DSB

two currently leading theories for the origin of the solar system—those of Kant and Laplace. According to Kant, comets formed as part of the primeval solar nebula, while according to Laplace they originated independently from the solar system. Newton found that the distribution of comets' aphelia and inclinations agrees better with the latter theory, although he noted that the problem was unsettled.

These calculations included considerations of the effect of large planetary perturbations on the distribution of cometary orbits; such studies culminated in 1891 in his most famous paper on perturbations. During the 1870's and 1880's Newton accumulated statistical data that indicated that long period comets could be captured by Jupiter, shortening their periods.

Newton devoted the last decade of his research to Biela's comet and meteor shower, to fireballs, and to meteorites. At his death he was probably the foremost American pioneer in the study of meteors.

Besides his scientific research, Newton was active in teaching and educational reform, especially about the metric system. He was a founder of the American Metrological Society, and he persuaded many manufacturers of scientific instruments and publishers of school arithmetic texts to adopt the system.

In 1868 the University of Michigan awarded Newton an honorary LL.D. After joining the American Association for the Advancement of Science in 1850, he served as the vice-president of its Section A in 1875, and as president of the Association in 1885. He was a president of the Connecticut Academy of Arts and Sciences, a member of the American Philosophical Society, and one of the original members of the National Academy of Sciences. In 1888 the National Academy awarded him its J. Lawrence Smith Gold Medal in recognition of his research on meteoroids. At his death he was the vice-president of the American Mathematical Society and an associate editor of the *American Journal of Science*.

Aside from societies in the United States, he was elected in 1860 corresponding member of the British Association for the Advancement of Science, in 1872 associate of the Royal Astronomical Society of London, in 1886 foreign honorary fellow of the Royal Philosophical Society of Edinburgh, and in 1892 foreign member of the Royal Society of London.

Newton's association with Yale and New Haven was long and rich. He directed the Yale mathematics department and also the observatory, which he helped organize in 1882, and he helped build the extensive collection of meteorites in the Peabody Museum. He also provided considerable assistance to poor students who wanted to attend Yale. For a time he was the only Democrat on the Yale faculty and became alderman in the strongly Republican first ward of New Haven.

BIBLIOGRAPHY

I. ORIGINAL WORKS. Newton published approximately seventy papers, an extensive bibliography of which is included in the memoir by Gibbs that is cited below. Newton's most significant writings included the following: "Explanation of the Motion of the Gyroscope," in *American Journal of Science*, **24** (1857), 253–254; "On the Geometrical Construction of Certain Curves by Points," in *Mathematics Monthly*, **3** (1861), 235–244, 268–279; "On November Star-Showers," in *American Journal of Science*, **37** (1864), 377–389; **38** (1864), 53–61; "On Shooting Stars," in *Memoirs of the National Academy of Sciences*, **1** (1866), 291–312; *The Metric System of Weights and Measures* (Washington, 1868); "On the Transcendental Curves Whose Equation Is $\sin y \sin my - a \sin x \sin nx + b$," in *Transactions of the Connecticut Academy of Arts and Sciences*, **3** (1875), 97–107, written with A. W. Phillips; "On the Origin of Comets," in *American Journal of Science*, **16** (1878), 165–179; "The Story of Biela's Comet," *ibid.*, **31** (1886), 81–94; and "On the Capture of Comets by Planets, Especially Their Capture by Jupiter," in *Memoirs of the National Academy of Sciences*, **6** (1891), 7–23.

II. SECONDARY LITERATURE. An article on meteors that gives a critique of Newton's work is M. Faye, in *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, **64** (1867), 550. Biographical sketches, which were written about the time of Newton's becoming president of the American Association for the Advancement of Science, are in *Science*, **6** (1885), 161–162; in *Popular Science Monthly*, **27** (1885), 840–843; and in James Grant Wilson and John Fisk, eds., *Appleton's Cyclopedia of American Biography*, IV (New York, 1888), 506–507.

Obituaries on Newton are William L. Elkin, in *Astronomische Nachrichten*, **141** (1896), 407; unsigned writers, in *Popular Astronomy*, **4** (1896), 236–240; in *Monthly Notices of the Royal Astronomical Society*, **57** (1897), 227–231; and in *New York Times* (13 Aug. 1896), 5. Biographical articles that were written after his death were J. Willard Gibbs, in *Biographical Memoirs. National Academy of Sciences*, **4** (1902), 99–124, which includes a bibliography; Anson Phelps Stokes, in *Memorials of Eminent Yale Men* (New Haven, 1914), 48–54; and David Eugene Smith, in Dumas Malone, ed., *Dictionary of American Biography*, XIII (New York, 1934), 470–471.

RICHARD BERENDZEN

NEWTON, ISAAC (*b.* Woolsthorpe, England, 25 December 1642; *d.* London, England, 20 March 1727), *mathematics, dynamics, celestial mechanics, astronomy, optics, natural philosophy.*

Isaac Newton was born a posthumous child, his father having been buried the preceding 6 October.

NEWTON

Newton was descended from yeomen on both sides: there is no record of any notable ancestor. He was born prematurely, and there was considerable concern for his survival. He later said that he could have fitted into a quart mug at birth. He grew up in his father's house, which still stands in the hamlet of Woolsthorpe, near Grantham in Lincolnshire.

Newton's mother, Hannah (née Ayscough), remarried, and left her three-year-old son in the care of his aged maternal grandmother. His stepfather, the Reverend Barnabas Smith, died in 1653; and Newton's mother returned to Woolsthorpe with her three younger children, a son and two daughters. Their surviving children, Newton's four nephews and four nieces, were his heirs. One niece, Catherine, kept house for Newton in the London years and married John Conduitt, who succeeded Newton as master of the Mint.

Newton's personality was no doubt influenced by his never having known his father. That he was, moreover, resentful of his mother's second marriage and jealous of her second husband may be documented by at least one entry in a youthful catalogue of sins, written in shorthand in 1662, which records "Threatning my father and mother Smith to burne them and the house over them."[1]

In his youth Newton was interested in mechanical contrivances. He is reported to have constructed a model of a mill (powered by a mouse), clocks, "lanthorns," and fiery kites, which he sent aloft to the fright of his neighbors, being inspired by John Bate's *Mysteries of Nature and Art*.[2] He scratched diagrams and an architectural drawing (now revealed and preserved) on the walls and window edges of the Woolsthorpe house, and made many other drawings of birds, animals, men, ships, and plants. His early education was in the dame schools at Skillington and Stoke, beginning perhaps when he was five. He then attended the King's School in Grantham, but his mother withdrew him from school upon her return to Woolsthorpe, intending to make him a farmer. He was, however, uninterested in farm chores, and absent-minded and lackadaisical. With the encouragement of John Stokes, master of the Grantham school, and William Ayscough, Newton's uncle and rector of Burton Coggles, it was therefore decided to prepare the youth for the university. He was admitted a member of Trinity College, Cambridge, on 5 June 1661 as a subsizar, and became scholar in 1664 and Bachelor of Arts in 1665.

Among the books that Newton studied while an undergraduate was Kepler's "optics" (presumably the *Dioptrice*, reprinted in London in 1653). He also began Euclid, which he reportedly found "trifling," throwing it aside for Schooten's second Latin edition of Descartes's *Géométrie*.[3] Somewhat later, on the occasion of his election as scholar, Newton was reportedly found deficient in Euclid when examined by Barrow.[4] He read Descartes's *Géométrie* in a borrowed copy of the Latin version (Amsterdam, 1659–1661) with commentary by Frans van Schooten, in which there were also letters and tracts by de Beaune, Hudde, Heuraet, de Witt, and Schooten himself. Other books that he studied at this time included Oughtred's *Clavis*, Wallis' *Arithmetica infinitorum*, Walter Charleton's compendium of Epicurus and Gassendi, Digby's *Two Essays*, Descartes's *Principia philosophiae* (as well as the Latin edition of his letters), Galileo's *Dialogo* (in Salusbury's English version)—but not, apparently, the *Discorsi*—Magirus' compendium of Scholastic philosophy, Wing and Streete on astronomy, and some writings of Henry More (himself a native of Grantham), with whom Newton became acquainted in Cambridge. Somewhat later, Newton read and annotated Sprat's *History of the Royal Society*, the early *Philosophical Transactions*, and Hooke's *Micrographia*.

Notebooks that survive from Newton's years at Trinity include an early one[5] containing notes in Greek on Aristotle's *Organon* and *Ethics*, with a supplement based on the commentaries by Daniel Stahl, Eustachius, and Gerard Vossius. This, together with his reading of Magirus and others, gives evidence of Newton's grounding in Scholastic rhetoric and syllogistic logic. His own reading in the moderns was organized into a collection of "Questiones quaedam philosophicae,"[6] which further indicate that he had also read Charleton and Digby. He was familiar with the works of Glanville and Boyle, and no doubt studied Gassendi's epitome of Copernican astronomy, which was then published together with Galileo's *Sidereus nuncius* and Kepler's *Dioptrice*.[7]

Little is known of Newton's friends during his college days other than his roommate and onetime amanuensis Wickins. The rooms he occupied are not known for certain; and we have no knowledge as to the subject of his thesis for the B.A., or where he stood academically among the group who were graduated with him. He himself did record what were no doubt unusual events in his undergraduate career: "Lost at cards twice" and "At the Taverne twice."

For eighteen months, after June 1665, Newton is supposed to have been in Lincolnshire, while the University was closed because of the plague. During this time he laid the foundations of his work in mathematics, optics, and astronomy or celestial mechanics. It was formerly believed that all of these discoveries were made while Newton remained in seclusion at Woolsthorpe, with only an occasional excursion into nearby Boothby. During these "two plague years of 1665 & 1666," Newton later said, "I was in the prime of my age for invention & minded Mathematicks & Philosophy more then at any time since." In fact, however, Newton was back in Cambridge on at least one visit between March and June 1666.[8] He appears to have written out his mathematical discoveries at Trinity, where he had access to the college and University libraries, and then to have returned to Lincolnshire to revise and polish these results. It is possible that even the prism experiments on refraction and dispersion were made in his rooms at Trinity, rather than in the country, although while at Woolsthorpe he may have made pendulum experiments to determine the gravitational pull of the earth. The episode of the falling of the apple, which Newton himself said "occasioned" the "notion of gravitation," must have occurred at either Boothby or Woolsthorpe.[9]

Lucasian Professor. On 1 October 1667, some two years after his graduation, Newton was elected minor fellow of Trinity, and on 16 March 1668 he was admitted major fellow. He was created M.A. on 7 July 1668 and on 29 October 1669, at the age of twenty-six, he was appointed Lucasian professor. He succeeded Isaac Barrow, first incumbent of the chair, and it is generally believed that Barrow resigned his professorship so that Newton might have it.[10]

University statutes required that the Lucasian professor give at least one lecture a week in every term. He was then ordered to put in finished form his ten (or more) annual lectures for deposit in the University Library. During Newton's tenure of the professorship, he accordingly deposited manuscripts of his lectures on optics (1670–1672), arithmetic and algebra (1673–1683), most of book I of the *Principia* (1684–1685), and "The System of the World" (1687). There is, however, no record of what lectures, if any, he gave in 1686, or from 1688 until he removed to London early in 1696. In the 1670's Newton attempted unsuccessfully to publish his annotations on Kinckhuysen's algebra and his own treatise on fluxions. In 1672 he did succeed in publishing an improved or corrected edition of Varenius' *Geographia generalis*, apparently intended for the use of his students.

During the years in which Newton was writing the *Principia*, according to Humphrey Newton's recollection,[11] "he seldom left his chamber except at term time, when he read in the schools as being Lucasianus Professor, where so few went to hear him, and fewer that understood him, that ofttimes he did in a manner, for want of hearers, read to the walls." When he lectured he "usually staid about half an hour; when he had no auditors, he commonly returned in a 4th part of that time or less." He occasionally received foreigners "with a great deal of freedom, candour, and respect." He "ate sparingly," and often "forgot to eat at all," rarely dining "in the hall, except on some public days," when he was apt to appear "with shoes down at heels, stockings untied, surplice on, and his head scarcely combed." He "seldom went to the chapel," but very often "went to St Mary's church, especially in the forenoon."[12]

From time to time Newton went to London, where he attended meetings of the Royal Society (of which he had been a fellow since 1672). He contributed £40 toward the building of the new college library (1676), as well as giving it various books. He corresponded, both directly and indirectly (often through Henry Oldenburg as intermediary), with scientists in England and on the Continent, including Boyle, Collins, Flamsteed, David Gregory, Halley, Hooke, Huygens, Leibniz, and Wallis. He was often busy with chemical experiments, both before and after writing the *Principia*, and in the mid-1670's he contemplated a publication on optics.[13] During the 1690's, Newton was further engaged in revising the *Principia* for a second edition; he then contemplated introducing into book III some selections from Lucretius and references to an ancient tradition of wisdom. A major research at this time was the effect of solar perturbations on the motions of the moon. He also worked on mathematical problems more or less continually throughout these years.

Among the students with whom Newton had friendly relations, the most significant for his life and career was Charles Montague, a fellow-commoner of Trinity and grandson of the Earl of Manchester; he "was one of the small band of students who assisted Newton in forming the Philosophical Society of Cambridge"[14] (the attempt to create this society was unsuccessful). Newton was also on familiar terms with Henry More, Edward Paget (whom he recommended for a post in mathematics at Christ's Hospital), Francis Aston, John Ellis (later master of Caius), and J. F. Vigani, first professor of chemistry at Cambridge, who is said to have eventually been banished from Newton's presence for having told him "a loose story about a nun." Newton was active in defending the rights of the university when the Catholic monarch James II tried to mandate the admission of the Benedictine monk Alban Francis. In 1689, he was elected by the university constituency to serve as Member of the Convention Parliament.

While in London as M.P., Newton renewed contact

NEWTON

with Montague and with the Royal Society, and met Huygens and others, including Locke, with whom he thereafter corresponded on theological and biblical questions. Richard Bentley sought Newton's advice and assistance in preparing the inaugural Boyle Lectures (or sermons), entitled "The Confutation of Atheism" and based in part on the Newtonian system of the world.

Newton also came to know two other scientists, each of whom wanted to prepare a second edition of the *Principia*. One was David Gregory, a professor at Edinburgh, whom Newton helped to obtain a chair at Oxford, and who recorded his conversations with Newton while Newton was revising the *Principia* in the 1690's. The other was a refugee from Switzerland, Nicolas Fatio de Duillier, advocate of a mechanical explanation of gravitation which was at one time viewed kindly by Newton. Fatio soon became perhaps the most intimate of any of Newton's friends. In the early autumn of 1693, Newton apparently suffered a severe attack of depression and made fantastic accusations against Locke and Pepys and was said to have lost his reason.[15]

In the post-*Principia* years of the 1690's, Newton apparently became bored with Cambridge and his scientific professorship. He hoped to get a post that would take him elsewhere. An attempt to make him master of the Charterhouse "did not appeal to him"[16] but eventually Montague (whose star had risen with the Whigs' return to power in Parliament) was successful in obtaining for Newton (in March 1696) the post of warden of the mint. Newton appointed William Whiston as his deputy in the professorship. He did not resign officially until 10 December 1701, shortly after his second election as M.P. for the university.[17]

Mathematics. Any summary of Newton's contributions to mathematics must take account not only of his fundamental work in the calculus and other aspects of analysis—including infinite series (and most notably the general binomial expansion)—but also his activity in algebra and number theory, classical and analytic geometry, finite differences, the classification of curves, methods of computation and approximation, and even probability.

For three centuries, many of Newton's writings on mathematics have lain buried, chiefly in the Portsmouth Collection of his manuscripts. The major parts are now being published and scholars will shortly be able to trace the evolution of Newton's mathematics in detail.[18] It will be possible here only to indicate highlights, while maintaining a distinction among four levels of dissemination of his work: (1) writings printed in his lifetime, (2) writings circulated in manuscript, (3) writings hinted at or summarized in correspondence, and (4) writings that were published only much later. In his own day and afterward, Newton influenced mathematics "following his own wish," by "his creation of the fluxional calculus and the theory of infinite series," the "two strands of mathematical technique which he bound inseparably together in his 'analytick' method."[19] The following account therefore emphasizes these two topics.

Newton appears to have had no contact with higher mathematics until 1664 when—at the age of twenty-one—his dormant mathematical genius was awakened by Schooten's "Miscellanies" and his edition of Descartes's *Géométrie*, and by Wallis' *Arithmetica infinitorum* (and possibly others of his works). Schooten's edition introduced him to the mathematical contributions of Heuraet, de Witt, Hudde, De Beaune, and others; Newton also read in Viète, Oughtred, and Huygens. He had further compensated for his early neglect of Euclid by careful study of both the *Elements* and *Data* in Barrow's edition.

In recent years[20] scholars have come to recognize Descartes and Wallis as the two "great formative influences" on Newton in the two major areas of his mathematical achievement: the calculus, and analytic geometry and algebra. Newton's own copy of the *Géométrie* has lately turned up in the Trinity College Library; and his marginal comments are now seen to be something quite different from the general devaluation of Descartes's book previously supposed. Rather than the all-inclusive "Error. Error. Non est geom." reported by Conduitt and Brewster, Newton merely indicated an "Error" here and there, while the occasional marginal entry "non geom." was used to note such things as that the Cartesian classification of curves is not really geometry so much as it is algebra. Other of Newton's youthful annotations document what he learned from Wallis, chiefly the method of "indivisibles."[21]

In addition to studying the works cited, Newton encountered the concepts and methods of Fermat and James Gregory. Although Newton was apparently present when Barrow "read his Lectures about motion," and noted[22] that they "might put me upon taking these things into consideration," Barrow's influence on Newton's mathematical thought was probably not of such importance as is often supposed.

A major first step in Newton's creative mathematical life was his discovery of the general binomial theorem, or expansion of $(a + b)^n$, concerning which he wrote, "In the beginning of the year 1665 I found the Method of approximating series & the Rule for reducing any dignity [power] of any Binomial into such a series. . . ."[23] He further stated that:

> In the winter between the years 1664 & 1665 upon reading Dr Wallis's *Arithmetica Infinitorum* & trying to interpole his progressions for squaring the circle [that is, finding the area or evaluating ${}_0\int^1 (1-x^2)^{\frac{1}{2}}\,dx$], I found out another infinite series for squaring the circle & then another infinite series for squaring the Hyperbola. . . .[24]

On 13 June 1676, Newton sent Oldenburg the "Epistola prior" for transmission to Leibniz. In this communication he wrote that fractions "are reduced to infinite series by division; and radical quantities by extraction of roots," the latter

> . . . much shortened by this theorem,
>
> $$\overline{P+PQ}^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ + \frac{m-2n}{3n}CQ + \frac{m-3n}{4n}DQ + \cdots \&c.$$
>
> where $P+PQ$ signifies the quantity whose root or even any power, or the root of a power, is to be found; P signifies the first term of that quantity, Q the remaining terms divided by the first, and m/n the numerical index of the power of $P+PQ$, whether that power is integral or (so to speak) fractional, whether positive or negative.[25]

A sample given by Newton is the expansion

$$\sqrt{(c^2+x^2)} \quad \text{or} \quad (c^2+x^2)^{\frac{1}{2}} = c + \frac{x^2}{2c} - \frac{x^4}{8c^3} + \frac{x^6}{16c^5} - \frac{5x^8}{128c^7} + \frac{7x^{10}}{256c^9} + \text{etc.}$$

where

$$P = c^2, \quad Q = x^2/c^2, \quad m = 1, \quad n = 2, \quad \text{and}$$

$$A = P^{\frac{m}{n}} = (c^2)^{\frac{1}{2}} = c, \quad B = (m/n)\,AQ = x^2/2c,$$

$$C = \frac{m-n}{2n}BQ = -x^4/8c^3,$$

and so on.

Other examples include

$$(y^3 - a^2y)^{-\frac{1}{3}}$$

$$(c^5 + c^4x - x^5)^{\frac{1}{5}},$$

$$(d+e)^{-\frac{3}{5}}.$$

What is perhaps the most important general statement made by Newton in this letter is that in dealing with infinite series all operations are carried out "in the symbols just as they are commonly carried out in decimal numbers."

Wallis had obtained the quadratures of certain curves (that is, the areas under the curves), by a technique of indivisibles yielding ${}_0\int^1 (1-x^2)^n\,dx$ for certain positive integral values of n (0, 1, 2, 3); in attempting to find the quadrature of a circle of unit radius, he had sought to evaluate the integral ${}_0\int^1 (1-x^2)^{\frac{1}{2}}\,dx$ by interpolation. He showed that

$$\frac{4}{\pi} = \frac{1}{{}_0\int^1 (1-x^2)^{\frac{1}{2}}\,dx} = \frac{3\cdot3\cdot5\cdot5\cdot7\cdot7\cdots}{2\cdot4\cdot4\cdot6\cdot6\cdot8\cdots}.$$

Newton read Wallis and was stimulated to go considerably further, freeing the upper bound and then deriving the infinite series expressing the area of a quadrant of a circle of radius x:

$$x - \frac{\frac{1}{2}x^3}{3} - \frac{\frac{1}{8}x^5}{5} - \frac{\frac{1}{16}x^7}{7} - \frac{\frac{5}{128}x^9}{9} - \cdots.$$

In so freeing the upper bound, he was led to recognize that the terms, identified by their powers of x, displayed the binomial coefficients. Thus, the factors $\frac{1}{2}, \frac{1}{8}, \frac{1}{16}, \frac{5}{128}, \ldots$ stand out plainly as $\binom{q}{1}, \binom{q}{2}, \binom{q}{3}, \binom{q}{4}, \ldots$, in the special case $q = \frac{1}{2}$ in the generalization

$$\int_0^x (1-x^2)^q\,dx = X - \binom{q}{1}\cdot\frac{1}{3}X^3 + \binom{q}{2}\cdot\frac{1}{5}X^5 - \binom{q}{3}\cdot\frac{1}{7}X^7 + \frac{q}{5}\cdot\frac{1}{9}X^9 + \cdots,$$

where

$$\binom{q}{n} = \frac{q(q-1)\cdots(q-n+1)}{n!}.$$

In this way, according to D. T. Whiteside, Newton could begin with the indefinite integral and, "by differentiation in a Wallisian manner," proceed to a straightforward derivation of the "series-expansion of the binomial $(1-x^p)^q$ . . . virtually in its modern form," with "$|x^p|$ implicitly less than unity for convergence." As a check on the validity of this general series expansion, he "compared its particular expansions with the results of algebraic division and square-root extraction ($q = \frac{1}{2}$)." This work, which was done in the winter of 1664–1665, was later presented in modified form at the beginning of Newton's *De analysi*.

He correctly summarized the stages of development of his method in the "Epistola posterior" of 24 October 1676, which—as before—he wrote for Oldenburg to transmit to Leibniz:

NEWTON

At the beginning of my mathematical studies, when I had met with the works of our celebrated Wallis, on considering the series, by the intercalation of which he himself exhibits the area of the circle and the hyperbola, the fact that in the series of curves whose common base or axis is x and the ordinates

$$(1-x^2)^{\frac{0}{2}},\ (1-x^2)^{\frac{1}{2}},\ (1-x^2)^{\frac{2}{2}},\ (1-x^2)^{\frac{3}{2}},\ (1-x^2)^{\frac{4}{2}},\ (1-x^2)^{\frac{5}{2}},$$

etc., if the areas of every other of them, namely

$$x,\ x-\tfrac{1}{3}x^3,\ x-\tfrac{2}{3}x^3+\tfrac{1}{5}x^5,\ x-\tfrac{3}{3}x^3+\tfrac{3}{5}x^5-\tfrac{1}{7}x^7,\quad \text{etc.}$$

could be interpolated, we would have the areas of the intermediate ones, of which the first $(1-x^2)^{\frac{1}{2}}$ is the circle. . . .[26]

The importance of changing Wallis' fixed upper boundary to a free variable x has been called "the crux of Newton's breakthrough," since the "various powers of x order the numerical coefficients and reveal for the first time the binomial character of the sequence."[27]

In about 1665, Newton found the power series (that is, actually determined the sequence of the coefficients) for

$$\sin^{-1}x = x + \tfrac{1}{6}x^3 + \tfrac{3}{40}x^5 + \cdots,$$

and—most important of all—the logarithmic series. He also squared the hyperbola $y(1+x) = 1$, by tabulating

$$\int_0^x (1+t)^r\,dt$$

for $r = 0, 1, 2, \cdots$ in powers of x and then interpolating

$$\int_0^x (1+t)^{-1}\,dt.$$[28]

From his table, he found the square of the hyperbola in the series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \frac{x^9}{9} - \frac{x^{10}}{10} + \cdots,$$

which is the series for the natural logarithm of $1 + x$. Newton wrote that having "found the method of infinite series," in the winter of 1664–1665, "in summer 1665 being forced from Cambridge by the Plague I computed the area of the Hyperbola at Boothby . . . to two & fifty figures by the same method."[29]

At about the same time Newton devised "a completely general differentiation procedure founded on the concept of an indefinitely small and ultimately vanishing element o of a variable, say, x." He first used the notation of a "little zero" in September 1664, in notes based on Descartes's *Géométrie*, then extended it to various kinds of mathematical investigations. From the derivative of an algebraic function $f(x)$ conceived ("essentially") as

$$\lim_{o\to \text{zero}} \frac{1}{0}\,[f(x+0) - f(x)]$$

he developed general rules of differentiation.

The next year, in Lincolnshire and separated from books, Newton developed a new theoretical basis for his techniques of the calculus. Whiteside has summarized this stage as follows:

> [Newton rejected] as his foundation the concept of the indefinitely small, discrete increment in favor of that of the "fluxion" of a variable, a finite instantaneous speed defined with respect to an independent, conventional dimension of time and on the geometrical model of the line-segment: in modern language, the fluxion of the variable x with regard to independent time-variable t is the "speed" dx/dt.[30]

Prior to 1691, when he introduced the more familiar dot notation ($\dot{x}$ for dx/dt, $\dot{y}$ for dy/dt, $\dot{z}$ for dz/dt; then $\ddot{x}$ for d^2x/dt^2, $\ddot{y}$ for d^2y/dt^2, $\ddot{z}$ for d^2z/dt^2), Newton generally used the letters p, q, r for the first derivatives (Leibnizian dx/dt, dy/dt, dz/dt) of variable quantities x, y, z, with respect to some independent variable t. In this scheme, the "little zero" o was "an arbitrary increment of time,"[31] and op, oq, or were the corresponding "moments," or increments of the variables x, y, z (later these would, of course, become $o\dot{x}$, $o\dot{y}$, $o\dot{z}$).[32] Hence, in the limit ($o \to$ zero), in the modern Leibnizian terminology

$$q/p = dy/dx \qquad r/p = dz/dx,$$

where "we may think of the increment o as absorbed into the limit ratios." When, as was often done for the sake of simplicity, x itself was taken for the independent time variable, since $x = t$, then $p = \dot{x} = dx/dx = 1$, $q = dy/dx$, and $r = dz/dx$.

In May 1665, Newton invented a "true partial-derivative symbolism," and he "widely used the notation $\bar{p}$ and $\hat{p}$ for the respective homogenized derivatives $x(dp/dx)$ and $x^2(d^2p/dx^2)$," in particular to express the total derivative of the function

$$\sum_i (p_i y^i) = 0$$

before "breaking through . . . to the first recorded use of a true partial-derivative symbolism." Armed with this tool, he constructed "the five first and second order partial derivatives of a two-valued function" and composed the fluxional tract of October 1666.[33]

Extracts were published by James Wilson in 1761, although the work as a whole remained in manuscript until recently.[34] Whiteside epitomizes Newton's work during this period as follows:

> In two short years (summer 1664–October 1666) Newton the mathematician was born, and in a sense the rest of his creative life was largely the working out, in calculus as in his mathematical thought in general, of the mass of burgeoning ideas which sprouted in his mind on the threshold of intellectual maturity. There followed two mathematically dull years.[35]

From 1664 to 1669, Newton advanced to "more general considerations," namely that the derivatives and integrals of functions might themselves be expressed as expansions in infinite series, specifically power series. But he had no general method for determining the "limits of convergence of individual series," nor had he found any "valid tests for such convergence."[36] Then, in mid-1669, he came upon Nicolaus Mercator's *Logarithmotechnica*, published in September 1668, of which "Mr Collins a few months after sent a copy . . . to Dr Barrow," as Newton later recorded.[37] Barrow, according to Newton, "replied that the Method of Series was invented & made general by me about two years before the publication of" the *Logarithmotechnica* and "at the same time," July 1669, Barrow sent back to Collins Newton's tract *De analysi*.

We may easily imagine Newton's concern for his priority on reading Mercator's book, for here he found in print "for all the world to read . . . his [own] reduction of $\log(1 + a)$ to an infinite series by continued division of $1 + a$ into 1 and successive integration of the quotient term by term."[38] Mercator had presented, among other numerical examples, that of $\log(1.1)$ calculated to forty-four decimal places, and he had no doubt calculated other logarithms over which Newton had spent untold hours. Newton might privately have been satisfied that Mercator's exposition was "cumbrous and inadequate" when compared to his own, but he must have been immeasurably anxious lest Mercator generalize a particular case (if indeed he had not already done so) and come upon Newton's discovery of "the extraction of roots in such series and indeed upon his cherished binomial expansion."[39] To make matters worse, Newton may have heard the depressing news (as Collins wrote to James Gregory, on 2 February 1668/1669) that "the Lord Brouncker asserts he can turne the square roote into an infinite Series."

To protect his priority, Newton hastily set to work to write up the results of his early researches into the properties of the binomial expansion and his methods for resolving "affected" equations, revising and amplifying his results in the course of composition. He submitted the tract, *De analysi per aequationes infinitas*, to Barrow, who sent it, as previously mentioned, to Collins.

Collins communicated Newton's results to James Gregory, Sluse, Bertet, Borelli, Vernon, and Strode, among others.[40] Newton was at that time unwilling to commit the tract to print; a year later, he incorporated its main parts into another manuscript, the *Methodus fluxionum et serierum infinitarum*. The original Latin text of the tract was not printed until long afterward.[41] Among those who saw the manuscript of *De analysi* was Leibniz, while on his second visit to London in October 1676; he read Collins' copy, and transcribed portions. Whiteside concurs with "the previously expressed opinions of the two eminent Leibniz scholars, Gerhardt and Hofmann," that Leibniz did not then "annex for his own purposes the fluxional method briefly exposed there," but "was interested only in Newton's series expansions."[42]

The *Methodus fluxionum* provides a better display of Newton's methods for the fluxional calculus in its generality than does the *De analysi*. In the preface to his English version of the *Methodus fluxionum*, John Colson wrote:

> The chief Principle, upon which the Method of Fluxions is here built, is this very simple one, taken from the Rational Mechanicks; which is, That Mathematical Quantity, particularly Extension, may be conceived as generated by continued local Motion; and that all Quantities whatever, at least by analogy and accommodation, may be conceived as generated after a like manner. Consequently there must be comparative Velocities of increase and decrease, during such generations, whose Relations are fixt and determinable, and may therefore (problematically) be proposed to be found.[43]

Among the problems solved are the differentiation of any algebraic function $f(x)$; the "method of quadratures," or the integration of such a function by the inverse process; and, more generally, the "inverse method of tangents," or the solution of a first-order differential equation.

As an example, the "moments" $\dot{x}o$ and $\dot{y}o$ are "the infinitely little accessions of the flowing quantities [variables] x and y": that is, their increase in "infinitely small portions of time." Hence, after "any infinitely small interval of time" (designated by o), x and y become $x + \dot{x}o$ and $y + \dot{y}o$. If one substitutes these for x and y in any given equation, for instance

$$x^3 - ax^2 + axy - y^3 = 0,$$

"there will arise"

$$x^3 + 3\dot{x}ox^2 + 3\dot{x}^2oox + \dot{x}^3o^3$$
$$- ax^2 - 2a\dot{x}ox - a\dot{x}^2oo$$
$$+ axy + a\dot{x}oy + a\dot{y}ox + a\dot{x}\dot{y}oo$$
$$- y^3 - 3\dot{y}oy^2 - 3\dot{y}^2ooy - \dot{y}^3o^3 = 0.$$

The terms $x^3 - ax^2 + axy - y^3$ (of which "by supposition" the sum $= 0$) may be cast out; the remaining terms are divided by o, to get

$$3\dot{x}x^2 + 3\dot{x}^2ox + \dot{x}^3oo - 2ax\dot{x} - a\dot{x}^2o + a\dot{x}y$$
$$+ a\dot{y}x + a\dot{x}\dot{y}o - 3\dot{y}y^2 - 3\dot{y}^2oy - \dot{y}^3oo = 0.$$

"But whereas o is suppos'd to be infinitely little, that it may represent the moments of quantities, consequently the terms that are multiplied by it will be nothing in respect of the rest."[44] These terms are therefore "rejected," and there remains

$$3x^2\dot{x} - 2a\dot{x}x + a\dot{x}y + a\dot{y}x - 3\dot{y}y^2 = 0.$$

It is then easy to group by $\dot{x}$ and $\dot{y}$ to get

$$\dot{x}(3x^2 - 2ax + ay) + \dot{y}(ax - 3y^2) = 0$$

or

$$\frac{\dot{y}}{\dot{x}} = -\frac{3x^2 - 2ax + ay}{ax - 3y^2},$$

which is the same result as finding dy/dx after differentiating

$$x^3 - ax^2 + axy - y^3 = 0.^{45}$$

Problem II then reverses the process, with

$$3\dot{x}x^2 - 2a\dot{x}x + a\dot{x}y + a\dot{y}x - 3\dot{y}y^2 = 0$$

being given. Newton then integrates term by term to get $x^3 - ax^2 + axy - y^3 = 0$, the validity of which he may then test by differentiation.

In an example given, o is an "infinitely small quantity" representing an increment in "time," whereas, in the earlier *De analysi*, o was an increment x (although again infinitely small). In the manuscript, as Whiteside points out, Newton canceled "the less precise equivalent 'indefinitè' (indefinitely)" in favor of "infinitely."[46] Certainly the most significant feature is Newton's general and detailed treatment of "the converse operations of differentiation and integration (in Newton's terminology, constructing the 'fluxions' of given 'fluent' quantities, and vice versa)," and "the novelty of Newton's . . . reformulation of the calculus of continuous increase."[47]

NEWTON

Other illustrations given by Newton of his method are determining maxima and minima and drawing tangents to curves at any point. In dealing with maxima and minima, as applied to the foregoing equation, Newton invoked the rule (Problem III):

> When a quantity is the greatest or the least that it can be, at that moment it neither flows backwards nor forwards: for if it flows forwards or increases it was less, and will presently be greater than it is; and on the contrary if it flows backwards or decreases, then it was greater, and will presently be less than it is.

In an example Newton sought the "greatest value of x" in the equation

$$x^3 - ax^2 + axy - y^3 = 0.$$

Having already found "the relation of the fluxions of x and y," he set $\dot{x} = o$. Thus, $\dot{y}(ax - 3y^2) = 0$, or $3y^2 = ax$, gives the desired result since this relation may be used to "exterminate either x or y out of the primary equation; and by the resulting equation you may determine the other, and then both of them by $-3y^2 + ax = 0$." Newton showed how "that famous Rule of *Huddenius*" may be derived from his own general method, but he did not refer to Fermat's earlier method of maxima and minima. Newton also found the greatest value of y in the equation

$$x^3 - ay^2 + \frac{by^3}{a + y} - xx\sqrt{ay + xx} = 0$$

and then indicated that his method led to the solution of a number of specified maximum-minimum problems.

Newton's shift from a "loosely justified conceptual model of the 'velocity' of a 'moveing body' . . ." to the postulation of "a basic, uniformly 'fluent' variable of 'time' as a measure of the 'fluxions' (instantaneous 'speeds' of flow) of a set of dependent variables which continuously alter their magnitude" may have been due, in part, to Barrow.[48] This concept of a uniformly flowing time long remained a favorite of Newton's; it was to appear again in the *Principia*, in the scholium following the definitions, as "mathematical time" (which "of itself, and from its own nature, flows equably without relation to anything external"), and in lemma 2, book II (see below), in which he introduced quantities "variable and indetermined, and increasing or decreasing, as it were, by a continual motion or flux." He later explained his position in a draft review of the *Commercium epistolicum* (1712),

> I consider time as flowing or increasing by continual flux & other quantities as increasing continually in time & from the fluxion of time I give the name of

fluxions to the velocitys with which all other quantities increase. Also from the moments of time I give the name of moments to the parts of any other quantities generated in moments of time. I expose time by any quantity flowing uniformly & represent its fluxion by an unit, & the fluxions of other quantities I represent by any other fit symbols & the fluxions of their fluxions by other fit symbols & the fluxions of those fluxions by others, & their moments generated by those fluxions I represent by the symbols of the fluxions drawn into the letter o & its powers o^2, o^3, &c: viz[t] their first moments by their first fluxions drawn into the letter o, their second moments by their second fluxions into o^2, & so on. And when I am investigating a truth or the solution of a Probleme I use all sorts of approximations & neglect to write down the letter o, but when I am demonstrating a Proposition I always write down the letter o & proceed exactly by the rules of Geometry without admitting any approximations. And I found the method not upon summs & differences, but upon the solution of this probleme: *By knowing the Quantities generated in time to find their fluxions.* And this is done by finding not prima momenta but primas momentorum nascentium rationes.

In an addendum (published only in 1969) to the 1671 *Methodus fluxionum*,[49] Newton developed an alternative geometrical theory of "first and last" ratios of lines and curves. This was later partially subsumed into the 1687 edition of the *Principia*, section 1, book I, and in the introduction to the *Tractatus de quadratura curvarum* (published by Newton in 1704 as one of the two mathematical appendixes to the *Opticks*). Newton had intended to issue a version of his *De quadratura* with the *Principia* on several occasions, both before and after the 1713 second edition, because, as he once wrote, "by the help of this method of Quadratures I found the Demonstration of Kepler's Propositions that the Planets revolve in Ellipses describing . . . areas proportional to the times," and again, "By the inverse Method of fluxions I found in the year 1677 the demonstration of Kepler's Astronomical Proposition. . . ."[50]

Newton began *De quadratura* with the statement that he did not use infinitesimals, "in this Place," considering "mathematical Quantities . . . not as consisting of very small Parts; but as describ'd by a continued Motion."[51] Thus lines are generated "not by the Apposition of Parts, but by the continued Motion of Points," areas by the motion of lines, solids by the motion of surfaces, angles by the rotation of the sides, and "Portions of Time by a continual Flux." Recognizing that there are different rates of increase and decrease, he called the "Velocities of the Motions or Increments *Fluxions*, and the generated

Quantities *Fluents*," adding that "Fluxions are ver nearly as the Augments of the Fluents generated i equal but very small Particles of Time, and, to speal accurately, they are in the *first Ratio* of the nascen Augments; but they may be expounded in any Line which are proportional to them."

As an example, consider that (as in Fig. 1) area *ABC*, *ABDG* are described by the uniform motion o

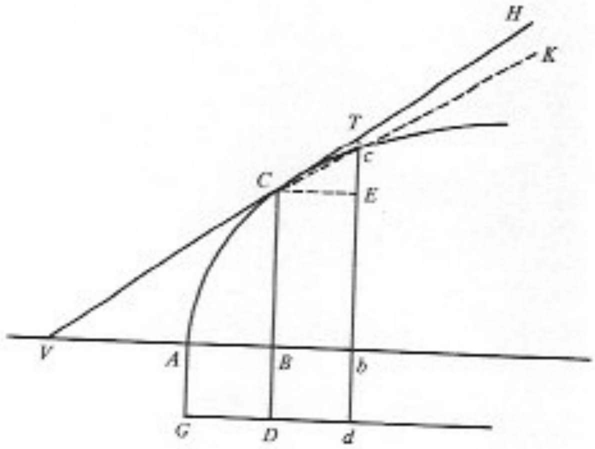

FIGURE 1

the ordinates *BC*, *BD* moving along the base in the direction *AB*. Suppose *BC* to advance to any new position *bc*, complete the parallelogram *BCEb*, draw the straight line *VTH* "touching the Curve in *C*, and meeting the two lines *bc* and *BA* [produced] in *T* and *V*." The "augments" generated will be: *Bb*, by *AB*; *Ec*, by *BC*; and *Cc*, by "the Curve Line *ACc*." Hence, "the Sides of the Triangle *CET* are in the *first Ratio* of these Augments considered as nascent." The "Fluxions of *AB*, *BC* and *AC*" are therefore "as the Sides *CE*, *ET* and *CT* of that Triangle *CET*" and "may be expounded" by those sides, or by the sides of the triangle *VBC*, which is similar to the triangle *CET*.

Contrariwise, one can "take the Fluxions in the *ultimate Ratio* of the evanescent Parts." Draw the straight line *Cc*; produce it to *K*. Now let *bc* return to its original position *BC*; when "*C* and *c* coalesce," the line *CK* will coincide with the tangent *CH*; then, "the evanescent Triangle *CEc* in its ultimate Form will become similar to the Triangle *CET*, and its evanescent Sides *CE*, *Ec*, and *Cc* will be *ultimately* among themselves as the sides *CE*, *ET* and *CT* of the other Triangle *CET*, are, and therefore the Fluxions of the Lines *AB*, *BC* and *AC* are in this same Ratio."

Newton concluded with an admonition that for the line *CK* not to be "distant from the Tangent *CH* by a small Distance," it is necessary that the points *C* and c not be separated "by any small Distance." If the points C and c do not "coalesce and exactly coincide," the lines CK and CH will not coincide, and "the ultimate Ratios in the Lines CE, Ec, and Cc" cannot be found. In short, "The very smallest Errors in mathematical Matters are not to be neglected."[52]

This same topic appears in the mathematical introduction (section 1, book I) to the *Principia*, in which Newton stated a set of lemmas on limits of geometrical ratios, making a distinction between the limit of a ratio and the ratio of limits (for example, as $x \to 0$, lim. $x^n/x \to 0$; but lim. x^n/lim. $x \to 0/0$, which is indeterminate).

The connection of fluxions with infinite series was first publicly stated in a scholium to proposition 11 of *De quadratura*, which Newton added for the 1704 printing, "We said formerly that there were first, second, third, fourth, *&c.* Fluxions of flowing Quantities. These Fluxions are as the Terms of an infinite converging series." As an example, he considered z^n to "be the flowing Quantity" and "by flowing" to become $(z + o)^n$; he then demonstrated that the successive terms of the expansion are the successive fluxions: "The first Term of this Series z^n will be that flowing Quantity; the second will be the first Increment or Difference, to which consider'd as nascent, its first Fluxion is proportional . . . and so on *in infinitum*." This clearly exemplifies the theorem formally stated by Brook Taylor in 1715; Newton himself explicitly derived it in an unpublished first version of *De quadratura* in 1691.[53] It should be noted that Newton here showed himself to be aware of the importance of convergence as a necessary condition for expansion in an infinite series.

In describing his method of quadrature by "first and last ratios," Newton said:

> Now to institute an Analysis after this manner in finite Quantities and investigate the *prime* or *ultimate* Ratios of these finite Quantities when in their nascent or evanescent State, is consonant to the Geometry of the Ancients: and I was willing [that is, desirous] to show that, in the Method of Fluxions, there is no necessity of introducing Figures infinitely small into Geometry.[54]

Newton's statement on the geometry of the ancients is typical of his lifelong philosophy. In mathematics and in mathematical physics, he believed that the results of analysis—the way in which things were discovered—should ideally be presented synthetically, in the form of a demonstration. Thus, in his review of the *Commercium epistolicum* (published anonymously), he wrote of the methods he had developed in *De quadratura* and other works as follows:

NEWTON

> By the help of the new *Analysis* Mr. *Newton* found out most of the Propositions in his *Principia Philosophiae*: but because the Ancients for making things certain admitted nothing into Geometry before it was demonstrated synthetically, he demonstrated the Propositions synthetically, that the Systeme of the Heavens might be founded upon good Geometry. And this makes it now difficult for unskilful Men to see the Analysis by which those Propositions were found out.[55]

As to analysis itself, David Gregory recorded that Newton once said "Algebra is the Analysis of the Bunglers in Mathematicks."[56] No doubt! Newton did, nevertheless, devote his main professorial lectures of 1673–1683 to algebra,[57] and these lectures were printed a number of times both during his lifetime and after.[58] This algebraical work includes, among other things, what H. W. Turnbull has described as a general method (given without proof) for discovering "the rational factors, if any, of a polynomial in one unknown and with integral coefficients"; he adds that the "most remarkable passage in the book" is Newton's rule for discovering the imaginary roots of such a polynomial.[59] (There is also developed a set of formulas for "the sums of the powers of the roots of a polynomial equation.")[60]

Newton's preference for geometric methods over purely analytical ones is further evident in his statement that "Equations are Expressions of Arithmetical Computation and properly have no place in Geometry." But such assertions must not be read out of context, as if they were pronouncements about algebra in general, since Newton was actually discussing various points of view or standards concerning what was proper to geometry. He included the positions of Pappus and Archimedes on whether to admit into geometry the conchoid for the problem of trisection and those of the "new generation of geometers" who "welcome" into geometry many curves, conics among them.[61]

Newton's concern was with the limits to be set in geometry, and in particular he took up the question of the legitimacy of the conic sections in solid geometry (that is, as solid constructions) as opposed to their illegitimacy in plane geometry (since they cannot be generated in a plane by a purely geometric construction). He wished to divorce synthetic geometric considerations from their "analytic" algebraic counterparts. Synthesis would make the ellipse the simplest of conic sections other than the circle; analysis would award this place to the parabola. "Simplicity in figures," he wrote, "is dependent on the simplicity of their genesis and conception, and it is not its equation but its description (whether

NEWTON

geometrical or mechanical) by which a figure is generated and rendered easy to conceive."[62]

The "written record of [Newton's] first researches in the interlocking structures of Cartesian co-ordinate geometry and infinitesimal analysis"[63] shows him to have been establishing "the foundations of his mature work in mathematics" and reveals "for the first time the true magnitude of his genius."[64] And in fact Newton did contribute significantly to analytic geometry. In his 1671 *Methodis fluxionum*, he devoted "Prob. 4: To draw tangents to curves" to a study of the different ways in which tangents may be drawn "according to the various relationships of curves to straight lines," that is, according to the "modes" or coordinate systems in which the curve is specified.[65]

Newton proceeded "by considering the ratios of limit-increments of the co-ordinate variables (which are those of their fluxions)."[66] His "Mode 3" consists of using what are now known as standard bipolar coordinates, which Newton applied to Cartesian ovals as follows: Let x, y be the distances from a pair of fixed points (two "poles"); the equation $a \pm (e/d)x - y = 0$ for Descartes's "second-order ovals" will then yield the fluxional relation $\pm(e/d)\dot{x} - \dot{y} = 0$ (in dot notation) or $\pm em/d - n = 0$ (in the notation of the original manuscript, in which m, n are used for the fluxions $\dot{x}$, $\dot{y}$ of x, y). When $d = e$, "the curve turns out to be a conic." In "Mode 7," Newton introduced polar coordinates for the construction of spirals; "the equation of an Archimedean spiral" in these coordinates becomes $(a/b)x = y$, where y is the radius vector (now usually designated r or ρ) and x the angle (ϑ or ϕ).

Newton constructed equations for the transformation of coordinates (as, for example, from polar to Cartesian), and found formulas in both polar and rectangular coordinates for the curvature of a variety of curves, including conics and spirals. On the basis of these results Boyer has quite properly referred to Newton as "an originator of polar coordinates."[67]

Further geometrical results may be found in *Enumeratio linearum tertii ordinis*, first written in 1667 or 1668, and then redone and published, together with *De quadratura*, as an appendix to the *Opticks* (1704).[68] Newton devoted the bulk of the tract to classifying cubic curves into seventy-two "*Classes, Genders*, or *Orders*, according to the Number of the Dimensions of an Equation, expressing the relation between the *Ordinates* and the *Abscissae*; or which is much at one [that is, the same thing], according to the Number of Points in which they may be cut by a Right Line."

In a brief fifth section, Newton dealt with "The Generation of Curves by Shadows," or the theory of projections, by which he considered the shadows produced "by a luminous point" as projections "on an infinite plane." He showed that the "shadows" (or projections) of conic sections are themselves conic sections, while "those of curves of the second genus will always be curves of the second genus; those of the third genus will always be curves of the third genus; and so on *ad infinitum*." Furthermore, "in the same manner as the circle, projecting its shadow, generates all the conic sections, so the five divergent parabolae, by their shadows, generate all the other curves of the second genus." As C. R. M. Talbot observed, this presentation is "substantially the same as that which is discussed at greater length in the twenty-second lemma [book III, section 5] of the *Principia*, in which it is proposed to 'transmute' any rectilinear or curvilinear figure into another of the same analytical order by means of the method of projections."[69]

The work ends with a brief supplement on "The Organical Description of Curves," leading to the "Description of the Conick-Section by Five Given Points" and including the clear statement, "*The Use of Curves in Geometry is, that by their Intersections Problems may be solved*" (with an example of an equation of the ninth degree). Newton in this tract laid "the foundation for the study of Higher Plane Curves, bringing out the importance of asymptotes, nodes, cusps," according to Turnbull, while Boyer has asserted that it "is the earliest instance of a work devoted solely to graphs of higher plane curves in algebra," and has called attention to the systematic use of two axes and the lack of "hesitation about negative coordinates."[70]

Newton's major mathematical activity had come to a halt by 1696, when he left Cambridge for London. The *Principia*, composed in the 1680's, marked the last great exertion of his mathematical genius, although in the early 1690's he worked on porisms and began a "Liber geometriae," never completed, of which David Gregory gave a good description of the planned whole.[71] For the most part, Newton spent the rest of his mathematical life revising earlier works.

Newton's other chief mathematical activity during the London years lay in furthering his own position against Leibniz in the dispute over priority and originality in the invention of the calculus. But he did respond elegantly to a pair of challenge problems set by Johann [I] Bernoulli in June 1696. The first of these problems was "mechanico-geometrical," to find the curve of swiftest descent. Newton's answer was brief: the "brachistochrone" is a cycloid. The second problem was to find a curve with the following property, "that the two segments [of a right line drawn from a given point through the curve], being

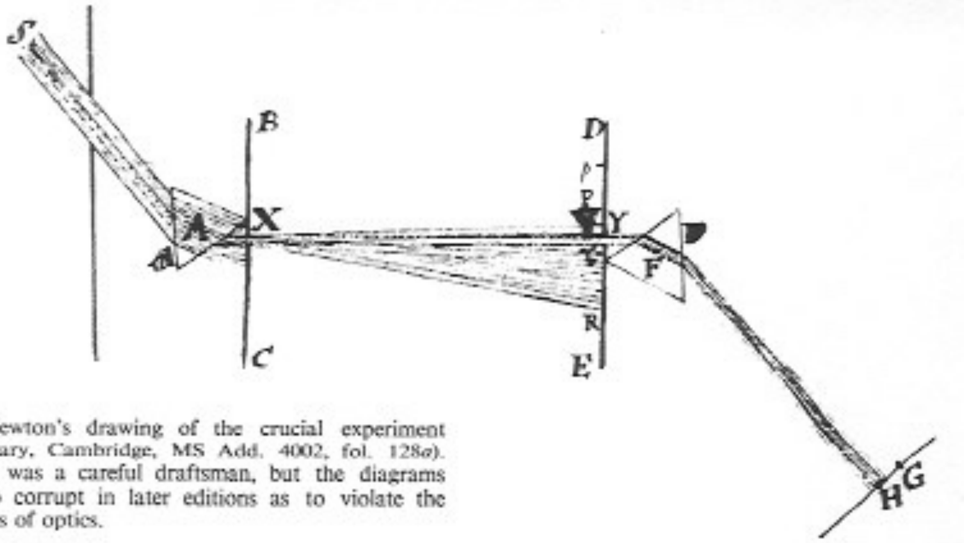

FIGURE 2. Newton's drawing of the crucial experiment (University Library, Cambridge, MS Add. 4002, fol. 128*a*). Newton himself was a careful draftsman, but the diagrams have become so corrupt in later editions as to violate the fundamental laws of optics.

raised to any given power, and taken together, may make everywhere the same sum."[72]

Newton's analytic solution of the curve of least descent is of particular interest as an early example of what became the calculus of variations. Newton had long been concerned with such problems, and in the *Principia* had included (without proof) his findings concerning the solid of least resistance. When David Gregory asked him how he had found such a solid, Newton sent him an analytic demonstration (using dotted fluxions), of which a version was published as an appendix to the second volume of Motte's English translation of the *Principia*.[73]

Optics. The study of Newton's work in optics has to date generally been limited to his published letters relating to light and color (in *Philosophical Transactions*, beginning in February 1672), his invention of a reflecting telescope and "sextant," and his published *Opticks* of 1704 and later editions (in Latin and English). There has never been an adequate edition or a full translation of the *Lectiones opticae*. Nor, indeed, have Newton's optical manuscripts as yet been thoroughly studied.[74]

Newton's optical work first came to the attention of the Royal Society when a telescope made by him was exhibited there. Newton was elected a fellow shortly thereafter, on 11 January 1672, and responded by offering the Society an account of the discovery that had led him to his invention. It was, he proudly alleged, "the oddest if not the most considerable detection yet made in the operations of nature": the analysis of dispersion and the composition of white light.

In the published account Newton related that in 1666 ("at which time I applyed myself to the grinding of Optick glasses of other figures than *Spherical*") he procured a triangular glass prism, "to try therewith the celebrated *Phaenomena of Colours*." Light from a tiny hole in a shutter passed through the prism; the multicolored image—to Newton's purported surprise—was of "an *oblong* form," whereas "according to the received laws of Refraction, I expected [it] should have been *circular*." To account for this unexpected appearance, Newton looked into a number of possibilities, among them that "the Rays, after their trajection through the Prisme did not move in curve lines," and was thereby led to the famous "experimentum crucis."[75] In this experiment Newton used two prisms: the first was employed to produce a spectrum on an opaque board (*BC*) into which a small hole had been drilled; a beam of light could thus pass through the hole to a second board (*DE*) with a similar aperture; in this way a narrow beam of light of a single color would be directed to a second prism, and the beam emerging from the second prism would project an image on another board (Fig. 2). Thus, all light reaching the final board had been twice subjected to prismatic dispersion. By rotating the first prism "to and fro slowly about its Axis," Newton allowed different portions of the dispersed light to reach the second prism.

Newton found that the second prism did not produce any further dispersion of the "homogeneal" light (that is, of light of about the same color); he therefore concluded that "Light it self is a *Heterogeneous mixture of differently refrangible Rays*"; and asserted an exact correspondence between color and "degree of Refrangibility" (the least refrangible rays being "disposed to exhibit a *Red* colour," while those of greatest refrangibility are a deep violet). Hence, colors "are not *Qualifications* of Light, derived from Refractions, or Reflections of natural Bodies," as commonly believed, but "*Original* and *connate properties*," differing in the different sorts of rays.[76]

The same experiment led Newton to two further conclusions, both of real consequence. First, he gave up any hope of "the perfection of Telescopes" based on combinations of lenses and turned to the principle of the reflector; second, he held it to be no longer a subject of dispute "whether Light be a Body." Observing, however, that it "is not so easie" to determine specifically "what Light is," he concluded, "I shall not mingle conjectures with certainties."[77]

Newton's letter was, as promised, read at the Royal Society on 6 February 1672. A week later Hooke delivered a report in which he criticized Newton for asserting a conclusion that did not seem to Hooke to follow necessarily from the experiments described, which—in any event—Hooke thought too few. Hooke had his own theory which, he claimed, could equally well explain Newton's experimental results.

In the controversy that followed with Hooke, Huygens, and others, Newton quickly discovered that he had not produced a convincing demonstration of the validity and significance of the conclusions he had drawn from his experiments. The objection was made that Newton had not explored the possibility that theories of color other than the one he had proposed might explain the phenomena. He was further criticized for having favored a corporeal hypothesis of light, and it was even said that his experimental results could not be reproduced.

In reply, Newton attacked the arguments about the "hypothesis" that he was said to have advanced about the nature of light, since he did not consider this issue to be fundamental to his interpretation of the "experimentum crucis." As he explained in reply to Pardies[78] he was not proposing "an hypothesis," but rather "properties of light" which could easily "be proved" and which, had he not held them to be true, he would "rather have . . . rejected as vain and empty speculation, than acknowledged even as an hypothesis." Hooke, however, persisted in the argument. Newton was led to state that he had deliberately declined all hypotheses so as "to speak of *Light* in *general* terms, considering it abstractly, as something or other propagated every way in straight lines from luminous bodies, without determining what that Thing is." But Newton's original communication did assert, "These things being so, it can be no longer disputed, whether there be colours in the dark, nor . . . perhaps, whether Light be a Body." In response to his critics, he emphasized his use of the word "perhaps" as evidence that he was not committed to one or another hypothesis on the nature of light itself.[79]

One consequence of the debate, which was carried on over a period of four years in the pages of the *Philosophical Transactions* and at meetings of the Royal Society, was that Newton wrote out a lengthy "Hypothesis Explaining the Properties of Light Discoursed of in my Several Papers,"[80] in which he supposed that light "is something or other capable of exciting vibrations in the aether," assuming that "there is an aetherial medium much of the same constitution with air, but far rarer, subtler, and more strongly elastic." He suggested the possibility that "muscles are contracted and dilated to cause animal motion," by the action of an "aethereal animal spirit," then went on to offer ether vibration as an explanation of refraction and reflection, of transparency and opacity, of the production of colors, and of diffraction phenomena (including Newton's rings). Even "the gravitating attraction of the earth," he supposed, might "be caused by the continual condensation of some other such like aethereal spirit," which need not be "the main body of phlegmatic aether, but . . . something very thinly and subtilly diffused through it."[81]

The "Hypothesis" was one of two enclosures that Newton sent to Oldenburg, in his capacity of secretary of the Royal Society, together with a letter dated 7 December 1675. The other was a "Discourse of Observations," in which Newton set out "such observations as conduce to further discoveries for completing his theory of light and colours, especially as to the constitution of natural bodies, on which their colours or transparency depend." It also contained Newton's account of his discovery of the "rings" produced by light passing through a thin wedge or layer of air between two pieces of glass. He had based his experiments on earlier ones of a similar kind that had been recorded by Hooke in his *Micrographia* (observation 9). In particular Hooke had described the phenomena occurring when the "lamina," or space between the two glasses, was "*double concave*, that is, thinner in the middle then at the edge"; he had observed "various coloured rings or lines, with differing consecutions or orders of Colours."

When Newton's "Discourse" was read at the Royal Society on 20 January 1676, it contained a paragraph (proposition 3) in which Newton referred to Hooke and the *Micrographia*, "in which book he hath also largely discoursed of this . . . and delivered many other excellent things concerning the colours of thin plates, and other natural bodies, which I have not scrupled to make use of so far as they were for my purpose."[82] In recasting the "Discourse" as parts 1, 2, and 3 of book II of the *Opticks*, however, Newton omitted this statement. It may be assumed that he had carried these experiments so much further than Hooke, introducing careful measurements and quantitative analysis, that he believed them to be his own. Hooke, on the other hand, understandably thought that he deserved more credit for his own contributions —including hypothesis-based explanations—than Newton was willing to allow him.[83] Newton ended the resulting correspondence on a conciliatory note when he wrote in a letter of 5 February 1676, "What Des-Cartes did was a good step. You have added much in several ways, and especially in taking the colours of thin plates into philosophical consideration. If I have seen further it is by standing on the shoulders of Giants."[84]

The opening of Newton's original letter on optics suggests that he began his prism experiments in 1666, presumably in his rooms in Trinity, but was interrupted by the plague at Cambridge, returning to this topic only two years later. Thus the famous eighteen months supposedly spent in Lincolnshire would mark a hiatus in his optical researches, rather than being the period in which he made his major discoveries concerning light and color. As noted earlier, the many pages of optical material in Newton's manuscripts[85] and notebooks have not yet been sufficiently analyzed to provide a precise record of the development of his experiments, concepts, and theories.

The lectures on optics that Newton gave on the assumption of the Lucasian chair likewise remain only incompletely studied. These exist as two complete, but very different, treatises, each with carefully drawn figures. One was deposited in the University Library, as required by the statutes of his professorship, and was almost certainly written out by his roommate, John Wickins,[86] while the other is in Newton's own hand and remained in his possession.[87] These two versions differ notably in their textual content, and also in their division into "lectures," allegedly given on specified dates. A Latin and an English version, both based on the deposited manuscript although differing in textual detail and completeness, were published after Newton's death. The English version, called *Optical Lectures*, was published in 1728, a year before the Latin. The second part of Newton's Latin text was not translated, since, according to the preface, it was "imperfect" and "has since been published in the *Opticks* by Sir Isaac himself with great improvements." The preface further states that the final two sections of this part are composed "in a manner purely Geometrical," and as such they differ markedly from the *Opticks*. The opening lecture (or section 1) pays tribute to Barrow and mentions telescopes, before getting down to the hard business of Newton's discovery "that . . . Rays [of light] in respect to the Quantity of Refraction differ from one another." To show the reader that he had not set forth "Fables instead of Truth," Newton at once gave "the Reasons and Experiments on which these things are founded." This account, unlike the later letter in the *Philosophical Transactions*, is not autobiographical; nor does it proceed by definitions, axioms, and propositions (proved "by Experiment"), as does the still later *Opticks*.[88]

R. S. Westfall has discussed the two versions of the later of the *Lectiones opticae*, which were first published in 1729;[89] he suggests that Newton eliminated from the *Lectiones* those "parts not immediately relevant to the central concern, the experimental demonstration of his theory of colors." Mathematical portions of the *Lectiones* have been analyzed by D. T. Whiteside, in Newton's *Mathematical Papers*, while J. A. Lohne and Zev Bechler have made major studies of Newton's manuscripts on optics. The formation of Newton's optical concepts and theories has been ably presented by A. I. Sabra; an edition of the *Opticks* is presently being prepared by Henry Guerlac.

Lohne finds great difficulty in repeating Newton's "experimentum crucis,"[90] but more important, he has traced the influence of Descartes, Hooke, and Boyle on Newton's work in optics.[91] He has further found that Newton used a prism in optical experiments much earlier than hitherto suspected—certainly before 1666, and probably before 1665—and has shown that very early in his optical research Newton was explaining his experiments by "the corpuscular hypothesis." In "Questiones philosophicae," Newton wrote: "Blue rays are reflected more than red rays, because they are slower. Each colour is caused by uniformly moving globuli. The uniform motion which gives the sensation of one colour is different from the motion which gives the sensation of any other colour."[92]

Accordingly, Lohne shows how difficult it is to accept the historical narrative proposed by Newton at the beginning of the letter read to the Royal Society on 8 February 1672 and published in the *Philosophical Transactions*. He asks why Newton should have been surprised to find the spectrum oblong, since his "note-books represent the sunbeam as a stream of slower and faster globules occasioning different refrangibility of the different colours?" Newton must, according to Lohne, have "found it opportune to let his theory of colours appear as a Baconian induction from experiments, although it primarily was deduced from speculations." Sabra, in his analysis of Newton's narrative, concludes that not even "the 'fortunate Newton' could have been fortunate enough to have achieved this result in such a smooth manner." Thus one of the most famous examples of the scientific method in operation now seems to have been devised as a sort of scenario by which Newton attempted to convey the impression of a logical train of discovery based on deductions from experiment. The historical record, however, shows that Newton's great leap forward was actually a consequence of implications drawn from profound scientific speculation and insight.[93]

In any event, Newton himself did not publish the *Lectiones opticae*, nor did he produce his planned annotated edition of at least some (and maybe all) of his letters on light and color published in the *Philosophical Transactions*.[94] He completed his English *Opticks*, however, and after repeated requests that he do so, allowed it to be printed in 1704, although he withheld his name, save on the title page of one known copy. It has often been alleged that Newton released the *Opticks* for publication only after Hooke—the last of the original objectors to his theory of light and colors—had died. David Gregory, however, recorded another reason for the publication of the *Opticks* in 1704: Newton, Gregory wrote, had been "provoked" by the appearance, in 1703, of George Cheyne's *Fluxionum methoda inversa* "to publish his [own tract on] Quadratures, and with it, his Light & Colours, &c."[95]

In the *Opticks*, Newton presented his main discoveries and theories concerning light and color in logical order, beginning with eight definitions and eight axioms.[96] Definition 1 of book I reads: "By the Rays of Light I understand its least Parts, and those as well Successive in the same Lines, as Contemporary in several Lines." Eight propositions follow, the first stating that "Lights which differ in Colour, differ also in Degrees of Refrangibility." In appended experiments Newton discussed the appearance of a paper colored half red and half blue when viewed through a prism and showed that a given lens produces red and blue images, respectively, at different distances. The second proposition incorporates a variety of prism experiments as proof that "The Light of the Sun consists of Rays differently refrangible."

The figure given with experiment 10 of this series illustrates "two Prisms tied together in the form of a Parallelopiped" (Fig. 3). Under specified conditions, sunlight entering a darkened room through a small hole F in the shutter would not be refracted by the parallelopiped and would emerge parallel to the incident beam FM, from which it would pass by refraction through a third prism IKH, which would by refraction "cast the usual Colours of the Prism upon the opposite Wall." Turning the parallelopiped about its axis, Newton found that the rays producing the several colors were successively "taken out of the transmitted Light" by "total Reflexion"; first "the Rays which in the third Prism had suffered the greatest Refraction and painted [the wall] with violet and blew were . . . taken out of the transmitted Light, the rest remaining," then the rays producing green, yellow, orange, and red were "taken out" as the parallelopiped was rotated yet further. Newton thus experimentally confirmed the "experimentum crucis," showing that the light emerging from the two prisms "is compounded of Rays differently Refrangible, seeing [that] the more Refrangible Rays may be taken out while the less Refrangible remain." The arrangement of prisms is the basis of the important discovery reported in book II, part 1, observation 1.

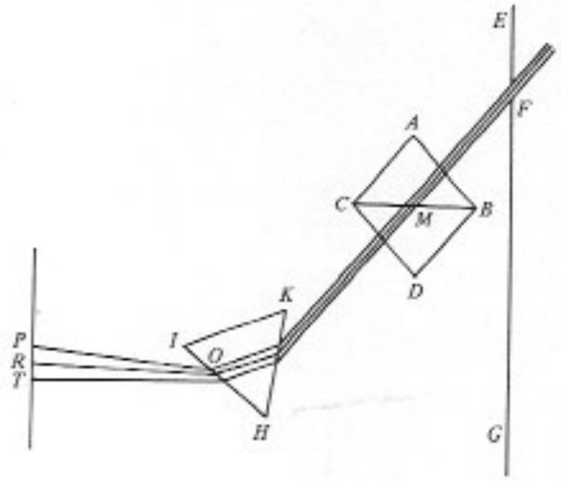

FIGURE 3

In proposition 6 Newton showed that, contrary to the opinions of previous writers, the sine law actually holds for each single color. The first part of book I ends with Newton's remarks on the impossibility of improving telescopes by the use of color-corrected lenses and his discussion of his consequent invention of the reflecting telescope (Fig. 4).

In the second part of book I, Newton dealt with colors produced by reflection and refraction (or transmission), and with the appearance of colored objects in relation to the color of the light illuminating them. He discussed colored pigments and their mixture and geometrically constructed a color wheel, drawing an analogy between the primary colors in a compound color and the "seven Musical Tones or Intervals of the eight Sounds, *Sol, la, fa, sol, la, mi, fa, sol. . . .*"[97]

Proposition 9, "Prob. IV. By the discovered Properties of Light to explain the Colours of the Rain-bow," is devoted to the theory of the rainbow. Descartes had developed a geometrical theory, but had

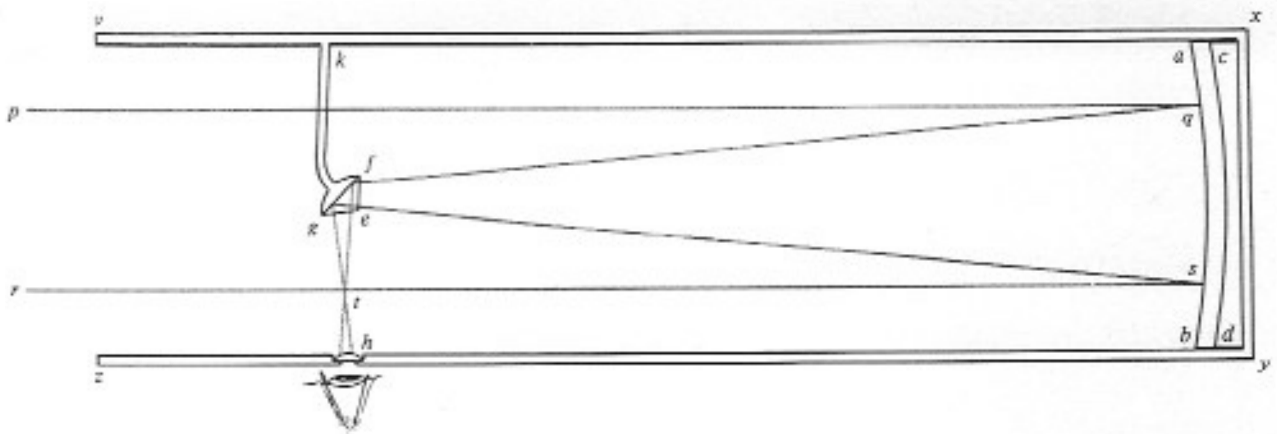

FIGURE 4. Newton's method "To shorten Telescopes": *efg* represents the prism, *abcd* the speculum, and *h* the lens.

used a single index of refraction (250:187) in his computation of the path of light through each raindrop.[98] Newton's discovery of the difference in refrangibility of the different colors composing white light, and their separation or dispersion as a consequence of refraction, on the other hand, permitted him to compute the radii of the bows for the separate colors. He used 108:81 as the index of refraction for red and 109:81 for violet, and further took into consideration that the light of the sun does not proceed from a single point. He determined the widths of the primary and secondary bows to be 2°15′ and 3°40′, respectively, and gave a formula for computing the radii of bows of any order n (and hence for orders of the rainbow greater than 2) for any given index of refraction.[99] Significant as Newton's achievement was, however, he gave only what can be considered a "first approximation to the solution of the problem," since a full explanation, particularly of the supernumerary or spurious bows, must require the general principle of interference and the "rigorous application of the wave theory."

Book II, which constitutes approximately one third of the *Opticks*, is devoted largely to what would later be called interference effects, growing out of the topics Newton first published in his 1675 letter to the Royal Society. Newton's discoveries in this regard would seem to have had their origin in the first experiment that he describes (book II, part 1, observation 1); he had, he reported, compressed "two Prisms hard together that their sides (which by chance were a very little convex) might somewhere touch one another" (as in the figure provided for experiment 10 of book I, part 1). He found "the place in which they touched" to be "absolutely transparent," as if there had been one "continued piece of Glass," even though there was total reflection from the rest of the surface; but "it appeared like a black or dark spot, by reason that little or no sensible light was reflected from thence, as from other places." When "looked through," it seemed like "a hole in that Air which was formed into a thin Plate, by being compress'd between the Glasses." Newton also found that this transparent spot "would become much broader than otherwise" when he pressed the two prisms "very hard together."

Rotating the two prisms around their common axis (observation 2) produced "many slender Arcs of Colours" which, the prisms being rotated further, "were compleated into Circles or Rings." In observation 4 Newton wrote that

> To observe more nicely the order of the Colours . . . I took two Object-glasses, the one a Plano-convex for a fourteen Foot Telescope, and the other a large double Convex for one of about fifty Foot; and upon this, laying the other with its plane side downwards, I pressed them slowly together, to make the Colours successively emerge in the middle of the Circles, and then slowly lifted the upper Glass from the lower to make them successively vanish again in the same place.

It was thus evident that there was a direct correlation between particular colors of rings and the thickness of the layer of the entrapped air. In this way, as Mach observed, "Newton acquired a complete insight into the whole phenomenon, and at the same time the possibility of determining the thickness of the air gap from the known radius of curvature of the glass."[100]

Newton varied the experiment by using different lenses, and by wetting them, so that the gap or layer was composed of water rather than air. He also studied the rings that were produced by light of a single color,

NEWTON

separated out of a prismatic spectrum; he found that in a darkened room the rings from a single color extended to the very edge of the lens. Furthermore, as he noted in observation 13, "the Circles which the red Light made" were "manifestly bigger than those which were made by the blue and violet"; he found it "very pleasant to see them gradually swell or contract accordingly as the Colour of the Light was changed." He concluded that the rings visible in white light represented a superimposition of the rings of the several colors, and that the alternation of light and dark rings for each color must indicate a succession of regions of reflection and transmission of light, produced by the thin layer of air between the two glasses. He set down the latter conclusion in observation 15: "And from thence the origin of these Rings is manifest; namely that the Air between the Glasses, according to its various thickness, is disposed in some places to reflect, and in others to transmit the Light of any one Colour (as you may see represented . . .) and in the same place to reflect that of one Colour where it transmits that of another" (Fig. 5).

Book II, part 2, of the *Opticks* has a nomogram in which Newton summarized his measures and computations and demonstrated the agreement of his analysis of the ring phenomenon with his earlier conclusions drawn from his prism experiments—"that whiteness is a dissimilar mixture of all Colours, and that Light is a mixture of Rays endued with all those Colours." The experiments of book II further confirmed Newton's earlier findings "that every Ray have its proper and constant degree of Refrangibility connate with it, according to which its refraction is ever justly and regularly perform'd," from which he argued that "it follows, that the colorifick Dispositions of Rays are also connate with them, and immutable." The colors of the physical universe are thus derived "only from the various Mixtures or Separations of

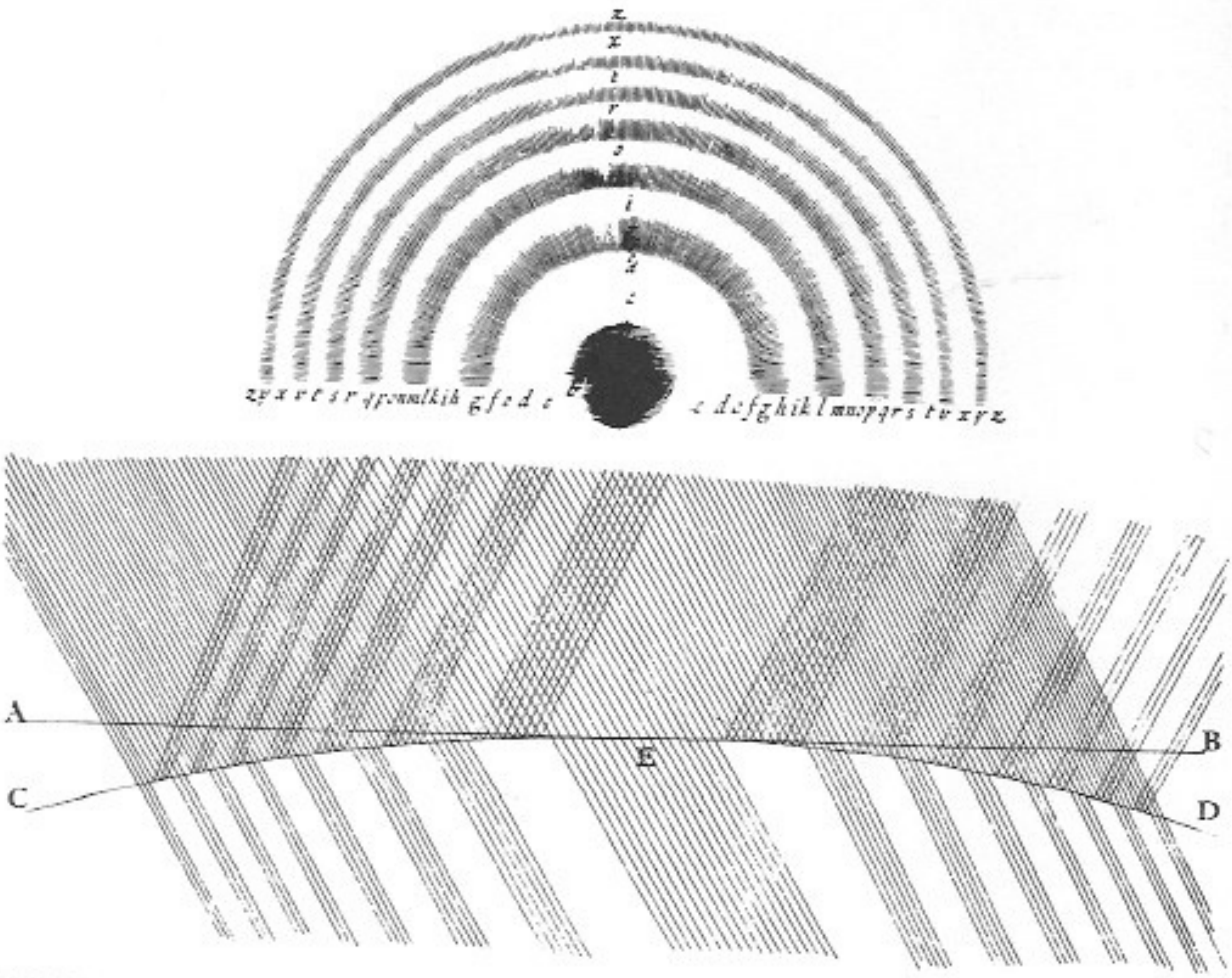

FIGURE 5. Two drawings from book II, part 1, plate 1 of the 1704 edition of the *Opticks*, illustrating Newton's studies of what are now called Newton's rings.

Rays, by virtue of their different Refrangibility or Reflexibility"; the study of color thus becomes "a Speculation as truly mathematical as any other part of Opticks."[101]

In part 3 of book II, Newton analyzed "the permanent Colours of natural Bodies, and the Analogy between them and the Colours of thin transparent Plates." He concluded that the smallest possible subdivisions of matter must be transparent, and their dimensions optically determinable. A table accompanying proposition 10 gives the refractive powers of a variety of substances "in respect of . . . Densities." Proposition 12 contains Newton's conception of "fits":

> Every Ray of Light in its passage through any refracting Surface is put into a certain transient Constitution or State, which in the progress of the Ray returns at equal Intervals, and disposes the Ray at every return to be easily transmitted through the next refracting Surface, and between the returns to be easily reflected by it.

The succeeding definition is more specific: "The returns of the disposition of any Ray to be reflected I will call its *Fits of easy Reflection*, and those of its disposition to be transmitted its *Fits of easy Transmission*, and the space it passes between every return and the next return, the *Interval of its Fits*."

The "fits" of easy reflection and of easy refraction could thus be described as a numerical sequence; if reflection occurs at distances 0, 2, 4, 6, 8, · · · , from some central point, then refraction (or transmission) must occur at distances 1, 3, 5, 7, 9, · · · . Newton did not attempt to explain this periodicity, stating that "I do not here enquire" into the question of "what kind of action or disposition this is." He declined to speculate "whether it consists in a circulating or a vibrating motion of the Ray, or of the Medium, or something else," contenting himself "with the bare Discovery, that the Rays of Light are by some cause or other alternately disposed to be reflected or refracted for many vicissitudes."

Newton thus integrated the periodicity of light into his theoretical work (it had played only a marginal part in Hooke's theory). His work was, moreover, based upon extraordinarily accurate measurements—so much so that when Thomas Young devised an explanation of Newton's rings based on the revived wave theory of light and the new principle of interference, he used Newton's own data to compute the wavelengths and wave numbers of the principal colors in the visible spectrum and attained results that are in close agreement with those generally accepted today.

In part 4 of book II, Newton addressed himself to "the Reflexions and Colours of thick transparent polish'd Plates." This book ends with an analysis of halos around the sun and moon and the computation of their size, based on the assumption that they are produced by clouds of water or by hail. This led him to the series of eleven observations that begin the third and final book, "concerning the Inflexions of the Rays of Light, and the Colours made thereby," in which Newton took up the class of optical phenomena previously studied by Grimaldi,[102] in which "fringes" are produced at the edges of the shadows of objects illuminated by light "let into a dark Room through a very small hole." Newton discussed such fringes surrounding the projected shadows of a hair, the edge of a knife, and a narrow slit.

Newton concluded the first edition of the *Opticks* (1704) with a set of sixteen queries, introduced "in order to a further search to be made by others." He had at one time hoped he might carry the investigations further, but was "interrupted," and wrote that he could not "now think of taking these things into farther Consideration." In the eighteenth century and after, these queries were considered the most important feature of the *Opticks*—particularly the later ones, which were added in two stages, in the Latin *Optice* of 1706 and in the second English edition of 1717–1718.

The original sixteen queries at once go beyond mere experiments on diffraction phenomena. In query 1, Newton suggested that bodies act on light at a distance to bend the rays; and in queries 2 and 3, he attempted to link differences in refrangibility with differences in "flexibility" and the bending that may produce color fringes. In query 4, he inquired into a single principle that, by "acting variously in various Circumstances," may produce reflection, refraction, and inflection, suggesting that the bending (in reflection and refraction) begins before the rays "arrive at the Bodies." Query 5 concerns the mutual interaction of bodies and light, the heat of bodies being said to consist of having "their parts [put] into a vibrating motion"; while in query 6 Newton proposed a reason why black bodies "conceive heat more easily from Light than those of other Colours." He then discussed the action between light and "sulphureous" bodies, the causes of heat in friction, percussion, putrefaction, and so forth, and defined fire (in query 9) and flame (in query 10), discussing various chemical operations. In query 11, he extended his speculations on heat and vapors to sun and stars. The last four queries (12 to 16) of the original set deal with vision, associated with "Vibrations" (excited by "the Rays of Light") which cause sight by "being propagated along the solid Fibres of the optick Nerves into the Brain." In query 13 specific wavelengths are associated with each of several colors. In query 15 Newton discussed binocular vision, along with other aspects of seeing, while in query 16 he took up the phenomenon of persistence of vision.

Newton has been much criticized for believing dispersion to be independent of the material of the prism and for positing a constant relation between deviation and dispersion in all refractive substances. He thus dismissed the possibility of correcting for chromatic aberration in lenses, and directed attention from refraction to reflecting telescopes.[103]

Newton is often considered to be the chief advocate of the corpuscular or emission theory of light. Lohne has shown that Newton originally did believe in a simple corpuscular theory, an aspect of Newton's science also forcibly brought out by Sabra. Challenged by Hooke, Newton proposed a hypothesis of ether waves associated with (or caused by) these corpuscles, one of the strongest arguments for waves probably being his own discovery of periodicity in "Newton's rings." Unlike either Hooke or Huygens, who is usually held to be the founder of the wave theory but who denied periodicity to waves of light, Newton postulated periodicity as a fundamental property of waves of (or associated with) light, at the same time that he suggested that a particular wavelength characterizes the light producing each color. Indeed, in the queries, he even suggested that vision might be the result of the propagation of waves in the optic nerves. But despite this dual theory, Newton always preferred the corpuscle concept, whereby he might easily explain both rectilinear propagation and polarization, or "sides." The corpuscle concept lent itself further to an analysis by forces (as in section 14 of book I of the *Principia*), thus establishing a universal analogy between the action of gross bodies (of the atoms or corpuscles composing such bodies), and of light. These latter topics are discussed below in connection with the later queries of the *Opticks*.

Dynamics, Astronomy, and the Birth of the "Principia." Newton recorded his early thoughts on motion in various student notebooks and documents.[104] While still an undergraduate, he would certainly have studied the Aristotelian (or neo-Aristotelian) theory of motion and he is known to have read Magirus' *Physiologiae peripateticae libri sex*; his notes include a "Cap:4. De Motu" (wherein "Motus" is said to be the Aristotelian ἐντελέχεια). Extracts from Magirus occur in a notebook begun by Newton in 1661;[105] it is a repository of jottings from his student years on a variety of physical and non-physical topics. In it Newton recorded, among other extracts, Kepler's third law, "that the mean distances of the primary Planets from the Sunne are in sesquialter proportion to the periods of their revolutions in time."[106] This and other astronomical material, including a method of finding planetary positions by approximation, comes from Thomas Streete's *Astronomia Carolina*.

Here, too, Newton set down a note on Horrox' observations, and an expression of concern about the vacuum and the gravity of bodies; he recorded, from "Galilaeus," that "an iron ball" falls freely through "100 braces Florentine or cubits [or 49.01 ells, perhaps 66 yards] in 5" of an hower." Notes of a later date—on matter, motion, gravity, and levity—give evidence of Newton's having read Charleton (on Gassendi), Digby (on Galileo), Descartes, and Henry More.

In addition to acquiring this miscellany of information, making tables of various kinds of observations, and supplementing his reading in Streete by Wing (and, probably, by Galileo's *Sidereus nuncius* and Gassendi's epitome of Copernican astronomy), Newton was developing his own revisions of the principles of motion. Here the major influence on his thought was Descartes (especially the *Principia philosophiae* and the Latin edition of the correspondence, both of which Newton cited in early writings), and Galileo (whose *Dialogue* he knew in the Salusbury version, and whose ideas he would have encountered in works by Henry More, by Charleton and Wallis, and in Digby's *Two Essays*).

An entry in Newton's Waste Book,[107] dated 20 January 1664, shows a quantitative approach to problems of inelastic collision. It was not long before Newton went beyond Descartes's law of conservation, correcting it by algebraically taking into account direction of motion rather than numerical products of size and speed of bodies. In a series of axioms he declared a principle of inertia (in "Axiomes" 1 and 2); he then asserted a relation between "force" and change of motion; and he gave a set of rules for elastic collision.[108] In "Axiome" 22, he had begun to approach the idea of centrifugal force by considering the pressure exerted by a sphere rolling around the inside surface of a cylinder. On the first page of the Waste Book, Newton had quantitated the centrifugal force by conceiving of a body moving along a square inscribed in a circle, and then adding up the shocks at each "reflection." As the number of sides were increased, the body in the limiting case would be "reflected by the sides of an equilateral circumscribed polygon of an infinite number of sides (i.e. by the circle it selfe)." Herivel has pointed out the near equivalence of such results to the early proof mentioned by Newton at the end of the scholium to proposition 4, book I, of the