



# **Ab-initio simulations and their application to modelling solute drag during the austenite-to-ferrite transformation**

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# Density Functional Theory (DFT)

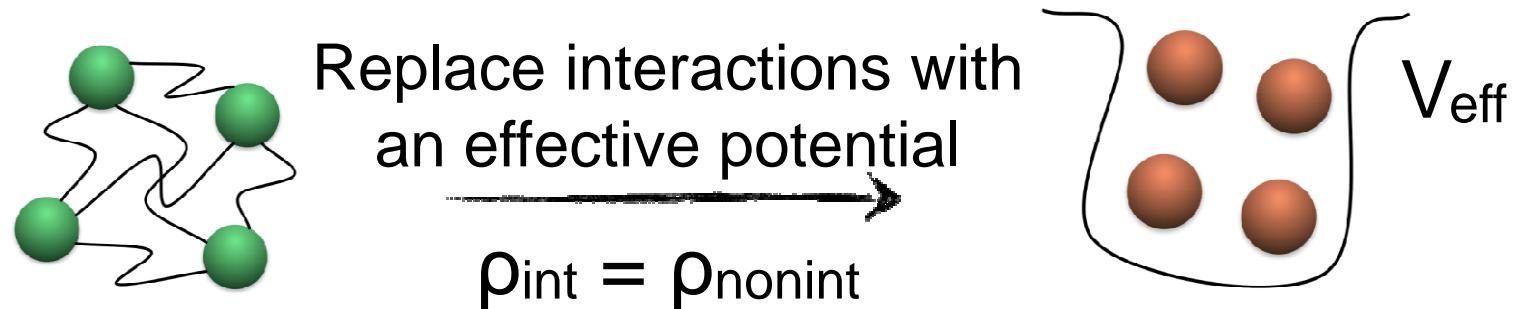
Hohenberg and Kohn :

$$\rho(\vec{r}) \xrightarrow{F(\rho(\vec{r}))} E_{Total}$$

Electron density

Total energy

Khon and Sham:



Since  $\rho$  is the same, the total energy must be the same too.

Need to find approximations for  $V_{eff}$  using theory of interacting electron gas (LDA, GGA).

**Note: Ground state theory –  $T = 0$  K, can consider  $\sim 100$  atoms**

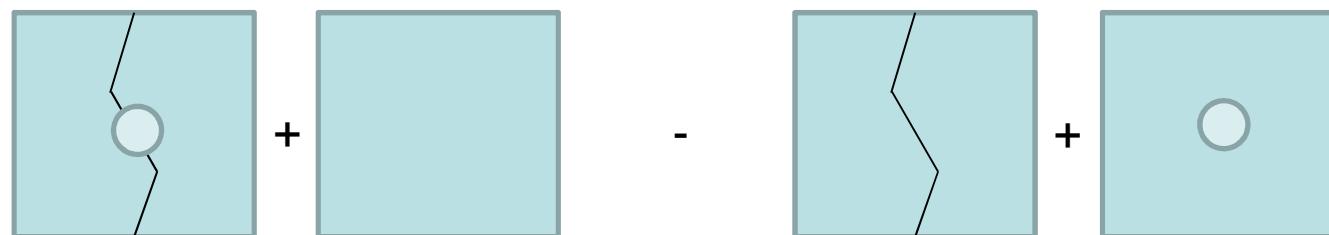


# DFT Simulations for Interfaces



- DFT calculations using the VASP code
- Periodic boundary conditions: Can only consider special boundaries (e.g. low  $\Sigma$  boundaries) with small unit cell
- Solute-grain boundary (GB) binding energies (segregation energy)—cost to move solute from the  $i$ th GB site to the bulk:

$$E_s = (E_{X+GB} + E_{ref}) - (E_{GB} + E_X)$$

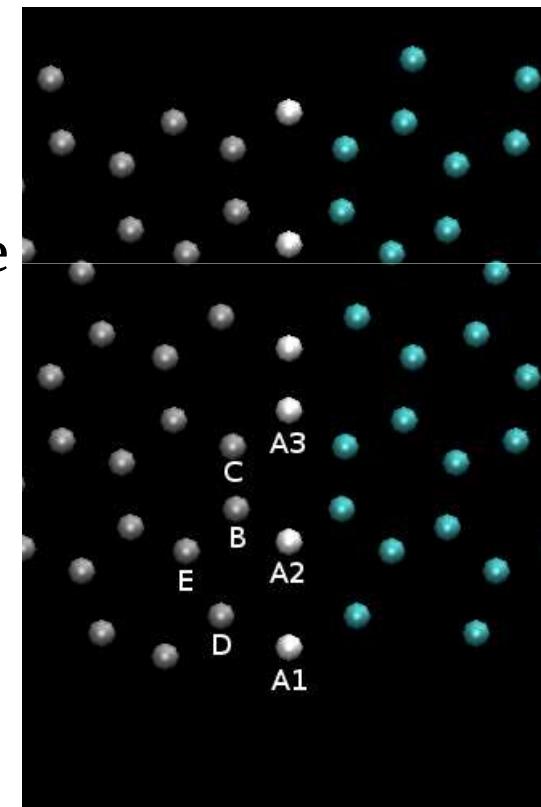




# $\Sigma 7$ Grain Boundary in Magnesium

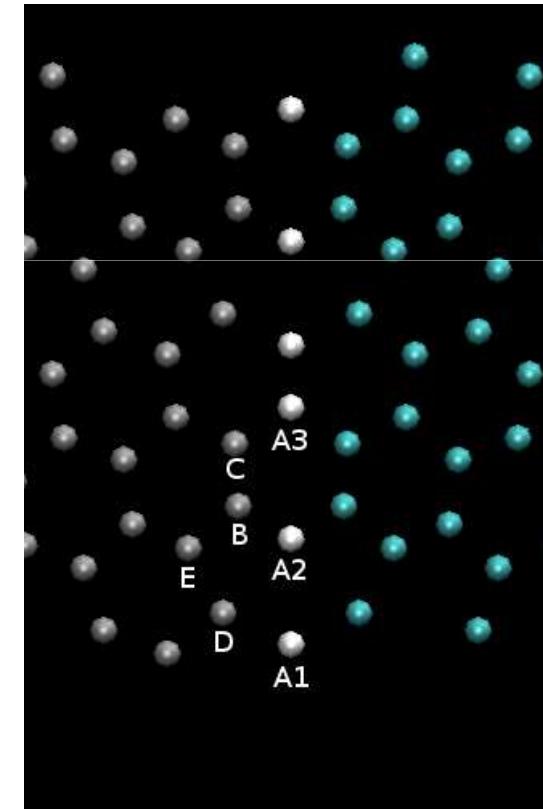
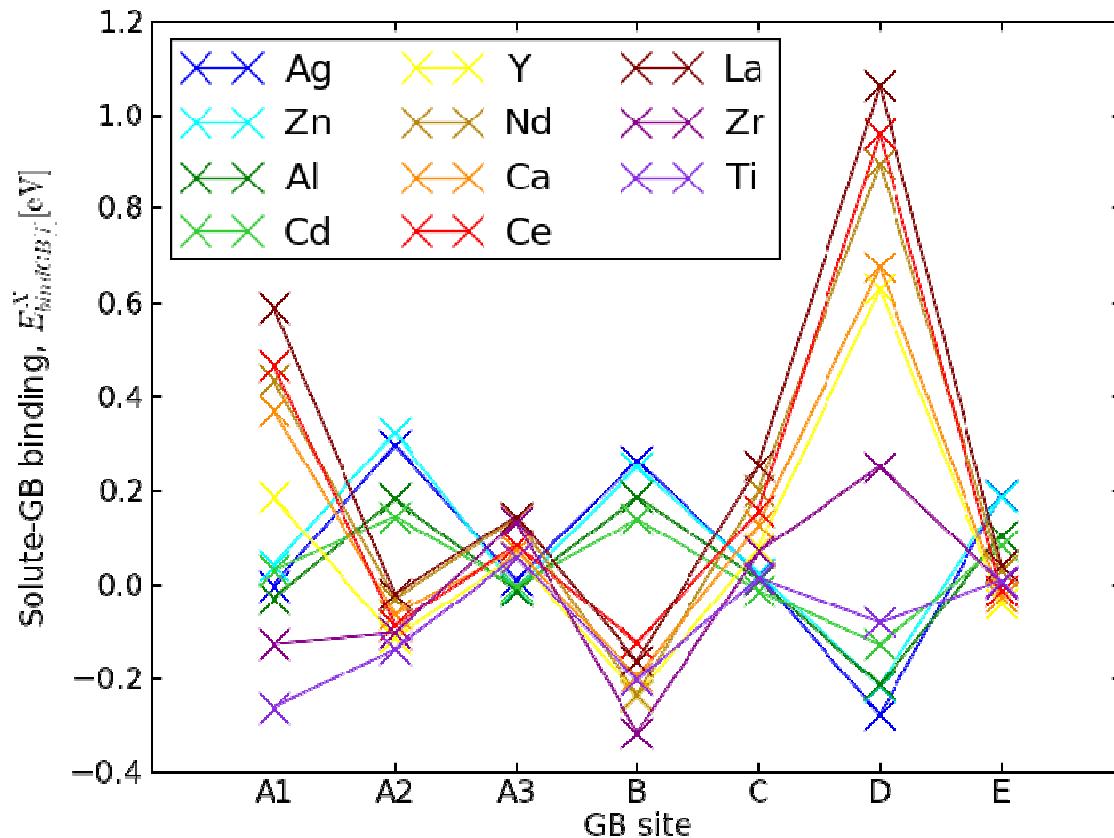


- $\Sigma 7$  tilt ( $21.8^\circ$  on  $0001$  axis) boundary with two conformations (A- and T-type)
  - Repeats over short enough distance to be used in DFT
  - 232 (184) atom cells used for A-(T-)type boundary
- A-type grain boundary looking down  $c$ -axis
  - Grain 1 in grey, grain 2 in green, and coincident sites in white
  - Sites labelled by distance from GB plane



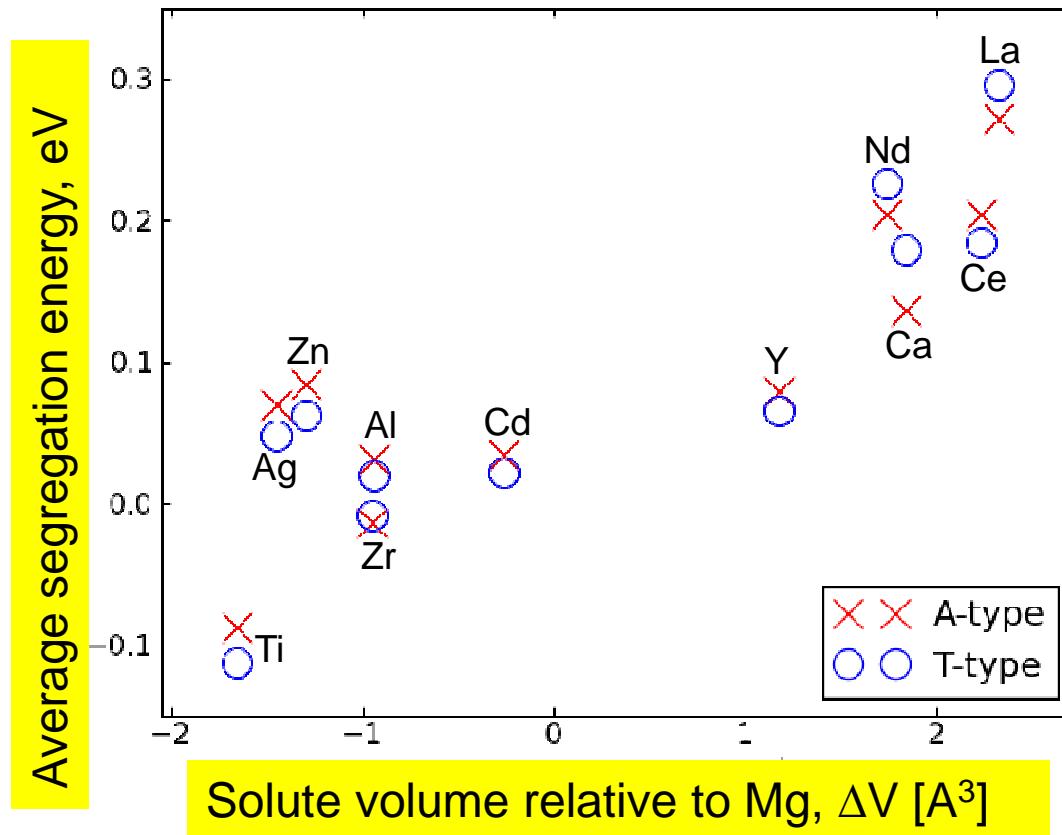
# Solute Segregation Energies at $\Sigma 7$ in Mg

- Three families of behaviour: La-like, Al-like, Zr-like



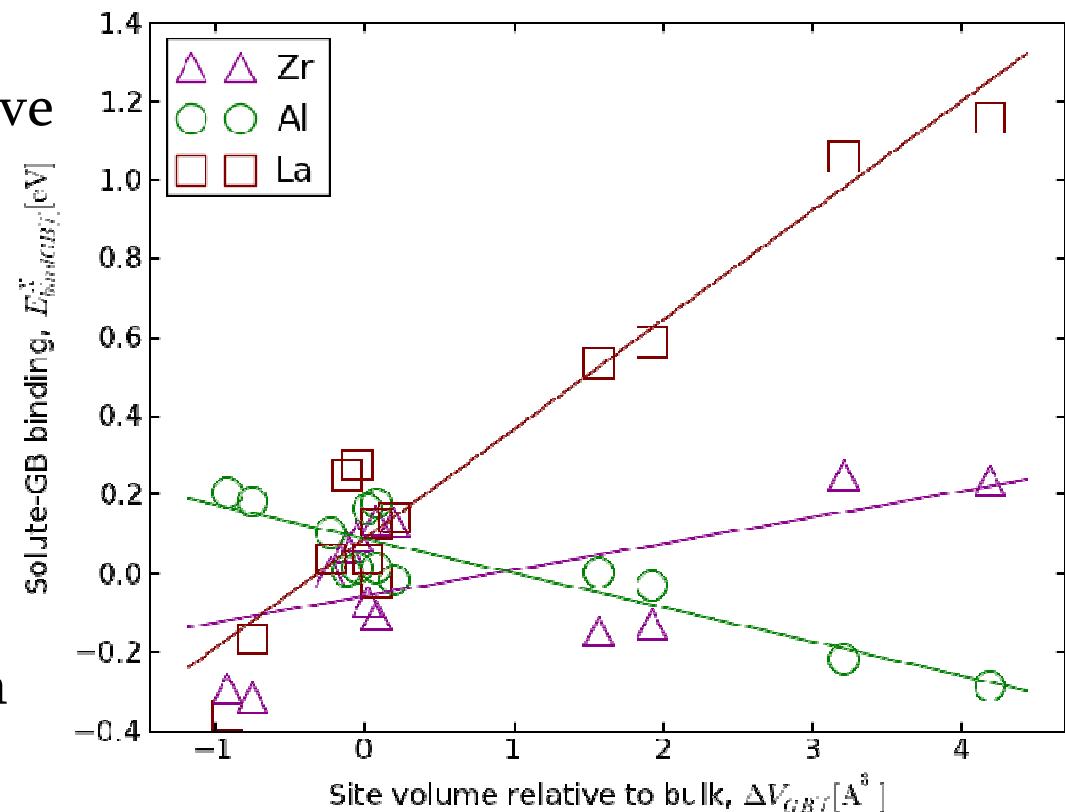
# Average Segregation Energy at $\Sigma 7$ in Mg

- A- and T-type boundaries similar
- In general, larger solutes with stronger binding



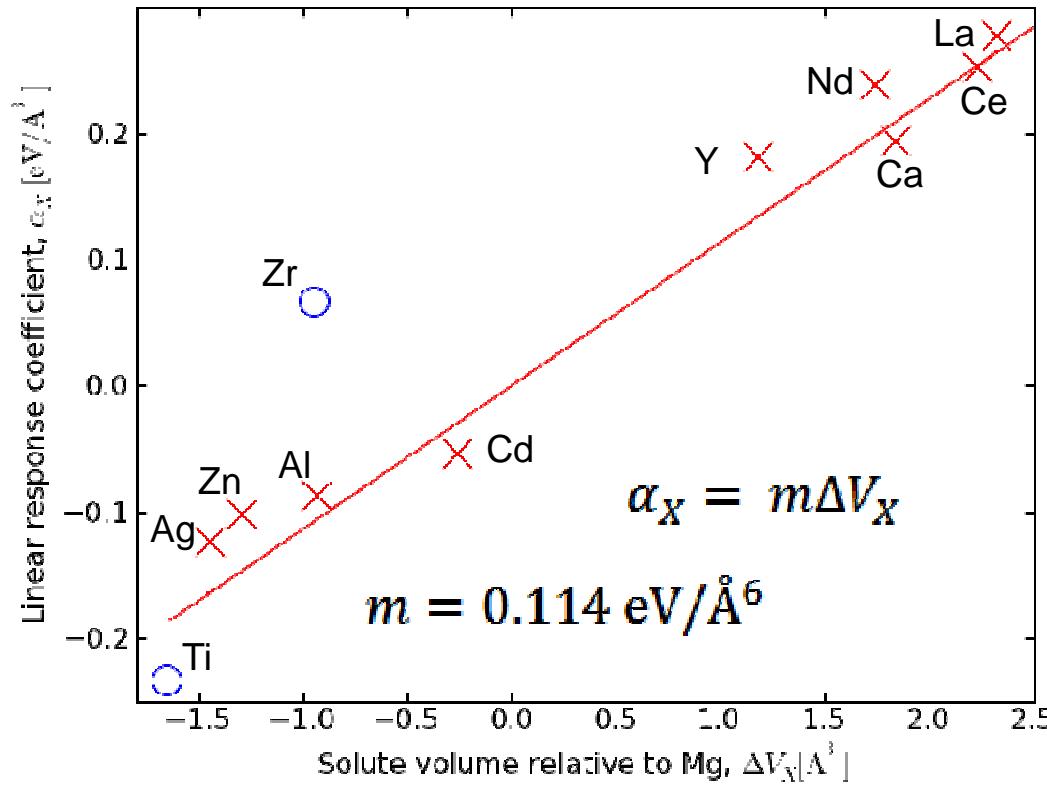
# Elastic Analysis

- GB sites may have different volume compared to bulk sites—what is binding dependence?
  - Al, La, and Zr representative of their families.
  - Al and La: simple linear model works well
- $$E_{bind\ GB[i]}^X = \alpha_X \Delta V_{GB[i]} + b_X$$
- Zr family: poor correlation with excess site volume



# Elastic Analysis

- Linear elastic response as function of solute volume (relative to Mg)

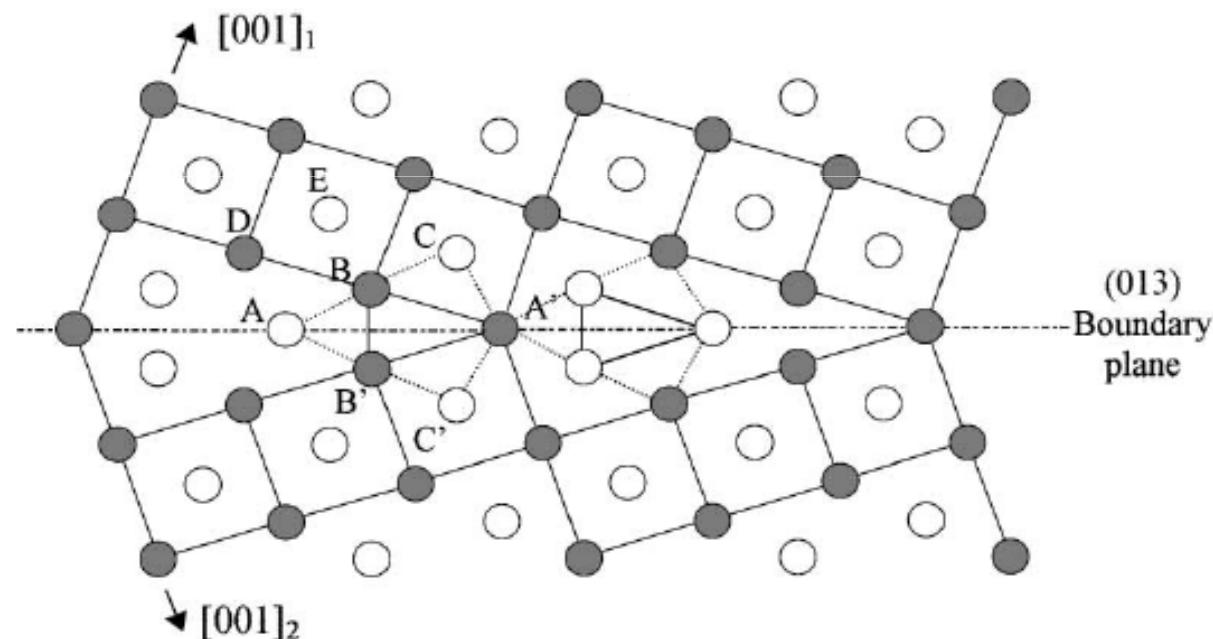


- Most solutes of interest in Mg elastically dominated solutes
- May be able to extend to random boundaries using molecular mechanics



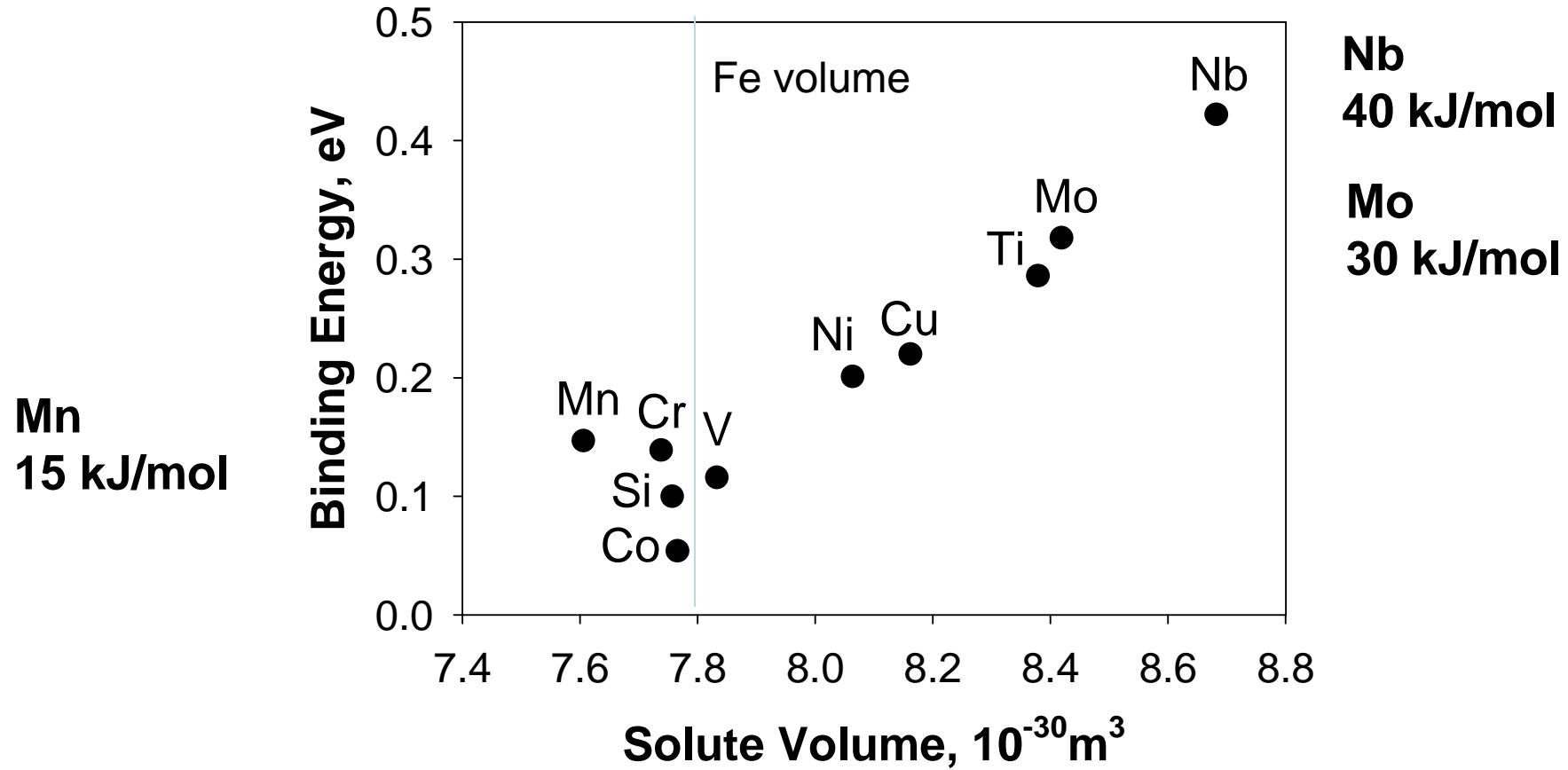
# $\Sigma 5$ Grain Boundary in BCC-Iron

$\Sigma 5$  [001] {013} Tilt Grain Boundary in BCC-Fe





# Average Binding Energies at $\Sigma 5$ in BCC-Fe

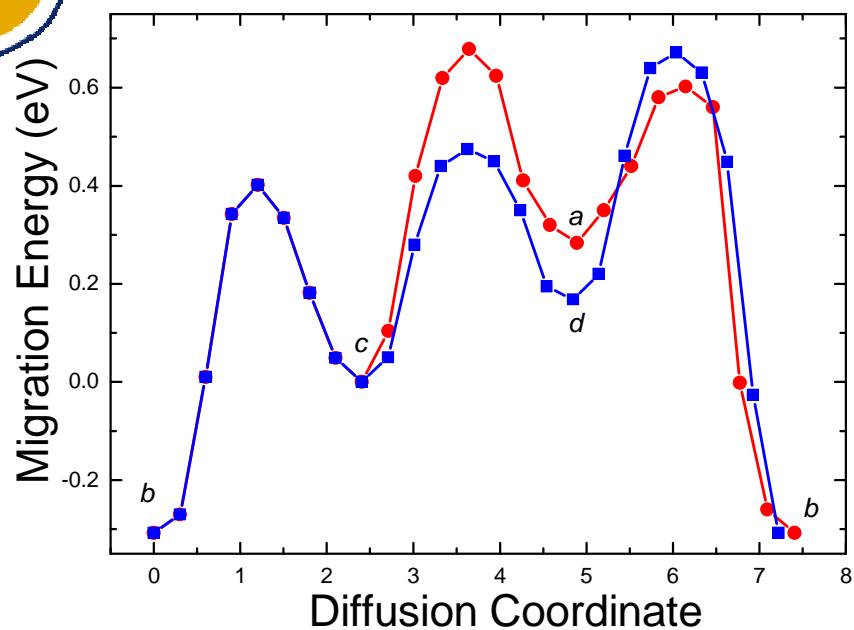


Binding energy increases with size of solute atom  
but also magnetic contributions (see e.g. Cr and Mn)

Nb with higher binding energy compared to Mo, Ti, Mn, Si and V  
→ strong interaction of Nb with grain boundaries (consistent with recrystallization delay)



# Grain Boundary Diffusion



Determine activation energies  
for individual jumps from DFT

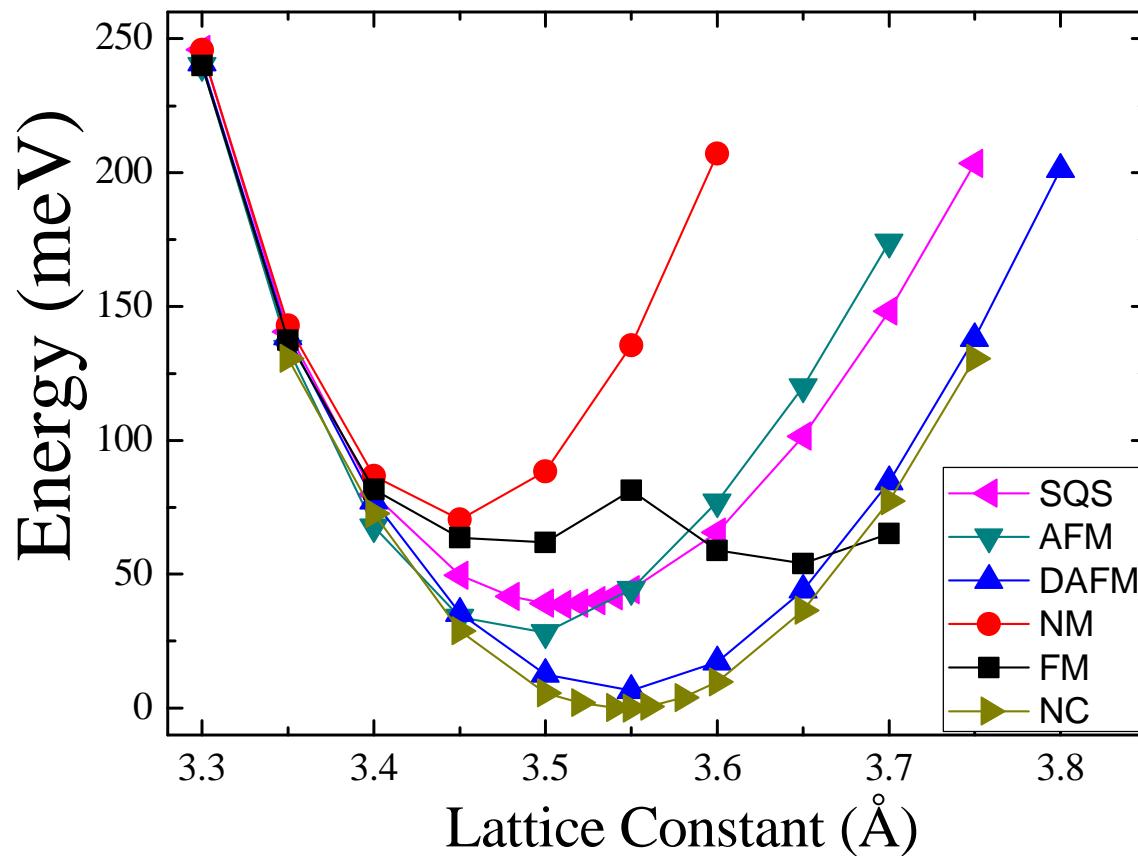
Kinetic Monte Carlo  
simulation to obtain effective  
grain boundary diffusivity:  
 $D_b = D_0 \exp(-Q_b/RT)$

	Fe	V	Mn	Cu	Ti	Nb	Mo
$Q_b$ (eV)	2.24	2.05	1.97	2.11	1.92	1.88	2.09
$Q_{bulk}$ (eV)	2.84	2.6	2.37	2.49	2.35	2.21	2.53
$Q_b/Q_{bulk}$	0.79	0.79	0.83	0.85	0.82	0.85	0.83

Activation energies at  $\Sigma 5$  boundary  
approximately 80-85% of those for bulk diffusion



# Magnetic Configurations for FCC-Fe

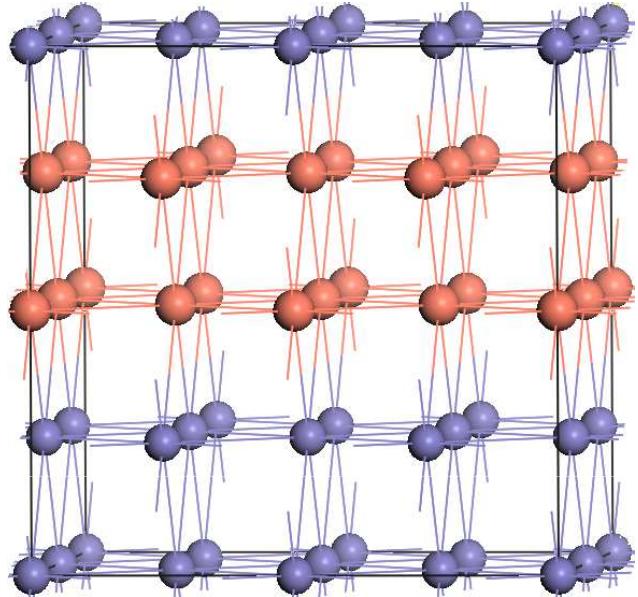


Non-Magnetic (NM), Ferro-Magnetic (FM), Anti-FM (AFM), Special Quasi-random Structure (SQS), DAFM (double-layer AFM), and non-collinear (NC) configurations vs. the lattice constant in fcc Fe.

Note:: 3.56 Å lattice constant concluded from experimental studies



# DAFM Structure for FCC-Fe



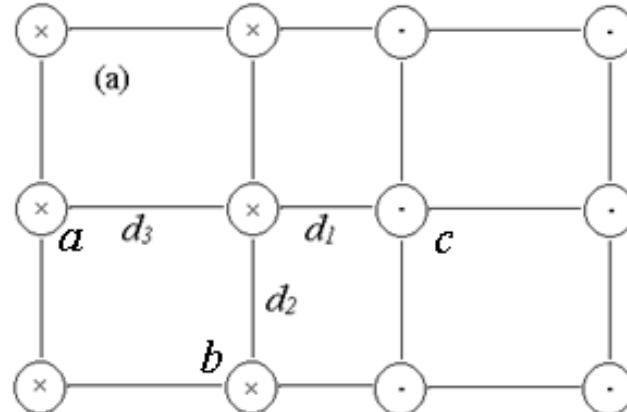
Schematics of DAFM structure for fcc Fe

Vacancy formation energy:

DAFM state: 1.72eV

Experiments [1]:  $1.70 \pm 0.2$ eV

[1] Kim SM, Buyers WJL. Journal of Physics F-Metal Physics 1978;8:L103



For non-relaxed pure bulk lattice:

$$d_1 = d_2 = d_3 = 2.51 \text{ \AA}$$

For relaxed pure bulk lattice:

$$d_1 = 2.46 \text{ \AA}, d_2 = 2.51 \text{ \AA}, d_3 = 2.56 \text{ \AA}$$

Activation energy:

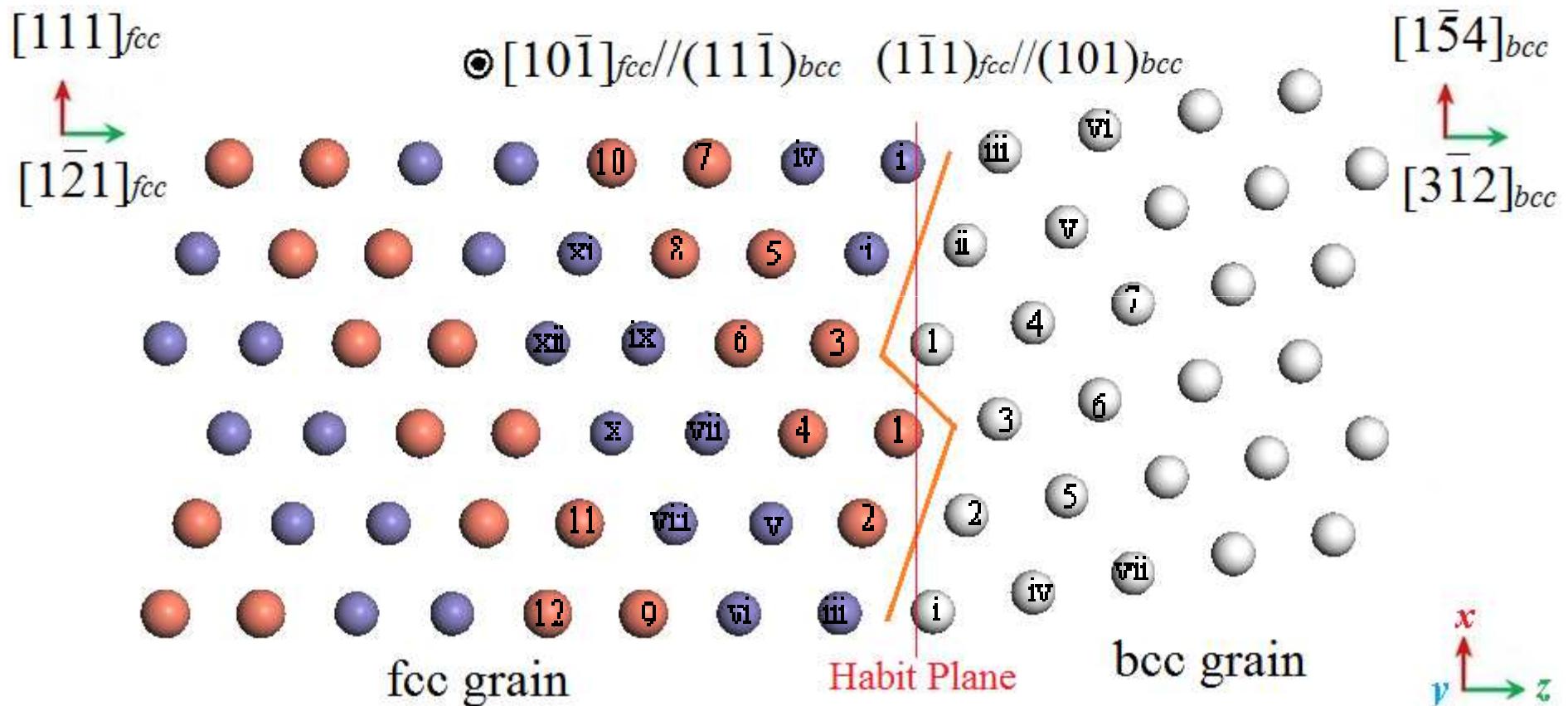
DAFM state: 2.81eV, 2.46eV, 2.65eV at  $a$ ,  $b$ ,  $c$  positions

Experiments: 2.80~2.94 eV;  
Temp. 1223~1473 K



# $\alpha$ - $\gamma$ (bcc-fcc) Interface (K-S OR)

Kurdjumov-Sachs (K-S) OR:  $(111)_{fcc} \parallel (101)_{bcc}$ ,  $[10\bar{1}]_{fcc} \parallel [1\bar{1}\bar{1}]_{bcc}$



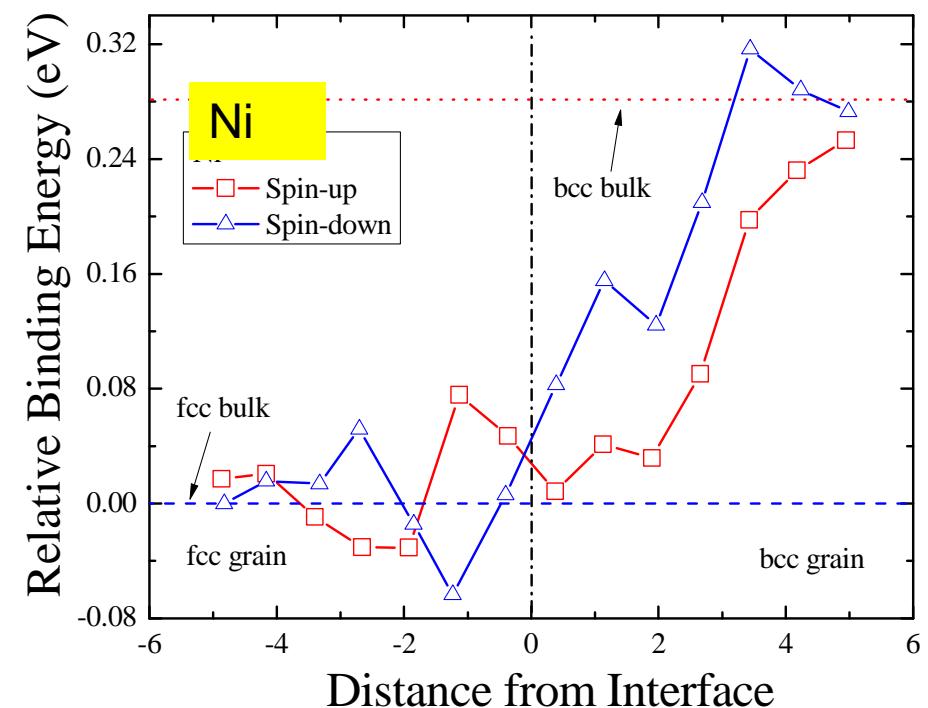
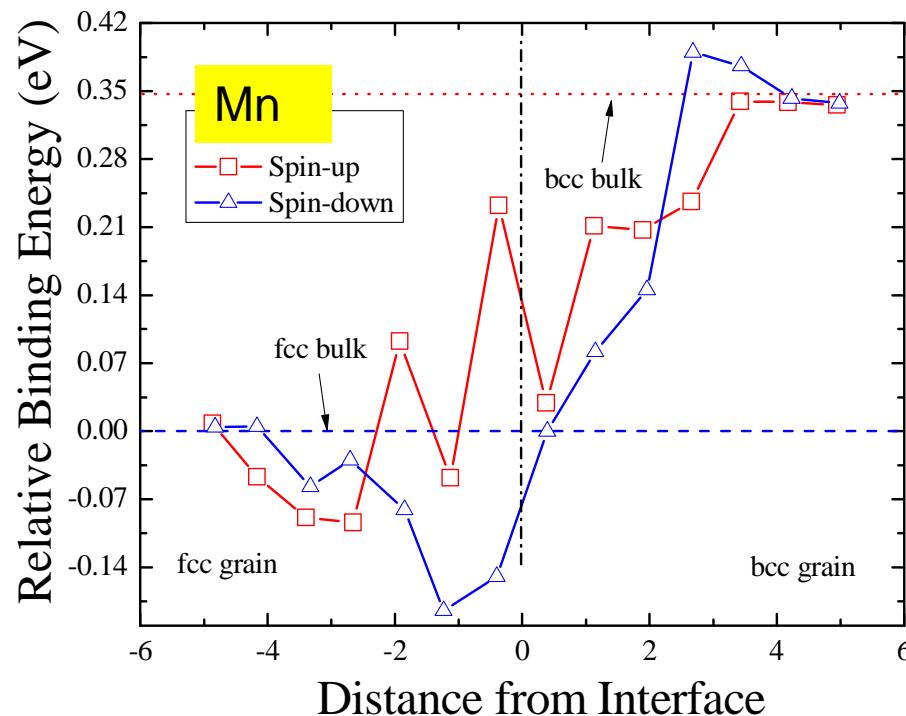


# Relative Binding Energies for Solutes with $\alpha - \gamma$ Interface

$$E_b = (E_{I+X} - E_I) - (E_{\gamma+X} - E_\gamma)$$

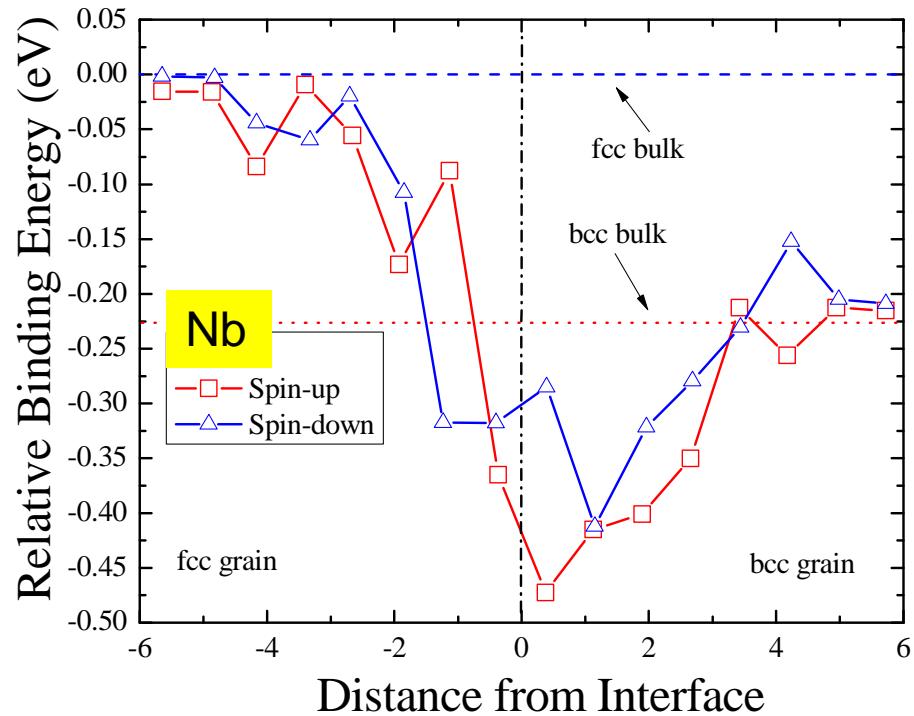
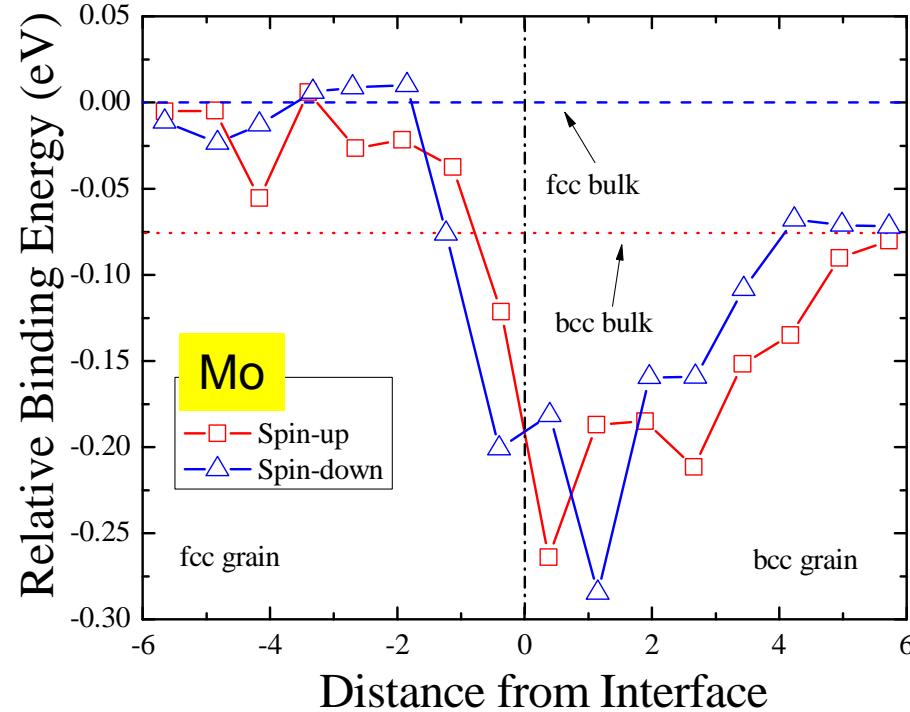
where  $E_{I+X}$  and  $E_I$  are the total energies of bcc-fcc interface with and without solute.

$E_{\gamma+X}$  and  $E_\gamma$  are the total energies of fcc bulk with and without solute.





# Relative Binding Energies for Solutes with $\alpha - \gamma$ Interface



Similar binding energies predicted for Nb, Mo and Mn as at  $\Sigma 5$ :

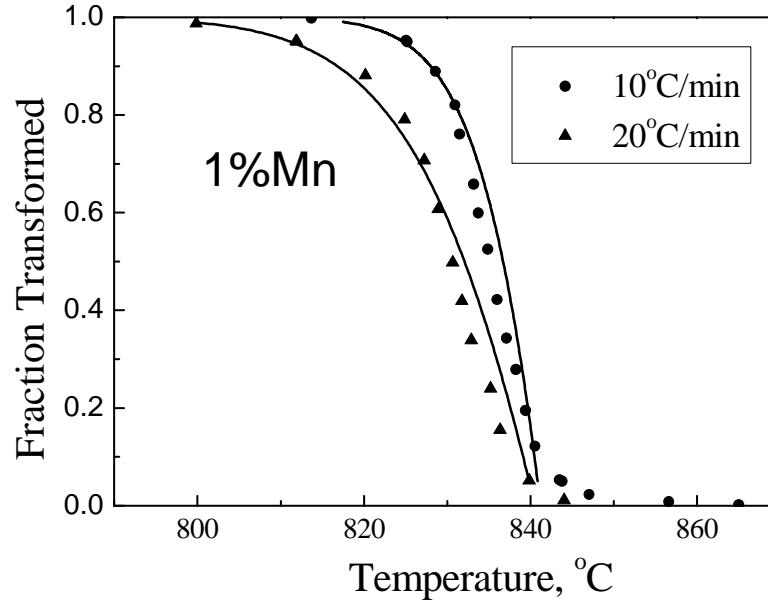
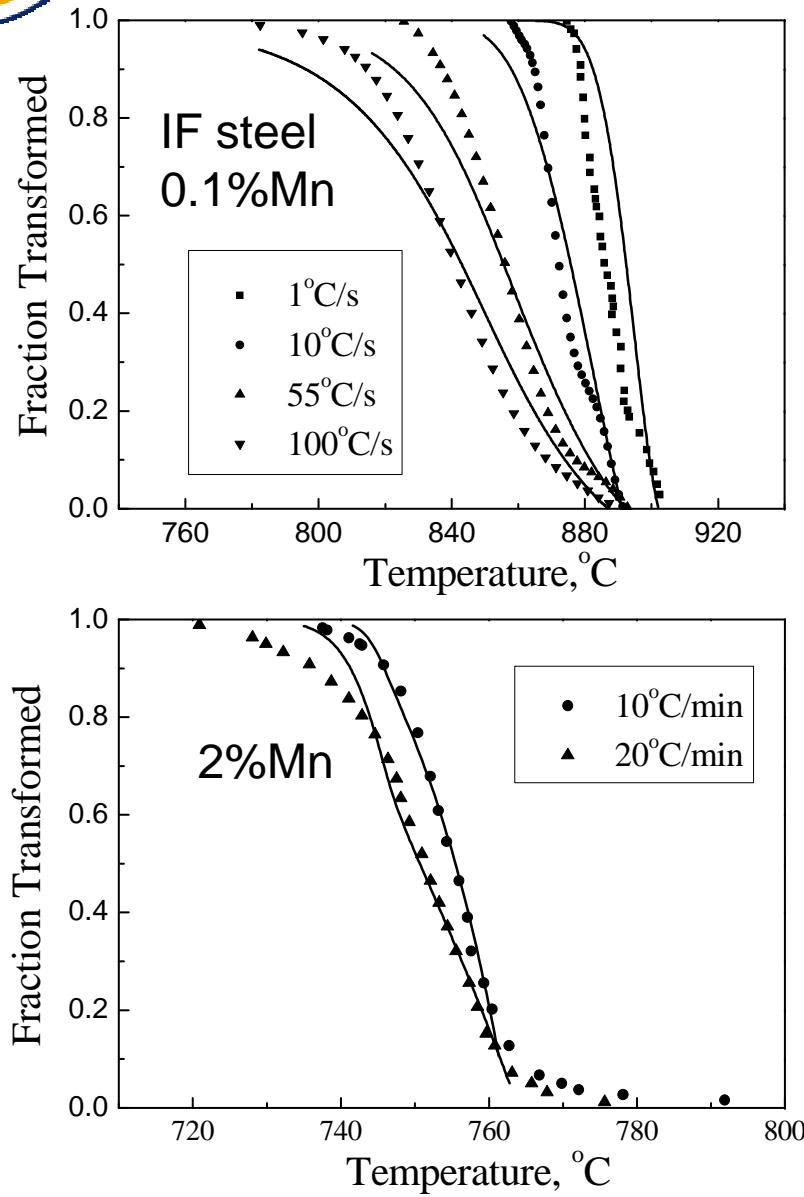
**Nb: 40 kJ/mol**

**Mo: 20-30 kJ/mol**

**Mn: 10-20 kJ/mol**

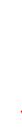


# CCT in IF Steel and Fe-Mn Alloys



Krielaart & van der Zwaag (1998)

Mn %	$M_0$ (cm mol/Js)	$Q$ (kJ/mol)	$Q_b$ (kJ/mol)	$E_0$ (kJ/mol)
0.1				16
1	450	140	216	11
2				6



94% of  
BCC bulk value



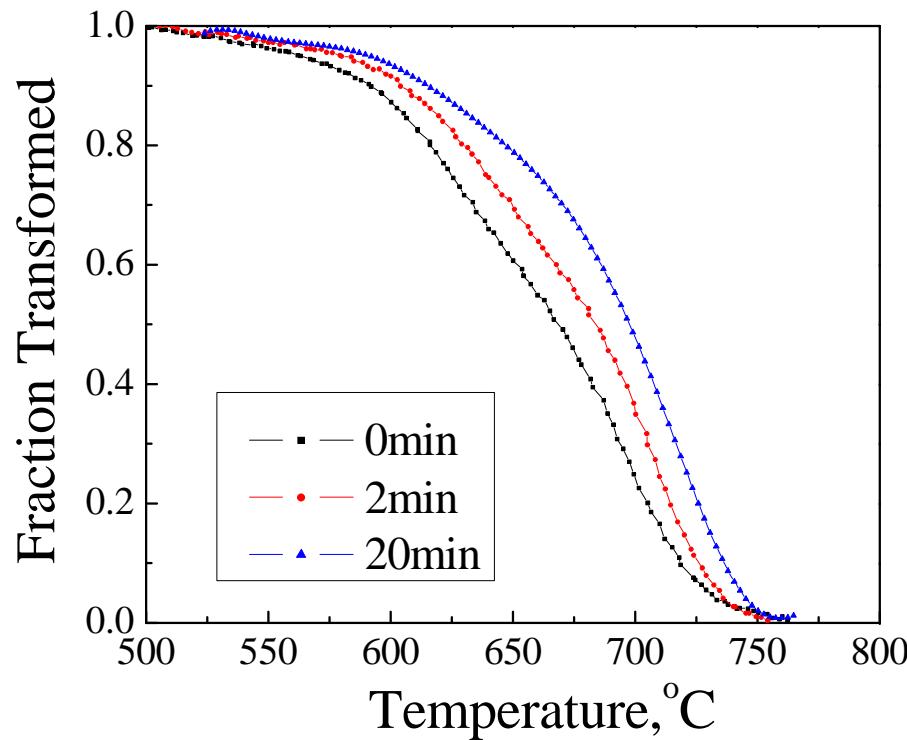
# Effect of Nb on Phase Transformation



X65 steel

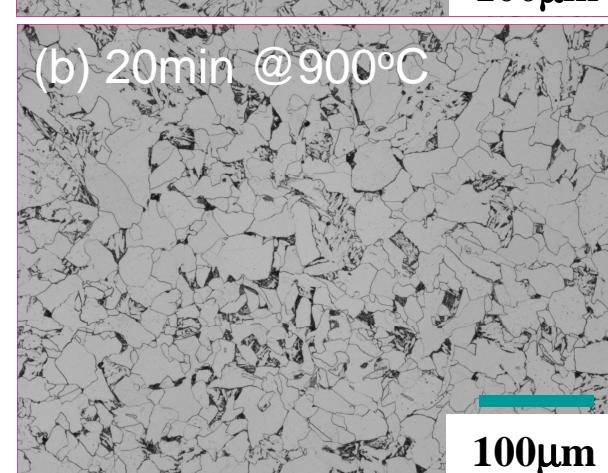
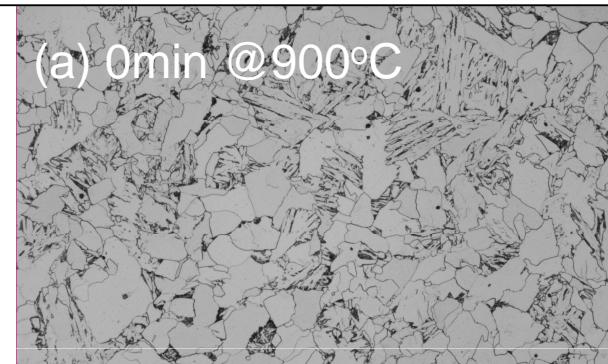
C	Mn	Si	P	S	Al	Nb	N	$A_{e3}$
0.06	1.49	0.2	0.009	0.002	0.038	0.047	0.0094	839°C

Continuous Cooling – 1°C/s



0min 2min 20min

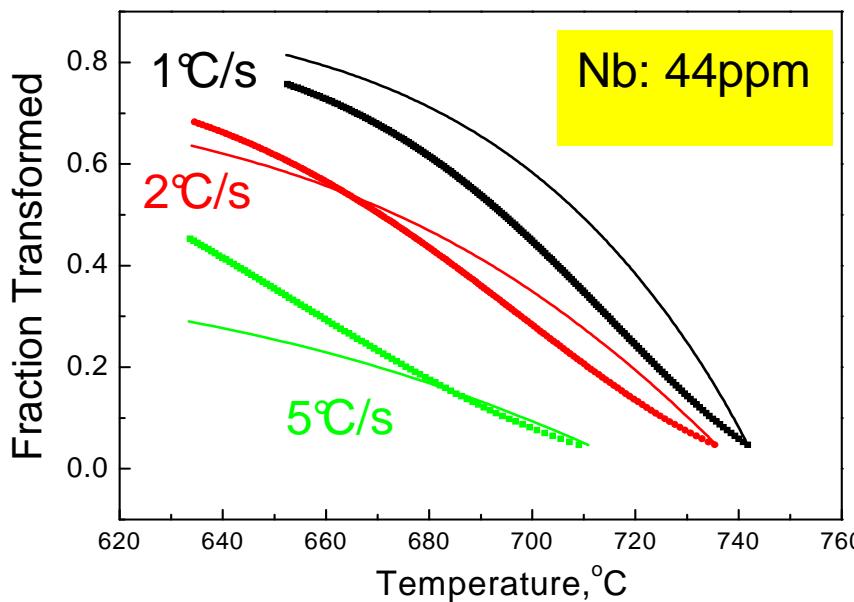
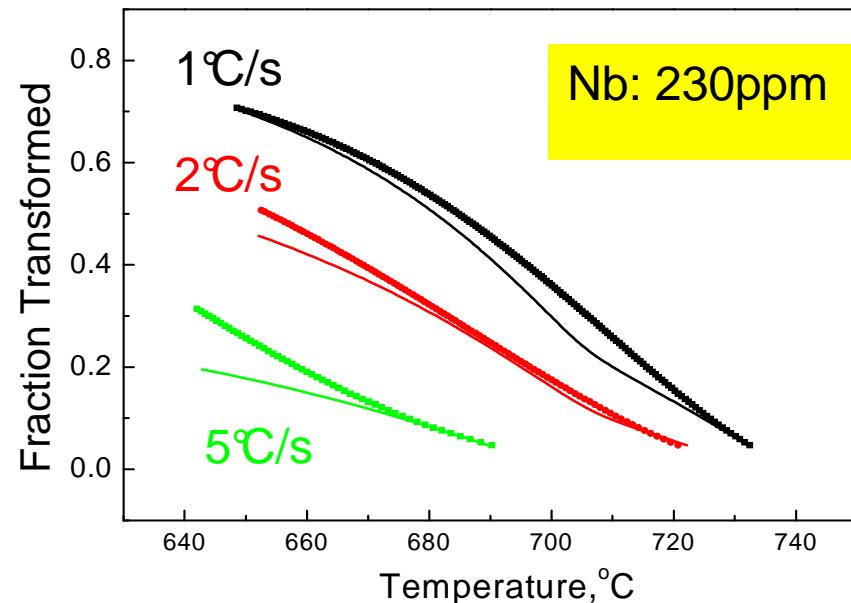
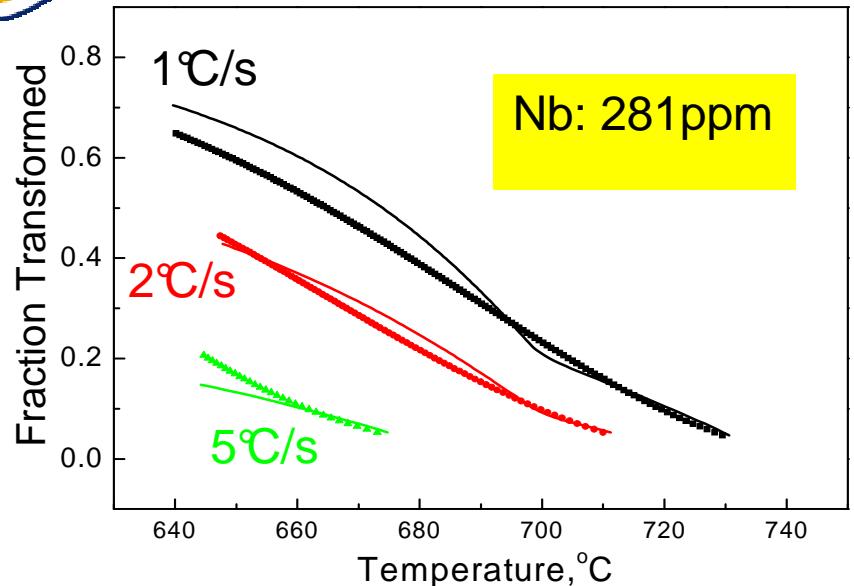
C<sub>Nb</sub>(ppm) 281 230 44



J.S. Park, Metall. Mater. Trans. A, 2009, 560.



# Solute Drag based Transformation Model



**Mixed-mode transformation  
(Fe-C-Nb system)**

$$M_0 = 2.4 \text{ cm mol/Js}$$

$$Q = 140 \text{ kJ/mol}$$

$$Q_b = 245 \text{ kJ/mol}$$

$$E_0 = 48 \text{ kJ/mol}$$

between FCC &  
BCC bulk value



## Conclusion & Outlook



- Promise of multi-scale modelling approach to account for effect of solutes on austenite decomposition kinetics
- Reliable trend predictions but still need a limited number of physically well defined fit parameters
- Need to extend current model from binary/ternary systems to more complex steel chemistries, e.g. Nb-Mo interaction
- Challenges: Ab-initio simulation of paramagnetic FCC-Fe and extension to bainite transformation