As to the quantity of water taken off, I find it to amount, upon the neareſt calculation, to twenty-four pints at each operation; for though the firſt time produced only twelve pints, and in ſeveral of the latter operations the quantity fell ſhort of twenty-four pints, yet I may venture to ſtate it at leaſt at twenty-four pints or three gallons on an average, as in many of the operations I took off from twenty-eight to thirty pints. The number of times I tapped her was in all 155, which brings out in the whole 3720 pints, being 465 gallons, not far ſhort of ſeven hogſheads and an half. As to the authenticity of the whole, your connections with the family, and frequent opportunities of ſeeing this young lady during her illneſs, will put it beyond a doubt. I have therefore no more to add, than my wiſh that the caſe may prove acceptable to the Society.

I am, &c.

VII. *Problems concerning Interpolations.* *By* Edward Waring, *M. D. F. R. S. and of the Inſtitute of* Bononia, Lucaſian *Profeſſor of Mathematics in the Univerſity of* Cambridge.

Read Jan. 9, 1779.

MR. BRIGGS was the firſt perſon, I believe, that invented a method of differences for interpolating logarithms at ſmall intervals from each other: his principles were followed by REGINALD and MOVTON in France. Sir ISAAC NEWTON, from the ſame principles, diſcovered a general and elegant ſolution of the abovementioned problem: perhaps a ſtill more elegant one on ſome accounts has been ſince diſcovered by Meſſ. NICHOLE and STIRLING. In the following theorems the ſame problem is reſolved and rendered ſomewhat more general, without having any recourſe to finding the ſucceſſive differences.

THEOREM I.

Aſſume an equation $a+bx+cx^2+dx^3 \ldots x^{n-1}=y$, in which the co-efficients a, b, c, d, e, &c. are invariable;

 let

let $\alpha, \beta, \gamma, \delta, \epsilon$, &c. denote n values of the unknown quantity x, whofe correfpondent values of y let be reprefented by $s^\alpha, s^\beta, s^\gamma, s^\delta, s^\epsilon$, &c. Then will the equation $a + bx + cx^2 + dx^3 + ex^4 \ldots x^{n-1} = y =$

$$\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}\times s^\alpha + \frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}\times s^\beta$$

$$+\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times\&c.}\times s^\gamma + \frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\epsilon}\times\&c.}{\overline{\delta-\alpha}\times\overline{\delta-\beta}\times\overline{\delta-\gamma}\times\overline{\delta-\epsilon}\times\&c.}\times s^\delta$$

$$+\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\&c.}{\overline{\epsilon-\alpha}\times\overline{\epsilon-\beta}\times\overline{\epsilon-\gamma}\times\overline{\epsilon-\delta}\times\&c.}\times s^\epsilon + \&c.$$

DEMONSTRATION.

Write α for x in the equation $y =$

$$\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}\times s^\alpha + \frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}\times s^\beta +$$

&c.; and all the terms but the firft in the refulting equation will vanifh, for each of them contains in its numerator a factor $x-\alpha=\alpha-\alpha=o$; and the equation will become $y=\frac{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}\times s^\alpha = s^\alpha$. In the fame manner, by writing $\beta, \gamma, \delta, \epsilon$, &c. fucceffively for x in the given equation it may be proved, that when x is equal to $\beta, \gamma, \delta, \epsilon$, &c. then will y become refpectively $s^\beta, s^\gamma, s^\delta, s^\epsilon$, which was to be demonftrated.

2. Affume $y=ax^r+bx^{r+s}+cx^{r+2s}+dx^{r+3s}\ldots x^{r+\overline{n-1}s}$; and when x becomes $\alpha, \beta, \gamma, \delta, \epsilon$, &c. let y become refpectively

$s^\alpha, s^\beta, s^\gamma, s^\delta, s^\epsilon$, &c.; then will $y =$

$$\frac{x^r\times\overline{x^s-\beta^s}\times\overline{x^s-\gamma^s}\times\overline{x^s-\delta^s}\times\overline{x^s-\epsilon^s}\times\&c.}{\alpha^r\times\overline{\alpha^s-\beta^s}\times\overline{\alpha^s-\gamma^s}\times\overline{\alpha^s-\delta^s}\times\overline{\alpha^s-\epsilon^s}\times\&c.}\times s^\alpha$$

$$+\frac{x^r\times\overline{x^s-\alpha^s}\times\overline{x^s-\gamma^s}\times\overline{x^s-\delta^s}\times\overline{x^s-\epsilon^s}\times\&c.}{\beta^r\times\overline{\beta^s-\alpha^s}\times\overline{\beta^s-\gamma^s}\times\overline{\beta^s-\delta^s}\times\overline{\beta^s-\epsilon^s}\times\&c.}\times s^\beta$$

$$+\frac{x^r\times\overline{x^s-\alpha^s}\times\overline{x^s-\beta^s}\times\overline{x^s-\delta^s}\times\overline{x^s-\epsilon^s}\times\&c.}{\gamma^r\times\overline{\gamma^s-\alpha^s}\times\overline{\gamma^s-\beta^s}\times\overline{\gamma^s-\delta^s}\times\overline{\gamma^s-\epsilon^s}\times\&c.}\times s^\gamma + \&c.$$

This may be demonftrated in the fame manner as the preceding theorem, by writing $\alpha, \beta, \gamma, \delta, \epsilon$, &c. fucceffively for x.

PROBLEM.

Let there be n values $\alpha, \beta, \gamma, \delta, \epsilon$, &c. of the quantity x, to which the n values $s^\alpha, s^\beta, s^\gamma, s^\delta, s^\epsilon$, &c. of the quantity y correfpond; fuppofe thefe quantities to be found by any function X of the quantity x; let $\pi, \varrho, \sigma, \tau$, &c. be values of the quantities x, to which $s^\pi, s^\varrho, s^\sigma, s^\tau$, &c. values of the quantity y correfpond: for x fubftitute its abovementioned values $\pi, \varrho, \sigma, \tau$, &c. in the function X, and let the quantities refulting be $s^\pi, s^\varrho, s^\sigma, s^\tau$, &c. not equal to the preceding $s^\pi, s^\varrho, s^\sigma, s^\tau$, &c. refpectively; to find a quantity which added to the function X fhall not only give the true values of the quantity y correfponding to the values $\alpha, \beta, \gamma, \delta, \epsilon$, &c. of the quantity x, but alfo cor-

correſponding to the values $\pi, \varrho, \sigma, \tau$, &c. of the above-mentioned quantity x.

Aſſume $s^{\pi}-S^{\pi}=T^{\pi}$, $s^{\varrho}-S^{\varrho}=T^{\varrho}$, $s^{\sigma}-S^{\sigma}=T^{\sigma}$, $s^{\tau}-S^{\tau}=T^{\tau}$, &c.; then the errors of the function X will be reſpectively T^{π}, T^{ϱ}, T^{σ}, T^{τ}, &c.; and the correcting quantity ſought may be

$$\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\varepsilon}\times\&c.}{\overline{\pi-\alpha}\times\overline{\pi-\beta}\times\overline{\pi-\gamma}\times\overline{\pi-\delta}\times\overline{\pi-\varepsilon}\times\&c.}\times\frac{\overline{x-\varrho}\times\overline{x-\sigma}\times\overline{x-\tau}\times\&c.}{\overline{\pi-\varrho}\times\overline{\pi-\sigma}\times\overline{\pi-\tau}\times\&c.}\times T^{\pi}$$

$$+\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\varepsilon}\times\&c.}{\overline{\varrho-\alpha}\times\overline{\varrho-\beta}\times\overline{\varrho-\gamma}\times\overline{\varrho-\delta}\times\overline{\varrho-\varepsilon}\times\&c.}\times\frac{\overline{x-\pi}\times\overline{x-\sigma}\times\overline{x-\tau}\times\&c.}{\overline{\varrho-\pi}\times\overline{\varrho-\sigma}\times\overline{\varrho-\tau}\times\&c.}\times T^{\varrho}$$

$$+\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\varepsilon}\times\&c.}{\overline{\sigma-\alpha}\times\overline{\sigma-\beta}\times\overline{\sigma-\gamma}\times\overline{\sigma-\delta}\times\overline{\sigma-\varepsilon}\times\&c.}\times\frac{\overline{x-\pi}\times\overline{x-\varrho}\times\overline{x-\tau}\times\&c.}{\overline{\sigma-\pi}\times\overline{\sigma-\varrho}\times\overline{\sigma-\tau}\times\&c.}\times T^{\sigma}$$

$$+\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\varepsilon}\times\&c.}{\overline{\tau-\alpha}\times\overline{\tau-\beta}\times\overline{\tau-\gamma}\times\overline{\tau-\delta}\times\overline{\tau-\varepsilon}\times\&c.}\times\frac{\overline{x-\pi}\times\overline{x-\varrho}\times\overline{x-\sigma}\times\&c.}{\overline{\tau-\pi}\times\overline{\;-\;}\times\overline{\tau-\sigma}\times\&c.}\times T^{\tau}$$

+ &c.

Aliter.

Let $\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\varepsilon}\times$ &c. $\times\overline{x-\pi}\times\overline{x-\varrho}\times\overline{x-\sigma}\times\overline{x-\tau}\times$ &c.$=$N; $\overline{\pi-\alpha}\times\overline{\pi-\beta}\times\overline{\pi-\gamma}\times\overline{\pi-\delta}\times\overline{\pi-\varepsilon}\times$ &c. $\times\overline{\pi-\varrho}\times\overline{\pi-\sigma}\times\overline{\pi-\tau}\times$ &c.$=\Pi$; $\overline{\varrho-\alpha}\times\overline{\varrho-\beta}\times\overline{\varrho-\gamma}\times\overline{\varrho-\delta}\times\overline{\varrho-\varepsilon}\times$ &c. $\times\overline{\varrho-\pi}\times\overline{\varrho-\sigma}\times\overline{\varrho-\tau}\times$ &c. $=$ P; $\overline{\sigma-\alpha}\times\overline{\sigma-\beta}\times\overline{\sigma-\gamma}\times\overline{\sigma-\delta}\times\overline{\sigma-\varepsilon}\times$ &c. $\times\overline{\sigma-\pi}\times\overline{\sigma-\varrho}\times\overline{\sigma-\tau}\times$ &c. $=\Sigma$; $\overline{\tau-\alpha}\times\overline{\tau-\beta}\times\overline{\tau-\gamma}\times\overline{\tau-\delta}\times\overline{\tau-\varepsilon}\times$ &c. $\times\overline{\tau-\pi}\times\overline{\tau-\varrho}\times\overline{\tau-\sigma}\times$ &c.$=$T, &c.; then may the correcting quantity ſought be N $\left(\frac{T^{\pi}}{\Pi\times\overline{x-\pi}}+\frac{T^{\varrho}}{P\times\overline{x-\varrho}}+\frac{T^{\sigma}}{\Sigma\times\overline{x-\sigma}}+\frac{T^{\tau}}{T\times\overline{x-\tau}}+\&c.\right)$.

This

This problem may be demonſtrated in the ſame manner as the preceding theorems, by writing for x in the correcting quantity ſucceſſively its values $\pi, \varrho, \sigma, \tau$, &c.

2. For the correcting quantity ſought may be aſſumed the quantity

$$\frac{\overline{x^l-\alpha^l}\times\overline{x^l-\beta^l}\times\overline{x^l-\gamma^l}\times\overline{x^l-\delta^l}\times\&c.\times x^r\times\overline{x^l-\varrho^l}\times\overline{x^l-\sigma^l}\times\overline{x^l-\tau^l}\times\&c.}{\overline{\pi^l-\alpha^l}\times\overline{\pi^l-\beta^l}\times\overline{\pi^l-\gamma^l}\times\overline{\pi^l-\delta^l}\times\&c.\times\pi^r\times\overline{\pi^l-\varrho^l}\times\overline{\pi^l-\sigma^l}\times\overline{\pi^l-\tau^l}\times\&c.}\times T^{\pi}+\frac{\overline{x^l-\alpha^l}\times\overline{x^l-\beta^l}\times\overline{x^l-\gamma^l}\times\overline{x^l-\delta^l}\times\&c.\times x^r\times\overline{x^l-\pi^l}\times\overline{x^l-\sigma^l}\times\overline{x^l-\tau^l}\times\&c.}{\overline{\varrho^l-\alpha^l}\times\overline{\varrho^l-\beta^l}\times\overline{\varrho^l-\gamma^l}\times\overline{\varrho^l-\delta^l}\times\&c.\times\varrho^r\times\overline{\varrho^l-\pi^l}\times\overline{\varrho^l-\sigma^l}\times\overline{\varrho^l-\tau^l}\times\&c.}\times T^{\varrho}+\&c.$$

3. In general, let z be any quantity which is $=o$, when x becomes either $\alpha, \beta, \gamma, \delta, \varepsilon$, &c.: let z become ſucceſſively A, B, C, D, &c. when x becomes $\pi, \varrho, \sigma, \tau$, &c. reſpectively. When x either $=\varrho, \sigma, \tau$, &c. let $\Pi=o$; but if $x=\pi$, let $\Pi=p$: in the ſame manner when x either $=\pi, \sigma, \tau$, &c. let $P=o$; but when $x=\varrho$ let $P=r$: and ſimilarly, let $\Sigma=o$ when x is either π, ϱ, τ, &c.; but when $x=\sigma$ let $\Sigma=s$: and likewiſe, when x is either π, ϱ, σ, &c. let $T=o$; but when $x=\tau$ let $T=t$: &c. then for the correcting quantity ſought may be aſſumed $\frac{z}{A}\times\frac{\Pi}{p}\times T^{\pi}+\frac{z}{B}\times\frac{P}{r}\times T^{\varrho}+\frac{z}{C}\times\frac{\Sigma}{s}\times T^{\sigma}+\frac{z}{D}\times\frac{T}{t}\times T^{\tau}+$ &c.

THE-

THEOREM.

Aſſume (n) quantities $\alpha, \beta, \gamma, \delta, \epsilon$, &c. then will the ſum of all the (n) quantities of the following kind

$$\frac{\alpha^m}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times \&c.}+\frac{\beta^m}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times \&c.}$$

$$+\frac{\gamma^m}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times \&c.}+\frac{\delta^m}{\overline{\delta-\alpha}\times\overline{\delta-\beta}\times\overline{\delta-\gamma}\times\overline{\delta-\epsilon}\times \&c.}$$

$+\frac{\epsilon^m}{\overline{\epsilon-\alpha}\times\overline{\epsilon-\beta}\times\overline{\epsilon-\gamma}\times\overline{\epsilon-\delta}\times \&c.} + \&c. = o$, if m be any whole number leſs than $n-1$; but if $m=n-1$, then will the above mentioned ſum $=1$. In general, the ſum of the n terms $\frac{\alpha^m(\beta\gamma\delta \&c.+\beta\gamma\epsilon \&c.+\beta\delta\epsilon+\gamma\delta\epsilon \&c.+\&c.)}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times \&c.}+$

$$\frac{\beta^m(\alpha\gamma\delta \&c.+\alpha\gamma\epsilon \&c.+\alpha\delta\epsilon \&c.+\gamma\delta\epsilon \&c.+\&c.)}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times \&c.}+$$

$$\frac{\gamma^m(\alpha\beta\delta \&c.+\alpha\beta\epsilon \&c.+\alpha\delta\epsilon \&c.+\beta\delta\epsilon \&c.+\&c.)}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon} \&c.}+$$

$$\frac{\delta^m(\alpha\beta\gamma \&c.+\alpha\beta\epsilon \&c.+\alpha\gamma\epsilon \&c.+\&c.)}{\overline{\delta-\alpha}\times\overline{\delta-\beta}\times\overline{\delta-\gamma}\times\overline{\delta-\epsilon}\times \&c.}+$$

$$\frac{\epsilon^m(\alpha\beta\gamma \&c.+\alpha\beta\delta \&c.+\alpha\gamma\delta \&c.+\beta\gamma\delta \&c.+\&c.)}{\overline{\epsilon-\alpha}\times\overline{\epsilon-\beta}\times\overline{\epsilon-\gamma}\times\overline{\epsilon-\delta}\times \&c.}+\&c.=o,$$

if m be leſs than n, and $m+r$ not equal to $n-1$, where r is equal to the number of letters contained in each of the contents above mentioned $\beta\gamma\delta$, &c. $\beta\gamma\epsilon$, &c. $\beta\delta\epsilon$, &c. $\gamma\delta\epsilon$, &c. &c. &c. reſpectively: but if $m+r=n-1$, then will the above mentioned ſum $=\pm 1$; it will be $+1$ if r be an even number, otherwiſe -1.

DEMONSTRATION.

Suppoſe $a+b\alpha+c\alpha^2+d\alpha^3+e\alpha^4+ \&c. = S^\alpha$,

$a+b\beta+c\beta^2+d\beta^3+e\beta^4+ \&c. = S^\beta$,

$a+b\gamma+c\gamma^2+d\gamma^3+e\gamma^4+ \&c. = S^\gamma$,

$a+b\delta+c\delta^2+d\delta^3+e\delta^4+ \&c. = S^\delta$,

$a+b\epsilon+c\epsilon^2+d\epsilon^3+e\epsilon^4+ \&c. = S^\epsilon$, multiply theſe equations into A, B, C, D, E, &c. unknown co-efficients to be inveſtigated, and there reſult

$A\times S^\alpha = Aa+Ab\alpha+Ac\alpha^2+Ad\alpha^3+Ae\alpha^4+\&c.$

$B\times S^\beta = Ba+Bb\beta+Bc\beta^2+Bd\beta^3+Be\beta^4+\&c.$

$C\times S^\gamma = Ca+Cb\gamma+Cc\gamma^2+Cd\gamma^3+Cd\gamma^4+\&c.$

$D\times S^\delta = Da+Db\delta+Dc\delta^2+Dd\delta^3+Dd\delta^4+\&c.$

$E\times S^\epsilon = Ea+Eb\epsilon+Ec\epsilon^2+Ed\epsilon^3+Ee\epsilon^4+\&c. \&c. \&c.$

Now ſuppoſe $AS^\alpha+BS^\beta+CS^\gamma+DS^\delta+ES^\epsilon+\&c.=a+bx+cx^2+dx^3+ex^4+\&c.$ and the correſpondent parts reſpectively equal to each other; that is, $a\,(A+B+C+D+E+\&c.)=a$; $b\,(A\alpha+B\beta+C\gamma+D\delta+E\epsilon+\&c.)=bx$; $A\alpha^2+B\beta^2+C\gamma^2+D\delta^2+E\epsilon^2+\&c.=x^2$; $A\alpha^3+B\beta^3+C\gamma^3+D\delta^3+E\epsilon^3+\&c.=x^3$; $A\alpha^4+B\beta^4+C\gamma^4+D\delta^4+E\epsilon^4+\&c.=x^4$, &c.: But it follows from Theorem 1. that (if $AS^\alpha+BS^\beta+CS^\gamma+DS^\delta+ES^\epsilon+\&c.$ $=a+bx+cx^2+dx^3+ex^4+\&c.$) $A=\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}$,

$\text{B}=\frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}$, $\text{C}=\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times\&c.}$,

$\text{D}=\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\epsilon}\times\&c.}{\overline{\delta-\alpha}\times\overline{\delta-\beta}\times\overline{\delta-\gamma}\times\overline{\delta-\epsilon}\times\&c.}$, $\text{E}=\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\&c.}{\overline{\epsilon-\alpha}\times\overline{\epsilon-\beta}\times\overline{\epsilon-\gamma}\times\overline{\epsilon-\delta}\times\&c.}$,

&c.: fubftitute thefe values for A, B, C, D, E, &c. refpectively in the preceding equations ($\text{A}+\text{B}+\text{C}+\text{D}+\text{E}+\&c.=1$, $\text{A}\alpha+\text{B}\beta+\text{C}\gamma+\text{D}\delta+\text{E}\epsilon+\&c.=x$, $\text{A}\alpha^2+\text{B}\beta^2+\text{C}\gamma^2+\text{D}\delta^2+\text{E}\epsilon^2+\&c.=x^2$, $\text{A}\alpha^3+\text{B}\beta^3+\text{C}\gamma^3+\text{D}\delta^3+\text{E}\epsilon^3+\&c.=x^3$, &c.) and there refult the equations (1) $\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}$

$$+\frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}+\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times\&c.}+\&c.=1;$$

$$(2)\ \alpha\times\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}+\beta\times\frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}$$

$$+\gamma\times\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times\&c.}+\&c.=x;$$

$$(3)\ \alpha^2\times\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}+\beta^2\times\frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}$$

$+\gamma^2\times\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times\&c.}=x^2$; and in general,

$$\alpha^m\times\frac{\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\alpha-\beta}\times\overline{\alpha-\gamma}\times\overline{\alpha-\delta}\times\overline{\alpha-\epsilon}\times\&c.}+\beta^m\times\frac{\overline{x-\alpha}\times\overline{x-\gamma}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\beta-\alpha}\times\overline{\beta-\gamma}\times\overline{\beta-\delta}\times\overline{\beta-\epsilon}\times\&c.}$$

$$+\gamma^m\times\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\delta}\times\overline{x-\epsilon}\times\&c.}{\overline{\gamma-\alpha}\times\overline{\gamma-\beta}\times\overline{\gamma-\delta}\times\overline{\gamma-\epsilon}\times\&c.}+\delta^m\times\frac{\overline{x-\alpha}\times\overline{x-\beta}\times\overline{x-\gamma}\times\overline{x-\epsilon}\times\&c.}{\overline{\delta-\alpha}\times\overline{\delta-\beta}\times\overline{\delta-\gamma}\times\overline{\delta-\epsilon}\times\&c.}$$

$+\&c.=x^m$, whatever may be the values of the quantities x; $\alpha, \beta, \gamma, \delta, \epsilon$, &c.: reduce all thefe fractions into terms, proceeding according to the dimenfions of the quantity x, and it is evident, that the fum of all the fractions multiplied

tiplied

tiplied into any dimenfion of x not equal to m will be $=0$; but the fum of all the fractions multiplied into x^m will be $=1$: from this propofition the theorem is eafily deduced.

I have invented and demonftrated from different principles to the preceding the firft part of this theorem, a particular cafe of which was publifhed by me many years ago.

From this theorem may eafily be deduced feveral others of a fimilar nature.