

Introduction To Whole Numbers Unit #1

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Professor Weissman's Algebra Classroom

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I'm going to make Algebra so simple, anyone can do it; so interesting, everyone can enjoy it!



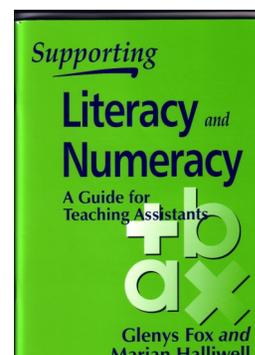
What are Literacy & Numeracy?

Literacy: is the ability to read and write. **Numeracy:** is the ability to use and understand numbers.

What is a Set?

All of us save things. Perhaps, they're CD's or baseball cards or photos. Whatever they are we usually put them in something, like a box, or an envelope. What do Mathematicians collect? Just about anything you would collect and more. And what do they put them in? They list them, separated by commas and put them between braces. Braces look like curly parentheses. Then they give the set a nickname, usually using a capital letter. For example, a set of symbols used in Mathematics might be called S and look like this:

$$S = \{ +, -, /, \pi, <, >, \pm, \sqrt{\ } \}$$



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What is the Set of Letters?

Letters are the building blocks of words. Words become sentences, paragraphs. Before you know it you have stories, essays, articles.

All words are made up of the 26 letters of the alphabet. We'll call the set

$$L = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$$

What Are the Counting Numbers?

How many different types of numbers do you know? Surely, you've heard of odd and even numbers. Fractions and decimals are other types.

The most basic reason for numbers is to count.

What is The Set of Natural Numbers?

Mathematicians have another name for the Counting Numbers. They call them the Natural Numbers. We'll call the set of Natural Numbers 'N'

The 3 dots at the end are called 'ellipses' and mean 'and so on'. The set is said to be infinite.

$$N = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots \}$$

What is the Set of Digits?

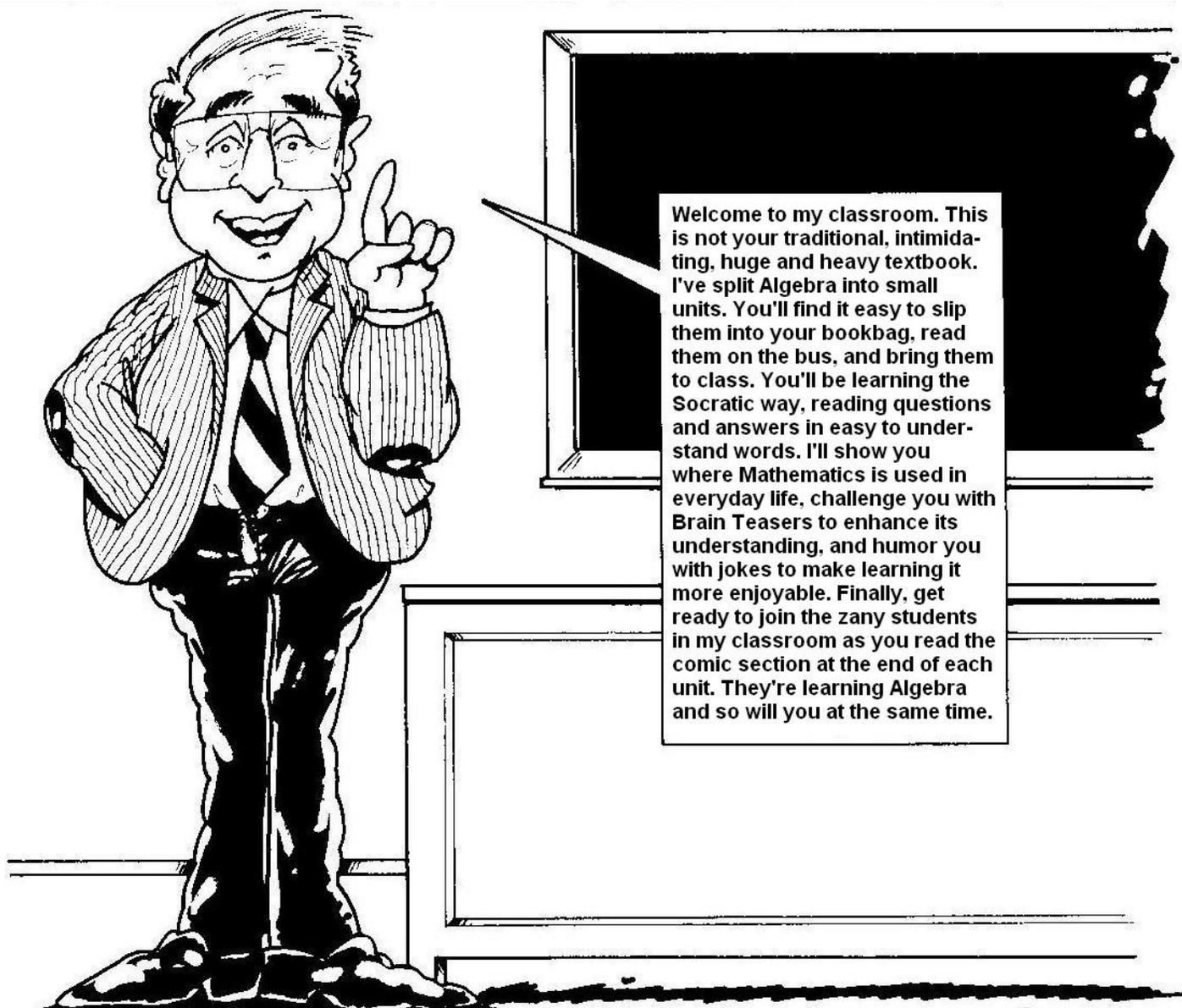
Digits are the building blocks of numbers. We will see how numbers become expressions then equations.

All numbers are made up of just 10 digits. We'll call them D and list them in set notation using braces, which look like curly parentheses.

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

And when we count, for example, the number of students in the class we start with the number 1.

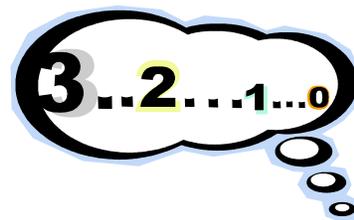
So what should we call the numbers: 1, 2, 3, 4, 5, 6 and so on? Why not, the "counting numbers.?"



Why Is The Number Zero Important?

At Blastoff, the countdown sounds like 5,4,3,2,1,0 blastoff! Is zero a number? Absolutely yes! Without the number zero it would be difficult to do much of Mathematics.

Probably the most important use of zero is as a placeholder. The number 407 would just not be the same without the zero in the middle.



What Are The Whole Numbers?

Let's add zero to the Natural Numbers. Our collection of numbers is changed. It needs a new name. We call it the set of whole numbers. Let's simply call it 'W.' Here's what it looks like:

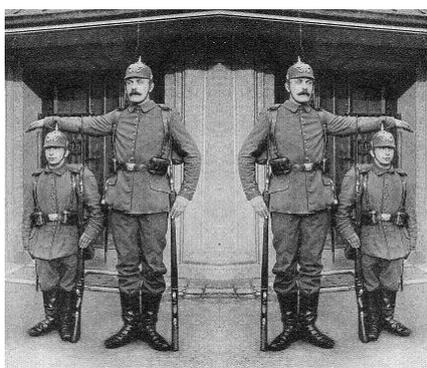
$$W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots\}$$

What are Symbols of Inequality?

When 2 numbers are not equal we could use bol. Note that the '<' symbol looks like the letter 'L' the first letter of the word 'Less.'

$$4 \neq 6 \text{ or } 6 \neq 4$$

Better yet we can use two other symbols that compare the numbers. These are the 'is less than' or '<' symbol and the 'is more than' or '>' sym-



Another point worth noting is that whichever symbol you use always point to the smaller number, just like the tall soldier is always pointing to the small soldier.

$$4 < 6$$

$$6 > 4$$

What Is A Number Line?



A thermometer is an every day use of a number line.

The Number Line is a very useful tool when we want to show the relative positions of numbers.

When comparing 2 numbers, just look at the number line. The number that is more to the left is always smaller than the number that is more to the right.

Comparing 3 and 7 we see that 3 is more to the left. So, we say, 3 is less than 7 and we write:

$$3 < 7$$



At the same time we can say that 7 is more to the right. That means 7 is more than 3 and we write:

$$7 > 3$$

The number line is also useful to show what happens when we add or subtract numbers.

Moving the left is like subtracting and moving to the right is like adding.

We can Graph or locate the number 5 on the number line with a black circle.

What number is 2 units to its left? It's 3. That shows the subtraction:

$$5 - 2 = 3$$

Again, locate the number 5 on the number line. What number is 2 units to the right? That would be 7. This illustrates the addition problem:

$$5 + 2 = 7$$

How Do We Read Numbers?

The proper way to read numbers is to emphasize place values.

Often, when people are in a rush a number is not read properly.

768

The number 768 read (or written) properly is: "Seven hundred sixty-eight."

Note the hyphen or dash between the words sixty and eight.

Numbers between 21 and 99 would have a hyphen. Except, of course, the numbers that end with a zero, such as, 30, 40, 50 and so on.

Some mistakes that are made are writing or saying:

MISTAKE:Seven sixty-eight (omitting the word hundred)

MISTAKE:Seven hundred and sixty-eight (don't use the 'and' word after the word hundred, we will see later that the AND word is reserved for the decimal.

The Continental Divide in the Americas is an imaginary line that divides the flow of water

between the Pacific Ocean and Atlantic Ocean. Rain or snow that drains on the east side of the Continental Divide flows toward the At-



lantic Ocean while precipitation on the west side drains and flows toward the Pacific Ocean. At

the spot where the picture was taken the elevation was 10,424 or ten thousand four hundred twenty-four feet above sea level.

What Is Rounding ?

Rounding replaces an exact number with an approximate number.

What Is Rounding Numbers Down?



You paid \$631 for the computer. When asked how much did it cost? You respond \$600.

\$ 631

You just rounded \$631 to the nearest \$100

Using the number line we can visualize why \$631 is rounded to \$600. We see that it is closer to \$600 than to \$700.

Why is it closer to \$600? Because the next digit (the 3 in the tens place) is less than 5.

What are Standard & Expanded Forms?

The number 777 uses the 7 digit three times. However each 7 represents a different amount and has a different value.

777 is said to be in 'Standard Form.' It's written the way we usually write numbers.

If we want to show the different values for each 7 we can change the number to 'Expanded Form.'

777 written in Expanded Form looks like this:

$$700 + 70 + 7$$

Writing a number in Expanded Form emphasizes the column that each digit is in or its 'Place Value.'

$$700 = 7 \times 100 \text{ Hundreds}$$

$$70 = 7 \times 10 \text{ Tens}$$

$$7 = 7 \times 1 \text{ Ones}$$

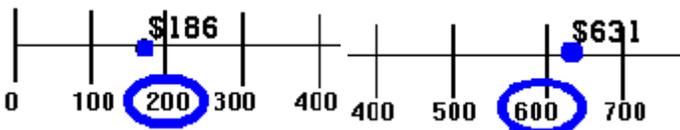
What Is Rounding Numbers Up?



\$186

The printer that you bought for \$186 is rounded to \$200 to the nearest \$100.

On the number line we see that \$186 is closer to \$200. We say that \$186 was rounded UP to \$200



When a number is rounded and the result is LARGER than the original number, we say that it is rounded UP.

When a number is rounded and the result is SMALLER than the original number, we say that it is rounded DOWN.

WHOLE NUMBERS						
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
3	6	8	2	1	4	5

How Do We Round Whole Numbers?

Here's how to round a whole number:

#1 Locate the 'round' digit in the place with the value that you want to round to and draw a cutoff line after that digit.

#2 Look at the decision digit which is the digit after the cutoff line. **#3**

- If the decision digit is 5,6,7,8 or 9 add one to the round digit.
- If the decision digit is 0,1,2,3 or 4 leave the round digit unchanged.

#3 Replace all digits after the cutoff line with

zeros.

Rounding 631 to nearest 100

6 is in the hundred's place. Cutoff after the 6 digit

6|31

The Decision Digit is 3

6|00 Since the decision digit < 5, keep the 6 and change digits after cutoff to zeros.

631 —> 600

Rounding 186 to nearest 100

1 is in the hundred's place. Cutoff after the 1 digit

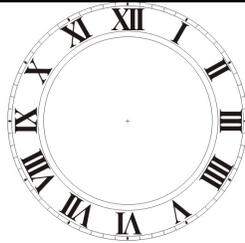
1|86

The Decision Digit is 8

2|00 Since the decision digit > 4, add one to the round digit



Note: Always round from the original value, not from a rounded number. The \$149 lawn mower to the nearest \$10 would be \$150. To Round \$149 to the nearest \$100, don't use the \$150. Instead, look at \$149 and round DOWN to \$100.



Clock face



What number is XLI ?



What Are Roman Numerals?

The Romans needed numbers for business and commerce. Around the 1st century AD they invented a system that used letters as symbols for numbers. The set of letters used for Roman Numerals looks like this: R={ I, V, X, L, C, M }

I= 1 V=5

X=10 L=50

C=100 M=1000

Using just these 6 symbols we can write any number. But, it's not so easy!

The easiest way to show how much was to make marks .

So, I means 1, II means 2, III means 3.

However, four lines seemed like too many. So the Romans decided to use a V as a symbol for 5.

Placing I in front of the V (or placing any smaller number in front of any larger number) means subtraction). So IV means 4.

Placing I in after the V (or placing any smaller number after any larger number) means addition). So IV means 4.

VI means 6, VII means 7, VIII means 8 .

X means 10.

But wait — what about 9? Remember? IX means to subtract I from X, leaving 9.

Numbers in the teens, twenties and thirties follow the same form as the first set, only with X's indicating the number of tens. So XXXI is 31, and XXIV is 24.

L means 50. How should we write 40? If you guessed XL, you're right 40 = 10 subtracted from 50. Also 60, 70, and 80 are LX, LXX and LXXX.

C is the symbol for 100. The subtraction rule means 90 is

written as XC and the addition rule means CX=110.

Like the X's and L's, the C's are tacked on to the beginning of numbers to indicate how many hundreds there are: CCCLXIX is 369.

D stands for 500. As you can probably guess by this time, CD means 400. So CDXLVIII is 448.

M is 1,000.

Can you understand now, why we stopped using Roman Numerals and switched to our present system whose numerals are called 'Arabic Numerals?'

In The News

The Mercedes car in the ad has 8500 miles and costs \$17,375. The original price was \$34,750.

Each of the 3 numbers could have been rounded.

The most likely rounded number is the 8500 miles. The 2 zeros at the end of the number make it suspect.

Maybe it has 8529 miles or 8479 miles. In fact, it could have been rounded from any number between 8450 miles

What 2 numbers can you sandwich the mileage of 31,000 for the Ford Freestar?

_____ and _____

Round the Being Sold For Price of the Ford Freestar of \$7995 to the nearest:

1. Ten Thousand _____
2. Thousand _____

EXAMPLE OF SAVINGS
2006 MERCEDES C230
Automatic, power windows, power door locks, leather, sunroof, 8500 miles, Stock#24197



\$34,750 ORIGINAL MSRP

BEING SOLD FOR: **\$17,375..**

EXAMPLE OF SAVINGS
2005 FORD FREESTAR
Automatic, power steering, power brakes, air conditioning, 31,000 miles, Stock#14054



\$12,995 NADA RETAIL BOOK VALUE

BEING SOLD FOR: **\$7,995.**

and 8549 miles.

We can 'sandwich' 8500 between 8450 and 8549.

Round the sales price \$17,375 to the nearest:

1. Ten Thousand _____
 2. Thousand _____
 3. Hundred _____
 4. Ten _____
3. Hundred _____
 4. Ten _____

Use symbols of inequality to show two relations between the sales prices of the two cars.

_____ < _____
_____ > _____

Circle The Numbers That Were Probably Rounded

THEIR FINEST PAYDAY

BY ERNIE NASPRETTO, KIRSTEN DANIS
and LISA L. COLANGELO
DAILY NEWS STAFF WRITERS

NYPD SERGEANTS will get raises of more than 24% — boosting their top pay to more than \$94,000 — under a tentative six-year deal unveiled yesterday.

When other payroll benefits are added on to their base salary, veteran sergeants will take home \$103,000 a year in 2010.

"The men and women of the Sergeants Benevolent Association play a vital part in keeping New York the safest big city in the nation — they deserve our gratitude and more," Mayor Bloomberg said yesterday.

"This is very important for the future of the department."

The proposed pact raises starting pay of sergeants from \$61,000 to \$73,000.

Newer sergeants promoted on or after April 1 this year will reach top pay after four years, while veteran sergeants will max out after three years. The union represents about 4,600 sergeants.

The agreement could put pressure on the

NYPD sergeants set for 24% wage hike in 6-yr. contract bid

Patrolmen's Benevolent Association, the city's largest police union, which is locked in a bitter contract dispute with City Hall.

"Everybody's reading everybody else's settlements and this could give them reason to think about what could be done," Bloomberg said.

The PBA's last contract expired three years ago, leaving rookie cops with a starting salary of \$21,500, which was imposed by an arbitration panel.

Police Commissioner Raymond Kelly has blamed the paltry starting pay for undermining the NYPD's recruiting.

A PBA spokesman said the union would not comment on another union's contract.

Bloomberg said he and Kelly hope the new pact with the sergeants union will encourage more cops to seek a promotion to the rank.

"We think this is a structure and a level that will encourage police officers to take on the added responsibility, do the added work to study and pass the test," Bloomberg said.

In April, the Daily News revealed that the number of cops taking the sergeant exam dropped from 7,154 in December 2003 to 3,856 in February this year. That drop corresponded with the sergeants' past contract, which slashed starting pay from \$67,355 to \$61,093.

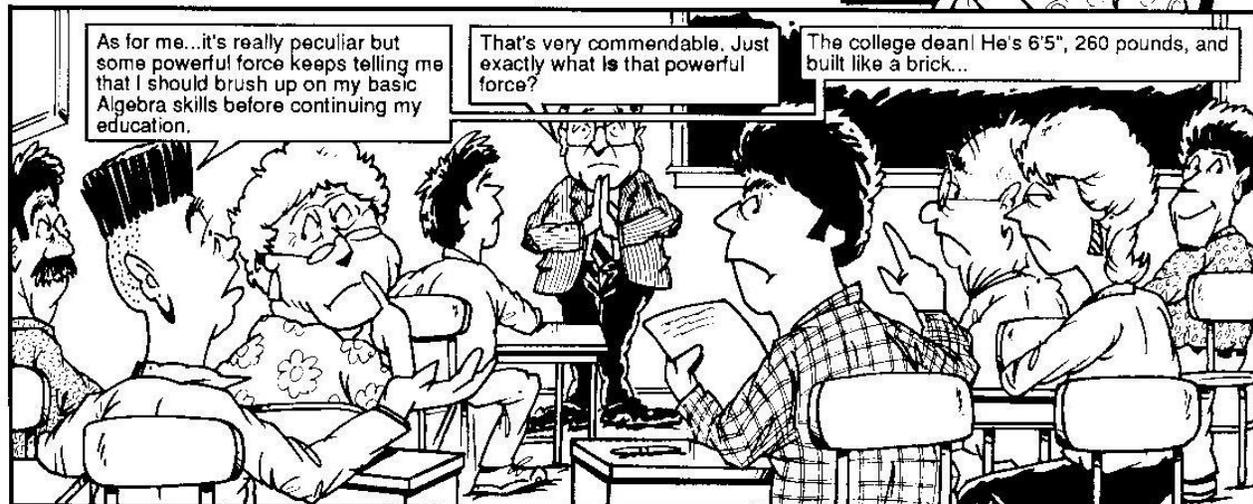
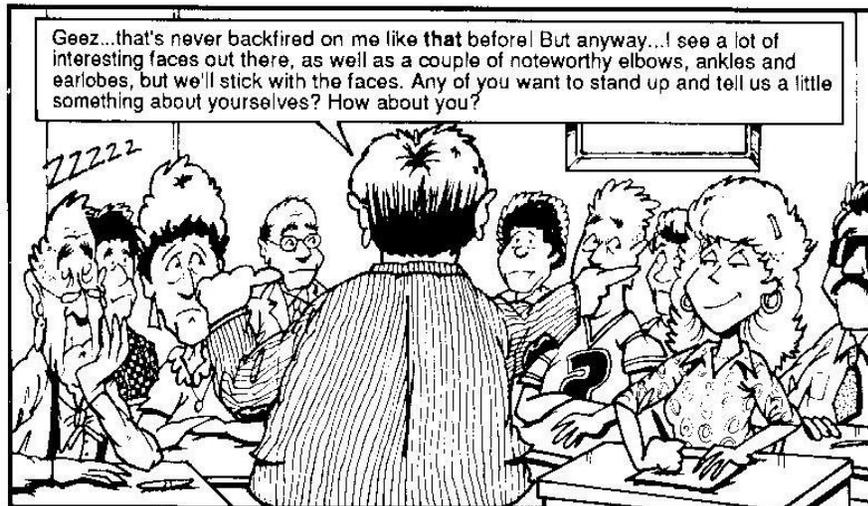
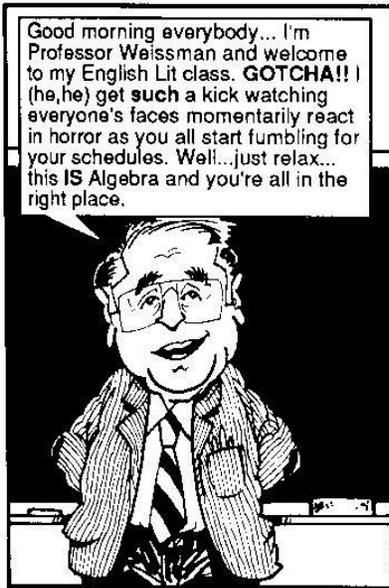
SBA President Edward Mullins said the new pact is "important because it's now achieving six-figure salaries for sergeants" — closing the salary gap with their counterparts in the suburbs.

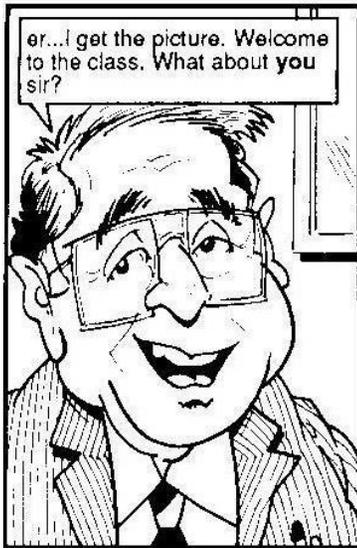
Several cops told The News yesterday that they planned to take the exam now.

"Without a doubt, I will take the next one and pass it," said a 33-year-old Queens officer with five years on the job.

enaspretto@nydailynews.com

Professor Weissman Meet His Class For The First Time





er...I get the picture. Welcome to the class. What about you sir?



I never got da chance to graduate but I plan on taking the GED and I need to learn dis here Algebro stuff.

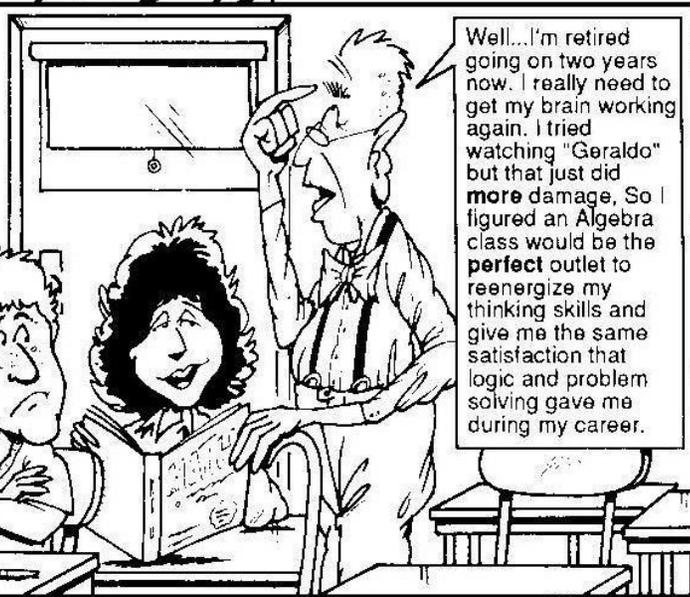


That's Algebra!

Hey look everybody...I'm joining already! You're okay dere professor.



Thanks...I try. Anybody else?



Well...I'm retired going on two years now. I really need to get my brain working again. I tried watching "Geraldo" but that just did more damage. So I figured an Algebra class would be the perfect outlet to reenergize my thinking skills and give me the same satisfaction that logic and problem solving gave me during my career.



Problem solving? You must have been some kind of big business tycoon or computer troubleshooter...right?



Actually...I sold knishes, but I never could decide which mustard to keep in that big dispenser jar... the yellow one or that spicy brown kind!



OHhhh...KAY! I think it's time we move on.

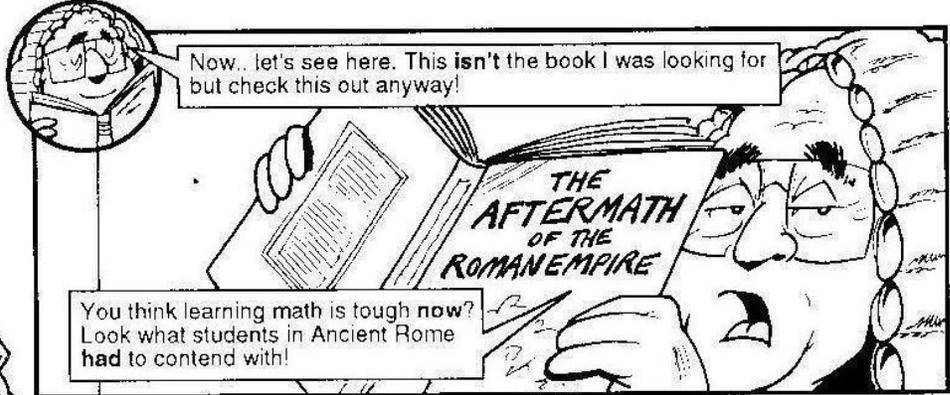


Hereyee... Hereyee!
All rise! The Honorable
Judge Weissman
presiding!



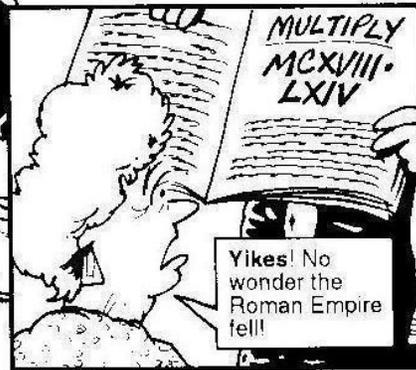
Here come da judge!
Here come da judge!
Boy... is the Professor
committed to his job!

He should be "committed" alright!



Now.. let's see here. This **isn't** the book I was looking for
but check this out anyway!

You think learning math is tough now?
Look what students in Ancient Rome
had to contend with!



Yikes! No
wonder the
Roman Empire
fell!



Oh goody...
here's the
book I was
looking for.

The Romans were active in trade and commerce, and from the time of learning to write they needed a way to indicate numbers. The system they developed lasted many centuries, and still sees some specialized use today. Roman numerals traditionally indicate the order of rulers i.e. Queen Elizabeth II and are still used for dates, on cornerstones and gravestones. The big differences between Roman and Arabic numerals (the ones we use today) are that Romans didn't have a symbol for zero, and that numeral placement within a number can sometimes indicate subtraction (if on the left) and addition (if on the right).
IX=10-1=9 XI=10+1=11



My car is so old that its
Vehicle ID Number is in
Roman Numerals !!!

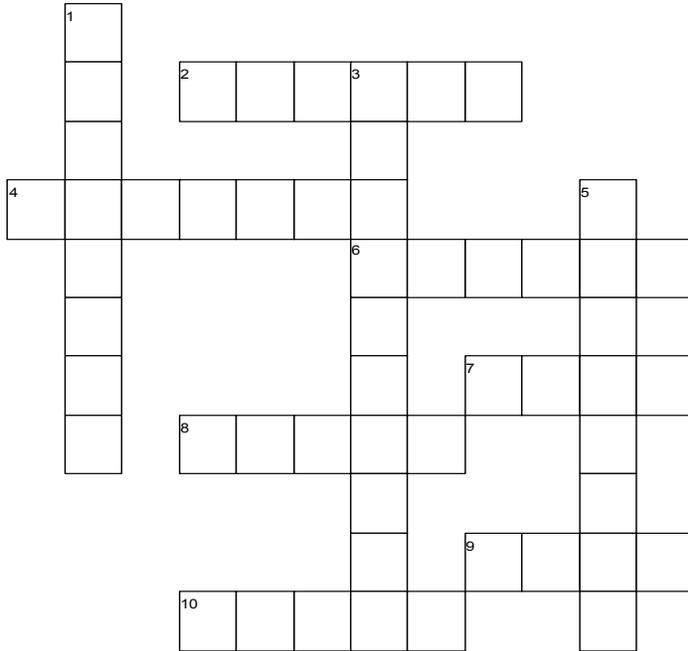


I'm sorry
Prof. but it's
all Greek to
me!

Exercise Set 1

1. Graph the number 7 on the number line.
 - a. ... 5 less than 7?
 - b. ... 3 units to the right of 7?
 - c. ... 1 more than 7?
 - d. ... 3 units to the left of 7?
2. On the number line which number is ...
 - a. ... 5 less than 7?
 - b. ... 3 units to the right of 7?
 - c. ... 1 more than 7?
 - d. ... 3 units to the left of 7?
3. What is the correct symbol, $>$ or $<$, that goes between the two numbers?
 - a. 23 and 32
 - b. 8 and 0
 - c. 0 and 5
 - d. 504 and 540
4. Write these numbers in words.
 - a. 78
 - b. 8
 - c. 561
 - d. 8,345
 - e. 7,002
 - f. 16,800
 - g. 908
 - h. 444
 - i. 165,165
 - j. 7,654,321
 - k. 5,004,003
 - l. 8,800
5. Write the numbers in Standard Form.
 - a. seventeen
 - b. seventy
 - c. sixty-six
 - d. five hundred eighty-one
 - e. five hundred one
 - f. one thousand four
 - g. one thousand forty
 - h. seventeen thousand three hundred thirty-four
 - i. seventy thousand thirty-four
 - j. three hundred nineteen thousand three hundred nineteen
 - k. two hundred three thousand
 - l. two thousand two hundred
6. Write these numbers in Expanded form.
 - a. 678
 - b. 307
 - c. 7,153
 - d. 8,060
 - e. 34,217
 - f. 342,017
 - g. 1,234,567
7. Round to the given place value.
 - a. 387 (100)
 - b. 129 (100)
 - c. 125 (10)
 - d. 387 (10)
 - e. 4,876 (1000)
 - f. 596 (100)
 - g. 596 (10)
 - h. 888 (1000)
 - i. 49 (100)
 - j. 34,567 (10,000)
 - k. 34,667 (1000)
 - l. 99,999 (100)
8. Sets
 - a. What are braces?
 - b. Name this set: $\{0,1,2,3,4,5,6,7,8, \dots\}$
 - c. Write the set of Natural Numbers using set notation
 - d. How many digits are there?
 - e. Using braces write the set of digits.
 - f. What are ellipses?
9. Roman Numerals
 - a. Write the set of the first six symbols used for Roman Numerals.
 - b. What are IX and XI using Roman Numeral? Explain.
 - c. What are the values of these symbols: C, I, M, L, and V in Roman Numerals?
 - d. How do you write 40 and 60 using Roman Numerals?
 - e. How do you write 90 and 110 using Roman Numerals?

Crossword #1



www.CrosswordWeaver.com

ACROSS

- 2 there are 10 of these
- 4 numbers; 1,2,3 4 and so on
- 6 the number 80
- 7 place value of 7 in the number 978
- 8 the number 40
- 9 smallest whole number
- 10 change 387 to 400

DOWN

- 1 form of $900+70+8$
- 3 < symbol (3 words)
- 5 form of number 978

Jokes Set #1

A student visiting a museum was admiring a Tyrannosaurus fossil, and asked the guide how old it was. "That skeleton's sixty million and eighteen days old," the employee replied.

"How can you know it that well?"

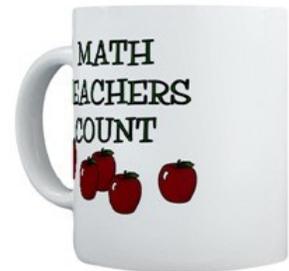
"Well, when I started working

here, I asked a scientist the exact question, and he said it was sixty million years old – and that was eighteen days ago."

There are 3 types of people. Those that can count and those that can not count.

A college student from Trinity Listed all the numbers to infinity But it gave him the fig-dets To write down all those digits, So he dropped Math and

took up divinity.



Brain Teasers Set #1

#1 Find the next letter in the sequence O,T,T,F,F,S,S, ?

#2 . Why are 1966 pennies worth almost twenty dollars?

#3 Using the digits 4, 8, 3, 9 write the largest counting number that is smaller than 5,000 and the smallest counting number that is greater than 5,000

#4 Sally's Mom had three daughters.

1. April
2. May
3. What was the third one's name?

#5 Complete this "Magic Square" using the missing digits: 2,4,5 and 8. All rows, columns and diagonals should add to 15.

	7	6
9		1
	3	

Crossword #1 Solution



Brain Teaser #1 Answers

#1 The letters represent the first letter of each of the counting numbers One, Two, Three, Four, Five, Six, Seven, ... So of course the next letter of the sequence would be "E" for the number 8.

#2 1996 pennies equals \$19 and 96 cents which is about \$20

#3 The largest number smaller than 5,000 is 4,983 and the smallest number larger than 5,000 is

8,349

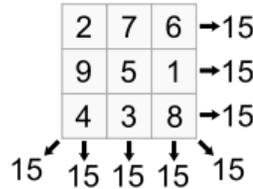
#4 Sally 's mother had three daughters.

1. April
2. May
3. Sally

Answers to Exercise Set 1

- 1.
2. a) 2 b) 10 c) 8 d) 4
3. a) < b) > c) < d) <
4. a) seventy-eight b) eight
 - c) five hundred sixty-one
 - d) eight thousand three hundred forty-five
 - e) seven thousand two
 - f) sixteen thousand eight hundred
 - g) nine hundred eight
 - h) four hundred forty-four
 - i) one hundred sixty-five thousand one hundred sixty-five
 - j) seven million six hundred fifty-four thousand three hundred twenty-one
 - k) five million four thousand three
 - l) eight thousand eight hundred
5. a) 17 b) 70 c) 66
 d) 581 e) 501 f) 1,004
 g) 1,040 h) 17,344 i) 319,319
 j) 203,000 k) 2,200
6. a) 600+70+8 b) 300+00+7
 c) 7000+100+50+3 d) 8000+00+60+0
 e) 30000+4000+200+10+7
 f) 30000+40000+2000+00+10+7
 g) 1000000+200000+30000+4000+500+00+7
7. a) 400 b) 100 c) 130
 d) 390

#5



- e) 5000 f) 600 g) 600 h) 1000
- i) 0 j) 30000 k) 35000
- l) 1000000
8. a) Curly braces { and } used to enclose a set
 b) The set of whole numbers
 c) $N = \{ 1, 2, 3, 4, 5, 6, 7, \dots \}$
 d) There are 10 digits
 e) $D = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$
 f) ... three dots used to mean 'and so on'
9. a) { I, V, X, L, C, M }
 b) I=1, X=10. I before X means subtract 10-1=9
 I after X means add 10+1=11
 c) 100, 1, 1000, 50, 5
 d) XL=40, LX=60 e) XC=90, CX=110

