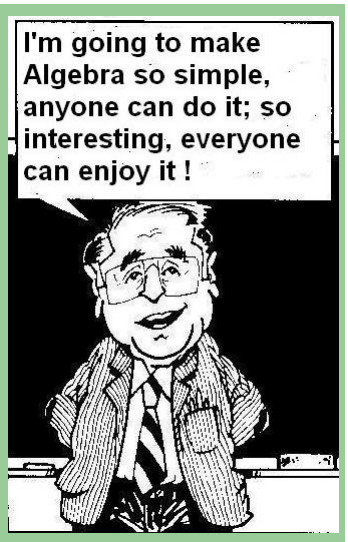
Lesson

Unit #7

Multiplying and Dividing Integers Order Of Operations

Professor Weissman's Algebra Classroom

Martin Weissman, Jonathan S. Weissman. & Tamara Farber



Inside this issue:

How Do I Multiply Signed Numbers? What is the odd/ even rule?

There's a simple rule for multiplying signed numbers. Every 2 negatives make a positive. You can overlook the positives. Just count the negatives.

- (-3)(+4)(-5) = +60 [2 negatives]
- (-3)(-4)(-5) = - 60 [3 negatives]
- (+3)(-4)(+5) = - 60 [1 negative]
- (+3)(+4)(+5) = +60 [0 negatives]

In each case you multiply the absolute values. (3)(4)(5)=60.

If the amount of negatives is EVEN, or 0,2,4,6,... then the product is positive.

If the amount of negatives is ODD then the product is negative.

How Do I Divide Signed Numbers?

The rule for dividing is the same as that for Multiplying. Every 2 negatives make a positive. You can overlook the positives. Just count the negatives.

$$\frac{+3}{+4} = +\frac{3}{4} \qquad \frac{-3}{-4} = +\frac{3}{4}$$

$$\frac{+3}{-4} = -\frac{3}{4} \qquad \frac{-3}{+4} = -\frac{3}{4}$$

$$+\frac{-3}{-4} = +\frac{3}{4} \qquad -\frac{-3}{+4} = +\frac{3}{4}$$

$$-\frac{+3}{-4} = +\frac{3}{4} \qquad +\frac{-3}{-4} = +\frac{3}{4}$$

$$-\frac{-3}{-4} = -\frac{3}{4} \qquad +\frac{+3}{+4} = +\frac{3}{4}$$

0=positive 1=negative 2=positive 3=negative

Twice the number 5 increased by one. Is it 11 or 12?

The given sentence is ambiguous. An ambiguity is an unclear expression. If there would be a comma in the sentence it would become clear.

Twice the number 5, increased by one would mean double five then add one. It's 11.

2(5)+1=10+1=11

Twice, the number 5 increased by one would mean add 1 to 5 then double it.

2(5+1)=2(6)=12

It's 12. To avoid ambiguity in Algebra we use parentheses and follow what is called the "order of operations." we have five basic operations: combine (add/subtract), multiply, divide and raise to a power.

What is order of operations?

Most students have heard of the 'word' or acronym PEMDAS. Each letter stands for an operation in Mathematics and the order of the letters reminds us of the order that the Math operations are done.

P=Parentheses
E=Exponents
M=Multiplication
D=Division A=Addition
S=Subtraction.

Do we still use PEMDAS in Algebra?

It's even easier in Algebra. Since we combine Add and Subtract PEMDAS becomes PEMDC

P=Parentheses
E=Exponents
M=Multiplication
D=Division C=Combine

Remember to first separate the expression into terms . Next you can use PEMD on each term and C will be your last step.

Simplify
$-2(4) - 7^2 - (-4) + (8-11) + 10$
Separate into terms
$-2(4) \mid - 7^2 \mid - (-4) \mid + (8-11) \mid + 10$
Simplify each term
-8 -49 +4 -3 + 10
Combine positives and negatives
+ 14 –60
Combine the two sums: – 46

How is an expression with brackets and parentheses simplified?

Usually, the parentheses are the innermost grouping. Simplify inside the parentheses first.

Then, simplify inside the brackets.

Simplify: -7 [2 - (5-8)]

Simplify inside (), keep () -7 [2 - (-3)]
Simplify using—outside (), drop () -7 [2 + 3]
Combine inside [], keep [] -7 [+5]
Simplify using –7 outside [], drop [] -35

The MD in PEMDC, of course means Multiplication and Division, but just because M is before D does **NOT** mean that multiplication comes before Division. Whichever comes first do first. For example, here division is done first.

$60 \div 10 \cdot 2$

$6 \cdot 2$

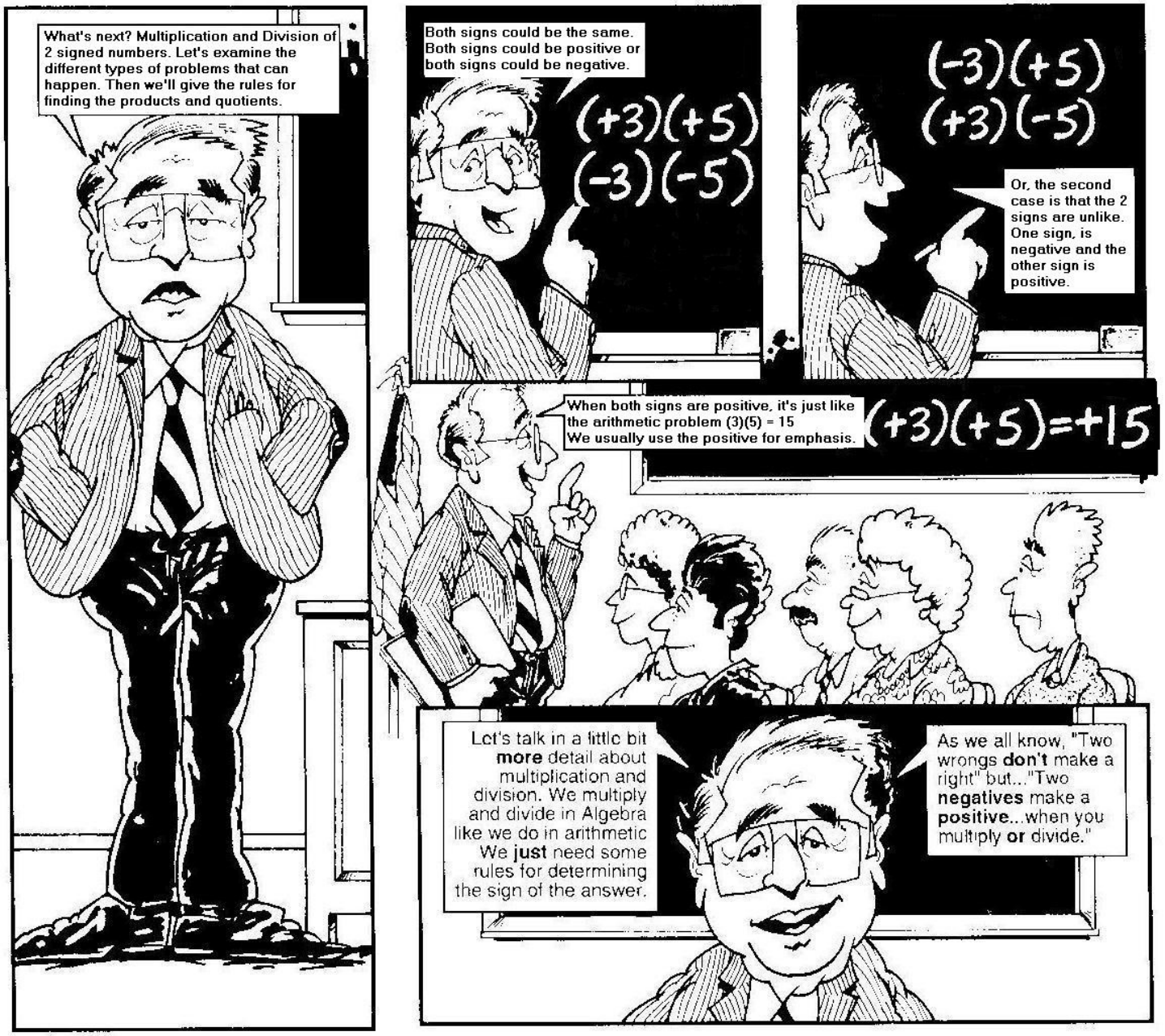
12

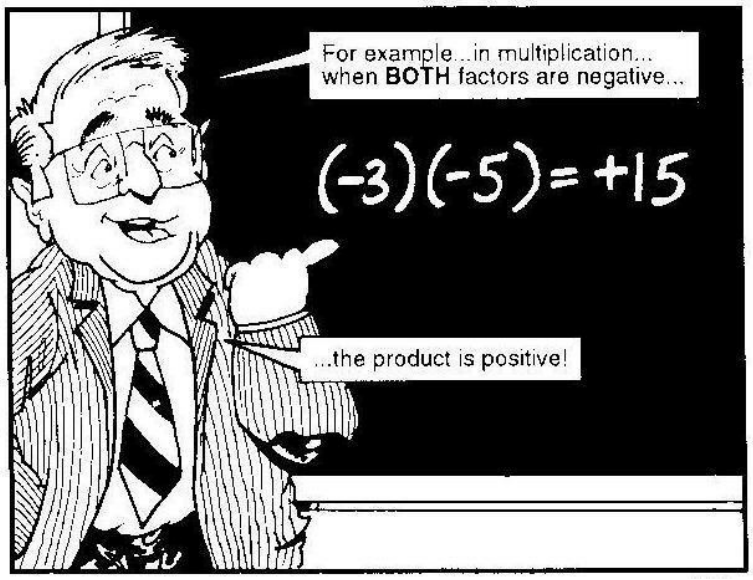
The P in the acronym PEMDC means simplify the expression ***INSIDE*** parentheses but **KEEP** the parentheses. For example, -5(3-10) does **not** mean –5 –7, rather we keep the parentheses and evaluate this way:

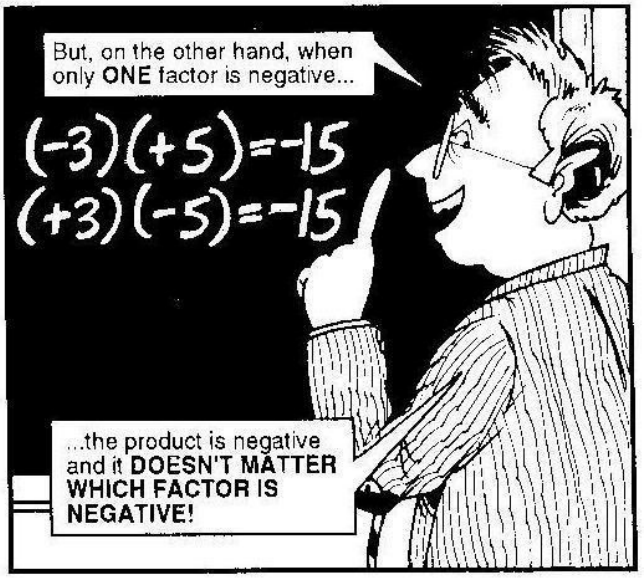
-5(3—10)

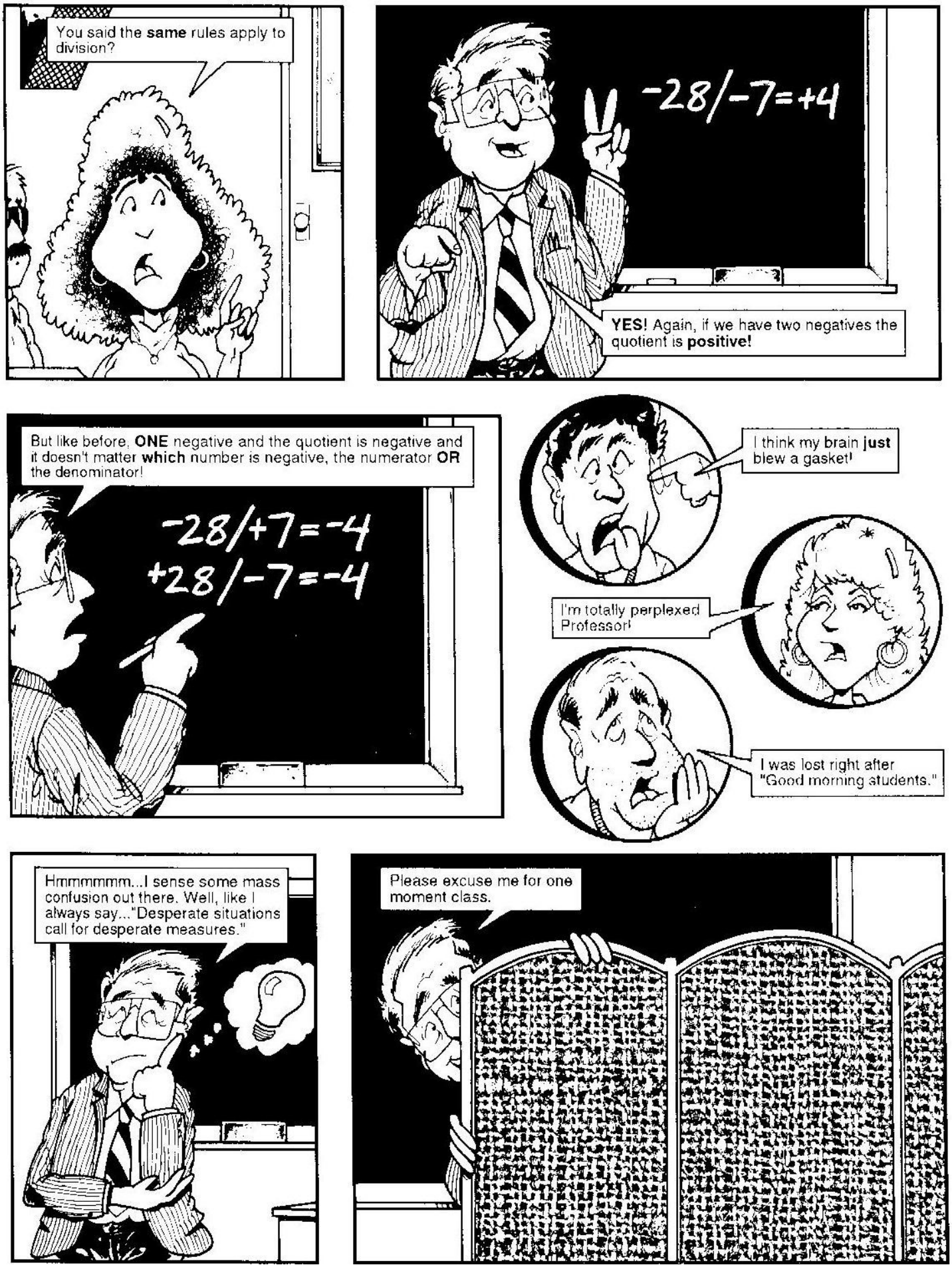
—5(—7)

+35


What's next? Multiplication and Division of 2 signed numbers. Let's examine the different types of problems that can happen. Then we'll give the rules for finding the products and quotients.
Both signs could be the same. Both signs could be positive or both signs could be negative.
(+3)(+5)
(-3)(-5)
(-3)(+5)
(+3)(-5)
Or, the second case is that the 2 signs are unlike. One sign, is negative and the other sign is positive.
When both signs are positive, it's just like the arithmetic problem (3)(5) = 15
We usually use the positive for emphasis.
(+3)(+5)=+15
Let's talk in a little bit **more** detail about multiplication and division. We multiply and divide in Algebra like we do in arithmetic We **just** need some rules for determining the sign of the answer.
As we all know, "Two wrongs **don't** make a right" but..."Two **negatives** make a **positive**...when you multiply **or** divide."

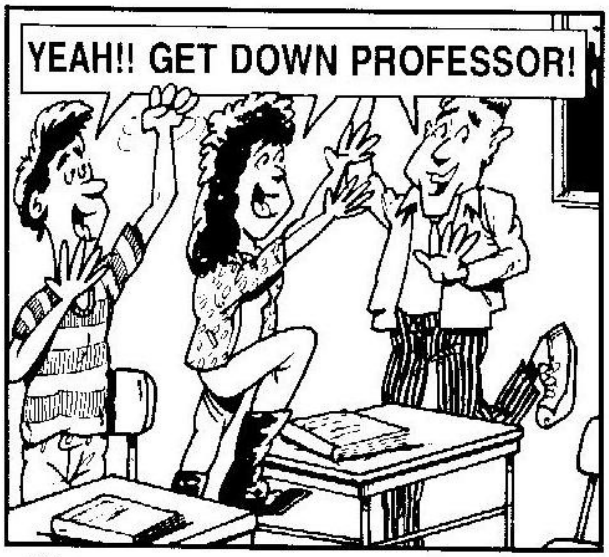

For example...in multiplication... when **BOTH** factors are negative...
(-3)(-5)= +15
...the product is positive!

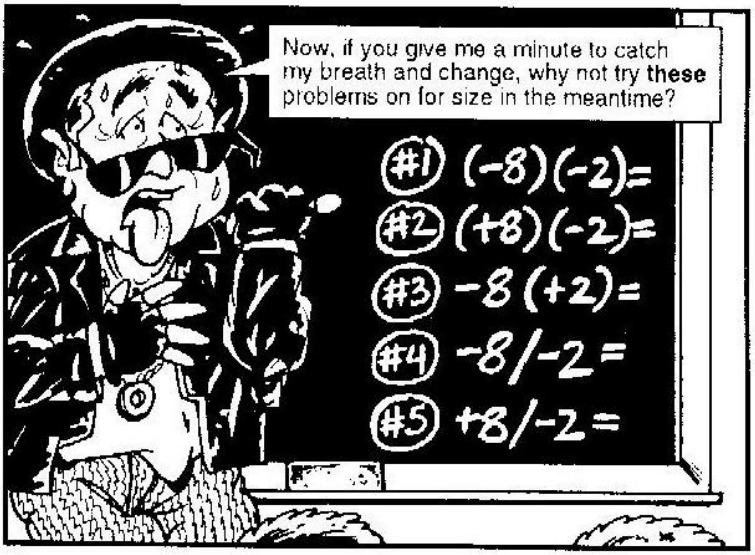

But, on the other hand, when only **ONE** factor is negative...
(-3)(+5)=-15
(+3)(-5)=-15
...the product is negative and it **DOESN'T MATTER WHICH FACTOR IS NEGATIVE!**


You said the **same** rules apply to division?
-28/-7=+4
YES! Again, if we have two negatives the quotient is **positive!**
But like before, **ONE** negative and the quotient is negative and it doesn't matter **which** number is negative, the numerator **OR** the denominator!
-28/+7=-4
+28/-7=-4
I think my brain **just** blew a gasket!
I'm totally perplexed Professor!
I was lost right after "Good morning students."
Hmmmmmm...I sense some mass confusion out there. Well, like I always say..."Desperate situations call for desperate measures."
Please excuse me for one moment class.

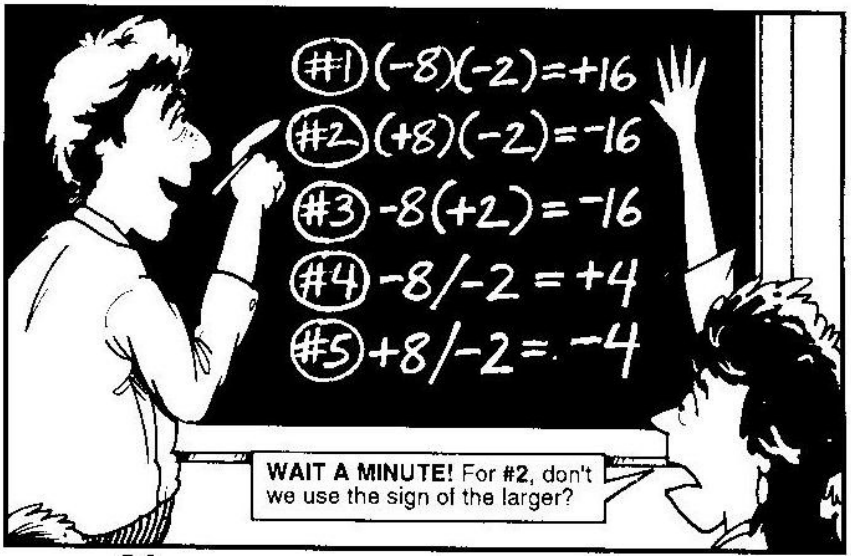

TA DA
I'M M.C. PROFESSOR AND I'M HERE TO EXPLAIN MUL-TEEPLYING! DEE-VIDING! ENRICHING YOUR BRAIN!
WHEN YOU DO EITHER ONE JUST REMEMBER THIS RULE "2 NEGATIVES MAKE A POSITIVE" AND YOU WON'T BE NO FOOL!
BOOMA
BOOM
IF BOTH FACTORS ARE NEGATIVES WHEN WE TRY TO MUL-TEEPLY, THE PRODUCT IS A POSITIVE NO MATTER WHAT WE TRY!
BOOM BOOM
IF ONLY ONE FACTOR'S NEGATIVE IT DON'T MATTER WHICH THEN THE PRODUCT'S ALSO NEGATIVE LET ME FINISH OFF MY PITCH!

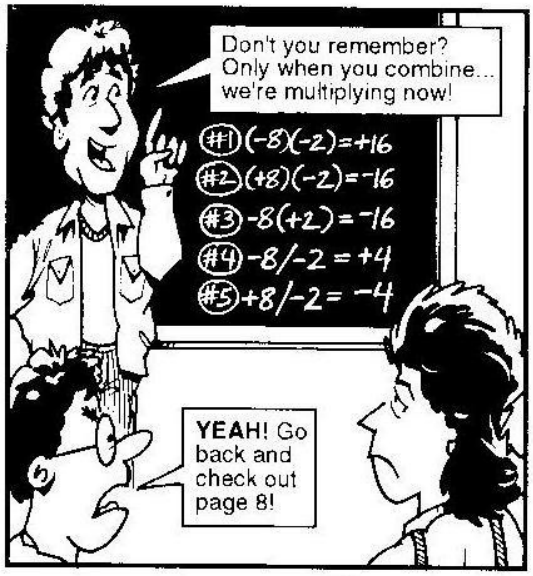

WORKING WITH DEE-VISION
THE RULES ARE QUITE THE SAME
THE QUOTIENT IS A POSITIVE
IF TWO NEGS ARE IN THE GAME!
SCRATCHA SCRATCH
SCRATCHA SCRATCH
AND LIKE BEFORE, ONE NEGATIVE
WHICH ONE, DON'T MAKE A DIFF
GIVES A QUOTIENT THAT'S A NEGATIVE.
NOW CHECKOUT MY DEEJAY RIFF!

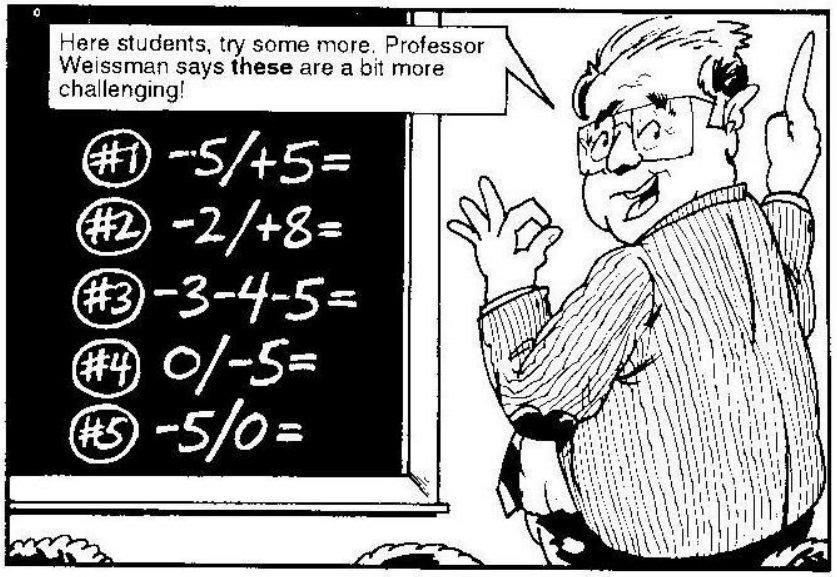

YEAH!! GET DOWN PROFESSOR!


Now, if you give me a minute to catch my breath and change, why not try these problems on for size in the meantime?
#1 (-8)(-2)=
#2 (+8)(-2)=
#3 -8(+2)=
#4 -8/-2=
#5 +8/-2=

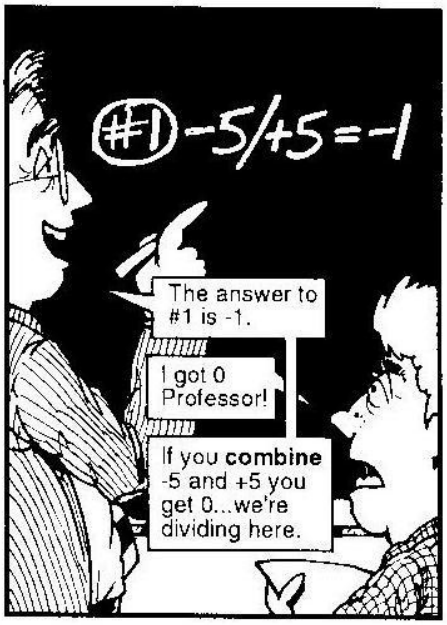

TWO EXHAUSTED MINUTES LATER
OK! Now that I've returned to normal, relatively speaking, anybody wanna give these a try? How about you?

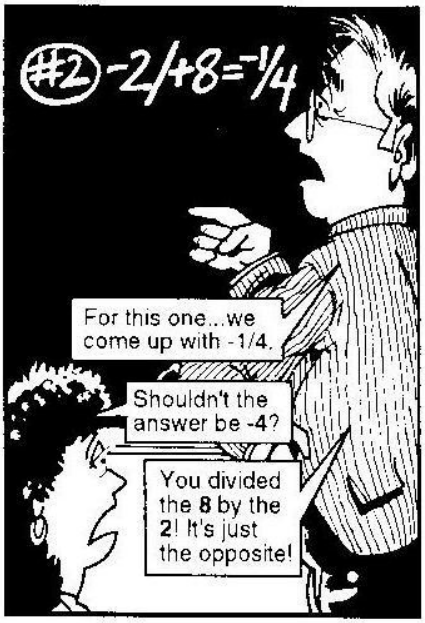

#1 (-8)(-2)=+16
#2 (+8)(-2)=-16
#3 -8(+2)=-16
#4 -8/-2=+4
#5 +8/-2=-4
WAIT A MINUTE! For #2, don't we use the sign of the larger?

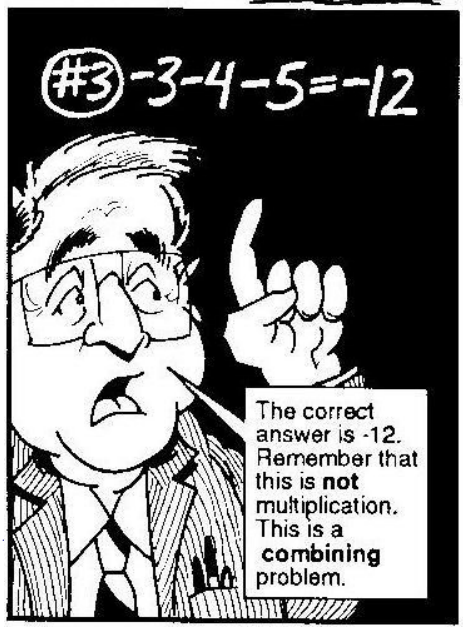

Don't you remember? Only when you combine... we're multiplying now!
#1 (-8)(-2)=+16
#2 (+8)(-2)=-16
#3 -8(+2)=-16
#4 -8/-2=+4
#5 +8/-2=-4
YEAH! Go back and check out page 8!

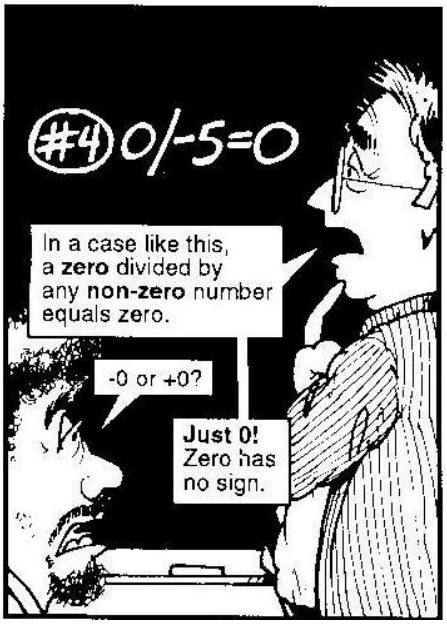

Here students, try some more. Professor Weissman says these are a bit more challenging!
#1 -5/+5=
#2 -2/+8=
#3 -3-4-5=
#4 0/-5=
#5 -5/0=

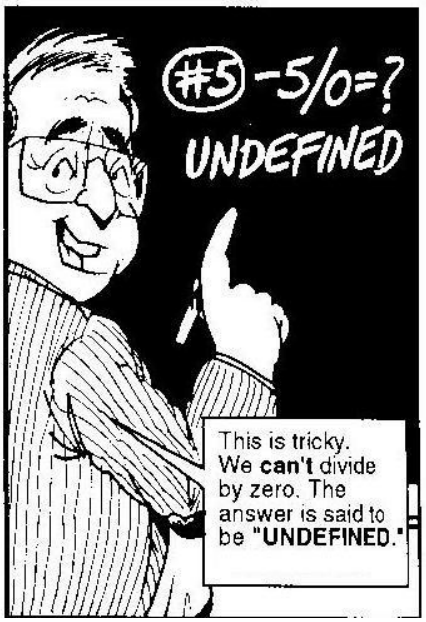

Now, let me go over each answer and explain how we arrived there.


#1 -5/+5=-1
The answer to #1 is -1.
I got 0 Professor!
If you combine -5 and +5 you get 0...we're dividing here.


#2 -2/+8=-1/4
For this one...we come up with -1/4.
Shouldn't the answer be -4?
You divided the 8 by the 2! It's just the opposite!


#3 -3-4-5=-12
The correct answer is -12. Remember that this is not multiplication. This is a combining problem.


#4 0/-5=0
In a case like this, a zero divided by any non-zero number equals zero.
-0 or +0?
Just 0! Zero has no sign.


#5 -5/0=?
UNDEFINED
This is tricky. We can't divide by zero. The answer is said to be "UNDEFINED."

Exercise Set 7

1. Evaluate

a. $-7 \bullet 9$

b. $-5 \bullet 11$

c. $-7(-2)$

d. $7 - (-2)$

e. $0(-5)$

f. $8(-5)$

g. $(-6)(-9)$

h. $-7 \bullet -7 \bullet 2$

i. $(-2)(-3)(-4)$

j. $-2-3-4$

k. $(-1)(-2)(-3)(-4)$

l. $-1-2-3-4$

m. $(-1)^{100}$

n. -3^2

o. $(-3)^2$

p. Find the product of -1 and -6

q. What is twice -3?

r. $(-1)^{21}$

2. Translate, then evaluate for $a=-3$, $b=+5$

a. The product of a and b

b. Twice a

c. -6 multiplied by a

d. 7 times a

3. Evaluate for these values: $a=-5$, $b=4$, $c=-1$

a. ab

b. ba

c. abc

d. $-abc$

e. $3ac$

f. $a(-c)$

g. $-6b$

h. $b-c$

i. $-a^2$

j. a^2-c^2

4. Is the value a solution to the equation? Justify your answer.

a. $5a=-15$ $a=-3$

b. $-2b= -8$ $b=4$

c. $36 = -4c$ $c=9$

d. $-100d=-2$ $d=50$

5. Divide

a. $(-24)\div(-6)$

b. $(24)\div(-3)$

c. $(-24)\div(3)$

d. $(-24)\div 0$

e. $0\div(-6)$

f. $100/-2$

g. $-75/-15$

h. $-3/24$

6. Translate three different ways: The quotient of x and y

7. Find the average of -11, $+10$, -4, $+1$, and -1

8. Is the given value a solution to the given equation?

a. $28/x=-7$ $x=-4$

b. $x/20 = -5$ $x=4$

c. $-3=-12/x$ $x=4$

d. $-6=x/18$ $x=-3$

e. $0/x = 0$ $x=-1$

f. $-50/x =-5$ $x=10$

9. Simplify using the order of operations. But, first separate the expression into terms.

a. $5 \cdot 7 + 2$

b. $5 \cdot (7 + 2)$

c. $10 \div 5-3$

d. $10 \div (5-3)$

e. $8 + 2 \bullet 5$

f. $15-5 \div 5$

g. $5 + 4^2$

h. $(5+4)^2$

i. $10 \cdot 4-6 \cdot 5$

j. $20 \div 5 - 4 \cdot 4$

k. $5 - 4^2$

l. $4 \cdot 5^2$

m. $80-2^3$

n. $-5(3)-6(-1)$

o. $3^2 - (-3)^2+10$

p. $(-7)^2-4(-5)(2)$

q. $10 \div 5 \bullet 5$

r. $-2[-7(6-11)]$

s. $-5(8-10)$

t. $5 - (8-10)$

10. Substitute and simplify

a. $a=1$ $b=-1$ $c=-6$

$$b^2 - 4ac$$

b. $a=1$ $b=1$ $c=-6$

$$\sqrt{b^2 - 4ac}$$

c. $x_1=-4$ $x_2=3$ $y_1=-1$ $y_2=1$

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Jokes Set #7

The Math Professor told his students to compose a daily time-table. A student showed his to the Professor.

wake up get ready for school :2 hours school: 5 hours part time job 6 hours homework 3 hours sleep 9 hours

The Professor looked at it and said:
- Very well, but your day consists of 25 hours.
- What a pity! Now I must wake up one hour earlier.

A cow gives 20 liters of milk a day. How many liters of a milk will it give in a week ?
- But we don't know, how many days a week a cow works.

Mental Math

Squaring a 2 digit number ending in 5 can be so easy with a little bit of practice you could do it in your head.

For example: 35^2

Look at the first digit 3

Multiply 3 by the number after 3, 4

3(4)=12

12 is the first part of the square

Since the second digit is a 5 then the last 2 digits of the answer will be 25.

25 is the second part of the square. Now put them together:

1225

Here's another example

75^2 =?

7(8)=56 5^2 =25

Put the results together : 75^2 = 5625

Answers to Exercise Set 7

1a. -63
b. -55
c. 14
d. 9
e. 0
f. -40
g. 54
h. 98
i. -24
j. -9
k. 24
l. -10
m. 1
n. -9
o. 9
p. 6
q. -6
r. -1

2a. ab -15
b. 2a -6
c. -6a 18
d. 7a -21

3a. -20
b. -20
c. 20
d. -20
e. 15
f. -5
g. -24
h. 5
i. -25
j. 24

4a. yes
b. yes
c. no
d. no

5a. 4
b. -8
c. -8
d. undefined
e. 0
f. -50
g. 5
h. -1/8

6a. x/y
b. x : y
c. x ÷ y

7. -1

8a. yes
b. no
c. yes
d. no
e. yes
f. yes

9a. 37
b. 45
c. -1
d. 5
e. 18
f. 14
g. 21
h. 81
i. 10
j. -12
k. -11
l. 100
m. 72
n. -9
o. 10
p. 89
q. 10
r. -70
s. 10
t. 7

10a. 25
b. 5
c. 2/7