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LECTURES ON
TEN BRITISH PHYSICISTS
OF THE NINETEENTH CENTURY

Babbage

BY

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few not conferred; he is still open, I believe, to receive some distinguished mark of recognition from the geologists.

Lord Kelvin has been twice married, but there is no direct heir to inherit either his genius or title. Notwithstanding the fact that he has long been the acknowledged leader of science in Great Britain, and indeed in Europe, his disposition has remained simple and kindly. A multitude of honors, and great fame and power has not spoiled the grandson of the small Irish farmer. He is still active in the production of scientific papers, and although now nearly 78 years of age is making preparations to again cross that ocean which has been the scene of so many of his exploits, and which is now much more safely navigated through the instrumentality of his inventions.*

* Lord Kelvin died on December 17, 1907, in the 84th year of his age. His activity in scientific discussions did not diminish with age. He revised the lectures on the wave-theory of light which he had delivered at Johns Hopkins University and published them in 1904. In that year also he was elected Chancellor of the University of Glasgow. He continued to take an active part in the work of scientific societies; only a few months before his death he delivered at the meeting of the British Association a long and searching address on the electronic theory of matter. He was buried in Westminster Abbey a few feet south of the grave of Newton.—EDITORS.

CHARLES BABBAGE *

(1791–1871)

CHARLES BABBAGE was born at Totnes in Devonshire on December 26, 1791. His father was a banker and was able to give his son a moderate fortune. Being a sickly child he received a somewhat desultory education at private schools, first at Alphington near Exeter, and later at Endfield near London. It appears that he instructed himself in the elements of Algebra, and that he early manifested a great fondness for it.

When he entered Trinity College, Cambridge, in 1810, he was already acquainted with the text books of Lacroix and other French writers; he had also read the book of Woolhouse which aimed at introducing into Cambridge the Leibnitzian notation for the differential calculus. Among his contemporary graduates he found congenial spirits in Peacock and Herschel, and the three friends, along with some juniors such as Whewell, were wont to breakfast together each Sunday morning and discuss philosophical subjects. At one of these philosophical breakfasts the "Analytical Society" was formed, the object of which as stated by Babbage was "to advocate the principles of pure *d*-ism in opposition to the *dot*-age of the University. Babbage was skillful in getting up what the politicians call a good cry. It was while he was yet an undergraduate that an idea occurred to him which ruled the whole of his subsequent career. One evening he was sitting in the rooms of the Analytical Society at Cambridge, his head leaning forward on the table in a dreamy mood, with a table of logarithms lying open before him. Another member, coming into the room and seeing him half asleep, called out "Well, Babbage, what are you dreaming

* This Lecture was delivered on April 21, 1903.—EDITORS.

about?" to which he replied "I am thinking that all these mathematical tables might be calculated by machinery."

In the last year of his undergraduate career, he migrated from Trinity College to Peterhouse, and did not compete for honors, believing Herschel sure of the first place, and not caring to come out second. He took merely a pass degree in 1815, and thereafter resided in London, where philosophical breakfasts continued to be a feature of his house. In the year following the text-book of Lacroix *Differential and Integral Calculus*, translated by Herschel, Peacock, and Babbage, was published by the Analytical Society; and four years later a volume of *Examples on the Calculus*. Lacroix had also written on the calculus of Finite Differences, and both Herschel and Babbage were attracted to the subject. The latter immediately contributed three papers on "The Calculus of Functions" to the Royal Society and he was elected a Fellow at the age of twenty-five.

He married, and made a tour of the Continent. He visited Paris and studied the details of the arrangement by which the celebrated French tables had been computed under the direction of Prony; and he copied the logarithms to fourteen places of figures of every 500th number from 10,000 to 100,000 from the manuscript tables deposited in the observatory at Paris. These tables were computed at the time of the Revolution, in order to facilitate the application of the decimal division of the degree which had been adopted. In executing the task Prony received a valuable hint from Smith's *Wealth of Nations* where the "division of labor" is exemplified. He adopted the idea; appointed three classes of mathematical workers; first, five or six analyists to investigate the best formulæ; second, seven or eight mathematicians to calculate arithmetical values at suitable intervals; and third, sixty or eighty arithmeticians (said to have been tailors on a strike) to compute intermediate values by the method of differences. The tables thus computed fill seventeen large folio volumes.

On his return to London he was encouraged by Wollaston (a pioneer in electrical science) to set about the realization

of his idea of a difference machine for computing tables. What is the fundamental idea of the method of differences? Write down the square numbers in the first column, the differences

Squares	First Differences	Second Differences	Third Differences
1	3	2	0
4	5	2	0
9	7	2	0
16	9	2	0
25	11	2	
36	13	2	
49		2	

between the successive squares in the second column, and the differences of the first difference in the third column; these last are constant, consequently the next differences are all zero. To compute a table of squares, then, it is only necessary to add to a square the preceding first and second differences, thus $49+13+2=64$, etc. In the case of logarithms and other transcendental functions there is no difference which becomes zero, but when a certain number of figures only are required, there is a difference which is zero within a certain range. Hence within that range the same process of calculation may be applied as for a function which has a certain order of differences constant. To calculate tables by a machine only a device for adding is required; to insure accuracy in the printed tables Babbage thought it necessary that the machine which computes the results should also print them.

By 1822 Babbage had constructed a small model having two orders of differences and applicable to computing numbers of from six to eight places. It could compute squares, triangular numbers, values of x^2+x+41, and values of any function of which the second difference was constant and not greater than about 1000. He exhibited this model to the Royal Astronomical Society and was subsequently awarded a gold medal on account of it. He also wrote a public letter to Sir Humphrey Davy, then president of the Royal Society, explaining the utility of his invention. Through what had been published the

Government was induced to apply to the Royal Society for an opinion on the merits and utility of the invention; it appointed a committee which reported favorably. The Government advanced £1500 and work was started in 1823. Babbage superintended the work, and he employed a mechanical engineer, named Clement, whose workshop was in Lambeth, to execute his plans. The construction of a Difference Engine was begun having six orders of differences, each consisting of about twenty places of figures, and provided with mechanism to print the results. It was called an Engine, because after being started with the proper differences for computing a table the results would be produced merely by power applied to a shaft.

Three years later (1826) the Lucasian professorship of mathematics at Cambridge became vacant. There were three candidates; French, who was the head of one of the colleges; Airy, afterwards astronomer royal; and Babbage. The appointment is made by the heads of the colleges, and in this case they were quite prepared to appoint a candidate from their own number who was more proficient in divinity and Hebrew than in mathematics. This was Newton's chair, but since his time mathematics had declined at Cambridge and was only now beginning to revive. Babbage threatened legal proceedings, with the result French retired and Airy was elected. Airy resided and lectured, the first Lucasian professor who had done so for many years; two years later he changed to the professorship of Astronomy, and his former rival Babbage was elected. This was in 1828; although Babbage held this professorship until 1839 he did not reside or lecture; his mind was completely absorbed with anxiety about the success and fame of his computing machine. However, with a view of delivering a course of lectures, he collected the material and published a book called *Economy of Machinery and Manufacture* which he dedicated to the University of Cambridge. The object of the volume is to point out the effects and the advantages which arise from the use of machines; to endeavor to classify their modes of action and to trace the consequences of applying machinery to supersede the skill and power of the human arm. Babbage wrote

many books, but this is considered his most finished production; it has been described as a " hymn in honor of machinery."

The work on the Difference Engine went on for five years with little interruption, and the expenses had amounted to nearly £7000, of which the Government had advanced less than half, the remainder having come out of Babbage's pocket. Before proceeding further he wished to have a complete understanding with the Government, which was eventually reached after a delay of two years. The Government repaid Babbage what he had advanced, arranged to pay certified bills, leased a part of the grounds belonging to Babbage's house, and erected thereon a fireproof office and workshops. While these were in course of erection, the work continued for three years longer in Clement's workshop. At the end of this time (1833) a portion of the Difference Engine was assembled, and found to fulfill all Babbage's expectations and even more.

The Royal Society, like the University of Cambridge, had also declined as a scientific center since the days of Newton. The president had often been one of high rank rather than eminent in science. At this time the reforming party put up Sir John Herschel as a candidate for the presidency in opposition to the Duke of Sussex, but the royal candidate was successful. Babbage was one of the leading reformers; he prepared and printed a book called *The Decline of Science in England* which proved highly beneficial in that it led in a short time to the foundation of the British Association for the Advancement of Science.

After the drawings and parts of the computing machine were removed to the fireproof premises adjoining Babbage's house, the engineer Clement made a claim for compensation for the removal of his business from Lambeth, a claim which Babbage declined to entertain as being extravagant. Whereupon Clement stopped the work on the machine, disbanded the specially trained workmen, and carried off all the tools, including those specially designed by Babbage and paid for by the Government. This he could do according to English law; he offered to sell the special tools to Babbage but the latter declined purchasing. Notwithstanding this bad break, the Government were willing to proceed; and the construction was actually in an advanced state. Among the workmen discharged by Clement was Joseph (afterwards Sir Joseph) Whitworth who later amassed a fortune by utilizing as a mechanical engineer the training which he got from Babbage.

While the work was suspended owing to change of workshop, Babbage experimented much with the portion of the engine which had been assembled; and his inventive mind conceived the idea of a much more general machine which he called an Analytical Engine. He immediately set to work to plan how it could be realized, and he considered that he had hit upon a much simpler mechanical invention for adding than the one adopted in the Difference Engine. Unfortunately instead of proceeding to complete the Difference Engine as the plans adopted and followed for ten years, as the Government desired him to do, he waited for an opportunity to explain about his new invention. However superior his new ideas might be, he ought to have perceived that the heads of the Government—and that Government frequently changing—were not capable of appreciating their value, and that they would judge of the matter from the business point of view, saying "You wish us to abandon the construction of an engine which has cost £17,000 and wish us to undertake a new and more elaborate engine; we cannot justify such expenditure to the House of Commons."

It was very unfortunate that Babbage did not see the practical necessity of completing the first engine on the plans adopted. By the course he adopted he gave his scientific enemies a chance to defeat the realization of his great invention. The matter was not finally settled until 1842—nine years after the construction was suspended. He was then informed by the Premier and the Chancellor of the Exchequer that they abandoned the undertaking, and that he might have what had been constructed for his own property. Babbage declined to accept it; the portion assembled was placed in a museum; the loose parts sold or melted down. Babbage appears to have thought that

the Ministers acted on their own judgment, but it was not so; Airy, the astronomer at Greenwich, records in his *Autobiography* that he was consulted and that he pronounced the Difference Engine to be worthless. Naturally the ministry attached great weight to this opinion, for the immediate value of this engine was claimed to be the construction of astronomical and nautical tables.

The portion of the Difference Engine which was put together has been exhibited at various Expositions in London, and is now in the Science and Art Museum at South Kensington; I saw it, and heard it explained, on the occasion of the Loan Exhibition of Scientific Apparatus in 1876. It consists of three columns; each column contained six cages, each cage one figure wheel. Each figure wheel has the numbers 0 to 9 placed around the circumference and may be set by hand at any one of the numbers. The right-hand column is for the resulting number, the middle column for the first difference, and the left-hand column for the second difference. Suppose any sets of proper numbers to be placed upon the three columns, then the mechanism is such that four half turns of the handle—two backwards and two forwards—causes the first difference to be added to the previous result and the second difference to be added to the first difference; hence if the machine printed the results, mere turning of the handle would produce the entire table of numbers or all the results requiring to be interpolated between two given values. To make the portion assembled more useful, slight departures from the general plan were adopted. The three upper wheels of the left-hand column were separated from the rest of the machine and employed to count the natural numbers, that is, to register the number of calculations made and give the numbers corresponding with the terms of the table computed. A wheel at the top of the central column indicated when each calculation is complete and also the position of the handle when the figure wheel was to be adjusted.

About this time (1829) the Earl of Bridgewater died, leaving a sum of £10,000 to trustees to be expended in the production of books "on the Power, Wisdom, and Goodness of God as manifested in the Creation," the writers to be selected by the President of the Royal Society. He, acting with certain bishops, selected eight authors, assigned to each a portion of the subject with an honorarium of £1250. Babbage was not one of the number, but in 1837, after the eight treatises had appeared, he published a volume entitled *The Ninth Bridgewater Treatise.* In design the book is grand and much superior to the regular treatises, but in execution it is like many others of Babbage's works, a magnificent torso. He was moved to write the book by a chapter in Whewell's Bridgewater volume where it is maintained that long application to mathematical and physical reasoning disqualifies the mind from duly appreciating the force of that kind of reasoning which alone can be adduced in favor of Natural Theology. Babbage thought that such reasoning tended to promote the prejudice that the pursuits of exact science are unfavorable to religion; he shows on the contrary that his pursuits had led him to new views of the truths of Natural Theology.

The most remarkable part of Babbage's book is where he takes up Hume's conception of a law of nature, and the consequences as to miracles which he deduced from it. According to Hume cause and effect are nothing more than invariable sequence; and a law of nature rests upon experience or repeated observation just as the reliability of a witness does. Babbage points to his Difference Engine (that is, the part completed) and remarks that it may be adjusted to produce the natural numbers. He asks a supposed observer how often a natural number must be produced to infer that this is the whole law of the machine; one hundred times? one thousand times? one million times? Babbage answers that according to the constitution and given adjustment of the machine it will produce the natural number up to 1,000,001; but after that it will give the triangular numbers and that after 2761 turns a further complexity will be introduced. These additional complexities are necessary consequences of the nature and given adjustment of the machine; and no amount of mere induction from given instances could detect the inner necessary connection. Hence casual connection and repeated sequence are not the same thing. He went on to prove by his Analytical Engine (existing only in drawings) that "It is more probable that any law, at the knowledge of which we have arrived by observation, shall be subject to one of those violations, which, according to Hume's definition, constitutes a miracle, than that it should not be so subjected." He rests this proposition on the statement that his Analytical Engine could be set to compute the successive terms of a given algebraic law, but so that one chosen term would be different, and then to resume the production of the true terms ever after. Provision could be made by the maker of the machine for a single suspension of the law at a given point.

Babbage devoted 37 years of his life to perfecting the invention of the Analytical Engine and no inconsiderable part of his fortune was spent thereon. This invention must be carefully distinguished from the Difference Engine; they are often popularly confounded but are confused in some scientific writings. When the fragment of the Difference Engine was put together in 1833, Babbage found that, as he had anticipated, it possessed powers beyond those for which it was intended, something in the same way as algebra displays powers beyond those of generalized arithmetic for which it was designed. Babbage saw that, by interposing a few connecting wheels, the column of Result could be made to influence the last Difference, and he proposed to arrange the axes circularly so that these columns should be near each other. He called this arrangement "the engine eating its own tail." This soon led to the idea of controlling the engine by entirely independent means, and to the idea of an engine which could calculate the numerical values of any function which the mathematician can express in a series of integral powers.

To realize the first idea—that is, to make the adjustment of the engine automatic—he had recourse to the device of punched cards similar to those invented by Jacquard for the weaving loom. The machine was to consist of three parts; first, the store; second, the mill; third, the cards. The store was to consist of 100 columns each of fifty wheels for indicating the given numbers, intermediate numbers, and resulting number. The mill was to consist of mechanism which would add two numbers, subtract a less number from a greater, multiply two numbers, or divide one number by another, according to the kind of gearing brought into operation. The cards were of three kinds; Number cards to communicate given numbers to the store; Directive cards to transfer numbers from the store to the mill and from the mill to the store; Operation cards to call for addition, subtraction, multiplication, division. For example, to compute numerical values of $(ab+c)d$, seventeen cards in all were required, as follows:

Number Card	Directive Card	Operation Card	
1			Places a on column 1 of store.
2			Places b on column 2 of store.
3			Places c on column 3 of store.
4			Places d on column 4 of store.
	1		Brings a from store to mill.
	2		Brings b from store to mill.
		1	Multiplies a and $b=p$.
	3		Takes p to column 5 of store.
	4		Brings p into mill.
	5		Brings c into mill.
		2	Adds p and $c=q$.
	6		Takes q to column 6 of store.
	7		Brings d into mill.
	8		Brings q into mill.
		3	Multiplies d and $q=r$.
	9		Takes r to column 7 of store.
	10		Takes r to printing apparatus.

Each form of calculation would require a special set of cards strung together in proper order; just as the particular pattern for a woven fabric requires its own set of Jacquard cards, and they would be applied to the calculating machine in the same manner. The great improvement in the construction of the engine proper was the invention of the principle of the Chain, by which the carriage of the tens is anticipated. This part of the design was actually constructed. For subtraction the adding rotations were reversed; multiplication was to be effected by successive additions, and division by successive subtractions. It is obvious that the machine could treat of

transcendental functions only when expressed in a series of powers. Irrational quantities would be represented approximately.

To express the complicated relations among the various parts of the machine, Babbage invented what he called a "mechanical notation" explained in a paper published in the *Philosophical Transactions* for 1826, entitled "On a method of expressing by signs the action of machinery." It consists of three divisions; first, Notation for the parts; second, Representation of trains; third, Representation of cycles. He denoted pieces and points of the frame by upright letters, the former capitals and the latter small letters; movable pieces and their points by slant letters, capitals and small letters respectively. On account of the great number of movable pieces he employed indices, placing them to the left above the letters. The train is designed to show how motion is transmitted from the prime motor to the final driven piece. The several pieces are marked on a diagram by trial so that each pair of driver (point) and driven (piece) may be connected by arrows; after a number of trials the pieces are so placed as to make the connecting arrows the shortest. In a cycle he aimed at representing the time during which each piece moved and the time of action of each of its working points. The period of the machine is represented by a vertical line divided into proper subdivisions on the nature of the machine; to each piece and to each working point is allotted a parallel line, and those portions of the period are marked off during which there is no movement of the piece or the point, thus giving a synoptic view of the motion of the machine. To make drawings, perfect the notations, and test mechanical contrivances, he turned his coach house into a forge and foundry, transformed his stables into a workshop, and expended a large sum in employing skilled workmen.

In 1840 he received a letter from M. Plana, nephew of Lagrange, urging him to come to a meeting of Italian philosophers which was to be held in Turin. Babbage went, furnished with models, drawings, and notations of his Analytical Engine, and explained them to the Italian mathematicians, among whom was M. Menabréa. Subsequently Menabréa wrote an account of the invention in French, which was afterwards translated into English and embellished with notes by Lady Lovelace, née Augusta Ada Byron, daughter of the poet Byron. This lady did not inherit the poetic genius of her father, but was remarkable for exact mathematical attainments, which were also possessed by her mother.

Babbage himself never wrote an extended account of the Analytical Engine; the memoir of Menabréa with the notes of Lady Lovelace gives the most complete account regarding it. In 1848 he made drawings for a new Difference Engine in which the adding was to be effected by his new contrivance. He was anxious to discharge whatever imagined obligation might be supposed to rest upon him in connection with the original undertaking, and an entirely practicable proposal was laid before the Premier (Lord Derby) by Lord Rosse, a mathematical nobleman. The Premier turned the matter over to his Chancellor of the Exchequer, Benjamin Disraeli, who gave an adverse decision. The wrath of Babbage at the novelist was unbounded; he denounced him as the Herostratus of Science. A few years later a Difference Engine, suggested by Babbage's plans, was actually constructed in Sweden by a printer named Scheutz; it performed successfully the kind of work for which it was designed. The original Scheutz machine was bought by the Dudley Observatory at Albany, N. Y., and was used to a slight extent about 1878; a copy of it constructed for the English Government has been used for the calculation of insurance tables.

After the death of Babbage in 1871 what he had accomplished on the Analytical Engine was transferred for safe-keeping to the Museum at South Kensington. The British Association appointed a committee to examine it; in 1878 they reported that the part assembled was only a small portion of the mill sufficient to show the methods of addition and subtraction; that the drawings were complete in exhibiting every movement essential to the design of the machine. They concluded that the labors of Babbage, first on the Difference Engine, and afterwards on the Analytical Engine were a marvel of mechanical

ingenuity; that the realization of the latter would be of utility; that the complete design is not more than a theoretic possibility; and that the mill portion of it might be constructed at reasonable expense.

Babbage was distinguished for his skill in solving ciphers. He wrote a paper "On the properties of letters occurring in various languages" and it appears that these researches gave the keys which he used. In 1851 he communicated to the Trinity House a note respecting occulting lights in lighthouses. His idea of making each lighthouse publish its own name was forthwith adopted by the English and American Governments. The application of the same idea to solar light led to the invention of the heliograph, first brought into practice by the Russians at Sebastopool and which figured so prominently in the siege of Ladysmith.

Babbage's last book, published in 1864, was a kind of autobiography entitled *Passages from the Life of a Philosopher*. Like many of his works it was brilliant in conception but incomplete in execution. In his later years he came before the public as the implacable foe of organ grinders. He estimated that one-fourth of his entire working hours had been wasted through audible nuisances to which his highly strung nerves rendered him peculiarly sensitive.

Charles Babbage died on October 18, 1871 in the 80th year of his age. To the public he was known as an eccentric and irritable person, as a crank on the subject of calculating machines. But his books show true nobility of nature; his engines exhibit marvelous mechanical ingenuity. He sowed many valuable seeds which less able but more thrifty minds turned to advantage. As a reformer he accomplished much for exact science, especially in the foundation of the Astronomical Society, the British Association, and the Statistical Society. The money expended by the Government on his machine was fully repaid, according to Lord Rosse, by the improvement in mechanical tools which he made incidentally in his designs. The main defect in his character was a want of persistence and an imperfect adjustment of his aims to what was practicable.