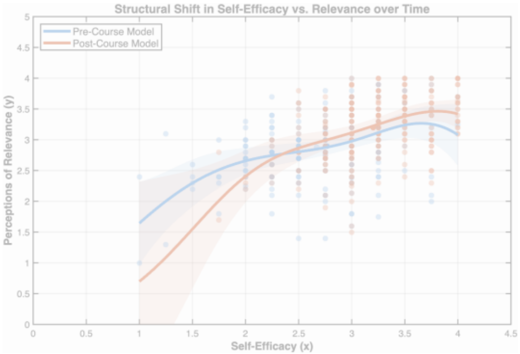
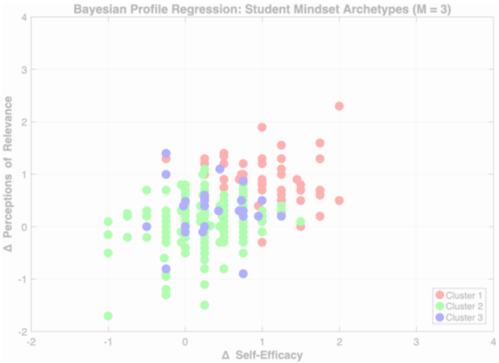
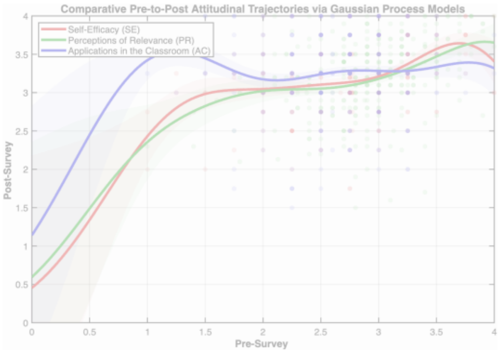
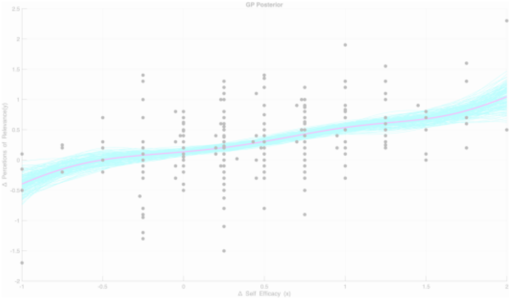






A Bayesian Approach to Self-Efficacy and Perceptions of Relevance in Math Course Redesign



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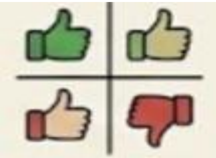
Background

Data were collected from 285 undergraduate students who enrolled in a 100-level mathematics course.

Survey questions are based on self efficacy and perceptions of relevance.

Pre-Survey
(Beginning of the semester)

Post-Survey
(End of the semester)



The responses were recorded according to these categories.

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Foundations of the Bayesian Approach

Frequentist Framework

Treats model parameters θ as **fixed, static constants**. Uncertainty is derived strictly from hypothetical infinite sampling repetitions.

- Relies on Likelihood L(θ)
- Requires large-sample assumptions
- Binary "reject/fail to reject" logic

Bayesian Framework

Treats unknowns as **random variables** governed by probability distributions. Uncertainty is quantified directly through the conditional posterior.

- Yields Credible Intervals (Posterior)
- Incorporates domain-specific **Priors**
- Iterative updating of certainty

Our model follows a **sequential updating scheme**. We transition from "what we knew" (Prior) to "what we learned" (Posterior) using empirical evidence.

$$\underbrace{p(\theta|w)}_{\text{posterior}} \propto \underbrace{p(w|\theta)}_{\text{likelihood}} \underbrace{p(\theta)}_{\text{prior}}$$

$$\text{Prior} \xrightarrow{\text{Data}} \text{Posterior}$$

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Solving the Education Data Problem

- **Non-Linear Flexibility:** Gaussian Processes let the data reveal "S-curves" and "Plateaus" without forcing rigid straight lines.
- **Regularization:** Priors ensure the model remains smooth in sparse regions (e.g., students with extremely low pre self efficacy).
- **Small-Sample Robustness:** Valid inferences can be made for sub-groups (clusters) without relying on infinite-sampling assumptions.

Gaussian Process

Traditional **Parametric** models (like linear regression) force data into rigid shapes, assuming a specific functional form from the start.

- **Non-Parametric Flexibility:** Gaussian Processes allow the data to speak for itself.
- **Infinite Parameters:** Instead of fixing a line, a GP considers a **distribution over all possible functions**.
- **Unbiased Discovery:** This prevents us from missing non-linear trends like "plateaus" in student growth.

In a Bayesian GP, the **Kernel (Covariance Function)** acts as our mathematical "Prior" on the shape of growth.

- **Encoding Smoothness:** We can formally assume that psychological growth is smooth and continuous.
- **Regularization:** This prevents the model from "overfitting" or creating wild, erratic lines in areas with few students.
- **Local vs. Global:** It allows nearby data points to influence each other, creating a realistic local trend.

My model uses a Gaussian Process, that map self efficacy x to Perceptions of relevance y to represent the correlation between them.

$$f(.) \sim GP(h(.), C(., .))$$
$$y_n | f(.) \sim Normal(f(x_n), 0.1) \qquad n = 1, ..., N$$

For the Gaussian process prior, I choose a mean h(x) = 0 and a covariance kernel with the form

$$C(a, b) = \lambda^2 exp(-(\frac{a-b}{l})^2)$$

to model a profile that remains relatively smooth over a distance l = 1. To make my prior profile highly uncertain, I use λ = 1.

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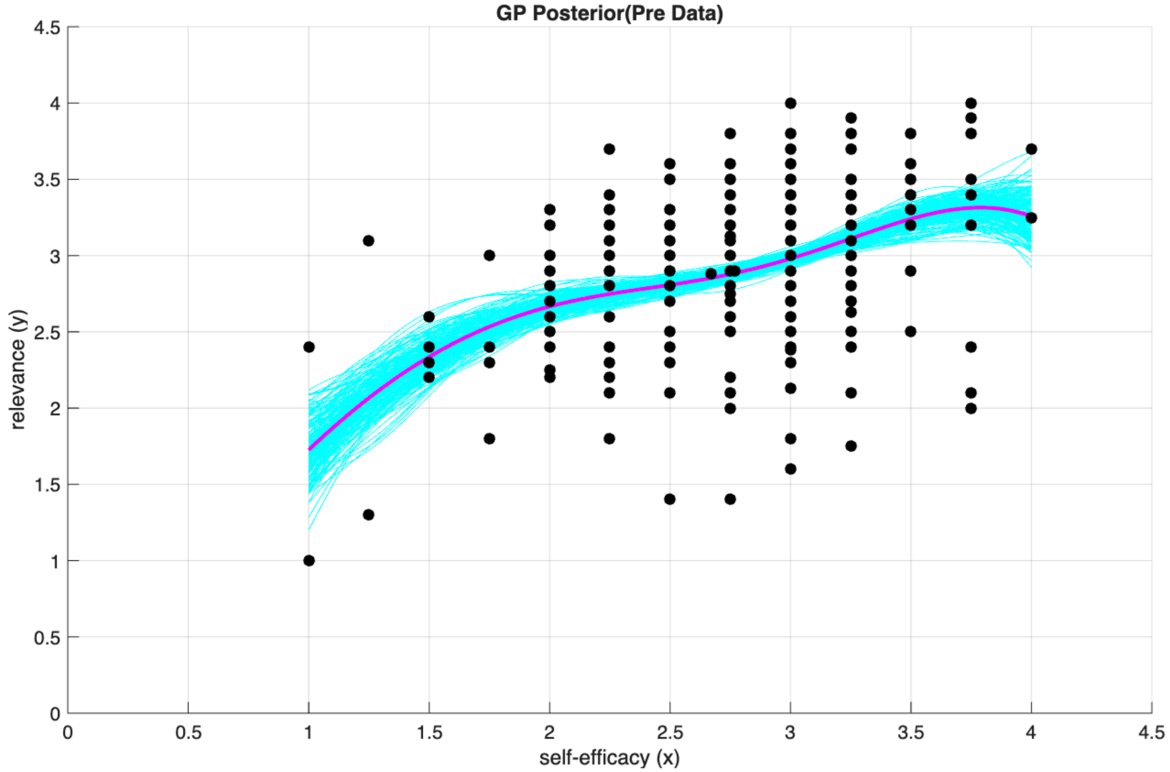
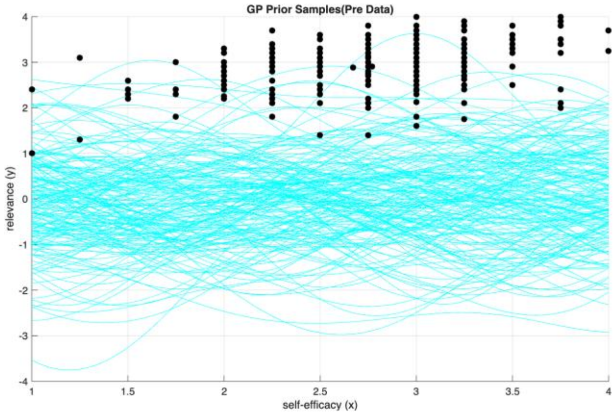
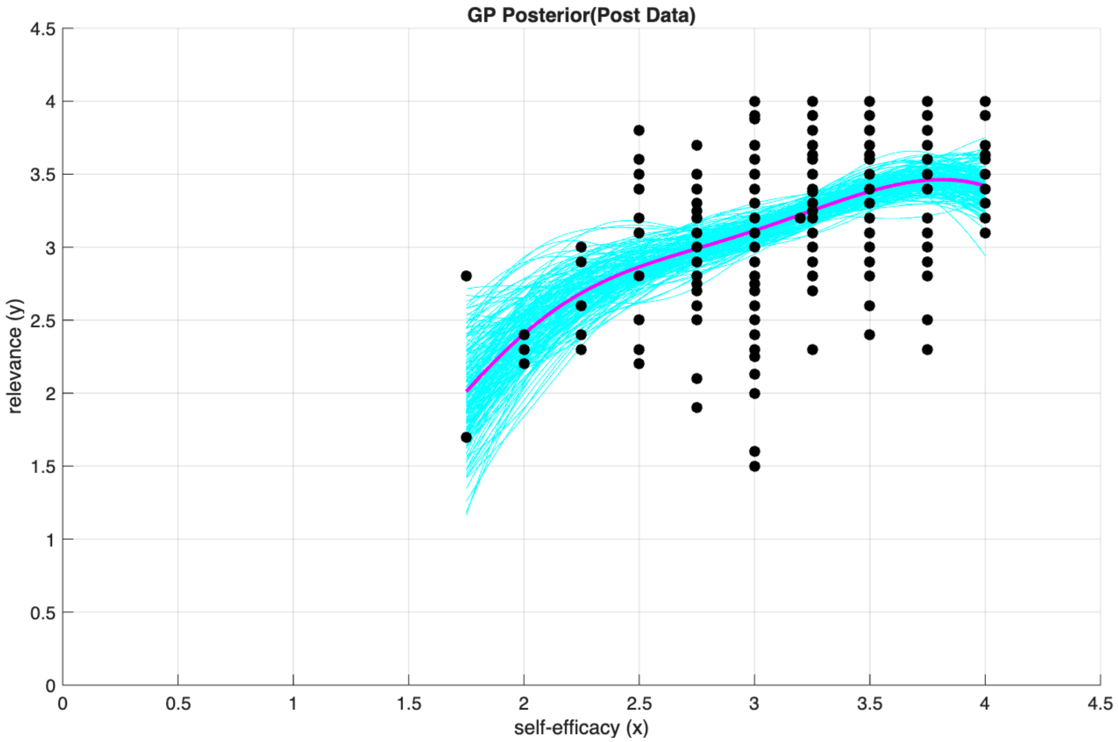
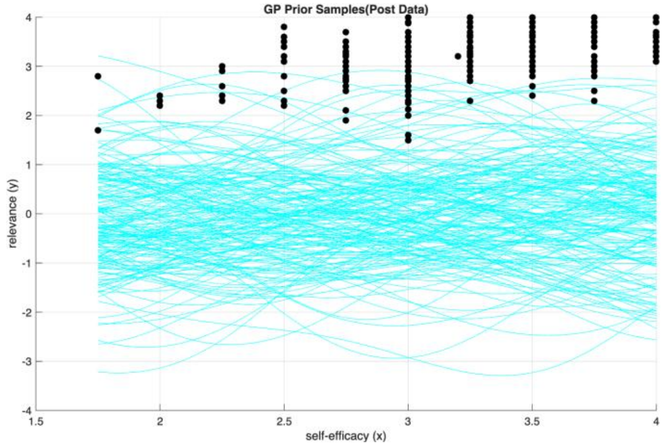

GP Posterior(Pre Data)
relevance (y)
self-efficacy (x)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

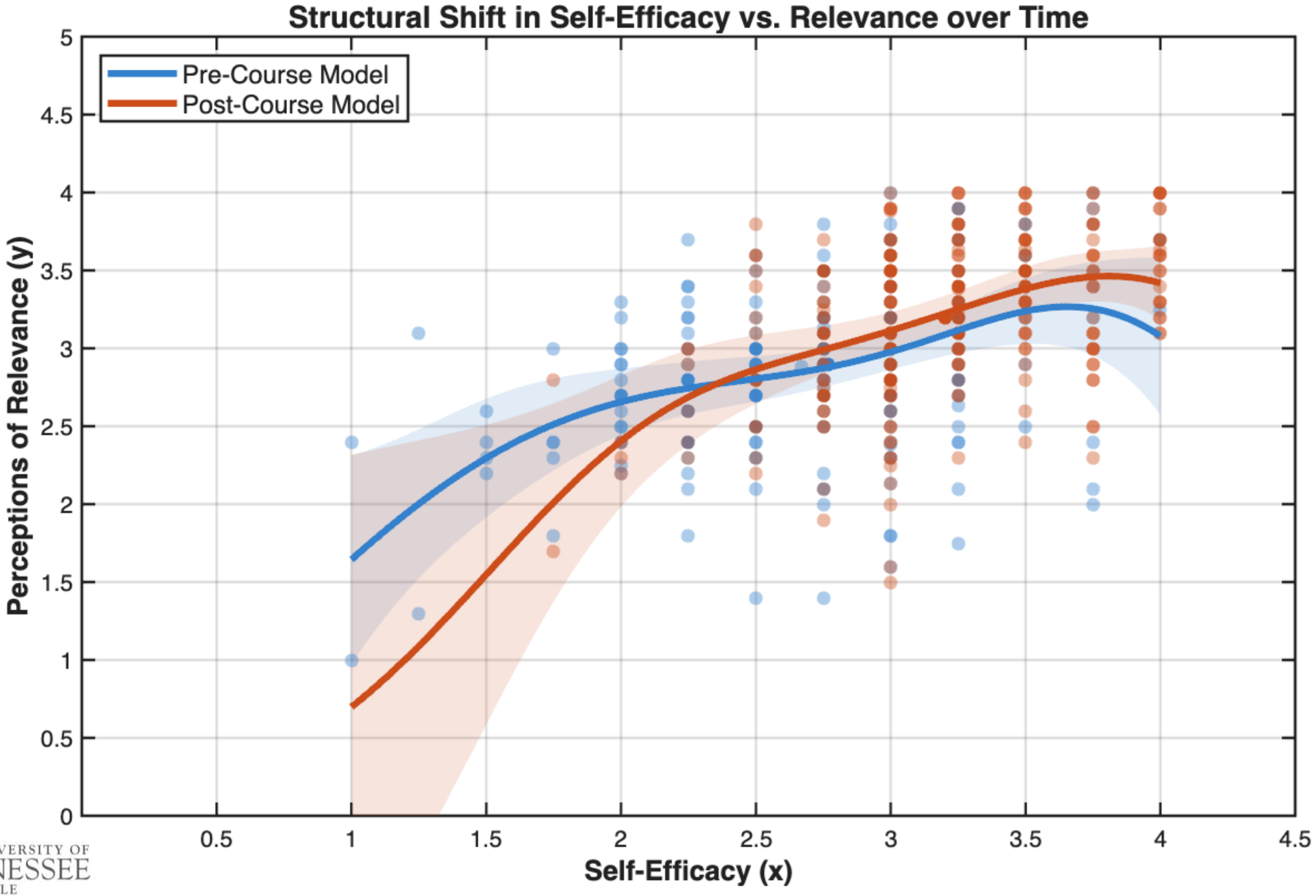
Pre-Survey


GP Prior Samples(Pre Data)
relevance (y)
self-efficacy (x)


GP Posterior(Post Data)
relevance (y)
self-efficacy (x)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Post-Survey


GP Prior Samples(Post Data)
relevance (y)
self-efficacy (x)



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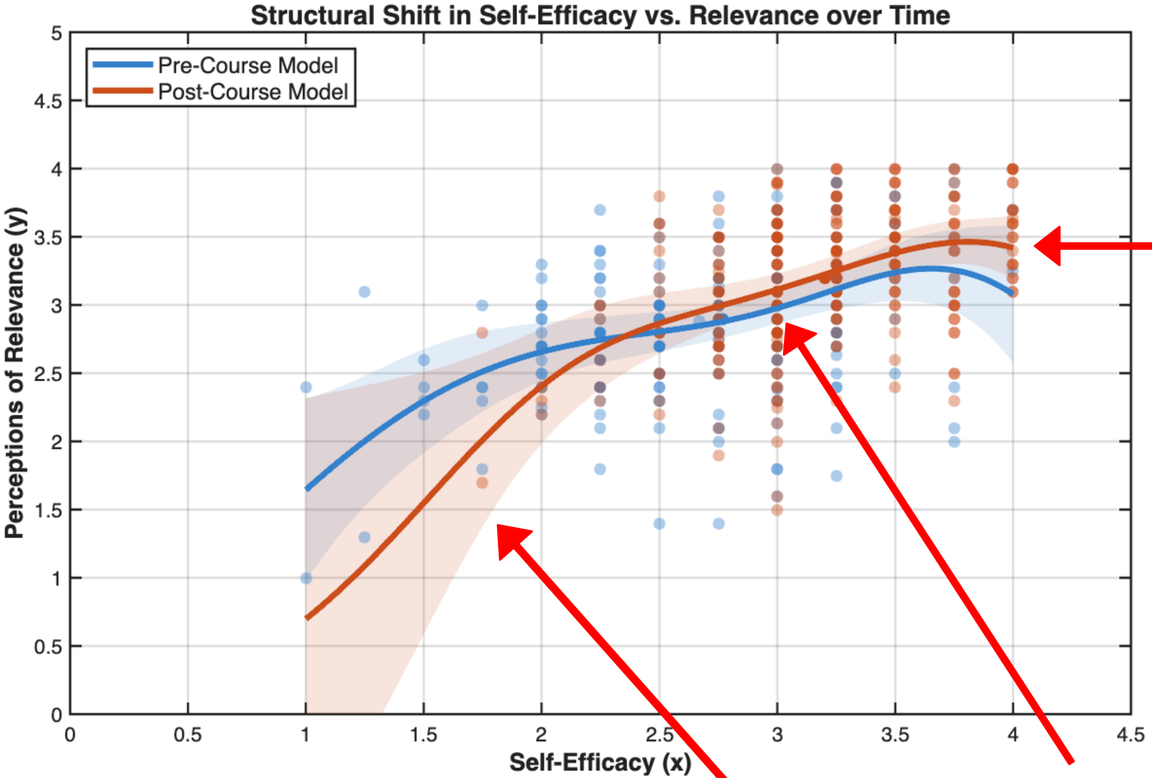


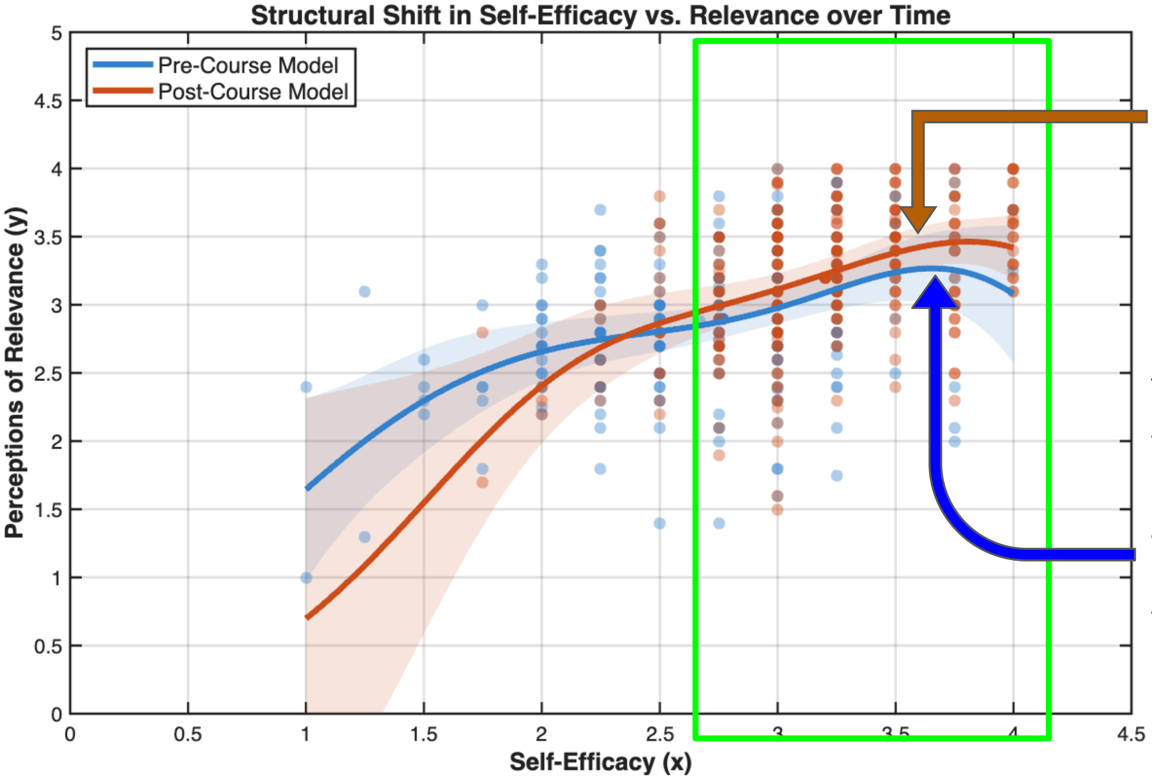
Each dot on this graph is an individual student.
The blue dots are where students started at the beginning of the semester (Pre-Survey), and the orange dots are where they ended up (Post-Survey)

Instead of forcing a rigid, **straight line through the data,** I used a flexible modeling method called a Gaussian Process. This method lets the data naturally bend to show the true, complex relationship.

Where the bands are narrow, a lot of data and are very certain about the trend.

The **light blue and light orange shaded bands** show our statistical confidence.

Where the bands widen out at the far edges, data is sparse, so the model is naturally expressing more caution.



For the students on average who had high self efficacy(x≥2.5), their perceptions of relevance was **higher at the end of the semester (Orange Line)** than it was at the beginning (Blue Line).

At the start of the semester, having the highest self efficacy did not automatically translate to having the highest perceptions of relevance.

The trend climbs steadily to a peak at high self-efficacy (x≈3.6), but turns downward for the group with the absolute highest self efficacy scores (x=4.0).

Students have high self efficacy

This downward turn is eliminated. The trend continues upward and stabilizes, showing that higher self efficacy consistently matches higher perceptions of relevance by the end of the term.

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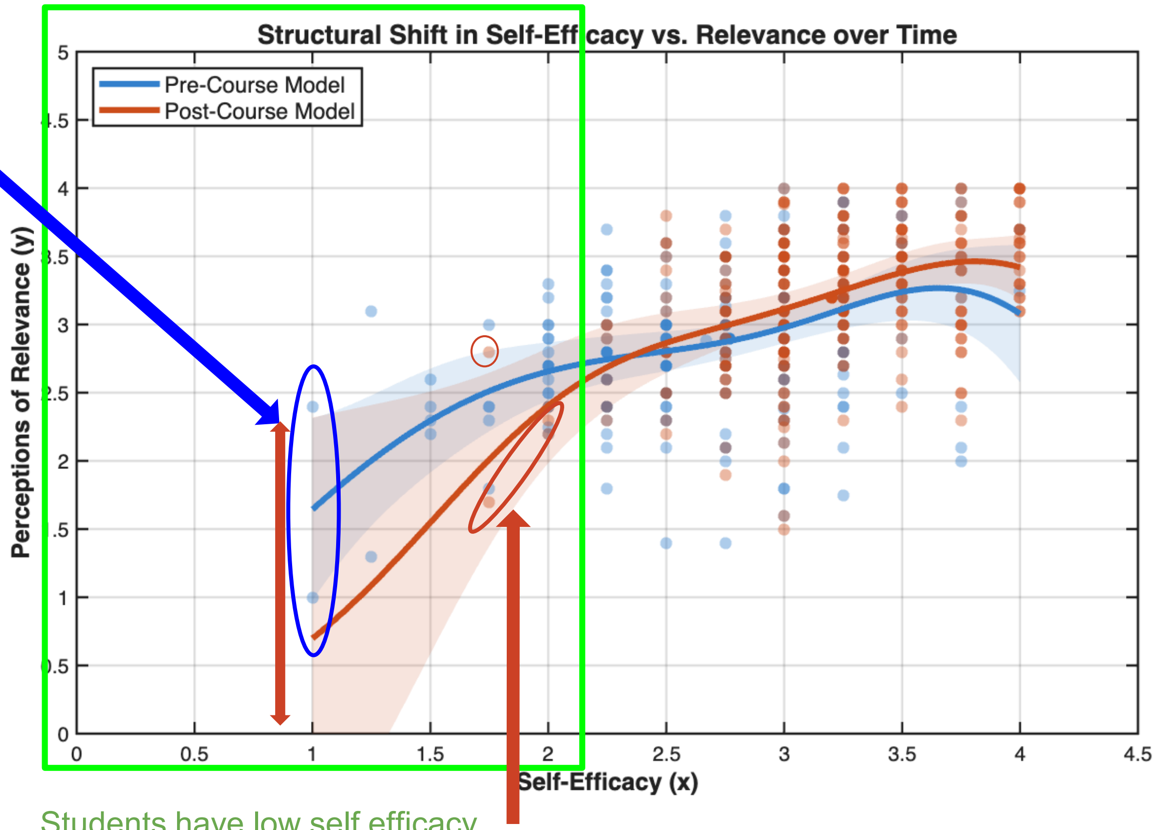
At the start of the semester,the group of students entering with a low self-efficacy score of 1.0 still rated the perceptions of relevance around 1.65.



The orange shaded cloud flares out dramatically here. This visually represents model uncertainty due to a very low density of data points.

Students have low self efficacy

Very few students actually finished the course with low self efficacy,which is a positive sign for the classroom overall.

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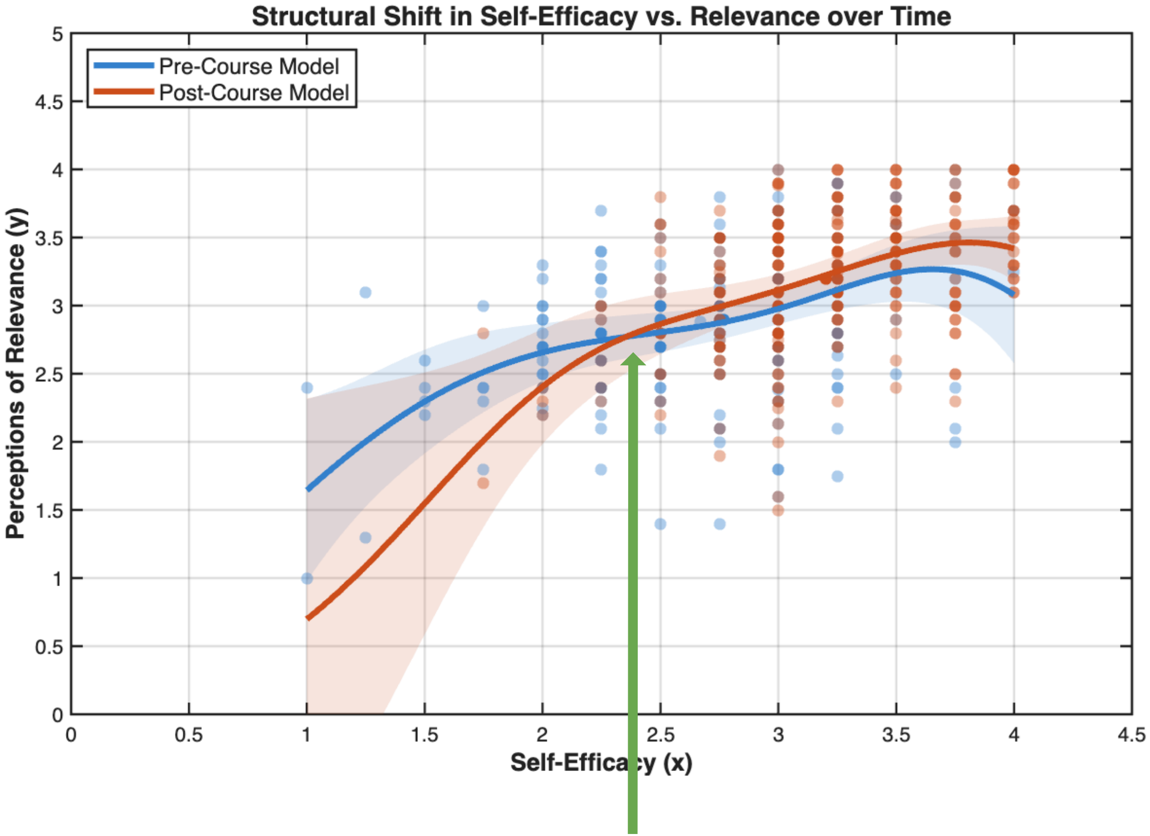
For the very small subgroup of students with low confidence, the post-course curve sits lower, though the wide shaded bands show the model has higher uncertainty here due to limited data.



To the right of this point where the vast majority of the data actually sits.
The orange line clearly climbs above the blue. This shows that for the main body of the classroom, the curriculum successfully shifted the trend upward, leaving them with a higher perceptions of relevance than they started with.

This intersection point is highly significant because it marks the psychological threshold where the overall trend shifts.

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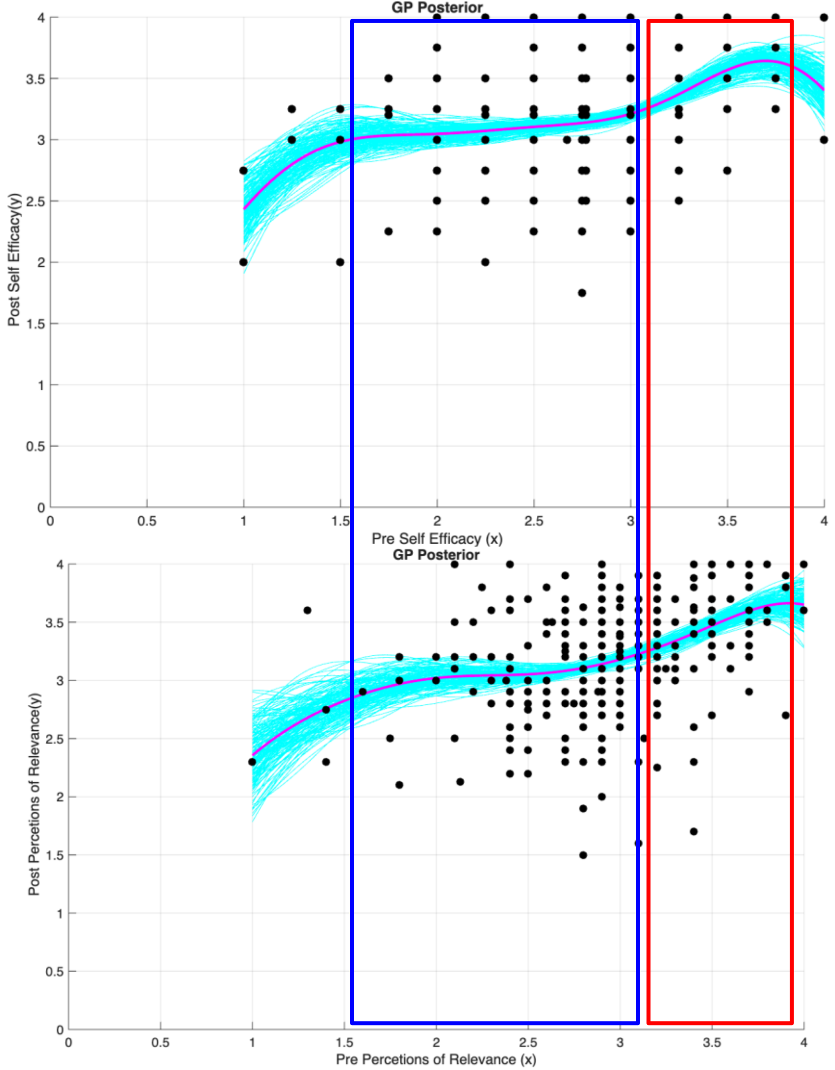
Are pre and post survey results correlated with one another?

A Bayesian Gaussian Process (GP) regression was conducted to capture localized, non-linear dependencies

Self Efficacy

Perceptions of Relevance

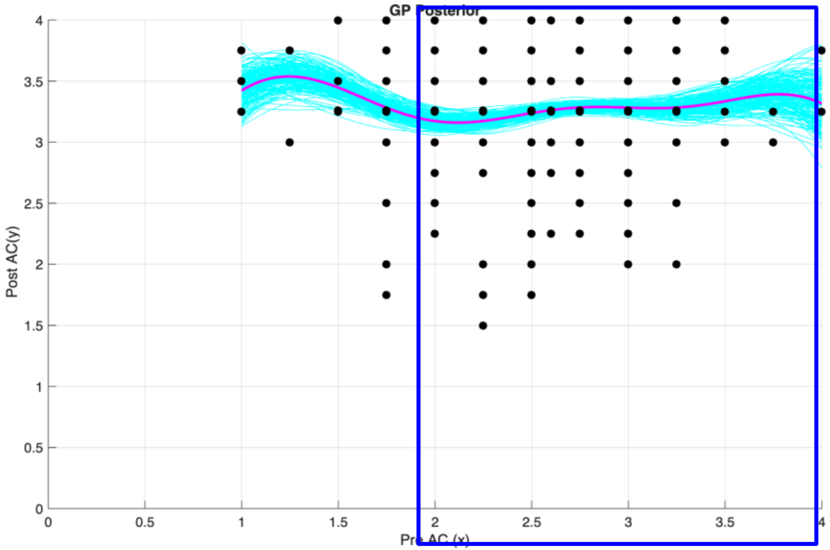


The GP posterior mean function reveals that the positive relationship is not uniform across the data continuum.

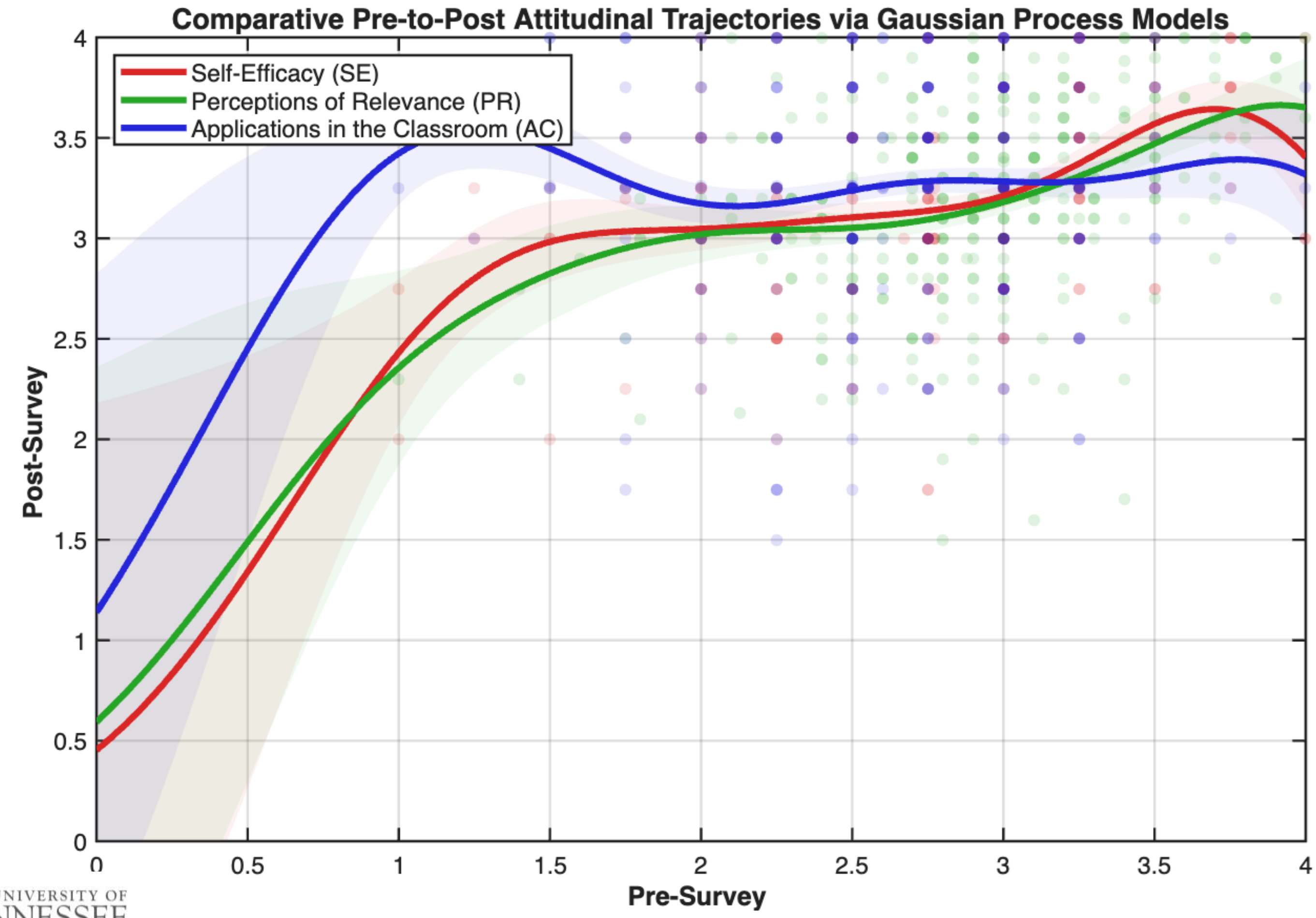
In the middle section, the curve flattens out right around a score of 3.0. This flat shape shows this course brings them all up to the same solid, positive score of 3.0.

Rather than flattening out completely or dropping at the high end, the curve takes a final upward turn toward the highest rated score. This confirms that the course structure remained engaging and did not hold back students who already had high self efficacy and perceptions of relevance.

Applications in the Classroom



The horizontal, flat nature of the trend line indicates that there is **no meaningful relationship** between a student's initial attitude about the applications of math and their final perception about it after taking the course.



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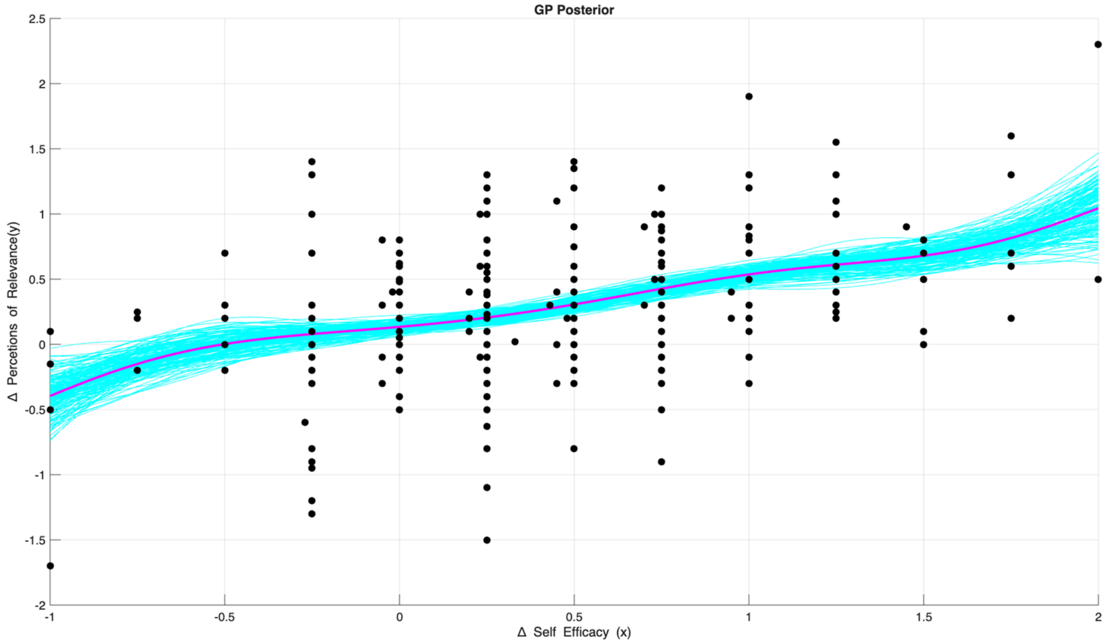
To determine **if the students who gained Self-Efficacy (SE) also tended to gain Perceptions of Relevance (PR)**,
A correlation analysis was conducted on the calculated pre-to-post change scores ($\Delta SE = SE\_post - SE\_pre$ and $\Delta PR = PR\_post - PR\_pre$).



The steady, upward trend shows a clear direct correlation: as a student's confidence grows, their appreciation for the math's relevance grows right alongside it.

Psychological growth isn't isolated. Building a student's confidence in their own math skills naturally unlocks their ability to see the value and relevance of what they are learning.

If a curriculum can successfully move a student forward in self-efficacy, it acts as a catalyst that pulls their perception of relevance up along with it.

Bayesian clustering

If we look at our class using traditional averages, it masks what actually happened in the classroom. A single average assumes every student had a similar experience.

Instead, we used a **probabilistic clustering technique** that treats the class as a mixture of different groups. It automatically sorted the students into two distinct archetypes based on where they started the semester, how much their attitudes changed, and their final grades

We have a dataset $w_{1:N}$. Each datapoint w_n in the dataset is generated by one out of M clusters which we label with $\sigma_{1:M}$. Our goal is to identify which cluster generated each w_n.

- We represent the clusters by probability distributions.

$$F_{\sigma 1}, F_{\sigma 2}, \cdots, F_{\sigma M}$$

- We represent the cluster features by the parameters of these distributions.

A Bayesian model consists of

$$s_n \sim \mathrm{Categorical}_{\sigma_1,\sigma_2,\ldots,\sigma_M}(\pi_{\sigma_1}, \pi_{\sigma_2}, \cdots, \pi_{\sigma_M})$$

$$w_n | s_n \sim F_{s_n}$$

$$\text{for } n = 1, \ldots, N$$

In our model, s_n is the identity of the cluster that generated w_n .

		Column 1	Column 2	Column 3	Column 4	Column 5
		Pre SE	**Pre PR**	**ΔSE=Post SE-Pre SE**	**ΔPR=Post PR-Pre PR**	**Mean Exam Grade**
Row 1	**Student 1**					
Row 2	**Student 2**					
. . .	**. . .**					
Row 285	**Student 285**					

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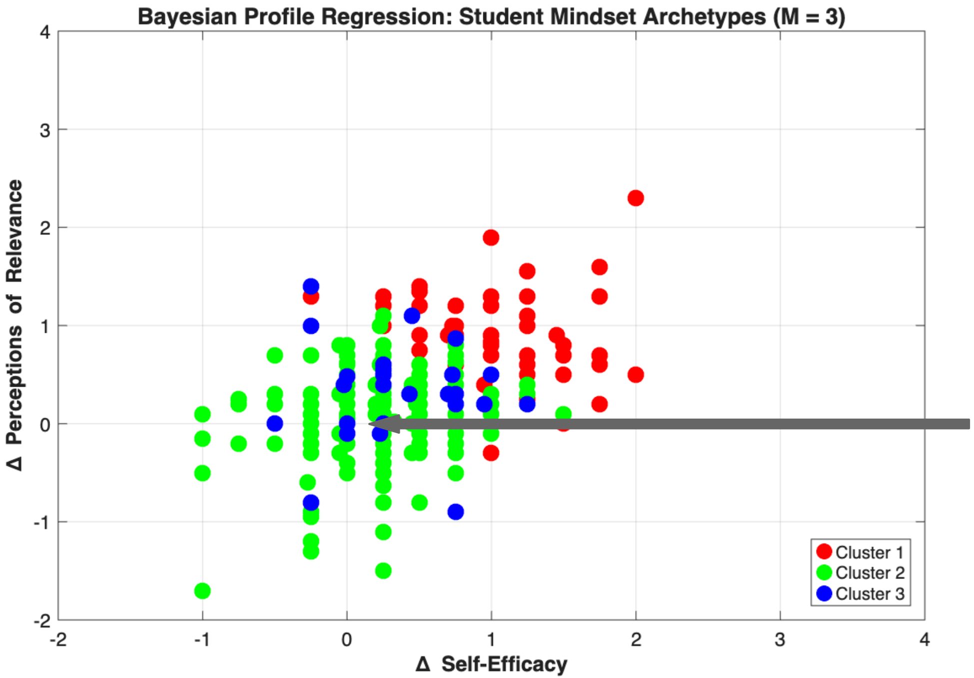
Now that we know who the groups are, this scatter plot maps exactly how their mindsets shifted over the semester.

The vertical axis is growth in seeing the relevance of perception.



The point (0,0) in the center means a student didn't change at all from the beginning of the semester.

The horizontal axis is growth in self efficacy

	Pre SE	Pre PR	ΔSE	ΔPR	Mean Exam Grade	Population size(%)
Cluster 1	2.26	2.51	0.96	0.78	84.39	25.74%
Cluster 2	3.00	3.09	0.21	0.10	86.45	64.70%
Cluster 3	2.70	2.79	0.40	0.31	56.67	9.56%

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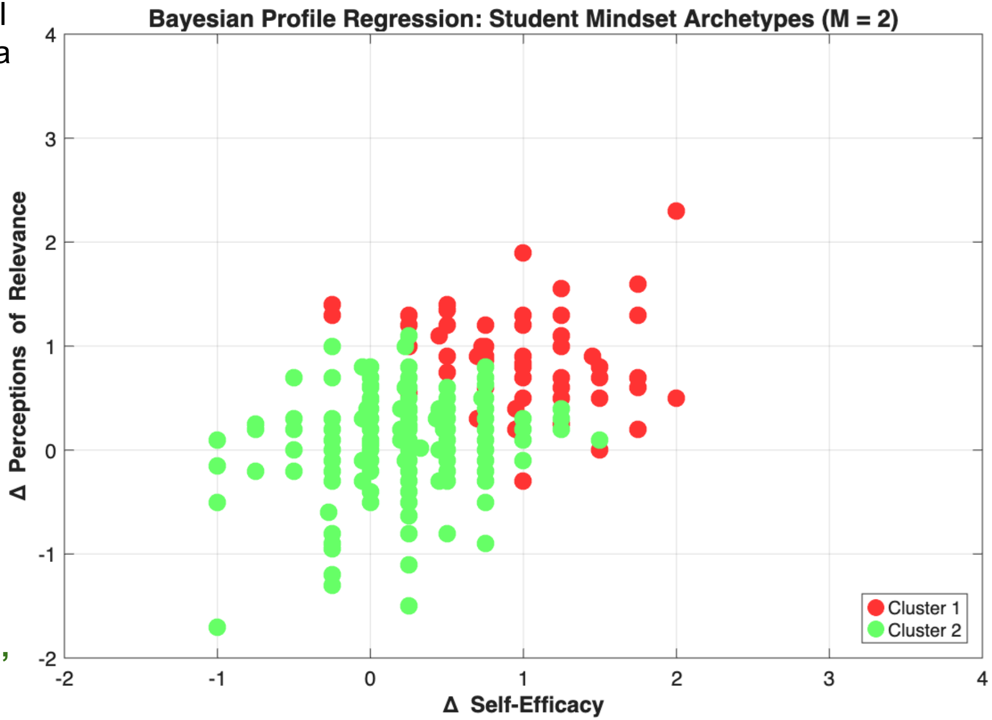
Looking at the above scatter plot, the data naturally wants to split into **two** main behavioral groups rather than three

The big, tight central cluster of students who had natural, modest shifts during the semester.

The distinct trail of students stretching up and to the right, who showed massive positive gains in both self efficacy and perceptions of relevance.

The blue points (Cluster 3) are completely mixed in with the green points. They don't form their own distinct shape or separate cloud on this chart, which means forcing a third cluster isn't adding any real value.

Moving from a 3-cluster model to a **2-cluster model** provides a cleaner, more statistically sound interpretation of the data.



A distinct, high-growth group of students who showed massive positive leaps in both confidence and perceived relevance.

A massive, stable group of students experiencing typical, steady progress

	Pre SE	Pre PR	ΔSE	ΔPR	Mean Exam Grade	Population size(%)
Cluster 1	2.29	2.53	0.91	0.74	80.71	28.64%
Cluster 2	2.98	3.06	0.22	0.12	84.22	71.36%

Cluster 1: The "Low-Baseline, High-Growth Responders" Group (28.64%)

This group started the semester with much lower initial self efficacy and lower perceptions of relevance

However, they were the "hidden success story" of the curriculum, showing massive positive growth by the end of the term

	Pre SE	Pre PR	ΔSE	ΔPR	Mean Exam Grade	Population size(%)
Cluster 1	2.29	2.53	0.91	0.74	80.71	28.64%
Cluster 2	2.98	3.06	0.22	0.12	84.22	71.36%

The "High-Baseline, Steady Progress" Group (71.36%) * These students entered the course already feeling relatively confident and valuing the material.

They maintained an exceptionally strong, stable trajectory throughout the term and achieved the highest overall academic performance

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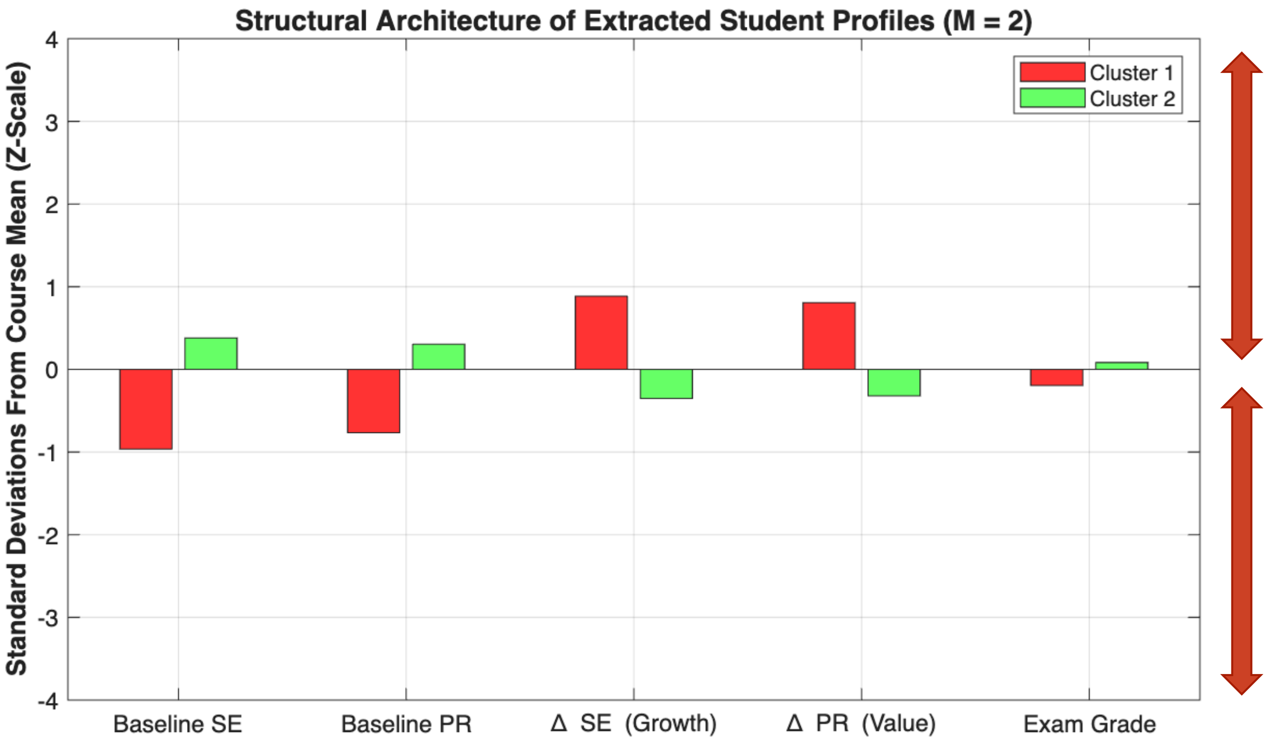
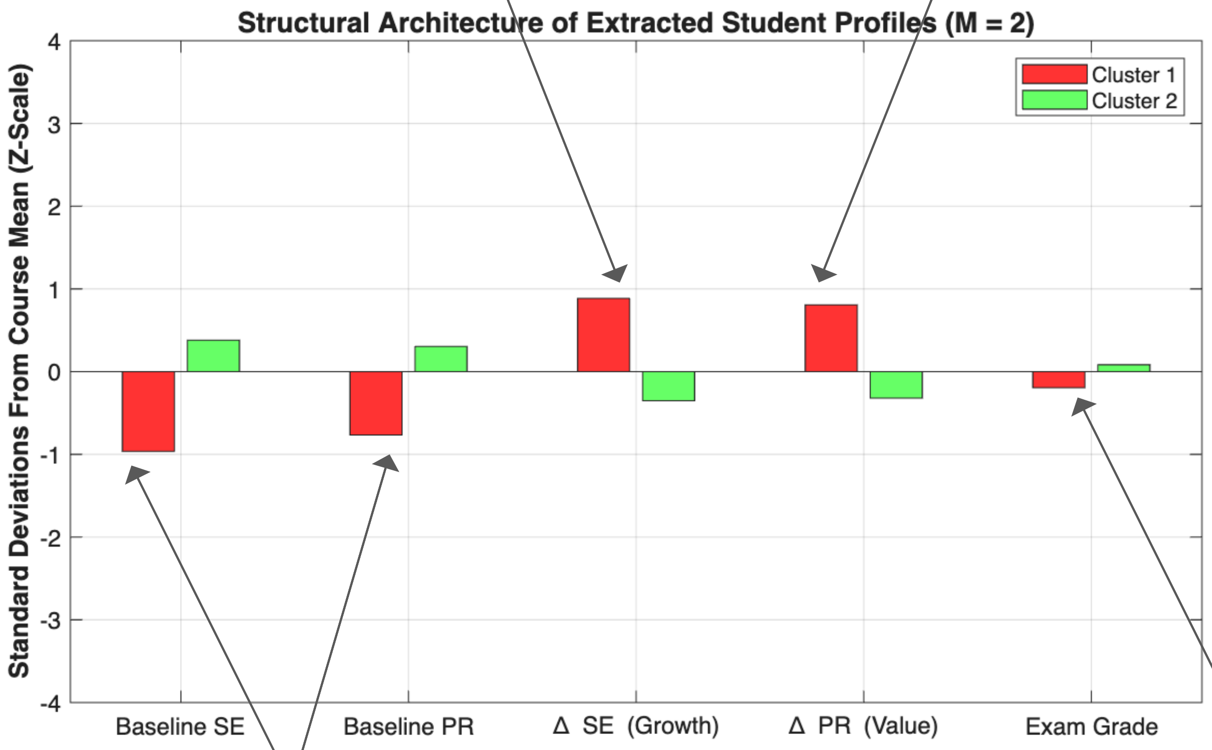


Bars stretching upward mean a group scored above the class average

The baseline zero-line represents the class average.

Bars stretching downward mean they scored below it.

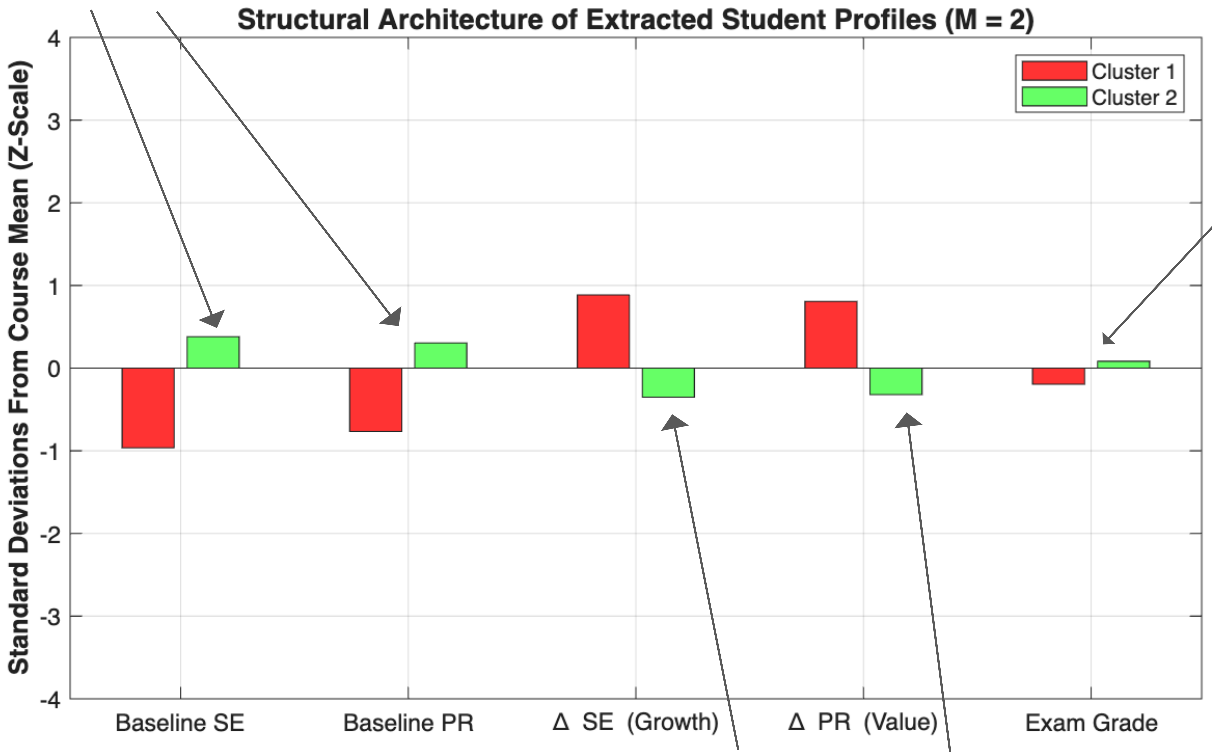
Despite the low start, they showed the most dramatic transformation, with growth scores (Δ) significantly higher than the course average.



The massive mindset shift in Cluster 1 demonstrates that psychological growth can occur even while academic performance is still catching up.

This group of students entered the course with lower initial attitudes, starting nearly a full standard deviation below the mean in self efficacy and perceptions of relevance.

This group entered with high baseline scores.



The students who started with high SE and PR, still holds the edge in final grades.

The green group didn't actually get worse. They just started out so high that they stayed where they were, while the red group caught up to them.

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Summary

Analyzed longitudinal data (N=285) from a 100-level math course redesign evaluating **Self-Efficacy (SE)** and **Perceptions of Relevanc(PR)**.

A direct, positive correlation was discovered between changes in confidence and value (ΔSE & ΔPR). Building a student's self-efficacy acts as a direct catalyst for unlocking their appreciation of mathematical relevance.

Gaussian Process Regression: Employed a non-parametric Bayesian GP to capture complex, non-linear trajectories without imposing rigid linear constraints.

Bayesian Cluster Analysis: Used probabilistic clustering to segment the student population dynamically rather than relying on masking classroom averages.

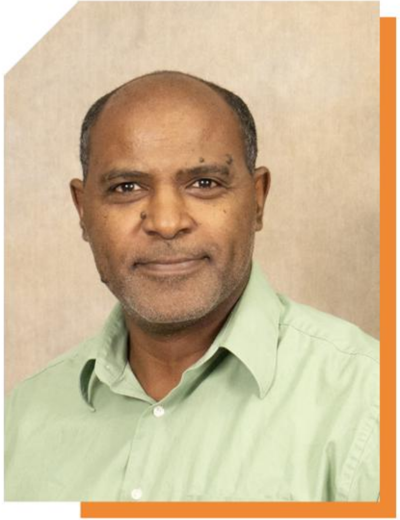
Optimal 2-Cluster Solution: While a 3-cluster model was tested, transitioning to a 2-cluster model provided the cleanest, most statistically sound representation of student archetypes:

1. **High-Baseline, Steady Progress (71.36%)**:
2. **Low-Baseline, High-Growth Responders (28.64%)**: However, they served as the primary success story of the curriculum, exhibiting dramatic attitudinal growth by the end of the term

Psychological vs. Academic Growth: The substantial mindset transformation among the low-baseline group proves that significant psychological growth can occur even while hard academic performance (exam grades) is still catching up.

Curriculum Success: The redesigned curriculum successfully shifted attitudinal trajectories upward for the main body of the classroom, stabilizing high self-efficacy into consistently high perceptions of relevance.

Thank you



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