Charles Babbage as an Algorithmic Thinker

I. GRATTAN-GUINNESS

While the career of Charles Babbage (1791-1871) shows a remarkable range of interests, strong threads bind together several of the principal ones: algorithmic thinking, with intimate links to algebra and to semiotics. The links connect especially his mathematical researches in functional equations with his work on mathematical tables and on calculating machines, but they are evident also in some of his social and industrial concerns. Evidence is presented to show that Babbage was consciously aware of at least some of these links. Attention to them casts light upon his achievements.

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General Terms: Algorithms.
Additional Terms: Babbage, Algebra, Semiotics, Mathematical Tables.

...the whole of analysis discovered is dependent in great part upon modified algorithms of certain fixed quantities...

Leonhard Euler, 1764[1]

This opinion was quoted in 1822 by Babbage, in a paper on notation which was related to functional equations (p. 1/344).[2] In this article I put forward a general thesis about Babbage's work as a mathematician, engineer, and scientist, of which several elements are exemplified by the quotation. My claim is as follows:

- Although Babbage's interests covered a remarkable and indeed polymathic range, *many of the most important ones exhibit common threads*, in both the choice of problem and the manner(s) of their investigation.
- The common threads center around algorithmic procedures in mathematics, science, and other walks of life.
- Babbage was consciously aware of some of these common features, especially in his deployment of analogy from one theory in another.
- Attention to these common features casts much light upon the character of his achievements, for it shows links within an apparently eclectic range of interests.

The content of this thesis will become clearer in and after the discussion (in the next section) of certain mathematical trends in Babbage's time, but some preliminary expansion on the word "algorithm" is necessary here. I intend it to refer, in a very general range of contexts, to ideas, theories, or procedures in which prominence is given to successive repetitions of a process or maneuver, its reversal, its compounding with other processes, and/or its substitution into itself. In mathematical contexts the words "iteration" and "combination" will also be used (and indeed, quoted).

In addition, two related notions have to be included. First, "algebra" refers both to the branch of mathematics in which Babbage worked (specifically, functional equations) and to certain features of algebraic thinking and proof which also arise elsewhere in his activities. Second, "semiotics" denotes theories of signs, symbols, and notations *as such* (in mathematics and elsewhere), in which is stressed their importance in a theory and in its philosophical basis. The word was not used in Babbage's time,* but it can be applied to several of his concerns and those of some contemporaries.

The thesis, then, is that Babbage consciously followed an algorithmic/algebraic/semiotic approach in his choice and solution of many of his problems and deployment of analogies, and that his historians should give it proper attention. For convenience I shall coin the word "algorithmism" to refer in general to this characteristic.

This thesis is explored in approximate chronological order of the development of Babbage's pertinent interests. The section "Mathematical orientations" covers the part

* Some unclear lines of influence are involved here. As they are only partly pertinent to Babbage, I grant them merely this short note. The word "semiotics," in the form "semeiotiki," is due to the philosopher John Locke. In his *Essay Concerning Human Understanding* (1690), it designated his third branch of knowledge, "The doctrine of signs." His considerable influence on philosophy[3] extended to Condillac, whose ideas affected French and then English algebraists in ways outlined later in the article. However, the word "semeiotic" does not seem to have come through; it was adopted as late as the end of the nineteenth century (by C.S. Peirce), and has become popular only very recently (partly because of the rise in importance of computing), and in the spelling "semiotic."

Charles Babbage (1791-1871): Some principal career details

		Age
1812-14	Founder member of Analytical Society (including also J. Herschel and G. Peacock); author in *Memoirs* (1813)	21
1814-27	Main researches in functional equations, in close collaboration with Herschel	
1816	Translation (with Herschel and Peacock) of S.F. Lacroix, *Calculus*	
1816-19	Possibility of publishing de Prony's tables	
1820	First ideas and models for calculating engine	
1822	Letter to Humphry Davy, on the possibilities of an engine	29
1825	Studies magnetism by rotation with Herschel	
1826	*A Comparative View of the Various Institutions for the Assurance of Lives*	35
1827	*Tables of Logarithms of the Natural Numbers* (corrected version, 1831)	
1829	Government backing for First Difference Engine secured	
1832	*On the Economy of Machinery and Manufactures* (new editions 1832, 1833, 1835)	
1832	Part of First Difference Engine completed	
1834	D. Lardner publishes article on this engine	41
1834-37	Main development of ideas for Analytical Engine	
1837	Meets A. Lovelace; she assists him in his work on engines	
1837	*The Ninth Bridgewater Treatise: A Fragment* (2nd edition 1838)	44
1840	Visits Italy; arouses interest in his engines, especially L. Menabrea	
1841-43	Lovelace and Menabrea publish articles on his engines	
1847	Sketches out Second Difference Engine	56
1850-53	Develops system of flashing signals for lighthouses	
1850s	Works on cryptography	
1862	Meets G. Boole	
1864	*Passages from the Life of a Philosopher* (autobiography of sorts)	71

that Babbage played in the reform of mathematics at Cambridge and his researches in functional equations and some other areas of mathematics. The section "Calculations by hand and by handle" starts with mathematical tables and moves on to the Difference and Analytical Engines. "Industry and science" notes a miscellany of other examples in manufacturing, cryptography, and physics. The final section draws some conclusions about the importance of algorithmism in Babbage (including a contrast with Boole) and speculates upon its origins. Reference is made in places to "the figure," which appears in the last section.

I give details of the principal pertinent events in or related to Babbage's life in the box. For more details, I refer the reader to A. Hyman's excellent biography.[4] I rely almost entirely upon Babbage's published writings and on certain manuscripts that appeared posthumously, for enough material is to be found there for my purpose. Many unpublished documents reinforce the thesis, and a few have been cited. The references will be found in the references list, but in the text I cite by volume and page number from M. Campbell-Kelly's fine new edition.[5] For example, in the reference at the start of this article to an 1822 publication, "p. 1/344" cites page 344 of Volume 1 of the new edition, but the superscript "2" cites the original publication. Dates associated in the text with items are normally those of first publication. A page range *not* preceded by a volume number and a slash refers to the work indicated by the superscript number that precedes it.

Mathematical orientations

...the dots of Newton, the d's of Leibnitz, or the dashes of Lagrange.

Babbage, 1864 (p. 11/19)[6]

British reforms: traditions in the calculus

One part of Babbage's life is well known; he played a major part in the conversion of British mathematics by the Analytical Society from Newton's fluxional calculus to the Continental notation. However, this "fact" is not a fact; it is also misleading in connotation. A revised account will be briefly summarized here.*

First, before that Society set to work in 1812, reforms in calculus teaching had been under way, at least among the staff, in various British institutions: in Scotland, in the circle around J. Playfair and also W. Spence; in Ireland, at Trinity College, Dublin, in moves initiated in 1812 by H. Lloyd; and in the Home Counties of England, at the Royal Military College and the Royal Military Academy (with P. Barlow, O. Gregory, C. Hutton, J. Ivory, W. Leybourn, and W. Wallace). At Cambridge itself, R. Woodhouse had become acquainted with, and even the current occupant of Newton's chair of mathematics, I. Milnor (a quite insignificant math-

* The main source for this outline is N. Guicciardini's book,[7] especially Chapters 7 through 9. The reading in J.M. Dubbey's book,[8] Chapters 2 and 3, is not recommended.

ematician), had been buying Continental mathematical books.* The single most important stimulus for change had been the publication of the first four volumes of P.S. Laplace's *Traité du mécanique céleste* (1799-1805). While the young men who founded the Analytical Society in 1812 made themselves remarkably familiar with Continental mathematics, they may not have been aware of all of these developments in Britain. Thus, while their movement certainly led to the most profound changes in British teaching and research in mathematics, it was not the first such initiative.

Second, the Society lasted as such only for a little over a year, while its founders (principally Babbage, J.F.W. Herschel, and G. Peacock) were undergraduates at Cambridge.[9,10] However, their intentions were maintained afterward, and I use the word "Society" in this looser sense when referring to the later activities of its former members.

Third, the change was not simply of notation but principally *of theory*. Fourth, there was no single Continental theory into which change could be effected; on the contrary, as Babbage himself indicated when referring to the pertaining notations in the quotation above, three theories were in competition:[11] limits (although not usually formulated in Newton's manner); the differential and integral version (dx, $\int x$, $\int y\,dx$ as an area, and so on), proposed by G.W. Leibniz but then used in the developed version largely created by Euler; and an algebraic approach introduced by J.L. Lagrange.

One did not necessarily have to make a choice. In particular, S.F. Lacroix, the chief textbook writer of the day, was a disciple of M.J. Condorcet and followed the eighteenth-century "*encyclopédiste*" philosophical tradition of presenting all available traditions in his writings. His principal text was the great *Traité du calcul différentiel et du calcul intégral*,[12] which had appeared in three volumes at the end of the eighteenth century. Babbage had bought a copy of it in 1811 (p. 11/19),[6] and he and his colleagues would have become very well acquainted with Continental traditions from it alone.** At all events, he and Herschel showed their erudition when publishing in 1813 their preface[13] to the first (and only) volume of the *Memoirs* of the Society; it is quite a comprehensive survey of Continental calculus over the previous 30 years.

Out of the possibilities available to them the Analytical Society chose the Lagrangian approach (the word "Analytical" was then often used in mathematics to refer to algebraic principles). The origin of this decision is not clear; it seems most likely that a general consensus among the members was made. The main principles of this approach are explained in the next subsection. The strength of their adhesion to it was made evident in the preface to the Society's translation of the second (1802) edition of Lacroix's shorter treatise on the calculus, published in 1816. As a wandering *encyclopédiste*, Lacroix had allowed himself to shift his penchant somewhat to limits; but the Young Turks from Cambridge admonished their senior *citoyen* for this sad preference "in place of the more correct and natural method of Lagrange."[14]† In an 1827 paper on notation in mathematics, Babbage praised in a similar vein the (attempted) expression of mechanics in algebraic theories that Lagrange had attempted to effect in his 1788 treatise *Méchanique analitique* (p. 1/397).[15]

The "analytical" algebraization of mathematical theories in France in the late eighteenth century related to a growing interest there in semiotics. The abbé Condillac and his semifollowers, the "idéologues," had stressed the importance of signs, especially in or from his textbook, the (so-called) *Logique* (1780). Indeed, the word "idéologie" originally denoted ideas, their reference, and means of signifying them. Further, for Condillac (common) algebra was the (semi-) formal language *par excellence*; a posthumous book on it called *La langue des calculs* was published in 1808.[16] While Condillac did not influence Lagrange personally to a notable extent, the general connection with algebra was then important — and not only in France, as we shall soon see.

Consequences of Lagrange's algebraized calculus

Lagrange had developed an algebraic version of the calculus, based on the assumption that every function $f(x + h)$ of a real variable x could be expanded in a Taylor series for every value of x (apart from values of x when f misbehaved, such as taking an infinite value):

$$f(x+h) = f(x) + hf'(x) + h^2 f''(x)/2! + \ldots \qquad (1)$$

He also claimed that the derivatives could be *defined*, by purely algebraic means, as the coefficients of the appropriate powers of the incremental variable h. He introduced the dash notation for derivatives, which was mentioned by Babbage at the head of the previous subsection; it denoted a functorial operator, going from the function $f(x)$ to the function $f'(x)$.

This approach is based on a clear program — doubtless one of the sources of attraction to the members of the Analytical Society in 1812. Babbage retained his enthusiasm for it even after belief in the Taylor series (Equation 1) was refuted by A.L. Cauchy in 1820 with counterexamples such as $\exp(-1/x^2)$ when $x = 0$. Indeed, Babbage did not realize and perhaps did not know of Cauchy's work, for in 1827 he spoke of the time that "has been required to fix permanently the foundations on which the calculus of Newton and Leibnitz shall rest," with a clear allusion to Lagrange's approach (p. 1/371).[15] (The independent discovery of this

* This fact was recently discovered at Queen's College Cambridge, where Milnor's inventory of his collection was discovered (communication to me from the library).

** Babbage told Lacroix in a letter of November 28, 1820, that he hoped for an English translation of the *Traité* (*Bibliothèque de l'Institut*, Paris, ms. 2396). He stressed the importance of Lacroix's work in letters of that time to J.B. Biot (ms. 4895).

† This sentiment may have belonged more to Babbage and Herschel than to Peacock; for the Leibniz-Euler form was preferred in teaching at Cambridge, doubtless due to its superior educational utility. This may have been Peacock's decision, as he was the only one of the three main founders of the society who stayed at the university after graduation.

counterexample by W.R. Hamilton in the 1830s also escaped Babbage's attention.) However, according to J.M. Dubbey[8] (p. 90), Babbage had sent three of his papers on functional equations to Cauchy in 1820, and to that topic we now turn.

In the course of pursuing his approach, especially from the late 1790s, Lagrange gave considerable impetus to the development of two new algebras: differential operators, using D (:= d/dx) as an algebraic object; and functional equations, in which the function was itself treated as the object. These algebras flowered especially with a group around the Alsatian L.F.A Arbogast, who developed the operational aspects by "separating the symbols" (a phrase of the time). They detached d/dx from y in the derivative dy/dx, and f from x in $f(x)$. Thereby they extended the realm of algebra by considering objects which were not numbers or geometrical sizes.

Laws and rules of manipulation of these new algebras had to be found. A notable contribution was made by F.J. Servois in 1814:[17] Seeking the fundamental properties of both algebras, especially functions, he characterized f as "distributive" and f and g as "commutative with each other" if, respectively,

$$f(x+y+\ldots)=f(x)+f(y)+\ldots \text{ and } f(g(x))=g(f(x)) \quad (2)$$

This is the origin of these standard words in algebra.

Babbage on functional equations and the calculus of functions

> The term function has long been introduced into analysis with great advantage, for the purpose of designating the result of every operation that can be performed on quantity...
> It is this inverse method with respect to functions, which I at present propose to consider.
>
> Babbage, 1815 (p. 1/93)[18]

Functional equations, in one and several variables, were Babbage's main mathematical interest from 1813 until the early 1820s. He wrote nine papers in or around them, which were published between 1813 and 1827. (They are republished in Babbage's works, Volume 1.[5]) Herschel also worked in this area, mostly on the special cases of difference equations, and with related summation of series; the two men corresponded intensively. It is not my intention to describe Babbage's methods in detail;* suffice it to indicate some principal concerns, especially the algebraic and semiotic aspects.

A simple example of a functional equation is

$$f(x+y)=f(x)f(y) \quad (3)$$

(which is an equation in *two* variables, because of the form of the left-hand side). The task is to find functions f which

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satisfy the stated property for all values of x and y, or some specified range of them. (The definitions in Equation 2 could be reinterpreted as functional equations.) Babbage and Herschel were drawn to functional equations not only by Lagrange's program but also by certain solution methods developed by G. Monge and Laplace in the 1770s.

In his first paper (published in 1815) and later, Babbage gave special attention to

$$f(x)=f(g(x)), \text{ and also } f^n(x)=g(x) \ (n \text{ integral}) \quad (4)$$

to solve for f in terms of a given g. (In the important circumstance when $g(x)=x$ in the second equation of Equations 4, f was said to be "periodic" of order n.) His most general equation in one variable was

$$F(x, f(x), f^2(g_1(x)), f^3(g^2(x)), \ldots, f^n(g_{n-1}(x)))=0 \quad (5)$$

to solve for f given F and the $\{g_r\}$ (p. 1/120).[18] Apart from some examples from the geometry of curves, he did not consider many applications, explaining in 1816 that "my object has been to direct the attention of the analyst to a new branch of the science" and stressing that "the doctrine of functions is of so general a nature, that it is applicable to every part of mathematical enquiry" (p. 1/193).[24]

At the time, the subject was often called "the calculus of functions," referring to the determination of particular functions ($f^n(x)$, say) even if no equations were involved; for them the phrase "functional equations" was used. Babbage's methods of determination and solution, which followed the French to some extent, were rather freewheeling. He manipulated functions and series, used self-substitutions of functions into equations, and deployed cunning changes of variable. He tried to study the difficult question

* Until M. Panteki's thesis[19] (see especially Chapters 2 and 3) is generally available, J.M. Dubbey's analysis,[8] Chapters 2 through 4, can be used, although the links with Herschel are not established. For example, their vast correspondence (mostly held at the Royal Society Library, London) is not used at all. This collection, together with some further letters and many letters from other correspondents in the British Library, Add. Mss. 37182-37183, are major manuscript sources for Babbage's mathematical researches, and also for the founding of the Analytical Society.

Some general background on operators can be obtained from E. Koppelman's 1971 article,[20] although mostly on differential operators. S. Pincherle[21] provides a valuable survey of operator methods, although largely after Babbage's time. Unfortunately, the survey of functional equations by J. Dhombres[22] almost entirely ignores Babbage's work — and also De Morgan's 1836 essay,[23] mentioned later in this article — even given his restriction to functions of several variables.

of the complete solution of an equation, and made very clever use of symmetric functions (where $h(x, y) = h(y, x)$ for all x and y) to build up an iterative sequence of particular solutions. These methods show very well his enthusiasm for algorithmic and iterative procedures. Like his predecessors, he did not normally consider conditions for existence and uniqueness; Cauchy was soon to focus on such questions.

As the quotation at the head of this subsection shows, Babbage was also very concerned with the inverse function. In this connection he made much use of the form $g^{-1}fg$ of solution* — perhaps the first example of this significant "conjugate" form (as it became known) in mathematics. Although he was more chary than Herschel of treating functions as objects, he exploited well the algorithmic power of this algebra: to handle a function f, its iterations $ff\ (= f^2)$, f^3, ..., the inverse function(s) f^{-1} and their iterations f^{-2}, ..., compounds with other functions fg, ..., and so on.

Babbage also tackled related topics, which had been little studied, such as ordinary and partial differential functional equations, integral equations, and fractional differentiation.** In 1817 he explicitly compared "the calculus of functions with other modes of calculation with which mathematicians have been long acquainted" (p. 1/216).[27] Summation of (in)finite series was a special concern, in which he followed Euler and others in seeking formal relationships between functions and their series expansions — indeed, the kind of procedure which Euler designated as an algorithm in the quotation at the head of this article.† One of his methods is described in a special algorithmic context in the last section.

These methods belonged to a tradition in British algebras which started principally with Woodhouse and was to become best known with Peacock. In an 1830 book on algebra,[30] and elsewhere, Peacock proposed "the principle of permanence of equivalent forms." It put forward conditions under which a mathematical expression or equation could be interpreted outside its "respectable" domain of interpretation (such as the sum of a divergent series, or a relation in which negative numbers were accepted as legitimate mathematical objects). In its emphasis on form, the principle carried something of a semiotic ring. But the principle was applied mostly to common algebra, and Babbage did not discuss it in his papers on functional equations, although he came close to it in some of his manuscripts.††

Semiotics in Babbage's mathematics

> What is there in a name? It is merely an empty basket, until you put something into it.
>
> Babbage, 1864 (p. 11/1)[6]

In 1816 Babbage introduced some good notations in developing his methods: underbars for homogeneous functions, so that "$\psi(\underline{x}, \underline{y}, z)_{p,q}$" indicated a function of degrees p in x and y together and q in y and z together (in a somewhat unclear passage (p. 1/144)[24]); and superscripts for iterative substitutions, with overbars for symmetric cases, such as (p. 1/125)[24]

$$\psi^{2,1}(x, y) := \psi(\psi(x, y), y) \tag{6}$$

$$\psi^{\bar{2},\bar{2}}(x, y) := \psi(\psi(x, y), \psi(x, y)) \tag{7}$$

Later he even took the general case $\psi^{\bar{m},\bar{n}}(x, y)$ as a mathematical problem of notations, for he found the numbers of occurrences of y and of x within it by forming and solving simple difference equations (pp. 1/348-349).[2]

Such points are evidence only of a (well-developed) normal desire of a mathematician to use good notations; but Babbage went much further to show his semiotic side also, considering families of symbols, and symbolism in general. Unlike its algebraic mathematics, French semiotics did not come over strongly to Britain, either in the revival of mathematics or in that of logic (which dated from the mid-1820s, with the publication of R. Whately's book *The Elements of Logic*[32]). Nevertheless, Babbage was aware of it. In the early 1820s he wrote an encyclopedia article on "Notation"[33] and, more importantly, a paper on "the influence of signs in mathematical reasoning,"[15] which were published in 1830 and 1827 respectively. There is much material in common between the two pieces.

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* The form $g^{-1}fg$ made its debut in Babbage's 1815 paper (p. 1/112).[18] Later he asked De Morgan to mention that it "was suggested to him by W.H. Maule Esq."[23] (p. 318). (See also De Morgan's letter of confirmation on this point to Babbage on July 10, 1835 (postmark), in the British Library, Add. Ms. 37189, no. 142.) A colleague in the Analytical Society, Maule communicated to Babbage in a letter of May 12, 1814, the special case

$$f(x) = \phi^{-1}((-1)^{2n}\phi(x)),\ \phi \text{ arbitrary}$$

as a solution of the periodic case of the first equation of Equation 4. He also pondered on the generality of the form $\phi^{-1}(-\phi(x))$ (37182, fol. 28).

Senior Wrangler and (the first) Smith's Prizeman at Cambridge in 1810, Maule was made a Fellow of Trinity College the following year, where he served for some years as a coach. But then he exhibited a typically British career preference in entering the bar, becoming eventually a judge.[25]

** See, in turn, Babbage in 1816 (pp. 1/182-184)[24] and 1820 (pp. 1/314-319);[26] integral equations, formulated in a very general way by Babbage and Herschel in 1813 (p. 1/55),[13] and tried in special cases such as by Babbage in 1816 (pp. 1/179-181);[24] and fractional differentiation (pp. 1/185-186),[24] learnt from Lacroix's large *Traité*. Normally Babbage did not separate functions from their arguments; Herschel was freer. I am currently examining Herschel's mathematics, for a meeting mounted by the Royal Society of London in May 1992 to celebrate his bicentenary in 1992.

† Following Dubbey on p. 139,[8] editor M. Campbell-Kelly adds a note in Volume 1, p. 266,[5] to Babbage's paper to describe the results obtained by Babbage as "mathematical howlers" for involving divergent series. Within the framework of rigor about to be established by Cauchy (including for functional equations), this remark is correct, but it does not represent the tradition within which Euler and Babbage were working. Babbage added a note to the end of that paper (p. 1/278)[28] reporting a recent conversation with S.D. Poisson (in Paris) on the dangers of divergent series, but his remark contradicts earlier parts of the paper[28] (see especially pp. 1/265-266), and neither man had a clear view on the matter. On the difference between the two traditions in the context of contemporary German mathematics, see H.N. Jahnke's 1987 article.[29]

†† See especially Babbage's manuscripts kept at the British Library, Add. Ms. 37202; they are usefully discussed by Dubbey in Chapter 5.[8] For the parentage of this algebraic tradition in Woodhouse, see H. Becher's 1980 article.[31]

Babbage noted the work of both the French philosophers and mathematicians: He quoted from the book *Des signes et de l'art de penser* (1819) written by the *idéologue* J.M. Degérando (pp. 1/374, 376),[15] and also explained the overbar and overarc notations $\overline{A\ B}$ and $\overset{\frown}{AB}$ used by L. Carnot to represent respectively directed straight and curved lines (p. 1/404).[15] He did not develop the "ideological" link, but variants of Carnot's notations were to be used in computing, as we shall see later.

Babbage also laid out various desiderata for notations, including one of almost iconic character: "*all notation should be so contrived as to have its parts capable of being employed separately*" (p. 1/418).[33] He gave as an example possible notations for the sine-squared function: He preferred "$(\sin\theta)^{2}$" — as found in the "excellent work" of Arbogast, or even better "$\sin\theta^{2}$" for avoiding brackets — by contrast, "$\sin.^{2}\theta$" (including the period) was "by far the most objectionable of any, and is completely at variance with strong analogies" (p. 1/422).[33]

In holding this opinion, Babbage was close to his friend Herschel, who had discussed in 1813[34] (p. 25) the use of the indices to denote powers of functions, suggesting the novelty "$\cos.^{-1}e$" for the inverse trigonometric function. This notation (without the period) has become standard, but the positive powers of these functions are always normally represented by the form which Babbage criticized — rightly.

The same fate awaited most of Babbage's (and Herschel's) work on functional equations and on notations. The French took some note of it; in 1821 J.D. Gergonne translated part of Babbage's paper[26] in his own mathematics journal,[35] while the Baron Ferrusac's abstracting *Bulletin* for science included routine short notices of his papers published in the period 1824 to 1831 of its run. Lacroix noted Babbage's 1816 paper[24] in 1819, in the second edition of his large *Traité*[36] (p. 595, and a note of Herschel on p. 732). Surprisingly, Babbage seems never to have consulted this new edition of a work that had helped him so much in his youth, and so, for example, seems never to have discovered Servois's 1814 paper,[17] to which Lacroix also gave publicity[36] (pp. 726-727).

Functional equations fell rather into the doldrums for several decades, and the only substantial use of Babbage's contributions was made in 1836 by De Morgan,[23] in the first systematic study of the subject. However, although published as an article in an important encyclopedia of the time, unfortunately it did not make the impact that it deserved.* Similarly, Babbage's concern with notations failed to raise the interest it deserved, although again De Morgan was a commentator.

Babbage's other mathematical writings

Some evidence of algorithmic concerns can be found elsewhere in Babbage's mathematics. His occasional writings on probability were concerned with combinatorial cases, including an 1821 paper[37] on the "martingal" in connection with successive betting, the estimation of mortality for the calculation of annuities in an 1826 book (pp. 6/91-96),[38] and the interpretation of apparent miracles in terms of some higher law unknown to man in his unofficial 1838 Bridgewater treatise (pp. 9/73-80).[39] In the latter case his algorithmic and analogical inclinations stood him in especially good stead. He gave as examples an iteration executed by his engine which followed a mathematical law and then "miraculously" contravened it at some stage, which, however, had been prepared deliberately by the operator (pp. 9/52-55).[39] (See also Babbage's *Passages from the Life of a Philosopher*,[6] pp. 11/292-293.)

> **"What is there in a name? It is merely an empty basket, until you put something into it."**
> **—Babbage, 1864**

A striking example of algorithmism comes from the mathematics of chess: "During the first part of my residence at Cambridge, I played at chess very frequently," he recalled (p. 11/25),[6] and in an 1817 paper[40] Babbage followed his hero Euler in studying the iteration of the knight's move so as to cover every square of the board. A further feature of this paper is displayed in the figure on pages 42-43.

Calculations by hand and by handle

In this section, I treat Babbage's concerns with computation and computing.

The appearance of mathematical tables

For about five years from 1826 Babbage concerned himself with the production of logarithmic tables.[41] He drew on existing tables for the basic numerical data and did not introduce any major new idea about their calculation. But concerning their physical appearance he showed his semiotic side. For ease of reading he spaced out the arrays of digits into five-row bands** and chose a font where all numerals were of the same height, yielding clean rows of digits. For clarity and compactness, he printed out digits only after the second decimal place, indicating by small zeros in the third place those cases when "1" had to be added to the second; and at the final seventh place he indicated rounding-up by setting a dot under the digit (explanations and sample pages from 1827 are given in Babbage's *Tables of Logarithms of the Natural Numbers* (pp. 2/72-107).[43] Babbage used a few of these principles also for the tables in his 1826 book on assurance (pp. 6/104-127).[38] In 1831 he printed some of the logarithmic tables with a variety of colored inks on papers of different colors, to compare various possibilities for clarity of reading (pp. 2/115-117).[44] Part of a page is contained in the figure on pages 42-43.

* An interesting example of De Morgan's extension of Babbage's ideas concerned the case of Equation 7 from 1822, where 0 substitutions in the function created a sort of identity operator: $\psi^{0,0}(v, v) = v$ (p. 1/345).[2] Babbage did not perceive the conceptual interest of this case; De Morgan called it the "*zero-function*"[23] (p. 278).

** This procedure was not novel with Babbage: it can be found in, for example, Barlow's tables[42] of 1814. But it was unusual at that time, when the rows were normally separated by lines.

In the end, Babbage's findings were not of major significance. But they were unusual, and reflect clearly the semiotic cast of his mind.

The Difference Engines

> What algebra is to arithmetic, the notation we now allude to is to mechanism.
>
> Lardner, 1834 (p. 2/174)[45]

Prior to these semiotic ruminations on tables, Babbage had brought to their algorithmic side a remarkable insight which was to influence his whole life: that logarithmic tables "might be calculated by machinery." In his autobiography he gave two occasions for this reflection — in 1812-1813 and 1819 (p. 11/31)[6] — and the latter is of particular significance.

During the 1790s a large set of logarithmic and trigonometric tables had been produced in Paris under the direction of the French engineer scientist G. Riche de Prony. The work was planned out according to Adam Smith's principles of the division of labor, and a large number of unemployed hairdressers were used to fill out the numbers on the sheets by adding and subtracting. Although the tables were completed in 1801, their size made publication a costly task and it was never done, despite the fact that printing was started more than once and various efforts were made over the years to find finance.[46]

Now one of these occasions occurred during the late 1810s, when the detente between Britain and France after the fall of Napoleon opened up the possibility that the British government might share the expenses. Although the plans did not come to fruit, they were proposed just at the time when Babbage was thinking about mechanical computation for the second time and must have remained in his mind, for he described de Prony's project in his "letter" of 1822 which secured the original governmental grant (pp. 2/10-12).[47] It seems that that project, with its extension *ad absurdum* of manual computation, helped Babbage to conceive of the need for an automated alternative.* Further, de Prony so conceived his tables that the hairdressers only had to add and subtract over differences of various orders. Babbage's variant was his projected Difference Engine no. 1 (as he later named it), working the same way mechanically up to Δ^7 and back again, "either proceeding backwards or forwards," as Dionysius Lardner put it in 1834 (p. 2/167).[45]**

The analogies here extend not only to de Prony; f and f^{-1} (see the earlier subsection on functional equations) also readily come to mind. Another analogy is shown in the figure (in the last section), concerning the layout of the wheels. Babbage himself reported yet another, in a paper on mechanical computation. He posed there a question concerning the determination of the digit in any place in the array (rather like the numbers of x's and y's in Equation 7), and in 1822 he came up with a difference equation "which had impeded my progress several years since, in attempting the solution of a problem connected with the game of chess" (p. 2/33).[52]

In 1848 Babbage thought out a simplified version of this engine. He denoted it in his autobiography as "Difference Engine no. 2" (pp. 11/75-85).[6] It has recently been constructed, as a new-old machine.[53] Little of the relevant paperwork has been published so far; I would expect it to provide further evidence supporting the claim.

The Analytical Engine

> The whole of arithmetic now appeared within the grasp of mechanism.
>
> Babbage, 1864 (p. 11/85)[6]

In 1834 Babbage came to his next great idea: a machine that would give commands as well as execute them (p. 11/46).[6] The quotation above suggests that with this extended algorithmism he had a glimmering of mechanical recursion, as it was to be conceived (in electrical and electronic contexts) a century later, the *whole* of arithmetic, numbers and operations with them. Thus was born the Analytical Engine.[54]

Many analogies were brought into play when Babbage developed this engine. For example, the Jacquard loom cards which ran it were of three kinds, for numbers, variables, and algebraic operations — as one would expect from a mathematician who distinguished f from x.† They were used to give compound instructions (like functions fg, ...), and they were able "to revolve *backwards* instead of *forwards*," in the words of Lady Lovelace in 1843 (p. 3/135)[49] so close to Lardner's quoted in the previous subsection. Again, presumably drawing on the experience of printing tables, in 1837 Babbage represented the four basic arithmetic operations on cards of four different colors (p. 3/52).[56]

In the mechanism Babbage distinguished between the "store," which held operands and results between operations, and the "mill," where they were sent to execute arithmetic operations (see A.G. Bromley's article,[54] p. 198). Another mathematical analogy comes readily to mind:

$$\underset{\text{store}}{x} \to \underset{\text{mill}}{f \text{ (or } fg \text{ or ...)}} \to \underset{\text{store}}{f(x) \text{ (or } f(g(x)) \text{ or ...)}}$$

Babbage did not miss this analogy. On the contrary, in his autobiography he foresaw actual *applicability* of the engine to functional equations (p. 11/325):[6]

* Compare Menabrea (pp. 3/63-66),[48] translation by Lovelace (pp. 3/94-97).[49] Strange it is, then, that Babbage did not apply Smith's principle of the division of labor to his own computer projects. One source of his failure seems to have been his own poor project planning.

** A less happy point of influence from de Prony may have been Babbage's penchant for calculating large numbers of digits. "The reason for this is not clear"[50] (p. 126). Now de Prony perplexes in a similar way, for some of his tables went up to around 30 decimal places, without explanation. A contemporary of Babbage, also aware of de Prony's use of differences, was the mathematician Thomas Knight; see his 1817 paper.[51]

† This classification of cards is often mentioned; see, for example, Lovelace (p. 3/115),[49] Babbage (pp. 11/89-91),[6] and his son (pp. 3/192).[55] It is criticized by A.G. Bromley[50] (p. 128) on modern metalogical grounds.

> *Calculus of Functions.*
> This was my earliest step, and is still one to which I would willingly recur if other demands on my time permitted... It is very remarkable that the Analytical Engine adapts itself with singular facility to the development and numerical working out of this vast department of analysis.

With her usual acuteness, Lady Lovelace had also stressed this possibility in 1843 (p. 3/116):[49]

> In studying the action of the Analytical Engine, we find that the peculiar and independent nature of the considerations which in all mathematical analysis belong to *operations*, as distinguished from the *objects operated upon* and from the *results* of the operations performed upon those objects, is very strikingly defined and separated.*

Menabrea had gone a little too far in his 1842 account of the engine. In a fit of outdatedness he invoked Lagrange's belief (Equation 1) in the Taylor series to stress the (alleged) generality of its range (p. 3/76).[48] Neither Lovelace nor Babbage picked up this detail in her translation (p. 3/107),[49] and earlier we saw that Babbage may not have known of Cauchy's refutation of the belief.

Semiotics in Babbage's engines

In an 1826 paper on "expressing by signs the action of machinery" (akin to the paper on mathematical notation[15] cited earlier, incidentally), Babbage varied his use of Carnot's curved and straight overbars by deploying vertical lines and curled left brackets to distinguish different types of motion of parts of Difference Engine no. 1 (pp. 3/215-216).[61] Later, in his account of the Analytical Engine, he used Lagrange-like predashes as in Equation 1 to distinguish the axes; for example, in 1837, *F*, '*F*, and ''*F* were used (p. 3/17).[56] Later, in the pamphlet[62] of 1851, he lettered parts of the engines on his working drawings in a manner extending this practice to sub-, super-, and all-over-the place indices, which indicated the type of part involved and its relationship to other parts.

Most important of all for semiotics, Babbage developed a "mechanical Notation" for all his engines, by means of which "the drawings, the times of action, and the trains for the transmission of force, are expressed in a language at once simple and concise" (p. 11/79).[6] It included rules on using upright, italic, and small-font letters for different kinds of referents (pp. 11/107-110).[6] Had he managed to construct his engines as envisaged, he might well have developed these ideas further in producing the envisaged printing mechanisms. There are obvious cross-influences between these concerns and his printing of mathematical tables (discussed in the earlier subsection on mathematical tables).

"It is not a bad definition of *man* to describe him as a *tool-making animal.*"
—Babbage, 1851

Industry and science

Babbage's style is evident in concerns other than mathematics and computing, as we shall now see.

The processes of manufacture

> It is not a bad definition of *man* to describe him as a *tool-making animal.*
>
> Babbage, 1851 (p. 10/104)[63]

To the modern view it is an irony that Babbage, much concerned as he was with various applications of probability and statistics, failed to notice their place in production engineering. Instead, the semioticist won: "Nothing is more remarkable, and yet less unexpected, than the *perfect identity* of things manufactured by the same tool," he wrote in 1835 (p. 8/47, italics inserted).[64] *We may have a clue here about his failure to complete any of his engines*: A lack of understanding of production processes led him to waste time and money on the excessively precise manufacture of some of their parts.

Babbage's statement was made in the most influential book that he published, *On the Economy of Machinery and Manufactures* (the 1835 edition is cited here). There was much concern at that time, especially among engineers, with the mathematical analysis of economic questions, especially concerning optimization[65] and equilibrium[66] (Chaps. 2-3). However, in his usual lateral *and algorithmic* way, Babbage focused instead upon the processes that take place: To the extent that optimization is treated, it is usually in the form of time-saving or time-consuming. A wide variety of production procedures and problems was given in the book, of which one is worth noting here: an extensive account of Adam Smith's principles of the division of labor and their

* On Lovelace's evident use of course-of-values recursion for calculating the Bernoulli numbers (pp. 3/166-167)[49] and recent criticisms, see R. Gandy's paper[57] (pp. 57-60). She took an overly modest view of her 1843 translation and notes[49] on Menabrea, for in the last of four letters sent to her publisher Richard Taylor during August 1843 she stated (in a stilted third-person mode): "She had not been without apprehensions that the *nature* and *content* of the Notes placed them somewhat beyond the bounds prescribed by the plan and objects of that publication [*Taylor's Scientific Memoirs*], at the same time that she could by no means flatter herself that they possessed any *intrinsic* value sufficient to justify an *exception* being made in their favour, or to compensate for the *many weeks of delay* in the appearance of the Number which she fears they caused." (Letter of August 16, 1843, Taylor & Francis Archives, St. Bride Printing Library, London; these letters complement the contemporary ones published in the excellent study by the Huskeys[58] on the relationship between Babbage and Lovelace.) The remark in the above extract about the "bounds" of the *Memoirs* carries more point than is at first apparent. Taylor had created the series of English translations of foreign scientific works as a positive reaction to Babbage's complaints[59] in 1830 of "the decline of science in England" (see W.H. Brock and A.J. Meadows' book,[60] pp. 89-92).

Figure. Babbage's iterative arrays. The illustrations are taken from the original publications. (a) From the 1817 paper on the knight's move in chess (Plate II, at p. 1/243).[40] (b) Part of a typical page from his logarithmic tables[44] of 1831, from the white-page printing. (c) A typical array of wheels for Difference Engine no. 1, from Lardner's 1834 article (p. 2/147).[45] (d) Part of his representation of a cypher in 1854 (p. 5/71).[69]

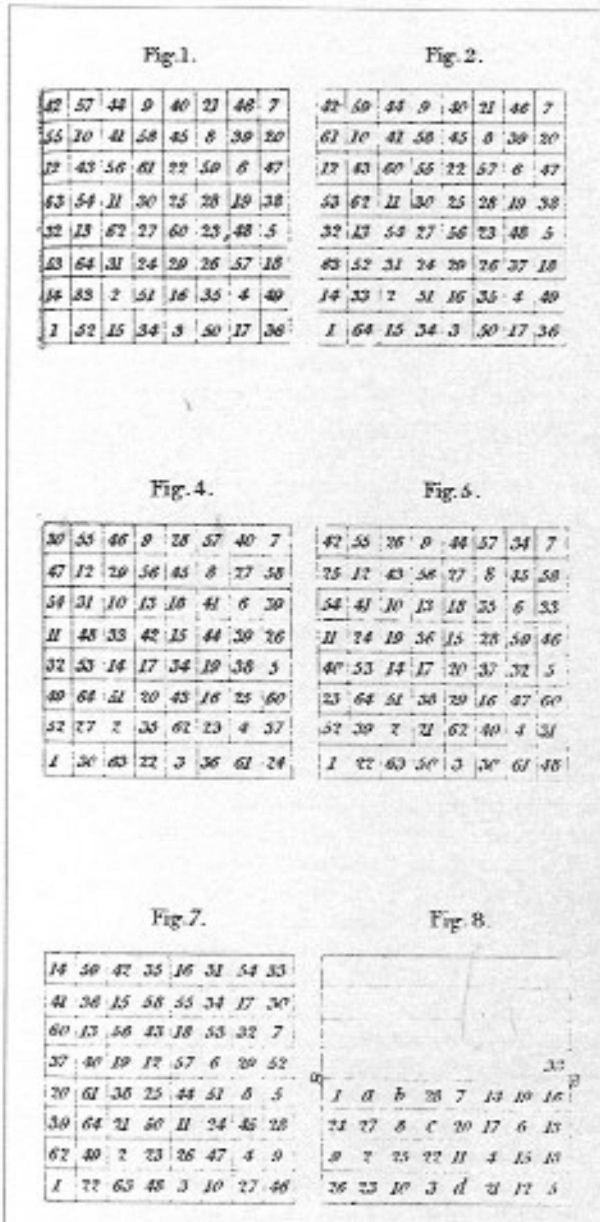
use by de Prony to manufacture his logarithmic and trigonometric tables (pp. 8/124-126, 135-139).[64]

Algorithmic thinking is evident in Babbage's ideas on the postal services. His proposal "for transmitting letters enclosed in small cylinders, along wires suspended from posts, and from towers or from church steeples" (p. 11/447)[6] has the characteristics of compounding and reversal, and the little model that he made around the mid-1820s in his own house shows that he took it seriously. Again, in his book on manufactures, he criticized the poor way in which letter boxes were indicated and advocated a semiotically much superior system: "at each letter-box, to have a light frame of iron projecting from the house over the pavement, and carrying the letters G.P., or T.P., or any other distinctive sign" (pp. 8/32-33).[64]

Repetitions and decoding

"I was much struck with the announcement" of F. Arago's researches of 1824 "on the magnetism manifested by various substances during rotation," recalled Babbage in his autobiography (p. 11/339),[6] and he and Herschel reported their own researches in a joint paper[67] of 1825 and Babbage in one of his own[68] a year later. One may presume that the repeated oscillations inherent in the phenomena attracted the attention of this natural algorithmist.

In 1851 Babbage published his study of another case of repetition, this time an optical one: He proposed occulting systems for lighthouses, in which "It would only be necessary to apply a mechanism which should periodically pull down an opaque shade over the glass cylinders of the argand [sic] burners." Further, a lighthouse could identify itself by exhibiting its identification number in a temporally semiotic

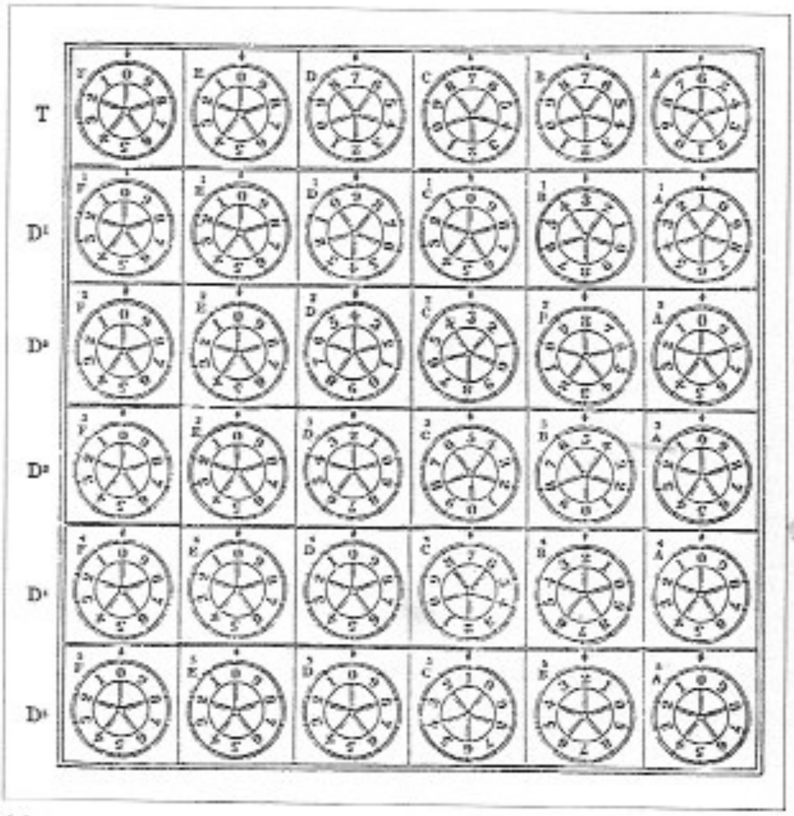


(a)

Log. 406. N. 255.

0° 42′	7° 5′	Num.	0	1	2	3	4	5	6	7	8	9	Diff.
30′	5′	2550	4065402	5572	5742	5913	6083	6253	6424	6594	6764	6934	4 68
	10	1	7105	7275	7445	7615	7786	7956	8126	8296	8466	8637	5 85
	20	2	8807	8977	9147	9317	9487	9658	9828	9998	o168	o338	6 102
	30	3	4070508	0678	0848	1018	1189	1359	1529	1699	1869	2039	7 119
	40	4	2209	2379	2549	2719	2889	3059	3229	3399	3569	3739	8 136
35′	50	5	3909	4079	4249	4419	4589	4759	4929	5099	5269	5439	9 153
	6′	6	5608	5778	5948	6118	6288	6458	6628	6798	6968	7137	
	10	7	7307	7477	7647	7817	7987	8156	8326	8496	8666	8836	
	20	8	9005	9175	9345	9515	9684	9854	o024	o194	o363	o533	
	30	9	4080703	0873	1042	1212	1382	1551	1721	1891	2060	2230	

(b)



(c)

	j	u	b	u	w	h	q	f	x
A	r	g	z	g	e	t	k	v	d
B	s	h	a	h	f	u	l	w	e
C	t	i	b	i	g	v	m	x	f
D	u	j	c	j	h	w	n	y	g
E	v	k	d	k	i	x	o	z	h
F	w	l	e	l	j	y	p	a	i
G	x	m	f	m	k	z	q	b	j
H	y	n	g	n	l	a	r	c	k
I	z	o	h	o	m	b	s	d	l
J	a	p	i	p	n	c	t	e	m
K	b	q	j	q	o	d	u	f	n
L	c	r	k	r	p	e	v	g	o
M	d	s	l	s	q	f	w	h	p
N	e	t	m	t	r	g	x	i	q
O	f	u	n	u	s	h	y	j	r
P	g	v	o	v	t	i	z	k	s
Q	h	w	p	w	u	j	a	l	t
R	i	x	q	x	v	k	b	m	u
S	j	y	r	y	w	l	c	n	v
T	k	z	s	z	x	m	d	o	w
U	l	a	t	a	y	n	e	p	x
V	m	b	u	b	z	o	f	q	y
W	n	c	v	c	a	p	g	r	z
X	o	d	w	d	b	q	h	s	a
Y	p	e	x	e	c	r	i	t	b
Z	q	f	y	f	d	s	j	u	c

(d)

manner, in terms of the appropriate numbers of occultations for each digit. For example, the lighthouse numbered 253 would signal

[time →]
0000000000 · 0 · 000 · 0 · 0 · 0 · 0 · 000 · 0 · 0 · 0000000000...

where the raised points indicate the (2, then 5, then 3) interruptions of shining of the light (pp. 10/62-65).[63]

Another late interest of Babbage lay in cryptography; it exhibits algorithmism and semiotics very clearly, especially in the transposition and rearrangement of the letters of the alphabet. In attending to the coding and *de*coding, he thought once again of going forward and backward. He did not publish much on it (principally an 1854 article[69]), but O.I. Franksen[70] has shown recently from the manuscripts that he gave it a great deal of attention. It also provides another feature for the figure.

Diagnosis

> The next morning I breakfasted with Humboldt. On the previous day I had mentioned that I was making a collection of the signs employed in map-making.
>
> Babbage, 1864 (p. 11/149)[6]

Babbage's choices

Babbage's eclecticism is not as random as it might appear: The constant concern with algorithms, semiotics, and algebraic thinking functioned together and gave his work far more interconnections than are obvious at first. This is the thesis proposed in the first section of this article, and it is strengthened not only by the *content* of the cases exhibited but also by their *choice*: That is, Babbage preferred to work on problems and contexts in which they were prominent rather than on the numerous other situations in which they were not (so) evident.

A further common factor can be pointed out, as I fulfill the promise of providing the figure. It shows four illustrations of what I call an "iterative array": that is, some kind of repeatable process or its product. Further, one passage from a paper of 1819 on infinite series (see the subsection on functional equations) proposed a mathematical analog: namely, "the *method of expanding horizontally and summing vertically*" when a sequence of series were to be summed together (pp. 1/248-249, 268).[28] The wide range of contexts highlights the strength of his liking for algorithmic thought.

A contrast with Boole

Babbage's position can be clarified by contrasting it with that of his contemporary George Boole, another major algebraist and a pioneer of logic. One might expect to find that the two enjoyed fruitful contact, but in fact it never developed, a point that has surprised some historians (see, for example, D. MacHale's book,[71] pp. 234-235).

The two men did meet, in 1862, probably at the September meeting at Cambridge of the British Association for the Advancement of Science.* Babbage explained the Difference Engine and recommended that Boole study Menabrea's 1842 article[48] and learn about the loom-card system. Boole hoped to meet Babbage in London after the Cambridge meeting, but he was called back urgently to his institution (Queen's College, Cork).** No significant contact developed between them, but Boole enclosed an offprint of a paper on probability theory, and this may have been the stimulus on Babbage around that time to read Boole's major paper[74] of 1844 on differential operators. "It related to the separation of symbols of operation from those of quantity, a question peculiarly interesting to me, since the Analytical Engine contains the embodiment of that method," Babbage noted (p. 11/105).[6] Nevertheless, despite the quality of the paper, "There was no ready, sufficient, and simple mode of distinguishing letters which represented quantity from those which indicated operation" (p. 11/105).[6]

Boole's work on logic began to appear with a short book[75] of 1847. Babbage read it and annotated it with the marginal remark "This is the work of a real *thinker*"[4] (p. 244). This praise is itself noteworthy; for unlike Boole's contributions to differential operators, his work on logic did not arouse strong interest until the early 1860s, when Stanley Jevons was the first to give it detailed scrutiny.† However, Babbage did not make use of Boole's ideas developing his engines. To a modern view this is a pity, for his thinking, especially on the Analytical Engine, was rather weak on logical matters.[50] However, there are intimate and explicit links between Boole's work on differential operators and on logic,[78] and it seems likely that Babbage would have found (or did find) similar and thus unsatisfactory conflations in Boole's logic, with the same symbol (deliberately) serving both for the operation of selecting of objects to form a class and for the (objectual) class itself. Indeed, while his criticisms of Boole's 1844 paper are rather overstated, they are quite accurate (when meant as criticisms) on Boole's algebra of logic.

Conversely, although Boole praised in 1860 Babbage's contributions to functional equations[79] (p. 208), he would have found repellent the mechanical aspects of Babbage's engines. He had stated in his 1847 book[75] (p. 2)

> To supersede the employment of common reason, or to subject it to the rigour of technical forms, would be the last desire of one who knows the value of that intellectual toil and warfare which imparts to the mind an intellectual vigour, and teaches it to contend with difficulties and to rely upon itself in emergencies.

Babbage's motives

> I believe my early perception of the immense power of signs in aiding the reasoning faculty contributed much to whatever success I may have had.
>
> Babbage, 1864 (p. 11/364)[6]

Algorithmic theories and methods occupied Babbage from his youth to his dying days; *they constitute a most unusual group of concerns for a scientist.* Their origins and drives must have been powerful.

The case of algebra is particularly instructive, for it came to him very young. He recalled that he was "passionately fond of algebra" when still a schoolboy (p. 11/18),[6] and soon afterward he was advocating Lagrange's approach at Cambridge. For a research area he chose functional equations, a perfectly reasonable choice but *by no means* a frontline topic at the time. Much more orthodox would have been, say, differential equations and applications to mechanics. Yet he went to that algorithmic theory and stayed there for several years of productive work. Indeed, he never lost or regretted his interest in functional equations, as we saw in his testimony in the subsection on the Analytical Engine and his attention to Boole.

What kind of explanation can we offer for the strong place of algorithmism in Babbage? Sociological elements make some contribution, in that from Woodhouse through Babbage and Boole right to the end of the century with A. Cayley and J.J. Sylvester, English mathematics showed a marked concern with algebras of one kind or another. But such factors are very limited, for an equal number of *non*algebraic English (near-) contemporaries and successors can be specified: the later work of Herschel, for example, W. Whewell, G.B. Airy, G.G. Stokes, and so on. The most similar case is De Morgan, whose fondness for algebras started out in the common versions and then passed through functional equations (see the subsection on semiotics in Babbage's mathematics) to the mathematical analysis of syllogistic logic.†† Peacock is another, less significant, example.

* Babbage would have attended this 1863 meeting as an "annual subscriber"; Boole delivered a lecture[72] on differential equations and also heard W.H.L. Russell[73] praise his contributions to this subject.

** The sole source of information on this meeting is Boole's letter sent to Babbage on October 15, 1862, from Cork, recalling the occasion, mentioning Cambridge, and apologizing for the failure to resume contacts (British Library, Add. Ms. 37198, no. 414). This letter is noted by Hyman[4] (p. 249); none survive from Babbage to Boole, if any were written.

† On the contacts between Boole and Jevons, see an article I published elsewhere.[76] In 1870 Jevons developed his own version of Boole's algebra into a mechanical inference machine.[77] There is some parallel with Babbage but seemingly no influence, although Jevons was influenced in general terms by Babbage's 1835 book[64] on manufactures.

†† On these mathematical links in De Morgan, see M. Panteki's dissertation[78] (Chaps. 3, 6). They are largely overlooked in D.D. Merrill's otherwise excellent recent study[80] of his logic. Babbage seems not to have taken note of De Morgan's contributions to logic, despite the kinship of some of them to functional equations; maybe they were mentioned in the teaching that De Morgan gave to Lady Lovelace.[58]

Thus a personal explanation seems to be required, centered primarily on Babbage himself — 'psychological,' maybe, though the scare quotes are there to scare off the psychohistorians.* For some reason algebra came naturally to Babbage, and the contexts of the time extended that inclination into a lifelong interest in matters algorithmic and semiotic.** In the quotation set at the head of this subsection, he stressed the importance of the semiotic aspects of his early orientations. The quotation "What is there in a name?" is the *first sentence* of his autobiography.[6] And the quotation on "tool-making animal" — serving as not a bad definition of man — is certainly a very good definition of Babbage himself. Can we see him as a naturally algorithmic/algebraic/semiotic mathematical thinker patched into a practically oriented personality? Relative to the French background, was he a fusion of the interests of Lagrange and de Prony? ■

Acknowledgments

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* Another and later "psychological" case in the context of algorithmic thinking is the American logician E.L. Post. He envisaged recursion in terms of states of mind of people at successive moments of time; and he had mental problems, which forced him to control to the minute the amount of time devoted to research.[81]

** In 1909 Babbage's autopsist V. Horsley found him to have developed his locutory and graphic functions more than the sensorial.[82]

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