

## 17<sup>th</sup> Alemi Meeting



# Modelling the kinetics of grain growth – Triple junctions and topology of grain arrangement

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# Contents

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➤ **Motivation**

- Materials at extreme conditions
- Fine and homogenously grained microstructures,
- Desired mechanical properties.

➤ **Theory**

- Grain growth – a brief introduction
- Kinetics of triple junctions – an application of the extremum principle

➤ **Results**

- Migration of grain boundaries meeting at triple junctions
- Shrinkage of a 4-sided grain
- Influence of topology

➤ **Conclusions**

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## Motivation I: Materials at extreme conditions

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➤ Line pipe steels



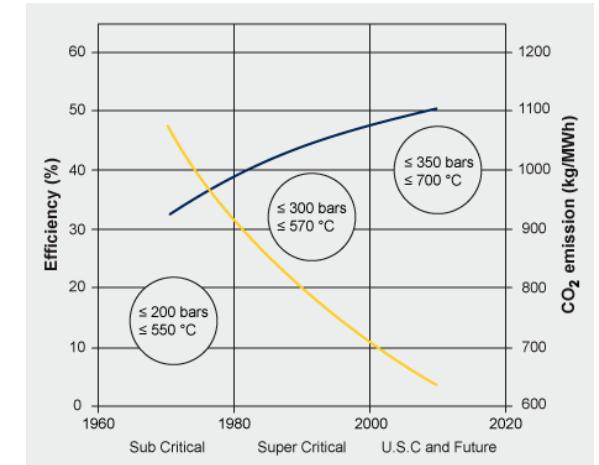
[turkstream.info](http://turkstream.info)

➤ Large steam turbines

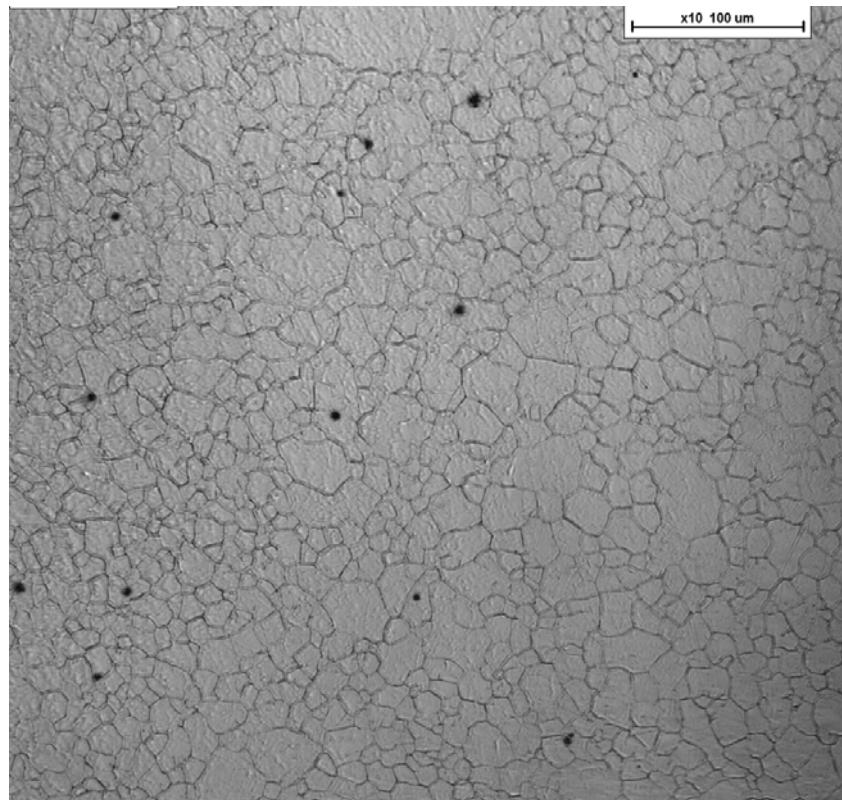
(e.g. high temperature exposed, creep-resistant steels)



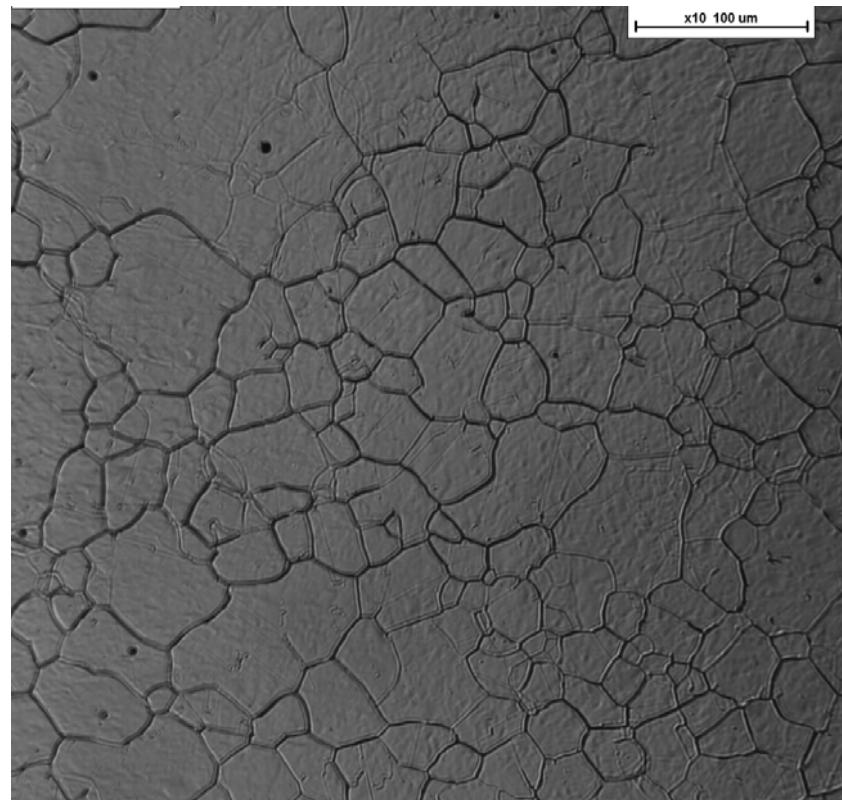
$$\eta_C = 1 - \frac{T_2}{T_1}$$



## Motivation II: An example for grain growth



$\vartheta = 1050^\circ\text{C}$  at  $t = 243\text{s}$



$\vartheta = 1250^\circ\text{C}$  at  $t = 183\text{s}$

Mass fractions·100:  $w_{\text{C}} = 0.06$ ,  $w_{\text{Mn}} = 1.65$ ,  $w_{\text{Nb}} = 0.034$ ,  $w_{\text{Ti}} = 0.012$ ,  $w_{\text{Mo}} = 0.24$ ,  $w_{\text{N}} = 0.005$

# Grain growth – a brief introduction

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## Grain growth in single phased materials

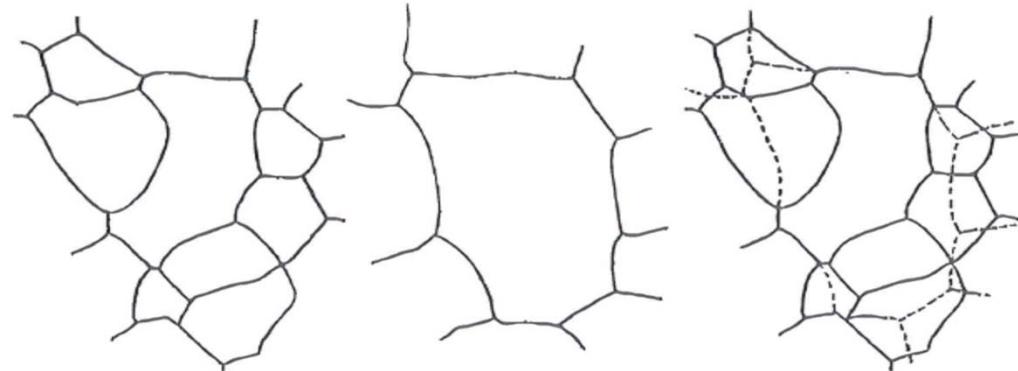
Curved grain boundaries tend to migrate to the center of the curvature.

Driving force: decrease in Gibbs energy due to reduction of grain boundary surface.

See . e.g [M. Hillert: Analytical treatments of normal grain growth: Materials Science Forum Vols 204-206 (1996) pp. 3-18.]

$$\mu \propto \gamma / r_m$$

$$r_m^2 - r_0^2 = kt$$



- Relation of curvature  $\kappa$  to mean grain size  $r_m \Rightarrow$  Steady state distribution  
*(no topological information)*
  - Calculating migration of grain boundaries connected by triple junctions
-

# Theory I: Gibbs energy and constraints

Gibbs energy  $G_i$  of grain boundary  $i$  per unit length  $L$  of the cylinder:

$$G_i = \int \gamma_i ds_i$$

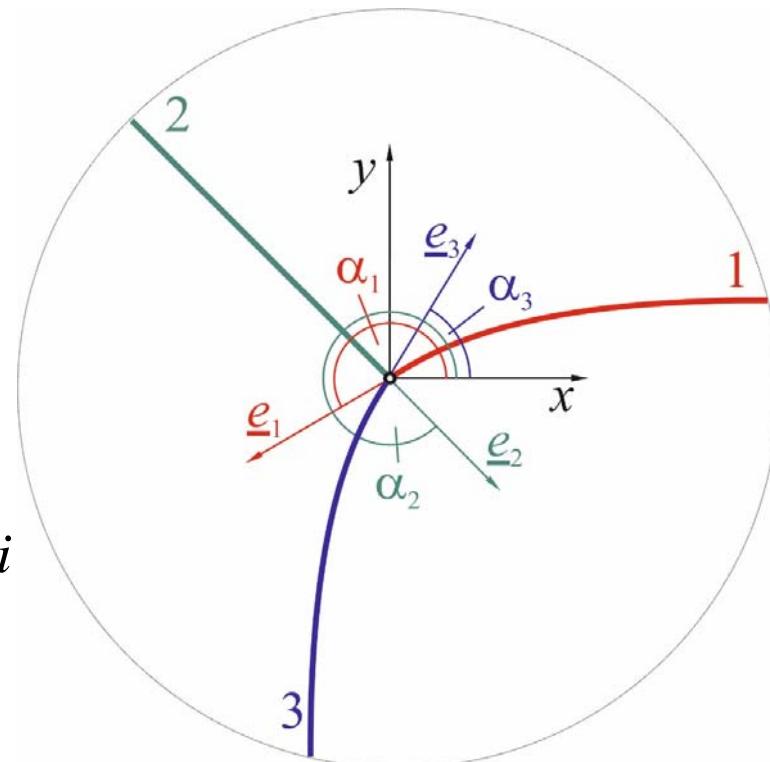
$\gamma_i$  ... specific grain boundary energy

Compatibility at triple junction

$$v_i^n = \underline{v}_T \cdot \underline{n}_i$$

Rate of change of length  $v_i^t$  of grain boundary  $i$

$$v_i^t = \underline{v}_T \cdot \underline{e}_i$$



From: F. D. Fischer, J. Svoboda, K. Hackl: "Modelling the kinetics of a triple junction", Acta Mater. 60 (2012) 4704-4711

## Theory II : Rate of Gibbs energy and dissipation

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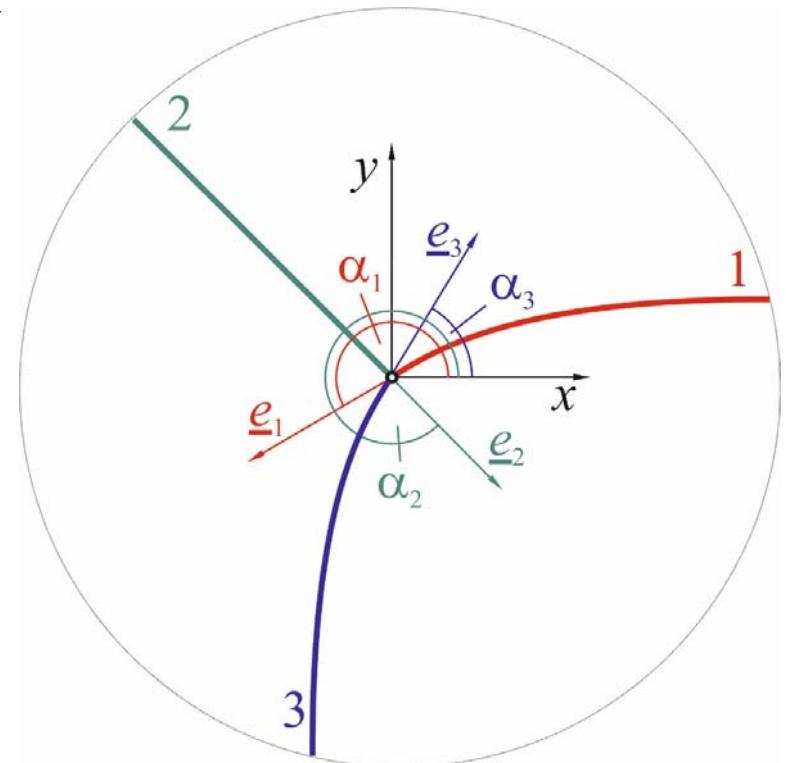
Rate of Gibbs energy  $\dot{G}_i$  of grain boundary  $i$

$$\dot{G}_i = \int_{S_i} \gamma_i \kappa_i(s_i) v_i^n(s_i) ds_i + \gamma_i \underline{v}_T \cdot \underline{e}_i$$

$$\dot{G} = \sum_{i=1}^3 \dot{G}_i$$

Dissipation  $Q$

$$Q = \sum_{i=1}^3 \left( \int_{S_i} \frac{(v_i^n(s_i))^2}{m_i} ds_i + \frac{\underline{v}_T^2}{m_T} \right)$$




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From: F. D. Fischer, J. Svoboda, K. Hackl: "Modelling the kinetics of a triple junction", Acta Mater. 60 (2012) 4704-4711

# Theory III

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Lagrangian for the system

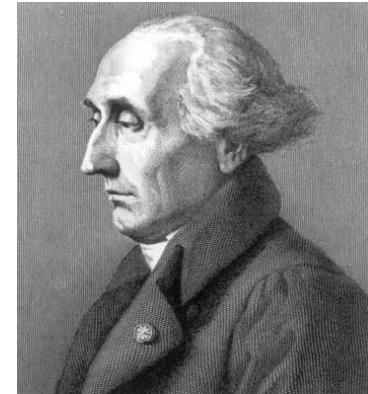
$$L = Q + \lambda(\dot{G} + Q) + \sum_{i=1}^3 \beta_i [v_i^n - \underline{v}_T \cdot \underline{n}_i]$$

Variation or derivation with respect to the kinetic parameters  $v_i^n(s_i)$ ,  $v_i^n$ ,  $\underline{v}_T$  yields the **evolution equations**:

$$v_i^n(s_i) = -m_T \gamma_i K_i(s_i)$$

$$\underline{v}_T = -m_T \sum_{i=1}^3 \gamma_i \underline{e}_i$$

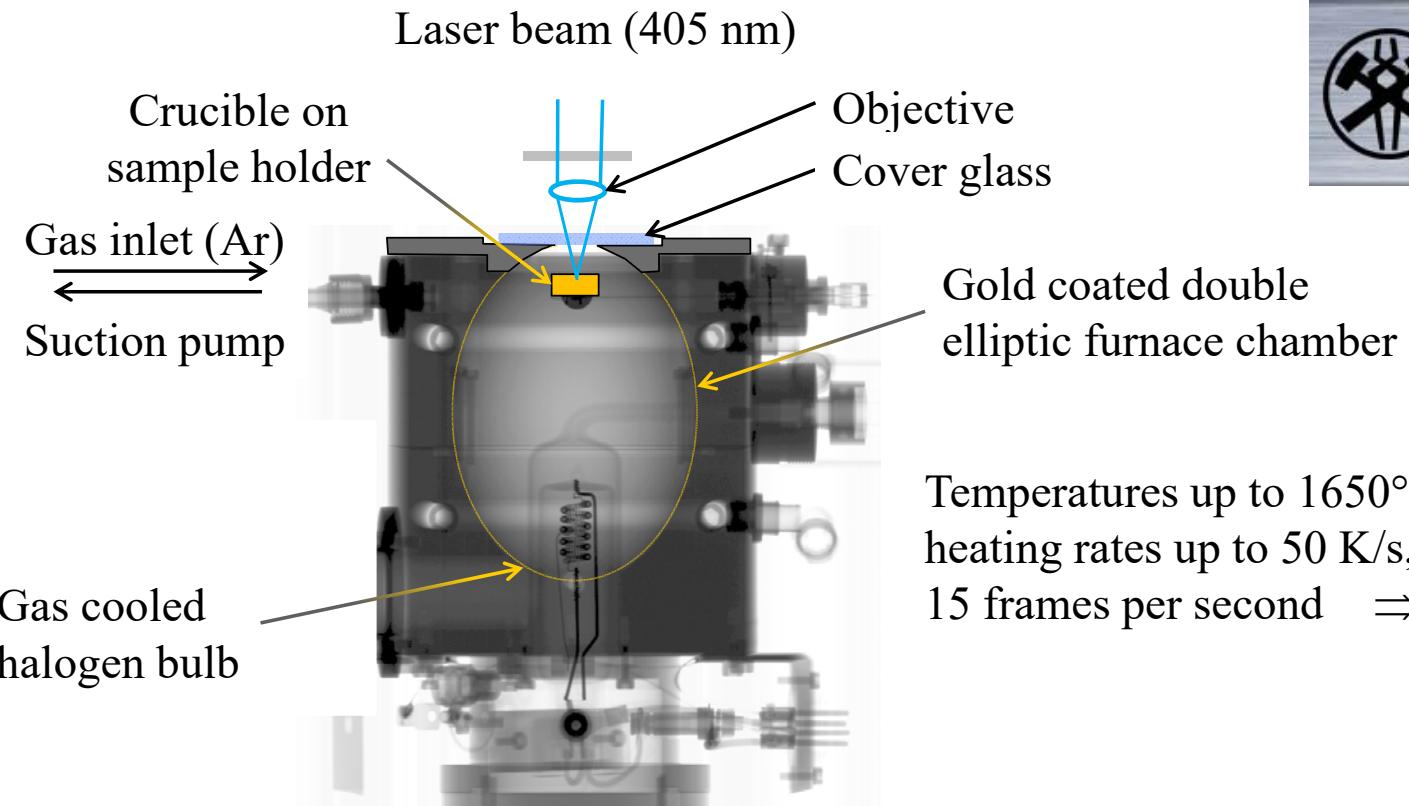
$$v_i^n = -m_i \gamma_i K_i$$



Joseph-Louis Lagrange  
[wikipedia.org](https://en.wikipedia.org)

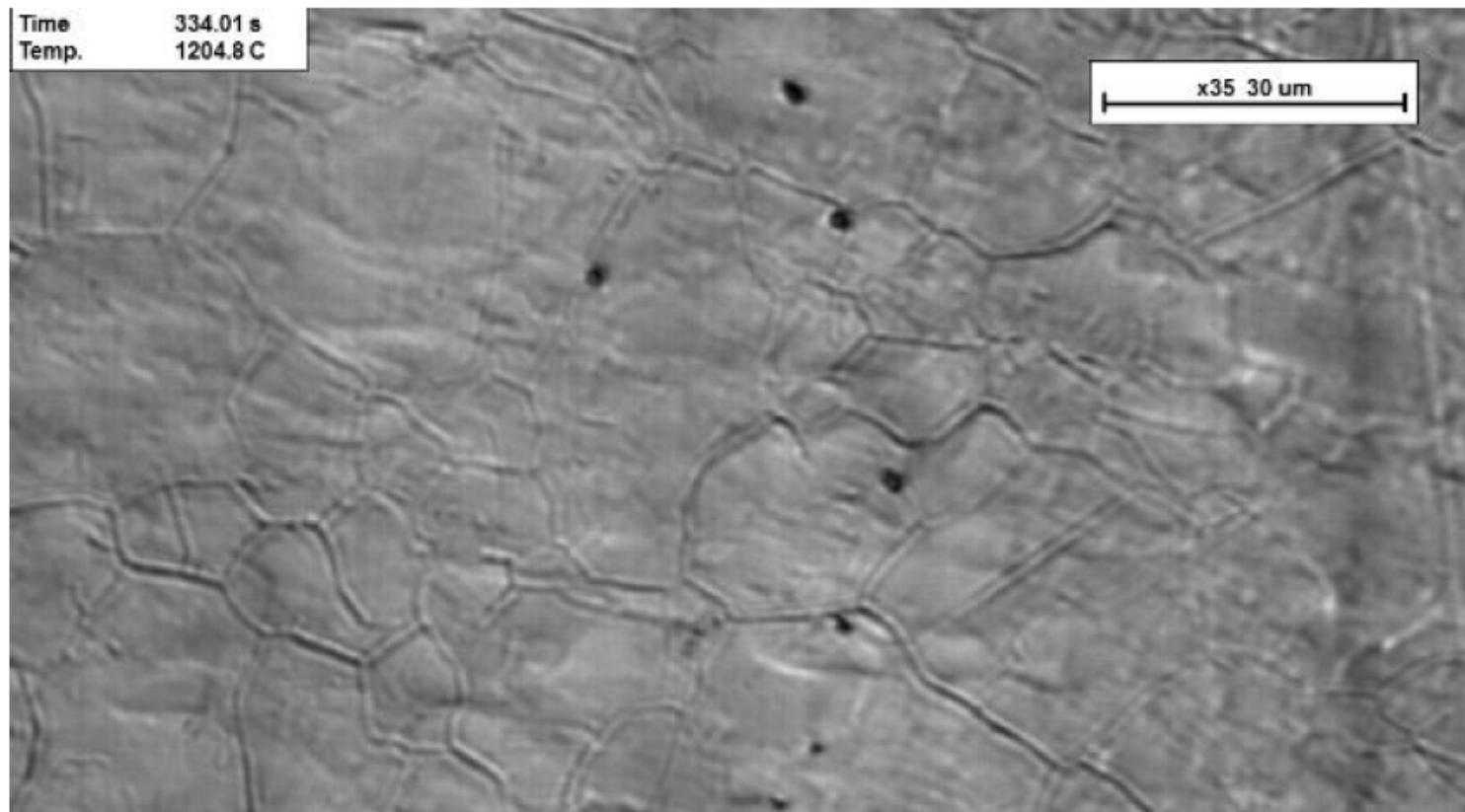
# HT-LSCM

High temperature laser scanning confocal microscopy



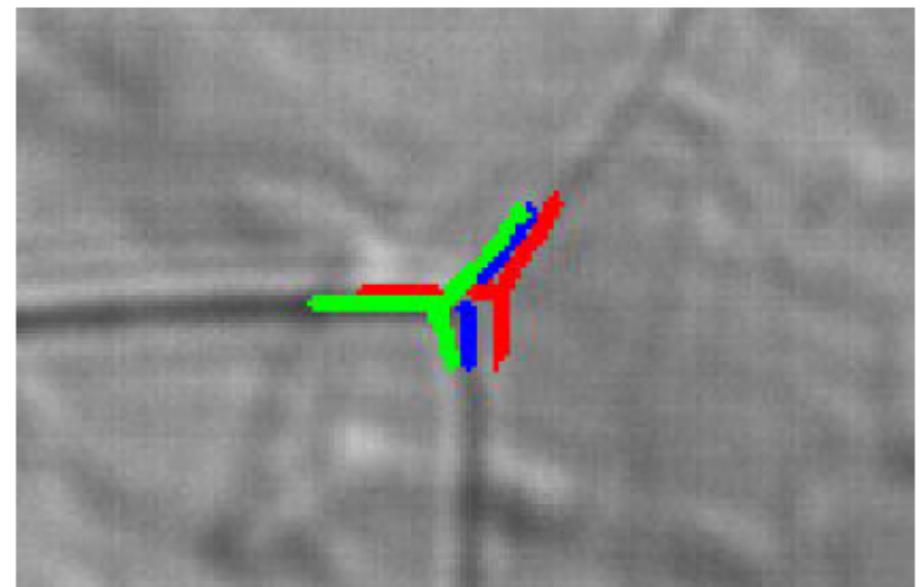
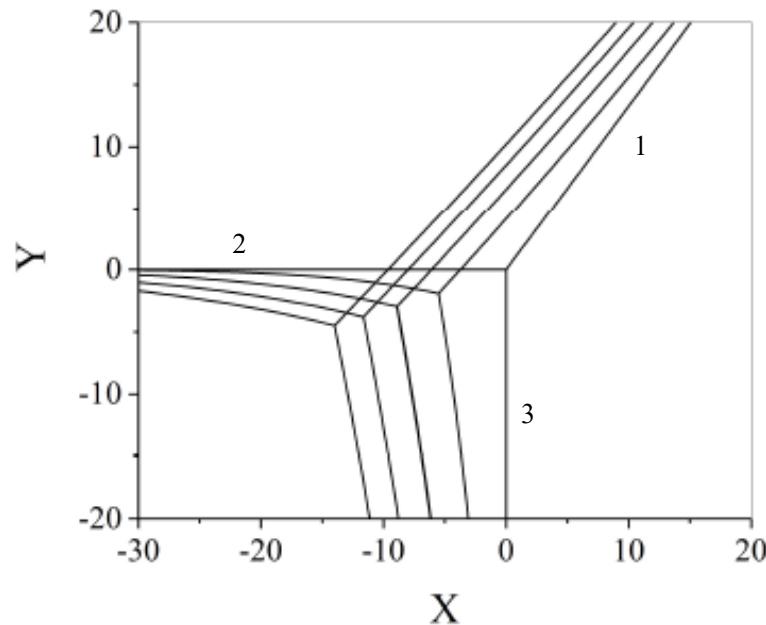
Radiographie Jördis Rosc/ÖGI

## Austenite grain boundaries revealed by HT-LSCM



## Motion of regions close to triple junctions I

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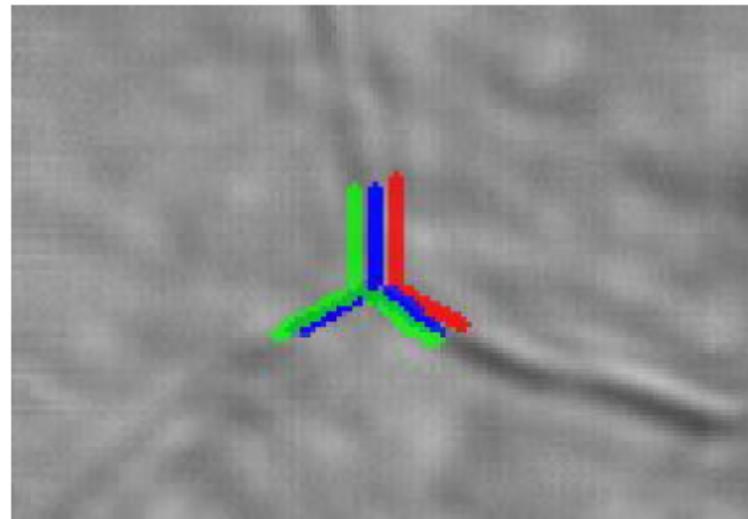
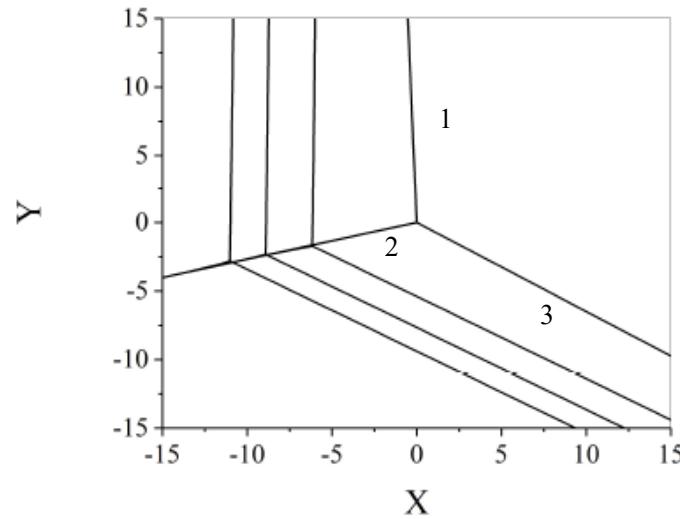
Temperature  $\vartheta = 1204.8^\circ\text{C}$ , duration of the motion **8s**.

**Best result:**  $\overline{M}_1 = 2, \overline{M}_2 = 0.267, \overline{M}_3 = 2.67, \overline{M}_t = 0.06$

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## Motion of regions close to triple junctions II

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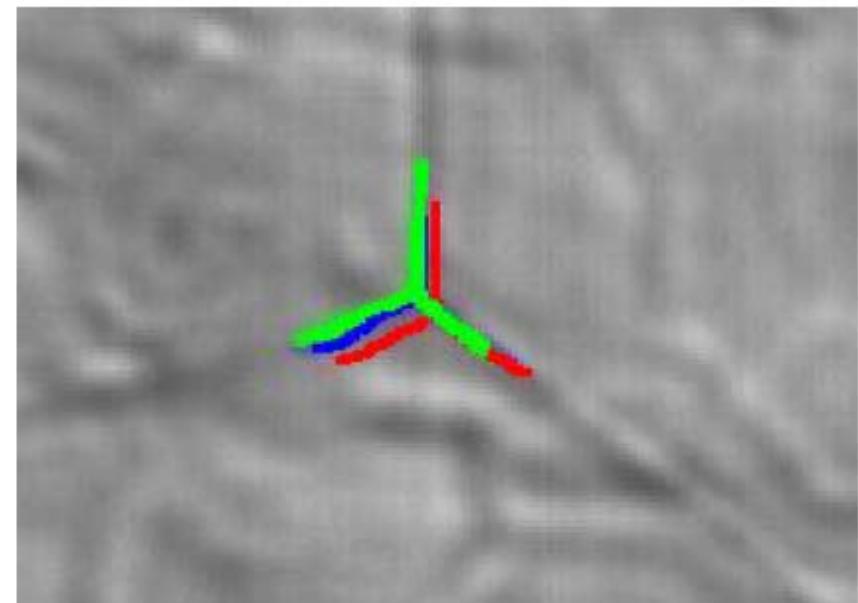
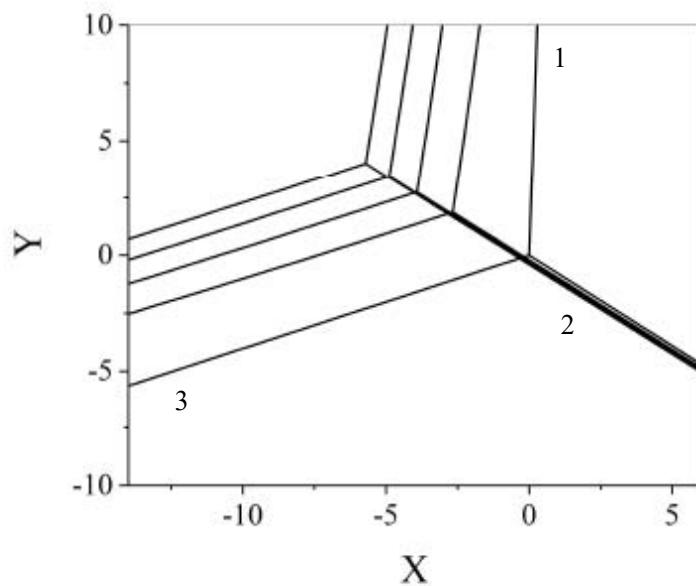
Temperature  $\vartheta = 1204.8^\circ\text{C}$ , duration of the motion **8s**.

**Best result:**  $\overline{M}_1 = 0.33$ ,  $\overline{M}_2 = 0.000067$ ,  $\overline{M}_3 = 0.267$ ,  $\overline{M}_t = 0.033$

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## Motion of regions close to triple junctions III

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Temperature  $\vartheta = 1204.8^\circ\text{C}$ , duration of the motion **8s**.

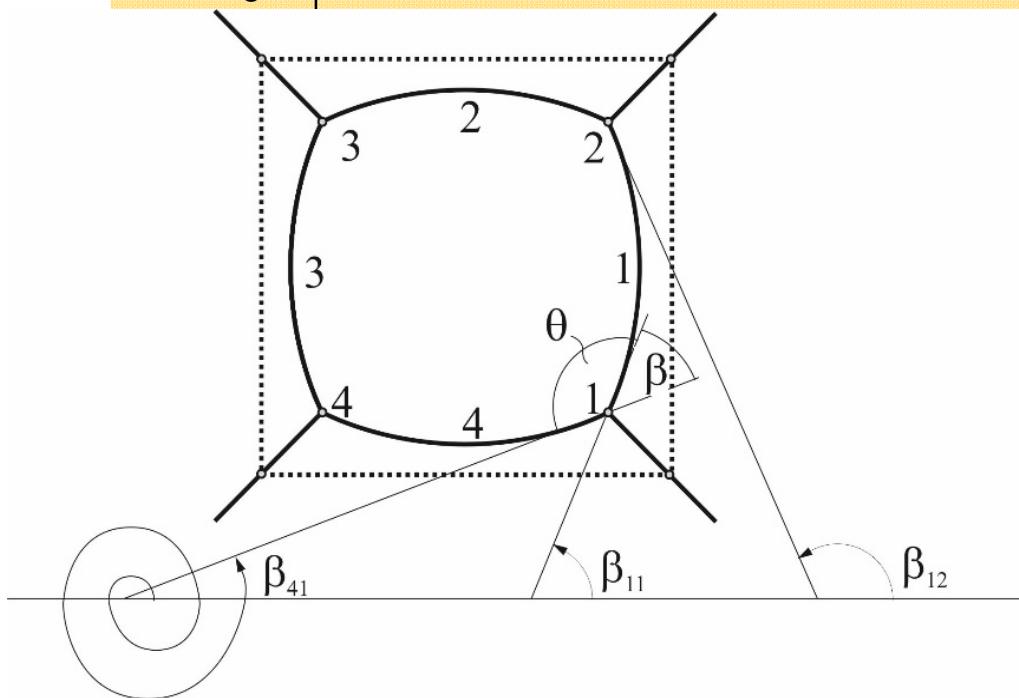
**Best result:**  $\overline{M}_1 = 3, \overline{M}_2 = 0.3, \overline{M}_3 = 3, \overline{M}_t = 0.06$

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# Theory IV

Rate of area change  $\dot{A}$

$$\dot{A} = m_{gb}\gamma \left[ (\beta_{11} - \beta_{n1}) + (\beta_{22} - \beta_{12}) + (\beta_{33} - \beta_{23}) + \dots + (\beta_{nn} - \beta_{(n-1)n}) \right]$$



From: W. W. Mullins, Two dimensional motion of idealized grain boundaries, J. Appl. Phys. 27 (1956) 900-904

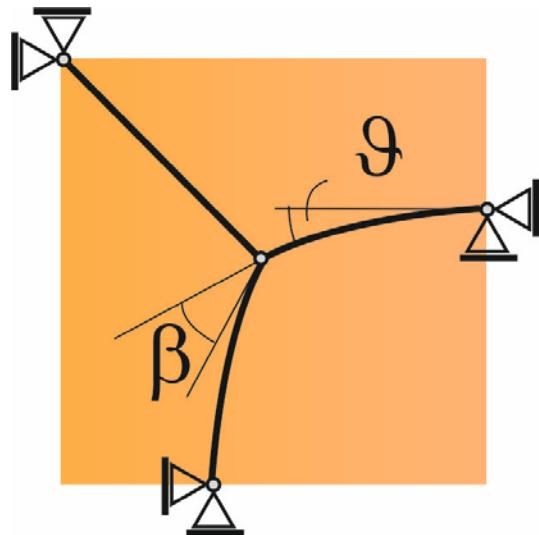
# Comparison of infinite and finite grains

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Rate of area change  $\dot{A}$  for  $n$ -sided polygons

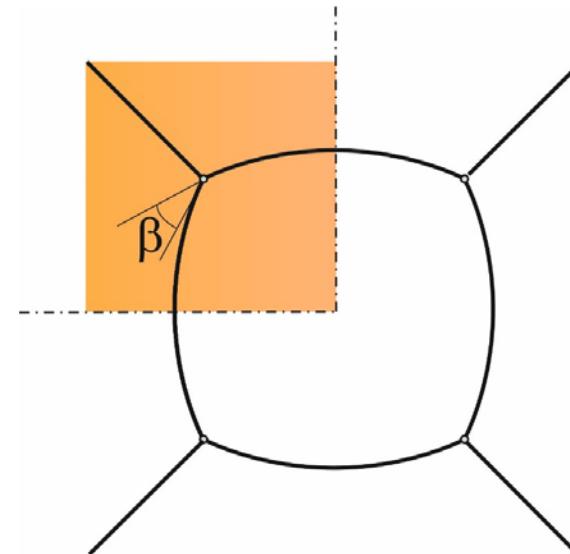
Case I: Infinite grains

$$\dot{A}_1 = m_{gb}\gamma \left[ n\beta - (2\pi - n\vartheta) \right]$$



Case II: Finite grains

$$\dot{A}_2 = m_{gb}\gamma(n\beta - 2\pi)$$



## Normalized quantities

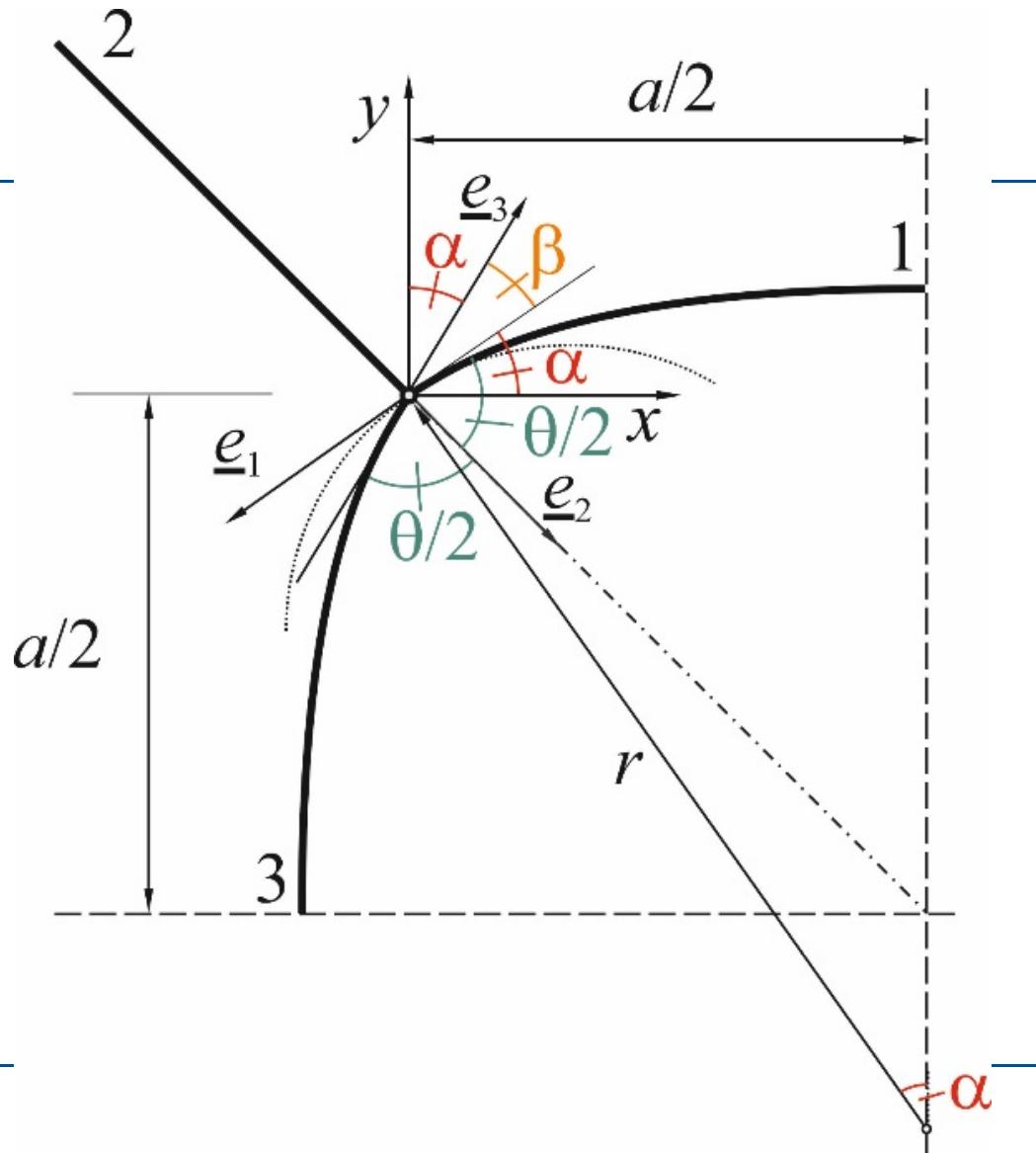
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$$\bar{\Lambda} = \frac{\Lambda}{a} a_0 = \frac{m_{\text{T}}}{m_{\text{gb}}} a_0$$

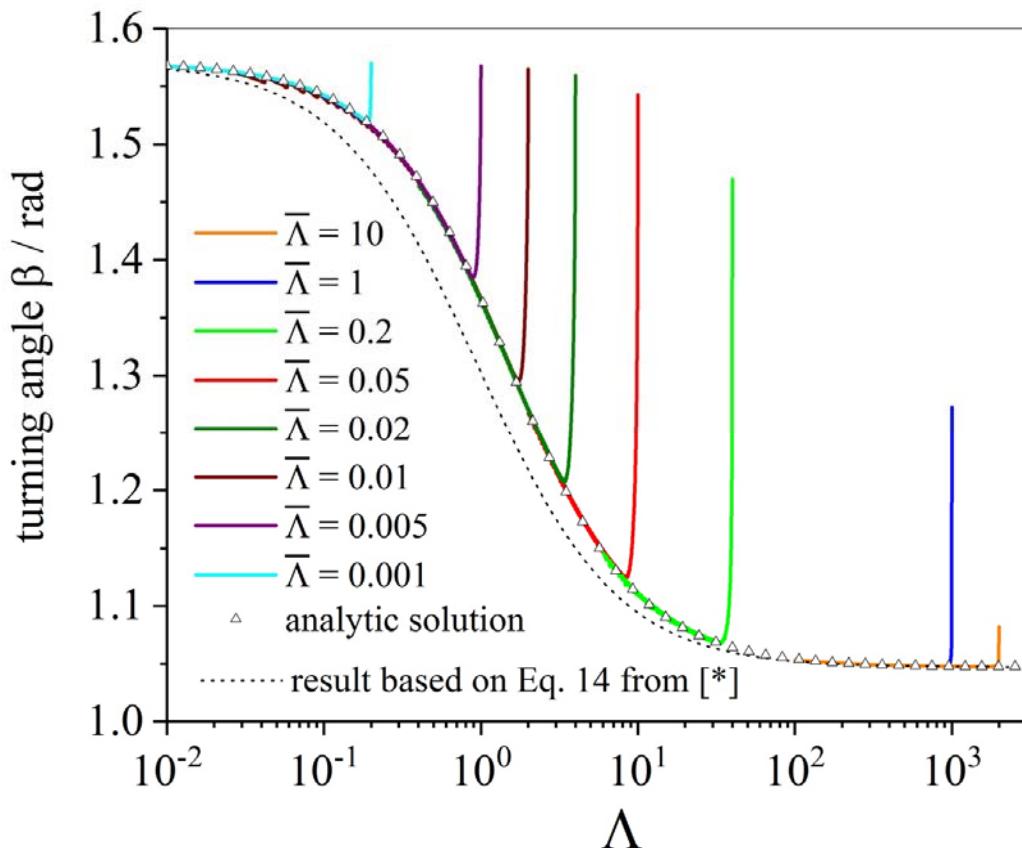
$$\Lambda = \frac{m_{\text{T}} a}{m_{\text{gb}}}$$

$$\tau = \frac{A_0}{m_{\text{gb}} \gamma}$$

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# Shrinkage of a quadratic grain I

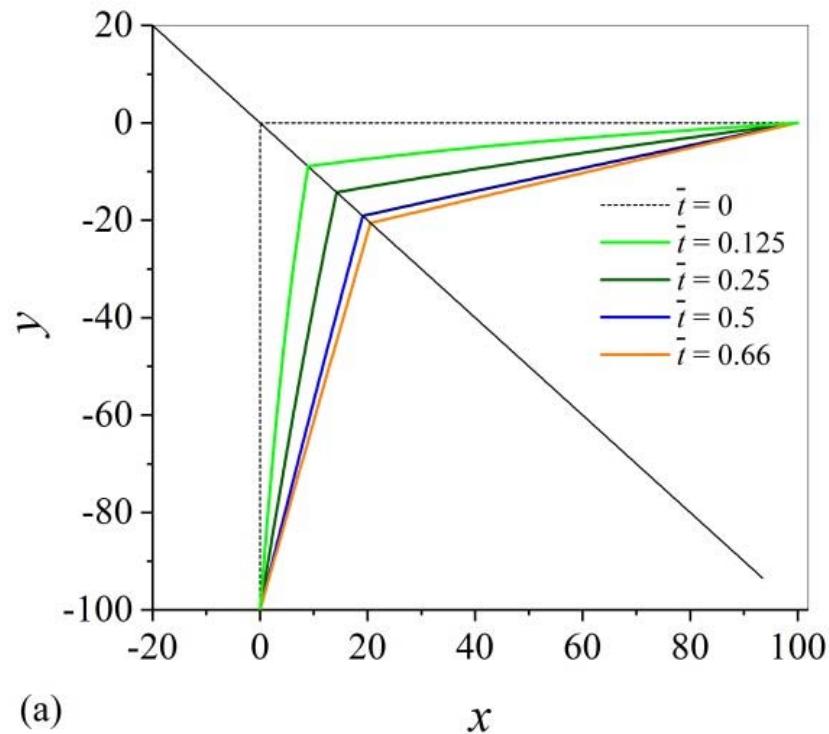


$$\Lambda = \frac{m_T a}{m_{gb}}$$

[\*] L.A. Barrales-Mora, G. Gottstein, L.S. Shvindlerman, Effect of a finite boundary junction mobility on the growth rate of grains in two-dimensional polycrystals, Acta Mater. 60 (2012) 546–555.

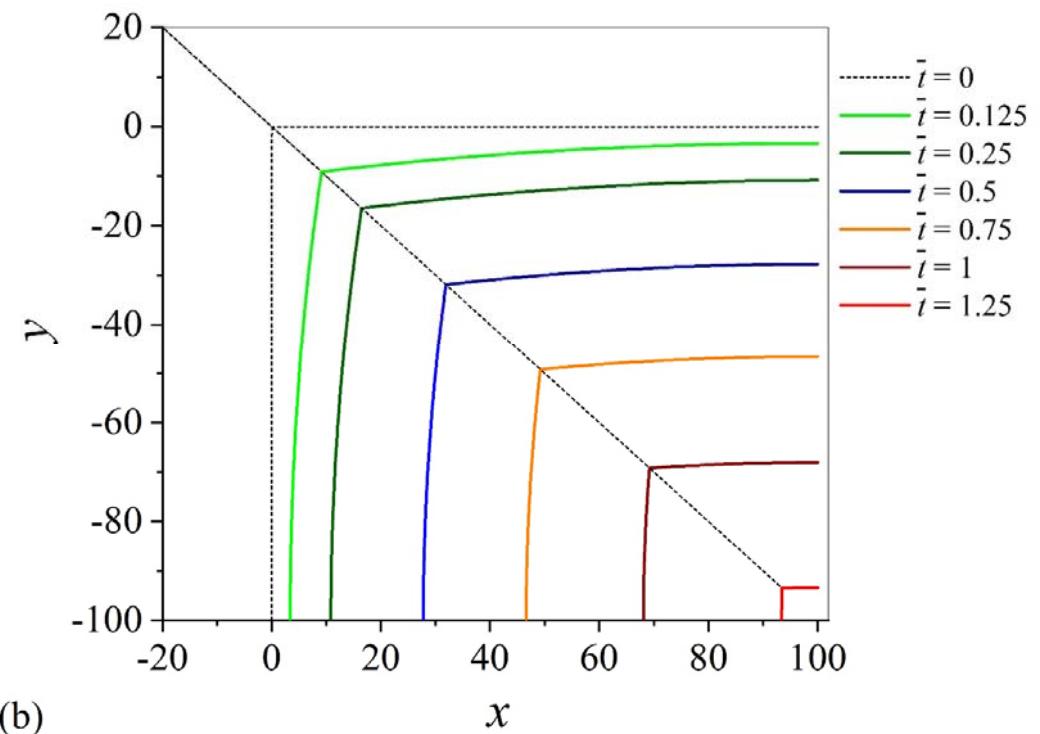
## Shrinkage of a quadratic grain II

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(a)

(a) Case I: Infinite grain

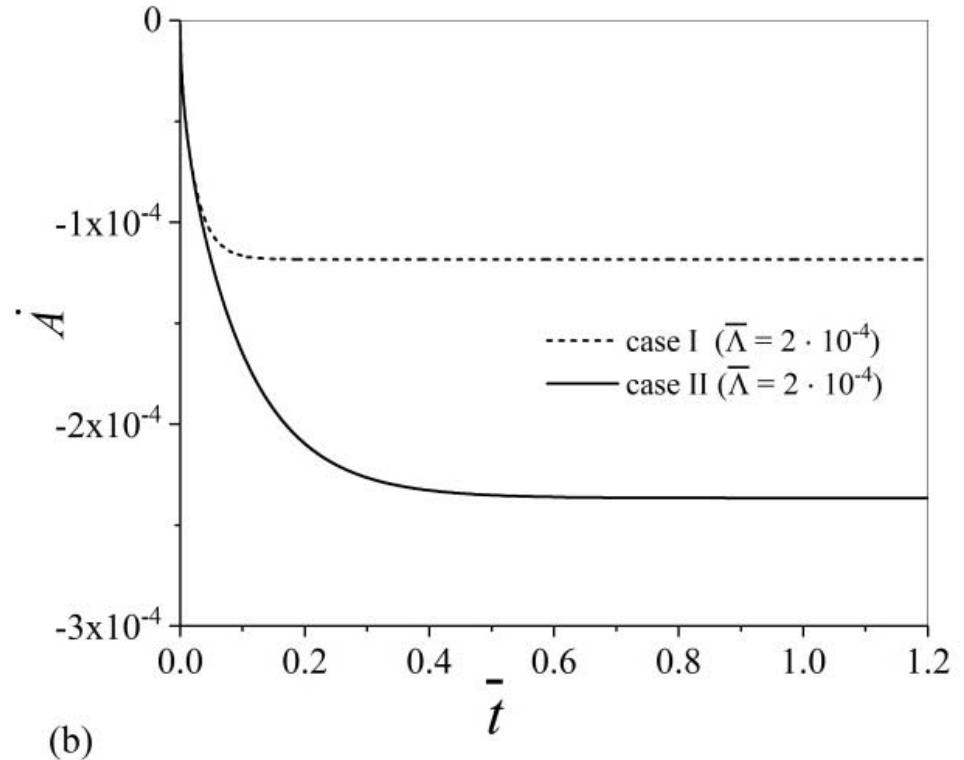
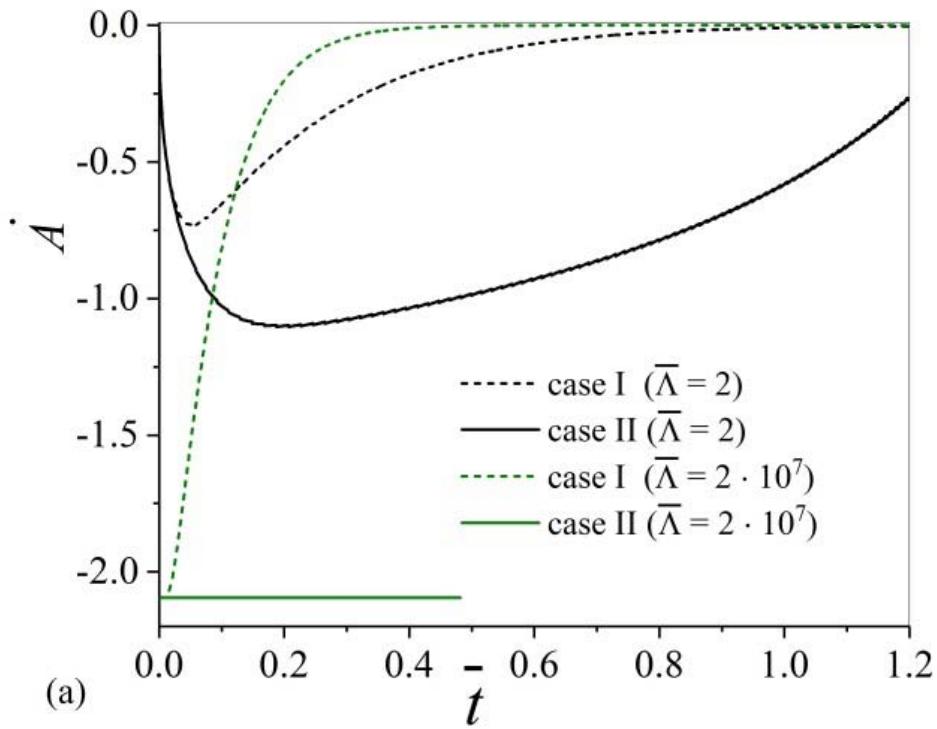


(b)

(b) Case II: Finite grain

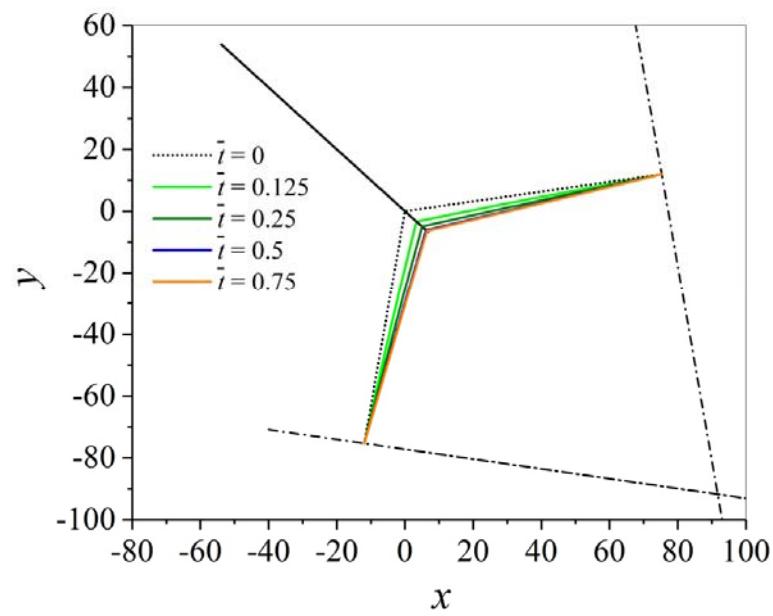
# Shrinkage of a quadratic grain III

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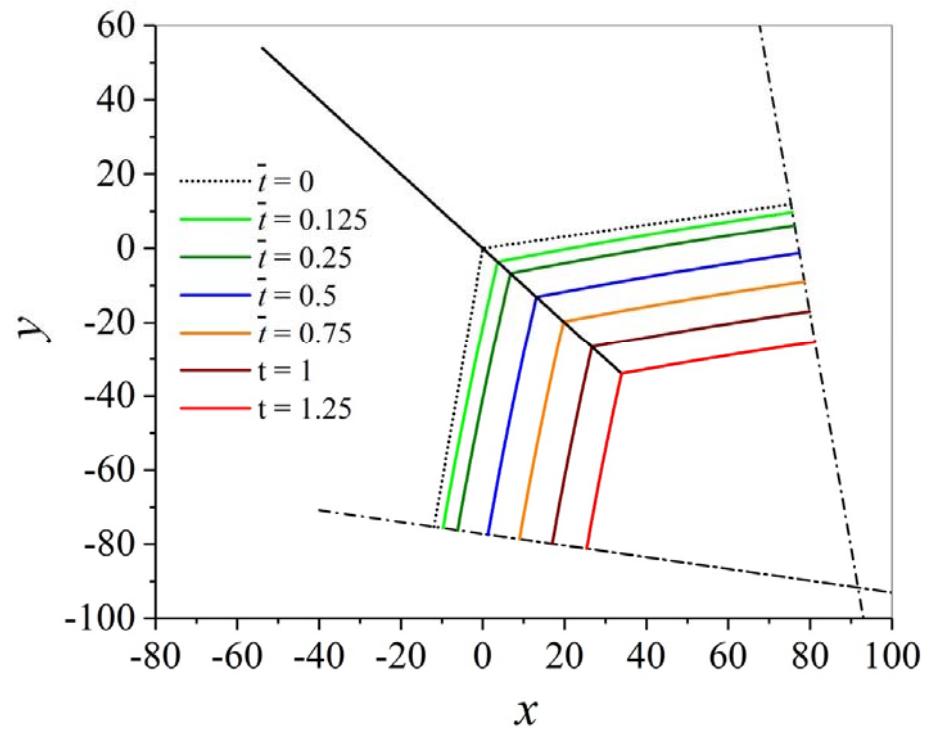


## Shrinkage of a pentagon

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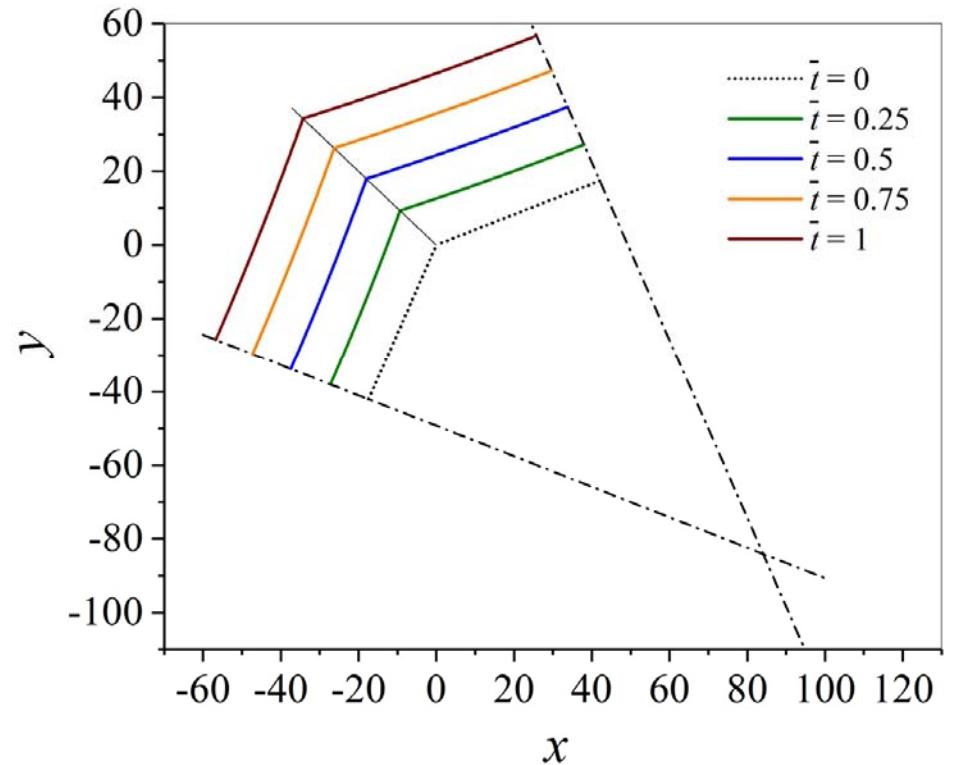
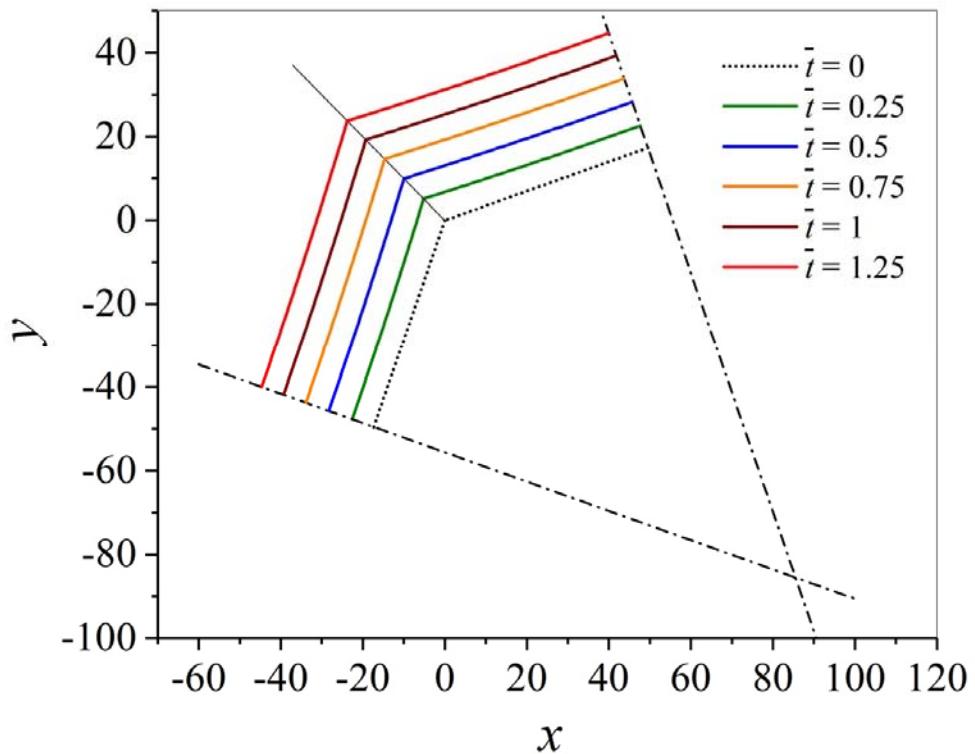
(a) Case I: Infinite grain



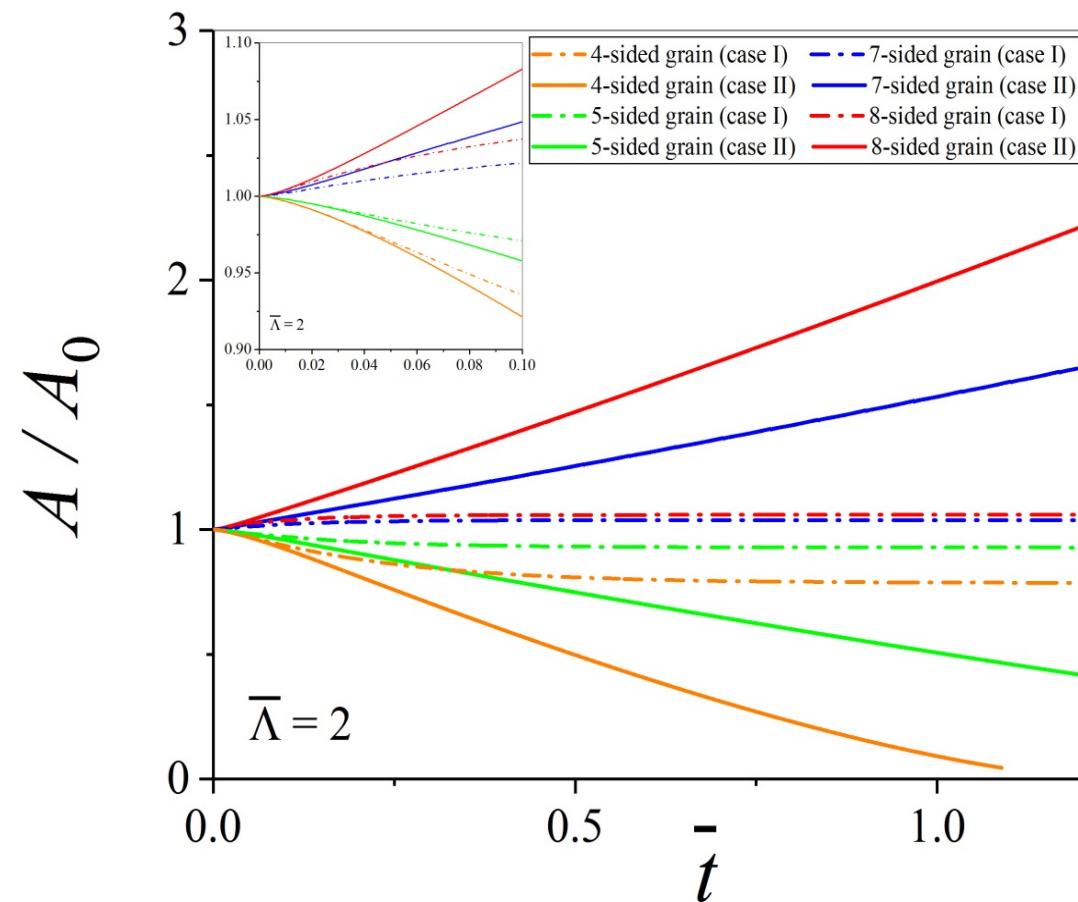
(b) Case II: Finite grain

## Growth of a heptagon and an octagon

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## Growth or shrinkage of n-sided grains



# Conclusions and Outlook

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- ***2dim-grain growth by motion of triple junctions***
    - Motion of triple junction – comparison to in-situ experiments
    - Influence of the topology
    - Initially quadratic grain
      - Change of the turning angle
      - Dependence on the ratio  $\bar{\Lambda}$
    - Growth and shrinkage of n-sided grains
  - ***Outlook***
    - Comparison to in-situ micrographs – influence of alloying elements on migrating grain boundaries
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**Thank you for your attention!**

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