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WOODHOUSE, BABBAGE, PEACOCK, AND MODERN ALGEBRA

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SUMMARIES

In a recent article, J. M. Dubbey [Historia Mathematica *4 (1977), 295-302*] *showed that George Peacock's* A Treatise on Algebra *(1830) was similar to an unpublished work written by Charles Babbage in 1821. Evidently perplexed about the absence of a dispute over priority, Dubbey concluded that Peacock had unconsciously assimilated Babbage's ideas, and that Babbage was too busy with other activities to be concerned. The thesis of this article is that the innovative aspects of the work of both Babbage and Peacock are extensions of ideas put forth in 1803 by Robert Woodhouse, and that probably neither Babbage nor Peacock was overly concerned with acknowledgments because their approach to algebra was not unique at Cambridge.*

In einem jüngst publizierten Aufsatz zeigte J. M. Dubbey [Historia Mathematica *4 (1977), 295-302* *dass George Peacock's* A Treatise on Algebra *(1830) Ähnlichkeit mit einer unveröffentlichten Arbeit von Charles Babbage aus dem Jahr 1821 besitzt. Offenbar überrascht über das Ausbleiben eines Prioritätstreites schloss Dubbey, Peacock habe unbewusst Ideen von Babbage übernommen, während dieser mit anderen Dingen zu beschäftigt gewesen sei, als dass er von dieser Übernahme ein Aufheben gemacht habe. Demgegenüber wird die These vertreten, die innovatorischen Aspekte der Arbeiten von Babbage wie von Peacock seien Weiterführungen jener Ideen, die Robert Woodhouse 1803 geäusset habe; vermutlich hätten weder Babbage noch Peacock sich übermässig um entsprechende Hinweise gekümmert, weil sich ihr Verständnis von Algebra von dem damals in Cambridge vorherrschenden nicht wesentlich unterschied.*

Dans un article récent, J. M. Dubbey [Historia Mathematica *4 (1977) a fait ressortir les similitudes existant entre le* A Treatise on Algebra *(1830) de George Peacock et une oeuvre inédite de Charles Babbage datant de 1821. Apparemment préoccupé par l'absence de querelle de priorité, Dubbey en vint à la conclusion que Peacock avait inconsiemment assimilé les*

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idées de Babbage, alors que ce dernier, trop actif par ailleurs, se désintéressait de cette question. Nous soutenons dans cet article que les innovations de Babbage et de Peacock étendent certaines idées émises dés 1803 par Robert Woodhouse et que, pour ceux-là, la reconnaissance d'antécédents n'apparaissait pas particulièrement importante étant donné que leur façon de concevoir l'algèbre n'était pas nouvelle à Cambridge.

Historians have placed Charles Babbage and George Peacock among the founders of modern algebra [Boyer 1968, 621-623, 633; Dubbey 1977; Koppelman 1971, 175-187, 229; Laita 1977, 165-172; Novy 1973, 187-194]. Although the algebra of Robert Woodhouse prefigures that of Babbage and Peacock [Koppelman 1971, 176-177, 183, 185, 187, 229], the connection between Woodhouse and his younger contemporaries at Cambridge has not been brought fully into view. As a result, J. M. Dubbey, in analyzing and comparing the work of Babbage and Peacock, concluded that it would be "idle to put forward any theory to account for the astonishing similarity" between Babbage's unpublished algebra of 1821 and Peacock's textbook on the subject in 1830 [Dubbey 1977, 302]. However, it is possible to account for this striking similarity by a simple suggestion: both Babbage and Peacock were expanding upon ideas promulgated by Woodhouse.

The similarity between Woodhouse's algebra and that of Babbage and Peacock is easily established. Woodhouse redefined certain aspects of algebra in *The Principles of Analytical Calculation* [1803]. In doing so, he expressed ideas similar to those contained in Dubbey's characterization of what was innovative in and common to the work of Babbage and Peacock:

> *(1) Algebra had previously been considered only as a modification of arithmetic. (2) Algebra consists of the manipulation of symbols in a way independent of any particular interpretation. (3) Arithmetic is only a special case of Algebra--a "Science of Suggestion" as Peacock put it. (4) The sign "=" is to be taken as meaning "is algebraically equivalent to." (5) The principle of the permanence of equivalent forms* [Dubbey 1977, 298].

Of the five principles, Woodhouse paid least attention to the first. To a great extent, he left the reader to discern what was new in his publication, but he did write of previous mathematicians' "erroneous opinion of the necessity of the existence of an arithmetical equality" between a function and its expansion. More importantly, he argued that this "erroneous opinion" caused mathematicians to "reject the notion of the extensions of demonstrated forms," [Woodhouse 1803, 54]. Woodhouse himself employed this "notion" repeatedly throughout the book. It is the prototype of the "principle of the permanence of equivalent forms" (point 5 of Dubbey). For example, Woodhouse extended the procedures he had already established for unknowns raised to positive integral powers to unknowns raised to negative and fractional powers [Woodhouse 1803, 15-20]. Also, having established for the binomial expansion that a rational exponent indicates the "operation" determining the "law of coefficients," Woodhouse "extended" the "form" of the binomial theorem to cases where the exponents were irrational and to exponential functions, functions of the form $y = b^x$ where b is a fixed positive number and x is an irrational independent variable [Woodhouse 1803, 29, 35-37, 53-54]. He similarly employed the principle of extension to establish the use of arithmetic operations on imaginary numbers:

> *In their simplest meaning the symbols + - × designate additions, subtractions, multiplications, to be made on the supposition that the characters connected by these symbols, can be resolved into units; and on this supposition, the first rules for transposition and multiplication are demonstrated; but subsequently to the extension of the rules, by which equations of no direct meaning and symbols incapable of being arithmetically computed are introduced, these symbols take more extensive signification: thus,* $a \pm b\sqrt{-1} + c \pm 2b\sqrt{-1}$ *is put* $a + c \pm 3b\sqrt{-1}$ *where the symbols* $b\sqrt{-1}$, $2b\sqrt{-1}$ *are connected together, in the same manner, as the signs of real quantities are, that is, of quantities that admit numerical computation: again* $(a + b\sqrt{-1}) \times (c + d\sqrt{-1}$ $= ac + ad\sqrt{-1} + cd\sqrt{-1} - bd$, *where the connecting sign × indicates an operation to be performed: what that operation is, we know from having previously established its nature, in those cases where the symbols employed, were supposed to represent collections of units* [Woodhouse 1803, 9-10].

The last sentence indicates that Woodhouse viewed arithmetic as a "science of suggestion" (see Dubbey's point 3 above) and he left no doubt that arithmetic was only a special case of algebra (point 3). But "suggestion" from arithmetic taken too literally could lead to error: "certain difficulties and paradoxes that have been proposed concerning periodic and diverging series ... have arisen from the arithmetical operations ... having been confounded with the algebraical.... The arithmetical and algebraical operations are indeed similar in their process, but the former are purposely instituted to give results within certain limits of numerical exactness, whereas in the latter, numerical equality or approximation not being the object, nor flowing from the nature of the operation, such equality, when numbers are substituted for the algebraical symbols, need not *necessarily* happen" [Woodhouse 1803, 63]. Thus, "the symbols + - may appear in analytical operations with a different meaning from what they have, in such equations as, $x + a = b - c$, for since $1/(x + 1) = 1 - x + x^2 - x^3 + \cdots$ by rule for transposition $1/(1 + x) - (1 - x + x^2 - x^3 + \cdots) = 0$ which *equation* is not to be explained by saying, that when numbers are substituted for x, the first part is numerically equal to the second" [Woodhouse 1803, 4]. Woodhouse contended that all expansions of functions were valid because algebra was a more generalized science than arithmetic. Hence, "the series $1 - 1 + 1 - 1 + 1 \cdots$; $1 - 2 + 2^2 - 2^3 \cdots$; $1 - 3 + 3^2 - 3^3 \cdots$; $1 - 1/2 + 1/2^2 - 1/3^2 + \cdots$ are true expansions because they are the results of certain operations, the object of which operations is not arithmetical equality" [Woodhouse 1803, 13]. In this context, he concluded also that the order of the symbols had to be taken into consideration: $1/(1 + x) \neq 1/(x + 1)$, $(1 + x)^m \neq (x + 1)^m$, $\phi(x + 0) \neq \phi(0 + x)$ [Woodhouse 1803, 57-58]. Similarly, the algebrist need not be concerned with "the different values" of $f(x + i)$ because $f(x + i)$ was a "mere symbol" and $fx + pi + qi^2 + \cdots$ was simply its "expanded value when produced according to a certain process," [Woodhouse 1803, **xx**]. By viewing algebra in this way, as the arbitrary manipulation of symbols independent of interpretation (point 2 of Dubbey), Woodhouse achieved a major objective. He repeatedly relegated the problem of achieving convergence to the realm of arithmetic, a problem "useless" in and "quite distinct from" algebra [Woodhouse 1803, 15, 46]. He could regard "all series produced according to the rules of certain operations ... as equally true, whether the terms of the series decrease or not ..." [Woodhouse 1803, 41].

Woodhouse based his exposition on the redefinition of "=" (point 4 of Dubbey). Even if an arithmetical inequality existed between a function and its expansion, such as in the case where $1/(1 + x) = 1 - x + x^2 - \cdots$, Woodhouse argued that the equation is nevertheless valid

> *whatever numerical inequality appears between the two sides of the* equation *when for* x *is substituted any number: the symbol = in these cases serves merely to* connect *the involved expression and the result of the operation: it is with this signification of the symbol =, that the deductive processes in works on analytical science, are to be understood: they are not logically exact when = is restricted to denote numerical equality. It is here in my power to fix its meaning by definition, and I think it more simple and commodius to use the symbol = with its extended signification...than to limit its signifcation and invent another symbol somewhat similar to =...* [Woodhouse 1803, 3-4].

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Thus, in 1803 Woodhouse had argued, as Babbage (in 1821) and Peacock (in 1830) were later to do that although arithmetic, as a specialized branch of algebra, had a heuristic value for algebraic developments, algebra was more than symbolized arithmetic; they viewed algebra as independent of arithmetic interpretations and limitations. From this foundation, they concluded that arithmetic operational symbols and the equal sign could be given more general definitions when employed in algebra. In addition, all three mathematicians explicitly stated that operational procedures verified under specific conditions could be defined as valid for cases involving more general conditions, although the validity of the latter could not be derived from the former or proved independently.

The uniformity, or similarity, of the arguments found in the work of these three mathematicians suggests a link between them. In fact, Babbage had read the *Principles* even before matriculating at Cambridge [Babbage 1864/1969, 26], and the book was in his library at his death [Babbage 1872, 60]. In 1833, Peacock acknowledged that Woodhouse, in a presentation to the Royal Society in 1802, had made "a very near approach" to his own principle of the permanence of equivalent forms [Peacock 1834, 233-234; Woodhouse 1802b]. At the same time, Peacock eulogized Woodhouse as "a very acute and able scrutinizer of the logic of analysis" and as "an author whose careful and bold examination of the first principles of analytical calculation entitle his opinion to the greatest consideration" [Peacock 1834, 233-234]. The allusion to *The Principles* is obvious.

Peacock was probably acquainted with Woodhouse's algebra by 1833. Babbage brought *The Principles* with him to Cambridge; he and Peacock were close friends, both as students and after graduation [Cannon 1978, 34, 42; Babbage 1864/1969, 29, 39-40; Whewell papers, 0.15.46[17]]. Moreover, other publications by Woodhouse were well known to Peacock. Like Babbage, he cited approvingly Woodhouse's textbooks on the calculus of variations and trigonometry [Babbage and Herschel 1813, ii, iv-v; Peacock 1820, Preface; Lacroix 1816, 643-644; Peacock 1834, 295]. In answering "bookwork" questions (those answered in textbooks or circulated manuscripts) for the Tripos from 1801 to 1820, Woodhouse's writings were cited 128 times as sources for solutions [Wright 1825, II, 717-747], and 19 times for help in solving "problems" (more difficult exercises designated to require some originality in solution), [Wright 1825, I, 52, 94-96, 213, 226-227, 229, 478-480, 488, 493; II, 146, 148, 591-593, 606, 609, 613, 617, 623-626, 628]. Since Woodhouse's works figured so prominently in the Tripos, Peacock would surely have studied them as a student; as a Moderator of the Tripos in 1817 and 1819, Peacock even posed 17 questions based upon examples from Woodhouse's textbooks [Moderators 1821, 338-385, 396-402; Wright 1825, II, 717-747]. It is not likely that Peacock could have been so familiar with

Woodhouse's other writings, while completely ignoring *The Principles*, to which he had easy access.

In fact, Babbage and Peacock were affiliated with Woodhouse while he was at the peak of a lifelong career at Cambridge which ended only with his death in December of 1827. Woodhouse was a Fellow of Caius College from 1798 to 1823 and a Moderator of the Tripos in 1799, 1800, 1803, 1804, 1807, and 1808. He became Lucasian Professor of Mathematics in 1820 and Plumian Professor of Astronomy and Experimental Philosophy in 1822, positions which made him one of the Examiners for the Smith's Prizes. Peacock entered Cambridge in 1809, graduated in 1813, became a Fellow of Trinity in 1814, and was a tutor there until 1839. Babbage matriculated in 1810. Although he left Cambridge after graduating in 1814, he maintained a close association with his Cambridge friends, returning as Lucasian Professor in 1828. Peacock was instrumental in reestablishing the University Observatory [Cannon 1978, 36], of which Woodhouse became the first Superintendent. Peacock and Woodhouse were among the founders of the Cambridge Philosophical Society, and Babbage was one of its original Fellows [Hall 1969, 7-8]. And, of course, all three were members of the Royal Society, Woodhouse from 1802, Babbage from 1816, and Peacock from 1818. Thus, both Babbage and Peacock were associates of Woodhouse, initially as students and later as colleagues.

Yet, except for Peacock's belated remarks in 1833, neither he nor Babbage acknowledged Woodhouse as a precursor in algebra. Perhaps this omission can be understood in the context of the times; matters more pressing than the creation of a new algebra faced mathematicians in Cambridge. In 1800, English mathematics was trapped in the doldrums of the fluxional notation and of an intuitive geometric-physical approach to mathematics designed to prepare the student for reading Newton's *Principia* [Woodhouse 1801, 81; Woodhouse 1803, 31, 40; Woodhouse 1809, ii-iii, v, 82; Wainewright 1815, 40-53, 60-67, 80-83; Dealtry 1816, i-iv; Anonymous 1816, 98; D. M. Peacock 1819, 1-13, 69, 85, 94-95, *passim*]. Students became wranglers by solving the traditional 18th-century problems that dominated the Tripos. As wranglers, they became the tutors, textbook authors, and Moderators of the Tripos, passing this heritage on to the next generation of wranglers. The study of any mathematics not pertinent to the traditional questions of the Tripos was not only ignored, but actually discouraged [Babbage 1969, 27]. Cambridge was isolated, and its students remained ignorant of continental developments. They were, as one critic wrote, "stopped at the first page of Euler or d'Alembert ... not from the difference of the fluxionary notation ... but from want of knowing the principles and the methods which they [the continental mathematicians] take for granted as known to every mathematical reader" [Anonymous 1800, 493].

Woodhouse set out to rectify the situation by bringing continental analysis to Cambridge. *The Principles* was not algebra in the modern sense of the word; rather, it was an attempt to introduce the differential notation and calculus to Cambridge. In it, Woodhouse attempted to lead the reader from arithmetic through the binomial theorem to Lagrange's definition of the derivatives of a function as the coefficients of the terms of its expansion in a Taylor series, and, thereby, to establish the calculus upon "common algebra" [Woodhouse 1803, 45, 72, 212]. *The Principles* did not gain acceptance within the Cambridge curriculum. The fluxional notation and geometric-physical foundations continued to be exclusively employed in textbooks [Dealtry 1810/1816] and in the Tripos until 1817 [Dubbey 1963, 45-46]. *The Principles* was cited only once between 1803 and 1820 as an aid for solving problems in the Triposes and not at all for bookwork questions [Wright 1825, I, 52]. At a time when algebra and analysis, as fields of study, were synonomous [Woodhouse 1802b, 87], the five principles of modern "algebra" (see p. 2 of the present paper) were the foundations of an unsuccessful calculus book.

When Babbage and Peacock came to Cambridge, fluxions still prevailed. In 1812, with a number of other undergraduates including John F. W. Herschel and Edward Ffrench Bromhead, they formed the Analytical Society to promote the replacement of fluxions with the calculus [Dubbey 1963, 39-40]. In an age in which there were numerous starting points for the calculus, none of them universally accepted, the Analytical Society followed Woodhouse in (1) believing that the foundations and procedures of the calculus had to be freed from all "foreign" devices borrowed from geometry and mechanics; (2) rejecting the theory of limits as the best basis for the calculus; and (3) turning to Lagrange's derived functions--without explicit reference to Woodhouse's five-point modifications [Woodhouse 1802b, 124-125; Woodhouse 1803, i-xxiv, 1-2, 102, 107-103, 209-218; Babbage and Herschel 1813, i-ii, iv-vi, xxi; Spence 1820, xii-xiv, 295; Peacock 1820, 122; Lacroix 1820, iii-iv, 581-621, 654]. Peacock still echoed Woodhouse when he rejected the "arithmetical" basis of the calculus in 1833, and showed that the Taylor series "may be so exhibited as to comprehend all those cases in which it is said to fail." Furthermore, he argued that "the rejection of diverging series from analysis ... is altogether inconsistent with the spirit and principles of symbolical algebra ..." [Peacock 1834, 246-248]. By 1833, however, Peacock was beginning to develop axiomatic algebra, as distinct from the analysis then being established by continental mathematicians on the basis of a theory of limits. In this context Peacock justifiably viewed Woodhouse's work, written twenty-seven years earlier, as an untested pioneering thrust, not a new algebra. "Such a generalization [as Woodhouse's extension of demonstrated forms]," Peacock wrote, "could not be considered legitimate, without much preparatory theory and without considerable modifications of our views respecting nearly all the fundamental operations and signs of arithmetical algebra ..." [Peacock 1834, 234].

Nonetheless, it appears that the objective Herschel had ascribed to Woodhouse had been achieved. Through his papers to the Royal Society for publication in the *Philosophical Transactions* [Woodhouse 1802a, b], and *The Principles*, Woodhouse fulfilled "the desire to propagate forward to other minds the rising impulse of his own" [Herschel 1857, 32]. Babbage, Bromhead, Herschel, and Peacock all struggled to provide a generalized algebra. In 1816, Bromhead wrote to Babbage: "You talk of some very new views on the foundations of analysis. I am on the same subject and have an idea wholly divesting it of any connection with number or quantity, but making it such that it may be applicable to any thing" [Babbage n.d., 214]. Babbage found that some of Bromhead's proposals were the same as his own [Babbage n.d., 216]. Discussions and work produced, as Babbage noted in 1817, a "mania Analytica" [Babbage n.d., 267; Whewell papers, 0.15.46[17]]; but, although Peacock requested that Herschel write an algebra textbook in 1816 [Cannon 1978, 34], no such publication appeared.

The new algebra was not brought into print. Instead, the *mania analytica* was dissipated in the varied undertakings of the members of the Analytical Society. Some devoted their energies to the calculus of functions and finite differences [Koppelman 1971, 181-188], others to bringing the continental calculus to Cambridge via a translated and annotated French textbook [Lacroix 1816] and a companion book (three volumes in one) of examples [Babbage 1820; Herschel 1820; Peacock 1820]. Although his publications have earned him recognition as one of the founders of modern algebra [Koppelman 1971, 181-187; Laita 1977, 165, 172], Herschel wrote of having lost by 1821 the "keen relish for abstract mathematical studies ... he once felt" [Sutton 1974, 44]. Babbage immersed himself in calculating engines, and neither he nor Bromhead published their algebra. The task of bringing the new algebra together was left to Peacock, busy with his duties as a tutor and Fellow of Trinity College.

In the meantime, Woodhouse's ideas and *The Principles* continued to exert their influence. In 1820, Peacock distinguished "algebraical" and "arithmetical" equality [Peacock 1820, 96]. D. M. Peacock, a fluxionist critical of the calculus textbook translated by the Analytical Society, found the differences between the fluxional and the differential notation "trifling" [Peacock 1819, 3, 61, 68-69]. Nevertheless he attacked the Analytical Society's principles, which certainly reflected those of Woodhouse. D. M. Peacock also attacked George Peacock's rejection of prime and ultimate ratios, and the limit theory upon which they were based. He also opposed the latter's introduction of Lagrange's derived functions as the foundation of the calculus and his contention that the calculus should not be separated from "common algebra" [Peacock 1819, 2-7, 40-42, 54-57]. D. M. Peacock also showed that Lacroix's definition of a single variable function was not adequate for multivariable functions. While he acknowledged that "the advocates of Lacroix will no doubt here answer" that Lacroix simply "extended" his original definition to multivariable functions, this was not, he argued a valid procedure [Peacock 1819, 48].

John Bonnycastle, in a textbook on algebra, cited *The Principles* and endorsed Woodhouse's view that, historically, the law of coefficients for the binomial expansion had to have been "expressed in general terms, without which it could never have been extended to those cases where the index is fractional, or negative" [Bonnycastle 1820, 166]. He also endorsed Woodhouse's argument that the derivation of the binomial theorem must have originated in the "simple rules of multiplication, division, extraction of roots, etc.," rather than in the method of increments, DeMoivre's multinomial theorem, fluxions, or some other "high origin" [Bonnycastle 1820, 168-169]. J. F. M. Wright, a "late scholar of Trinity" (Peacock's college), cited *The Principles*, indicating that he was acquainted with it [Wright 1825, title page, I, 52]. Authors of a derivation (published in 1827) of the binomial theorem, one of whom was Thomas Tylecote (seventh wrangler of 1821 and a Fellow of St. Johns from 1824 to 1838), insisted that the equal sign denoted only arithmetical equality and, therefore, could not be defined as the connecting symbol between a function and its expansion, a restriction which, of course, George Peacock rejected [Peacock 1834, 249]. When a new edition of the most successful Cambridge algebra textbook of the 19th century appeared [Wood 1830], a reviewer criticized the author's proof of the validity of employing the negative sign as follows: "That this should be considered a proof by any Cambridge writer, after the observations of Professor Woodhouse on this subject, surprises us" [Anonymous 1832, 280-281]. The same reviewer accepted Peacock's algebra textbook as a supplement to Wood's within the Cambridge curriculum, although it was criticized for introducing "new characters" and "strange terms" but no "new conclusions" [Wright 1830-1831, II, 51]. Given the acceptance of Peacock's textbook, Cambridge mathematicians must have been prepared beforehand to receive the five principles. Had they not found Peacock's presentation lucid, it would have met the same fate as Woodhouse's *The Principles*.

The Principles had failed to dislodge fluxions from the Cambridge curriculum. When the Analytical Society succeeded in doing so more than a decade later, *The Principles*, along with the fluxionist textbooks, became a relic of Cambridge's discredited past. Nonetheless, the ideas expressed in *The Principles* influenced Bromhead, Babbage, Herschel, and Peacock in their struggle to create a new algebra. It is likely that in the *mania analytica* of the times, as these young mathematicians elaborated upon Woodhouse's ideas and exchanged their own, recognition of Woodhouse's priority, indeed all claims of priority, became moot. That the concepts first articulated by Woodhouse continued to appear regularly in the literature and in the algebra of both Babbage and Peacock suggests that they had become common knowledge at Cambridge. Viewed in this context, Babbage's failure to charge Peacock with plagiarism, although the latter had read the former's work [Dubbey 1977, 302], and Peacock's failure to acknowledge either Babbage or Woodhouse as a precursor are perhaps more easily understood. Linking Woodhouse to Babbage and Peacock is neither a worthless exercise in determining priority nor an attempt to prove that Woodhouse was a great mathematician. On the contrary, one must admit that in the context of the history of mathematics generally, and for the development of continental analysis specifically, Woodhouse must be seen as a failure [Grattan-Guiness 1970, 71]. Even in his own time, *The Principles* was an elementary textbook which was neither lucid nor a satisfactory introduction to higher analysis or current research. It took the reader no further than the elementary procedures of the calculus, doing so by means that can only be described as arbitrary and with definitions contradicting the accepted dicta of the time. Yet these definitions laid down by Woodhouse became the foundations of modern algebra, and for this, in particular, his work is worthy of note.

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