the expression

$$\mathbf{P} = (\varepsilon - 1)(\mathrm{E} + \mathbf{u}\times\mathbf{B}), \tag{119}$$

where the magnetic term is a consequence of the magnetic force acting on the polarization electrons, which move nearly at the velocity $\mathbf{u}$. The plane-wave solutions of equations (113), (114), (117), and (119) led to the Fresnel coefficient.[129]

127. Ibid., 557–558.

128. Ibid., 565.

129. Ibid., 575–577. The equations are the same as those used by Reiff to derive the Fresnel coefficient (ref. 119).

Larmor also dealt with the general insensitivity of optical phenomena to global motion and with the Michelson-Morley experiment. From Lorentz' memoir he borrowed the local time, the contraction of lengths, and the notion of corresponding states. The only difference concerned the justification of the Lorentz contraction. Lorentz had to suppose that molecu- lar forces transformed like electromagnetic ones. To Larmor the supposition was a consequence and a proof of the fact that all matter was made out of singularities in the same ether.[130]

From a British point of view, Larmor's theory had two other major advantages, it was based on an explicit picture of ether and it was thoroughly dynamical. No doubt, Larmor had had to give up the idea of a macroscopic dynamical approach based on current elements. But the dynamical foundation of MacCullagh's ether remained, and that of the elec- trons' motion could be improved. Larmor offered a derivation of the Lorentz force which formally resembled Heaviside's earlier treatment of charge convection and, more closely, the dynamical foundation that Lorentz gave to his own theory in 1892.[131] However, Lorentz and Larmor used the dynamical method for different purposes. For Lorentz, the method provided a mechanical foundation for the coupling between ether and electrons without specification of the structure of ether or the coupling mechanism. For Larmor, the method was a means to constrain a preexisting picture and to extract from it more precise mathematical relations.

Larmor's new dynamical method improved on his previous derivation of the forces between two electrons, which was limited to low velocity. The starting point was still the expression for the kinetic energy of the medium,

$$T = \frac{1}{2}\int\left(\frac{\partial\xi}{\partial t}\right)^2 d\tau = \frac{1}{2}\int B^2 d\tau. \tag{120}$$

Introducing the auxiliary vector **A**, such that $\mathbf{B} = \nabla\times\mathbf{A}$, an integration by parts yields

$$T = \frac{1}{2}\int \mathbf{A}\cdot(\nabla\times\mathbf{B})d\tau = \frac{1}{2}\int\mathbf{A}\cdot\mathbf{J}d\tau. \tag{121}$$

130. Larmor (ref. 121), *LaMPP1*, 566. That Larmor relied on Lorentz is clearly stated in Larmor to Lodge, 29 May 1895, quoted in Warwick (ref. 3), 56: "I have just found, develop- ing a suggestion by Lorentz, that if there is nothing else than electrons..., then movement of a body...through the ether *does actually* change its dimensions." I believe that Larmor's and Lorentz' interpretations of the "corresponding states" were identical. For a different view, cf. Warwick, ibid., 63.

131. O. Heaviside, "On the electromagnetic effects due to the motion of electrification through a dielectric," *PM* (1889), also in Heaviside, *Electrical papers* (New York, 1892), 2, 504–518; Lorentz (ref. 24).

In an anachronistic notation, the microscopic total current **J** is

$$\mathbf{J} = \frac{\partial\mathbf{D}}{\partial t} + \sum_i e_i\mathbf{v}_i\delta(\mathbf{r}-\mathbf{r}_i), \tag{122}$$

where the term i of the sum indicates the contribution of the charge e_i with the position $\mathbf{r}_i$ and the velocity $\mathbf{v}_i$. For the part of the energy dependent on the motion of a given electron, Larmor gave

$$T_i = \frac{1}{2}m_i v_i^2 + e_i\mathbf{v}_i\cdot\mathbf{A}(\mathbf{r}_i), \tag{123}$$

where m_i, the electromagnetic mass, depended on the structure of the singu- larity representing the electron.[132]

In the sequel, Larmor regarded **A** as the potential of the magnetic field of the other electrons and considered no self-interaction effect other than the electromagnetic mass. As he later acknowledged, this could be a valid approximation only if the electron under consideration emitted no radia- tion.[133] Otherwise, the value of T_i in equation (123) proved sufficient for most purposes.

To derive the equations of motion of electrons and ether Larmor used the effective Lagrangian

$$L = T - \frac{1}{2}\int\mathbf{E}\cdot\mathbf{D}d\tau + \int\psi\nabla\cdot\mathbf{D}d\tau - \sum_i e_i\psi(\mathbf{r}_i). \tag{124}$$

The second term is the potential energy of the medium; the two last terms, with the Lagrange parameter ψ, allow **D** and the position of the various electrons to be treated as independent variables despite the constraint

$$\nabla\cdot\mathbf{D} = \sum_i e_i\delta(\mathbf{r}-\mathbf{r}_i). \tag{125}$$

The variation with respect to **D** yields the equation of electromotive force:

$$\mathbf{E} = -\frac{\partial\mathbf{A}}{\partial t} - \nabla\psi. \tag{126}$$

For the variation with respect to $\mathbf{r}_i$, Larmor retained only the part T_i of T, that gives the equation of motion of an electron as:[134]

132. Larmor (ref. 121), 568.

133. Larmor, "On the theory of moving electrons and electric charges," *PM* (1896), also in *LaMPP1*, 615–618, on 618.

134. Larmor (ref. 121), 568–570. For an improved argument, see *Aether and matter* (ref. 125), chapt. 6.

$$m\ddot{\mathbf{r}} = e\left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla\psi + \mathbf{v}\times\mathbf{B}\right). \qquad (127)$$

The basic equations of Larmor's theory were now closely related to, but not quite the same, as Lorentz'. A first difference was that Larmor did not eliminate the potentials **A** and ψ, whose dynamical significance he wanted to stress. Another important difference concerned the self-interaction of the electron. In Lorentz' approach, the charge density of the electron was a meaningful concept, and permitted an exact calculation of electromagnetic inertia and radiation damping force. In Larmor's view, the electron was a fault in the rotational ether and its self-interaction was not clearly defined.

With his dynamical method, Larmor could isolate the self-mass but had no direct access to the radiation damping force. He did not mention the existence of this force in his memoir of 1895; and some of his earlier remarks on atomic electrons suggest that he initially overlooked the generality of radiation damping. He wrote: "A system of electrons moving steadily across the medium, or rotating steadily round a centre would thus carry a steady configuration of strain along with it; and no radiation will be propagated away except when this steady state of motion is disturbed." In 1896, replying to a criticism by Morton, he acknowledged that his expression of the force acting on an electron could not be complete and that it could only be a valid approximation when the electron emitted no radiation. But Larmor never gave the precise expression of the damping force, even though he was the first to determine exactly the rate of radiation by an accelerated electron.[135]

Vindication

Facing or anticipating criticism, Larmor struggled to make his theory appear necessary. For this purpose he offered an historical reconstruction based on the failure of the concept of the dynamical current element. This concept, he said, was a consequence of Maxwell's theory, if consistently developed; it had been contradicted by experiment; and this failure had "led to the introduction of the mobile electron." Here Larmor temporarily espoused the empiricist scheme of theory change. His reconstruction obviously did not jibe with the true sequence of events. What prompted Larmor's introduction of the electron was the failure of the vortex

135. Larmor (ref. 92) (Aug. 1894), *LaMPP1*, 517; ref. 133, *LaMPP1*, 618; "On the theory of the magnetic influence on spectra; and on the radiation from moving ions," *PM* (1897), also in *LaMPP2*, 140–149. See Warwick, "Frequency, theorem and formula: Remembering Joseph Larmor in electromagnetic theory," Royal Society of London, *Notes and records*, *47* (1993), 49–60.

conception of atomic currents; the idea of the dynamical current element developed only later.[136]

Moreover, Larmor's assertion that Maxwell's theory implied the unwanted electrodynamic tension in conductors cannot be maintained. Maxwell's dynamical method, if correctly extended to three-dimensional currents, leads to Ampère's electrodynamic forces without the Larmor tension. Larmor's conclusion followed from a mathematical error concerning the variation of a flux during a deformation of the medium.[137] Although the mistake does not seem ever to have been noticed, Larmor's "proof" that Maxwell's theory was incompatible with Ampère's electrodynamics sounded suspicious and did not convince Maxwellians.[138]

Similar comments can be made about another of Larmor's "proofs" of the failure of Maxwell's theory. He argued that Rowland's experiment, if performed with a uniformly charged rotating disk, as well as Röntgen's experiment of the rotating polarized dielectric, eluded Maxwell's theory, whereas the new electron theory offered a satisfactory explanation.[139] In fact, in his memoir of 1894, Larmor did not regard Rowland's and Röntgen's experiments as a threat to Maxwell's theory, nor to his own theory in its pre-electronic stage. Instead, he believed that Rowland's effect had only been established for disks with radial lines of division and that Röntgen's experiment was not conclusive.[140] Only after the introduction of electrons did Larmor present these experiments as a proof of the necessity of electrons.

As in the case of the Larmor tension, Larmor's argument was not only historically wrong but logically flawed. He took for granted that in the experiments with uniformly charged or uniformly polarized rotating disks the displacement field was unaltered by the rotation, so that there was no displacement current and no magnetic field. But it is not true in Maxwell's electrodynamics of moving bodies as completed by Hertz and Heaviside. Helmholtz and Hertz had shown that the motion of the medium in the transitory layer between the rotating disk and the unmoved part of ether implied displacement currents and therefore magnetic effects.[141] The presumable

136. Larmor, "A dynamical theory of the electric and luminiferous medium. Part III: relations with material media" (abstract), *PRS*, *41* (1897), 272–285; also in *LaMPP1*, 624–639, on 627.

137. Larmor assumed (see equation 104) that $\int \mathbf{J} \cdot d\mathbf{r}$ was invariant during a deformation of the medium, while the true invariant is $\int \mathbf{J} \cdot d\mathbf{S}$.

138. Cf. Hunt (ref. 1), 226.

139. Larmor (ref. 121), *LaMPP1*, 583. As Larmor later noted (ref. 87, 31n), the explanation of Röntgen's experiment requires a correction in the expression (110) of the polarization current.

140. Larmor (ref. 92), *LaMPP1*, 466–467.

141. Helmholtz, "Bericht betreffend Versuche über die electrodynamische Wirkung elektrischer Convection, ausgefürt von Hrn. Henry Rowland" (1876), in Helmholtz, *Wissenschaft-*

source of Larmor's misinterpretation was that in his own theory ether discontinuities occurred at the surface of the disk. Moreover, in the rotational ether there was no place for a motion of ether controlled by matter: the velocity of ether was nothing but the magnetic field.

Even though Larmor's proofs of his own theory were questionable, the idea of electrons as the basic constituents of matter had a good chance of acceptance. Ionic theories were growing more and more popular. What needed defense, was the rotational ether. As Larmor admitted, "it would be possible to ignore the existence of an aether altogether, and simply hold that actions are propagated in time and space from one molecule of matter to the surrounding ones in accordance with the system of mathematical equation which are usually associated with that medium." This was not, however, an attractive possibility to a British or Irish physicist. As Larmor wrote, "the idea of an aethereal medium supplies so overwhelmingly natural and powerful an analogy as for purposes of practical reason to demonstrate the existence of the aether."[142]

A concept of aether according to Larmor implied a simple, vivid picture that would "ease the intuitive grasp of relations." In the past the most common picture had been that of the elastic solid and all basic physical actions had been regarded as contact actions. Larmor knew that for many of his colleagues the failure to provide a picture of that kind would be perceived as a weakness; that is why he spent much time, both in the third part of his "dynamical theory" and in *Aether and matter*, justifying his departure from the elastic-solid approach. Ether, he argued, did not have to imitate any form of macroscopic matter, it had only to be a picturable, dynamic, unifying basis for all known phenomena:[143]

> The main problem of transcendental physics is to assign the nature of the ultimate medium or scheme of relations which combines physical phenomena into a unity, in whose relations [the fundamental] dynamical notions [relating inertia and force] have their scope: and it is only the prejudice of education that would keep, in this wider field, too close to the ideal of mechanical transmission in a homogeneous elastic solid.

Larmor improved the concrete model of the rotational ether and the model of the electron he had sketched before. He imagined an aggregation

liche Abhandlungen (Leipzig, 1882), *1*, 791–797; Hertz, "Über die Gleichungen der Elektrodynamik für bewegte Körper," *AP*, *41* (1890), 369–399, also in *Gesammelte Werke* (Leipzig, 1894–1895), 2, on 274–275.

142. Larmor (ref. 87), 13–14.

143. Larmor, "A dynamical theory of the electric and luminiferous medium. Part II: Theory of electrons" (abstract), *PRS*, *58*, (1895), 222–228; also in *LaMPP1*, 536–542, on 541; Abstract of "Part III: Relations with material media," *PRS*, *61* (1897), 272–285, also in *LaMPP1*, 625–639, on 629–630.

of frictionless solid spheres studded by a number of frictionless spikes. Such an aggregation behaved as a perfect fluid, the only effect of the spikes being to coordinate the rotations of contiguous spheres. Now, suppose that each sphere contains a gyrostat pivoted on a ring whose perpendicular diameter is itself pivoted on the inner walls of the sphere. Owing to the gyrostatic effect, the rotation of each sphere is elastically resisted. The rotation of a volume element of the fluid, containing a large number of such spheres, is also elastically resisted because, thanks to the studding, each sphere participates in this rotation. A medium thus constituted is perfectly fluid with respect to local translatory motion, but elastic with respect to local rotation, as befits a rotational ether.[144]

In this model, an electron pair is created by the carving of a thin channel AB in the medium. A rotation of a given angle around the axis of the channel is imparted to the channel's walls and the channel is filled again with studded spheres. In the new equilibrium configuration of the medium, the points A and B are permanent centers of radial twist representing a positive and a negative electron.[145]

As ingenious as it was, the model suffered from several defects. It could only represent ether during a finite stretch of time since the gyrostatic resistance to rotation implied a loss of energy of the flywheels. More embarrassingly, the model provided no intuitive explanation of the forces between electrons. These could only be derived by the abstract principle of stationary action. In order to explain how electric forces could be propagated through a medium without deriving from contact action, Larmor had nothing better to offer than a crude analogy with the forces between vortices in a perfect fluid. Yet, in his opinion, all these limitations of the models were natural, because ether, being the ultimate, unpalpable thing, could not be expected to behave like a macroscopic mechanism.[146]

As Larmor himself noted, an instructive comparison can be made with Maxwell's early cellular model of the electromagnetic field.[147] Maxwell's model was more literally mechanistic, for it involved a system of magnetic stresses (and, implicitly, a system of electric stresses). Nevertheless, Maxwell found his combination of elastic rotating cells and idle wheels too awkward and gave it up in favor of the more abstract Lagrangian method. Larmor's own model was hardly simpler than Maxwell's and offered no stress mechanism for the ponderomotive forces. Larmor regarded his handiwork as a compromise in which some fundamental physical relations

144. Larmor (ref. 87), 15–17.

145. Ibid., 17–19.

146. Ibid., 18, 20–23.

147. Ibid., 19.

received a mechanistic illustration while others could only be deduced by dynamical considerations. He disliked the idea of an abstract dynamical foundation in the style of Maxwell's *Treatise* and he disliked even more the idea of postulating fundamental equations, as Lorentz had done in his *Versuch*.

Larmor's attitude was not self-evident, even to British physicists weaned on mechanical illustrations. He felt the need of a philosophical justification of his concept of ether. For this purpose he seems to have taken a Kantian position. According to Kant, a physical theory is obtained by presenting the external world to our intuition and subsuming the resulting picture to our categories of understanding, like causality and permanence of substance. Newtonian mechanics was the archetype of this theory formation, since it applied dynamical principles to a system after determining its kinematics, that is, its spatio-temporal relations. Kant's conception easily extended to other parts of physics, if only a Newtonian mechanism, visible or not, underlay every phenomenon.

There was a more general possibility, in which a mechanism did not have to exist, but a spatio-temporal picture might still be available. What made up for the lack of mechanism was a more abstract dynamical principle like the principle of stationary action. Although Kant's name did not occur in his writings, Larmor clearly regarded his rotational ether as an enlargement of Kant's notion of intuition in the spririt of the reformers of geometry: "The sole spatial relations of the aether itself, on which its dynamics depend, those namely of incompressibility and rotational elasticity," he wrote, "are to be classed along with the existing Euclidian relations of measurements in space (which also might *a priori* be different from what they are) as part of the ultimate scheme of mental representation of the actual physical world."[148]

Larmor completed the picture by introducing point singularities in the rotational ether, and deduced all physical laws, no matter how complex, from the dynamics of the picture. The ordinary mechanics of concrete matter was no longer foundational, it was derived, together with optics and electromagnetism from the ultimate, dynamical medium: ether.

Unlike Kant, Larmor acknowledged the role of hypotheses or empirical data in shaping intuition. However, he believed, like Kant, in invariable laws of the mind: "The uniformities which it is customary to call laws of nature," he wrote, "are often just as much laws of the mind: they form an expression of the implication between mind and matter, by means of which material phenomena are mentally grasped." As evidence for this authority of the mind, he pointed to the formal analogies connecting very distinct

148. Larmor (ref. 87), *LaMPP2*, 24.

branches of physics and concluded with a Kantian aphorism: "The mind sees in all things that which it brings with it the faculty of seeing."[149]

In his own theory, he wished to present the concept of ether with point singularities as almost necessary. For example, he regarded, *post factum*, the introduction of electrons as a logical necessity commanded by the redefinition of space as ether: "As soon as we...cease to regard space as mere empty geometrical continuity, the atomic constitution of matter...is raised to a natural and necessary consequence of the new standpoint." In the same vein, he referred to his failed concept of dynamical current elements as a proof "that no other conception of electricity than the atomic one is logically self-consistent." Larmor intended to offer no less than a new transcendental aesthetics: "It might be held that this conception of discrete atoms and continuous aether really stands, like those of space and time, in intimate relation with our modes of mental apprehension, into which any consistent picture of the external world must of necessity be fitted."[150]

Improvements

Once Larmor had in hand the fundamental picture and equations of the electron theory, he bent his efforts to developing the consequences for forces acting on polarized matter, for the thermodynamics of electromagnetic systems, and for magneto-optical effects. Maxwell's account of the electromagnetic stresses in material media could not be maintained since they depended on the rejected picture of a single, continuous medium with variable permittivity and permeability. The electron hypothesis reduced all electromagnetic forces, including stresses within matter, to forces acting on the electrons. Thermodynamic effects naturally entered the picture, since a finite temperature involved a thermal agitation of the electrons. Many of Larmor's considerations about thermodynamics were innovative, but they have no place here.[151]

In his memoir of 1897 and in his *Aether and matter* of 1900, Larmor improved Lorentz' technique of corresponding states. He found that his conception of electrons as singularities in ether allowed a great simplification. In establishing the correspondence, neither the equation of motion of the electrons nor the source terms in the microscopic field

149. Larmor, *Aether and matter* (ref. 125), 70.

150. Ibid., 76; "On the ascertained absence of effects of motion through the aether, in relation to the constitution of matter, and on the FitzGerald-Lorentz hypothesis," *PRS*, *19* (1904), also in *LaMPP2*, 274–280, on 278; "The methods of mathematical physics" (British Association address, 1900), *LaMPP2*, 192–216, on 202.

151. Larmor (ref. 87).

equations needed to be considered. It was sufficient to find corresponding states for the free electromagnetic field. What allowed the simplification was "the completeness of the aethereal scheme," which Larmor took to imply that "the electron *taken by itself* must be in any conceivable theory a simple singularity of the aether whose movements when it is free, and interactions with other electrons if it can be constrained by matter, are traceable through the differential equations of the surrounding free aether alone."[152]

Belief in this completeness was an act of faith. The existence of a consistent theory of matter based on the rotational ether had not been proved. However, Larmor's simplification of the technique of corresponding states was a true gain. Consider a system of electrons belonging to the molecules of a body moving with the uniform velocity **u**. With respect to a system of axes attached to the moving body, the differential equations of the free ether in the space between the electrons read, in electrostatic units:

$$\nabla\times\mathbf{D} = -\frac{1}{c}\left(\frac{\partial}{\partial t} - \mathbf{u}\cdot\nabla\right)\mathbf{B}, \quad \nabla\times\mathbf{B} = \frac{1}{c}\left(\frac{\partial}{\partial t} - \mathbf{u}\cdot\nabla\right)\mathbf{D}. \qquad (128)$$

Suppose that a linear transformation

$$(\mathbf{r},t) \to (\mathbf{r}',t'),$$
$$\mathbf{E}(\mathbf{r},t),\ \mathbf{B}(\mathbf{r},t) \to \mathbf{E}'(\mathbf{r}',t'),\ \mathbf{B}'(\mathbf{r}',t') \qquad (129)$$

Larmor observed that the system enjoyed three properties:[153]

α) The equations $\nabla'\times\mathbf{E}' = -\frac{1}{c}\frac{\partial\mathbf{B}'}{\partial t'}, \quad \nabla'\times\mathbf{B}' = \frac{1}{c}\frac{\partial\mathbf{E}'}{\partial t'}$, (130)

hold whenever equations (128) hold for the corresponding fields **E** and **B**.

β) Wherever $\nabla\cdot\mathbf{D}$ vanishes, so does $\nabla'\cdot\mathbf{D}'$ at the corresponding point

γ) For any two surfaces closely surrounding corresponding singularities,

$$\oint\mathbf{D}\cdot d\mathbf{S} = \oint\mathbf{D}'\cdot d\mathbf{S}' \qquad (131)$$

The properties α) and β) imply that **D**′ and **B**′ satisfy the usual equations of the free ether except for point singularities; the property γ) implies that corresponding point singularities have the same strength. Then the following theorem holds: If the field **D**, **B** describes a possible evolution of the system of electrons in the moving body, the field **D**′, **B**′ will evolve with respect to **r**′ and t' as would the field of a system of electrons belonging to a body at rest.

152. Larmor, *Aether and matter* (ref. 125), 165, 171–172; cf. also (ref. 87), 41.
153. The conditions β) and γ) are found only in *Aether and matter* (ref. 125), 174–176.

Like Lorentz, Larmor regarded the two corresponding systems of electrons as identical except for their different velocity with respect to ether. For the stationary system of electrons constituting a solid body, the identity implied that the dimensions of the body and the period of oscillation of atomic electrons were altered by motion at the rates given by the transformation $(\mathbf{r}',t')\to(\mathbf{r},t)$. Accordingly, Larmor approved Lorentz' contraction of lengths and, more originally, predicted the dilatation of atomic periods, that is, the second-order Doppler effect.[154]

From 1897 on, Larmor's transformations of the coordinates of space and time, with the x-axis lying along **u**, were

$$x' = \gamma x,\quad y' = y,\quad z' = z,$$
$$t' = \gamma^{-1}t - \gamma ux/c^2, \qquad (132)$$

where

$$\gamma = (1 - u^2/c^2)^{-\frac{1}{2}}. \qquad (133)$$

This transformation is the same as the one applied by Lorentz in 1892 to the d'Alembertian operator save for a rescaling of time by the factor γ^{-1}. Once combined with the transformation from axes at rest to the moving axes, it gives the modern Lorentz transformation.[155]

For the fields, in 1897 Larmor replaced the transformation

$$\mathbf{D}' = \mathbf{D} + \frac{1}{c}\mathbf{u}\times\mathbf{B}, \quad \mathbf{B}' = \mathbf{B} - \frac{1}{c}\mathbf{u}\times\mathbf{D}, \qquad (134)$$

which Lorentz had given in 1892 and 1895, by

$$\mathbf{D}' = (\gamma^{-1},1)(\mathbf{D} + \frac{1}{c}\mathbf{u}\times\mathbf{B}),\ \mathbf{B}' = (\gamma^{-1},1)(\mathbf{B} - \frac{1}{c}\mathbf{u}\times\mathbf{D}). \qquad (135)$$

As shown by Lorentz in 1899 and by Larmor himself in 1900, the latter transformation satisfied the condition α) *exactly*, although Larmor initially restricted his attention to the second order in u/c.[156]

But the condition γ) is not met. Larmor mended this in 1900 by globally rescaling the field transformation according to

154. Larmor (ref. 87), *LaMPP2*, 41; *Aether and matter* (ref. 125), 179.
155. Larmor (ref. 87), 39–40; *Aether and matter* (ref. 125), 172–173.
156. Larmor (ref. 87), *LaMPP2*, 39.

$$\mathbf{D}' = (1,\gamma)(\mathbf{D} + \frac{1}{c}\mathbf{u}\times\mathbf{B}),\ \mathbf{B}' = (1,\gamma)(\mathbf{B} - \frac{1}{c}\mathbf{u}\times\mathbf{D}). \tag{136}$$

Because the strength of a singularity in the accented field does not change in time, it is sufficient to check that the condition γ) holds for an electron at rest with respect to the moving body. Then the corresponding singularity in the accented field is also at rest and the magnetic induction $\mathbf{B}'$ around this singularity is infinitely smaller than the displacement $\mathbf{D}'$. Using the transformation (136), we therefore have

$$D_x = D_x',\ D_y = \gamma D_y',\ D_z = \gamma D_z'. \tag{137}$$

Since the surface element transforms according to

$$dydz = dy'dz',\ dxdz = \gamma^{-1}dx'dz',\ dxdy = \gamma^{-1}dx'dy', \tag{138}$$

the product $\mathbf{D}\cdot d\mathbf{S}$ is the same as $\mathbf{D}'\cdot d\mathbf{S}'$ in the immediate vicinity of the corresponding singularities, so that the property γ) holds exactly.[157]

As Larmor noted,[158] the condition γ) determines the scale of the transformed field only if the coordinate transformation is given. Otherwise, a simultaneous rescaling of coordinates and fields is still possible, as Lorentz' showed in 1899. Lorentz managed to eliminate the resulting indetermination in 1904 within a specific dynamics of a *finite* electron and at the cost of much effort. Having no such possibility, Larmor temporarily avoided the issue by assuming, without proof, that the original coordinate transformation (132) did not need further rescaling.[159]

In *Aether and matter* Larmor proved that property β) holds to second order in u/c.[160] Yet it is easily seen to hold to any order. Equations (132), (136), (128) imply

$$\nabla'\cdot\mathbf{D}' = \gamma\nabla\cdot\mathbf{D}, \tag{139}$$

which leads to the property β).

With his correspondence theorem proved to second order in u/c, Larmor analyzed the effect of motion on optical phenomena. His reasoning and conclusions were very similar to Lorentz'. He discussed the Doppler effect, aberration, the Lorentz contraction, and the general absence of effects of the earth's motion on terrestrial optical experiments.[161] The only point of

157. Larmor, *Aether and matter* (ref. 125), 175–176.

158. Ibid., 176.

159. In Larmor's theory the ambiguity cannot be removed, because for linear aethereal equations and point-electrons the theory is scale invariant, as noted in *Aether and matter* (ref. 125), 189–190.

160. Larmor, *Aether and matter* (ref. 125), 175.

161. Ibid., 179.

disagreement with Lorentz concerned Mascart's result, the independence of the rotatory power of quartz from the earth's motion. In the *Versuch*, Lorentz had investigated this problem on the basis of macrosopic field equations with a modified relation between electric force and polarization:

$$\mathbf{E} = \chi^{-1}\mathbf{P} + j\nabla\times\mathbf{P} + k\dot{\mathbf{P}}\times\mathbf{u}. \tag{140}$$

He found that a first-order modification of the rotatory power was to be expected, unless a relation existed between the coefficients j and k, for which he saw no ground. However, Larmor expected no modification, on the basis of his version of the correspondence theorem. In *Aether and matter* he also proposed a macroscopic calculation similar to Lorentz', but leading to the null-result. As Lorentz showed in 1902, Larmor's calculation was defective and, once corrected, implied a modification of the rotatory power: the fundamental equations were the same as Lorentz', with $k=0$. Nevertheless, by that time Lorentz agreed with Larmor that molecular forces behaved like electronic forces with respect to the correspondence theorem, so that the absence of first-order effects of the earth's motion on optical experiments had to be quite general.[162]

Larmor did not expect *all* terrestrial experiments to be insensitive to the earth's motion. *Aether and matter* proposed several first and second order effects. One of them dealt with the interaction between an electrically charged body and a magnet sharing the earth's motion. According to Lorentz, there could be no net force acting on the charged body because a compensatory induced charge appeared at the surface of the magnet. Larmor speculated that the compensation would not occur for a non-conducting magnet, if such a thing could exist.[163]

Another effect predicted by Larmor was a second-order increase of the electric conductivity in a moving conductor. According to the theorem of corresponding states, Larmor reasoned, the value of dx'/dt' for an ion moving in a moving conductor had to be the same as the migration velocity w of an ion in a conductor at rest exposed to the same electromotive force. Consequently, the velocity dx/dt of the ion with respect to the moving conductor (which, for Larmor, was the velocity with respect to ether minus u) is

$$\frac{dx}{dt} = \frac{w}{1 + uw/c^2},$$

162. Lorentz (ref. 35), *LCP5*, 114–119; Larmor, *Aether and matter* (ref. 125), 211–218; Lorentz, "The rotation of the plane of polarization in moving media," *PRA* (1902), also in *LCP5*, 156–166.

163. Larmor, *Aether and matter* (ref. 125), 66.

which follows from the expression (132) of the coordinate transformation. If only one kind of ion exists, or if ions with opposite signs do not move with the same velocity, this alteration of velocity implies a second-order modification of the conductivity.[164]

These examples show the great distance between the correspondence theorem and Relativity. For Lorentz and Larmor, the transformed (accented) coordinates and fields had no direct physical meaning. Most crucially, they did not refer to the properties of the moving bodies themselves, but only to these bodies once brought to rest. Therefore, further reasoning was required to deduce the physical properties of moving bodies from the properties of the transformed states. And this reasoning could easily be mistaken. Larmor's case of altered conductivity is especially instructive. He took for granted that the velocity dx/dt determined the measured flux of ions in the moving conductor, that is, the electric current. In fact, as we know from relativity theory, the velocity dx'/dt' plays this role.

Larmor's error here should be imputed to his attitude toward the relativity principle. Like Lorentz and nearly all physicists, he believed that the principle applied to matter *and* ether, not to matter by itself. He therefore expected that motion with respect to ether ought to be detectable, even though previous attempts had failed. He was as blind as Lorentz to the fact that "corresponding states" were the measurable states in the moving body. Also, he and Lorentz failed to see that their correspondence theorem held exactly at any order in u/c, the former by simple neglect, the latter because of a misconceived transformation for velocities and currents.

Larmor's theory came slightly closer to the future relativity theory than Lorentz' in that it did not require any assumption about the internal structure of electrons or about the behavior of molecular forces. To determine corresponding states, Larmor had only to consider the ether field and the corresponding equations. However, his dynamical method for deriving the motion of electrons from the free-field equations was only approximate, and, in this respect, inferior to Lorentz' and other models of a finite electron. Moreover, Larmor's conception rested on the conjecture that all properties of matter, including stability and homogeneity, could be explained on the basis of ether structures only.

Larmor's own doubts in this regard obtrude in *Aether and matter*. He noted there that his theory was scale-invariant, and inferred that the measure of the electrons' charge was arbitrary. He also recognized that atoms made of rotating positive and negative electrons would lose energy by radiation, and therefore be unstable, unless their configuration was so symmetrical that the total electric moment vanished. Last but not least, Larmor

164. Ibid., 184.

deplored that his theory had done nothing to explain forces of cohesion. He did not believe, however, that these shortcomings were fatal to his theory:[165]

> The aim of theoretical physics is not a complete and summary conquest of the *modus operandi* of natural phenomena: that would be hopelessly unattainable if only for the reason that the mental apparatus with which we conduct the search is itself in one of its aspects a part of the scheme of Nature which it attempts to unravel....The object of scientific explanation is in fact to coordinate mentally, but not to exhaust, the interlaced maze of natural phenomena.

3. CONCLUSION

Lorentz and Larmor shared the ambition of integrating optics, electrodynamics, and the atomic structure of matter (also gravitation!) in the same united framework. From a practical point of view, the resulting electron theories had much in common: the basic electrodynamical equations as well as the means to derive macroscopic laws from these equations. However, the two theories differed in several respects that reveal their author's methodological and cultural orientations and throw into relief the character of mathematical physics at the turn of the last century.

In the first place, Lorentz and Larmor disagreed about the relative roles of optics and electrodynamics. Lorentz shared Maxwell's and Helmholtz' opinion that optics had to be derived from electrodynamics; his preference appeared as early as his dissertation, where he deduced the laws of reflexion and refraction of light from "Maxwell's theory." Instead, Larmor, like William Thomson, denied that Maxwell had succeeded in founding optics on electrodynamics. In their view, theories of the optical ether were more satisfactory than Maxwell's theory of electromagnetism and more likely to provide a basis for a unified physical theory. This difference of judgement conditioned the manner in which the two theorists introduced electrons as the universal, sub-atomic carriers of electricity.

For Lorentz, electrons (or ions) were a necessity of the electromagnetic reduction of optics, when optical phenomena involving the atomic structure of matter were taken into account. He judged that anomalous dispersion implied a coupling between the propagation of light and atomic vibrators; the electromagnetic interpretation of this coupling suggested that the atomic vibrator should be thought of as an elastically bound charged particle. The idea was even more natural for one who, like the young Lorentz, used

165. Larmor, *Aether and matter* (ref. 125), 189–190 (scale), 223 (radiation drain), 166 (quotation).

Helmholtz' version of Maxwell's theory and therefore conceived electric polarization as a local displacement of charge.

Larmor's first theory, based on MacCullagh's optical ether and vortex rings, had no organic necessity for subatomic charged particles. Ions were required to explain electrolysis and perhaps chemical bonding, but only as secondary entities, obtained by straining ether vortices. In general, ions did not figure in polarization and conduction, which derived from intrinsic properties of ether—its rotational elasticity and ability to sustain vortical motion. What convinced Larmor to introduce electrons was the ultimate failure of the vortex explanation of macroscopic and microscopic currents. Hence, his program could integrate electrons only after a major internal crisis. Larmor thenceforth judged the electrons to be an *a priori* necessity, while for Lorentz they were just the most natural link in the electromagnetic interpretation of some optical phenomena. What one wins against oneself one is likely to regard as transcendental truth.

Larmor's starting from an optical ether had other consequences. Contrary to Lorentz', Larmor's electrons were not independent entities, but specific ether constructs. Larmor's theory was essentially monistic, Lorentz' clearly dualistic. The stationarity of ether had different justifications in the two theories. For Larmor, ether was stationary because it could not be otherwise. Singularities in MacCullagh's medium moved without dragging the medium. Moreover, a velocity of the medium would have meant a magnetic field much too strong to have remained undetected. For Lorentz, the stationarity of ether was nothing but the simplest hypothesis, and the most directly compatible with the optics of moving bodies.

Larmor's reluctance to found optics on Maxwell's theory followed from his dissatisfaction with Maxwell's dynamical considerations. In his opinion, Maxwell's Lagrangian treatment of a system of currents did not provide a sufficient dynamical foundation because it lacked any picture of the hidden motion occurring in the magnetic field. In this regard, previous optical theories fared better, since they were usually based on the analogy between the ether and an elastic solid. In his own theory Larmor wished to maintain the essence of MacCullagh's ether and even to provide a dynamical illustration for its rotational elasticity. It did not matter that ether differed altogether from any known substance. In Larmor's eyes, its strangeness was just one more sign of its ultimate character.

Hamilton's principle still played an essential role in Larmor's derivation of the dynamical equations of the system made out of MacCullagh's ether and singularities; but it did not act as a substitute for a definite picture. Lorentz was more open to the abstract dynamical foundation found in Maxwell's *Treatise*. He made a great effort to generalize it and to adapt it to the new electron theory. However, he finally agreed with Larmor that the mere possibility of a mechanical picture was hardly satisfactory. But there they parted ways. In subsequent work Lorentz was tempted to drop the idea of a mechanical foundation and to start directly from the field equations and the Lorentz force.

The electron theories of Larmor and Lorentz both represented a radical departure from Maxwell's original conceptions of charge and current. However, the history and the final form of the break differed in each case. Lorentz initially used Helmholtz' reformulation of Maxwell's theory, in which charge and current were primitive concepts defined through their actions at a distance. In this context, polarization was a second-level concept, a microscopic displacement of charge, whereas in Maxwell's theory (or in Faraday's conception), polarization was a primitive concept, from which the concepts of charge and current derived. Since Lorentz' first ionic theory of dispersion belonged to this framework, his venturing beyond Maxwell's conceptions of charge and current essentially reflected Helmholtz' earlier betrayal of Maxwell. Only the atomistic conception of charge needed to be supplemented.

After Hertz' experiments, Lorentz paid more attention to Maxwell's ideas, and grasped important aspects of it. He understood that for Maxwell electricity was an incompressible fluid; that electric displacement was the elastically resisted flow of this fluid in a dielectric; that an electrification could not be created without conferring conductivity on the medium; and that the notion of electric charge derived from the notion of field. However, in his view this conception could only apply at the microscopic level of electrons and free ether. Macroscopic displacement in material dielectrics involved microscopic shifts of elastically bound electrons; macroscopic conduction involved a nearly free circulation of electrons. In the end there was no advantage in maintaining two different intuitions of charge and current, a Maxwellian one that applied to purely imaginary sub-electronic processes and a Weberian one that applied at the macroscopic level. In later writing, Lorentz maintained only Weber's concept. As a result, the theory was clearly dualistic, based on the field and on microscopic charges no longer thought of as field constructs.

Larmor initially agreed with Maxwell that electric charge was a discontinuity in displacement, and electric conduction a relaxation of displacement. His first theory, with the vortex interpretation of electric currents, confirmed this view. It shared with Maxwell's the idea that displacement was a state of elastic constraint and that conduction currents and charges could only result from a breakdown of the elastic property of the medium. However, Larmor did not adopt Maxwell's interpretation of displacement as an elastically resisted shift of imaginary fluid, presumably because this picture could only be an illustration of the form of the total current and could not provide a basis for a dynamical theory of the field. Larmor temporarily considered Helmholtz' polarization theory, which had the advantage of being more definite though even less dynamical. In the end, the interpretation of displacement as a twist in MacCullagh's ether provided Larmor with the desired dynamical picture.

After the elimination of vortices and the introduction of electrons, Larmor's concepts of charge and current lost much of their Maxwellian flavor. He traced all conduction currents back to the convection of electrons and did away with the irreversible ruptures of the elastic properties of the medium. However, Larmor remained in a sense closer to Maxwell than Lorentz: his electron theory was still a monistic, ether-based theory, in which electric charge emerged from ether surgery.

Regarding the electrodynamics and optics of moving bodies, Larmor's theory was still rudimentary when he read Lorentz' *Versuch*. As a consequence, he adopted Lorentz' averaging methods and the technique of corresponding states. The two theorists shared the same interpretation of the corresponding states, with the same limitations. They both overlooked the essential point that those states were the measured ones in the moving frame of reference and that the correspondence theorem held exactly at every order in u/c. At the root of their oversight was their belief that the principle of relativity applied to matter *and* ether, not to matter by itself. Both expected that motion with respect to ether would be detected someday.

However, Lorentz and Larmor did not apply the correspondence theorem in the same way. Since Larmor conceived matter as built out of point singularities in ether, he could do without Lorentz' assumptions regarding the internal structure of electrons and the behavior of molecular forces. To determine corresponding states, he had only to consider the ether field and the corresponding field equations. The simplification was considerable, but not likely to impress continental electron theorists, who did not think that Larmor had proved his concept of matter.

Finally, there was a glaring difference in form and outlook between Lorentz' and Larmor's theories. Lorentz denied the possibility of an ultimate foundation of physical theory and required only that the basic assumptions be as simple and clear as possible. Larmor tended to regard the rotational ether as the ultimate thing and the ensuing electron theory as part of a new transcendental aesthetics. This philosophy of ether ran against the methodological soberness of previous leading Maxwellians like Heaviside and Hertz and increased the contrast between British and continental styles in theoretical physics at the turn of the century.

KARL HUFBAUER*

Artificial eclipses: Bernard Lyot and the coronagraph, 1929–1939

EVEN THOUGH EUROPE was in turmoil, the early months of 1939 must have been satisfying to Bernard Lyot (1897–1952). In January the Royal Astronomical Society of London, despite its bias against instrumentalists, announced the choice of the forty-two-year-old Parisian astrophysicist as its Gold Medallist.[1] And in March the Académie des Sciences of Paris, despite

*Department of History, University of California at Irvine, Irvine, CA 92717. I began thinking about this study when Ron Doel informed me that there might well be sufficient materials in Paris to write an archivally based account of Lyot's coronagraphic research. For further information, documents, research leads, and insights concerning Bernard Lyot and the Parisian astronomical, physics, and optical communities between the World Wars, I am especially indebted to his son Gérard Lyot, and his former collaborators Audouin Dollfus, Maurice Françon, and Jean Rösch. I also acknowledge the help of the following: Barbara Becker, Claudine Billoux, Andrew Butrica, André Chenevez, Emmanuel Davoust, Suzanne Debarbat, Michael Dennis, David DeVorkin, Clark Elliott, Jack Evans, Charles Fehrenbach, Roland Greiss, J.L. Heilbron, Klaus Hentschel, Michael Hoskin, Sally Hufbauer, Peggy Kidwell, Henri Meyer, Mary Jo Nye, Jean Luc Olivié, Donald Osterbrock, Alex Soojung-Kim Pang, Dominique Pestre, Jean Ritz, Dorothy Schaumberg, Paul Simon, Tom Williams, Jacques Vulmière, and Marina Zuccoli.

The following abbreviations are used: AN, Archives Nationales, Paris; *CR*, Académie des Sciences, Paris, *Comptes rendus*; DM, Donald Menzel General Correspondence, Harvard University Archives, Cambridge, MA; *DSB*, *Dictionary of scientific biography*; EPA, École Polytechnique Archives, Bibliothèque, Palaiseau; *IAUT*, International Astronomical Union, *Transactions*; *JBAA*, British Astronomical Association, *Journal*; *JPR*, *Journal de physique et le radium*; LOA, Lowell Observatory Archives, Flagstaff, AZ; *MNRAS*, Royal Astronomical Society, *Monthly notices*; MWOC, Mount Wilson Observatory Collection, Huntington Library, San Marino, CA; OPA, Observatoire de Paris Archives, Paris; *PAAS*, American Astronomical Society, *Publications*; RRM, Robert R. McMath Papers, Bentley Historical Library, University of Michigan, Ann Arbor, MI; SHMA, Sources for the History of Modern Astrophysics, American Institute of Physics, College Park, MD; *VAG*, Astronomische Gesellschaft, *Vierteljahrschrift*; *ZfA*, *Zeitschrift für Astrophysik*.

1. H.H. Plaskett and D.H. Sadler, "Meeting of the Royal Astronomical Society, Friday, 1939 January 13," *Observatory*, *62* (1939), 29–43, on 29. Seven of the ten other medallists chosen from Jan 1930 through Jan 1940 were oriented primarily to observation—J.S. Plaskett, R.G. Aitken, V.M. Slipher, H. Shapley, H. Kimura, W.H. Wright, and E.P. Hubble—and three to theory—W. de. Sitter, E.A. Milne, and H. Jeffreys. Cf. A.S. Eddington, "Physical and optical societies twenty-first annual exhibition of apparatus: Opening address delivered by the President," Physical Society, *Proceedings*, *43* (1931), 119–123.