MATHEMATICS AND PHILOSOPHY: WALLIS, HOBBES, BARROW, AND BERKELEY*

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While there has been considerable interest in similarities between George Berkeley's mathematical philosophy and twentieth-century counterparts, few scholars have looked back into the seventeenth and early eighteenth centuries for the roots of Berkeley's mathematical views.[1] This want of perspective permits a somewhat inflated impression of Berkeley's novelty, and inhibits scholars from fully appreciating vital connections between his mathematical and general philosophy. The present paper fills this gap by analyzing Berkeley's mathematical views against the background of the English mathematical world of the seventeenth and early eighteenth centuries. It shows how Berkeley (1685-1753) struggled with the pressing problems of his mathematical contemporaries: the ranks of geometry and arithmetic within mathematics, the status of the "impossible numbers" (including the negatives and imaginaries), and the legitimacy of reasoning on symbols. He shared interest in these questions especially with John Wallis (1616-1703), Thomas Hobbes (1588-1679), and Isaac Barrow (1630-77), and, as demonstrated below, interwove their insights with his distinctive philosophical tenets to produce his mathematical philosophy.

While setting Berkeley's mathematical views in this larger context, the present paper highlights the close interrelationship between the mathematics and philosophy of his period. The debate of Wallis and Hobbes on the merits of algebra, for example, was at the same time a discussion of the larger question of one's ability to reason on signs without consideration of their meaning; similarly, the problem of the "imaginary" numbers raised the question of reasoning on signs for which there were absolutely no corresponding ideas. Thus the paper suggests that the English mathematical ferment of the seventeenth and early eighteenth centuries provided Berkeley not only with elements of his mathematical philosophy but also with problems and insights which dominated his general philosophy.

In addition, the paper sheds new light on Berkeley's mathematical

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[1] The major exception is G. A. Johnston, whose suggestive account of the mathematical background of Berkeley's work was limited by dependence on secondary histories of mathematics. See Johnston, *The Development of Berkeley's Philosophy* (New York, 1923), especially 75-93, 209-23, and 261-81, and "The Influence of Mathematical Conceptions on Berkeley's Philosophy," *Mind*, 25 (1916), 177-92.

philosophy. It argues that Berkeley recognized a tripartite division of mathematics into (1) geometry (the highest mathematical science which was based on sense perceptions), (2) arithmetic and algebra (formal sciences involving reasoning on mere signs), and (3) analysis (a method applied to geometry). This tripartite approach to mathematics demands careful delineation as a corrective to the mutually inconsistent monolithic interpretations which have labeled Berkeley variously as a materialist, conventionalist, formalist, and realist philosopher of mathematics.[2] The paper also refines the work of those earlier scholars who, realizing the invalidity of a strictly monolithic interpretation, claimed that the young Berkeley distinguished between perceptible geometry and the formal sciences of arithmetic and algebra while the mature philosopher embraced a formalist view of all mathematics.[3] Rather, as shown below, the tripartite approach remained the cornerstone of Berkeley's mathematical philosophy through the publication of his last major mathematical work, *The Analyst*, and, in fact, the approach lay behind his inability to come to terms with the calculus.

I. Berkeley entered the mathematical arena shortly after the close of the seventeenth century during which geometry had been pitted against arithmetic and algebra in the mathematical facet of the battle of the ancients against the moderns. According to some of the century's mathematical conservatives, geometry stood above arithmetic and algebra as the mathematics of the ancients and hence the sounder science; arithmetic enjoyed less prestige partially because of its association with algebra and thus with the moderns. Other supporters of the ancient mathematics, moreover, fell under the seventeenth-century empiricist spell and dismissed arithmetic primarily because of its want of a definite physical basis.

Pursued as an extension of arithmetic in early modern Europe, the new algebra differed significantly from both arithmetic and geometry. It involved reasoning on symbols (which were taken as signs of numbers). Furthermore, because of the blossoming of the theory of equations in the seventeenth century, the new algebra admitted not merely the numbers of traditional arithmetic—whole numbers and their fractions—but negative and imaginary numbers as well. Added to these differences concerning objects reasoned upon were differences between algebra and geometry concerning methods used. Geometry was primarily deductive

[2] For a sketch of the main interpretations, see Robert J. Baum, "The Instrumentalist and Formalist Elements of Berkeley's Philosophy of Mathematics," *Studies in History and Philosophy of Science*, 3 (1972), 119-20, and for the materialist interpretation, Bruce Silver, "Berkeley and the Mathematics of Materialism," *New Scholasticism*, 46 (1972), 427-38.

[3] See, e.g., J. O. Urmson, *Berkeley* (Oxford, 1985), 67-68, and G. J. Warnock, *Berkeley* (Baltimore, 1969), 205-11.

or synthetic, while seventeenth-century algebra was primarily analytic.[4] Following the synthetic method, the geometer derived logical conclusions from axioms and postulates. But the algebraist, as in the solving of equations, often proceeded analytically: he began with an assumed relationship between known and unknown quantities, determined the unknown, and, only at the end of his calculation, demonstrated that the newly determined value(s) satisfied the original relationship.

Although the methodological differences were mentioned, the seventeenth-century debate over the relative merits of geometry versus arithmetic and algebra centered on the questions of the ability of the human mind to reason on symbols, the legitimacy of the negative and imaginary numbers, and finally the ultimate source of the numbers of arithmetic. The symbolism of algebra was touted above all as a means of abbreviating mathematical processes. For example, while earlier mathematicians had described in prose the process of multiplying a number by itself a certain number of times, the new algebraists moved towards simply recording the number raised to the appropriate exponent.[5] Thus the number a multiplied by itself six times became, in the new symbolism, a^6. Led by Descartes, seventeenth-century mathematicians also began to transform geometrical problems into algebraic ones. In this new analytic geometry, for example, (a, b) could stand for a point in a plane whose abscissa was a and ordinate, b, and the equation $x^2 + y^2 = 9$ could stand for a circle with a radius of 3.

But, it was asked, did the use of symbols really shorten mathematics? Was the human mind able to reason on symbols, or, when faced with symbols, did the mathematician need to translate them back into appropriate prose expressions or into the ideas for which they stood? Could, for example, a mathematician correctly manipulate a^6 without recalling that he was multiplying the number a by itself six times? Similarly, could the mathematician draw accurate conclusions about $x^2 + y^2 = 9$ without recalling that the curve under consideration was a circle with all the usual geometrical properties of that figure? If the latter was impossible, as Hobbes believed, then algebraic symbolism actually wasted the mathematician's time by forcing him to translate ideas or prose descriptions into symbols and then back again, and so on.

The problem of symbolic reasoning was complicated by algebra's appeal to "impossible numbers," which seemed to defy suitable definition and mental conception. These numbers included the negatives, defined at this time as "quantities less than nothing," and the imaginaries, num-

[4] For the traditional distinction between the two methods, see *The English Works of Thomas Hobbes of Malmesbury*, ed. Sir William Molesworth (11 vols.; London, 1839-45), I: 310 (hereafter abbreviated as E.W.).

[5] J. F. Scott, *The Mathematical Work of John Wallis, D.D., F.R.S. (1616-1703)* (London, 1938), 137-38.

bers which, when squared, produce negative numbers. The best English mathematicians of the seventeenth through early nineteenth centuries struggled to define or otherwise legitimate the negative and imaginary numbers.[6] Preoccupation with these numbers was precipitated in the seventeenth century by their increased use in algebra and by the century's emphasis on clarity of conception. The negative and imaginary numbers were forced upon mathematicians by advances in the theory of equations. The seventeenth century witnessed not only new solutions of polynomial equations but also formulation of the fundamental theorem of algebra. The fundamental theorem states that: a polynomial equation (with complex coefficients) of *n*th degree has *n* (complex) roots. For some equations, of course, the *n* roots include negative or imaginary numbers. Thus the simple equation $x^2 + 1 = 0$ has two roots ($\sqrt{-1}$ and $-\sqrt{-1}$), both of which are imaginary. In short, acceptance of negative and imaginary roots (and hence numbers) proved a prerequisite for further development of a theory of equations. Ironically, such acceptance was urged upon mathematicians in the century when "clarity of conception, which Descartes emphasized, and upon which Hobbes insisted as much as did the members of the Royal Society, was one of the key values."[7] The negative and imaginary numbers thus set up a dilemma: the choice between mathematical usefulness and philosophical clarity. Mathematicians themselves saw the dilemma. They described these numbers variously as "impossible," "imaginary," and "unintelligible;" a few even called for abandonment of the negatives and imaginaries as "a parcel of algebraick quantities, of which our understandings cannot form any idea."[8]

While the problems of symbolic reasoning and impossible numbers were specifically algebraic problems, arithmetic also came under direct attack. Major seventeenth-century English thinkers were committed not only to clear thinking but also to empiricism. Francis Bacon, Thomas Hobbes, Isaac Barrow, and John Locke, for example, all rejected Platonic ideas and stressed experience as a source of human knowledge. This empiricist strain affected the English understanding of and outlook on mathematics. Geometry fit easily into the various philosophical schemes constructed around the empiricist tenet, since geometry, it was argued, dealt ultimately with physical objects. An empirical explanation of arithmetic, however, proved more difficult. While an empiricist mathematician could point to a line drawn on paper as an object of geometry, the

[6] See Ernest Nagel, " 'Impossible Numbers': A Chapter in the History of Modern Logic," *Studies in the History of Ideas*, 3 (1935), 429-74.

[7] Richard Foster Jones, *Ancients and Moderns: A Study of the Rise of the Scientific Movement in Seventeenth-Century England* (2nd ed.; St. Louis, 1961), 169.

[8] Francis Maseres, "A Method of Extending Cardan's Rule for Resolving One Case of a Cubick Equation . . . ," *Philosophical Transactions of the Royal Society of London*, 68 (1778), 947.

physical universe failed to offer any particular objects of which he could say: "there are the numbers one, two, three, ..." As shown below, Hobbes (to some extent), Barrow, and Berkeley were troubled by this want of definite physical referents for the whole numbers. Consequently Hobbes and Barrow subordinated arithmetic to geometry. Eventually, however, following hints from Hobbes and Barrow that numbers were "names," "marks," or "signs," Berkeley salvaged arithmetic as a formal science—one which, however, he persisted in ranking just below the sense-based geometry.

II. The mathematical phase of the battle of the ancients against the moderns, from which some of the key elements of Berkeley's mathematical philosophy seem to have emerged, began with the extensive mathematical writings of John Wallis. A strong supporter of the mathematics of the moderns, Wallis expressed no qualms about symbolic reasoning, accepted the negative and imaginary numbers, introduced new algebraic symbols, boldly generalized existing symbols and concepts, applied algebra to geometry, and even suggested the subordination of geometry to arithmetic.

Wallis's major algebraic publication was his *Treatise of Algebra*, published in 1685 as its author neared his seventieth birthday. This work offered Wallis's mature description and endorsement of algebra as "specious arithmetick," which by assigning

> Notes or *Symbols* (which he [Vieta] calls *Species*) to Quantities both known and unknown, doth (without altering the manner of demonstration, as to the substance,) furnish us with a short and convenient way of Notation; whereby the whole process of many Operations is at once exposed to the Eye in a short Synopsis.[9]

Thus Wallis focused on the use of symbols and the mathematical economy achieved thereby as the distinguishing characteristics of the new algebra, and expressed complete satisfaction that algebraic reasoning preserved the "substance" of mathematical demonstration.

Even he, however, admitted that algebra's appeal to the negative and imaginary numbers was troublesome. Still, in chapter 66 of the *Treatise*, Wallis urged acceptance of these numbers based on their mathematical usefulness and physical applicability. He first admitted that the imaginaries were "Impossible . . . as to the first and strict notion of what is proposed. For it is not possible, that any Number (Negative or Affirmative) Multiplied into itself, can produce (for instance) −4. . . ." Then he quickly drew a parallel between the impossibility of the imaginaries and that of the negatives. "But it is also Impossible," he declared, "that

[9] John Wallis, *A Treatise of Algebra, both Historical and Practical* (London, 1685), preface, n. p.

any Quantity (though not a Supposed Square) can be *Negative*. Since that it is not possible that any *Magnitude* can be *Less than Nothing*, or any *Number Fewer than None*." Thus, he implied, if mathematicians were bound by "strict notions," which included traditional, meaningful definitions, then they ought necessarily to reject the negatives along with the imaginaries. Such strict interpretation, however, was not for Wallis; the "supposition" of negative numbers, he continued, was neither "un-useful" nor "absurd." The usefulness of the negatives, especially for the theory of equations, was beyond question. But Wallis felt obliged to clarify his denial of their absurdity. He explained that a negative number was susceptible of "Physical Application . . . [and] denotes as Real a Quantity as if the Sign were $+$; but to be interpreted in a contrary sense." An example reinforced his point that the negatives were mathematically acceptable because, even if they defied definition, they could be interpreted or applied in the physical world. If positive numbers are used to represent land gained from the sea, he noted, then negatives represent that lost to the sea. In addition, Wallis offered a geometrical interpretation for the imaginary numbers: he held that the "true notion" of the imaginary root $\sqrt{-\mathrm{bc}}$ was a mean proportional between $+\mathrm{b}$ and $-\mathrm{c}$ or $-\mathrm{b}$ and $+\mathrm{c}$. In short, Wallis emphasized utility and physical applicability over "strict notions" or traditional definitions.[10]

Wallis not only defended the new algebra and its problematic numbers but also argued that algebra was applicable to geometry and perhaps mathematically prior to it as well. Following Descartes, he used algebra extensively in the study of geometry. In *Conic Sections: New Method Exposed*, for example, he defined the parabola, ellipse, and hyperbola through equations. As he boasted, the treatise demonstrated that these conic sections could be treated strictly algebraically, that is, without traditional development as slices of a cone.[11] In *Mathesis Universalis*, Wallis implied that geometry was subordinate to arithmetic. He noted that continuous magnitude, the special subject of geometry, was measured by number, and thus hinted that number was antecedent to, and in some respects governed, continuous magnitude. The key passage, often quoted by Wallis's critics, was: "Simply because a line of two feet added to a line of two feet makes a line of four feet, it does not follow that two and two make four; on the contrary, the former follows from the latter."[12]

III. While Wallis represented the vanguard of modern mathematics

[10] *Ibid.*, 264-66.

[11] Scott, 21-22.

[12] John Wallis, *Mathesis Universalis* (Oxford, 1657), 69. The translation is taken from Florian Cajori, "Controversies on Mathematics between Wallis, Hobbes, and Barrow," *Mathematics Teacher*, 22 (1929), 147.

in seventeenth-century England, Thomas Hobbes led the defense of the mathematics of the ancients. He favored geometry over arithmetic and synthesis over analysis. He also condemned as a waste of time the new algebra's heavy reliance on symbols and symbolic reasoning. Yet Hobbes was no naive defender of the old mathematics; his mathematical views were shaped by seventeenth-century respect for experience as a source of knowledge and his nominalist inclinations.

The materialist component of Hobbes's mathematical philosophy has already been summarized by George Croom Robertson. "The prominence he [Hobbes] assigns to Geometry, as if it were equivalent to Mathematics," Robertson observed, "follows from his original position that nothing but extended body exists as the subject of science."[13] Indeed, Hobbes maintained throughout his career that extended body was the subject of geometry,[14] and in his *Six Lessons to the Savilian Professors of Mathematics* of 1656 he declared that geometry was demonstrable—that is, based on knowledge of causes rather than effects—precisely because "the lines and figures from which we reason are drawn and described by ourselves" (E.W. VII, 184).

The preceding statement can be taken literally, for Hobbes's theory of geometry drew as heavily on his rejection of universal abstract ideas as on his emphasis on sense experience. He thought that the geometer reasoned on specific lines and figures, or on their universal names, but never on universal abstract ideas of such. This view of geometry fit in with the nominalist leanings expressed in his *Elements of Philosophy*, but it also put him at odds with the traditional Euclidian approach. In the *Elements*, especially in his discussion of triangles, Hobbes turned to universal names rather than universal abstract ideas to explain man's ability to generalize. Here he offered the case of a mathematician who studied the properties of a given triangle "without any use of words either understood or expressed." Even if the mathematician determined that the sum of the interior angles of the given triangle was equal to two right angles, Hobbes contended, in the absence of the universal name "triangle" he would not know that this particular property prevailed in other triangles. Rather the mathematician "would be forced, as often as a different triangle were brought before him (and the difference of triangles is infinite) to begin his contemplation anew; which he would have no need to do if he had the use of names, for every universal name denotes the conceptions we have of infinite singular things" (E.W. I, 79-80).

Rejection of universal ideas made a geometrical maverick of Hobbes. Hobbes was more conservative than Wallis, for example, in maintaining

[13] George Croom Robertson, *Hobbes* (Edinburgh and London, 1910), 104-5.

[14] Emphasis on extended body as geometry's subject prevailed in the *Six Lessons*, where however Hobbes suggested time as a related geometrical subject (E.W. VII, 194).

the superiority of geometry over arithmetic. But, when the two men debated various mathematical issues, Wallis seems to have taken the traditional Euclidian view of geometry, while Hobbes argued for his nominalist view. Specifically, as explained in the *Six Lessons*, Wallis followed Euclid's definition of a line as "length without breadth," and Hobbes insisted that a line was "a body whose length is considered without its breadth" (E.W. VII, 202). In short, according to Hobbes, geometry dealt not with the abstract idea of line, but rather with sensible lines whose breadth was ignored by mathematicians. Generalizations about such lines depended on the use of the general name "line" to denote all specific lines.

Hobbes's theory of arithmetic also embodied his rejection of universal ideas and stress on sense observations. But this theory was not as well-developed and clear-cut as that of geometry. Hobbes seemed torn between viewing arithmetic as dependent on geometry and, alternatively, developing it as a somewhat independent theory of names. This tension was evident in the *Elements of Philosophy*. In that work's major section on quantity, which contained one of his lengthier discussions of arithmetic, the philosopher avoided the question of the nature of numbers and, working from his materialist conception of science, asked rather how number could be "exposed" or "set before" the senses. "Quantity exposed," he explained, "must be some standing or permanent thing, such as is marked out in consistent or durable matter; or at least something which is revocable to sense" (E.W. I, 40). Number could be exposed in two ways:

> either by the exposition of points, or of the names of number, *one, two, three, &c*; and those points must not be contiguous, so as that they cannot be distinguished by notes, but they must be so placed that they may be *discerned* one from another; for, from this it is, that number is called *discrete quantity*.... But that number may be exposed by the names of number, it is necessary that they be recited by heart and in order, as one, two, three, &c.... (E.W. I, 141)

The first method of exposition implied a dependence of arithmetic on geometry. While Hobbes did not explicitly identify numbers as geometrical points (defined in the *Six Lessons* as "bod[ies] ... whose quantity is not considered" [E.W. VII, 201]), he at least suggested that such points could serve the senses as representatives of numbers. But still, Hobbes admitted, there was a second, nongeometrical way of exposing numbers, depending on a view of numbers as names and requiring only their recitation in the right order.

Moreover, there is evidence in another section of the *Elements* that the problem of the negative numbers pushed Hobbes towards the latter view of numbers as essentially names. While whole numbers were susceptible of representation as points, he could find no physical referents for the number zero or negative numbers. Indeed a theory of numbers as names—which included names standing for no specific extended bodies—seemed essential for explanation of zero and the negatives. "Nor, indeed," Hobbes opined,

> is it at all necessary that every name should be the name of something. . . . [T]his word *nothing* is a name, which yet cannot be the name of any thing: for when, for example, we substract 2 and 3 from 5, and so nothing remaining, we would call that substraction to mind, this speech *nothing remains,* and in it the word *nothing* is not unuseful. And for the same reason we say truly, *less than nothing* remains, when we substract more from less; for the mind feigns such remains as these for doctrine's sake, and desires, as often as is necessary, to call the same to memory. (E.W. I, 17-18)

Thus, denying universal ideas, unable to find physical representations for all numbers, and yet mindful (along with Wallis) of the usefulness of zero and the negative numbers, Hobbes sketched a new approach to arithmetic as a science of names.

While Hobbes's views on geometry and arithmetic thus evidenced considerable novelty, his almost uniform condemnation of algebra earned him a place of honor on the side of the ancients. He viewed algebra differently from geometry and arithmetic. Geometry and arithmetic were, in his opinion, sciences; algebra—which he saw as essentially symbolic reasoning—was an art of invention. Algebra was unsuitable for mathematical demonstration, because demonstration required names or words, not merely symbols.

Hobbes criticized algebra in a section of the *Elements* dealing with geometrical analysis and in his *Six Lessons*. In the *Elements* he expressed a preference for synthesis over analysis, distinguished algebra from analysis, and hinted at reservations concerning the algebraist's dependence on symbols (E.W. I, 314-17). In his *Six Lessons* he issued a stronger attack on algebra. Inspired at least partially by his theory of names, he focused on the uselessness of symbols in mathematical demonstration. The attack began as Hobbes noted that the ancients had used analysis but not symbols. "Had Pappus," he asked, ". . . not proceeded analytically in a hundred problems . . . and never used symbols? Symbols are poor unhandsome, though necessary, scaffolds of demonstration; and ought no more to appear in public, than the most deformed necessary business which you do in your chambers" (E.W. VII, 248). In invention, Hobbes then admitted, symbols provided a shorthand to record the mathematician's progress. As, however, the mathematician tried to communicate his results to others, symbolic formulation prolonged mathematical demonstration rather than shortening it. For communication of mathematics required translation of symbols into the things they represented. Thus, concerning Wallis's algebraic restatements of some of Hobbes's geometrical results, the latter retorted:

you show me how you could demonstrate the sixth and seventh articles a shorter way. But though there be your symbols, yet no man is obliged to take them for demonstration. And though they be granted to be dumb demonstrations, yet when they are taught to speak as they ought to do, they will be longer demonstrations than these of mine (E.W. VII, 281-82).

IV. Isaac Barrow, the first Lucasian Professor of Mathematics at Cambridge, also generally sided with the ancients. Although more productive in mathematics than Hobbes, he was a more conservative critic of arithmetic and algebra. Like Hobbes, Barrow emphasized geometry's connection with the sensible; he also questioned Wallis's subordination of geometry to arithmetic. But unlike Hobbes, Barrow explicitly denied arithmetic an existence independent of geometry and, when discussing the new algebra, confused it with the method of analysis and dismissed it as a part of logic.

The mathematical philosophies of Hobbes and Barrow began with a common rejection of Platonic ideas. As Barrow explained: "there is no need, at least in Speculative Sciences, for supposing any *physical Anticipations, common Notions, or congenite Ideas.*" Rather than embracing Hobbes's theory of universal names, however, Barrow developed a theory of universal ideas which assumed the accuracy of sense observations and saw ideas as products of human reason applied to those observations. Showing his faith in the senses, Barrow asked: "Perhaps I am ignorant of the Manner of perceiving by my Senses, but do I not therefore see what is before mine Eyes?" He soon thereafter declared that "the *Sense* then, when right and perfect, as it is naturally in most Men of a sound Constitution, discerns many Objects *certainly.*" Human reason, Barrow continued, produced universal ideas from accurate sense observations: "The Mind, from the Observation of the Things objected, takes Occasion of framing like Ideas, which, as soon as it clearly perceives to agree with the Things that may exist, it affirms and supposes; then appropriating Words to them forms Definitions. . . ."[15]

Having suggested mind-created ideas in place of the Platonic, Barrow still wrestled with the role of universals in science. He appeared wary of universal ideas, whatever their source, and warned against setting up too great a distance between the sensible and the intelligible. Taking geometry as his example, he attributed its excellence to the accuracy of the perceptions on which it was based, and urged mathematicians to keep particular geometrical objects before their eyes even as they reasoned on geometrical ideas. The mathematical sciences were ones of "excellent

[15] Isaac Barrow, *The Usefulness of Mathematical Learning Explained and Demonstrated: Being Mathematical Lectures Read in the Publick Schools at the University of Cambridge*, tr. John Kirkby (London, 1734), 115, 69, 70, 115. These lectures, read by Barrow between 1664 and 1666, were originally published in Latin in 1683 and reissued in 1684 and 1685.

demonstration," he argued, precisely "because we do clearly conceive, and readily obtain distinct Ideas of the Things which these Sciences contemplate; they being Things the most simple and common, such as lie exposed to Senses." Asking in illustration what straight lines, triangles, squares, circles, and the like were, Barrow responded: "Things which we perceive clearly and distinctly."[16] He contended, moreover, that geometry's edge as a demonstrative science could be perserved only as long as mathematicians permitted sensible objects to inform their reasonings on geometrical ideas. Somewhat blurring the distinction between the particular and the universal, he explained that mathematical

> objects are at the same time both intelligible and sensible in a different respect; intelligible as the Mind apprehends and contemplates their universal . . . Ideas, and sensible as they agree with several particular Subjects occurring to the Sense. . . . Why *ex. gr.* should one Science treat of an intelligible Sphere, and another of a sensible one? when these, as to the Verity of the Thing, are altogether the same, and as to the Action of the Mind subordinate; nor can any thing be attributed to the intelligible Sphere (i.e. one understood universally) which does not perfectly agree with the sensible (i.e. with every particular one). . . .[17]

In short, for Barrow, geometry was the science of universal ideas which were derived by the mind from sensible objects and which, according to the above, were possibly little or no more than particular objects "understood universally."

Arithmetic did not fit as easily into Barrow's philosophy. While Hobbes had wavered between the geometrical exposition of numbers and his theory of names, Barrow took a different tack. Arguing against Wallis's claim of geometry's subordination to arithmetic, he denied numbers any independent existence and presented them instead as mere "notes" or "signs" of magnitude. This treatment of numbers was a curious one—conservative in its emphasis on geometry over arithmetic to the extent of recognizing geometry as the only mathematical science, and yet progressive in its foreshadowing of Berkeley's view of numbers as arbitrary characters.

Barrow specifically attacked Wallis's interpretation of the addition of a pair of two-feet-long lines. He challenged Wallis to consider the addition of a line of two feet to a line of two palms (the palm being a unit of measurement based on the length of the human hand). As Barrow noted, the sum here was not "a line of four Feet, four Palms, or four of any Denomination."[18] Yet, he argued, if (as Wallis held) two feet plus two feet gave four feet simply because $2 + 2 = 4$, then two of any unit of length plus two of any other unit of length ought to give four of some

[16] *Ibid.*, 53-54.

[17] *Ibid.*, 19. Here Barrow referred to Aristotle's *Metaphysics* (II, 3) and *Analytics Posteriori* (I, 24).

[18] *Ibid.*, 37.

unit of length. Every student of arithmetic, however, knows that the units of measurement must be the same for such additions to work. Barrow admitted as much and used the admission to make his general points that numbers did not stand for any particular bodies, that numbers were relative, and that they were denominations assigned to quantities (such as lines) according to human "Pleasure." Thus he

> remark[ed] that no Number of itself signifies any thing distinctly, or agrees to any determinate Subject, or certainly denominates anything. For every Number may with equal Right denominate any Quantity: and in like manner any Number may be attributed to every Quantity. For instance, Any Line A may be indifferently called *One*, *Two*, *Three*, *Four*, or any other Number . . . as it remains undivided, may be cut into, or compounded of, two, three, four, or any other Number of Parts. . . . Whence it appears that no Number can design any thing certain and absolute, but may be applied to any Quantity at Pleasure.[19]

Specifically, a line of two feet, referred to in Wallis's example, may be called one, if viewed as undivided; two, if divided into feet; twenty-four, according to inches; and so on.

Denied a Platonic ideal basis and absolute sensible referents, then, numbers became in Barrow's philosophy mere "notes," "signs," or "symbols" expressing the various ways in which the human mind viewed magnitude as either divided or undivided. "I say that a *Mathematical Number*," Barrow summarized,

> . . . is only a kind of *Note* or *Sign* of Magnitude considered after a certain Manner; *viz.* as we conceive it either as altogether incomplex, or as compounded of certain homogeneous equal Parts. . . . For in order to expound and declare our Conception of a Magnitude, we design it by the *Name* or *Character* of a certain Number, which consequently is nothing else but the *Note* or *Symbol* of such Magnitude so taken.[20]

If arithmetic emerged subservient to geometry in Barrow's scheme of mathematics, algebra fared worse. Barrow generally ignored algebra and, in his few references to the subject, refused to recognize it as a science and presented it instead as an instrument of logic. In his discussion of irrational numbers, for example, Barrow characterized algebra as "yet . . . no Science."[21] Elsewhere he confused algebra with the method of analysis. "I am wholly silent about that which is called *Algebra* or the

[19] *Ibid.*, 34-35.

[20] *Ibid.*, 41. Barrow's philosophy of arithmetic failed to explain the use of whole numbers to count nongeometrical objects like men and angels. Therefore, he maintained that there were two kinds of numbers: mathematical (the notes of magnitude, described in the quoted passage) and "metaphysical" or "transcendental" (which, although nonmathematical, were prior and essential to mathematics, such as in the definition of a triangle as a polygon with three sides) (Barrow, 38-39).

[21] Barrow, 44.

Analytic Art," he commented, ". . . [b]ecause indeed *Analysis* . . . seems to belong no more to *Mathematics* than to *Physics, Ethics*, or any other Science. For this is only a Part or Species of *Logic*, . . . [and] an Instrument subservient to the Mathematics. . . ."[22]

V. A serious, far-ranging British intellectual, Berkeley could hardly have ignored the mathematical preoccupations of the seventeenth and early eighteenth centuries. As G. A. Johnston has already observed: "Mathematical conceptions form[ed] the warp and woof of the thought of the day; and Berkeley, like everybody else, was exposed to their influence."[23] Berkeley's early mathematical publications and his *Philosophical Commentaries*, which consisted of notes written in preparation for *An Essay towards A New Theory of Vision* of 1709 and the *Principles of Human Knowledge* of 1710, revealed specifically acquaintance with the mathematical speculations of Wallis, Hobbes, and Barrow. For example, in his early mathematical publications Berkeley mentioned three of Wallis's major mathematical books.[24] In addition the *Commentaries* acknowledged the controversy between Wallis and Hobbes and included the young Berkeley's note to inquire into that dispute when he "treat[ed] of Mathematiques" (W.G.B. I, 99-100). Finally, multiple entries of the *Commentaries* displayed Berkeley's careful critical study of the mathematical lectures which Barrow delivered at Cambridge between 1664 and 1666.[25]

Berkeley's philosophy of mathematics began with rejection of universal abstract ideas, shared with Hobbes, and with rejection of matter with an existence independent of mind. In his *Principles of Human Knowledge* he attacked John Locke's conceptualism, which held that the human mind formed general ideas through abstraction from particulars, and

[22] *Ibid.*, 28.

[23] Johnston, "The Influence of Mathematical Conceptions," 178. For Johnston's suggestion of a link between mathematics and Berkeley's "metaphysical theory of signs," see 179.

[24] George Berkeley, *The Works of George Berkeley Bishop of Cloyne*, ed. A. A. Luce and T. E. Jessop (9 vols.; London, 1948-57), IV: 171, 213, 236 (hereafter abbreviated as W.G.B.).

[25] Scholars have hitherto failed to fully appreciate the influence on Berkeley of Barrow's general mathematical (as opposed to his geometrical and optical) lectures. But entry 334, e.g., referring to "Barrow Lect" (W.G.B. I, 40), is clearly Berkeley's note to see (probably reread) Barrow's discussion of the certainty of mathematics notwithstanding mathematical controversies (see Barrow, *Usefulness of Mathematical Learning*, 224-44). Entries 384 and 462 state that "Barrow owns the Downfall of Geometry . . ." and that "Barrows arguing against indivisibles, lect. I. p. 16 is a petitio principii . . ." (W.G.B. I, 46, 57). Both entries capture the critical response of Berkeley, who supported indivisibles, to Lecture IX of Barrow's mathematical lectures (see especially Barrow, *Usefulness of Mathematical Learning*, 156, 153). (Entry 462 refers to Barrow's Lecture IX as Lect. I, apparently because this was the first lecture read by Barrow in 1665.)

suggested that such views impeded mathematical progress. Locke had given a mathematical example of a general idea—the famous one of the triangle—and in so doing had expressed enough hesitancy that Berkeley was able to turn Locke's own words against abstract general ideas. According to Locke,

> when we nicely reflect upon them, we shall find that *general ideas* are fictions and contrivances of the mind. . . . For example, does it not require some pains and skill to form the general idea of a triangle,. . . . for it must be neither oblique nor rectangle, neither equilateral, equicrural, nor scalenon; but all and none of these at once. In effect, it is something imperfect, that cannot exist; an idea wherein some parts of several different and inconsistent ideas are put together.[26]

Citing this passage at length in the *Principles* and admitting his own inability to imagine such a triangle, Berkeley declared that he "suspect[ed] the mathematicians are, as well as other men, concerned in the errors arising from the doctrine of abstract general ideas, and the existence of objects without the mind" (W.G.B. II, 95). In this and later writings, then, he sought to deliver mathematics from such errors through elaboration of a new theory based on neither of these premises, but rather on the tripartite approach which saw geometry as a science of perceptible finite extension, arithmetic and algebra as lesser sciences of signs, and analysis as a method applied to geometry (W.G.B. IV, 100).

Like Hobbes, Berkeley argued that geometry was the highest mathematical science because it dealt with extension. In the early *Principles*, he wrote that geometry concerned "*extension* . . . considered as relative" (W.G.B. II, 97) and, in the *Analyst* of 1734, that geometry's "object" was "the proportions of assignable extension" and its "end" was "measure[ment of] assignable finite extension" (W.G.B. IV, 96). Because he rejected matter existing independently of mind, however, Berkeley's position on geometry—while mathematically close to Hobbes's—differed in a fundamental philosophical way. For him the finite extension of geometry existed only in so far as it was perceptible. As he explained in the *Principles*: "Every particular finite extension, which may possibly be the object of our thought, is an *idea* existing only in the mind, and consequently each part thereof must be perceived" (W.G.B. II, 98). Sharing with Hobbes rejection of abstract general ideas and emphasis on extension as geometry's object, and in fact compounding his alienation from his contemporaries through rejection of independent matter, Berkeley was at least as much a geometrical maverick as Hobbes. Both men, for example, opposed the Euclidian definition of a straight line (W.G.B. I, 61); the Irish philosopher also rejected the infinite divisibility of lines (W.G.B. II, 98).

Like Hobbes, despite unconventional views, Berkeley followed the

[26] John Locke, *An Essay Concerning Human Understanding*, IV, 7, 9.

mathematical majority in maintaining the generality of geometrical arguments. He explained general arguments without universal abstract ideas in the *Principles* in his response to Locke's statement on the triangle (W.G.B. II, 32-33). Rejecting explicitly Locke's conceptualism, he offered an explanation similar to Barrow's. As Barrow had proposed that "the intelligible Sphere . . . [was] one understood universally,"[27] so Berkeley contended that the "universal" triangle was simply one triangle taken as a sign of all others. "The particular triangle I consider, whether of this or that sort it matters not," he stated, "doth equally stand for and represent all rectilinear triangles whatsoever, and is in that sense *universal*" (W.G.B. II, 34).

Despite the claims of J. O. Urmson and G. J. Warnock to the contrary, Berkeley's view of geometry as a science of finite extension remained relatively constant through the end of his career. The older Berkeley, Warnock has contended, "regard[ed] geometry itself as an abstract calculus, *applicable* (more or less roughly) to the physical world but not descriptive of its properties."[28] As mentioned by Warnock, in *De motu* Berkeley observed that "geometers for the sake of their *art* make use of many devices which they themselves cannot describe nor find in the nature of things" (W.G.B. IV, 41; my italics). Berkeley pointed here specifically to geometers' considering a curve "as consisting of an infinite number of straight lines, though in fact it does not consist of them" (W.G.B. IV, 48). The later *Analyst*, however, clarified Berkeley's position: while such devices might be part of the art of geometry, they were to be accorded no place in the science of geometry. Thus concerning a curve considered as an infinite number of straight lines, the *Analyst* asked: "whether it would not be righter to measure large polygons having finite sides, instead of curves, than to suppose curves are polygons of infinitesimal sides, a supposition neither true nor conceivable?" (W.G.B. IV, 96). Furthermore, as already noted, Berkeley continued in the *Analyst* to describe finite extension as geometry's object and in addition explained that geometry was at its best not when most abstract but "when from the distinct contemplation and comparison of figures, their properties are derived, by a perpetual well-connected chain of consequences, the objects being still kept in view, and the attention ever fixed upon them . . ." (W.G.B. IV, 65-66).

Ranking just below geometry, according to Berkeley, were the sciences of arithmetic and algebra. A man of his times, Berkeley shared Hobbes's and Barrow's concern for the lack of an immediate empirical basis for the numbers of arithmetic, and so followed their lead in developing arithmetic as a science of signs. Unlike Hobbes and Barrow, however, Berkeley embraced Wallis's enthusiasm for symbolic reasoning and al-

[27] See note 17.

[28] Warnock, 210. See also Urmson, 68.

gebra. In a stroke of synthetic genius, then, he presented algebra—the extension of arithmetic—also as a science of signs.

The origins of Berkeley's philosophy of arithmetic and algebra (from Barrow, Wallis, and most likely Hobbes as well) were complicated. The young Berkeley in fact claimed no originality for the theory of numbers as names. Rather entry 881 of the *Philosophical Commentaries* admitted: "It has already been observ'd by others that names are no where of more necessary use than in Numbering" (W.G.B. I, 104). Moreover, the ideas and terms incorporated into the young Berkeley's accounts of numbers and numbering revealed the definite influence of Barrow and the likely influence of Hobbes. Entries 104 and 110 were pithy statements of the main points about numbers made by Barrow in his mathematical lectures. Entry 104 noted: "Numbers not without the mind in any thing, because tis the mind by considering things as one that makes complex ideas of 'em tis the mind combines into one . . ." (W.G.B. I, 17). Entry 110 reiterated Barrow's points that numbers did not stand for any particular bodies but rather were assigned by the mind according to its pleasure. As Berkeley expressed it, "[n]umber not in bodies it being the creature of the mind depending entirely on its' consideration & being more or less as the mind pleases" (W.G.B. I, 18).[29]

While Berkeley relied heavily on Barrow's theory of numbers, his strong appreciation of algebra prevented him from concurring with Barrow's relegation of arithmetic to a part of geometry and almost complete rejection of algebra. As his writings attest, the young Berkeley had a refined algebraic taste, acquired at least partially from Wallis.[30] Entries 834 and 837 of the *Commentaries* referred to the controversy between Wallis and Hobbes, and the former entry proved that Berkeley understood Hobbes's main objection to symbolic reasoning (W.G.B. I, 99-100). But the content, profuse symbolism, and references of Berkeley's early mathematical writings affirmed that on the issue of symbolic reasoning and algebra he stood with Wallis and against Hobbes. As Johnston has noted, among Berkeley's earliest works were three mathematical pieces of 1707: *Miscellanea Mathematica*, which contained "a perfect orgy of symbols," *De Radicibus Surdis*, which proposed new symbolism for the irrational numbers, and *De Ludo Algebraico*, which looked forward to algebra's application "to the whole extent of mathematics, and every art and science, military, civil, and philosophical."[31] These pieces, moreover, referred specifically to Wallis's major mathematical books, *Mathesis Uni-*

[29] For Barrow's theory, see above. The description in the *Commentaries* of numbers as "names" (W.G.B. I, 92, 104) probably indicates Hobbes's influence as well.

[30] Malebranche's *Recherche de la vérité* possibly also contributed to the young Berkeley's favorable disposition towards algebra. See A. A. Luce, *Berkeley and Malebranche: A Study in the Origins of Berkeley's Thought* (Oxford, 1934), 15.

[31] (W.G.B. IV, 219). This English translation is taken from Johnston, *The Development of Berkeley's Philosophy*, 215. For Johnston's remarks on these pieces, see 213.

versalis (W.G.B. IV, 171), *Treatise of Algebra* (IV, 213), and *Arithmetica Infinitorum* (IV, 236). Most importantly, in siding with the innovative Wallis, Berkeley accepted a modern symbolic view of algebra—a view which gained substantial British support only in the early nineteenth century. Even then, Berkeley was no passive recipient of Wallis's ideas: charmed by Wallis's algebra with its negative and imaginary numbers, seeing algebra as a generalization of arithmetic, and accepting the insights of Hobbes and Barrow on the latter, Berkeley formulated his own distinctive theory of algebra (including the negative and imaginary numbers) as a science of signs.

As is well-known, his view of arithmetic and algebra was abstract and formal. Berkeley rejected both the contention that arithmetic was based on Platonic or other abstract general ideas and that it was immediately conversant about sensible objects. Instead he argued that numbers were "creatures of the mind,"[32] that arithmetic and algebra were sciences immediately conversant about signs rather than objects, and that their practitioners could reason without concern for the significance of the signs. He in fact used algebra to support the contention that reasoning could take place without concern for meaning and that some useful signs stood for no (particular or general) ideas.

The germs of these views were scattered throughout the early *Philosophical Commentaries*. In addition to the entries (which restated Barrow's claims about number) were others which pointed towards the general view of arithmetic and algebra as sciences of signs, while yet expressing concern about their want of speculative content. Thus, entry 763 declared: "Numbers are nothing but Names, never Words." The next entry highlighted the imaginary numbers as a stimulus to Berkeley's formulation of the new theory of arithmetic and algebra: "Mem: Imaginary roots to unravel that Mystery" (W.G.B. I, 92). Entry 767 asked: "Take away the signs from Arithmetic & Algebra, & pray w^t remains?" The response came in the following entry, as Berkeley admitted that arithmetic and algebra were "sciences purely Verbal, & entirely useless but for Practise in Societys of Men. No speculative knowledge, no comparing of Ideas in them" (W.G.B. I, 93).

In the *Principles of Human Knowledge* Berkeley developed the arithmetical insights of the *Commentaries*. He explained the relativity of number and attacked the doctrine of abstract general ideas as it related to arithmetic (W.G.B. II, 46, 95). Paralleling Wallis's position on the negative and imaginary numbers, he also stressed usefulness—rather than ideal content—as the justification of arithmetic. Arithmetic, he sum-

[32] Such was Berkeley's arithmetical theory at least through publication of the controversial *Siris*. It is perhaps significant that the latter work, in which he flirted with Platonism, declared: "Number is no object of sense: it is an *act* of the mind. The same thing in a different conception is one or many" (W.G.B. V, 134; my ital.).

marized here, "regard[s] not the *things* but the signs, which nevertheless are not regarded for their own sake, but because they direct us how to act with relation to things, and dispose rightly of them" (W.G.B. II, 97).

As Berkeley's mathematical and general philosophy intermingled in his theory of arithmetic, so too in the *Principles* and in *Alciphron, or the Minute Philosopher* he used algebra to illustrate the general point that strict reasoning could be conducted on words without consideration of their significates. Thus in the former work, he noted:

> a little attention will discover, that it is not necessary (even in the strictest reasonings) significant names which stand for ideas should, every time they are used, excite in the understanding the ideas they are made to stand for: in reading and discoursing, names being for the most part used as letters are in *algebra*, in which though a particular quantity be marked by each letter, yet to proceed right it is not requisite that in every step each letter suggest to your thoughts, that particular quantity it was appointed to stand for. (W.G.B. II, 37)

Siding with Wallis, then, Berkeley not only accepted symbolic algebra but also elevated it into a prime example of sound reasoning devoid of consideration of particular or general ideas. Going a step farther in *Alciphron*, he offered the algebra of the imaginary numbers as proof that men could engage in sound and useful reasoning on signs which stood for absolutely no ideas—either particular or general. This second discussion of algebra occurred as Berkeley developed the point that language does not aim solely at a comparison of ideas but sometimes at "something of an active operative nature, tending to a conceived good." Such a good, he continued,

> may sometimes be obtained, not only although the ideas marked are not offered to the mind, but even although there should be no possibility of offering or exhibiting any such idea to the mind: for instance, the algebraic mark, which denotes the root of a negative square, hath its use in logistic operations, although it be impossible to form an idea of any such quantity. And what is true of algebraic signs is also true of words or language, modern algebra being in fact a more short, apposite, and artificial sort of language. . . . (W.G.B. III, 307)

It is certainly striking that in the above crucial discussions of the mind's ability to reason on signs without concern for significates, Berkeley turned to algebraic examples. In doing so, he was simply following a pattern begun in his letter of December 8, 1709, to Molyneux. "We may very well, and in my Opinion often do," he told Molyneux, "reason without Ideas but only the Words us'd, being usd for the most parts as Letters in Algebra. . . . Numbers We can frame no Notion of beyond a certain Degree, and yet We can reason as well about a Thousand as about five, the Truth on't is Numbers are nothing but Names" (W.G.B. VIII, 25). The pattern seems convincing evidence that Berkeley's acceptance of symbolic algebra and the view of numbers as names lay behind his favorable disposition towards his general theories of signs and language—if not in fact behind his development of the theories. Above all the view of numbers as names seems to have been crucial to the theories. In the letter to Molyneux, for example, Berkeley's strongest example of reasoning on words without ideas came not from algebra but from arithmetic. As he himself pointed out, algebraic symbols, while standing for quantities, can be manipulated without continual concern for their significates; but numbers (and this was the crucial point) stand for no notions and "are nothing but Names." Similarly, the passage from *Alciphron*, which was the fulfillment of the promise of the *Commentaries* to use imaginaries as the key to the nominal essence of numbers, appealed not to algebra in general but rather to a particular type of number, the imaginary. Following this line of argument, Berkeley's general theories of signs and language can be traced at least partially back to Barrow's mathematical lectures.

Interestingly, however, Berkeley's enthusiasm for arithmetic and algebra as sciences of signs was qualified. In *Alciphron* and *The Analyst* he expressed ambivalence towards these sciences which, as he had noted in the *Commentaries*, lacked speculative content. Thus the section on algebra in *Alciphron* concluded with the observation that

> even the mathematical sciences themselves, . . . if they are considered, not as instruments to direct our practice, but as speculations to employ our curiosity, will be found to fall short in many instances of those clear and distinct ideas which, it seems, the minute philosophers of this age, whether knowingly or ignorantly, expect and insist upon in the mysteries of religion. (W.G.B. III, 307-8)

Also, some queries of *The Analyst* assumed that algebra, although a science, ranked somewhere below geometry. Query 38 questioned algebra's role in training men's mental powers. While the text of *The Analyst* extolled geometry as a subject which sharpened the mind (W.G.B. IV, 65-66), the latter query asked "Whether tedious calculations in algebra and fluxions be the likeliest method to improve the mind?" (W.G.B. IV, 99). Even more tellingly, query 45 described algebra as "allowed to be a science." Query 41, on the other hand, emphasized the scientific character of algebra. It asked whether algebraists "are obliged to the same strict reasonings as in geometry? And whether such their reasonings are not deduced from the same axioms with those in geometry? Whether therefore algebra be not as truly a science as geometry?" (W.G.B. IV, 100). Thus, even as he elaborated abstract theories of arithmetic and algebra, Berkeley felt the tension between the old emphasis on content which led Hobbes and Barrow to rank geometry above arithmetic and algebra, and the newer emphases on method and applicability, stressed by Wallis.

A stronger tension dominated Berkeley's reflections on the calculus. He was fundamentally skeptical of the latter subject although appreciative of its usefulness. As is well known, his skepticism concerned primarily the foundations of the calculus and secondarily its methods. Consideration of Berkeley's tripartite approach to mathematics reveals, moreover, that the foundational skepticism stemmed from his view of analysis as a method and of the calculus (the result of the application of analysis to geometry) as a refined part of geometry. Since he saw geometry as a science of perceptible extension, Berkeley expected the fundamental entities of the calculus—Leibniz's infinitesimals and Newton's fluxions—to stand for particular ideas or perceptible objects. The lack of such significates for these remarkably useful entities seemed, in his opinion, a paradox.

Two entries in the *Philosophical Commentaries* alluded to the core of Berkeley's problem with the calculus. Entry 337 raised the issue of the infinitesimals' "being nothings" (W.G.B. I, 41). But, why was this a problem? Could not Berkeley have simply developed the calculus as another science of signs, similar to arithmetic and algebra? Entry 354a cryptically addressed this very point. "[N]or can it be objected," the young philosopher wrote, "that we reason about Numbers w^{ch} are only words & not ideas, for these Infinitesimals are words, of no use, if not suppos'd to stand for Ideas" (W.G.B. I, 42). The hints of the *Commentaries* were clarified in *The Analyst*. Here Berkeley described Newton's fluxions as "the general key by help whereof the modern mathematicians unlock the secrets of Geometry" and analysis, as "a most excellent method" (W.G.B. IV, 66, 100); the mathematics based on application of this method, he characterized as "the abstruse and fine geometry" (W.G.B. IV, 88). In short Berkeley had set for himself a tension-filled course towards explanation of the calculus: he saw its fundamental entities as "nothings" and yet recognized the calculus as a part of geometry, a subject dealing with perceptible extension. Approaching the calculus from this position (and clearly not from a formalist one), Berkeley then repeatedly challenged mathematicians to explain the infinitesimals and the fluxions. For example, in an oft-quoted passage, he asked: "And what are these fluxions? The velocities of evanescent increments? And what are these same evanescent increments? They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them the ghosts of departed quantities?" (W.G.B. IV, 89). But these and similar questions were largely rhetorical, for Berkeley seemed quite sure of mathematicians' inability to relate the fluxions or the infinitesimals to perceptible objects. In fact early in *The Analyst* he wondered if his mathematical contemporaries were

> not wonderfully deceived and deluded by their own peculiar signs, symbols, or species. Nothing is easier than to devise expressions or notations, for fluxions

and infinitesimals of the first, second, third, fourth, and subsequent orders. . . . But if we remove the veil and look underneath, if, laying aside the expressions, we set ourselves attentively to consider the things themselves which are supposed to be expressed or marked thereby, we shall discover much emptiness, darkness, and confusion . . . (W.G.B. IV, 69)

The want of perceptible backing for fluxions and infinitesimals, coupled with Berkeley's view of analysis as a method applied to perceptible geometry, precluded his offering any easy resolution of the foundational problem sketched in *The Analyst*. Berkeley himself saw the dilemma and in the latter work alluded to the possible need for modifying his mathematical philosophy to accommodate the fluxions and infinitesimals. In particular, he suggested—but did not develop—revision of his theory of geometry. Thus he simply observed (without elaboration) that "there is indeed reason to apprehend that all attempts for setting the abstruse and fine geometry [the calculus] on a right foundation . . . will be found impracticable, till such time as the object and end of geometry are better understood than hitherto they seem to have been" (W.G.B. IV, 88). More indirectly, Query 27 asked:

Whether because, in stating a general case of pure algebra, we are at full liberty to make a character denote either a positive or a negative quantity, or nothing at all, we may therefore, in a geometrical case, limited by hypotheses and reasonings from particular properties and relations of figures, claim the same licence? (W.G.B. IV, 98)

VI. As Wallis, Hobbes, and Barrow had roamed freely over mathematics and philosophy, so Berkeley blended problems and insights from the two disciplines. He shared the period's empiricist tenor with Hobbes and Barrow, and saw geometry as the science of perceptible extension. Enduring through *The Analyst*, this view of geometry was a major stumbling block to Berkeley's acceptance of contemporary explanations of the foundations of the calculus. While the calculus was essentially "fine geometry," he contended, its fundamental entities stood for no perceptible objects and hence had no geometrical legitimacy. Mathematical entities devoid of significates, Berkeley continued, were admissible only in arithmetic and algebra—mathematical sciences towards which he, combining the daring algebraic insights of Wallis with the arithmetical speculations of Hobbes and Barrow, assumed a stance more modern and abstract than that of most of his mathematical contemporaries.

Berkeley's philosophy of arithmetic and algebra was remarkable not only for the fine algebraic taste and synthetic genius it evidenced but also for its reverberations in his general philosophy. His early acceptance of symbolic reasoning and the view of numbers as names seems to have paved the way towards his nominalism and theories of signs and language; algebra and the imaginary numbers provided his prime examples of reasoning on symbols without concern for significates. In short the English mathematical debate of the seventeenth and early eighteenth centuries was central to Berkeley's intellectual development. The debate spawned his distinctive tripartite mathematical philosophy[33] and, to a limited extent, even some major aspects of his general philosophy.

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[33] Recognition of Berkeley's tripartite approach to mathematics resolves the problem of the inconsistent interpretations of his philosophy of mathematics. Thus, this tripartite approach permitted Silver, who emphasized geometry, to see Berkeley as a materialist; Baum, who stressed arithmetic and algebra, to view him as a formalist and instrumentalist (see note 2); and Boyer, who concentrated on Berkeley's theory of calculus, to present him as a naive realist (Carl Boyer, *The History of the Calculus and Its Conceptual Development* [New York, 1949], 227).

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