NICHOLAS SAUNDERSON LL.D.
Lucasian Professor of Mathematicks in
the University of Cambridge
Died 19. Ap. 1739 Aged 56

I. Vanderbanck. pinx. 1718 — From the Original painted for Martin Folkes Esq. — G. Vander Gucht. Sculp.

Fig: I.

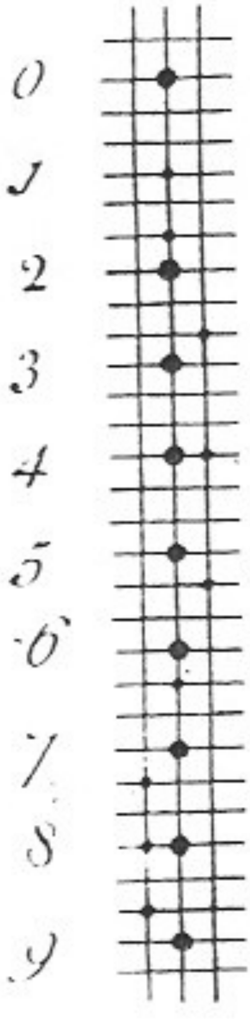
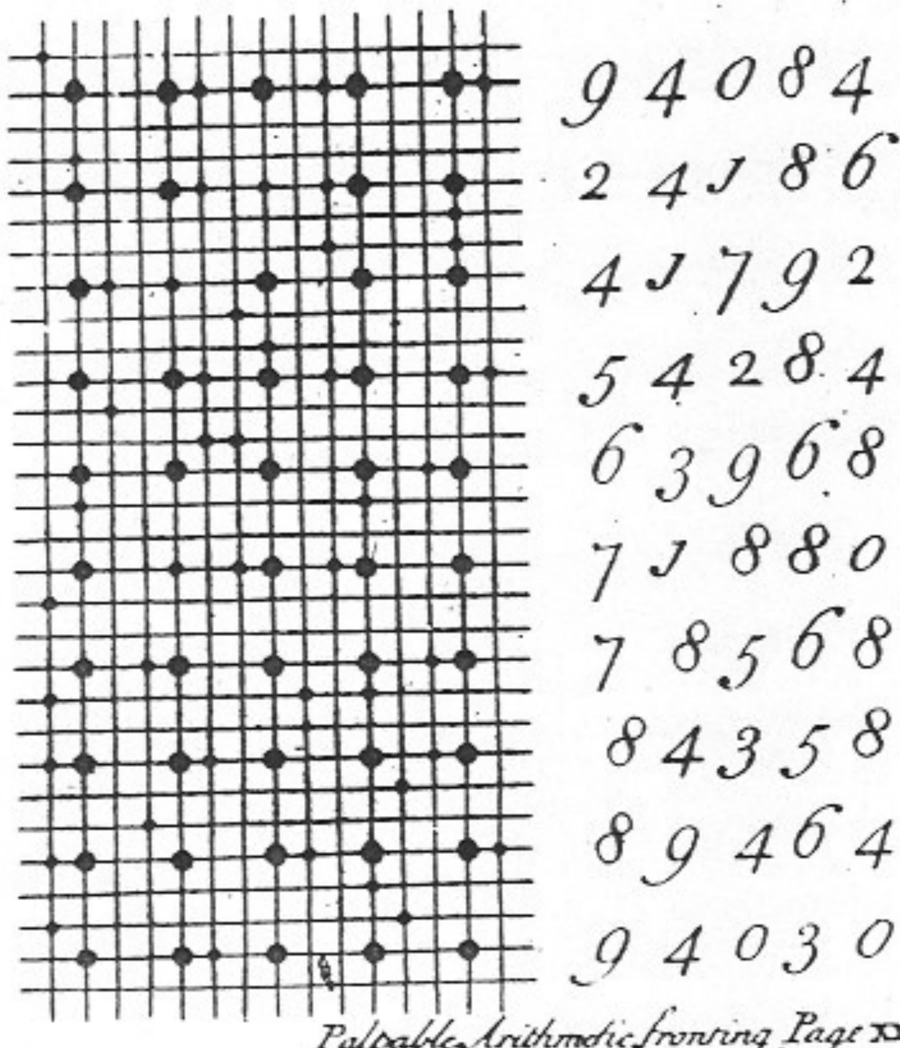


Fig: II.



Palpable Arithmetic fronting Page XXIV.

NICHOLAS SAUNDERSON

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THE

ELEMENTS of ALGEBRA,

IN TEN BOOKS:

By NICHOLAS SAUNDERSON *LL.D.*

Late *Lucasian* Professor of the Mathematics in the University of CAMBRIDGE, and Fellow of the Royal Society.

VOLUME THE FIRST,

Containing the Five first Books.

To which are prefixed

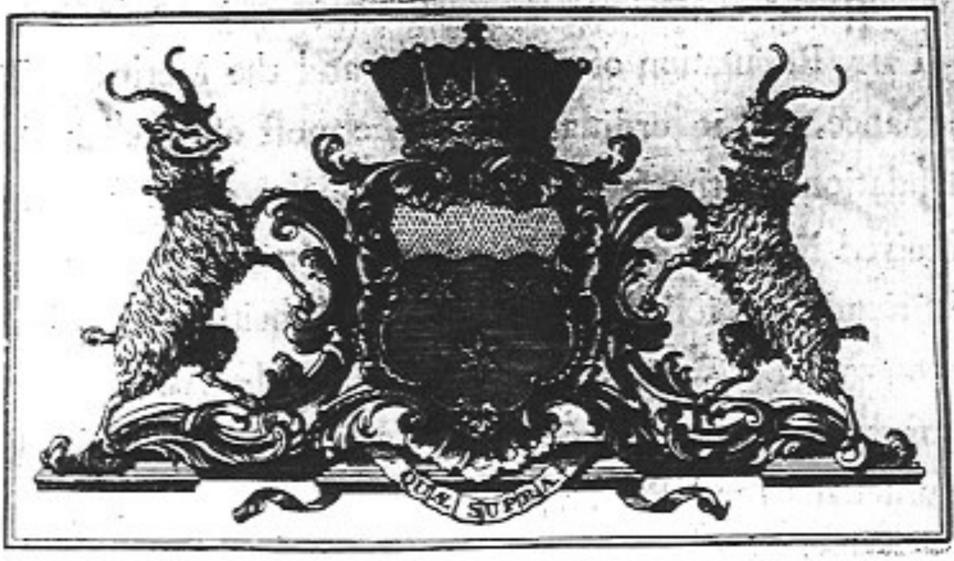
I. The LIFE and CHARACTER of the AUTHOR.

II. His PALPABLE ARITHMETIC Decyphered.

CAMBRIDGE,

Printed at the UNIVERSITY-PRESS:

MDCCXL.



To the Right Honourable

JOHN Earl of RADNOR,
Viſcount BODMYN,
Baron ROBARTES of TRURO, &c.

MY LORD,

THE Relation I bore to the Author of the following Work, gives me a Right to intereſt myſelf in the Succeſs of it. To recommend it to the Publick with all poſſible Advantage, is the leaſt I owe to the Memory of the beſt of Fathers.

The Reputation of an Author, and the Merit of a Performance, are in ordinary Cafes the moft effectual Recommendation to the favour of the Publick; and if I am not deceived by the Voice of Fame, nor flattered by the Report of Friends, the following Treatife wants neither of thefe Advantages. But whatever be the Cafe of the Author, or the Performance, the Subject itfelf, I am afraid, calls for fome Countenance and Protection.

The *Belles Lettres*, My Lord, lie open to all Mankind; every one that is capable of reading any thing, either has, or pretends to have fome Relifh for them. But Science lies more concealed, and more out of the reach of vulgar Apprehenfions: Some have not Capacities or a juft Tafte for Mathematical Truths; and of thofe that have, few have Refolution enough to purfue them with a proper Induftry: and be their Ufe and Importance ever fo great, they will hardly emerge under fuch Difadvantages without fome powerful Encouragement.

These, My Lord, are Difficulties arifing from the nature of the Subject; to remove which, and to infpire the Youth with a generous Emulation for fo noble a Science,

would

would be doing Service to the Publick; as well as Juftice to the Author.

The Province of Exhortation and Inftruction I leave to others, and content myfelf with the only Method that lies within my reach, of ufhering this Work into the World under the Authority and Protection of fome great Name; one whofe Birth and Quality may give a Dignity to Science, and by being a Mafter of it himfelf, has the jufter Title to patronize it in others; who does not meafure the worth of a Performance by the fulfom Incenfe of the Dedicator, but penetrates into the Work itfelf, and learns from thence the real Abilities of the Writer.

It is for thefe Reafons, My Lord, that I prefume to lay this Treatife at Your Feet. Suffer me to borrow Your Lordship's Name to throw a Luftre upon my Subject, to attract the Attention of the Lazy and the Prejudiced; and to invite them by fo great an Example to an Acquaintance with a Science, whofe Ufe is almoft as extenfive as Truth itfelf, and whofe Invention does Honour to Human Reafon.

Putting this Treatife under Your Lordship's Protection I look upon to be a Part of the Author's Will, and which

which it is incumbent upon me to execute. Had He lived to give it to the Publick, I am confident he would have ſought no other Patron. Your LORDSHIP was his firſt Acquaintance in the Univerſity and his firſt Scholar, and, (what he ever remembered with the ſincereſt Gratitude) his moſt generous Benefactor. A great Part of the Work was drawn up originally for Your LORDSHIP's Uſe, and was for many Years in your ſole Poſſeſſion. And ſurely, My LORD, there is ſomething of Juſtice in granting Protection to a Book, which firſt opened to You the Beauties of Mathematical Knowledge, and to which You are in ſome meaſure indebted for that excellent Taſt, that ſo eminently diſtinguiſhes Your LORDSHIP.

THE Author was honoured with the Friendſhip of moſt Perſons of Quality and Condition that ſtudied in the Univerſity, though it ended for the moſt Part with their Reſidence there. But Your LORDSHIP raiſes Friendſhips upon more laſting Principles; the Moment a Man of Merit knows You, he becomes your Friend, and when he is once ſo, he can never be otherwiſe. The Author had a Heart extremely ſenſible to true Worth; it received a very ſtrong and durable Impreſſion from the uncommon Share of it he obſerved in Your LORDSHIP; his Eſteem purſued You into the World,

World, and continued undiminifhed 'till he left it; he loved and honoured You fincerely and paffionately when living, and mentioned You with peculiar Tendernefs and Affection in his dying Moments.

I CANNOT recal with Indifference a Scene in which I bore fo confiderable a Part, though my Youth rendered me incapable of taking my juft Share of it. He was fnatched away at a time when I was unable to make a true Eftimate of the Lofs; when I had only experienced the Tendernefs of the Parent, without enjoying the more folid Benefit of the Friend and the Inftructor.

BUT I will fhut up a Subject, which can give YOUR LORDSHIP no Pleafure, and which muft give me the tendereft Concern. It is an Event that had been truly fatal to me, had not Your LORDSHIP's Goodnefs prevented it. This has taught me to bear a Misfortune which is the common Lot of all Men; and 'tis from this I am daily learning (as far as the Ties of Nature and filial Piety will permit) to forget it. Suffer me, MY LORD, to boaft (for there is fomething of Vanity as well as Gratitude in it) that in Your fingular Generofity and Indulgence I have found the fubftantial Comfort of the Friend and the Parent. It is with Pride and

Pleafure

Pleaſure that I proclaim to the World my Obligations to You; and it is with the warmeſt Gratitude, and the moſt perfect Regard that I profeſs myſelf,

My Lord,

Your Lordship's

moſt obliged and

devoted Servant

John Saunderson.

THE

ELEMENTS

OF

ALGEBRA,

IN TEN BOOKS:

By NICHOLAS SAUNDERSON *LL.D.*

Late *Lucasian* Professor of the Mathematics in the University of CAMBRIDGE, and Fellow of the Royal Society.

To which is prefixed,

An Account of the AUTHOR's LIFE and CHARACTER,

Collected from his oldest and most intimate ACQUAINTANCE.

CAMBRIDGE,

Printed at the UNIVERSITY-PRESS:

And Sold by Mrs. *Saunderson* at CAMBRIDGE, by *John Whiston* Bookseller at *Boyle's Head* in *Fleetstreet* LONDON, and *Thomas Hammond* Bookseller in YORK. MDCCXL.

ADVERTISEMENT.

THE learned Author of this Book having, in his paper of Propoſals, drawn up a ſhort Scheme of his Work, it is thought proper to reprint it here, in order to give the Reader in a few words an idea of his performance.

This Work *(ſays the Author)* is chiefly intended for the inſtruction of young beginners, and for the uſe of thoſe who have ſuch under their care. It is divided into ten books, with an introduction prefixed concerning vulgar and decimal fractions; the former being abſolutely neceſſary to a thorough knowledge of Algebra. In this introduction, all the reductions and operations of fractions are fully and clearly accounted for, and ſuch a fund of reaſoning eſtabliſhed thereby in the mind of the learner, as cannot fail to furniſh him with proper rules to work by in all caſes wherever fractions are concerned, without any further aſſiſtance.

The firſt book treats of the nature of Algebra and Algebraic quantities; of their addition, ſubtraction, multiplication and diviſion; of proportion, fractions, and extraction of roots in Algebra; and in the laſt place, of the manner of reſolving ſimple equations, illuſtrated by a conſiderable number of examples. In this book, under the head of multiplication is ſhewn, how by bare multiplication alone many uſeful theorems may be invented, and have been invented, both in Arithmetic and Geometry. Under the heads of diviſion and extraction of roots, ſome account is given of the riſe and continuation of infinite ſerieſes, and how they may be tried by involution; but they are here touched upon only ſo far as may be apprehended by the meaneſt capacities, all other conſiderations concerning them being reſerved to other parts of this treatiſe, where it may reaſonably be preſumed, the learner will be better prepared for them.

The ſecond book contains a great variety of entertaining queſtions producing ſimple equations, and ſolved partly by ſingle poſitions, and partly by more; where all the uſeful methods of extermination are explained.

The third book treats of quadratic equations, and of the manner of reſolving them, exemplified in various queſtions introduced for that purpoſe; where throughout the whole are occaſionally interſperſed many curious obſervations concerning the roots of equations, both poſſible and impoſſible.

The fourth book treats of pure Algebra, that is, where letters of the alphabet are uſed, not only to repreſent unknown quantities, but alſo ſuch as are known, which in tenderneſs to the learner has hitherto been avoided. Here ſeveral of the former problems are reſumed, more indefinitely propofed, and general folutions given them by general theorems or canons, firft traced out analytically, and afterwards demonftrated fynthetically, whereby the learner may make himfelf thorough mafter of both forts of demonftrations.

The fifth book gives the folutions of many curious queftions of that fort which admit of more anfwers than one, and fome even of an infinite number, by a general method very eafy to be comprehended, and (as the author conceives) entirely new. Here are demonftrated many elegant and ufeful theorems relating both to whole numbers and fractions; particularly that of Mr. *Cotes*, for finding the leaft numbers that will exprefs a given ratio to any given degree of exactnefs. Here alfo occafion is taken to introduce the reader into an acquaintance with the moft entertaining parts of *Euclid*'s doctrine of Incommenfurables, where he will meet with more fubtil and more refined reafoning than perhaps in any other part of the Mathematics whatever; and all levelled to the loweft capacity.

The fixth book is a choice collection of fuch queftions as are ufually known by the name of *Diophantine queftions*; the folutions of them here given are very eafy and intelligible, and in fome cafes perhaps may be thought preferable to any folutions of the fame problems in *Diophantus*, or in any of his commentators or followers: for as *Diophantus*'s porifms are entirely loft, he lies, in many cafes, at the mercy of his commentators.

The feventh book treats in the firft place of the doctrine of proportion as it is delivered in the fifth book of the Elements. Here it is fhewn that the common idea of proportionality may, without the leaft difguife, be fo enlarged, as to extend to Incommenfurables: and having thus eftablifhed an infallible and adequate criterion of proportionality, the fifth and feventh definitions to the fifth book of the Elements are fhewn to be no more than plain and natural confequences of it, but more proper upon many accounts for *Euclid*'s purpofe, in carrying on his fyftem of Geometry. Then all the propofitions of the fifth book of the elements are clearly and fuccinctly demonftrated in their order, and as near as poffible, in *Euclid*'s manner, fo as to lofe nothing either of the force or elegance of his demonftrations; and yet fuch an eafy and familiar turn is given to the whole, that it is to be hoped this book, which has always been looked upon as an unfurmountable rub in the way through the fix firft books of the Elements, will now be read with as much eafe to the imagination, nay more, than any other part of the Elements whatever. The latter part of this book gives a clear and diftinct account of the compofition and refolution of ratios, and of their great ufe in Natural and Mechanical Philofophy: infomuch that it is to be hoped, this part of the doctrine of proportion will be no longer a myftery to any one who will read it with the leaft degree of attention.

The

The eighth book applies Algebra to Geometry, and by the help of a few plain and eaſy problems, conveys to the mind of the learner the moſt ſublime myſteries of that ſcience. Here the compoſition of the Geometrical problems is firſt deduced from the *analyſis*, and then ſynthetical demonſtrations are formed from the conſtructions there given, without any regard to the *analyſis*. The ſecond part of this book contains the doctrine of ſolids, ſo far as it relates to priſms, cylinders, pyramids, cones, ſpheres, ſpheroids, &c; the principal properties whereof are taken out of *Euclid*, *Archimedes* and others, and demonſtrated after the ſimpleſt manner, without the leaſt ſtreſs laid upon the imagination.

The ninth book conſiſts of ſeveral miſcellaneous tracts; as firſt, of powers and their indexes; 2dly, of *Newton*'s method of evolving a binomial, conſidered in it's full extent; 3dly, of logarithms, their nature and uſe, and particularly of *Briggs*'s logarithms; 4thly, of logarithmotechny, or the method of computing logarithms, drawn from their ſimpleſt properties; 5thly, of *Newton*'s invention of diviſors; and 6thly, of the Arithmetic of ſurd quantities.

The tenth and laſt book treats firſt of equations in general, and then of cubic and biquadratic equations in particular, gives the beſt methods of reſolving them where they will admit of an accurate reſolution, and proper rules for approximations where they will not; particularly *Newton*'s method is here deſcribed and explained.

By the account here given it is eaſy to ſee, that the author intended this treatiſe, not as a courſe of Algebra only, but alſo to promote, as far as poſſible, the ſtudy of Geometry, by removing or explaining all thoſe difficulties which, from a long experience, he knows are apt to retard, if not diſcourage, young ſtudents in their progreſs through the Elements.

Though great care hath been taken to give the publick a correct edition of this excellent work, yet there are ſtill remaining ſeveral errors of the preſs, though moſt of them are of little conſequence. If the candid reader will take the trouble to correct the moſt material of thoſe that are mentioned in the catalogue of errata, *I hope and believe, he will find no others that do in the leaſt obſcure the ſenſe of the Author.*

There is one paſſage in the ninth book, relating to ſome matters of fact, wherein I conceive, the learned Author was miſtaken as to ſome few particulars; concerning which I had not an opportunity to diſcourſe with him, becauſe of his ſudden and unexpected ſickneſs and death. The paſſage is in page 620, *where the Author having recommended Mr.* Briggs's *ſyſtem of logarithms as the beſt accommodated for practice which are now in uſe, proceeds in the following words:* The Lord *Napeir*, a Scotch Nobleman, was the firſt inventer of logarithms: but our countreyman Mr. *Briggs*, Profeſſor of Geometry in *Greſham* College, was undoubtedly the firſt who thought of this

this ſyſtem, and propoſing it to the noble inventer, the Lord *Napeir*, he afterwards publiſhed it with that Lord's conſent and approbation.

Now in the firſt *place I obſerve, that our late worthy Profeſſor wrote the name of the celebrated inventer of logarithms differently in different parts of his manuſcript copy. Sometimes he is called* Nepier, *and ſometimes* Neper. *I have an engliſh book written by this illuſtrious author, and printed in the year* 1611, *which in the title page is ſaid to be* ſet foorth by *John Napeir* L. of Marchiſton: *accordingly I have every where in the following work, cauſed his name to be printed* Napeir.

Secondly, *whereas our author ſtiles him* The Lord *Napeir*, a Scotch Nobleman; *this, I think, ought to be underſtood in a qualified ſenſe. In* The Peerage of Scotland *written by* George Crawfurd *Eſq*; *and printed at* Edinburgh *in* 1716, *I find, page* 364, *that Sir* Archibald Napier, *eldeſt ſon to the foreſaid* John, *was the firſt of this family that was created a Peer of* Scotland, *being raiſed to that honour by King* Charles *the firſt in the year* 1627, *his father (the inventer of logarithms) having departed this life ſeveral years before, in the reign of King* James *the ſixth. I confeſs, Mr.* Briggs *writes him* Johannes Neperus Baro Merchiſtonii: *but if the account given of this family by Mr.* Crawfurd *be true, the inventer of logarithms can only be ſuppoſed to have been one of the inferior order of Barons in* Scotland, *and not a Peer of that Realm.*

Thirdly, *whereas Dr.* Saunderſon *ſtiles Mr.* Briggs, Profeſſor of Geometry in *Greſham* College; *it is true, he was ſo, when* Napeir *firſt publiſhed his* Canon mirificus logarithmorum; *but when Mr.* Briggs *publiſhed his own* Arithmetica logarithmica *in* 1624, *he was* Savilian *Profeſſor of Geometry at* Oxford, *as appears from the title page of that book.*

Fourthly, *whereas Mr. Profeſſor* Saunderſon *ſaith, that* our countreyman *Briggs* was undoubtedly the firſt who thought upon this ſyſtem of logarithms which is now in uſe; *I know not by what authority he aſſerts this. On the contrary, Dr.* Keill *in the preface to his treatiſe* de logarithmis *writes thus:* Aliam deinde magis commodam logarithmorum formam *Neperus* excogitavit, et communicato conſilio cum Domino *Henrico Briggio*, Geometriae in Academia *Oxonienſi* Profeſſore, hunc ſocium operis ſibi adjunxit, ut logarithmos in meliorem formam redactos complepleret. *See likewiſe* Briggs's *preface to his* Arithmetica logarithmica.

I have nothing further to add, but that the Reader is obliged to the ingenious Mr. Abraham de Moivre *for two excellent performances inſerted in this work: the former at the end of the fourth book, is his curious ſolution of two problems concerning proportionals; and the latter in an appendix to theſe Elements, is a noble diſcovery of a rule for extracting the cubick, or any other root of (what is called) an impoſſible binomial, ſuch as* $a+\sqrt{-b}$, *and alſo for extracting any root out of a given power thereof.*

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A LIST of fuch of the

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ERRATA.

ERRATA.

Note, The moſt material errors are marked with *.

* Pag. 4. in the anſwer to the tenth queſtion, *for* 15625 feet *read* 15625 ſquare feet.
Pag. 45. lin. 8. *read* as any. And lin. 22, 23. *read* decimal.
Pag. 57. lin. penult. *read* the two firſt products.
Pag. 64. lin. 6 from the bottom, *read* it was ſhewn.
* Pag. 138. lin. 12. *for* ſubſtitute *read* ſubſtituting.
* Pag. 239. lin. 4 from the bottom, for $s\times -q\overline{-qr}=-qs\overline{-qrs}$ read $s\times\overline{-q-qr}=\overline{-qs-qrs}$.
* And in the laſt line, for $-q\overline{-qr}$ read $\overline{-q-qr}$.

In the Second Volume.

Pag. 390. lin. 11. *dele* muſt.
* Pag. 536. lin. 14. for $x^2-a^2x^2$ read $x^4-a^2x^2$.
Pag. 551. lin. 21. *for* where *read* when.
Pag. 604. lin. ult. for x^{-+1} read x^3.
Pag. 658. lin. 4. *read* DEFINITIONS.
* Pag. 671. lin. 3. *for* binomial *read* binomial root.
Pag. 683. line 21. *read* left.
* Pag. 703. in the firſt line of the ſixth example, for $\frac{q}{4}$ read $\frac{qq}{4}$.
Pag. 738. line 13. *dele* a.

In the Table of the Contents.

* Art. 231. for *two numbers* read *two ſquare numbers*.
Art. 403. for *Napeirs* read Napeir's.

Errata in the Pointing.

Pag. 44. lin. 16. *after* .4375 *put a comma.*
Pag. 81. lin. 18. at the end of the line, after $2aa$, inſtead of a comma put a ſemicolon.
Pag. 91. at the end of the fifth example, put a full ſtop inſtead of a comma.
Pag. 124. lin. 8 from the bottom, at the end of the line, put a ſemicolon inſtead of a comma.
Pag. 167. lin. 25. *after* no ſquare number *put a ſemicolon inſtead of a comma.*
Pag. 178. line the laſt but one of the ſecond example, *after* ± 1 *put a ſemicolon.*
Pag. 199. lin. 16. after $x=\frac{32400-675x}{x}$ put a ſemicolon.
Pag. 238. lin. 7. put a full ſtop at the end of the paragraph.
Pag. 351. lin. 5. at the end of the paragraph, put a full ſtop inſtead of a comma.

In the Second Volume.

Pag. 542. lin. 5 from the bottom, *for* DEG: but *read* DEG. But.
Pag. 668. lin. 16. for *root: and* vice verſa read *root. And* vice verſa.
Pag. 738. lin. antepen. at the end of the line, after $xx+3x+2=0$ put a comma.

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DEVONSHIRE
MONTAGU
PEMBROKE
R. WALPOLE

WHEREAS ABIGAIL SAUNDERSON, Widow of the late NICHOLAS SAUNDERSON, Doctor of Laws, and Professor of Mathematicks in His Majesty's University of *Cambridge*, and JOHN SAUNDERSON, Son, and ANNE SAUNDERSON, Daughter of the said NICHOLAS SAUNDERSON, have humbly represented unto Us, that the said NICHOLAS SAUNDERSON, did, in his Life-time, with great Labour and Expence compose a Treatise, Intituled, *The Elements of Algebra in Ten Books*, now printing at *Cambridge* in Two Volumes *in Quarto*, with the *Memoirs of the Author's Life*, the Property of which Treatise and Memoirs is in the abovementioned ABIGAIL, JOHN and ANNE SAUNDERSON; and have farther represented unto Us, that the said NICHOLAS SAUNDERSON, did also compose several other Mathematical and Philosophical Treatises, the Property of which last mentioned Treatises, is in the aforesaid JOHN SAUNDERSON; and have therefore humbly besought Us to grant unto them the said ABIGAIL, JOHN and ANNE SAUNDERSON, His Majesty's Royal Privilege and License, for the sole printing and publishing the aforesaid Treatise and Memoirs, Intituled, *The Elements of Algebra in Ten Books*, with the *Memoirs of the Author's Life*, for the Term of Fourteen Years, and also unto the said JOHN SAUNDERSON His Majesty's Royal Privilege and Licence for the sole printing and publishing the rest of the Treatises abovementioned, for the said Term of Fourteen Years; We, being willing to give due Encouragement to Works of this Nature, that tend to the Advancement and Propagation of Learning, have thought fit to condescend to their Request, and do therefore in His Majesty's Name, by these Presents, so far as may be agreeable to the Statutes in that behalf made and provided, grant unto them the said ABIGAIL, JOHN, and ANNE SAUNDERSON, their Heirs, Executors and Assigns, His Majesty's Royal Privilege and Licence, for the sole printing and publishing the aforesaid Treatise and Memoirs, Intituled, *The Elements of Algebra in Ten Books*, with the *Memoirs of the Author's Life* for the Term of Fourteen Years; and also unto the said JOHN SAUNDERSON, his Heirs, Executors and Assigns for the sole printing and publishing the rest of the Treatises abovementioned for the said Term of Fourteen Years, to be computed from the Date hereof; strictly forbidding and prohibiting all His Majesty's Subjects, within His Majesty's Kingdoms and Dominions to reprint, abridge, or translate any of the abovementioned Treatises, either in the like, or any other Volume or Volumes whatsoever, or to import, buy, vend, utter or distribute any Copies of them, reprinted beyond the Seas, during the said Term of Fourteen Years, without the Consent and Approbation of the said ABIGAIL, JOHN and ANNE SAUNDERSON, their Heirs, Executors and Assigns, by Writing under their Hands and Seals first had and obtained, as they and every of them Offending herein, will answer the contrary at their Perils; Whereof the Commissioners and other Officers of His Majesty's Customs, the Master, Wardens and Company of Stationers of the City of *London*, and all other His Majesty's Officers and Ministers, whom it may concern, are to take notice, that due Obedience be given to these Presents.

Given at *Whitehall* the Twenty Ninth Day of *May*, One Thousand Seven Hundred and Forty, and in the Thirteenth Year of His Majesty's Reign.

By their EXCELLENCYS *Command*

ANDREW STONE.

MEMOIRS

OF THE

LIFE and CHARACTER

OF

Dr. *Nicholas Saunderson*,

Late *Lucasian* Professor of the Mathematics in the University of CAMBRIDGE.

THE following Treatise was entirely finished, before the Public was engaged in the Subscription; but the Author dying during the Impression of the Work, it is believed some short Account of a Person so remarkable and singular will be no unacceptable Entertainment to the Reader. Therefore the following Gentlemen, Dr. THOMAS NETTLETON of *Halifax*, Dr. RICHARD WILKES of *Woolverhampton*, the Rev. Mr. JOHN BOLDERO late Fellow of *Christ*'s College, the Rev. Mr. GERVAS HOLMES late Fellow of *Emmanuel* College, the Rev. Mr. GRANVILE WHEELER, and Dr. RICHARD DAVIES of *Shrewsbury* late Fellow of *Queen*'s College, who were the intimate Friends of the Deceased, in the different Parts of his Life from his Youth to the time of his Death, have communicated the following Particulars.

Mr. Nicholas Saunderson was born in *January* 1682, at *Thurlston* near *Penniston* in *Yorkshire*. His Father, besides a small Estate, had a Place in the Excise, which he enjoyed above Forty Years with good Repute. His eldest Son, of whom we are speaking, when Twelve Months old, was deprived by the Small Pox, not only of his Sight, but his Eyes also, for they came away in Abscess. A Sense so little enjoyed was soon forgot; he retained no more Idea of Light and Colours than if he had been born Blind.

He was sent early to the Free School at *Penniston*, and under the Instruction of Mr. Staniforth, laid the Foundation of that Knowledge of the Greek and Roman Languages, which he afterwards improved so far by his own Application to the Classic Authors, as to hear the Works of *Euclid*, *Archimedes* and *Diophantus* read in their original Greek. *Virgil* and *Horace* were his Favourites among the Roman Poets: His Memory was well stored with their most beautiful Passages, and he would frequently in Conversation quote them with great Propriety. He was well versed in the Writings of *Tully*, and dictated Latin in a familiar and elegant Style. He afterwards acquired a competent knowledge of the French Tongue.

As soon as he had gone through the Business of the Grammar School, his Father, whose Occupation led him to be conversant in Numbers, began to instruct him in the common Rules of Arithmetic. Here it was his Genius first appeared; he soon became able to work the common Questions, to make long Calculations by the strength of his Memory, and to form new Rules to himself for the more ready solving of such Problems, as are often proposed to Learners, more with a design to perplex than instruct: so that in all Difficulties, his Schoolfellows generally applied to him instead of their Master.

At

At the Age of Eighteen he was introduced to the acquaintance of RICHARD WEST of *Underbank* Esq; a Gentleman of Fortune and a Lover of the Mathematics: who observing Mr. SAUNDERSON's uncommon Capacity, took the Pains to instruct him in the Principles of Algebra and Geometry, and gave him every Encouragement in his Power to the Prosecution of these Studies: foreseeing of what Advantage to Letters so great a Genius might be. Soon after, he grew acquainted with Dr. NETTLETON; and it was to the great Pleasure these Gentlemen took in assisting and improving him in his Studies, that our Author owed his first Institution in the Mathematic Sciences. They furnished him with Books, and often read and expounded them to him: but he soon surpassed his Masters, and became fitter to teach, than learn any thing from them.

OUR Author's Passion for Learning grew with him, and his Father, willing to encourage this laudable Disposition, sent him to a private Academy at *Attercliff* near *Sheffield.* Logic and Metaphysics made up the principal Learning of this School. The former being chiefly the Art of Disputing in Mood and Figure, a dry Study, much conversant in Words, the latter dealing in such abstract Ideas as have not the Objects of Sense for their Foundation, were neither of them agreeable to the Genius of our Author; he therefore made but a short Stay here for Instruction.

AFTER he left this Place, he remained some time in the Country, prosecuting his Studies in his own way, without any Guide or Assistant: indeed he needed no other than a good Author, and some Person that could read it to him: by the Strength of his own Genius he could easily master any Difficulty that occurred therein. His Education had hitherto been carried on at the Expence of his Father, who having a numerous Family, grew uneasy under the Burden. His

Friends therefore began to think of fixing him in ſome way of Buſineſs, by which he might ſupport himſelf. His Inclination led him ſtrongly to the Univerſity of CAMBRIDGE, where he expected to meet with the beſt Opportunities of Improvement in his favourite Studies. But the great Expence of Education, the length of Time he muſt continue there to obtain his Degrees, and be duly qualified for any of the Liberal Profeſſions, were Difficulties not to be ſurmounted. At laſt it was reſolved he ſhould try his Fortune there, but in a way very uncommon; not as a Scholar but a Maſter: for his Friends, obſerving the extraordinary Proficiency which he had already made in Mathematical Learning, and a peculiar Felicity of Expreſſion in conveying his Ideas to others, were ſanguine in their Hopes, that he might teach the Mathematics with Credit and Advantage, even in the Univerſity. Or if this Deſign ſhould miſcarry, they promiſed themſelves Succeſs in opening a School for him in *London*.

Accordingly in the Year 1707, being now Twenty-five Years of Age, he was brought to CAMBRIDGE by Mr. JOSHUA DUNN, then a Fellow-Commoner of *Chriſt*'s College, where he reſided with his Friend, but was not admitted a Member of the College. The Society were extreamly pleaſed with ſo unuſual a Gueſt, allotted him a Chamber, the uſe of their Library, and indulged him in every Privilege that could be of Advantage to him. But many Difficulties obſtructed his Deſign: He was placed here without Friends, without Fortune, a Youth untaught himſelf, to be a Teacher of Philoſophy in an Univerſity where it then reigned in the greateſt Perfection. Mr. WHISTON was at this time in the Mathematical Profeſſor's Chair, and read Lectures in the manner propoſed by Mr. SAUNDERSON; ſo that an Attempt of this kind looked like an Encroachment on the Privileges of his Office. But as a good-natured Man and an Encourager of

Learning

Learning, he readily consented to the Application of Friends made in behalf of so extraordinary a Person. Mr. DUNN had been very assiduous in making known his Character; his Fame in a few Months had filled the University, so that Men of Learning and Curiosity grew ambitious and fond of his Acquaintance. His Lecture, as soon as opened, was attended by many from several of the Colleges, and in some time was so crowded, that he could hardly divide the Day among all who were desirous of his Instruction: few, whose Inclination led them to the severer Studies, but eagerly embraced the Opportunity of laying a Foundation in Mathematics and Philosophy under so great a Master.

Sir ISAAC NEWTON had left *Cambridge* several Years before Mr. SAUNDERSON came thither. His *Principia Mathematica* had been long since published, but were at first overlooked, and not sufficiently understood by the World. It was one Design of this Treatise to demolish the *Vortices* and other romantic Chimæras of DES CARTES: and now the Learned began to be sensible how much the Author had done towards a Reformation in Philosophy; which before had been founded upon very erroneous Principles, and *Hypotheses* feigned in the Closet, without one Experiment to shew their Reality in Nature. Sir ISAAC, ever studious of brevity, had drawn up his Demonstrations in the concisest manner possible; leaving the Mathematical Reader to furnish himself with every thing before known, and often to take large Steps alone. His Treatise of *Optics* and his *Arithmetica Universalis* were both written in the same masterly Style, and each contain great and peculiar Discoveries. Mr. SAUNDERSON made these several Pieces the Foundation of his Lecture; they afforded a noble Field to display his Genius in; and the Public Schools of the University did sufficiently testify his Success. For those wonderful *Phænomena* of Nature, whose Solution

lution was before attained with Difficulty by the beſt Mathematicians, became the *Theſes*, which the Youth of three or four Years ſtanding defended in their Diſputations for their firſt Degree in Arts. We every Year heard the Theory of the Tydes, the *Phænomena* of the Rainbow, the Motions of the whole Planetary Syſtem as upheld by Gravity, very well defended by ſuch as had profited by his Lectures.

IT will be matter of ſurpriſe to many that our Author ſhould read Lectures in Optics, diſcourſe on the Nature of Light and Colours, explain the Theory of Viſion, the Effect of Glaſſes, the *Phænomena* of the Rainbow, and other Objects of Sight: but if we conſider that this Science is altogether to be explained by Lines, and ſubject to the Rules of Geometry, it will be eaſy to conceive that he might be a Maſter of theſe Subjects.

AS Mr. SAUNDERSON was inſtructing the Univerſity Youth in the Principles of *Newtonian* Philoſophy, it was not long before he became acquainted with the incomparable Author, and enjoy'd his frequent Converſation concerning the more difficult Parts of his Works. Dr. HALLEY, Mr. DE MOIVRE and many of the moſt noted Mathematicians in *London* highly eſteemed his Friendſhip, and in deference to his ſtrong Reaſon and Judgment, frequently conſulted him concerning their Writings and Deſigns.

UPON the removal of Mr. WHISTON from his Profeſſorſhip, Mr. SAUNDERSON's Mathematical Merit was univerſally allowed ſo much ſuperior to that of any Competitor in the Univerſity, that an extraordinary Step was taken in his Favour, to qualify him with a Degree, which the Statutes require. Upon Application made by the Heads of Colleges to the Duke of SOMERSET their Chancellor, together with the Interceſſion of the Honourable FRANCIS ROBARTES Eſq; a Mandate was readily granted by the QUEEN, for conferring on him the Degree

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gree of Maſter of Arts. Upon which he was choſen *Lucaſian* Profeſſor of the Mathematics in *November* 1711. During this whole Tranſaction Sir Isaac Newton intereſted himſelf very much in his Favour.

At this time the ingenious Mr. Roger Cotes filled the *Plumian* Chair of Aſtronomy and Experimental Philoſophy; a Man of great ſweetneſs of Temper, and engaged to our Author in the ſtricteſt Friendſhip; of the ſame Age, of the ſame Genius and Inclination to the Mathematics, both approved and recommended to Profeſſorſhips by Sir Isaac Newton. No Univerſity could ever at one time boaſt of two ſo capable and ſo diſpoſed to promote the Study of Philoſophy among her Pupils. Had they lived to more mature Ages, mutually aſſiſting and inſpiring each other in the purſuit of Knowledge, what Glory might have accrued to our Univerſity, what Advancement to Science from their united Labours! But Mr. Cotes was hurried away by a Fever in the Flower of his Age, having only time to compoſe a few Pieces, as Specimens of his extraordinary Capacity, but of great value to the Learned. And our Author's Life, though longer, was ſo devoted to Lectures, that he now leaves to Poſterity as few Monuments of his Abilities.

Our Author's firſt Performance after he was ſeated in the Chair, was an Inauguration Speech, made in very elegant Latin, and a Style truly *Ciceronian:* it was delivered with ſuch juſt Elocution, and in a manner ſo graceful, as to gain him the univerſal Applauſe of his Audience. In it he firſt returned his Thanks to Her Majesty for the Royal Mandate, to the Chancellor for his ready Application to the Queen, and to the Electors and the reſt of his Friends for their good Opinion of his Abilities and Mathematical Knowledge. To theſe he added a long and noble Encomium on

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the Mathematics, ſhewing the Excellence and Advantage of this above every other Method of Reaſoning.

FROM this time he applied himſelf cloſely to the reading of Lectures, and gave up his whole time to his Pupils: ſo that his Friends ſoon loſt all the Pleaſure of his Converſation. He continued among the Gentlemen of *Chriſt's* College till the Year 1723, when he took a Houſe in *Cambridge*, and ſoon after married a Daughter of the Rev. Mr. WILLIAM DICKONS late Rector of *Boxworth* in the County of *Cambridge*; by whom he had a Son and a Daughter, both now living.

IN the Year 1728, when His preſent Majeſty King GEORGE the Second honoured the Univerſity of *Cambridge* with a Royal Viſit, He was pleaſed to ſignify his deſire of ſeeing ſo remarkable a Perſon. Accordingly our Profeſſor attended upon His Majeſty in the Senate Houſe, and was there created Doctor of Laws by his Royal Favour.

Dr. SAUNDERSON was naturally of a ſtrong, healthy Conſtitution; but being too ſedentary, and conſtantly confining himſelf to his Houſe, he became at length a Valetudinarian of a very Scorbutic Habit. For ſome Years he frequently complained of a Numbneſs in his Limbs, which in the Spring of the Year 1739 ended in a Mortification in his Foot. His Blood was in ſo ill a State that no Art or Medicines were able to ſtop its Progreſs. He died the 19th of *April* 1739; in the Fifty-ſeventh Year of his Age, and lies buried according to his laſt Requeſt in the Chancel at *Boxworth*.

AFTER his Life, it may be expected that ſome Account ſhall be given of his Character likewiſe; but I am at a loſs for Colours, ſtrong enough to paint a Character ſo bright and uncommon, and where to place it for View, in the trueſt Point of Light. A blind Man moving in the Sphere of a Mathema-

Mathematician ſeems a *Phænomenon* difficult to be accounted for; and has excited the Admiration of every Age in which it has appeared. *Tully* mentions it as a thing ſcarce credible in his own Maſter in Philoſophy *Diodotus*[a], "that "he exerciſed himſelf therein with more aſſiduity, after he "became blind: and what he thought next to impoſſible "to be done without Sight, that he profeſſed Geometry, de- "ſcribing his Diagrams ſo expreſsly to his Scholars, that they "could draw every Line in its proper Direction." St. *Jerom* relates a more remarkable Inſtance in *Didymus*[b] of *Alexandria*, who "though blind from his Infancy, and there- "fore ignorant of the very Letters, appeared ſo great a Mi- "racle to the World, as not only to learn Logic, but Geo- "metry alſo to perfection, which ſeems the moſt of any thing "to require the help of Sight." The Character of *Didymus* is celebrated, among other Hiſtorians, by *Caſſiodorus*; who makes mention alſo of one *Euſebius*[c] an *Aſiatic*, who according to his own Account of himſelf, "had been blind from "five Years old, and yet had treaſured up in his Mind all "kinds of Learning, and explained them likewiſe with the "greateſt clearneſs to others." And *Trithemius* gives a like Inſtance in one "*Nicaiſe*[d] of *Mechlin*, who though blind from

[a] Cic. Tuſc. Diſp. V, 39. *Diodotus Stoicus, cæcus multos annos, noſtræ domi vixit: is verò, quod credibile vix eſſet, cùm in Philoſophia multò etiam magis aſſiduè quam antea verſaretur,* —— *tum quod ſine oculis fieri poſſe vix videtur, Geometriæ munus tuebatur, præcipiens diſcentibus, unde, quo, quamque lineam ſcriberent.*

[b] Hieronymus de viris Illuſt. Cap. CIX. *Didymus Alexandrinus captus a parva ætate oculis, & ob id elementorum quoque ignarus, tantum miraculum ſui omnibus præbuit, ut Dialecticam quoque & Geometriam, quæ vel maximè viſu indiget, uſque ad perfectum didicerit.*

[c] Caſſiodorus de Inſt. Div. Liter. cap. 5. tradit *de partibus Aſiæ quendam ad nos veniſſe Euſebium nomine, qui ſe infantem quinque annorum ſic cæcatum eſſe narrabat, ut ſiniſtrum ejus oculum fuiſſe excavatum orbis profundiſſimus indicaret: dexter verò globus vitreo colore confuſus ſine videndi gratia infructuoſis niſibus volvebatur. Hic* —— *diſciplinas omnes & animo retinebat, & expoſitione planiſſima lucidabat.*

[d] Trithemius de Scriptoribus Eccleſ. N. DCCCLXXVI. *Nicaſius de Voerda, Mechlinienſis,* ——*captus à tertio ætatis ſuæ anno oculis,* —— *ſecundum noſtra ætate Didymum Alexandrinum exhibuit, dum in omni doctrina & ſcientia, tam divina quam humana eruditiſſimus evaſit. Nam in gymnaſio Colonienſi* —— *jura publicè docuit, libros utriuſque juris, quos nunquam vidit, audita didicit, tenuit mente, aperte recitavit.*

"the third Year of his Age, yet, like another *Didymus*, be-"came so great a Master of all Learning and Knowledge, "divine and human, that in the University of *Cologne*, he "publicly taught the Canon and Civil Law, openly reciting "Books which he had never seen, but had learnt by only "hearing them read to him." Beside these I have heard mention made of an *Hollander*, if not some others, who notwithstanding their Blindness have excelled in Mathematical Learning.

It is remarkable of the few, who have laboured under this Defect, and the still fewer, who had Genius enough to surmount the Difficulties attending it, that so many should be found to excel in Learning, and particularly in the Mathematics, as the two first above mentioned *certainly* did, and *probably* the others also. But if we consider that the Ideas of extended Quantity, which are the chief Objects of Mathematics, may as well be acquired from the Sense of Feeling as that of Sight; that a fixed and steady Attention is the principal Qualification for this Study, and that the Blind are by necessity more abstracted than others, we shall perhaps find Reason to think there is no other Branch of Science more adapted to their Circumstances. It is said of *Democritus* that he put out his Eyes, to enable him to think the more intensely; "imagining, says *Tully* [e], the Acuteness "of the Mind was taken off by the Sight of the Eye." And it was an Observation frequently made by our Professor, that Diagrams which are intended only as helps to the Imagination, are often the means of misleading the Judgment. It is certain, however useful they may be to the Learner, yet the Inventer must in all Cases proceed without them. The Scheme must be erected in his Imagination, in Circumstances as ge-

[e] Cic. Tusc. Disp. V, 39. *Democritus —— impediri etiam animi aciem aspectu oculorum arbitrabatur.*

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neral as the Propoſition, ſuch as cannot be delineated upon Paper. And I am confident, that any one who is deſirous of more than a general Knowledge of theſe Things, who would invent and improve upon what is to be learned from Books, will find his Mind greatly aſſiſted and enlarged, by accuſtoming himſelf to think and reaſon in the Circumſtances of a blind Man. But a Perſon who has the Misfortune to be ſuch, and is deprived of all the Pleaſures of Sight, will more frequently and more cloſely retire into himſelf, and finding few other Amuſements but in the purſuit of Truth, will be more likely to excel in theſe abſtract Sciences.

THE ſame Circumſtance may poſſibly contribute ſomething towards raiſing the Genius beyond its natural Pitch, in ſome other Arts, particularly Muſic and Poetry. The Poet indeed muſt firſt have his Imagination filled with all the beautiful Variety of Images in Art and Nature, which the Sight only can ſupply: if he then be deprived of that Senſe,

So much the rather may Celeſtial Light
Shine inward, and the Mind through all her Powers
Irradiate: —

as our blind Poet expreſſes it. Accordingly in the Catalogue of Epic (the ſublimeſt kind of) Poets, we find two *blind Bards*, ſurpaſſing all that any Age or any Nation have produced in the Flights of Fancy. And I cannot but wonder the very ingenious *Enquirer* into the Life and Writings of *Homer*, who endeavours to account for the great Genius, from a Concurrence of natural Cauſes, ſhould take no notice of that Circumſtance, which was ſo peculiar to his Poet.

IT was by the Senſe of Feeling our Author acquired moſt of his Ideas at firſt: and this he enjoyed in great Acuteneſs and Perfection, as it commonly happens to the Blind, whether by the kind Gift of Nature, or the Neceſſity of Application. Yet he could not, as ſome have imagined, (and as Mr. *Boyle* was made to believe of a blind Man at *Maeſtricht*) diſtinguiſh Colours by that Senſe; and having made repeated Tryals himſelf, he uſed to ſay, it was pretending to Impoſſibilities. But he could with great Nicety and Exactneſs diſcern the leaſt Difference of Rough and Smooth in a Surface, or the leaſt Defect of Poliſh. Thus he diſtinguiſhed in a Set of *Roman* Medals, the genuine from the falſe, though they had been counterfeited with ſuch Exactneſs as to deceive a *Connoiſſeur*, who had judged by the Eye. But ſays the Profeſſor, "I, who had not that Senſe to truſt to, could "eaſily feel a Roughneſs in the new-caſt, ſufficient to diſtin-"guiſh them by." His Senſe of Feeling was very accurate in diſtinguiſhing the leaſt Variation in the Atmoſphere. I have been preſent with him in a Garden, making Obſervations on the Sun, when he has taken notice of every Cloud that diſturbed our Obſervation, almoſt as juſtly as we could. He could tell when any thing was held near his Face, or when he paſſed by a Tree at no great Diſtance, provided the Air was calm, and little or no Wind: theſe he did by the different Pulſe of the Air upon his Face.

I wiſh I were capable of entertaining the Curious with the many Contrivances he had, to ſupply his Defect of Sight. He had a Board made with Holes bored at the equal Diſtance of half an Inch from each other: Pins were fixed in them, and by drawing a Piece of Twine round their Heads, he could more readily delineate all rectilinear Figures uſed in Geometry, than any Man could with a Pen. He had another Board with Holes made in right Lines for Pins of different Sizes. By the help of theſe he could calculate, and ſet down the Sums, Products, or Quotients in Numbers, as exactly as others could by Writing. By the help of an Armillary Sphere, the Schemes in Geometry that lie in different

Planes,

Planes, and the regular Solids cut in Wood, and the Form of ſeveral Curves made after the ſame manner, he was able on theſe Subjects to convey the cleareſt Ideas to his Pupils.

A refined Ear is what ſuch are commonly bleſſed with, who are deprived of their Eyes. Our Profeſſor was perhaps inferior to none in the Excellence of his: he could readily diſtinguiſh to the fifth Part of a Note, and by his Performance on the Flute, which he had learned as an Amuſement in his younger Years, diſcovered ſuch a Genius for Muſic, as would probably have appeared as wonderful, as his Excellence in the Mathematics; had he cultivated that Art with equal Application. By his Quickneſs in this Senſe he not only diſtinguiſhed Perſons, with whom he had ever once converſed, ſo long as to fix in his Memory the Sound of their Voice, but in ſome Meaſure Places alſo. He could judge of the Size of a Room into which he was introduced, of the Diſtance he was from the Wall: and if ever he had walked over a Pavement in Courts, Piazzas, &c, which reflected a Sound, and was afterwards conducted thither again, he could exactly tell whereabouts in the Walk he was placed, merely by the Note it ſounded.

THE Reader muſt greatly admire the Strength of his Memory, when aſſured that he could calculate in his Mind, multiply, divide, extract the ſquare or cube Root to many Places of Figures; could go along with any Calculator in working Algebraical Problems, Infinite Series, &c; and immediately correct the Slips of the Pen, as well in Signs as in Numbers. Thoſe who read to him had frequent Occaſions of admiring his great Sagacity and Quickneſs of Conception; with how much eaſe he followed any Track of Reaſoning, and with what Art he ſtored up in his Mind ſuch Parts, as would ſerve him to recollect and ruminate upon the whole. Indeed in the more abſtruſe Parts of Mathematics, where the

Scheme

Scheme was very intricate and perplexed, they often found it difficult to raiſe in his Imagination a clear and diſtinct Perception of it: but that once done, he ſeldom or never required any farther Aſſiſtance; his Mind retained ſo ſtrongly every Impreſſion that was once rightly made upon it. By the help of theſe ſtrong Faculties, a clear Imagination, tenacious Memory, and quick Reaſon, the Books of Mathematics lay ever open to him; he ſaw the whole in one View, every Dependency in the Chain of Truth. Thus he knew how to found every thing on the moſt eaſy Principles, and to compoſe with the juſteſt Symmetry and Order.

As in the Knowledge of the Mathematics he was exquiſite, and equal to any, ſo in the Addreſs of a Teacher, he was perhaps ſuperior to all. This Quality was conſpicuous at his firſt Appearance in the World, and muſt have been highly improved by long Uſe and Experience. He ſeemed perfectly to know what Difficulties young Minds are apt to be involved in, and how beſt to obviate or remove them. His Expreſſion was ſtrong and clear; and his Method ſo juſt and natural, that no one was at a loſs to follow him. He was very happy in all the Arts of facilitating a Demonſtration, in forming curious Poſitions to help the Imagination and obviate the Difficulties of Conception. I dare appeal to the following Sheets to determine, whether the ſeveral Propoſitions, which have paſſed through the Hands of *Euclid*, *Archimedes*, *Diophantus*, and the greateſt Maſters both ancient and modern, have not been greatly improved under his, by lowering the Ground-Work, and rendering the Structure more plain, yet more uſeful and ſubſtantial.

HIS Inclination led him to thoſe Parts of the Mathematics, which are not the moſt abſtracted, and end only in Contemplation. A Propoſition muſt have its Uſes, in order to engage his Attention. Either the Method of Enquiry muſt

help

help to form the Mind, and teach new Modes of Reafoning; or the Propofition itfelf muft tend to fome Good, to the Improvement of Life or Science. He confidered Mathematics as the Key to Philofophy, as the Clue to direct us through the fecret Labyrinths of Nature; and thought the Mind was more highly entertained as well as improved in unravelling Her Works, than inveftigating the moft fubtile Properties of abftract Quantity.

As to the *Geometric* and *Analytic* Methods of Reafoning, each of which have their Advocates and Favourers among the Mathematicians of the prefent Age: our Profeffor, I think, did Juftice to both, in allowing each the Advantage on different Occafions, and making ufe of that which feemed the moft proper for the prefent. The *Geometric* being the moft Intuitive, and conveying the ftrongeft and cleareft Ideas to the Mind, he allowed preferable, where equally obvious and eafy of Application. But as it was often otherwife, the *Analytic* advancing us in Science much fafter and farther than we could have gone by all the Methods of the Ancients, and being the very Art and Principle of Invention, He thought the Moderns were greatly affifted by the ufe of it.

Our Profeffor would not be induced by the Defires and Expectations of any, to engage in the War that was lately waged among Mathematicians, with no fmall Degree of Heat, concerning the *Algorithm or Principles of Fluxions*. Yet he wanted not the greateft Refpect for the Memory of Sir Isaac Newton, and thought the whole Doctrine entirely defenfible by the ftricteft Rules of Geometry. He owned indeed that the great Inventer, never expecting to have it canvaffed with fo much trifling Subtilty and Cavil, had not thought it neceffary to be guarded every where by Expreffions fo cautious as he might have otherwife ufed: for he wrote only for fuch fincere Lovers of Truth as himfelf was. But the general Averfion

Averſion he had to all Controverſial Writings withheld him from appearing in this. However, as he intended another Volume to his *Algebra* on the *Fluxionary* Part, he there deſigned to be particularly accurate and explicit upon the *Algorithm*; with an indirect View to the Controverſy on foot, and to obviate, the beſt he could, every Difficulty that had been ſtarted.

I cannot upon this Occaſion paſs by the Name of Sir Isaac Newton, without mentioning the profound Veneration paid to it by our Profeſſor. If he had ever differed in Sentiment from any of his Mathematical and Philoſophical Writings, upon more mature Conſideration, he ſaid, he always found the Miſtake to be his own. The more he read his Works, and obſerved upon Nature, the more Reaſon he found to admire the Juſtneſs and Care, as well as Happineſs of Expreſſion of that incomparable Philoſopher. He has left ſome valuable Comments on the *Principia*, which not only explain the more difficult Parts, but often improve upon the Doctrines, and which may in their preſent State be no unacceptable Preſent to the Public, though far ſhort of any thing he would himſelf have publiſhed on that Subject.

There was ſcarce any Part of the Mathematics, on which our Profeſſor had not wrote ſomething for the uſe of his Pupils. But he diſcovered no Intention of publiſhing any of his Works, till the Year 1733: when his Friends, alarmed by a violent Fever that had highly threatened his Life, and being unwilling that the Labours of ſo great a Man ſhould be loſt to the World, importuned him to ſpare ſome time from his Lectures, (which he then attended ſeven or eight Hours in a Day, to the great hazard and prejudice of his Health,) and to employ it in finiſhing ſome of his Works; which he might leave behind him, as a valuable Legacy both to his Family and the Public. He yielded ſo far to theſe Intreaties,

treaties, as to compose in a very short time the following Work, which he left perfect, and transcribed fair for the Press.

I NEED not say much of the Nature of it, but refer the Reader to the Contents, and to the Book itself. I will only observe, it was chiefly intended for the Instruction of young Beginners, and for the use of those that had such under their Care. The Author has therefore been very explicit and accurate in the exposition of every Part, and taken great care to express his Sentiments in the most simple and easy manner. His Design was not only to complete a Course of Algebra, but also, as far as possible, to promote the Study of Geometry, by removing or explaining all those Difficulties, which by his long Experience in teaching, he found were apt to retard, if not discourage young Students in their Progress through the Elements. And further, as Algebra is in it's own Nature an Art of Reasoning, and may be considered as the Logical Institutes of the Mathematician, the Author has been every where attentive to improve the Mind, and to furnish it with every Method of Reasoning that may be useful in our Researches into Nature. He has often exposed the same Truths to us in several Lights, as we arrive at them by different Methods of Enquiry: since this served to illustrate the Consistency of those Methods. He has also taken every Occasion to observe the Transition of Truth from one Law to another: to observe the Consistency of it's several Laws in the most intricate Cases, where they seem most to thwart and contradict each other, as if Nature were put to her Shifts to preserve that Consistency and Uniformity which is every where the Characteristic of Truth. Such Observations cannot but suggest to the Mind the most simple and natural Ways of discovering Truth, and must therefore be greatly instructive, as well as entertaining, to all who are engaged in these Enquiries.

 BUT

But the learned Mathematician must not wholly form his Judgment of the Author's Capacity from the following Work; which is of a Nature too low for the Exercise of so great a Genius. Yet the Reader will from hence learn to lament, with me, the Loss of a Life so valuable to the learned World, when he was just entering upon Designs, which, had he lived to execute, would have greatly enriched our Treasure of Mathematic Learning, and have proved Monuments to latest Posterity worthy of his Name. His Manuscripts were all left (for the Benefit of his Children) to the Care and Disposal of John Robartes of *Twickenham* Esquire, now Earl of Radnor; from whose Love and Esteem for Letters and learned Men, and particularly the learned Author, the Public may be well assured, that these Remains will be so disposed of, as to be most advantageous to Science, and honourable to their Author.

The Talents of Dr. Saunderson were not confined to the Study: when he put on the Companion, none supported Conversation with greater Wit and Elegance. His Discourse was so enlivened with frequent Allusions to Objects of Sight, that there appeared no Defect of the blind Man. Nothing was observed of that Disrelish of Humour, nothing of those Absences and Inattention to Discourse, which usually blemish and characterise Persons devoted to these severer Muses. His Judgment on the various Passions and Interests of Mankind was equally acute as on the Subjects of Philosophy. The Force and Spirit of his Expression surprised and fixed the Attention of all that heard him. But above all, the Mathematician's Reverence for Truth shone forth in every Circumstance of Life and Conversation, and added a Lustre to his most shining Qualities. His Sentiments on Men and Opinions, his Praises or Censures, his Friendship or Disregard were expressed without partiality or reserve. This Frankness

of

of Temper endeared him to all ſuch as were happy in his Acquaintance and Eſteem; but raiſed him up what Enemies he had, and betrayed him to ſeveral Animoſities, which Men of more Art and Complaiſance would have choſe to avoid, at the Expence of ſo ſcrupulous and ſo diſintereſted a Sincerity.

IT would be thought an Omiſſion in theſe Memoirs of the Life of Dr. SAUNDERSON, if no notice were taken of the manner in which he reſigned it. The Reverend Mr. GERVAS HOLMES informed him, that the Mortification gained ſo much ground, that his beſt Friends could entertain no hopes of his Recovery. He received this notice of his approaching Death with great Calmneſs and Serenity; and after a ſhort Silence, reſumed Life and Spirits, and talked with as much Compoſure of Mind as he had ever done in his moſt ſedate Hours of perfect Health. He appointed the Evening of the following Day to receive the Sacrament with Mr. HOLMES; but before that came, he was ſeized with a Delirium, which continued to his Death.

Dr. *SAUNDERSON*'s PALPABLE ARITHMETIC

DECYPHER'D.

THE Author of the following Piece, (who is Dr. SAUNDERSON'S *immediate Succeſſor in the Profeſſorſhip,) has been prevailed on to let it be inſerted here, as an Illuſtration to ſome of his Performances; though it was originally deſigned for another Place.*

THAT the learned and ingenious Dr. SAUNDERSON, late *Lucaſian* Profeſſor of Mathematics in the Univerſity of CAMBRIDGE, notwithſtanding the loſs of his Sight, was able to make long and intricate Calculations, both Arithmetical and Algebraical, is a Thing as certain as it is wonderful. This appears beyond all Contradiction, not only from his elaborate Treatiſe of Algebra now publiſhed, but from other undoubted Monuments ſtill in being. He had contrived for his own uſe, a commodious Notation for any large Numbers, which he could expreſs on his *Abacus*, or Calculating Table, and with which he could readily perform any Arithmetical Operations, by the Senſe of Feeling only; which therefore may be called his *Palpable Arithmetic*. As I have had an Opportunity, by the favour of Mrs. SAUNDERSON, of viewing and examining ſeveral Specimens of this Arithmetic, which by good fortune he had compleated and left behind him, though he has not left the leaſt Hint by which his Method might be diſcovered; I had the Curioſity to propoſe to myſelf

ſelf the decyphering (as it may be called) of theſe Specimens, in which I have ſucceeded to my own Satisfaction. And as others may have the ſame Curioſity, or as this Method may poſſibly be of uſe to other Perſons, whoſe Misfortune may place them in like Circumſtances, I ſhall here attempt to give a ſuccinct but particular Account of it.

His Calculating Table was a ſmooth thin Board, ſomething more than a Foot ſquare, raiſed upon a ſmall Frame ſo as to lie hollow; which Board was divided by a great Number of equidiſtant parallel Lines, and by others as many, at right Angles to the former. The Edges of the Table were diſtinguiſhed by Notches, at about half an Inch diſtance from one another, and to each Notch belonged five of the aforeſaid Parallels; ſo that every ſquare Inch was divided into an Hundred little Squares. At every Point of Interſection the Board was perforated by ſmall Holes, capable of receiving a Pin; for it was by the help of Pins, ſtuck up to the Head through theſe Holes, that he expreſſed his Numbers. He uſed two Sorts of Pins, a larger and a ſmaller Sort; at leaſt their Heads were different, and might eaſily be diſtinguiſhed by feeling. Of theſe Pins he had a large Quantity in two Boxes, with their Points cut off, which always ſtood ready before him when he calculated. And theſe were his Inſtruments, of which we muſt now ſee the uſe.

In order to this we may firſt obſerve, that to every numeral Figure a little Square was appropriated on the Table, conſiſting of four of the little contiguous Squares above deſcribed, and which therefore allowed a ſmall Interval between each Figure; and this numeral Figure was different, according to the different Magnitude or Situation of the one or two Pins, which always compoſed it. For which Purpoſe he had ſettled in his Mind, and ſtrictly obſerved, the following Analogy or Notation. A great Pin in the Center of the Square (which,

(which, and no other, was always its Place,) was a Cypher, or 0, and therefore I ſhall call it by that Name. Its chief Office was, to preſerve Order and Diſtance among his Figures and Lines. This Cypher was always preſent, except only in the Caſe of an Unit; to expreſs which the great Pin in the Center was changed into a little one. When 2 was to be expreſſed, the Cypher was reſtored to its Place, and the little Pin was put juſt over it. To expreſs 3, the Cypher remained as before, and the little Pin was advanced into the upper Angle on the right Hand. To expreſs 4, the little Pin deſcended, and immediately followed the Cypher. To expreſs 5, the little Pin deſcended to the lower Angle on the right Hand. To expreſs 6, the little Pin retreated, till it was juſt under the Cypher. To expreſs 7, the little Pin retreated into the lower Angle on the left Hand. To expreſs 8, the little Pin aſcended, till it was juſt before the Cypher. Laſtly, to expreſs 9, the little Pin aſcended into the upper Angle on the left Hand. And thus all the Digits were provided for, by an eaſy and uniform Notation, which might readily enough be apprehended and diſtinguiſhed by the Feeling. But to ſhew theſe Digits or Figures more diſtinctly, I ſhall endeavour to repreſent them by a Scheme. See *Fig.* I.

And thus he could write down, as we may ſay, any propoſed Number upon his Table, and by lightly running his Fingers over it, he could at any time readily read it, and know what it ſignified. The great Pins or Cyphers, which were always placed at the Centers of the little Squares, and moſt frequently at equal Diſtances from one another, were a ſure Guide to direct him to keep the Line, to aſcertain the Limits of every Figure, and to prevent any Ambiguity that might otherwiſe ariſe. As three of the erect Parallels were ſufficient for a ſingle Figure, ſo three of the tranſverſe Parallels would ſuffice for a Line of Figures, and the next three

for

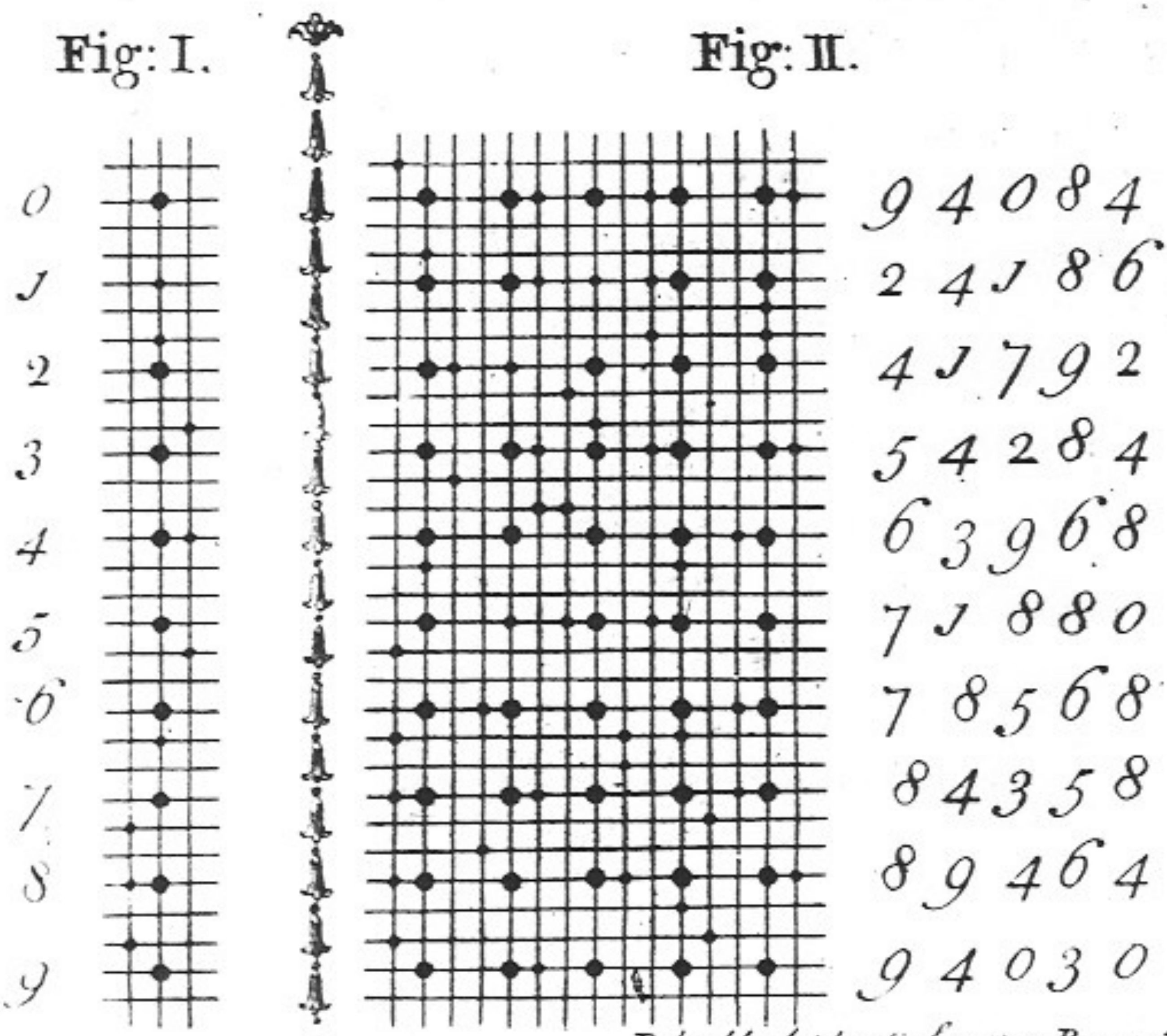
for another Line, and ſo on, without any Danger of interfering. And thus it is not hard to conceive, how he might have any Number of Lines of Figures upon his Table at the ſame time, in a deſcending order, or how he might derive one Number from another, or in a word, how he might make any Computations required. He could place and diſplace his Pins, as I have been informed, with incredible Nimbleneſs and Facility, much to the Pleaſure and Surprize of all the Beholders. He could even break off in the middle of a Calculation, and reſume it when he pleaſed, and could preſently know the Condition of it, by only drawing his Fingers gently over the Table. There is an obvious Expedient, which, eſpecially in long Calculations, would have made the Proceſs very expeditious, and therefore I queſtion not but he had often recourſe to it. And that is, to prepare the Table beforehand, (which any other might have done for him,) by filling every third Hole of every third parallel Line with large Pins, or Cyphers. Then when he intended to calculate, he would have nothing elſe to do, but to compleat every Figure, by adding a ſmall Pin in its proper Place. Except when an Unit was to be expreſſed, in which caſe he muſt have changed the large Pin into a ſmall one.

THE Specimens of this Arithmetic which I have peruſed, and reduced to common Numbers, are certain Arithmetical Tables, which he had computed and preſerved for his own uſe; but for what Purpoſes they were calculated does not eaſily appear. They ſeem to have ſome relation to the Tables of natural Sines, Tangents, and Secants; but their full uſe I muſt leave to future Enquiry. They are four Pieces of ſolid Wood, of the Form of rectangular Parallelepipeds, each about eleven Inches long, five and an half broad, and ſomething above half an Inch thick. The two oppoſite Faces of every one were divided into little Squares, after the manner

of

of the *Abacus* above deſcribed, but they were perforated only in the neceſſary Places, where the Pins were ſtuck faſt up to the Head. Each Face exhibited nine ſmall Arithmetical Tables, of ten Numbers each; and every Number, generally ſpeaking, conſiſted of five Places or Figures. For farther ſatisfaction I have delineated one of theſe ſmall Tables, both as I found it, and as I have interpreted it. See *Fig.* II.

BUT beſides this Arithmetical Uſe of his Table, which was indeed its principal and primary Uſe, he could deſcribe upon it very neat and perfect Geometrical Figures, conſiſting of right Lines, any how interſecting one another, of which I have ſeen ſome Inſtances. This he did two ways, either by Pins ſet in Rows, which exhibited the appearance of pricked Lines; or by Pins placed only at the Interſections. Then by winding a Piece of fine Thread or Silk about their Heads, he could very well exhibit any continued ſtrait Lines at pleaſure, or any Syſtem of ſuch Lines. Whether he had palpable Letters alſo, ſomething like Printing Types, to diſtinguiſh the ſeveral angular Points, and to aſſiſt in demonſtrating the Properties of thoſe Figures, does not now appear. If he had found any occaſion for ſuch Helps, his fruitful Genius would eaſily have ſupplied them. It is not very difficult to conceive likewiſe, how the ſame Table might poſſibly be applied to the repreſenting all kinds of Algebraical Equations, and to the ſeveral Reductions of ſuch Equations, eſpecially by the uſe of the Types aforementioned, or ſomething like it. And he might have Types, in the nature of Pins, for the common Algebraic Signs, and to inſinuate the ſeveral Operations; by which means his Table would bear a near reſemblance to a Printer's Form: which I queſtion not but he could have read by his feeling only, if he had ever applied himſelf to it. I have been well aſſured, that he could ſpell very well; that he knew the Shapes of the Letters, both Small and Capital, Fig: I. Fig: II.



Palpable Arithmetic fronting Page xxiv.

Capital, and would ſometimes amuſe himſelf, when Opportunity offered, by reading the Inſcriptions upon Tomb-Stones with his Fingers. He has been often heard to regret very much, that he did not apply himſelf to learn to write in his younger Years, which (as he affirmed) he made no queſtion but he could have eaſily accompliſhed. None of theſe things will appear incredible to thoſe, who have known him give his Judgment with as much Accuracy upon the goodneſs of a Mathematical Inſtrument, and the juſtneſs of its Diviſions, by examining it only with his Touch, as the moſt judicious Eye could have done; and he was often conſulted on this Occaſion. This at leaſt is evident, that he could manage all kinds of Equations, and other perplext Calculations, with great Skill and Sufficiency; but how far he relied upon the Strength of his Imagination, (which was certainly very great,) and when he had recourſe to Mechanical Contrivances to aſſiſt it, I will not undertake to determine.

FROM what I have already deſcribed, and from many other ingenious Devices of the like Nature, which I have ſeen, I ſhall conclude with this general Obſervation, that as the knowledge and uſe of Symbols, (or of ſenſible and arbitrary Signs of intellectual Ideas,) is of the greateſt Importance and Extent in all Parts of the Mathematics; ſo he had invented a new Species of Mathematical Symbols, unknown and unheard of before, which were particularly accommodated to his own Circumſtances. The ſenſible Symbols, commonly received and made uſe of, to repreſent Mathematical Ideas, and to convey them to our own, or to the Minds of others, are derived from two of our principal Senſes, and are either audible or viſible. The audible Symbols, it is true, he made good uſe of, and acquired much of his Knowledge by converſing with others, but chiefly by hearing others read all the beſt Mathematical Authors to him. But he was entirely

tirely deprived of the uſe of viſible Symbols, which to us we find are ſo abſolutely neceſſary, as that we always acquire the greateſt and moſt valuable Part of our Mathematical Knowledge by their Means. What did he do under this (as it ſhould ſeem) inſuperable Difficulty, in order to ſatisfy his great Thirſt after this kind of Knowledge? why, he had recourſe to another of his Senſes, which he had in great Perfection, and ſubſtituted Feeling in the place of Seeing; by inventing a Sort of Mathematical Symbols, which we may call palpable or tangible Symbols. Theſe he made uſe of, to convey thoſe Ideas to his Underſtanding, which were denied entrance through his Eyes. Thus by the aſſiſtance of theſe Symbols, imperfect and inadequate as they needs muſt be, and by the help of a quick Apprehenſion and obſtinate Perſeverance, he ſucceeded in theſe Sciences to Admiration. Theſe were the Inſtruments, as unfit as they may ſeem, by which he conveyed to his Mind the moſt abſtruſe and ſublime Mathematical Ideas, and by which he was enabled to deduce from them the moſt general and uſeful Concluſions. This we muſt all confeſs, if we have any degree of Candour, as well as Eyes and Underſtanding, to be very extraordinary and ſurprizing. I cannot but repreſent to my Imagination, during the whole courſe of his Studies, the Idea of a great Mathematical Genius, ſtruggling under the greateſt Diſadvantages, and labouring under the ſevereſt natural Inabilities and Diſcouragements. Yet whatever Difficulties he meets in his way, by a happy Sagacity and ſtubborn Induſtry, he finds as many Expedients to overcome them all; reſolving not to relinquiſh his purſuit, but ſteadily to perſiſt, till he is ſuperior to all Obſtacles, and till he has gratified the no mean Ambition, of placing himſelf in the firſt Claſs of Mathematicians.

A Second Advertisement.

SINCE the printing of the former Advertisement, I have been favoured with the use of an English *Translation of* Napeir's *Book, entitled* A Description of the admirable Table of Logarithmes, *perused and approved by the Author himself, and printed in the year* 1616. *From which Book it appears, that He himself, and not* Briggs, *was likewise the inventer of the System of Logarithms now in use, as I had before observed. For thus he writes,* book 1. chap. 4. sect. 9. pag. 19, 20. "*But because the addition and subtraction of these former numbers may seeme somewhat painfull, I intend (if it shall please God) in a second Edition, to set out such* Logarithmes *as shal make those numbers aboue written to fall upon decimal numbers, such as* 100 000 000, 200 000 000, 300 000 000, *&c. which are easie to be added or abated to or from any other number.*" *Another quotation to the same purpose, from the Author's* Rabdologia, *published in the year* 1617, *may be seen in Mr. Professor* WARD's Lives of the Professors of Gresham College, p. 122.

Amongst Professor SAUNDERSON's Questions for Exercise in the Rule of Three, *there are some which belong to the Double Rule of Three, where five numbers are given in order to find a sixth; as in the thirtysecond question, pag.* 13. If two men in three days will earn four shillings, how much will five men earn in six days? *Where the numbers given are* 2, 3, 4, 5, 6; *the three first of which are always in the conditional part of the question, and the other two in the remaining part, which moves the question. It is also to be noted, that in all questions belonging to the Double Rule of Three, the numbers are so to be placed, as that the first and the fourth, the second and the fifth, the third and the number sought, may be of the same denomination respectively; as in the question proposed:* 2 men, 3 days, 4 shillings; 5 men, 6 days, shillings required.

These things being premised, the Professor's first rule is this, pag. 14. In all questions of this nature, if the three last numbers be multiplied together, and the product be divided by the product of the two first, the quotient will give the number sought. *As in the question proposed,* $\frac{4\times5\times6}{2\times3}=20$, *the number sought. Again, in the thirtythird question,* If for the carriage of 3 hundred weight 40 miles I must pay 7 shillings and 6 pence, what must I pay for the carriage of 5 hundred weight 60 miles? *The terms are* 3 hundred weight, 40 miles, $7\frac{1}{2}$ shillings; 5 hundred weight, 60 miles. *Answ.* $\frac{7\frac{1}{2}\times5\times60}{3\times40}=18\frac{3}{4}$, *that is,* 18 *shillings and* 9 *pence.*

There is another case, wherein inverse proportion is partly concerned, as in the thirtyninth question, pag. 16. If two acres of land will maintain three horses four days, how long will five acres maintain six horses? *Here it is plain, that inverse proportion is concerned: for the same quantity of land will maintain six horses but half the time that it would maintain three. Wherefore for resolving questions of this nature, the Professor proposes a second rule, pag.* 17, 18. In all questions belonging to the Double Rule of Three inverse, where the numbers are supposed to be ordered as in the Double Rule of Three direct, if the three middle numbers be multiplied together, and the product be divided by the product of the two extremes, the quotient of this division will be the number sought. *As in the thirtyninth question, the order of the terms being* 2 acres, 3 horses, 4 days; 5 acres, 6 horses; *therefore* $\frac{3 \times 4 \times 5}{2 \times 6} = 5$, *the number of the days sought.*

But here it must be observed, that the terms of this question are capable of being ranged in a different order, thus: If three horses will eat up the grass of two acres of land in four days, how long will six horses be in eating up that of five acres? *Here the order of the terms given is,* 3 horses, 2 acres, 4 days; 6 horses, 5 acres; *according to which order, neither this rule nor the former will solve the question.*

Wherefore, to clear this difficulty, and to find out, in what order the Professor's second rule requires the terms given should be placed, it may be useful to investigate a general Theorem for resolving all questions in the Double Rule of Three. This I shall do in the method proposed by the late Mr. WARD *of* Chester, *in his* Introduction to the Mathematicks, part 1. chap. 7. sect. 3. pag. 96, *where he denotes the six quantities in questions belonging to the Double Rule of Three, by these six letters,* P, T, G, p, t, g; *the capitals* P, T, G *denoting the terms in the conditional part of the question, as the small letters* p, t, g *do the other terms, of the same denomination with their respective capitals.* P, p *signify the principal terms, or causes of the gain, loss, or expence mentioned in the question;* T, t *the times (as in quest.* 32 *and* 39*) or spaces (as in quest.* 33*) wherein the said gain, loss, or expence is produced; and* G, g *the gain, loss, or expence itself, arising from the principals in the aforesaid times or spaces.*

Now in order to form a Theorem for resolving questions in the Double Rule of Three direct, let us take any particular question, as for example, the thirty-second: If 2 men in 3 days will earn 4 shillings, how much will 5 men earn in 6 days? *Where* 2=P, 3=T, 4=G, 5=p, 6=t, *and* g *is the number sought. This question then may be resolved into two proportionalities, thus:* If 2 (P) men will earn 4 (G) shillings in a given time, 5 (p) men will earn 10 $\left(\frac{Gp}{P}\right)$ shillings in the same time. *Again,* If in 3 (T) days

10 $\left(\frac{Gp}{P}\right)$ shillings be earned, then in 6 (*t*) days 20 $\left(\frac{Gpt}{PT}\right)$ will be earned. *Therefore* $\frac{\text{Gpt}}{\text{PT}}=\text{g}$; *and multiplying both sides by* PT, *we have the Theorem* Gpt=gPT. *And since the thirtysecond question proposed, gives us five terms in this order*, P, T, G, p, t; *the term sought being* $\text{g}=\frac{\text{Gpt}}{\text{PT}}$, *it is manifest, that the term sought is found, by dividing the product of the three last terms given, by the product of the two first, according to the Professor's first rule.*

Let us now take the thirtyninth question; that we may thence form a Theorem for resolving questions of the same nature with it, which belong to the Double Rule of Three inverse. If 2 (*G*) acres of land will maintain 3 (*P*) horses 4 (*T*) days, how long will 5 (*g*) acres maintain 6 (*p*) horses? *Which question I resolve into two proportionalities: first into this inverse proportionality*, If a certain piece of ground will maintain 3 (*P*) horses 4 (*T*) days, it will maintain 6 (*p*) double the number of horses but half the time, that is, 2 $\left(\frac{PT}{p}\right)$ days: *Secondly, into this direct one*, If 2 (*G*) acres will maintain this double number of horses 2 $\left(\frac{PT}{p}\right)$ days, 5 (*g*) acres will maintain them 5 $\left(\frac{gPT}{Gp}\right)$ days. *Therefore* t, *the number sought, is* $\frac{\text{gPT}}{\text{Gp}}$. *And multiplying both sides by* Gp, *we have the same Theorem as before*, Gpt=gPT, *which therefore is a general Theorem for solving all questions in the Double Rule of Three, whether direct or inverse. And since the order of the terms given in the thirtyninth question is* G, P, T, g, p; *and since also* t, *the number sought, is found equal to* $\frac{\text{gPT}}{\text{Gp}}$, *therefore according to the Professor's second rule, if the product of the three middle terms be divided by the product of the two extremes, the quotient will be the term sought.*

But in the practising of this second rule, care must be always taken to make the term G *the first in order: and then, if* t *be sought, as in the thirtyninth question, the order of the terms given will be* G, P, T, g, p, *as before; but if* p *be sought, the order will be* G, T, P, g, t. *And lastly, I think I scarce need to observe, that if* g *be the term sought, the operation must always be made by the Professor's first rule, where the order of the terms is* P, T, G, p, t, *as in the thirtysecond and thirtythird questions; or else* T, P, G, t, p, *which comes to the same thing.*

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ERRATA.

In the laft Page of the former Advertifement, lin. 34, 35. *read* compleret.
* Pag. 120. Prob. 23. lin. 2. for 16 *years* read 15 *years.*

POSTU-

POSTULATA.

BEFORE I enter upon my province, it may not be amiſs to acquaint my young diſciple what preparations he is to make, and what qualifications I expect of him beforehand, that we may neither of us find ourſelves diſappointed afterwards. I expect then that he knows how to add, to ſubtract, to multiply, to divide, to find a fourth proportional, and to extract roots, eſpecially the ſquare root: nay I expect further, that he ſhall not only be able to perform all theſe operations exactly and readily, but alſo that he ſhall be able to apply them upon all common occaſions; in a word, I expect that he be tolerably well ſkilled in common Arithmetick, at leaſt ſo far as relates to whole numbers: for this reaſon it is that I have prefixed a few arithmetical queſtions, wherein he may firſt try his ſtrength and ſkill before he ventures any further; they are for the moſt part very eaſy, I cannot ſay indeed they are the beſt choſen, but they were ſuch as lay in my way when I firſt begun this work and was haſtening to matters of greater moment, and I do not ſee but they may, if ſtudied with care and attention, anſwer well enough the end they were intended for: If he finds no difficulty in theſe, he will have little reaſon to doubt of his ſucceſs afterwards; but if he does, he ought then at laſt to become ſenſible of his own defects and to endeavour to ſupply whatever is wanting, and to correct whatever is amiſs before he enters himſelf under my conduct; in the mean time he has my leave to hope that I ſhall be leſs upon the reſerve with him when he falls more immediately under my care.

N. B. The *praxis* of the rule of proportion, and of the rule for extracting the ſquare root, not being (properly ſpeaking) of the nature of ſimple *poſtulata*, but rather deducible from the four firſt; I ſhall not fail to demonſtrate theſe rules ſo ſoon as I ſhall find proper opportunities for that purpoſe.

Questions for exercise in Multiplication.

Multiplication is taking any one number called the multiplicand as often as is expressed by any other number called the multiplicator, and the number produced by this operation is called the product: whence it follows, that the product contains the multiplicand as often as there are units in the multiplicator, and that if a number of a greater denomination is to be reduced to an equivalent number of a less, it must be done by multiplication. As for example; In a pound sterling there are 20 shillings; therefore in every sum of money consisting of even pounds, there are twenty times as many shillings as there are pounds; therefore if any number of pounds be multiplied by 20, the product will be an equivalent number of shillings; and the same must be observed in all other cases.

QUEST. 1.

It is required to reduce 456 pounds, 13 shillings and 4 pence into shillings, pence and farthings.

Answer.	Shillings	9133
	Pence	109600
	Farthings	438400.

QUEST. 2.

A certain island contains 36 counties, every county 37 parishes, every parish 38 families, and every family 39 persons: I demand the number of parishes, families and persons in the whole island.

Answer.	Parishes	1332
	Families	50616
	Persons	1974024.

QUEST. 3.

In 1730 years, 42 weeks and 3 days, how many minutes?

N. B. A year consists of 365 days 6 hours, and an hour of 60 minutes.

Hours in one year	8766
In 1730 years	15165180
In 42 weeks 3 days	7128
In the whole	15172308
Minutes in the whole	910338480.

QUEST. 4.

There is a certain field 102004 *feet long, and* 102003 *feet broad: I demand the number of ſquare feet therein contained.*

Anſwer. 10404714012.

QUEST. 5.

There is a certain floor 24 *feet,* 4 *inches broad, and* 96 *feet,* 6 *inches long: I demand how many ſquare inches are therein contained.*

Anſwer. 338136 ſquare inches.

QUEST. 6.

A certain piece of wood 1 *foot,* 2 *inches thick,* 3 *feet,* 4 *inches broad, and* 5 *feet,* 6 *inches long, is to be cut into ſmall cubes like dies, each of which is to be a quarter of an inch every way: I demand into how many dies the whole may be reſolved.*

Anſwer. The whole may be reſolved into 2365440 dies.

QUEST. 7.

I demand the number of changes that may be rung on 12 *bells.*

Changes upon	2 bells	2
on	3 bells	6
on	4 bells	24
on	5 bells	120
on	6 bells	720
on	7 bells	5040
on	8 bells	40320
on	9 bells	362880
on	10 bells	3628800
on	11 bells	39916800
on	12 bells	479001600.

QUEST. 8.

How many different ways can four common dies come up at one throw?

Anſwer 1296 ways.

QUEST. 9.

Suppoſe one undertakes to throw an ace at one throw with four common dies; what probability is there of his effecting it?

Anſwer. By the laſt queſtion four dies can come up 1296 different ways with and without the ace; and by a like computation, they can come up 625 ways without the ace; therefore there are 671 ways wherein one or more of them may turn up an ace; therefore the undertaker has the better of the lay in the proportion of 671 to 625.

QUEST. 10.

There are two incloſures of the ſame circumference, that is, both incloſed with the ſame number of pales; but one is a ſquare whoſe ſide is 125 *feet, and the other an oblong or long ſquare,* 124 *feet in breadth, and* 126 *in length:* quære *which is the greater cloſe, that is, which,* cæteris paribus, *will bear moſt graſs.*

Anſwer. The ſquare: for that contains 15625 feet; whereas the other contains but 15624.

Queſtions for exerciſe in Diviſion.

The deſign of diviſion is to ſhew how often one number called the diviſor is contained in another called the dividend, and the number that ſhews this is called the quotient: whence, and from the definition of multiplication already given, I obſerve *1ſt*, That the diviſor multiplied by the quotient, and conſequently the quotient multiplied by the diviſor, will always be equal to the dividend, provided there be no remainder after the diviſion is over; but if there be, then this remainder added to, or taken into the product will give the dividend, which is the beſt proof of diviſion. *2dly*, That as the diviſor is ſuch a part of the dividend as is expreſſed by the quotient; ſo alſo is the quotient ſuch a part as is expreſſed by the diviſor. Thus 12 divided by 3 quotes 4; therefore 3 is a fourth part, and

4

4 a third part of 12. 3*dly*, Hence may a number be found that shall be divisible by any two given numbers whatever without remainders, to wit, by multiplying the two given numbers together. Thus if I would have a number that can be divided by both 6 and 9 without any remainders, I multiply 9 by 6, and the product 54 will answer both conditions; though 18 be the least number of that kind. 4*thly*, Multiplication and division by the same number are the reverse of each other, and so must necessarily have contrary effects: for whereas multiplication increases a number by taking it as often as is expressed by the multiplicator, division (on the contrary) lessens it, by taking only such a part of it as is expressed by the divisor. 5*thly*, Hence if a number of a lesser denomination be to be changed into an equivalent number of a greater, as farthings into pence, pence into shillings &c, it must be done by division, as the reverse is done by multiplication. 6*thly*, Whenever it is proposed to know how often one quantity of any kind is contained in another of the same kind, the numbers representing these quantities must be reduced to the same denomination before any division can take place. Thus if I would know how many thirteenpencehalfpennies there are in 20 shillings, I must not only reduce the thirteenpencehalfpenny to 27 halfpence, but also the 20 shillings into 480 halfpence; and then must enquire by division how often 27 halfpence are contained in 480 halfpence, that is, how often 27 is contained in 480; the quotient is 17, and the remainder 21, that is, 21 halfpence; for in all division, the remainder must be of the same denomination with the dividend whereof it is a part; therefore in 20 shillings there are 17 thirteenpencehalfpennies, and 10 pence halfpenny over.

QUEST. 11.

It is required to reduce 987654321 *farthings into pounds, shillings and pence.*

Answer. 987654321 farthings are equivalent to 246913580 pence and 1 farthing; or to 20576131 shillings, 8*d*. 1*q*; or to 1028806 pounds, 11*s*. 8*d*. 1*q*.

QUEST. 12.

One lends me 1296 *guineas when they were valued at* 1*l*. 1*s*. *and sixpence apiece: how many must I pay him when they are valued at* 1. 1*s*. *apiece?*

Answer. 1326 guineas, 18 shillings.

QUEST.

QUEST. 13.

A certain floor 24 feet 4 inches broad, 96 feet 6 inches long, is to be laid at the rate of 12 pence the ſquare foot: I demand what the whole charge will amount to.

Anſwer. The floor contains 338136 ſquare inches, or 2348 ſquare feet and 24 ſquare inches; therefore the whole charge amounts to 117 pounds, 8 ſhillings and two pence.

QUEST. 14.

There is a certain cooler 36 inches deep, 42 inches wide, and 72 inches long: I demand its ſolid content in Engliſh gallons.

Note. An ale gallon is 282 cubic inches.

Anſwer. The veſſel contains 108864 cubic inches, that is, 386 gallons and 12 cubic inches over.

QUEST. 15.

A cubic foot of water weighs 76 pounds, Troy or Roman weight; and air is 860 times lighter than water: I demand the weight of a cubic foot of air.

N. B. A pound Troy contains 12 ounces, one ounce 20 pennyweights, and one pennyweight 24 grains.

Anſwer. A cubic foot of air weighs Troy weight 1 *oz.* 1 *pwt.* 5 *gr.*

QUEST. 16.

The mean time of a lunation, that is, from new moon to new moon, is 29 days, 12 hours, 44 minutes and 3 ſeconds; and a Julian year conſiſts of 365 days, 6 hours: I demand then how many lunations are contained in 19 Julian years.

Hours in a Lunation	708
Minutes	42524
Seconds	2551443
Hours in 19 Julian years	166554
Minutes	9993240
Seconds	599594400

Lunations 235; and 1 hour, 28′, 15″ over.

QUEST. 17.

In what time may all the changes on 12 bells be rung, allowing 3 ſeconds to every round? See Queſt. the *7th*.

The number of changes on 12 bells 479001600
The time 1437004800 ſeconds,
or 23950080 minutes,
or 399168 hours,
or 45 years, 27 weeks, 6 days, 18 hours.

QUEST. 18.

A General of an army diſtributes 15 pounds, 19 ſhillings and 2 pence half-penny, among 4 captains, 5 lieutenants and 60 common ſoldiers, in the manner following: Every captain is to have 3 times as much as a lieutenant, and every lieutenant twice as much as a common ſoldier: I demand their ſeveral ſhares.

The ſhare of a common ſoldier	3*s*. 4*d*. $\frac{1}{4}$
of a lieutenant	6*s*. 9*d*. $\frac{1}{2}$
of a captain	1*l*. 0*s*. 4*d*. $\frac{1}{2}$

Queſtions for exerciſe in the Rule of Three.

And firſt in the Rule of Three Direct.

The rule of proportion, or rule of three, or by ſome the golden rule, is that which teacheth, having three numbers given to find a fourth proportional, that is, to find a fourth number that ſhall have the ſame proportion to ſome one of the numbers given, as is expreſſed by the other two; and therefore whenever a queſtion is propoſed wherein ſuch a fourth proportional is required, that queſtion is ſaid to belong to the rule of proportion. Now in queſtions of this nature, eſpecially where the numbers given are not merely abſtract numbers, but are applied to particular quantities, three things are uſually required, to wit, preparation, diſpoſition, and operation.

Firſt as to the preparation, it muſt be obſerved that of the three numbers given in the queſtion, two will always be of the ſame kind, and muſt be reduced to the ſame denomination, if they be not ſo already; and

and if the remaining number be of a mixt denomination, that alſo muſt be reduced to ſome ſimple one.

Secondly, in diſpoſing the numbers thus prepared, thoſe two that are of the ſame denomination muſt be made the firſt and third numbers in the rule of proportion, and conſequently the remaining number muſt be the ſecond. But here particular care muſt be taken, that of the two numbers that are of the ſame denomination, that be made the third in the rule of proportion, upon which the main ſtreſs of the queſtion lies, or to which the queſtion more immediately relates, or which contains the demand; and the place of this number being once known, the other two muſt take their places as above directed. This ordering of the numbers for the operation is commonly called, ſtating of the queſtion.

Laſtly, having thus ſtated the queſtion, multiply the ſecond and third numbers together; divide the product by the firſt, and the quotient thence ariſing will be the fourth number ſought; which fourth number, as well as the remainder, if there be any, muſt always be underſtood to be of the ſame denomination with the ſecond. As for example,

QUEST. 19.

A piece of plate weighing 3 pounds, 4 ounces and 5 pennyweights, Troy weight, is valued at 5 ſhillings and 6 pence an ounce; what is the value of the whole?

Here we have three quantities concerned in the queſtion, *viz.* 3 pounds, 4 ounces and 5 pennyweights; one ounce; and 5 ſhillings and 6 pence; whereof the two firſt, which are of the ſame kind, muſt be reduced to the ſame denomination, and the laſt to a ſimple one, thus: for one ounce I write 20 pennyweights; for 3 pounds, 4 ounces and 5 pennyweights, 805 pennyweights; and for 5 ſhillings and 6 pence, 66 pence; and ſo the numbers are ſufficiently prepared. In the next place I enquire which of the two numbers 20 and 805, which are of the ſame denomination, is that upon which the main ſtreſs of the queſtion lies, and I find it to be 805; for the main buſineſs of this queſtion is to enquire into the value of 805 pennyweights of plate; the reſt being no more than *data* in order to diſcover this: So I make 805 my third number, 20 which is a number of the ſame denomination my firſt, and 66 my ſecond; and ſtate the queſtion thus; *If 20 pennyweights of plate be worth 66 pence, what will 805 pennyweights of plate be worth?* Now to anſwer this queſtion, I multiply 805 by 66, and the product is 53130; this I divide by my firſt number 20, and the quotient is 2656, and there remains 10, that is, 10 pence; therefore to render my quotient more compleat, I bring the remaining 10 pence

pence into 40 farthings, and so divide again by 20, and find the quotient to be 2, that is, 2 farthings, without any remainder; so the value sought is 2656 pence, 2 farthings; that is, 11 pounds, 1 shilling and 4 pence half-penny.

A demonstration of this Praxis.

Case 1st. Now to demonstrate this manner of operation, I shall resume the foregoing question, but at first under a different supposition, as thus; *If one pennyweight of plate cost* 66 *pence, what will* 805 *pennyweights cost?* Here nobody doubts but that upon this supposition, 805 pennyweights will cost 805 times 66 pence, or 66 times 805, that is, 53130 pence; therefore in all instances of this kind, that is, where the first number in the rule of proportion is unity, the fourth number must be found by multiplying the second and third numbers together.

Case 2d. Let us now put the question as it was at first stated, to wit, *If* 20 *pennyweights of plate be worth* 66 *pence, what will* 805 *pennyweights be worth?* Now upon this supposition it is easy to see, that neither 1 pennyweight, nor consequently 805 pennyweights will be worth above a 20th part of what they were in the former case; and therefore we must not now say that 805 pennyweights are worth 53130 pence, but a 20th part of that sum, *viz.* 2656 pence 2 farthings: and as this way of reasoning will be the same in all other instances, it follows now, that *In the rule of proportion, let the numbers given be what they will, the fourth number must be had by multiplying the second and third numbers together, and dividing the product by the first.* Q. E. D.

QUEST. 20.

How far will one be able to travel in 7 *days* 8 *hours, at the rate of* 13 *miles every* 4 *hours, allowing* 12 *hours to a travelling day?*

Answer. 299 miles.

QUEST. 21.

What will 1296 *yards of walling amount to, at the rate of* 4 *shillings and* 5 *pence a rod, a rod being* 5 *yards and a half?*

Answer. 52 pounds, 8 pence, 3 farthings.

QUEST. 22.

In the mint of England *a pound of gold, that is,* 11 *ounces fine and* 1 *allay, is at this time coined into* 44 *guineas and an half: I demand how much ſterling a pound of pure gold is worth, obſerving that the allay is valued at nothing.*

Anſwer. 50 pounds, 19 ſhillings and 5 pence ½ penny

QUEST. 23.

What is the annual intereſt of 987 *pounds,* 6 *ſhillings and* 5 *pence at the rate of* 6 per cent?

Anſwer. 59 pounds, 4 ſhillings and 9 pence ¼ penny.

QUEST. 24.

The circumference of the earth according to the French menſuration is 123249600 *French feet: I demand the ſame in Engliſh miles.*

N. B. A thouſand French feet are equivalent to 1068 Engliſh feet; 3 feet make a yard, and 1760 yards make a mile.

Anſwer.	131630573	Engliſh feet,
or	43876857	yards and 2 feet,
or	24930	miles, 57 yards and 2 feet.

QUEST. 25.

Suppoſing all things as in the foregoing queſtion, I demand how long a ſound will be in paſſing from pole to pole, upon a ſuppoſition that a ſound paſſes over 1142 *feet in a ſecond of time.*

Anſwer. 16 hours and 32 ſeconds.

QUEST. 26.

Monſieur Huygens *found that at* Paris, *the length of a pendulum that ſwung ſeconds, was three feet,* 8 *lines and* ½: *I demand it's length in Engliſh meaſure.*

Note.

Note. A line is $\frac{1}{12}$ part of an inch, and 1000 French half lines are equivalent to 1068 English $\frac{1}{2}$ lines, as in the 24th queſtion.

Anſwer. The length in English meaſure of a pendulum that ſwings ſeconds, is 941 Engliſh $\frac{1}{2}$ lines; or 39 inches, 2 lines and $\frac{1}{2}$.

QUEST. 27.

I demand in how long a time, a pipe that diſcharges 15 pints in 2 minutes, 34 ſeconds, will fill a ciſtern that is 36 inches deep, 42 inches wide, and 72 inches long. (ſee queſtion the 14th.)

Anſwer. In 31707 ſeconds; or 8 hours, 48'. 27".

For as eight pints make a gallon, ſo alſo eight cubic half inches, that is, eight ſmall cubes of half an inch every way make one cubic inch; therefore a pint contains 282 cubic half inches, and fifteen pints 4230; but the whole veſſel contains 108864 cubic inches by queſt. 14; which are equivalent to 870912 cubic half inches; therefore this queſtion ought to be ſtated thus;

If 4230 cubic half inches be diſcharged in 154 ſeconds of time, in what time will 870912 cubic half inches be diſcharged? And the anſwer is,

In 8 hours, 48'. 27". as above.

QUEST. 28.

If a wall 6 feet thick, 9 feet high and 432 feet long, coſt 720 pounds in building, what will be the price of a wall of the ſame materials, that is 12 feet thick, 18 feet high and 576 feet long?

In the former wall are contained 23328 cubic feet; in the latter 124416; therefore the anſwer to this queſtion is 3840 pounds.

QUEST. 29.

A certain ſteeple projected upon level ground a ſhadow to the diſtance of 57 yards, when a four-foot ſtaff perpendicularly erected caſt a ſhadow of 5 feet 6 inches: what was the height of the ſteeple?

Anſwer. 41 yards, 1 foot, 4 inches.

QUEST. 30.

Two persons A *and* B *make a joint ſtock;* A *puts in* 372 *pounds, and* B 496 *pounds, for the ſame time; and they gain* 114 *pounds,* 2 *ſhillings: I demand each mans ſhare of the gain.*

Both their ſtocks make 868 pounds: ſay then, if 868 pounds ſtock, bring in 114 pounds, 2 ſhillings gain, what will 372 pounds, *A*'s part of the ſtock bring in? *Anſwer.* 48 pounds, 18 ſhillings for *A*'s ſhare of the gain; and this ſubtracted from the whole gain, leaves 65 pounds, 4 ſhillings, for *B*'s ſhare of the gain.

Note. If there be ever ſo many partners, their ſhares of the gain muſt all be found by the rule of proportion, except the laſt, which may be had by ſubtraction; but it would be better to find them all by the rule of proportion, becauſe then, if all the ſhares when added together, make up the whole gain, it will be an argument that the work is rightly performed.

QUEST. 31.

Two perſons A *and* B *make a joint ſtock;* A *puts in* 496 *pounds for* 2 *months, and* B 620 *pounds for* 3 *months; and they gain* 456 *pounds. What will be each mans ſhare of the gain?*

In order to give an anſwer to this queſtion, it muſt be conſidered, that it is the ſame in the caſe of trade, as it is in that of money let out to intereſt, where time is as good as money, that is, whoever lets out 496 pounds for 2 months, is intitled to the ſame ſhare of the whole gain, as if he had let out twice as much, that is, 992 pounds, for one month: in like manner, he that lets out 620 pounds for 3 months, has a right to the ſame ſhare of the gain, as if he had let out three times as much, that is, 1860 pounds, for 1 month: ſubſtitute therefore theſe ſuppoſitions inſtead of thoſe in the queſtion, which may ſafely be done without affecting the concluſion, and then this queſtion will be reduced to the form of the laſt, without any conſideration of the particular quantity of time, thus; *Two merchants* A *and* B *make a joint ſtock;* A *puts in* 992 *pounds, and* B 1860 *pounds for the ſame time; and they gain* 456 *pounds. What will be their reſpective ſhares of the gain?*

Anſwer. *A*'s ſhare will be 158 pounds, 12 ſhillings and 2 pence; and *B*'s, 297 pounds, 7 ſhillings and 10 pence.

QUEST. 32.

If two men in three days will earn 4 ſhillings, how much will 5 men earn in 6 days?

This and the following queſtion belong to that which they call the double rule of three, wherein 5 numbers are concerned: theſe numbers muſt always be placed as they are in this example, that is, the two laſt numbers muſt always be of the ſame denomination with the two firſt reſpectively, and the number ſought of the ſame denomination with the middle one; then may the queſtion be reduced to the ſingle rule of three two ways, either by expunging the firſt and fourth numbers, or the ſecond and fifth: if you would have the firſt and fourth numbers expunged, you muſt argue thus; two men will earn as much in three days, as one man in two times 3, or 6 days; alſo 5 men will earn as much in 6 days, as one man in 30 days; ſubſtitute therefore this ſuppoſition and this demand, inſtead of thoſe in the queſtion, and it will ſtand thus; If one man in 6 days will earn four ſhillings, how much will one man earn in 30 days? Which is as much as to ſay, *If in 6 days a man will earn 4 ſhillings, how much will he earn in 30 days?*

Anſwer. 20 ſhillings.

If you would have the ſecond and fifth numbers expunged, you muſt argue thus; two men will earn as much in three days, as 3 times two or 6 men in one day; alſo 5 men will earn as much in 6 days, as 30 men in one day; put then the queſtion this way, and it will ſtand thus; If 6 men in one day will earn 4 ſhillings, how much will 30 men earn in one day? That is, *If in any quantity of time 6 men will earn 4 ſhillings, how much will 30 men earn in the ſame time?*

Anſwer. 20 ſhillings, as before.

Whoſoever attends to both theſe methods of extermination, will eaſily fall into a third, which includes both the other, and in practice is much better than either of them; for at the concluſion of both operations, the number ſought was found by multiplying 30 by 4, and dividing the product by 6: Now if he looks back, and traces out theſe numbers, he will find that the number 30 came from the multiplication of the two laſt numbers 5 and 6 together, that 4 was the middle number in the queſtion, and that the diviſor 6 was the product of the two firſt numbers

2 and

2 and 3 multiplied together; therefore *In all questions of this nature, if the three last numbers be multiplied together, and the product be divided by the product of the two first, the quotient will give the number sought, without any further trouble.*

QUEST. 33.

If for the carriage of three hundred weight 40 *miles, I must pay* 7 *shillings and* 6 *pence, what must I pay for the carriage of* 5 *hundred weight* 60 *miles?*

Answer. 225 pence, or 18 shillings and 9 pence.

Questions in the rule of three Inverse.

Hitherto we have instanced in the rule of three direct; but there is also another rule of proportion, called the rule of three inverse; which as to the preparation, and disposition of it's numbers, differs nothing from the rule of three direct; but only in the operation; for whereas there, the fourth number was found, by multiplying the second and third numbers together, and dividing by the first; here it is found by multiplying the first and second numbers together, and dividing by the third. All that remains then, is to be able to distinguish, when a question belongs to one rule, and when to the other; in order to which, observe the following directions: *If more requires more, or less requires less, work by the rule of three direct; but if more requires less, or less requires more, work by the rule of three inverse.* The meaning whereof is, that if, when the third number is greater than the first, the fourth must be proportionably greater than the second; or if, when the third number is less than the first, the fourth must be proportionably less than the second, the question then belongs to the rule of three direct: But if, when the third number is greater than the first, the fourth must be less than the second; or when the third number is less than the first, the fourth must be greater than the second; in either of these cases, the question belongs to the rule of three inverse, and must be resolved as above directed.

As for example,

QUEST. 34.

If 12 *men will eat up a quantity of provision in* 15 *days, how long will* 20 *men be in eating up the same?*

This

This queſtion is of ſuch a nature, that more requires leſs; for 20 men will conſume the ſame proviſion in leſs time than 12; therefore the queſtion belongs to the rule of three inverſe; ſo I multiply the firſt and ſecond numbers together, and divide by the third, and the quotient 9, that is, 9 days, is an anſwer to the queſtion.

A demonſtration of the rule of three Inverſe.

If I was to anſwer this queſtion by pure dint of thought, without any rule to direct me, I ſhould reaſon thus: whatever quantity of proviſion laſts 12 men 15 days, the ſame will laſt 1 man 12 times as long, that is, 12 times 15, or 180 days; but if it will laſt 1 man 180 days, it will laſt 20 men but the 20th part of that time, that is, 9 days: here then the fourth number was found by multiplying the firſt and ſecond numbers together, and dividing the product by the third; and the reaſon is the ſame in all other caſes, wherever the rule of three inverſe is concerned. *Q. E. D.*

QUEST. 35.

One lends me 372 pounds for 7 years and 8 months, or 92 months: how long muſt I lend him 496 pounds for an equivalent?

Anſwer. 5 years, 9 months.

QUEST. 36.

If a ſquare pipe 4 inches and 5 lines wide, will diſcharge a certain quantity of water in one hour's time; in what time will another ſquare pipe, 1 inch and 2 lines wide, diſcharge the ſame quantity from the ſame current?

The orifice of a ſquare pipe 4 inches, 5 lines, or 53 lines wide, contains 2809 ſquare lines; and the orifice of a pipe 1 inch, 2 lines, or 14 lines wide, contains 196 ſquare lines. Say then, *If an orifice of 2809 ſquare lines will diſcharge a certain quantity of water in one hour; in what time will an orifice of 196 ſquare lines diſcharge the ſame?*

Anſwer. In 14 hours, 19′. 54″.

QUEST.

QUEST. 37.

If 3 men, or 4 women, will do a piece of work in 56 days, how long will one man and one woman be in doing the same?

Becauſe of the 3 men, or 4 women, ſome number muſt be found that is diviſible both by 3 and by 4 without remainder; ſuch a one is the number 12, which is the product of 3 and 4 multiplied together; (ſee obſervation the third upon the definition of diviſion:) make then 3 men or 4 women equivalent to 12 boys, and you will have 1 man equivalent to 4 boys, 1 woman to 3 boys, and 1 man and 1 woman to 7 boys, and the queſtion will ſtand thus; *If 12 boys will do a piece of work in 56 days, how long will 7 boys be in doing the ſame?*

Anſwer. 96 days.

QUEST. 38.

If 5 oxen, or 7 colts, will eat up a cloſe in 87 days, in what time will 2 oxen and 3 colts eat up the ſame?

Anſwer. In 105 days.

QUEST. 39.

If 2 acres of land will maintain 3 horſes 4 days, how long will 5 acres maintain 6 horſes?

This queſtion may perhaps at firſt ſight, be taken to be ſomewhat of the ſame nature with the 32d and 33d queſtions, which belonged to the double rule of three direct; but when it comes to be examined into more narrowly, it will be found to be of a very different nature: for we cannot ſay here as we did there, that 2 acres will laſt 3 horſes as long as 1 acre will laſt 6 horſes; this would be a very unjuſt way of thinking, and wherever it is ſo, the queſtion ought to be referred to another rule, which they call the double rule of three inverſe; the propriety or impropriety of this thought, being an infallible criterion whereby to diſtinguiſh, when a queſtion belongs to one rule, and when to the other. All queſtions belonging to this rule, as well as thoſe belonging to the other, may be reduced to the ſingle rule of three two ways; either by expunging the firſt and fourth numbers, or the ſecond and fifth; but then the methods of of extermination are different. In queſtions of this nature, if the firſt and fourth numbers are to be expunged, the 2 firſt numbers are to be multiplied by the fourth, and the 2 laſt by the firſt; but if the ſecond and fifth numbers are to be expunged, then the two firſt numbers are to be multiplied by the fifth, and the two laſt by the ſecond: thus in the queſtion before us, if we would exterminate the firſt and fourth numbers, we muſt multiply the two firſt numbers, that is, 2 and 3, by the fourth, that is, by 5, and ſay, that 2 acres will laſt 3 horſes juſt as long as 10 acres will laſt 15 horſes; we muſt alſo multiply the 2 laſt numbers, to wit, 5 and 6, by the firſt, that is, by 2, and ſay, that 5 acres will laſt 6 horſes as long as 10 acres will laſt 12 horſes: uſe now theſe numbers inſtead of thoſe in the queſtion, and it will be changed into this equivalent one; If 10 acres of land will maintain 15 horſes 4 days, how long will 10 acres maintain 12 horſes? Strike out of the queſtion the firſt and fourth numbers, which being equal, will be of no uſe in the concluſion, and then the queſtion will ſtand thus; *If 15 horſes will eat up a certain piece of ground in 4 days, how long will 12 horſes be in eating up the ſame?*

Anſwer. 5 days; for this queſtion belongs to the rule of three inverſe.

If we would exterminate the ſecond and fifth numbers out of the queſtion, we muſt multiply the two firſt numbers by the fifth, and ſay, that 2 acres will laſt 3 horſes juſt as long as 12 acres will laſt 18 horſes; we muſt alſo multiply the 2 laſt numbers by the ſecond, and ſay, that 5 acres will laſt 6 horſes as long as 15 acres will laſt 18 horſes: uſe theſe numbers inſtead of thoſe in the queſtion, and it will be changed into this equivalent one; If 12 acres will maintain 18 horſes 4 days, how long will 15 acres maintain 18 horſes? That is, (ſtriking out the ſecond and fifth numbers) *If 12 acres of land will maintain a certain number of horſes 4 days, how long will 15 acres laſt the ſame number?*

Anſwer. 5 days, as before; for this queſtion belongs to the rule of three direct.

In both theſe operations, the number ſought was at laſt found by multiplying 15 by 4, and then dividing the product by 12: now whoſoever looks back upon the foregoing reſolution, and obſerves how theſe numbers were formed, he will eaſily perceive, that the number 4 was the middle term in the queſtion; that the number 15 in both operations was the product of the numbers 3 and 5, which lay next the middle term on each ſide; and that the diviſor 12 was in both caſes the product of the extreme numbers 2 and 6: therefore *In all queſtions belonging*

longing to the double rule of three inverse, where the numbers are supposed to be ordered as in the double rule of three direct, if the three middle numbers be multiplied together, and the product be divided by the product of the two extremes, the quotient of this division will be the number sought. And thus may all the trouble of expunging be avoided, though I thought it proper to explain that method in the first place, in order to let the learner into the reason of this last theorem which is founded upon it.

Questions wherein the extraction of the square root is concerned.

QUEST. 40.

There is a certain field whose breadth is 576 yards, and whose length is 1296 yards: I demand the side of a square field equal to it.

Answer. This field will be equal to a square whose side is 864 yards.

QUEST. 41.

There is a certain inclosure 3 times as long as it is broad, whose area is 46128 square yards: I demand its breadth and length.

The breadth multiplied into the length, that is, the breadth multiplied into 3 times itself, is 46128; therefore the breadth multiplied into itself is 15376; therefore the breadth is 124, and the length 372.

QUEST. 42.

A certain society collect among themselves a sum amounting to 15 pounds, 5 shillings and a farthing, every one contributing as many farthings as there were members in the whole society: I demand the number of members.

Answer. 121 members.

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THE INTRODUCTION,

Concerning Vulgar and Decimal Fractions.

DEFINITIONS.

Art. 1. A FRACTION simply and abstractedly considered, is that wherein some part or parts of an unit are expressed: as if an unit be supposed to be divided into 4 equal parts, and three of these parts are to be expressed, it must be done by the fraction three fourths, to be written thus $\frac{3}{4}$: here the number 4, which shews into how many equal parts the unit is supposed to be divided, and so determines the true value, magnitude, or denomination of those parts, is called the denominator of the fraction; and the number 3, which shews how many of these parts are considered in the fraction, is called the numerator: thus in the fraction $\frac{1}{2}$ or one half, 1 is the numerator, and 2 the denominator; in $\frac{2}{2}$ or two halves, 2 is both numerator and denominator, *&c.*

When a fraction is applied to any particular quantity, that quantity is called the integer to the fraction: thus in $\frac{3}{4}$ of a penny, a penny is the integer; in three fourths of six, the number 6 is the integer; thus in three fourths of five sixths, the fraction five sixths is the integer; for though in an absolute sense it be a fraction, yet here with respect to the fraction three fourths, it is an integer: and thus may one and the same quantity, under different ways of conception, be both an integer and a fraction; as a foot is an integer, and a third part of a yard is a fraction, though they both signify the same thing. When the integer to a fraction is not expressed, unity is always to be understood: thus $\frac{3}{4}$ is $\frac{3}{4}$ of an unit; thus when we say, $\frac{1}{3}$ and $\frac{1}{4}$ make $\frac{7}{12}$, the meaning is, that if $\frac{1}{3}$ part of an unit, and $\frac{1}{4}$ part of an unit be added together, the sum will amount to the same as if that unit had been divided into 12 equal parts, and 7 of those parts had been taken; thus again, when we say that $\frac{2}{3}$ of $\frac{4}{5}$ are equivalent to $\frac{8}{15}$, we mean, that if an unit be divided into 5 equal parts, and 4 of them be taken, and then this fraction $\frac{4}{5}$ be again divided into 3 equal

 parts,

parts, and 2 of them be taken, the reſult will be the ſame, as if the unit had at firſt been divided into 15 equal parts, and 8 of them had been taken; and whatever is true in the caſe of unity, will be equally true in the caſe of any other integer whatever: thus if it be true that $\frac{1}{3}$ and $\frac{1}{4}$ of an unit are equal to $\frac{7}{12}$ of an unit, that is, if it be true in general that $\frac{1}{3}$ and $\frac{1}{4}$ added together are equal to $\frac{7}{12}$, it will be as true of any particular integer, ſuppoſe of a pound ſterling, that $\frac{1}{3}$ of a pound, and $\frac{1}{4}$ of a pound when added together, are equal to $\frac{7}{12}$ of a pound; again, if it be true in general that $\frac{2}{3}$ of $\frac{4}{5}$ are equal to $\frac{8}{15}$, it is as true in particular that $\frac{2}{3}$ of $\frac{4}{5}$ of a pound are equivalent to $\frac{8}{15}$ of a pound, &c.

Of proper and improper fractions; and of the reduction of an improper fraction to a whole or mixt number.

2. Fractions are of two ſorts, proper and improper; a proper fraction is that, whoſe numerator is leſs than the denominator, as $\frac{1}{2}$; therefore an improper one is that, whoſe numerator is equal to, or greater than the denominator, as $\frac{2}{2}$, $\frac{3}{2}$, &c.

OBJECTION.

But is there no abſurdity in the ſuppoſition of an improper fraction, as in three halves for inſtance, conſidering that an unit cannot be divided into more than two halves? *Anſwer:* no more than there is in ſuppoſing three half pence to be the price of any thing, conſidering that a penny cannot be divided into above two halfpence. Theſe fractions therefore are called improper, not from any abſurdity either in the ſuppoſition or in the expreſſion, but becauſe they may be more properly and more intelligibly expreſſed, either by a whole number, or at leaſt by a mixt number conſiſting of a whole number and a fraction; as for example, if the numerator of a fraction be equal to the denominator, as $\frac{4}{4}$, that fraction will always be equivalent to unity, as $\frac{4}{4}$ of an hour, that is, four quarters of an hour, are equivalent to one hour, $\frac{4}{4}$ of a penny, that is, four farthings, are equal to one penny, &c: and the reaſon is plain; for if an unit be divided into four equal parts, and four of theſe parts be expreſſed in a fraction, the whole unit is expreſſed in that fraction, that is, ſuch a fraction muſt always be looked upon as equal to an unit: therefore if the numerator be double of the denominator, as $\frac{8}{4}$, the fraction muſt be equal to the number 2, becauſe $\frac{8}{4}$ contain $\frac{4}{4}$ or 1 twice; in like manner $\frac{12}{4}$ are equal to, and may be more properly expreſſed by, the number 3; $\frac{16}{4}$ by the number 4, &c: and univerſally, as often as the numerator

numerator of a fraction contains the denominator, so many units is that fraction equivalent to: but to find how often the numerator contains the denominator, is to divide the numerator by the denominator; therefore if the numerator of an improper fraction be divided by the denomina-tor, the quotient, if nothing remains, will be the whole number by which the fraction may be expressed; but if any thing remains of this division, then the quotient together with a fraction whose numerator is that re-mainder, and denominator the divisor, will be a mixt number, expressing the fraction proposed: thus $\frac{24}{3}$ are equivalent to the whole number 8, but $\frac{25}{3}$ are equivalent to the mixt number 8 $\frac{1}{3}$, $\frac{26}{3}$ to the mixt number 8 $\frac{2}{3}$, just as 24 feet are equal to 8 yards, 25 feet to 8 yards and 1 foot, 26 feet to 8 yards and 2 feet, *&c:* and this is what we call the reduction of an improper fraction into a whole or mixt number.

The reduction of a whole or mixt number into an improper fraction.

3. As unity may be expressed by any fraction of any form or deno-mination whatever, provided the numerator be equal to the denominator, as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, *&c;* so the number 2 is reducible to any fraction whose nu-merator is double the denominator, as $\frac{4}{2}$, $\frac{6}{3}$, $\frac{8}{4}$, *&c;* and so is every num-ber reducible to any fraction, whose numerator contains the denominator as often as there are units in the number proposed: therefore whenever a whole number is to be reduced to a fraction whose denominator is given, it must be multiplied by that given denominator, and the product with that denominator under it, will be the equivalent fraction; thus if the number 5 is to be reduced into halves, that is, into a fraction whose de-nominator is 2, it must be multiplied by 2, and so you will have 5 equal to $\frac{10}{2}$, just as 5 pence are equivalent to 10 halfpence; if the number 8 is to be reduced into thirds, it must be multiplied by 3, and so you will have 8 equal to $\frac{24}{3}$, just as 8 yards are equal to 24 feet; lastly, if the number 2 is to be reduced into fourths, it will be equal to $\frac{8}{4}$, just as 2 pence are equal to 8 farthings. If the number to be reduced be a mixt number, consisting of a whole number and a fraction, the whole number must always be reduced to the same denomination with the fraction an-nexed, and the rule will be this: multiply the whole number by the de-nominator of the fraction annexed; add the numerator to the product, and the sum with the denominator under it will be the equivalent frac-tion: Thus the mixt number 5 $\frac{1}{2}$ is equivalent to $\frac{11}{2}$, just as 5 pence half-penny in money is equivalent to 11 halfpence: This operation carries it's own evidence along with it; for the number 5 itself is equal to $\frac{10}{2}$

as

as above; therefore $5\frac{1}{2}$ muſt be equivalent to $\frac{11}{2}$: again, the number $8\frac{2}{3}$ is equal to $\frac{26}{3}$, juſt as 8 yards and 2 feet over are equivalent to 26 feet; laſtly, $2\frac{3}{4}$ is reducible to $\frac{11}{4}$, juſt as 2 pence and 3 farthings are reducible to 11 farthings.

A Lemma.

4. *If any integer be aſſumed, as a pound ſterling, and alſo any fraction, as $\frac{3}{4}$, I ſay then, that $\frac{3}{4}$ parts of one pound, amount to the ſame as $\frac{1}{4}$ part of 3 pounds.*

To demonſtrate this Lemma (which ſcarce wants a demonſtration) I argue thus: If any quantity, greater or leſs, be always divided into the ſame number of parts, the greater or leſs the quantity ſo divided is, the greater or leſs will the parts be; thus $\frac{1}{4}$ of a yard is 3 times as much as $\frac{1}{4}$ of a foot, becauſe a yard is 3 times as much as a foot; and for the ſame reaſon $\frac{1}{4}$ of 3 pounds is 3 times as much as $\frac{1}{4}$ of 1 pound; but $\frac{3}{4}$ of 1 pound are alſo 3 times as much as $\frac{1}{4}$ of 1 pound; therefore $\frac{3}{4}$ of 1 pound are equal to $\frac{1}{4}$ of 3 pounds, becauſe both are juſt 3 times as much as $\frac{1}{4}$ of 1 pound. *Q. E. D.*

How to eſtimate any fractional parts of an integer in parts of a leſſer denomination, and vice verſâ.

5 This may be done various ways; but the ſhorteſt and ſafeſt, as I take it, is that which follows: Suppoſe I had a mind to know the value of $\frac{5}{6}$ of a pound; I ſhould argue as in the foregoing lemma, that $\frac{5}{6}$ of one pound, are the ſame as $\frac{1}{6}$ of 5 pounds; but the latter is more eaſily taken than the former; therefore I apply myſelf wholly to the latter, to wit, to find the ſixth part of 5 pounds, thus: 5 pounds, or 100 ſhillings, divided by 6, quote 16 ſhillings, and there remain 4 ſhillings; again, 4 ſhillings, or 48 pence, divided by 6, quote 8 pence, and there remains nothing; therefore the value of 1 ſixth of 5 pounds, or $\frac{5}{6}$ of 1 pound, is 16 ſhillings and 8 pence. Again, ſuppoſe I would know the value of $\frac{6}{7}$ of a pound, I find the value of $\frac{1}{7}$ of 6 pounds thus; 6 pounds, or 120 ſhillings divided by 7, give 17 ſhillings, and there remains 1 ſhilling; again, 1 ſhilling, or 12 pence, divided by 7, give 1 penny, and there remain 5 pence; again, 5 pence, or 20 farthings, divided by 7, give 2 farthings, and there remain 6 farthings; laſtly, a ſeventh part of 6 farthings is juſt as much as $\frac{6}{7}$ of 1 farthing, by the lemma: hence I conclude, that $\frac{6}{7}$ of a pound are 17 ſhillings, 1 penny, 2 farthings, and $\frac{6}{7}$ of a farthing: But the value of $\frac{6}{7}$ of a farthing is ſo near to one farthing, that if I would

rather

rather admit of a ſmall inaccuracy in my account, than a fraction, I ſhould make the value of $\frac{6}{7}$ of a pound to be 17 ſhillings, 1 penny and 3 farthings. Laſtly, ſuppoſe I would know the amount of $\frac{2}{3}$ parts of 17 ſhillings and ſixpence, I ſhould argue thus; $\frac{2}{3}$ parts of 17 ſhillings and ſixpence, are equivalent to $\frac{1}{3}$ part of twice as much, that is, to $\frac{1}{3}$ part of 35 ſhillings: but $\frac{1}{3}$ part of 35 ſhillings is 11 ſhillings and 8 pence; therefore $\frac{2}{3}$ parts of 17 ſhillings and ſixpence make 11 ſhillings and 8 pence.

Of the reverſe of this reduction, one ſingle inſtance will ſuffice: Let it then be required to reduce 1 ſhilling, 2 pence, 3 farthings, to fractional parts of a pound: here I conſider, that in 1 pound are 960 farthings; and in 1 ſhilling, 2 pence, 3 farthings, are 59 farthings; therefore 1 farthing is $\frac{1}{960}$ of a pound; and 1 ſhilling, 2 pence, 3 farthings, are $\frac{59}{960}$ of a pound.

Preparations for further reductions and operations of fractions.

6 All the operations and reductions of fractions, are mediately or immediately deducible from the following principle, which is; that *If the numerator of a fraction be encreaſed, whilſt the denominator continues the ſame, the value of the fraction will be encreaſed proportionably; and* vice verſâ. *On the other hand, if the denominator be encreaſed in any proportion, whilſt the numerator continues the ſame, the value of the fraction will be diminiſhed in a contrary proportion; and* vice verſâ. Thus $\frac{2}{3}$ are twice as much as $\frac{1}{3}$, and $\frac{1}{6}$ is but half as much.

From this principle it follows, that if the numerator and denominator of a fraction be both multiplied, or both divided by the ſame number, the value of the fraction will not be affected thereby; becauſe, as much as the fraction is encreaſed by multiplying the numerator, juſt ſo much again it will be diminiſhed by multiplying the denominator; and as much as the fraction is diminiſhed by dividing the numerator, juſt ſo much again it will be encreaſed by dividing the denominator: Thus the terms of the fraction $\frac{3}{4}$ being doubled, produce $\frac{6}{8}$, a fraction of the ſame value; and on the contrary, the terms of the fraction $\frac{6}{8}$ being halved, give $\frac{3}{4}$.

Hence it appears, that every fraction is capable of infinite variety of expreſſion, ſince there is infinite choice of multiplicators, whereby the numerator and denominator of a fraction may be multiplied, and ſo the expreſſion may be changed, without changing the value of the fraction: thus the fraction $\frac{1}{2}$, if both the numerator and denominator be multiplied by 2, becomes $\frac{2}{4}$; if by 3, $\frac{3}{6}$; if by 4, $\frac{4}{8}$; if by 5, $\frac{5}{10}$; and ſo on *ad infinitum*; all which are nothing elſe but different expreſſions of the ſame

fraction:

fraction: therefore in the midſt of ſo much variety, we muſt not expect that every fraction we meet with ſhould always be in it's leaſt or loweſt terms; but how to reduce them to this ſtate whenever they happen to be otherwiſe, ſhall be the buſineſs of the next article.

The reduction of fractions from higher to lower terms.

7. Whenever a fraction is ſuſpected not to be in it's leaſt terms, find out, if poſſible, ſome number that will divide both the numerator and denominator of the fraction without any remainder; for if ſuch a number can be found, and the diviſion be made, the two quotients thence ariſing will exhibit reſpectively, the numerator and denominator of a fraction, equal to the fraction firſt propoſed, but expreſſed in more ſimple terms: this is evident from the laſt article. As for example; let the fraction $\frac{10}{15}$ be propoſed to be reduced: here to find ſome number that will divide both the numbers 10 and 15 without any remainder, I begin with the number 2, as being the firſt whole number that can have any effect in diviſion; but I find 2 will not divide 15; 3 is the next number to be tried; but neither will that ſucceed, for it will not divide 10; as for the number 4, I paſs that by, becauſe if 2 would not divide 15, much leſs will 4 do it; the next number I try is 5, and that ſucceeds; for if 10 and 15 be divided by 5, the quotients will be 2 and 3 reſpectively, each without remainder; therefore the fraction $\frac{10}{15}$, after being reduced to it's leaſt terms, is found to be the ſame as $\frac{2}{3}$; that is, if an unit be divided into 15 equal parts, and 10 of them be taken, the amount will be the ſame, as if it had been divided into 3 equal parts, and 2 of them had been taken. Secondly, if the fraction propoſed to be reduced be $\frac{2520}{7560}$, divide it's terms by 2, and you will have the fraction $\frac{1260}{3780}$; divide again by 2, and you will have $\frac{630}{1890}$; divide again by 2, and you will have $\frac{315}{945}$; therefore all further diviſion by 2 is excluded: divide then theſe laſt terms by 3, and you will have $\frac{105}{315}$; divide again by 3, and you will have $\frac{35}{105}$; divide by 5, and you will have $\frac{7}{21}$; and laſtly divide by 7, and you will have $\frac{1}{3}$; ſo that the fraction $\frac{2520}{7560}$, after a common diviſion by 2, 2, 2, 3, 3, 5, 7, is found at laſt equal to $\frac{1}{3}$. Thirdly, the fraction $\frac{36}{48}$, after a continual diviſion by 2, 2, 3, becomes $\frac{3}{4}$. Fourthly, $\frac{56}{84}$ after a continual diviſion by 2, 2, 7, becomes $\frac{2}{3}$. Fifthly, $\frac{144}{180}$ after a conti-

continual diviſion by 2, 2, 3, 3, becomes $\frac{4}{5}$. Sixthly, $\frac{42}{126}$ after a continual diviſion by 2, 3, and 7, becomes $\frac{1}{3}$. Seventhly, $\frac{315}{840}$ after a continual diviſion by 3, 5, 7, becomes $\frac{3}{8}$. Eighthly, $\frac{35}{840}$ after a continual diviſion by 5 and 7, becomes $\frac{1}{24}$. Ninthly, $\frac{735}{245}$ after a continual diviſion by 5, 7, 7, becomes $\frac{3}{1}$, or 3.

Some perhaps may think themſelves helped in the practice of this rule by the following obſervations:

Firſt, that 2 will divide any number that ends with an even number, or with a cypher, as 36, 30, &c. and no other.

Secondly, that 5 will divide any number that ends with a 5, or with a cypher, as 75, 70, &c. and no other.

Thirdly, that 3 will divide any number, when it will divide the ſum of it's digits added together: thus 3 will divide 471, becauſe it will divide the number 12, which is the ſum of the numbers 4, 7 and 1.

Fourthly, if both the numerator and denominator have cyphers annexed to them, throw away as many as are common to both: thus $\frac{3500}{56000}$ is the ſame as $\frac{35}{560}$, or $\frac{7}{112}$, or $\frac{1}{16}$.

After all, there is a certain and infallible rule for finding the greateſt common diviſor of any two numbers whatever, that have one, whereby a fraction may be reduced to it's leaſt terms by one ſingle operation only. I ſhall be forced indeed to poſtpone the demonſtration of this rule to a more convenient place, not ſo much for want of principles to proceed upon, as for want of a proper notation; but the rule itſelf is as follows: Let a and b be two given numbers, whoſe greateſt common diviſor is required; to wit, a the greater, and b the leſs: then dividing a by b without any regard to the quotient, call the remainder c; divide again b by c, and call the remainder d; then divide c by d, and call the remainder e; then divide d by e, and call the remainder f; and ſo proceed on, till at laſt you come to ſome diviſor, as f, which will divide the preceding number e without a remainder: I ſay then, that this laſt diviſor will be the greateſt common diviſor of the two given numbers a and b. As for example; let a be 1344 and b 582: then to find the greateſt common diviſor of theſe numbers, I divide a (1344) by b (582) and there remains 180, which I call c; then I divide b (582) by c (180) and there remains 42, which I call d; then I divide c (180) by d (42) and there remains 12, which I call e; then I divide d (42) by e (12) and there remains 6, which I call f; laſtly I divide e (12) by f (6)

 and

and there remains nothing: whence I conclude that 6 is the greatest common divisor of the two numbers 1344 and 582; and as the quotients by 6 are 224 and 97, it follows, that the fraction $\frac{582}{1344}$, when reduced to it's least terms, will be $\frac{97}{224}$. If no common divisor can be found but unity, it is an argument that the fraction is in it's least terms already.

From this and the last article it follows, that all fractions that are reducible to the same least terms are equal; as $\frac{4}{6}$, $\frac{6}{9}$, $\frac{10}{15}$, &c, which are all reducible to $\frac{2}{3}$; though it does not follow *è converso*, that all equal fractions are reducible to the same least terms; this will be demonstrated in another place. (See *Elements of Algebra*, Art. 193.)

For the better understanding of the following article, it must be observed, that this mark × is a sign of multiplication, and is usually read *into*: thus 2 × 3 signifies 6, 2 × 3 × 4 signifies 24, 2 × 3 × 4 × 5 signifies 120, &c; and in some cases it will be better to put down these components or factors, than the character of the number arising from their continual multiplication, as in the following article. It ought also to be observed, that it matters not in what order these components are placed; for 2 × 3 × 4 × 5 signifies just the same as 4 × 5 × 2 × 3, &c.

The reduction of fractions of different denominations, to others of the same denomination.

8. There is another reduction of fractions, no less useful than the former; and that is, the reduction of fractions of different denominations to others of the same denomination, or which have the same denominator, without changing their values; which is done as follows: Having first put down the fractions to be reduced, in any order, one after another, and beginning with the numerator of the first fraction, multiply it by a continual multiplication, into all the denominators but it's own, and put down the product under that fraction; then multiply in like manner, the numerator of the next fraction into all the denominators but it's own, and put down the product under that fraction; and so proceed on through all the numerators, always taking care to except the denominator of that fraction, whose numerator is multiplied. Then multiplying all the denominators together, put down the product under every one of the products last found, and you will have a new set of fractions, all of the same denomination with one another, and all of the same values with their respective original ones. As for example; let it be proposed to reduce the following fractions to the same denomination, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{6}$, $\frac{7}{8}$: 1/!, The numerator of the first fraction is 1, and the denominators

nators of the reſt are 4, 6 and 8, and $1 \times 4 \times 6 \times 8$ gives 192; therefore I put down 192 under $\frac{1}{2}$. 2*dly*, The numerator of the ſecond fraction is 3, and the denominators of the reſt are 6, 8 and 2, and $3 \times 6 \times 8 \times 2$ gives 288; therefore I put down 288 under $\frac{3}{4}$. 3*dly*, $5 \times 8 \times 2 \times 4$ gives 320; therefore I put down 320 under $\frac{5}{6}$. 4*thly*, $7 \times 2 \times 4 \times 6$ gives 336; therefore I put down 336 under $\frac{7}{8}$. Laſtly, $2 \times 4 \times 6 \times 8$, or the product of all the denominators is 384: this therefore I put down under every one of the numerators laſt found, and ſo have a new ſet of fractions, *viz.* $\frac{192}{384}$, $\frac{288}{384}$, $\frac{320}{384}$, $\frac{336}{384}$, all of the ſame denomination, as appears from the operation itſelf; and all of the ſame value with their reſpective original ones, as will appear preſently; but firſt ſee the work:

$$\frac{1}{2}. \quad \frac{3}{4}. \quad \frac{5}{6}. \quad \frac{7}{8}.$$

$$\frac{192}{384}. \quad \frac{288}{384}. \quad \frac{320}{384}. \quad \frac{336}{384}.$$

A demonſtration of the rule.

All that is to be demonſtrated in this rule is, to prove from the nature of the operation itſelf, that the original fractions ſuffer nothing in their values by this reduction: in order to which, it will be convenient to put down the components of the new numerators inſtead of their proper characters, as in the laſt article; as alſo thoſe of the common denominator, and the work will ſtand thus:

$$\frac{1}{2}. \quad \frac{3}{4}. \quad \frac{5}{6}. \quad \frac{7}{8}.$$

$$\frac{1 \times 4 \times 6 \times 8}{2 \times 4 \times 6 \times 8}. \quad \frac{3 \times 6 \times 8 \times 2}{4 \times 6 \times 8 \times 2}. \quad \frac{5 \times 8 \times 2 \times 4}{6 \times 8 \times 2 \times 4}. \quad \frac{7 \times 2 \times 4 \times 6}{8 \times 2 \times 4 \times 6}.$$

By this method of operation it appears, that the numerator and denominator of the firſt fraction $\frac{1}{2}$, are both multiplied by the ſame number in the reduction, to wit, by $4 \times 6 \times 8$; and therefore that fraction ſuffers nothing in it's value, by art. 6. In like manner, the terms of the ſecond fraction $\frac{3}{4}$ are both multiplied by the ſame number $6 \times 8 \times 2$; therefore that fraction can ſuffer nothing in it's value; and the ſame may be ſaid of all the reſt. *Q. E. D.*

Other examples to this rule.

$$\frac{1}{2}. \quad \frac{1}{3}. \quad \frac{1}{4}. \quad \frac{1}{5}. \quad \frac{1}{6}. \qquad\qquad \frac{1}{3}. \quad \frac{1}{4}. \quad \frac{1}{5}. \quad \frac{1}{6}.$$

$$\frac{360}{720}. \quad \frac{240}{720}. \quad \frac{180}{720}. \quad \frac{144}{720}. \quad \frac{120}{720}. \qquad\qquad \frac{120}{360}. \quad \frac{90}{360}. \quad \frac{72}{360}. \quad \frac{60}{360}.$$

$\frac{1}{4}$

$\frac{1}{4}$. $\frac{1}{5}$. $\frac{1}{6}$. $\frac{1}{5}$. $\frac{1}{6}$.

$\frac{30}{120}$. $\frac{24}{120}$. $\frac{20}{120}$. $\frac{6}{30}$. $\frac{5}{30}$.

The uſe of this rule will ſoon appear in the addition and ſubtraction of fractions: in the mean time it may not be amiſs to obſerve, that it would be very difficult, if not impoſſible, to compare fractions of different denominations, without firſt reducing them to the ſame. As for inſtance; ſuppoſe it ſhould be asked, which of theſe two fractions is the greater, $\frac{3}{4}$, or $\frac{5}{7}$; in this view it would be difficult to determine the queſtion; but when I know that $\frac{3}{4}$ are the ſame with $\frac{21}{28}$, and that $\frac{5}{7}$ are the ſame with $\frac{20}{28}$, I know then, that $\frac{3}{4}$ are greater than $\frac{5}{7}$ by a twenty eighth part of the whole. We now proceed to the four operations of fractions, to wit, their addition, ſubtraction, multiplication and diviſion: and firſt,

Of the addition of fractions.

9. Whenever two or more fractions are to be added together, let them firſt be reduced to the ſame denomination, if they be not ſo already; and then adding the new numerators together, put down the ſum with the common denominator under it. In the caſe of mixt numbers, add firſt the fractions together, and then the whole numbers: but if the fractions when added together, make an improper fraction, reduce it by the 2d art. to a whole, or mixt number; and then putting down the fractional part, if there be any, reſerve the whole number for the place of integers.

To this rule might be referred (if it had not been taught already in the 3d art.) the reduction of a mixt number into an improper fraction, which is nothing elſe but adding a whole number and a fraction together, and may be done by conſidering the whole number as a fraction whoſe denominator is unity.

Examples of addition in fractions.

1ſt, $\frac{3}{10}$ and $\frac{4}{10}$ when added together make $\frac{7}{10}$, for juſt the ſame reaſon as 3 ſhillings and 4 ſhillings when added together make 7 ſhillings.

2dly, The fractions $\frac{1}{3}$ and $\frac{1}{4}$ when reduced to the ſame denomination by the laſt art. are $\frac{4}{12}$ and $\frac{3}{12}$, and theſe added together make $\frac{7}{12}$; therefore the fractions $\frac{1}{3}$ and $\frac{1}{4}$ when added together make up the fraction $\frac{7}{12}$.

For a better confirmation of theſe abſtract concluſions, but chiefly to inure the learner to conceive and reaſon diſtinctly about fractions, it will be be very convenient to apply these examples in some particular case, as for instance, in the case of a pound sterling; and if we do so here, we are to try, whether $\frac{1}{3}$ and $\frac{1}{4}$ of a pound when added together, amount to $\frac{7}{12}$ of a pound, or not: here then we shall find by division, that the third part of a pound is 6 shillings and 8 pence, and the fourth part 5 shillings; and these added together, make 11 shillings and 8 pence; therefore $\frac{1}{3}$ and $\frac{1}{4}$ of a pound when added together, make 11 shillings and 8 pence; but by the 5th art. it will be found that $\frac{7}{12}$ of a pound are also 11 shillings and 8 pence; therefore $\frac{1}{3}$ and $\frac{1}{4}$ of a pound, when added together, make $\frac{7}{12}$ of a pound; and the same would have been true in any other instance whatever.

3*dly*, $\frac{2}{5}$ and $\frac{3}{8}$, that is, $\frac{16}{40}$ and $\frac{15}{40}$, when added together, make $\frac{31}{40}$, which will also be true in the case of a pound sterling; for by the 5th art. $\frac{2}{5}$ of a pound are 8 shillings, $\frac{3}{8}$ of a pound are 7 shillings and 6 pence, and their sum is 15 shillings and 6 pence; which will also be found to be the value of $\frac{31}{40}$ of a pound; therefore $\frac{2}{5}$ and $\frac{3}{8}$ of a pound when added together, make $\frac{31}{40}$ of a pound.

4*thly*, $\frac{2}{3}$ and $\frac{4}{5}$, that is, $\frac{10}{15}$ and $\frac{12}{15}$, when added together, make $\frac{22}{15}$, an improper fraction; which being reduced to a mixt number, by the 2d art. is 1 and $\frac{7}{15}$: let us now try, whether $\frac{2}{3}$ of a pound, and $\frac{4}{5}$ of a pound when added together will make one pound and $\frac{7}{15}$ of a pound over, or not: now $\frac{2}{3}$ of a pound, or 13 shillings and 4 pence, added to $\frac{4}{5}$ of a pound, or 16 shillings, amount to 1 pound 9 shillings and 4 pence; and $\frac{7}{15}$ of a pound, are found to be 9 shillings and 4 pence; therefore $\frac{2}{3}$ and $\frac{4}{5}$ of a pound when added together, make one pound and $\frac{7}{15}$ of a pound over.

5*thly*, $\frac{3}{4}$ and $\frac{5}{6}$, that is, $\frac{18}{24}$ and $\frac{20}{24}$: when added together, make $\frac{38}{24}$, or $1\frac{7}{12}$, which will also be true in the case of a pound sterling.

6*thly*, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, that is, $\frac{360}{720}$, $\frac{240}{720}$, $\frac{180}{720}$, $\frac{144}{720}$, $\frac{120}{720}$, when added together, make $\frac{1044}{720}$; that is, $1\frac{9}{20}$: try it in money.

7*thly*, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$, that is, $\frac{360}{720}$, $\frac{480}{720}$, $\frac{540}{720}$, $\frac{576}{720}$ and $\frac{600}{720}$, when added together, make $\frac{2556}{720}$, that is, $3\frac{11}{20}$.

8*thly*, The sum of the mixt numbers $7\frac{1}{3}$ and $8\frac{1}{4}$ is $15\frac{7}{12}$; for the sum of the fractions is $\frac{7}{12}$ by the second example, and the sum of the whole numbers is 15.

9*thly*, $5\frac{2}{3}$ added to $7\frac{4}{5}$ gives $13\frac{7}{15}$; for the sum of the fractions is $1\frac{7}{15}$, by the fourth example; and the whole number 1, added to the whole numbers 5 and 7 gives 13.

10*thly*,

10*thly*, $8\frac{1}{2}$, $9\frac{2}{3}$, $10\frac{3}{4}$, $11\frac{4}{5}$, $12\frac{5}{6}$ added together make $53\frac{11}{20}$; for the fractions themſelves make $3\frac{11}{20}$ by the ſeventh example, and the whole number 3 added to the reſt makes 53.

11*thly*, The whole number 2 added to the fraction $\frac{3}{4}$ gives $\frac{11}{4}$; for the whole number 2 may be conſidered as a fraction, whoſe denominator is unity; now $\frac{2}{1}$ and $\frac{3}{4}$ when reduced to the ſame denomination, are $\frac{8}{4}$ and $\frac{3}{4}$, which added together make $\frac{11}{4}$.

Thus alſo may unity be added to any fraction whatever, when ſubtraction requires it; but better thus: unity may be made a fraction of any denomination whatever, provided the numerator be equal to the denominator, by art. 2d: ſuppoſe then I would add unity to $\frac{2}{3}$; I ſuppoſe unity equal to $\frac{3}{3}$, and this added to $\frac{2}{3}$ makes $\frac{5}{3}$: again, unity added to $\frac{3}{5}$ makes $\frac{8}{5}$, becauſe $\frac{5}{5}$ and $\frac{3}{5}$ make $\frac{8}{5}$.

Of the ſubtraction of fractions.

10. Whenever a leſs fraction is to be ſubtracted from a greater, they muſt be prepared as in addition; that is, they muſt be reduced to the ſame denomination, if they be not ſo already; then ſubtracting the numerator of the leſs fraction from that of the greater, put down the remainder with the common denominator under it. In the caſe of mixt numbers, ſubtract firſt the fraction of the leſſer number from that of the greater, and then the leſſer whole number from the greater: but if, as it often happens, the greater number has the leſſer fraction belonging to it, then an unit muſt be borrowed from the whole number and added to the fraction, as intimated in the cloſe of the laſt article.

Examples of ſubtraction in fractions.

1*ſt*, $\frac{3}{10}$ ſubtracted from $\frac{4}{10}$ leaves $\frac{1}{10}$, juſt in the ſame manner as 3 ſhillings ſubtracted from 4 ſhillings leaves 1 ſhilling.

2*dly*, $\frac{3}{4}$ ſubtracted from $\frac{5}{6}$, that is, $\frac{18}{24}$ ſubtracted from $\frac{20}{24}$, leaves $\frac{2}{24}$, or $\frac{1}{12}$. So $\frac{3}{4}$ of a pound, or 15 ſhillings, ſubtracted from $\frac{5}{6}$ of a pound, or 16 ſhillings and 8 pence, leaves $\frac{1}{12}$ of a pound, that is, 1 ſhilling and 8 pence.

3*dly*, $7\frac{1}{3}$ ſubtracted from $8\frac{1}{2}$, that is, $7\frac{2}{6}$ ſubtracted from $8\frac{3}{6}$, leaves $1\frac{1}{6}$.

4*thly*, $7\frac{3}{4}$ ſubtracted from $8\frac{1}{4}$, that is, $7\frac{3}{4}$ ſubtracted from $7\frac{5}{4}$, leaves $\frac{2}{4}$, or $\frac{1}{2}$; for here the greater number having the leſs fraction belonging to it, I borrow an unit from the whole number 8, and ſo reduce it to 7; and then this unit, under the name of $\frac{4}{4}$, I add to the fraction $\frac{1}{4}$, and ſo make it $\frac{5}{4}$.

concluſion, and we ſhall then have $9y - rr = 0$, and $y = \frac{rr}{9}$, and $\sqrt{y}$ or $e = \frac{r}{3}$: thus then at laſt we have found the ultimate ratio of r to e, which is that of r to $\frac{r}{3}$, that is, of 3 to 1, and the ultimate magnitude of the fraction $\frac{r}{e}$ is $\frac{3}{1}$ or 3 : this being known, we ſhall have $\frac{q-\frac{r}{e}}{2}$ or $f = -4$, and $\frac{q+\frac{r}{e}}{2}$, or $g = -1$, and the equations derived from this ſuppoſition of $e = 0$ will be $xx - 4 = 0$, and $xx - 1 = 0$.

How others may reliſh theſe mathematical myſteries I know not; but for my own part I muſt own, it is not without the utmoſt pleaſure and ſurprize that I obſerve, what admirable ſhifts nature hath to bring herſelf off upon all occaſions when ſhe ſeems to be ſurrounded with inſuperable difficulties; ſhifts ſo far beyond all human contrivance, that it is not always in the power of human underſtanding to trace out her ſteps and follow her, much leſs to preſcribe to her: but the ſublimer parts of Mathematics, and more particularly the doctrine of Fluxions, furniſh us with ſtill more frequent and pregnant inſtances of this kind, inſomuch that the ancient Sage (whether he knew it or not) had a great deal of reaſon on his ſide when he cried out as he did,

Magna eſt veritas, et praevalebit.

The Metaphyſicians tell us, that the material world is nothing elſe but an emanation from the ideal: and it may be ſo, for ought I know; but in the mean time I cannot but be ſurprized to hear Gentlemen talk at this high-flown rate, who know ſo little either of one world or the other. It is in Mathematics only that truth, that is, rational truth, appears moſt conſpicuous, and ſhines in her ſtrongeſt luſtre: in all other ſciences ſhe is either ſelf-evident, or lies ſo near the day as to afford but little pleaſure in the diſcovery; or if ſhe lies deeper and muſt be dug for, ſhe is ſeen for the moſt part through ſo much droſs, obſcurity and confuſion, that falſhood herſelf under a plauſible diſguiſe often paſſes for truth. Whoſoever then would be thoroughly acquainted with the nature, beauty and harmony of truth; whoſoever to the utmoſt of his finite capacity would ſee truth as it has actually exiſted in the mind of God from all eternity, he muſt ſtudy Mathematics more than Metaphyſics: in Mathematics there appears an uninterrupted vein of truth, which the ſearcher is at liberty to purſue, or any particular branch of it, as far as he pleaſes, making every known truth ſubſervient to the diſcovery of ſome other: but in moſt

other

other ſciences, all that beautiful analogy, all that harmonious connexion and conſiſtency is quite loſt; and thoſe truths that are diſcovered, appear ſo ſcattered, and ſo very independent one of another, that they ſeem to have no manner of relation one to another, though it is certain that all truths have: upon all theſe accounts it is no wonder that much greater advances have been made in and by the Mathematical ſciences, than in all the reſt put together without them.

As to the material world, if nature be not governed by conſtant and ſteady laws, all Philoſophy is vain and fruitleſs: but if on the contrary ſhe be always conſiſtent with herſelf, if ſhe will ſooner produce a monſter than deviate in the leaſt from any of her own laws, ſhe will then ſubmit in all caſes to be examined and tried by thoſe laws: whether they be ſuch as fall immediately under common obſervation and experience, or others leſs obvious that are only by reaſon deducible from theſe, we ſhall always find nature as much obliged by her own laws, as truth by the neceſſity and reaſon of things; ſo that we never need to doubt of our following nature, if we reaſon juſtly from things known to things unknown, whether it be from effects to their cauſes, or from cauſes to their effects. But then we ought to be cautious in the ſeveral ſteps we take, and enquire firſt, whether the cauſes ſuch and ſuch *Phænomena* are to be aſcribed to, have themſelves a real exiſtence in nature or not; and if they have, ſecondly, whether they are or can be naturally productive of thoſe effects; and if they are, laſtly, whether they be efficacious enough to produce them in ſuch quantities and degrees as we find them in nature: by this means we ſhall find many effects that were vulgarly aſcribed to different cauſes, actually proceeding from one and the ſame cauſe; and as our natural knowledge improves, the number of natural cauſes will be reduced, and nature, every ſtep we take, will appear more ſimple and uniform. But then theſe deductions, theſe enquiries cannot poſſibly be made to any degree of certainty without a thorough knowledge of quantity and proportion: and hence ariſes the very great uſe and even neceſſity of the Mathematical ſciences in natural Philoſophy; and I doubt not but this was one main end of the allwiſe Author of nature in beſtowing ſo ineſtimable a gift upon rational creatures; for this method of reaſoning, if duely cultivated and purſued, would bring us nearer to a true knowledge of God in his creation than all the metaphyſical jargon of the ſchools. This was the method the great *Newton* took; and how he ſucceeded in it is ſufficiently known to all the ſober and thinking part of the learned world.

It is not to be denied indeed but that Mathematicians may, and very often do fail in their reſearches after nature, ſometimes from a too raſh and inconſiderate application of their principles, but oftener from the difficulty of their ſubjects: but what then? becauſe theſe Gentlemen ſometimes fail in in their attempts, muſt other minute Philoſophers, that are wholly deſtitute of theſe main qualifications, think they have nature more in their power? Sure I am that theſe, as they have no foundation, ſo neither can they have the leaſt colour of pretence to any ſuch enquiries: all that theſe can do to keep their ignorance in countenance, muſt be ſeemingly to deſpiſe and heartily to rail at what they do not underſtand; as if they voluntarily declined a part of learning, which their want of application and genius have rendered them altogether unfit for.

But the undoubted experience of this and the laſt age have ſufficiently eſtabliſhed the uſe of Mathematical Learning in Mechanical and Natural Philoſophy beyond all that has been or perhaps can be ſaid againſt it. Nor have the Moderns, eſpecially thoſe of our own Nation, ſtopped here; for they have endeavoured to the utmoſt of their power to purge it from all thoſe metaphyſical diſputes and ſubtilties wherewith it hath ſo long been overrun, poiſoned and ſtifled in it's growth; ſubtilties which at beſt amount to little more than a ſort of legerdemain tricks, contrived by Philoſophers purely to cover their own ignorance by bewildering the minds of others, depraving their taſtes, and often giving them ſuch an unhappy turn of thought as utterly to diſqualify them for all ſober and rational enquiries: but as the evil we are here complaining of, and the malign influence it has upon the minds of thoſe who are in any meaſure tainted with it, begins now (God be thanked) to be pretty well known, it is to be hoped that all wiſe men will guard againſt a diſtemper ſo eaſily catched, and ſo very difficultly cured. I do not know whether my zeal for truth may not have carried me too far in this digreſſion: but I thought too much caution could not be given to all ſober and ingenuous Youths, eſpecially at their firſt ſetting out in their ſtudies, againſt a falſe taſte, which has done ſo much miſchief in the world by extinguiſhing that exquiſite and refined pleaſure which mankind naturally feel in the contemplation of truth, where it is genuine and uncorrupted by a falſe alloy; a pleaſure not founded in our appetites, paſſions or humours, but belonging to us purely as thinking, reaſoning creatures; and if ſtrictly enquired into, and it's abſtracted nature thoroughly weighed, will perhaps be found the only one the preſent ſtate of Man is capable of in common with ſuperior Beings.

APPENDIX

APPENDIX.

MR. *Abraham de Moivre* having with great ſagacity not only diſcovered a method for extracting the cube root of an impoſſible binomial, ſuch as $a+\sqrt{-b}$, but alſo rendered that method univerſal, by ſhewing how to extract any other root of the ſaid binomial, and likewiſe how to extract any root out of any given power thereof, hath been pleaſed to communicate it, in order to it's being publiſhed by way of Appendix to theſe Elements. This important diſcovery is a freſh inſtance of the penetration and ſkill of the ingenious Author, and cannot but be very acceptable to thoſe who deſire to improve themſelves in Algebra. Dr. *Saunderſon* had formerly conſulted Mr. *de Moivre* upon the ſubject of extracting the cube root of an impoſſible binomial in the following Letter, which Letter was principally deſigned to return him thanks for the ſolution of the latter of his two curious problems about proportionals, and for conſenting to have them both inſerted into his book; whereby alſo it appears, how highly our late Profeſſor eſteemed the great abilities of his learned Friend.

To Mr. *Abraham de Moivre.*

Dear Sir, *Cambridge, Septemb.* 26. 1738.

This is the firſt opportunity I could get to acknowledge the favour of your's, and to thank you for your ſolution of your ſecond problem, and for your having ſo frankly and without the leaſt reſerve, conſented to have them inſerted into my book, which will certainly be the greateſt ornament it can receive, and the greateſt recommendation of it to the world. In this laſt ſolution you have ſhewn (if poſſible) more art and penetration than in the former; but they are both maſter-pieces in their kind, and do a great deal of honour not only to yourſelf, but alſo to the art in general, by whoſe irreſiſtible (and I had almoſt ſaid, unlimited) power, ſuch great things can be effected, beyond the compaſs of all other ſciences, eſpecially when applied by one of your extraordinary ſagacity and genius. I find you and I have both hit upon the ſame thought in dividing one equation by another in order to obtain a more ſimple one; though you have made a much better uſe of it than I have done, or indeed had occaſion to do. Pray, when you write next, be ſo good as to let me know, whether you have any thing by you relating to the extrac-

tion

tion of the cube root of an impoſſible binomial, ſuch as $-5+\sqrt{-2}$, or $-5-\sqrt{-2}$, or whether in your reading you have met with any way of doing this with the ſame certainty as in the caſe of a poſſible binomial: for my own part, I have met with nothing to the purpoſe about it, not even in *Wallis* himſelf, who attempts it. I am

Dear Sir,

moſt affectionately your's,

N. Saunderſon.

P. S. I would not for all the world have been without this laſt ſolution, becauſe they both together more compleatly illuſtrate your method, than would either of them alone; a method wherein you have ſo well ſucceeded, and which I am ſure every one muſt make uſe of, that intends to penetrate any further into theſe difficulties.

To the Editor of Dr. *Saunderſon*'s Algebra.

Sir,

Among ſeveral letters which I have received from my late worthy Friend Dr. *Saunderſon*, there is one wherein he was pleaſed to put a queſtion to me, which was, whether I had any method of extracting the cube root of an impoſſible binomial, ſuch as $a+\sqrt{-b}$, adding, that what Dr. *Wallis* had writ upon that ſubject, did not ſatisfy him. I do not exactly remember the terms of my anſwer; but it was to this purpoſe, *viz.* that I had long ago demonſtrated, that the rule given by Dr. *Wallis* was no more than a *petitio principii*, or at beſt, a trial which could not ſucceed in many caſes; yet, that as my demonſtration lay in a heap of many other papers, I could not readily come at it: but now that I have recovered it, I hereby ſend it you.

But it is neceſſary to premiſe one thing; which is, that the Doctor's deſign was to ſhew, that in cubic equations there is no caſe inexplicable, notwithſtanding the common received opinion of the contrary. One of theſe caſes appears in the equation by him mentioned, $r^3-63r=162$; wherein if we take the value of r according to the rule commonly aſcribed to *Cardan*, we ſhall have $r=\sqrt[3]{81+\sqrt{-2700}}+\sqrt[3]{81-\sqrt{-2700}}$; which value conſiſts of two parts, each of which includes the imaginary quantity $\sqrt{-2700}$. Now Dr. *Wallis* in order to prove the raſhneſs of thoſe who had aſſerted, that in cubic equations there are ſome caſes irreducible, or (as he calls them) impracticable, ſays, that the cubic root of $81+\sqrt{-2700}$ may be extracted by another impoſſible binomial, *viz.* by by $\frac{9}{2}+\frac{1}{2}\sqrt{-3}$; and in the ſame manner, that the cubic root of $81-\sqrt{-2700}$ may be extracted, and ſhews it to be $\frac{9}{2}-\frac{1}{2}\sqrt{-3}$; from whence he infers, that $\frac{9}{2}+\frac{1}{2}\sqrt{-3}$, $+\frac{9}{2}-\frac{1}{2}\sqrt{-3}$, or barely 9, is one of the roots of the equation propoſed, *viz.* $r^3-63r=162$: he alſo finds the other two roots, -6, -3.

It muſt be owned, that $\frac{9}{2}+\frac{1}{2}\sqrt{-3}$ is the cubic root of $81+\sqrt{-2700}$, and that his inference is true: but thoſe who will conſult his Algebra, page 190 and 191, will plainly ſee, that the rule he gives is nothing but a trial, both in determining that part of the root which is out of the radical ſign, and that part which is within; and that if the original equation had been ſuch, as to have it's roots irrational, his trial would never have ſucceeded. But farther I ſhall prove, that the extracting the cube root of $81+\sqrt{-2700}$ is of the ſame degree of difficulty as that of extracting the root of the original equation $r^3-63r=162$, and that both require the triſection of an angle for a perfect ſolution: in order to which, let the following general problem be propoſed.

To extract the cubic root of the binomial a$+\sqrt{-}$b.

Solution.

Let the cube root of $a+\sqrt{-b}$ be $x+\sqrt{-y}$: therefore the cube of $x+\sqrt{-y}$ is equal to $a+\sqrt{-b}$; but the cube of it is $x^3+3xx\sqrt{-y}-3xy-y\sqrt{-y}$: make the rational parts equal to a, and the irrational equal to b: hence we have theſe two equations,

$$x^3-3xy=a,$$

$$\overline{3xx-y}\times\sqrt{-y}=\sqrt{-b};$$

and by ſquaring both equations, we ſhall have

$$x^6-6x^4y+9xxyy=aa,$$

and $$-9x^4y+6xxyy-y^3=-b:$$

then taking the difference of theſe two equations, we ſhall have

$$x^6+3x^4y+3xxyy+y^3=aa+b;$$

and extracting the cube root on both ſides, we ſhall have $xx+y=\sqrt[3]{aa+b}$: let (for ſhortneſs's ſake) $\sqrt[3]{aa+b}$ be ſuppoſed $=m$; we have therefore $y=m-xx$: but from the firſt equation, *viz.* $x^3-3xy=a$, we have $y=\frac{x^3-a}{3x}$; wherefore $m-xx=\frac{x^3-a}{3x}$, and $3mx-3x^3=x^3-a$, or $4x^3-3mx=a$: but it is known, that if r be the *radius*, l the coſine of an arc, and x the coſine of the third part of that arc, then the equation expreſſing the relation between l and x, will be $4x^3-3rrx=rrl$: which equation is of the ſame nature with the preceding, as appears by ſupposing $rr=m$, or $r=\sqrt{m}$, and $rrl=a$, or $ml=a$, or $l=\frac{a}{m}$: and therefore we may now draw this conclusion, that if with a *radius* $=\sqrt{m}$, we describe a circle, and take an arc, of which the cosine shall be $\frac{a}{m}$, then shall the cosine of the third part of that arc be the value of x; which value being known, the value of y will also be known, it being always equal to $m-xx$, as we have shewn before.

Still we cannot be sure that the equation $4x^3-3mx=a$ depends upon the trisection of an arc, unless we know previously that the cube of m is greater than the square of a: but we may soon be satisfied of it, if we consider that m has been supposed $=\sqrt[3]{aa+b}$; therefore $m^3=aa+b$, which by inspection appears bigger than aa.

But before we proceed any farther, it will be proper to observe that there are three different values of x, and as many of y: for let C represent the whole circumference of the circle, of which the *radius* is $=\sqrt{m}$, and A the arc, of which the cosine is $\frac{a}{m}$; then the cosines of $\frac{A}{3}$, $\frac{C-A}{3}$, $\frac{C+A}{3}$ will be so many different values of x; and therefore there will be so many different values of y, it having been proved before that y is always $=m-xx$.

But to apply this to the case proposed by Dr. *Wallis*, which was to extract the cube root of $81+\sqrt{-2700}$; make $a=81$, and $b=2700$; therefore $aa+b=9261$, and consequently $m=\sqrt[3]{9261}=21$, and therefore our *radius* is $\sqrt{21}$. Again, the cosine $\frac{a}{m}=\frac{81}{21}=\frac{27}{7}$: now it will be found by an easy trigonometrical calculation, that if $\sqrt{21}$ be the *radius* of a circle, and $\frac{27}{7}$ the cosine of an arc, then this arc will be $32^\circ.\ 42'$ nearly, representing the arc A; and therefore $C-A$ will be $327^\circ.\ 18'$, and $C+A=392^\circ.\ 42'$: then the thirds of those arcs will be $10^\circ.\ 54'$, $109^\circ.\ 06'$, $130^\circ.\ 54'$; whereof the first being less than a quadrant, it's cosine, that is, the sine of $79^\circ.\ 06'$ must be looked upon as positive, and the other two being greater than quadrants, their cosines, that is, the sines of $19^\circ.\ 06'$ and $40^\circ.\ 54'$, must be looked upon as negatives. This being laid down, it will be found by the rules of trigonometry that the sines of $79^\circ.\ 06'$, $19^\circ.\ 06'$, $40^\circ.\ 54'$, to *radius* $\sqrt{21}$, will respectively be 4.4999, -1.4999, -3.0000, or $\frac{9}{2}$, $-\frac{3}{2}$, -3; and consequently that the three values of y, which (as we have shewn

shewn before) are universally expressed by $m-xx$, will respectively be $21-\frac{81}{4}$, $21-\frac{9}{4}$, $21-9$, or $\frac{3}{4}$, $\frac{75}{4}$, 12; of which the square roots are $\frac{1}{2}\sqrt{3}$, $\frac{5}{2}\sqrt{3}$, $2\sqrt{3}$; and therefore the three values of $\sqrt{-y}$ will be $\frac{1}{2}\sqrt{-3}$; $\frac{5}{2}\sqrt{-3}$, $2\sqrt{-3}$; from all which we may conclude, that the three roots of $\sqrt[3]{81+\sqrt{-2700}}$ are $\frac{9}{2}+\frac{1}{2}\sqrt{-3}$, $-\frac{3}{2}+\frac{5}{2}\sqrt{-3}$, $-3+2\sqrt{-3}$: and by the same way of arguing we may prove that the three roots of $\sqrt[3]{81-\sqrt{-2700}}$ are $\frac{9}{2}-\frac{1}{2}\sqrt{-3}$, $-\frac{3}{2}-\frac{5}{2}\sqrt{-3}$, $-3-2\sqrt{-3}$.

But to illustrate what I have said, let it be proposed to find the equation which would result from the addition of the two binomials $\sqrt[3]{a+\sqrt{-b}}+\sqrt[3]{a-\sqrt{-b}}$ in order to free them from their radicality: which to find, let us suppose

$$1°,\ z^3=a+\sqrt{-b};$$
$$2°,\ v^3=a-\sqrt{-b};$$
$$3°,\ z+v=x.$$

By the two first equations it appears that $z^3+v^3=2a$; by the third, that $z+v=x$; therefore $\frac{z^3+v^3}{z+v}=\frac{2a}{x}$; but $\frac{z^3+v^3}{z+v}=zz-zv+vv$, and therefore we may conclude that $zz-zv+vv=\frac{2a}{x}$; but squaring the third equation, we have $zz+2zv+vv=xx$; then subtracting the first of these two last equations from the second, we shall have $3zv=xx-\frac{2a}{x}$; but if z^3 be multiplied by v^3, and $a+\sqrt{-b}$ by $a-\sqrt{-b}$, we shall have $v^3z^3=aa+b$; therefore $vz=\sqrt[3]{aa+b}$, and $3vz=3\sqrt[3]{aa+b}$; or supposing $\sqrt[3]{aa+b}=m$, then $3vz=3m$; but we had found before, that $3vz=xx-\frac{2a}{x}$; therefore $xx-\frac{2a}{x}=3m$, or $x^3-2a=3mx$, or $x^3-3mx=2a$; which to resolve, the trisection of an angle is required, it being of the same nature as the former was, *viz.* $4x^3-3mx=a$; for if in the equation $x^3-3mx=2a$, instead of x we write $2x$, we shall have $8x^3-6mx=2a$, or $4x^3-3mx=a$.

But to go still farther, let it be proposed to extract such root of the binomial $a+\sqrt{-b}$ as may be denominated by n, *viz.* $\sqrt[n]{a+\sqrt{-b}}$. Let $x+\sqrt{-y}$ be that root: then supposing $\sqrt[n]{aa+b}=m$, let a circle be described

scribed of which the *radius* shall be equal to $\sqrt{m}$: moreover supposing $\frac{n-1}{2}=p$, (no matter whether n be an odd or an even number, provided it be an integer,) take, in that circle, an arc, of which the cosine shall be $\frac{a}{m^p}$, and let that arc be supposed $=A$; let C represent the whole circumference; then the cosines of the arcs $\frac{A}{n}$, $\frac{C-A}{n}$, $\frac{C+A}{n}$, $\frac{2C-A}{n}$, $\frac{2C+A}{n}$ &c to the same *radius* $\sqrt{m}$, will be so many different values of x. But it is to be observed, that so many of those cosines are to be taken as there are units in n, and no more.

I have but one word to add, which is, that if it be required to extract any root out of any given power of the binomial $a+\sqrt{-b}$, for instance, the cube root of the square, the same may also be done thus: let $a+\sqrt{-b}$ be raised to it's square, and it will be $aa-b+2a\sqrt{-b}$; then supposing $aa-b=d$, and $2a\sqrt{-b}$ or $\sqrt{-4aab}=\sqrt{-e}$, nothing remains now to be done, but to extract the cube root of the binomial $d+\sqrt{-e}$, which may be done as before.

I think it would be needless to say any thing more upon this subject: wherefore I now subscribe myself

Sir, &c.

April 29. 1740.

A. De Moivre.

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a.	b.	c.	d.	e.	f.	g.	h.
41.	24.	17.	7.	3.	1.	0.	0.

a *by* b, *and let the remainder be* c; *then divide* b *by* c, *and let the remainder be* d; *then divide* c *by* d, *and let the remainder be* e, &c: *I ſay then that the ſeries* a, b, c, d, e &c *may be continued to terms leſs than any aſſignable quantity*, *as* q. 296
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 guineas

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roots

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440. Every cubic equation may be reduced to this form, to wit, $x^3 \pm 3aax = \pm 2aab$. 694

441. Preparation for enquiring into the roots of a cubic equation of this form, $x^3 \pm 3aax = \pm 2aab$. ibid.

442. Every cubic equation of this form, $x^3 - 3aax = \pm 2aab$ will have all it's roots possible, provided that b be not greater than a or less than $-a$, but lies between those two limits. 695

443. Setting aside the case of the last article, I say that in all other cases,

cases, the equation $x^3 \pm 3aax = \pm 2aab$ *can have only one root possible; which root will be affirmative or negative according as* b *is so, that is, according as* + *or* —2aab *is concerned in the equation.* 696

444. *The sum of the two last articles, supposing the several cases of cubic equations under the form of* $x^3 \pm 3aax = \pm 2aab$ *to be restored to their primitive form* $x^3 \pm px = \pm q$. 698

Of the resolution cubic equations. 699

445. *To resolve any cubic equation that hath but one possible root.* ibid.

Examples. 702

446. *Another way more simple and direct.* 704

Examples of this method of resolving cubic equations. 705

Extraction of the root of a cubic equation by the help of logarithms. 706

447. *All cubic equations might be resolved this way, if we could extract the cube root of an impossible binomial.* ibid.

448. *The last case of cubic equations considered, with a rule for finding the root true in three of it's first figures at least.* 708

449. Definitions. 710

450. *In every quadrilateral figure inscribed in a circle, the rectangle under the diagonals is equal to both the rectangles under the opposite sides taken together.* 711

451. A Lemma. (*Fig.* 64.) *Let* ABC *be an equilateral triangle inscribed in a circle; and from any one of it's angles* A, *let a line as* ADE *be drawn, cutting the opposite side* BC *in* D, *and the circle in* E; *and joyn the chords* BE, CE : *I say then that cord* AE *will be equal to the sum of the two chords* BE *and* CE *put together.* 712

452. A Problem. (*Fig.* 65, 66.) *Let* ABCD *be an arc of a given circle whose diameter is* AF, *and let the arc* AB *be a third part of the whole arc* ABCD : *It is required, having given the chord of the arc* ABD, *to find the chord of the arc* AB; *and that, whether the arc* ABD *be less than a semicircle as in the 65th figure, or greater as in the 66th.* ibid.

453. A Problem. *Allowing the trisection of a circular arc, it is required to construct a cubic equation of this form,* $x^3 - px = \pm q$, *where the cube of* $\frac{p}{3}$ *is supposed to be greater (or not less) than the square of* $\frac{q}{2}$. 713

454. A Problem. *Supposing all things as in the last article, let it now be required to resolve the foregoing equation by the help of a canon of sines.* 714
455. A Problem. *To find two mean proportionals between any two given numbers.* 717
456. *Mr* Cotes's *method of resolving cubic equations considered and demonstrated.* 718
457. *Another method of resolving cubic equations, which finds all the three roots at once; by Mr* Colson: *with examples.* 720
458, 459, 460. Newton's *method of approximation applied to the resolution of cubic equations.* 722

Of the resolution of biquadratic equations: *and first*

461, 462, 463. *The resolution of a biquadratic equation of this form,* $x^4+pax^3+qaaxx+pa^3x+a^4=0$. 728
464, 465. *A further illustration of the foregoing resolution by a geometrical problem out of* Pappus. 730
466, 467. *Of the resolution of all sorts of biquadratic equations by the mediation of cubics.* 735
468. *The composition of a biquadratic equation whose second term is wanting.* 737
469. *The resolution of a biquadratic equation (according to the foregoing method) whose second and fourth terms are both wanting.* 738
This article concludes the book, with a digression upon the excellency of the Mathematical sciences above all others for the discovery of truth; their use and necessity in mechanical and natural Philosophy; whence the study of them is recommended preferably to Metaphysical disputes and subtilties. 740

APPENDIX.

A Letter from Professor Saunderson *to Mr* Abraham de Moivre. 743
Mr de Moivre's *method of extracting the cubic or any other root of an impossible binomial, such as* $a+\sqrt{-b}$; *with an application of that method to the extraction of any root out of any given power of the said binomial.* 744