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LECTURES ON TEN BRITISH PHYSICISTS OF THE NINETEENTH CENTURY

BY

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SIR GEORGE GABRIEL STOKES *

(1819–1903)

GEORGE GABRIEL STOKES was born August 13, 1819, at Skreen, County Sligo, Ireland. His father was the rector of the parish, a clergyman of the Church of England in Ireland. When twelve years of age he was sent to a school in Dublin and two years later to Bristol College in the West of England. At this time the principal of that college was Dr. Jerrard whose researches on the solution of equations of the fifth degree were discussed by Sir William Rowan Hamilton at the Bristol meeting of the British Association in 1836. In 1837 young Stokes entered Pembroke College, Cambridge, and four years later graduated as senior wrangler, won the first Smith's prize, and was elected to a fellowship. During all his school and college years he had won distinction in mathematical studies.

He now did what was a great novelty in those days—turned one of his rooms into a physical laboratory. The University had no lecture rooms for its professors, far less laboratories or museums. Being a powerful analyst as well as a skillful experimenter he immediately entered on a period of fruitful scientific production. He chose as channels of publication the two institutions which had been recently inaugurated at Cambridge, namely, the Cambridge Philosophical Society, and the Cambridge and Dublin Mathematical Journal. To the former he contributed two papers on pure mathematical analysis, namely, "on the critical values of the sums of periodic series," based on Fourier's analysis of periodic functions; and another "On the numerical calculation of a class of definite integrals and infinite series," in which he was able to calculate the first

* This Lecture was delivered on April 28, 1903.—EDITORS.

fifty roots of an equation of which Airy had been able to calculate only two. Other memoirs followed: "On the theories of the internal friction of fluids in motion and of the equilibrium and motion of elastic solids," in which he shows for the first time how to take account of the equations of motion, of differences of pressure in different directions due to the viscosity of the fluid; and the resulting equations constitute the complete foundation of the hydrokinetics of the present day. "On the theory of oscillatory waves," in which he investigates the steep waves of the deep sea where the elevations are narrower than the hollows and the height of an elevation exceeds the depth of a hollow. "On the formation of the central spot of Newton's rings beyond the critical angle," his earliest investigation in the wave-theory of light. "On the dynamical theory of diffraction," containing the mathematical theory of the propagation of motion in a homogeneous elastic solid; also an experimental investigation from which he concluded that the plane of polarization is the plane perpendicular to the direction of vibration in plane-polarized light, agreeing with Fresnel's position as opposed to that of MacCullagh.

To the *Cambridge* and *Dublin Mathematical Journal* he contributed the following papers: "On the motion of a piston and of the air in a cylinder"; "On a formula for determining the optical constants of doubly refracting crystals"; "On attractions and on Clairault's theorem." A series of notes on hydrodynamics was prepared supplementary to a report on that subject which he presented to the British Association in 1846. Shorter papers he communicated to the Philosophical Magazine, two of which are the aberration of light and the constitution of the luminiferous ether viewed with reference to that phenomenon. On the theory of the emission of light, the explanation of aberration is simple; in these papers he attempts an explanation which shall be in accordance with the undulatory theory without making the startling supposition that the earth in its motion round the sun experiences no resistance from the ether.

In 1849 the Lucasian Chair of Mathematics at Cambridge fell vacant—the chair filled by Sir Isaac Newton 180 years earlier. From 1828 to 1839 this chair was occupied by Charles Babbage who neither lectured nor resided; his successor, Joseph King, seems also to have made it a sinecure. But now the electors—who are the heads of the colleges—saw in Stokes a young, talented, and enthusiastic investigator who might worthily follow in the steps of Newton. At the time of the election Peter Guthrie Tait was an undergraduate and twenty-five years later he recorded his impression of the event: "To us, who were mere undergraduates when he was elected to the Lucasian professorship, but who had with mysterious awe speculated on the relative merits of the men of European fame whom we expected to find competing for so high an honor, the election of a young, and to us unknown, candidate was a very striking phenomenon. But we were still more startled, a few months afterwards, when the new professor gave public notice that he considered it part of the duties of his office to assist any member of the University in difficulties he might encounter in his mathematical studies. Here was, we thought (in the language which Scott puts into the mouth of Richard Coeur de Lion) "a single knight fighting against the whole mêlée of the tournament." But we soon discovered our mistake, and felt that the undertaking was the effort of an earnest sense of duty or the conscience of a singularly modest but exceptionally able and learned man. And as our own knowledge gradually increased and we became able to understand his numerous original investigations, we saw more and more clearly that the electors had indeed consulted the best interests of the University, and that the proffer of assistance was something whose benefits were as certain to be tangible and real as any that mere human power and knowledge could guarantee."

Tait himself benefited by this proffer of assistance; so did Thomson and Clerk Maxwell. In fact Prof. Stokes is regarded as the principal founder of the Cambridge school of mathematical physicists, one of the main glories of the British mathematicians of the nineteenth century, the only other name having any claim to the position being that of William Hopkins who tutored them all. Thus at the age of 35 years Stokes was placed in the position where he was to do his life work. At that time the salary attached to the chair was small; the colleges collected all the revenues, and the University proper had very little for the payment of her officers.

Before his appointment to the Lucasian chair, Stokes had contributed a paper to the *Transactions* of the Royal Society of London: "On the theory of certain bands in the spectrum." He was now (1851) elected a Fellow of the Society. Two years later he was appointed one of the secretaries, an office which he continued to hold for thirty years. In 1852 he contributed an important paper "On the change of refrangibility of light" for which he received a Rumford medal, and which is considered his greatest contribution to science. Sir John Herschel had discovered a phenomenon, now called fluorescence, in the behavior of a solution of sulphate of quinine when a beam of light strikes on it. Viewed by transmitted light, the liquid appears colorless and transparent like water, but viewed by reflected light it exhibits a peculiar blue color. This blue color comes from a narrow stratum of the liquid adjacent to the surface by which the light enters. Light, which has once produced this effect, though unaltered apparently by transmission through the liquid, cannot produce the blue stratum in a posterior solution. Stokes reasoned that certain invisible rays in the beam are changed into visible rays—the blue rays; which means that certain waves of a length too small to be seen are, by incidence on the molecules of the solution, transformed into waves of greater length so as to become visible. How the change takes place is not known; but what Stokes did establish was that the appearance of the visible blue light was due to disappearance of certain invisible light rays. In the substances which Stokes examined, the change was in every case to greater wave-length; on which he based an induction that the change was always from smaller to greater wave-length, an induction which in more recent years has been overturned.

Soon after he contributed to the Cambridge Philosophical Society a paper "On the effect of the internal friction of fluids on the motion of pendulums." In this he investigates the motion of a pendulum which has for its bob a globe and moves in a viscous fluid contained in a spherical envelope concentric with the bob when at rest; and also the motion of a globe moving uniformly with a small velocity through a mass of viscous fluid. He applies the result of the second investigation to explain the suspension of clouds in the air; and determined from the known viscosity of air the terminal velocity of an exceedingly minute globule of water falling through it. Up to this time the motion of a pendulum had been corrected for buoyancy and for the inertia of the air; Stokes supplied the correction for viscosity.

In 1857 he married, and in consequence of the provision of the statute governing the colleges, his fellowship became vacant. On account of this diminished income he took more work, such as Lectures at the School of Mines in London. When the colleges were reformed (about 1875) fellows engaged in teaching in the University were allowed to retain their fellowships after marriage; and in the case of Stokes the provision was applied extro-actively, and he was reinstated a Fellow of Pembroke College. Professor Stokes not only lectured to the junior members of the University and advised the senior members in questions of applied mathematics, but he was also very helpful to scientists in general. He was in applied mathematics and physics what Cayley was in pure mathematics—a valuable referee and advisor in the work of others. He had the true spirit of a philosopher, more anxious to see science advance than that he should have priority in the advancement. Lord Kelvin has stated that before he removed from Cambridge in 1852, Stokes explained to him the principles of spectrum analysis upon which solar and stellar chemistry has been founded, a work which was afterwards carried out fully by Balfour Stewart and Kirchoff. The following is the account which Stokes himself gives.

In 1849 Foucault accidentally observed that in a solar beam which had traversed the electric arc between two carbon poles, the double dark line *D* appeared darker than usual, and the bright *D* line was seen in precisely the same place in the spectrum of light coming from the electric arc; Stokes was informed by Foucault of this observation a few years later. It seemed to Stokes that a dynamical illustration of how a medium could act both by emission and absorption for light of a definite refrangibility was not far to seek. He says: "I imagine a series of stretched wires, like pianoforte wires, all turned to the same note. The series, if agitated, suppose by being struck, would give out that note, which on the other hand it would be capable of taking up if sounded in air. To carry out the analogy, we have only to suppose a portion of the molecules constituting the vapor of the arc to be endowed with a capacity of vibrating in a definite manner, that is according to a definite time of vibration. But what were these molecules? It is well known that the bright *D* line in flames is specially characteristic of compounds of sodium, though from its very general occurrences some had doubted whether it were not really due to something else. But in what condition must we suppose the sodium in the arc to be? The compounds of sodium, such as common salt, carbonate of soda, etc., are colorless; and it would be contrary to the analogy of what we know as to the relation of gases and vapors to their liquids or solutions to suppose that a gas which does exercise absorption should be merely the vapor of a heated solid which does not. On this ground it seemed to me that the substratum which exercised the selective absorption in Foucault's experiment must be free sodium. This might be conceivably set free from its compounds in the intense actions which go on in the sun or in the electric arc; but I had not thought that a body of such powerful affinities would be set free in the gentle flame of a spirit lamp nor experienced that the fact of that flame emitting light of the indefinite refrangibility of *D* entails of *necessity* that it should absorb light of that same refrangibility."

In 1869 Stokes was president of the British Association at a meeting in Exeter. His address was devoted chiefly to recent progress in spectrum analysis to which Mr. Huggins had just applied Döppler's principle in the theory of sound and deduced that Sirius is receding from the Sun at the rate of 30 miles per second. Stokes closed his address with some observations on life and mind these being characteristic of his philosophical attitude which was that of the golden mean. He says "What this something which we call life may be, is a profound mystery. We know not how many links in the chain of secondary causation may yet remain behind; we know not how few. It would be presumptuous indeed to assume in any case that we had already reached the *last link*, and to charge with irreverence a fellow worker who attempted to push his investigations *yet one step* farther back. On the other hand, if a thick darkness enshrouds all beyond, we have no right to assume it to be impossible that we should have reached even the last link of the chain, a stage where further progress is unattainable, and we can only refer the highest law at which we stopped to the fiat of an Almighty Power. . . . When from the phenomena of life we pass on to those of mind we enter a region still more profoundly mysterious. We can readily imagine that we may here be dealing with phenomena altogether transcending those of mere life, in some such way as those of life transcend, as I have endeavored to infer those of chemistry and molecular attractions, or as the laws of chemical affinity in their turn transcend those of mere mechanics. Science can be expected to do but little to aid us here, since the instrument of research is itself the object of investigation. It can but enlighten us to the depth of our ignorance and lead us to look to a higher aid for that which most nearly concerns our well-being."

In 1880 the Cambridge University Press began the republication in collected form of Stokes' *Mathematical and Physical Papers*. In this publication he introduced for the first time the *solidus* notation for division, originally introduced by De Morgan in his article on the Calculus of Functions in the *Encyclopædia Metropolitana*. If a fraction like $\frac{a}{b}$, or a differential coefficient such as $\frac{dy}{dx}$, is mentioned in the text, the printing of such expres-

sions requires a good deal of "justification" on the part of the compositor. To avoid this expense and the loss of space Stokes introduced the linear notation a/b and dy/dx. The symbol: and ÷ likewise indicate division but he did not use them in the text. He did not use / in writing out centered equations, excepting where it is needed to simplify the index of an exponential function. He considered it convenient to enact that the solidus shall as far as possible take the place of the horizontal bar for which it stands and accordingly that the quantities immediately preceding and following shall be welded into one, the welding action to be arrested by a period. For example m^2-n^2/m^2+n^2 is to mean $(m^2-n^2)/(m^2+n^2)$, and a/bcd means $\frac{a}{bcd}$, but $a/bc.d$ means $\frac{a}{bc}d$.

This solidus notation for algebraic expressions occurring in the text has since been used in the Encyclopedia Brittanica, in Wiedemann's Annalen and quite generally in mathematical literature. The solidus may be viewed as a symbol of operation, denoting reciprocal in the same way as $\sqrt{\ }$ denotes square root and as $-$ denotes reverse. The expression $/a$ is a sufficient notation for the reciprocal of a; in $1/a$ the figure 1 is redundant, just as in $0-a$ the 0 is redundant. The horizontal bar serves the two-fold purpose of a vinculum and a sign for reciprocal. When the reciprocal idea is detached and denoted by /, rules for the manipulation of / can be enunciated; thus $1/a=a$; $(/a)(/b)=1ab$, just as $\sqrt{a}\sqrt{b}=\sqrt{ab}$. The notation of algebra is in fact planar; its complete reduction to a linear form is not a simple matter and was not tackled by Stokes, but this has been attempted by later writers, some of whom write *exp x* for e^x. One indeed has proposed to use $\diagdown$ for involution and | instead of a bracket so that $c|(d+e)^3$ would be written $c/|d+e\diagdown 3|$.

In the winters of 1883-4-5 Prof. Stokes delivered in Aberdeen, Scotland, three courses of lectures on Light, under the auspices of the Burnett trust. In 1784 John Burnett, a merchant of Aberdeen, died, bequeathing a portion of his property to establish prizes for the best and next best essay on the following subject: "That there is a Being, all powerful, wise, and good, by whom everything exists; and particularly to obviate difficulties regarding the wisdom and goodness of the Deity; and, this in the first place, from considerations independent of written revelation of the Lord Jesus; and from the whole to point out influences most necessary for and useful to mankind." The prizes were to be competed for at intervals of forty years; and awards were actually made on two occasions. On account of the length of the interval the trustees began to think that the endowment might be better applied, and they obtained authority to change the funds so as to appoint special lecturers who should be appointed for three years, the courses to be given at intervals of five years and to cover subjects with special regard to the object of the testator. Prof. Stokes was the first lecturer appointed.

The subject of his first lecture was the Nature of Light. He brings out prominently Newton's difficulty in the hypothesis of undulation—that light should produce rays and sharp shadows while sound does not; and Brewsters' difficulty that space should be filled with an ether in order that the light of yon twinkling star may come to us. And he concludes with this lesson: "It may be said, if the former emission theory is nowadays exploded, why dwell on it at all? Yet surely the subject is of more than purely historical interest. It teaches lessons for our future guidance in the pursuit of truth. It shows that we are not to expect to evolve the system of nature out of the depths of our inner consciousness, but to follow the painstaking inductive method of studying the phenomena presented to us, and be content gradually to learn new laws and properties of natural objects. It shows that we are not to be disheartened by some preliminary difficulties from giving a patient hearing to a hypothesis of fair promise, assuming of course that those difficulties are not of the nature of contradictions between the results of observation and experiment and conclusions certainly deducible from the hypothesis on trial. It shows that we are not to attach undue importance to great names, but to investigate in an unbiased manner the facts which lie open to an examination."

In his second course of lectures he treated of light as a means of investigation. One of the objects taken up was the nature of comets. He held that the nucleus consists, in its inner portions at least, of vapor of some kind in an incandescent state. To explain the cause of this incandescence he brings forward the "greenhouse theory." The glass of a greenhouse is transparent to the higher but opaque to the lower forms of radiation, and hence acts as a trap for the sun's rays. The nucleus of the comet he supposed to be surrounded by an envelope of some kind, transparent to the higher but opaque to the lower forms of radiation. Thus solar heat can get freely at the nucleus, but cannot escape until it has raised the nucleus, in part at least, to incandescence. The coma and tail are formed by the condensation of small quantities of this vapor, so that they are mere mists of excessive tenuity. Prof. Tait preferred his own "brickbat theory"; he considered that Stokes' theory made the comet resemble the huge but barely palpable Efreet of the *Arabian Nights*, who could condense himself so as to enter the bottle of brass with the seal of Solomon the son of David. (*Nature*, August 20, 1885.)

The third course of lectures treated of the beneficial effects of light. As regards the special application contemplated by Burnett, he concludes: "If we shut our eyes to the grandeur of Nature and do not attempt, through the things that are made, to acquire higher conceptions of the eternal power and Godhood of the Mâker, our conceptions of the Divine Being are apt to become too anthropomorphic. If on the other hand we confine our attention to the study of Nature in all its immensity, our conceptions of its Author are in danger of merging in a sort of pantheistic abstraction in which the idea of personality is lost." Tait remarked with reference to these sentences that the first Burnett lectures had set a noble example to successors, and that Stokes had supplied a valuable warning not only to them but "to the rapidly changing quaternion of neoteleologists that were soon to be set to work in the Scottish Universities." He referred to the new institution of the Gifford Lecturers.

The second volume of Stokes' *Mathematical and Physical Papers* was published in 1883; the third in 1901. This long delay was due to the fact that his time was engrossed by scientific business; in his later years he seems to have had little ambition in the direction of scientific publication. In 1885 Prof. Stokes after having discharged the duties of Secretary of the Royal Society for thirty years, was elected President, which office he held for the usual period of five years. For twenty years after 1887 he represented the University of Cambridge in Parliament. In 1889 he received the honor of a baronetcy.

In 1891 Sir George Stokes was made one of the changing quaternion to which Tait referred; he was appointed, by the University of Edinburgh, lecturer on the Gifford foundation. Lord Gifford, one of the judges of the High Court of Justice in Scotland, died about 1887, leaving by his will a sum of money to each of the Scottish Universities. The object of the endowment was to appoint for one or two years a thinker, who might not belong to any Christian denomination provided only he was a true and reverent inquirer after truth, to deliver a course of public lectures on some point bearing on Natural Theology, treating the subject just as any other science. Stokes delivered two courses of lectures in 1891 and 1893. Trained in Cambridge University where little attention was paid to philosophy, he seems to have felt a difficulty in treating Natural Theology "just as any other science" and he could not speak with the same authority as when he was discoursing on light.

Four years ago (in 1899) the University of Cambridge celebrated in brilliant style the jubilee of his professorship. Delegates were invited from the learned societies; sixty-eight of them, mostly British, were represented. The celebration of the jubilee began with the delivery of the Rede lecture by Prof. Cornu of the école polytechnique of Paris; the endowment for this lecture dates back to 1524. The subject was appropriate to the occasion: "The wave-theory of light, its influence on modern physics." At an evening reception a bust of Stokes was presented to Pembroke College and a replica to the University. The presentation was made by Lord Kelvin who

said that Sir George Stokes had published in his own name but a very small part of the good he had done to the world. At the principal function, which was held in the Senate house, the delegates were received by the Vice Chancellor of the University; they presented the addresses of which they were bearers and these were handed to Sir George. In reply he said that he often thought, in reviewing his long life, that he might have worked harder, and he attributed his longevity to his comparative idleness—a remark which was cheered by the undergraduates in the gallery. A special meeting of his early love, the Cambridge Philosophical Society, was held and the papers there presented are published in a memorial volume.

In the summer of 1902 he was elected to the mastership of Pembroke College. Later in the year he took part with Lord Kelvin in making the presentation of a portrait of Prof. Tait to St. Peter's College. He died on February 1, 1903, in the 84th year of his age. In many respects the life of Stokes resembles that of Newton. Both were skilled experimenters, especially in optics; of Stokes it used to be said that if you gave him sunlight and three-quarters of an hour, there was no experiment in optics he could not perform. Both Newton and Stokes filled the Lucasian chair of mathematics; both represented Cambridge University in Parliament; both filled the offices of Secretary and President of the Royal Society; both received the dignity of *Sir*; and both lived to an advanced age. They also resembled one another in type of mind and in religious views; but Stokes never sat down to produce a work at all commensurate in labor or in importance with the *Principia*.