So Trendy: Analyzing the growth of trends using Calculus

Suzanne Dorée, Stella Hofrenning, Jody Sorensen

1 About the Activity

- Course: Calculus I
- Partner Disciplines: Business and Economics
- Pedagogical Approach: Active learning, small group activity
- Required Technology: web browser, spreadsheet

2 Institutional and Course Contexts

- Type/size of institution: Small Comprehensive University.
- Size of Class: 28 or fewer
- Characteristics of Students: Undergraduates from diverse mathematical backgrounds. Some are first year students who took Precalculus or Calculus in high school, others came through Augsburg's Precalculus course.
- Mathematical Content: Applications of rates of change, limits, and derivative rules.
- Purpose/Goal of the Activity: This activity is designed for students to see logistic model, to see an application of Calculus in the social sciences, and to practice with derivative rules with parameters and in an applied context.
- After and Before: Students need to know how to take derivatives to complete this activity, including the product/quotient rules and the chain rule. They also need to be able to take limits. This project does not use the differential equation for the logistic model, so if you're going to cover that in your course it might be nice to do this beforehand.
- Other Prerequisites: Some familiarity with basic spreadsheet usage might be helpful.
- Inspiration for the Activity: Working with an economist, and discovering the interactive graphic we use.

3 Partner Discipline Background

The Bass Model of Diffusion:

Every year new products are developed in the market because consumer wants and tastes continually change. For example, consumers are becoming more health conscious and want products with low sugar so Coca-Cola Zero was introduced. Some consumers are more likely to adopt a new trend, product, or technology than others. These consumers are called "innovators". Innovators adopt a new trend or product independent of the decisions made by others. Innovators influence others to adopt the product through positive word-of-mouth. Those who purchase primarily because of the influence of innovators are called "imitators". An important and very difficult question to answer is how quickly a new product will be adopted by customers in the market. Every year 30,000 new products are introduced and 95% fail. Since there is no historical sales data it's difficult to examine and analyze the potential profitability of a new product.

The Bass Model of Diffusion (1969) helps to forecast the different adoption patterns of products and to use that information to predict market success. Some products may exhibit slow growth in the market where the sales gradually increase, while other products may experience rapid sales growth when first introduced in the market (for example iPods). Sometimes products have huge sales growth and then quickly fade (fads such as Beanie Babies or Pet Rocks). The Bass Model works well for a variety of products such as durable goods (such as refrigerators, air conditioners), computers and technology products, medical products and a large variety of consumer goods and services.

There are three important parameters that define a Bass Model for a product. The potential market M, the coefficient of innovation p, and the coefficient of imitation q. M refers to the total amount of customers in the market who will potentially adopt the product. The coefficient of innovation p captures the impact of the external influences of advertising. The coefficient q captures the influence of prior adopters. In other words, as more innovators purchase and talk about the product, the more others in the market will adopt the product. The model assumes that all potential buyers eventually purchase the product and that the maximum number of potential buyers or adopters is fixed.

4 Implementation Plan

- Formal Learning Objectives
  - Interpret the graph of a logistic function: identify points of inflection, identify that an increasing function has a positive derivative, estimate values of the derivative from a graph, tell the story of a function from its graph, and identify a graph that is not logistic
  - Use the general formula of a logistic function: calculate the limit as t goes to infinity, apply derivative rules to calculate the derivative, find parameter values for a logistic function through specfic points
  - Use a spreadsheet program: populate a column of a spreadsheet, create a scatterplot in a spreadsheet, graph a function in a spreadsheet, compare actual to model data graphically and numerically
- Materials and Supplementary Documents
  Access to website (https://ourworldindata.org/technology-adoption) and the Excel spreadsheet microwave.xls .
- Time Required
  This activity is designed to take about an hour, with students working in groups of 2 to 4. This should allow most students time to complete the activity and be ready to turn it in.
- Implementation Recommendations
  This activity is done in a weekly calculus lab period where students apply the content they've been learning in new situations. No introduction is necessary, just let them dig in and get down to it. They are asked to access a website (https://ourworldindata.org/technology-adoption), and to analyze some data in a spreadsheet program such as Excel or Google Sheets. They could be given or shown a detailed color picture of the graph from the website if internet access is a challenge. If students don't have access to technology, the instructor could work through and display the spreadsheet steps at the appropriate times.

  Students get to apply derivative rules in context on an example from the wider world. They are also exposed to the logistic model before coming across the differential equation, and have an intuitive example for when they do encounter that topic again.
- Alternative Solutions
- Common Errors and Questions
  Students typically do well with this activity. Questions 6 and 7 allow for quite varied responses so some students like a little bit of reassurance there. It can be a challenge for students to pull knowledge from different parts of the course and apply it in a new setting. This transference is a goal of our calculus labs.

- Tips to Handle Controversial Issues (if applicable)

5 Additional Information

This activity could be modified up or down. Students in Precalculus could work with the logistic equation and explore various logistic models by graphing. It could also be modified to include the logistic differential equation, for either a calculus or differential equations course. It's natural to explain why the growth rate would be proportional to both the people who have the new technology (so more people become aware of it) and to the people who do not (fewer people who can adopt the new technology). By choosing a product other than microwaves on which to focus, different versions could be created for different sections, or so that there was variation from year to year. The data used to create the graphic is easily downloadable, and seems to be regularly updated as well.

References:

Our World in Data website, ourworldindata.org, based at Oxford University.

Bass, Frank (1969). "A new product growth for model consumer durables". Management Science. 15 (5): 215–227. doi:10.1287/mnsc.15.5.215.