

Lesson Number 2

Whole Numbers Addition & Subtraction

©2007

Professor Weissman's Algebra Classroom

Martin Weissman, Jonathan S. Weissman. & Tamara Farber



What Is Arithmetic?

Arithmetic is a branch of mathematics that involves combining numbers by addition, subtraction, multiplication and division. For examples, consider the numbers 10 and 2. We can combine them 4 different ways.

$$10+2=12 \quad 10-2=8 \quad 10 \times 2=20 \quad 10 \div 2=5$$

Basic Arithmetic does not use symbols other than the 10 digits, the of 4 basic operations, the decimal point and the equal sign.

$$S=\{0,1,2,3,4,5,6,7,8,9, +, -, \times, \div, ., =\}$$

Inside this

Algebra	1
4 Operations	1
Translating	2
Addition Properties	2
Evaluating	3
Estimating	3
Perimeter	3

How Is Algebra Different From Arithmetic?

Algebra is a branch of mathematics which we can think of as a generalization and extension of arithmetic. In Algebra, we use letters to represent numbers. In arithmetic, we're concerned about specific calculations. A student earns \$8 an hour and works 4 hours, how much does he earn? In Algebra the question might look like this: A student earns \$8 an hour and works for x hours, how much does he earn?

Don't We Use Letters In Arithmetic Also?

Yes, Arithmetic does use letters, also, especially in formulas. You might say that over the years, some Algebra has crept into Arithmetic. It's become what we call, Pre Algebra. This is fine, since it will make it easier to learn Algebra itself. A word of caution is in order. Since Algebra uses letters in place of numbers, it can be confusing to use the letter X for multiplication. As we will see later, the letter X will be replaced with either parentheses or a raised dot.

$$10 \times 2=20 \text{ becomes } 10(2) \text{ or } 10 \cdot 2$$

What Skills Will Be Important To Learn Algebra?

Exercises	4
Fun Page	5
Solutions Page	6

We study Algebra to solve problems that occur in real life, and these problems are expressed in English. Therefore, we will need to translate the problems from English to Algebra, using numbers, letters and symbols.

How Do We Translate Between English And Algebra?

We have already learned the translation of 2 inequality symbols; the "Is less than" and "Is more than."

$$4 < 6 \quad 6 > 4$$

We continue with the 1st operation: addition

$$6 + 4$$

can be read 6 plus 4. However, it can be said several other ways in English.

- The sum of 6 and 4
- 6 increased by 4
- 4 more than 6
- 4 added to 6

It's important to know all of these translations because nearly all of the time we are translating from English to Algebra.

Note that in the last 2 translations the we say 4 before 6.

Translation Examples

Example #1: 10 more than 2.

Translation: $2+10$

Start with 2 then add 10: $2+10$

Note: The correct translation starts with 2. If you write $10+2$ then although it simplifies to the same answer (12), it still is the incorrect translation.

Example #2 7 more than 11

Translation: $11+7$



#3 How would you translate "add cream to coffee?"

coffee + cream

Start with the coffee then add the cream.

But Aren't 2+10 and 10+2 the same?

When simplified both are the same. Many Mathematics teachers actually will accept either translation. However, the only reason that they are equivalent is

that the operation involved is Addition. We say that addition has a

Commutative Property

which means that if we

need to add 2 numbers then the order that we add them is not important. Either way, we get the same answer.

$$2+10 = 10+2$$

What Other Properties Does Addition Have?

The operation of Addition has these 3 properties:

1. Commutative
2. Associative
3. Identity (Zero)

We've already discussed the commutative property. It means that the

order is not important when adding 2 numbers.

$$7+11 = 11+7$$

The Associative Property deals with adding 3 numbers. It says that the number in the middle can be grouped (or asso-

ciated) with the first number or the third number.

For example: $9+8+2$

You can say $(9+8)$ is 17 and $17+2$ is 19. Or, you can say, $(8+2)$ is 10 and $9+10$ is 19. Both answers are 19. The property looks

like this:

$$(9+8)+2=9 + (8+2)$$

The Identity (Zero) property is easy. It says that if you add zero to any number the result is the same (identical).

$$7+0=7$$

Does Subtraction Have Any Properties?

Subtracting zero from any number does not change the number. So there is an identity or zero property.

$$7-0 = 7$$

Is Subtraction commutative? That is, if we switch the order of the numbers, is the answer the same.

Clearly, NO!

$$10-7 \neq 7-10$$

$10-7=3$. However, $7-10$ can not be done in Arithmetic because the first number must be the larger. Later, we'll see that $7-10 = -3$, a Negative number.

Is subtraction Associative? Again, no, as this example will show with these 3 numbers: 10,4, and 3

$$(10-4)-3 \neq 10 - (4-3)$$

If we associate the middle number 4 with the first number 10, we get $(10-4)-3$ or $6-3=3$.

But if we group the 4 with the third number 3 we get $10 - (4-3)=10-1=9$. Different answers.

$$(10-4)-3 \neq 10 - (4-3)$$

$$6-3 \neq 10-1$$

$$3 \neq 9$$

How Is The Subtraction Symbol Translated?

The operation of Subtraction also has 5 ways to be translated:

$$6 - 4$$

Aside from the obvious 6 minus 4, we can say:

- The difference of 6 and 4
- 6 decreased by 4
- 4 less than 6

- 4 subtracted from 6
- Again in 2 translations the numbers are reversed. With subtraction, however, we have to be careful. Why? Because there is no Commutative Property for subtraction. That is $6-4$ and $4-6$ are not equal. In symbols we write:

$$6-4 \neq 4-6$$

So, don't translate 4 less than 6 as $4-6$.

Translating must be precise but can get very tricky.



Each of these 3 expressions has the word 'less' but each is translated differently:

- 3 is less than 7 $3 < 7$
- 3 less than 7 $7-3$
- 3 less 7 $3-7$



A fence goes around the perimeter of a yard.

$$P=2L+2W$$

What Does Evaluate Mean?

A number like 13 never changes its value. Look at it tomorrow and it's still 13. That's why it's called a **constant**. However, when we use a letter like x to represent a number it may have a value of 13 now and an hour later it may be some other value. That's why we call x a **variable**. In Algebra, when we say evaluate we want the value of an expression for specific values of the variables.

For example, the formula for the Perimeter of a rectangle is:

$P=2L+2W$. The Perimeter means the **distance around**. What is the perimeter of a rectangular yard whose length, L is 7 feet and whose Width W is 5 feet?

To evaluate an expression, $2L+2W$, we replace each letter with its value and then simplify the expression.

$$\begin{aligned} L=7 \quad W=5 \quad 2L+2W=? \\ 2(7)+2(5) \\ 14 + 10 \\ 24 \end{aligned}$$

The perimeter is 24 feet. You will need 24 feet of fence to enclose the yard.

Another important geometric shape is the triangle which has three sides. The formula for the Perimeter of a triangle with sides a , b , and c is:

$$P=a+b+c$$

If the 3 sides in feet are $a=6$, $b=8$, $c=10$ then here is how we evaluate and find the perimeter:

$$P=6+8+10 \quad P=24 \text{ feet}$$



What Is Estimating?

Estimating is very similar to rounding. Recall, that when rounding you are told to which place. For example to round the numbers 654, 73, and 3,166 you might be told 'to the nearest 100' and the results would be 700, 100, and 3,200.

To estimate, however, each number would be rounded to the highest place value. The results would be 700, 70, and 3,000.

One time you might estimate is when you are shopping and want to know if you're over your limit.

For example, let's say that you have several items in your cart. Their prices are \$83, \$174, and \$29. Let's estimate their total cost.

$$\begin{aligned} \$84 \text{ ---} > \$80 \\ \$179 \text{ ---} > \$200 \end{aligned}$$

$$\$29 \text{ ---} > \$30$$

$$80+200+30=310$$

The actual sum is:

$$84+179+29=292$$



Exercise Set 2

1. List 5 ways to say that you are adding.
 - a. Find the sum of 457 and 987
 - b. What is 7 increased by 12?
 - c. Find the total of 690, 4,888 and 234
 - d. What is 453 more than 1000?
 - e. What is 1,234 added to 9,876?
2. Translate then evaluate
 - a. Find the sum of 457 and 987
 - b. What is 7 increased by 12?
 - c. Find the total of 690, 4,888 and 234
 - d. What is 453 more than 1000?
 - e. What is 1,234 added to 9,876?
3. Estimate by rounding to the first digit, then find the exact answer.
 - a. $689 + 378$
 - b. $5,777 + 2,087$
 - c. $9,499 + 7,378 + 1,923$
 - d. $728 - 231$
 - e. $7,876 - 639$
4. List 5 ways to say that you are subtracting.
 - a. Find 124 decreased by 100
 - b. What is 50 minus 20?
 - c. What is the difference between 88 and 79?
 - d. What is 127 less than 200?
 - e. What is 50 less 30?
 - f. Subtract 47 from 98
5. Evaluate $x+y$ when
 - a. $x=3,567$ and $y=763$
 - b. $x=1,098$ and $y=12,765$
6. Evaluate $x-y$ when
 - a. $x=876$ and $y=459$
 - b. $x=8,989$ and $y=7,387$
7. Translate. Do not evaluate.
 - a. 6 increased by 9
 - b. 12 less 7
 - c. 2 less than 7
 - d. 2 is less than 7
 - e. 34 minus 12
8. Name the 3 properties of addition that start with the letters: CAI
9. Name the property
 - a. $7+3=3+7$
 - b. $1+(2+3)=(1+2)+3$
 - c. $6+0=6$
10. Complete the addition property then give its name.
 - a. $9+(2+8)=(9+2) \underline{\hspace{1cm}}$
 - b. $\underline{\hspace{1cm}}+7=7+8$
 - c. $33+\underline{\hspace{1cm}}=33$
 - d. $a+b=\underline{\hspace{1cm}}$
 - e. $a+(b+c)=\underline{\hspace{1cm}}+c$
 - f. $\blacktriangle + \blacksquare = \underline{\hspace{1cm}}$
11. Is the given value of x a solution to the equation?
 - a. $x=12$ $x+8=20$?
 - b. $x=30$ $48=78-x$?
 - c. $x=7$ $x-11=4$?
12. Geometry and perimeter
 - a. What is the formula for the perimeter of a rectangle with length L and width W ?
 - b. A rectangle has a length of 8 inches and a width of 5 inches. What is its perimeter?
 - c. A student's desktop is 2 feet by 3 feet. What is the perimeter of the desktop?
 - d. What is the formula for the perimeter of a triangle with sides $a, b,$ and c ?
13. The sum of two numbers
 - a. The sum of two numbers is 100. The first is 30, use the numbers 100 and 30 to represent the second.
 - b. The sum of two numbers is 100. The first is 60, use the numbers 100 and 60 to represent the second.
 - c. The sum of two numbers is 100. The first is x , use the numbers 100 and x to represent the second.
- e. Three sides of a triangle are 5 cm, 7 cm, and 10 cm. What is the perimeter?
- f. What is the distance around a rectangle called?

Jokes Set #2

A little boy was doing his math homework. He said to himself, "Two plus five, that son of a bitch is seven. Three plus six, that son of a bitch is nine...."

His mother heard what he was saying and gasped, "What are you doing?"

The little boy answered, "I'm doing my math homework,

Mom."

"And this is how your teacher taught you to do it?" the mother asked.

"Yes," he answered.

Infuriated, the mother asked the teacher the next day, "What are you teaching my son in math?"

The teacher replied, "Right now, we are learning addition."

The mother asked, "And are you teaching them to say two plus two, that son of a bitch is four?"

After the teacher stopped laughing, she answered, "What I taught them was, two plus two, THE SUM OF WHICH,

is four."

A woman in a bar tries to pick up a mathematician.

"How old, do you think, am I?" she asks coyly.

"Well, 18 by that fire in your eyes, 19 by that glow on your cheeks, 20 by that radiance of your face, and adding that up is something you can probably do for yourself..."

Brain Teasers Set #2

SEND
+MORE

MONEY

Hints:

M=1 O=0 E=5

Each letter represents a digit. If one letter is a certain number then all instances of that letter equals that number. What are the values of each letter? What would the addition problem look like with all the letters replaced with digits?

Answers to Exercise Set 2

1. sum, total, increased by, added to, more than, plus
- 2a. 457+987 ; 1,444
- b. 7+12 ; 19
- c. 690+4,888+234 ; 5,812
- d. 1000+453 ; 1,453
- e. 9,876+1,234 ; 11,110
- 3a. 1,100 ; 1,067
- b. 8,000 ; 7,864
- c. 18,000 ; 18,800
- d. 500 ; 497
- e. 7,400 ; 7,237
- 4a. subtract, decreased by, less than, less, difference, minus
- b. 24
- c. 9
- d. 73
- e. 20
- f. 51
- 5a. 4,330
- b. 13,863
- 6a. 417
- b. 1,602
- 7a. 6+9
- b. 12-7
- c. 7-2
- d. 2<7
- e. 34-12
8. Commutative, Associative, Identity
- 9a. Commutative
- b. Associative
- c. Identity
- 10a. $9+(2+8)=(9+2)+8$
- b. $8+7=7+8$
- c. $33+0=33$
- d. $a+b=b+a$
- e. $a+(b+c)=(a+b)+c$
- f. $\blacktriangle + \blacksquare = \blacksquare + \blacktriangle$
- 11a. yes
- b. yes
- c. no
- 12a. $P=2L+2W$
- b. 26 inches
- c. 10 feet
- d. $P=a+b+c$
- e. 22 cm.
- f. perimeter
- 13a. 100-30
- b. 100-60
- c. 100-x

Brain Teaser #2 Answers

Since M = 1, O = 0, and E = 5, that simplifies things a bit.

S5ND
+10R5
10N5Y

S must = 9 because of its location and the final answer is in the ten thousands.

95ND
+10R5
10N5Y

Now to solve for N. In the hundreds position, $5+0 = N$ and we know $E = 5$. There must be a carryover from the tens position, so $N = 6$.

Using the same sort of logic, $R = 8$. The result so far is:

956D
+1085
1065Y

Now for D. D could be any integer from 0-9. It can't be 1, 5, 6, 8, 9, 0 because they have been used. It can be 2, 3, 4, 7. We reason that D is greater than 5 because the tens column needs a carryover, therefore $D = 7$

9567
+1085

1065Y

which means $Y = 2$.

9567
+1085
10652

www.math911.com

mathprof@hotmail.com

Professor Weissman's

MATH911

#1 TOP RATED STEP BY STEP TUTORIALS

Free Math Tutorials !!

- INTRODUCTORY ALGEBRA
- INTERMEDIATE ALGEBRA
- COLLEGE ALGEBRA
- TRIGONOMETRY
- PRE-CALCULUS

Download Now: www.math911.com