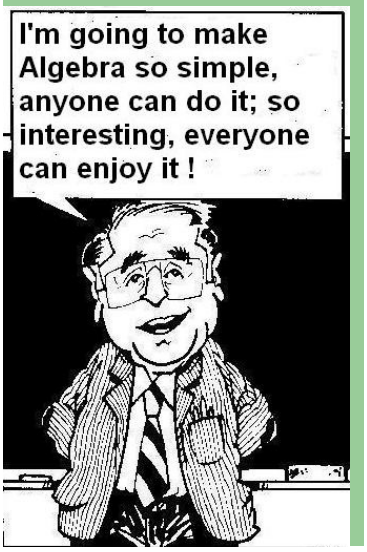
Lesson Number 3

Multiplication of Whole Numbers & Factors

Professor Weissman's Algebra Classroom

Martin Weissman, Jonathan S. Weissman. & Tamara Farber



Inside this issue:

What Are The Different Ways To Show Multiplication?

Before you see how we show Multiplying, let's just say that we are not going to use the letter X any more! That's because we use the letter X in Algebra to stand for a number. That being said, if we want to show 7 times 5 we have all these ways:

7(5) (7)5 (7) (5) Using Parentheses around either number or both

7•5 Using a '**dot**' , but be careful. The dot must be raised to the middle so that it will not be confused with a decimal point.

x (y) (x) y (x) (y) 7xy xy Neither parentheses nor a dot is really needed.

5(y) (5)y (5) (y) 5•y 5y Neither parentheses nor a dot is really needed.

Why is Multiplication Called A Shortcut?

Multiplication is a shortcut for addition. For example if the cost of a bus ride is $3 then the cost of 4 bus rides would be $12.

We can say that 4 threes are 8. What we mean is that is we ADD 4 threes then the sum is 12. The 4 tells us the number of times to ADD 3.

$$4 \cdot 3 = 3+3+3+3 = 12$$

What Are Powers Of Ten?

Some powers of ten are the numbers in the sequence 10, 100, 1000, 10000, etc.

These numbers are generated by using 10 as a base with exponents (or powers) of 1, 2, 3, 4, 5, etc. The exponent tells you how many zeros follow the 1.

$10^1=10$ $10^2=100$

$10^3=1000$ $10^4=10000$

$10^5=100000$

Why Is An Exponent Called A Shortcut?

An exponent is a shorthand way to show that we are MULTIPLYING with the same number. Suppose that we are multiplying with a 3, four times. That's 3•3•3•3. Here we can say that 4 threes are 81.

$$3^4=3 \cdot 3 \cdot 3 \cdot 3=81$$

3 is called the base. 4 is called the exponent.

What Are Factors?

In a multiplication problem each of the numbers being multiplied is called a factor. For example, 7ab would have 3 factors, 7, a, and b. The entire expression 7ab is called a product.

How many factors are in: $9x^2y$? When expanded it looks like this:

$9 \cdot x \cdot x \cdot y$ There are 4 factors.

How Are Powers Of Ten Used?

#1

Multiply 365 by 1000. There's really no need to set up the problem and multiply by with all those zeros.

Just write 365 and attach those 3 zeros that you see in 1000. 365 000 or 365,000 is the answer.

#2

Multiply 24 by 10^5

Simply write 24 and then attach 5 zeros (the exponent is 5).

24 00000 or 2,400,000

How Do I Estimate the Product When I Multiply?

To estimate a product, round each number to the first digit. The numbers will have zeros at the end. Multiply these numbers. The result is an estimate of what the actual product should be.

Example: Multiply 7,345 by 497

Round the numbers: 7,000 (500)

Multiply:3,500,000

Actual Product: 7,345(497) = 3,650,465

Estimating tells you whether or not your answer is "in the ballpark."

What Are Factors Of A Number?

The factors of a number are those numbers that divide exactly into the number . You might say that the factors go into the number 'evenly' with 'no remainder.'

Example #1 What are the factors of 12? What 2 numbers multiply to 12. Start with 1

1(12) = 12

2(6) = 12

3(4) = 12

The factors of 12 are:

1,2,3,4,6,12

Example #2 What are the factors of 20? What 2 numbers multiply to 20? Start with 1.

1(20) = 20

2(10) = 20

4(5) = 20

The factors of 20 are:

1,2,4,5,10,20

Example #3
Find the factors of 5.

1(5) = 5

The only factors of 5 are:
1 and 5

Example #4
Find the factors of 13

1(13) = 13

The only factors of 13 are:
1 and 13

How Do I Multiply Numbers With Zeros At the Ends?

There's a shortcut when multiplying numbers which end in zeros.

#3 Multiply 1300 by 40000

Just multiply 13 by 4 and then add 6 zeros. That's the total amount of zeros.

13(4)=52

52 000000= 52,000,000

What Properties Does Multiplication Have?

If you recall, Multiplication is repeated addition. Multiplication like addition is both Commutative and Associative.

Commutative: $7 \cdot 8 = 8 \cdot 7$

Associative: $3 \cdot (4 \cdot 5) = 3 \cdot (4 \cdot 5)$

Addition has an Identity, it's zero. Multiplication has an Identity. It's One. If you multiply any number by 1 you don't change it's value.

Identity (One): $8 \cdot 1 = 8$

What Are Primes And Composites?

To determine if a number is prime or composite, follow these steps:

1. Find all factors of the number.
2. A number is prime If the number has only two factors, 1 and itself
3. A number is composite if the number has more than 2 factors.

Here are some of the first the counting numbers broken into Primes (P) and Composites (C)
P= {2,3,5,7,11,13,17,19, ... }
C= {4,6,8,9,10,12,14,15,16, ...}

Lesson

How Do I Find The Area Of A Rectangle?

Use the formula Area=LW, to find the area of a rectangle.

A rectangle has 4 sides, but we only use 2 of them to find the Area (A)

A = LW

A = (3)(5)

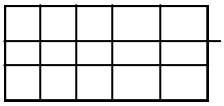
A = 15 square feet

Which side is the length which side is the width.?

The answer that is doesn't matter, because Multiplication is Commutative.

(3)(5)=(5)3)

Note that the answer includes the word square. This is because when we find the area we are looking for the amount of squares inside.



You'll need to find an area when you need to

- paint
- seed a lawn
- cover a floor, ceiling

Review What is the Perimeter of the rectangle?

P=2L+2W

P=2(3)+2(5)

P=6+10 =16 feet (no squares it's the distance around.

Example #1

How many one foot square tiles are needed to cover a floor that is 13 feet wide and 20 feet long?

A salesman might say that the floor is 13 by 20 and write 13 x 20. The word by and the symbol 'x' is a reminder that for area you multiply.

A = LWA = (13)(20) = 260 square feet

Since each tile is one square foot, 260 tiles will be needed. It's always a

Example #2

The label on a can of paint says that it will 50 square feet. The wall that will be painted is 8 feet high and 20 cover about feet wide. How many cans will be needed?

A = LWA=8(20)=160 square feet.

Each can covers 50 square feet. 3 Cans would cover 150 square feet. That's not enough. We'll need 4 cans. Not to worry, because it's a good idea to have extra paint. Why?

Example #3

A rectangular garden 24 feet by 40 feet is being constructed.

a. How many square feet of sod (grass) will be needed?

b. How many feet of fence will be needed to enclose the garden?

Solution.

a. A=LW A=24(40)=960 sq ft.

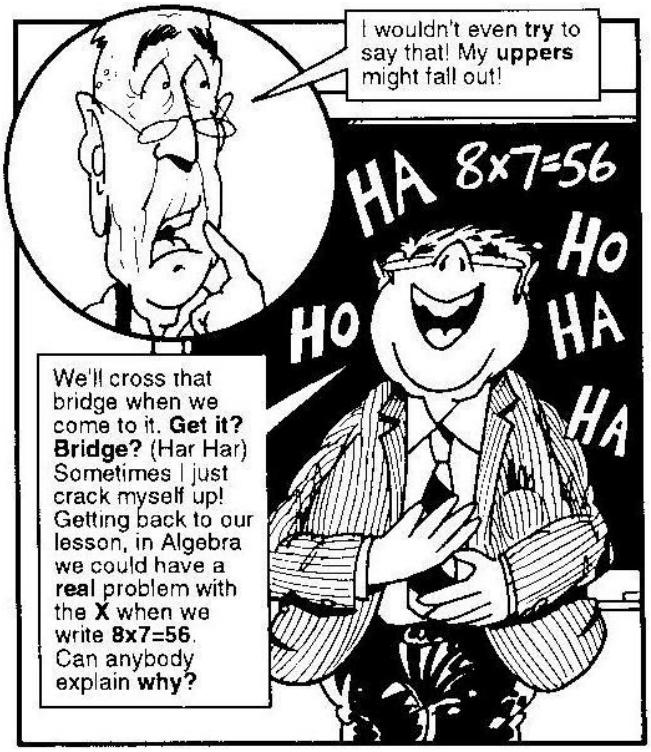
b. P=2L+2W

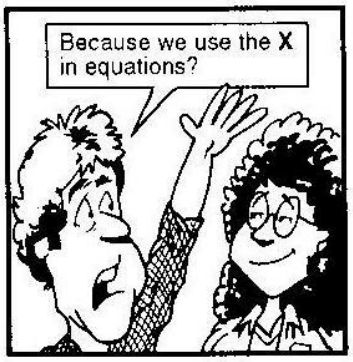
P = 2(24)+2(40)

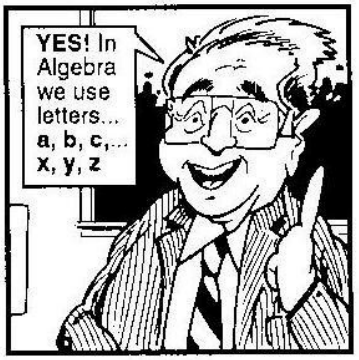
P = 48+80

P = 128 feet


Let's see. We've discussed Addition and Subtraction, the Commutative and Associative Properties. Zero is the Identity for both addition and Subtraction, ... OK, ...
Let's move on! Now we're ready to continue with multiplication and division. But before that, a little **nomenclature** is in order.
Who's Norman Clature?
NOMEN! NOMENCLATURE!
WHAT'S THAT?
8×7=56
Doesn't **anyone** have a dictionary?
Dictionary? I thought this was math!
Right here Professor Weissman! *"A set of names or terms as those used in a particular science or art."*
DICTIONARY
Correctamundo! This is some of the **"nomenclature"** we use in multiplication and division.
MULTIPLICATION
Factors
Product
DIVISION
Dividend (Numerator)
Divisor (Denominator)
Quotient
In this example, the 8 and 7 are **FACTORS** and the result, 56, is called the **PRODUCT.**
8×7=56
Factor X Factor=Product
Division is usually indicated by a fraction... either with the bar **or** a slash. In division, we get a **QUOTIENT** when we divide a **DIVIDEND (NUMERATOR)** by a **DIVISOR (DENOMINATOR).**
Dividend (Numerator)
60/12=5
OR
60/12=5
Quotient
Divisor (Denominator)
A little trick is to remember **"FACTORIES MAKE PRODUCTS."**
ACME
What's that you say..."we get a quotient when we divide a dividend by a divisor?" Geez...and I thought "she sells seashells by the sea shore" was a tough one!

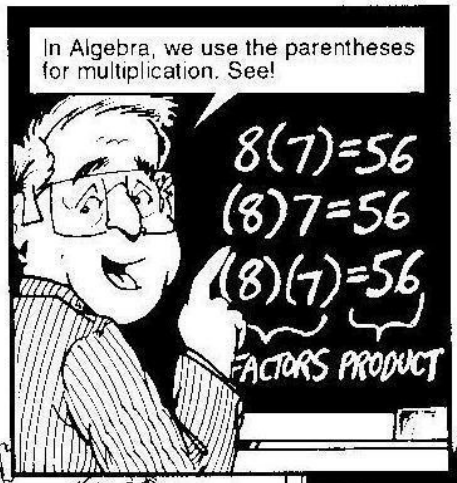

I wouldn't even **try** to say that! My **uppers** might fall out!
HA 8x7=56
HO
HO
HA
HA
We'll cross that bridge when we come to it. **Get it? Bridge?** (Har Har) Sometimes I just crack myself up! Getting back to our lesson, in Algebra we could have a **real** problem with the **X** when we write **8x7=56**. Can anybody explain **why?**

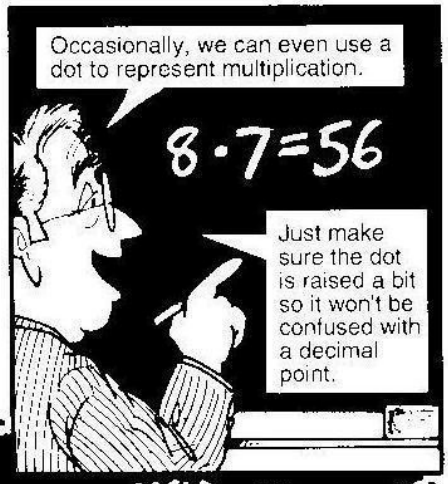

Because we use the **X** in equations?

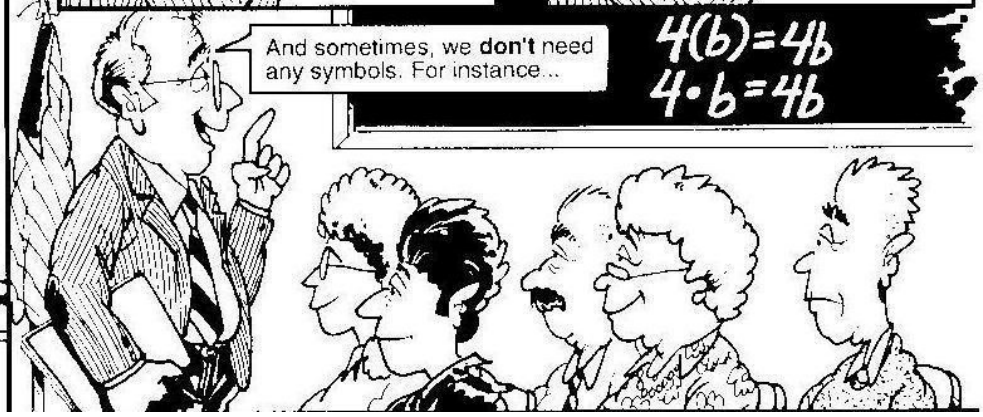

YES! In Algebra we use letters... **a, b, c,... x, y, z**

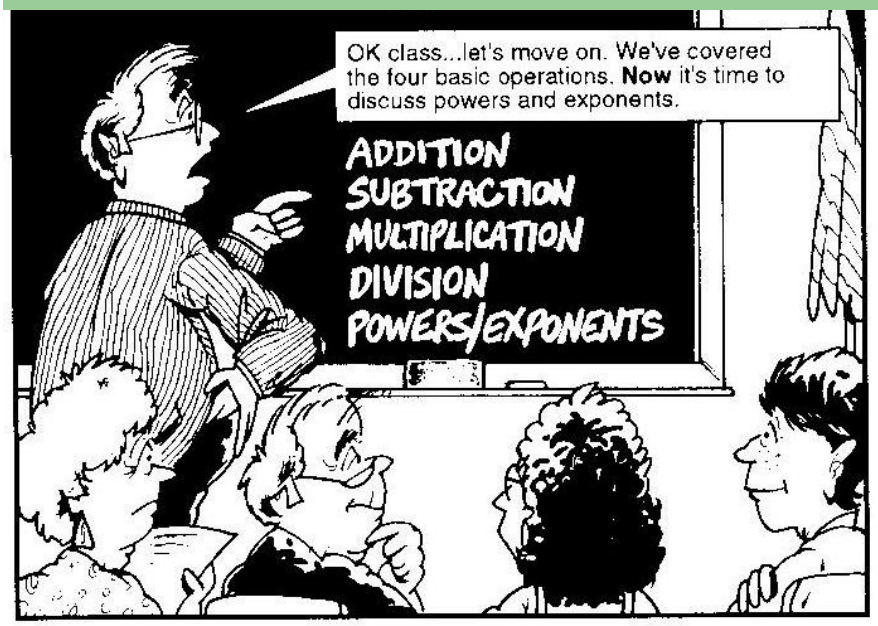

Oh great...now I'm back in nursery school! What's next Professor...**finger painting** or **nappy time**?

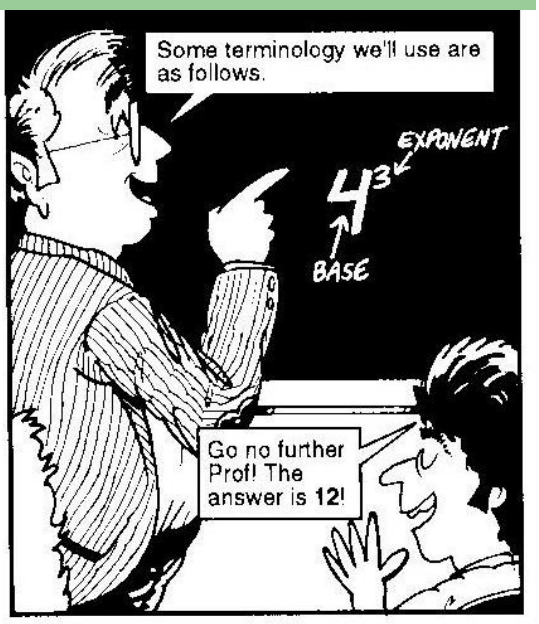

Allow me to explain. Since we might use **X** instead of a number, we **can't** use the letter **X** when we multiply?

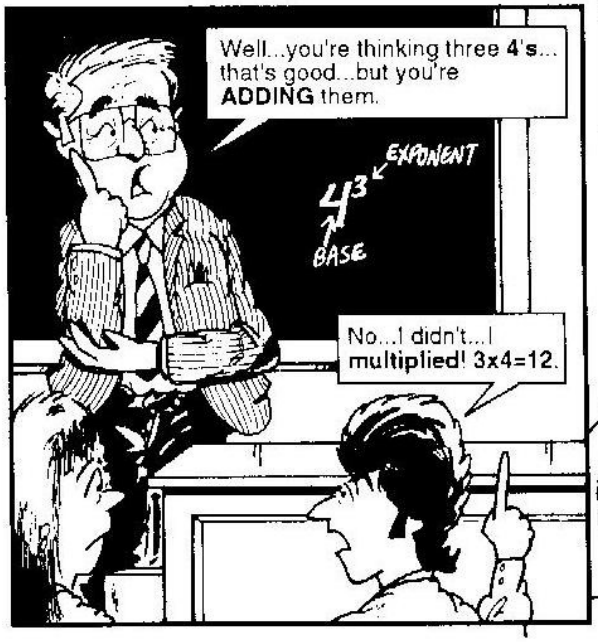

In Algebra, we use the parentheses for multiplication. See!
8(7)=56
(8)7=56
(8)(7)=56
FACTORS PRODUCT


Occasionally, we can even use a dot to represent multiplication.
8·7=56
Just make sure the dot is raised a bit so it won't be confused with a decimal point.

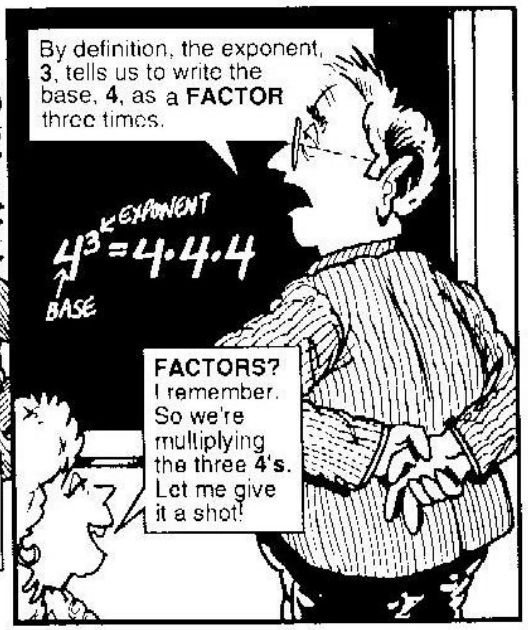

And sometimes, we **don't** need any symbols. For instance...
4(b)=4b
4·b=4b

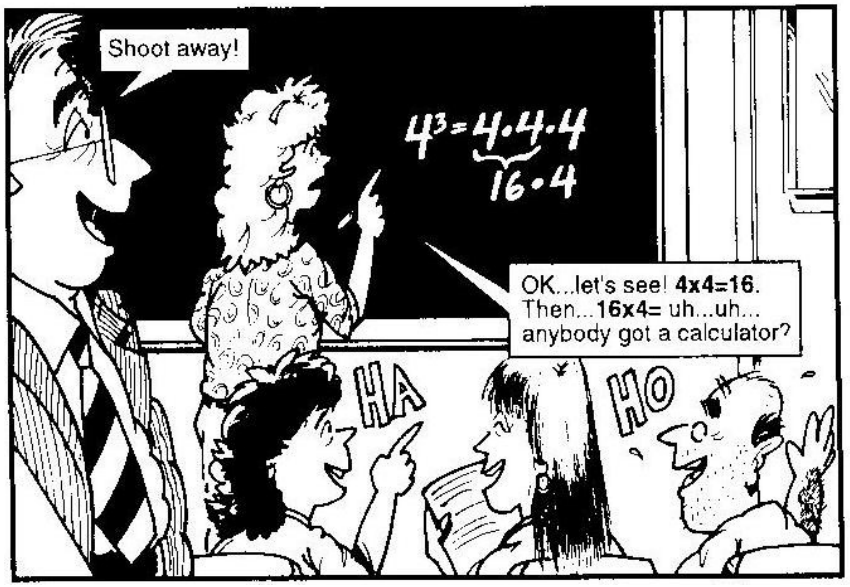

OK class...let's move on. We've covered the four basic operations. **Now** it's time to discuss powers and exponents.
ADDITION
SUBTRACTION
MULTIPLICATION
DIVISION
POWERS/EXPONENTS

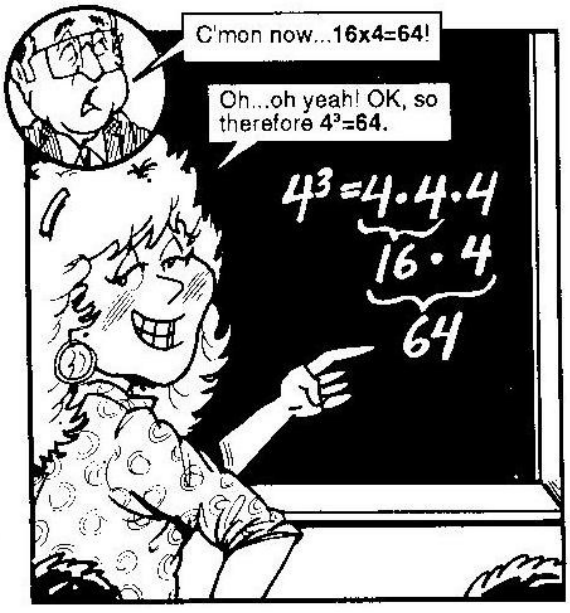

Some terminology we'll use are as follows.
EXPONENT
4^3
BASE
Go no further Prof! The answer is **12**!

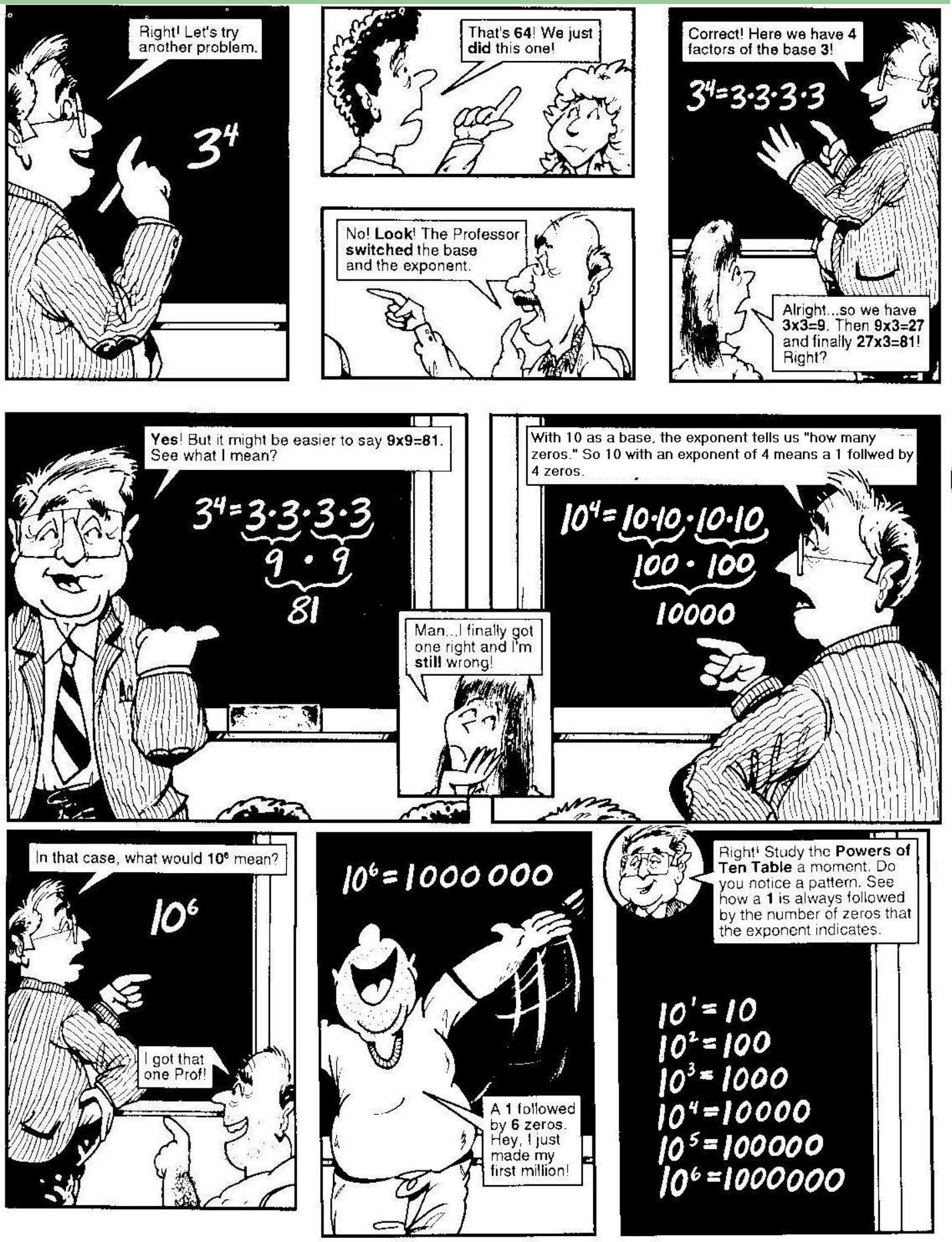

Well...you're thinking three **4's**... that's good...but you're **ADDING** them.
EXPONENT
4^3
BASE
No...I didn't...I **multiplied**! 3x4=12.

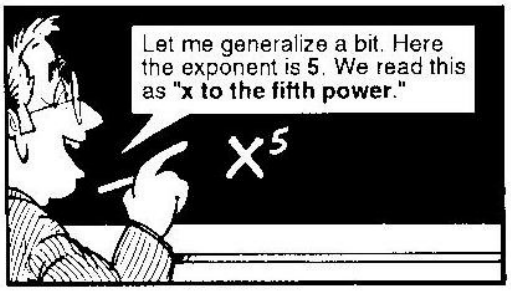

Multiplication **IS** repeated addition! **3x4=4+4+4=12** but exponents mean repeated **multiplication**. Let me show you!
Enlighten us oh great guru of Algebra.

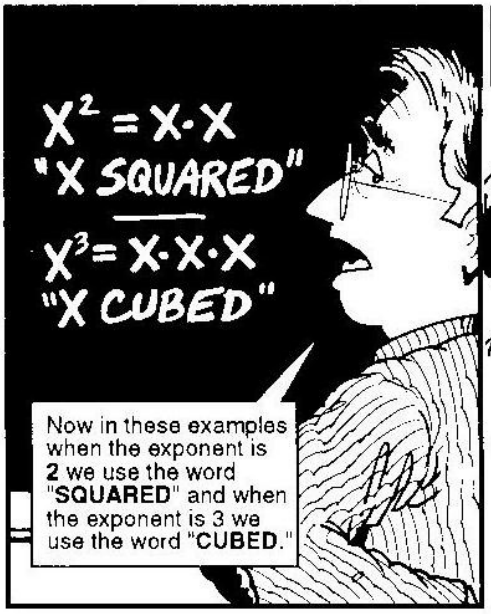

By definition, the exponent, **3**, tells us to write the base, **4**, as a **FACTOR** three times.
EXPONENT
$4^3 = 4 \cdot 4 \cdot 4$
BASE
FACTORS? I remember. So we're multiplying the three **4's**. Let me give it a shot!


Shoot away!
$4^3 = \underbrace{4 \cdot 4}_{16} \cdot 4$
OK...let's see! **4x4=16**. Then...**16x4=** uh...uh... anybody got a calculator?
HA
HO

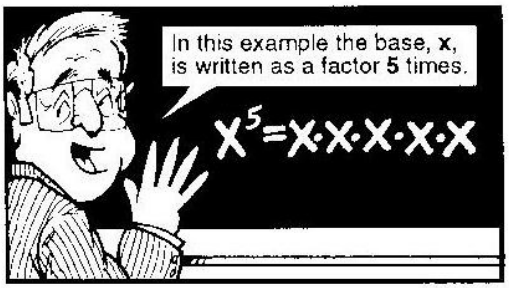

C'mon now...**16x4=64**!
Oh...oh yeah! OK, so therefore **4^3=64**.
$4^3 = \underbrace{\underbrace{4 \cdot 4}_{16} \cdot 4}_{64}$

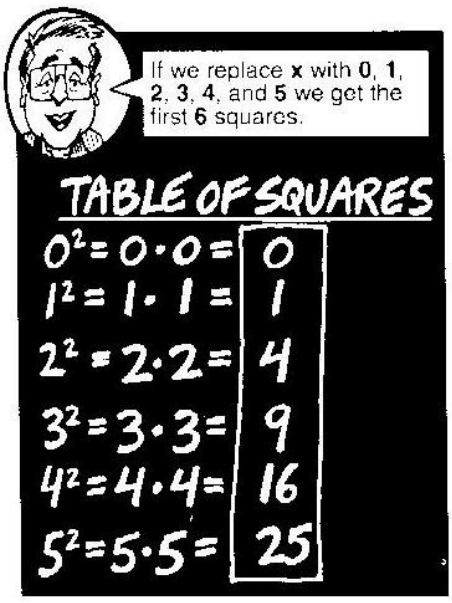

Right! Let's try another problem.
3^4
That's 64! We just did this one!
No! Look! The Professor switched the base and the exponent.
Correct! Here we have 4 factors of the base 3!
$3^4=3\cdot3\cdot3\cdot3$
Alright...so we have 3x3=9. Then 9x3=27 and finally 27x3=81! Right?
Yes! But it might be easier to say 9x9=81. See what I mean?
$3^4=3\cdot3\cdot3\cdot3$
$9\cdot9$
81
Man...I finally got one right and I'm still wrong!
With 10 as a base, the exponent tells us "how many zeros." So 10 with an exponent of 4 means a 1 follwed by 4 zeros.
$10^4=10\cdot10\cdot10\cdot10$
$100\cdot100$
10000
In that case, what would 10^6 mean?
10^6
I got that one Prof!
$10^6=1000000$
A 1 followed by 6 zeros. Hey, I just made my first million!
Right! Study the Powers of Ten Table a moment. Do you notice a pattern. See how a 1 is always followed by the number of zeros that the exponent indicates.
$10^1=10$
$10^2=100$
$10^3=1000$
$10^4=10000$
$10^5=100000$
$10^6=1000000$

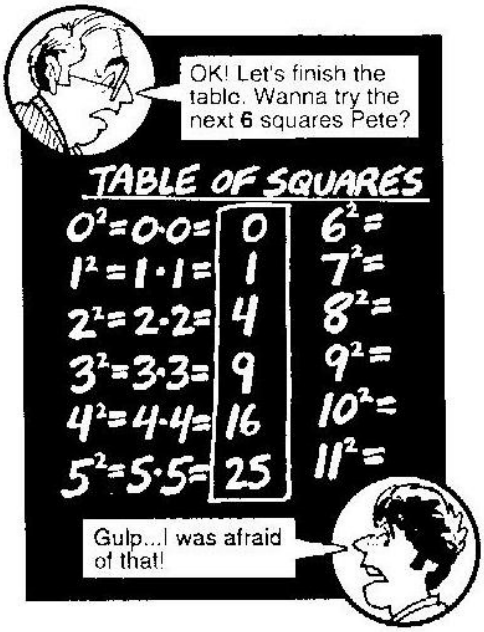

Let me generalize a bit. Here the exponent is 5. We read this as "x to the fifth power."
X^5

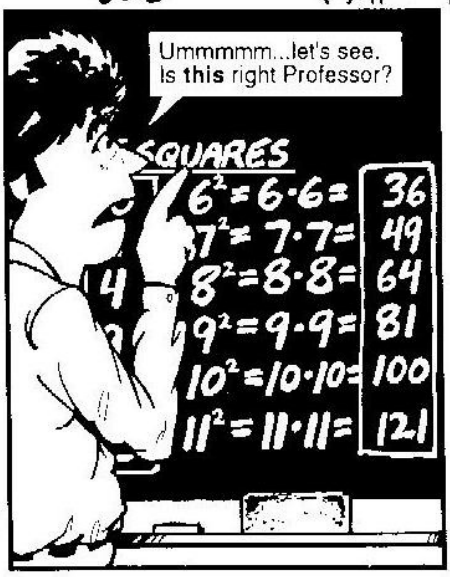

$X^2 = X \cdot X$
"X SQUARED"
$X^3 = X \cdot X \cdot X$
"X CUBED"
Now in these examples when the exponent is 2 we use the word "SQUARED" and when the exponent is 3 we use the word "CUBED."

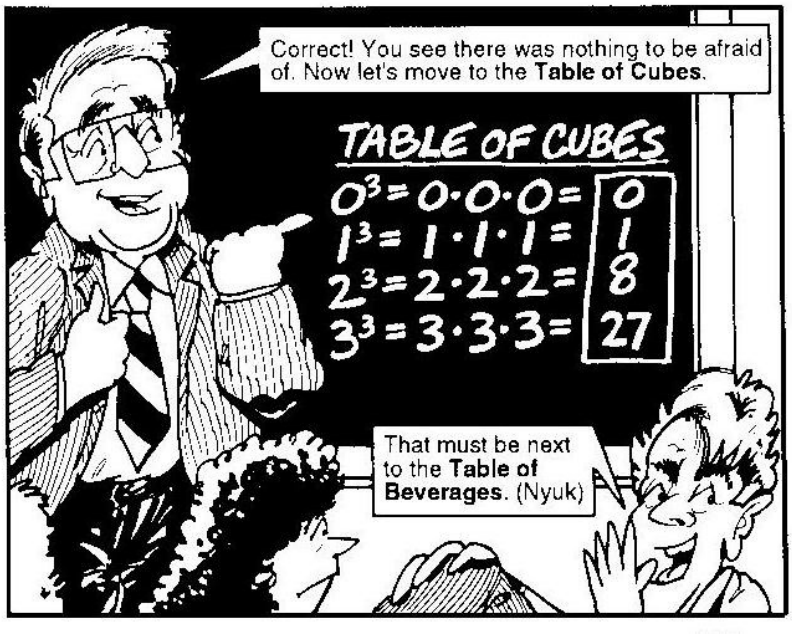

So what are the answers Prof?
We can't go any further unless we know the value of x. Let me show you.

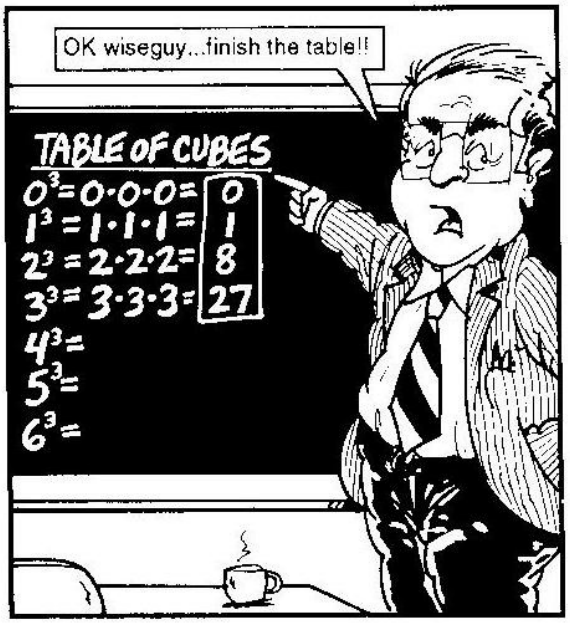

In this example the base, x, is written as a factor 5 times.
$X^5 = X \cdot X \cdot X \cdot X \cdot X$

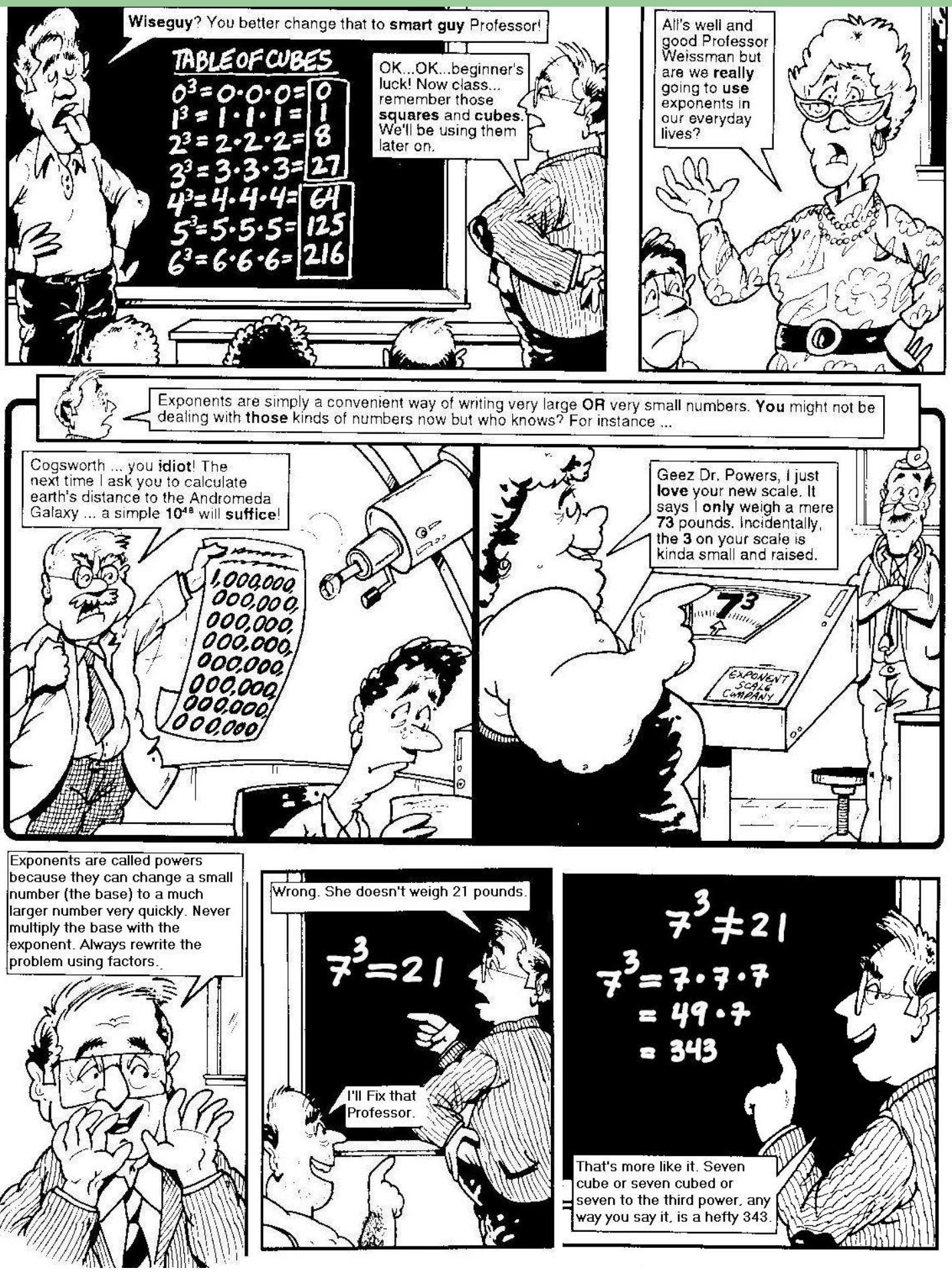

If we replace x with 0, 1, 2, 3, 4, and 5 we get the first 6 squares.
TABLE OF SQUARES
$0^2 = 0 \cdot 0 = 0$
$1^2 = 1 \cdot 1 = 1$
$2^2 = 2 \cdot 2 = 4$
$3^2 = 3 \cdot 3 = 9$
$4^2 = 4 \cdot 4 = 16$
$5^2 = 5 \cdot 5 = 25$


OK! Let's finish the table. Wanna try the next 6 squares Pete?
TABLE OF SQUARES
$0^2 = 0 \cdot 0 = 0$
$1^2 = 1 \cdot 1 = 1$
$2^2 = 2 \cdot 2 = 4$
$3^2 = 3 \cdot 3 = 9$
$4^2 = 4 \cdot 4 = 16$
$5^2 = 5 \cdot 5 = 25$
$6^2 =$
$7^2 =$
$8^2 =$
$9^2 =$
$10^2 =$
$11^2 =$
Gulp...I was afraid of that!


Ummmmm...let's see. Is this right Professor?
SQUARES
$6^2 = 6 \cdot 6 = 36$
$7^2 = 7 \cdot 7 = 49$
$8^2 = 8 \cdot 8 = 64$
$9^2 = 9 \cdot 9 = 81$
$10^2 = 10 \cdot 10 = 100$
$11^2 = 11 \cdot 11 = 121$


Correct! You see there was nothing to be afraid of. Now let's move to the Table of Cubes.
TABLE OF CUBES
$0^3 = 0 \cdot 0 \cdot 0 = 0$
$1^3 = 1 \cdot 1 \cdot 1 = 1$
$2^3 = 2 \cdot 2 \cdot 2 = 8$
$3^3 = 3 \cdot 3 \cdot 3 = 27$
That must be next to the Table of Beverages. (Nyuk)


OK wiseguy...finish the table!!
TABLE OF CUBES
$0^3 = 0 \cdot 0 \cdot 0 = 0$
$1^3 = 1 \cdot 1 \cdot 1 = 1$
$2^3 = 2 \cdot 2 \cdot 2 = 8$
$3^3 = 3 \cdot 3 \cdot 3 = 27$
$4^3 =$
$5^3 =$
$6^3 =$


Wiseguy? You better change that to smart guy Professor!
TABLE OF CUBES
$0^3 = 0 \cdot 0 \cdot 0 = 0$
$1^3 = 1 \cdot 1 \cdot 1 = 1$
$2^3 = 2 \cdot 2 \cdot 2 = 8$
$3^3 = 3 \cdot 3 \cdot 3 = 27$
$4^3 = 4 \cdot 4 \cdot 4 = 64$
$5^3 = 5 \cdot 5 \cdot 5 = 125$
$6^3 = 6 \cdot 6 \cdot 6 = 216$
OK...OK...beginner's luck! Now class... remember those squares and cubes. We'll be using them later on.
All's well and good Professor Weissman but are we really going to use exponents in our everyday lives?
Exponents are simply a convenient way of writing very large OR very small numbers. You might not be dealing with those kinds of numbers now but who knows? For instance ...
Cogsworth ... you idiot! The next time I ask you to calculate earth's distance to the Andromeda Galaxy ... a simple 10^{18} will suffice!
1,000,000 000,000 000,000 000,000 000,000 000,000 000,000 000,000
Geez Dr. Powers, I just love your new scale. It says I only weigh a mere 73 pounds. Incidentally, the 3 on your scale is kinda small and raised.
7^3
EXPONENT SCALE COMPANY
Exponents are called powers because they can change a small number (the base) to a much larger number very quickly. Never multiply the base with the exponent. Always rewrite the problem using factors.
Wrong. She doesn't weigh 21 pounds.
$7^3 = 21$
I'll Fix that Professor.
$7^3 \neq 21$
$7^3 = 7 \cdot 7 \cdot 7$
$= 49 \cdot 7$
$= 343$
That's more like it. Seven cube or seven cubed or seven to the third power, any way you say it, is a hefty 343.

Exercise Set 3

1. Multiplication

a. (8)(765)

b. 64(809)

c. 707(8)

d. 56●10

e. 56(100)

f. 56(1000)

g. 768(1000)

h. 60(700)

i. 78(567)(0)(888)

j. 9●8

k. What is the product of 4 and 5?

l. What is twice 15?

m. Write the product of x and y

2. Estimate each product then find the exact answer

a. 7,854(38)

b. 39,804(82)

3. Translate

a. The product of 5 and a

b. The square of 8

c. The cube of 2

d. The fifth power of 10

4. Evaluate the expression for the given values.

a. xy when x=8, y=9

b. 7x when x=5

c. 5xy when x=4, y=3

d. xyz when x=2, y=5, z=10

5. Name 3 properties of multiplication that start with the letters CAI.

6. Identify the property.

a. (7●9)●5=7●(9●5)

b. 5●1=5

c. 12●7=7●12

d. 8●0=0

7. Complete using a property of multiplication then name the property.

a. 7=7●___

b. 8●___=9●8

c. 66●____=0

d. (6●8)●11=6●_____

e. ab=_____

8. Solutions to equations

a. Is 7 a solution to the equation 6x=54?

b. Is 5 a solution to the equation 30=5y

9. Write in exponential form.

a. 5●5●5

b. 1●1●1●1●1

c. a●a●a

d. xxxyy

e. 7●7●2●2●2

f. 10●10●10●10

g. ☺ • ☺ • ☺

10. Write in expanded form and evaluate.

a. 2^5

b. 10^5

c. $5^2 \bullet 2^5$

d. 0^5

e. 1^5

11. Evaluate the expression for the given values.

a. y^3 y=7

b. y^5 y=2

c. y^6 y=10

d. a^2b^3 a=3 b=2

12. Geometry

a. What is the formula for the area of a rectangle with sides L and W?

b. What is the area of a rectangle with sides 5 inches and 7 inches?

c. What is the perimeter of a rectangle with sides 5 inches and 7 inches?

d. What is the area of a square with a side 5 inches?

13. Find all the factors of:

a. 4

b. 8

c. 12

d. 16

e. 24

f. 36

g. 48

h. 100

14. Break each number into its prime factors:

a. 4

b. 8

c. 12

d. 16

e. 24

f. 48

g. 100

h. 13

i. 17

j. 21

k. 49

l. 120

15a. Find all possible pairs of factors whose product is 12 then list those whose:

b. sum is 7

c. difference is 11

d. sum is 8

e. difference is 4

16a. Find all possible pairs of factors whose product is 18 then list those whose:

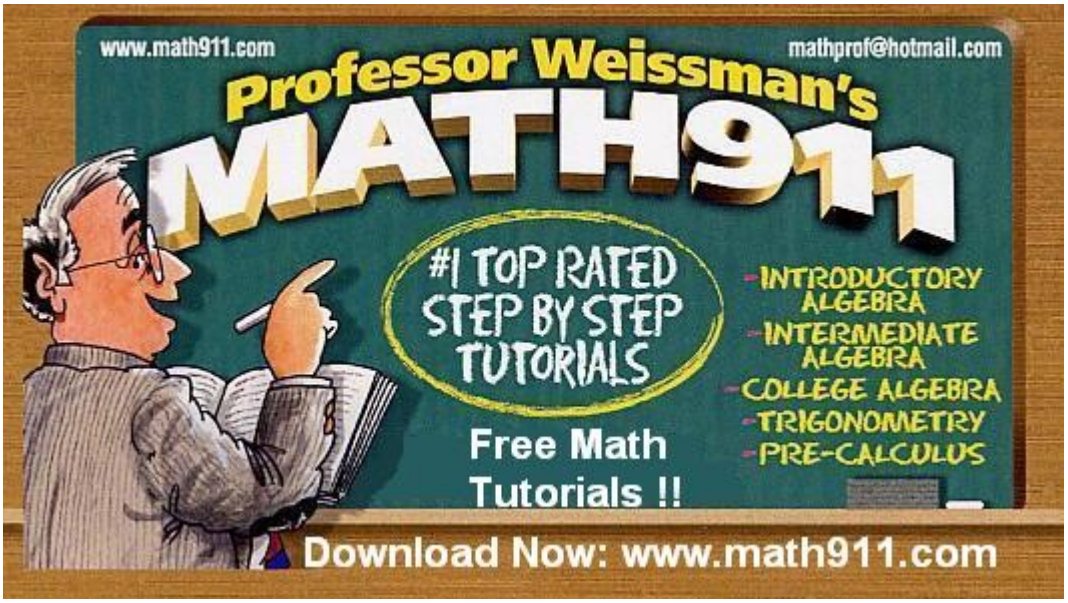
a. sum is 11

b. difference is 17

c. sum is 9

d. difference is 7

Free Software For All Mathematics Subjects Including Statistics



Jokes Set #3

Some engineers are trying to measure the height of a flag pole. They only have a measuring tape and are quite frustrated trying to keep the tape along the pole: It falls down all the time.
A mathematician comes along and asks what they are doing. They explain it to him.
"Well, that's easy..."
He pulls the pole out of the ground, lays it down, and measures it easily. After he has left, one of the engineers says:
"That's so typical of these mathematicians! What we need is the height - and he gives us the length!"

Brain Teasers Set #3

1. Three men go to a cheap motel, and the desk clerk charges them a sum of $30.00 for the night. The three of them split the cost ten dollars each. Later the manager comes over and tells the desk clerk that he overcharged the men, since the actual cost should have been $25.00. The manager gives the bellboy $5.00 and tells him to give it to the men. The bellboy, however, decides to cheat the men and pockets $2.00, giving each of the men only one dollar.

   Now each man has paid $9.00 to stay for the night, and 3 x $9.00 = $27.00. The bellboy has pocketed $2.00. But $27.00 + $2.00 = $29.00. Where is the missing $1.00?

Answers to Exercise Set 3

1a. 6,120

b. 51,776

c. 5,656

d. 560

e. 5600

f. 56,000

g. 768,000

h. 42,000

i. 0

j. 72

k. 20

l. 30

m. xy

2a. 320,000 ; 298,452

b. 3,200,000 ; 3,263,928

3a. 5a

a. 8^2

b. 2^3

c. 10^5

4a. 72

b. 35

c. 60

d. 100

5. Commutative, Associative, Identity (1), Zero (0)

6a. Associative

b. Identity

c. commutative

d. zero

7a. 7=7●1

b. b. 8●9=9●8

c. 66●0=0

d. (6●8)●11=6●(8●11)

e. ab=ba

8a. No

b. No.

9a. 5^3

b. 1^5

c. a^3

d. x^3y^2

e. $7^2 2^3$

f. 10^4

g. ☺ 3

10a. 2(2)(2)(2)(2)=32

b. (10)(10)(10)(10)(10)= 100,000

c. 5(5)(2)(2)(2)(2)(2) = 800

d. (0)(0)(0)(0)(0)=0

e. (1)(1)(1)(1)(1)=1

11a. 343

b. 32

c. 1,000,000

d. 72

12a. A=LW

b. 35 square inches

c. 24 inches

d. 25 square inches

13a. 1.2.4

b. 1,2,4,8

c. 1,2,3,4,6,12

d. 1,2,4,8,16

e. 1,2,3,4,6,8,12,24

f. 1,2,3,4,6,9,12,18,36

g. 1,2,3,4,6,8,12,16,24 ,48

h. 1,2,4,5,10,20,25,50, 100

14a. 2●2

b. 2●2●2

c. 2●2●3

d. 2●2●2●2

e. 2●2●2●3

f. 2●2●2●2●3

g. 2●2●5●5

h. 13

i. 17

j. 3●7

k. 7●7

l. 2●2●2●3●5

15a. All pairs are: 1,12 2,6 and 3,4

b. 3,4 c. 1,12

d. 2,6 e. 2,6

16a. All pairs are: 1,18 2,9 and 3,6

b. 2,9 c. 1,18

d. 3,6 d. 2,9

Brain Teaser #3 Answer

it's all in how you phrase the question.

The men paid $27 total. $25 to the manager, and $2 to the bellboy.

The fact that they also originally paid an extra $3 and then got it back again is entirely irrelevant.

What makes this puzzle seem so impossible?? it's the question they ask you at the end.

There is no missing dollar. it's a trick, a math trick. here's the answer.

the three men:

$30 (what they paid in the beginning)
-$5 (amount deducted)
+$3 (amount given back)
=$28

$28
+$2 (amount bellboy kept)
=$30

there's no missing dollar!!!!

The men paid $27
The motel got $25
The bellboy got $2

$25 + $2 = $27