Combine Like Terms

Unit #14

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Professor Weissman's Algebra Classroom

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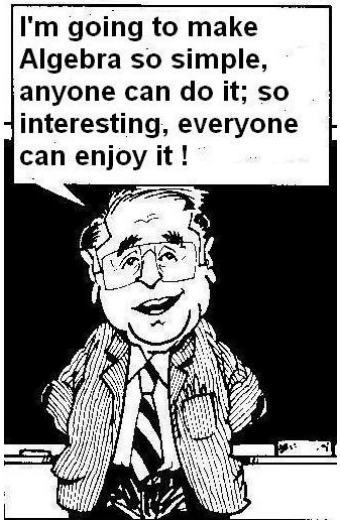

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What Does The Proverb "Oil And Water Don't Mix" mean?

Have you ever heard someone talking about a relationship of two people using the phrase "oil and water don't mix?"

What is trying to be conveyed is that this couple is not a good match. Certain of their qualities or personalities are incompatible.

Where Do We Learn That Oil And Water Don't Mix?

One of the most basic rules students learn in science class is that oil and water don't mix. Nowhere was this lesson more painfully learned than in Prince William Sound, Alaska, where the Exxon Valdez spilled several million gallons of crude oil into the water, damaging the pristine environment and killing thousands of seabirds and wildlife.

Many things in real life just don't mix or are kept apart for various reasons.

When shopping in the supermarket you don't put apples and oranges in the same plastic bag.

The literal meaning of the proverb is clear. You can not 'combine' oil and water. They will always be visible as distinct and different. from each other.

What is the Algebra Equivalent Of The Proverb?

In Algebra, we say that "Unlike terms don't combine; You can only combine LIKE TERMS."

What Are 'terms?'

A term is an expression made up of numbers and/or letters connected by only multiplication and/or division. A term starts with either a + or a — sign. A + or — sign separates an expression into terms.

What Are Like Terms?

Terms with the same variable factors are called like terms. 2n and 3n are like terms, but 4x and 3y are not like terms. So too, $2x^2$ and 3x are unlike terms. The both have x but the exponents are unlike.

What Are some Examples of Terms?

Examples of terms:

a is a term

3a is a term

$3a^2$ is a term

$3a^2 - 4$ are two terms

$3a^2 - 4a$ are two terms

$3a^2 - 4a + 7$ are three terms.

What Are The Three Terms of $3a^2 - 4a + 7$?

We'll separate the expression just before each + or — sign.with a line. Prefix a + sign at the beginning of the expression.

$$+3a^2 \mid - 4a \mid + 7$$

The First Term is: $+3a^2$

The Second Term is: $- 4a$

The Third Term is: $+ 7$

How are Terms Different From Factors?

Terms are combined, factors are multiplied. 7+x+y is an expression with three terms. 7xy is a term with three factors. Both terms and factors can be rearranged.

7 + x + y = y + 7 + x

7xy = 7yx

1862 July 8, Morrill Anti-Bigamy Law, signed by Abraham Lincoln.

First basic federal legislation by the Congress of the United States that was designed "to punish and prevent the practice of polygamy in the Territories of the United States".

What Is A Nomial?

A nomial is a name that we give to an expression with terms.

A trinomial has three terms. A binomial has 2 terms. A monomial has only one term.

The word polynomial is a general name used for expressions that have many terms. What is many? For some, 2 could mean a lot. So a binomial might also be called a polynomial.

We also use the prefixes mono, bi and poly with how many wives a man has. Monogamy means 1 wife, bigamy 2 wives and polygamy means many wives. As with terms, Polygamy might also mean 2 wives.

What Does Descending Powers Mean?

An exponent is a power. When an expression has many terms with different exponents, it is usually preferable to rearrange the terms so that the term with the highest exponent is first. We then continue with the next highest exponents in the same way. If there is a constant term it goes last.

Example: Rewrite in descending powers:

$7a - 4a^3 + 9 + 6a^2$

$-4a^3 + 6a^2 + 7a + 9$

The perimeter, P, of a rectangle is the distance around the rectangle. If we call the length L and the width W, then adding the 4 sides we get P = L + W + L + W. When we combine like terms we get P = 2L + 2W

If the Length=2 feet and the Width=4 feet, then

P=2(3 ft)+2(9 ft),

P=6 ft + 18 ft (combine like terms)

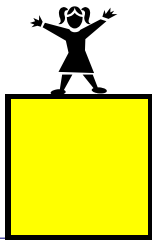
P= 24 ft.

The perimeter, P, of a square is the distance around the square. If we call the length of each side, S, then adding the 4 sides we get P = S + S + S + S. When we combine like terms we get P = 4S

If each side S=5 feet, then

P = 4(5 ft)

P= 20 ft



What Are Missing Terms?

Recall the use of the digit 0 as a placeholder in numbers. For example the digit 0 in the number 1,023 means 0 hundreds.

When an expression only has one variable, say x, and the terms are put in descending order it is sometimes obvious that a term is missing.

For example $3x + x^3 + 7$ when put in order looks like this: $x^3 + 3x + 7$

The expression is missing an x^2 term. We can show this missing term but we must use a coefficient of 0.

So, with the missing term the expression looks like this: $x^3 + 0x^2 + 3x + 7$

How Are Like Terms Combined?

Professor Weissman suggests that you combine like terms in a column. Let's see why.

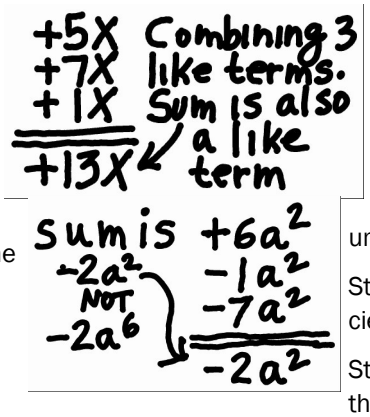
Combine: $5x + 7x + x$

Step #1 Put all the like terms in an x column.

Step #2 Combine the coefficients.

Step #3 Attach an x to the sum of the coefficients.

It's easy to make a mistake with exponents. By using an x column you can remember better that the sum and the terms combined are all like terms.



It's especially easy to make a mistake when combining like terms with exponents. It's very tempting to add the exponents.

Consider this example.

Combine: $6a^2 - a^2 - 7a^2$

Step #1 Put these three like terms in an a^2 column.

Step#2 Combine the coefficients

Step #3 The sum, like the three terms combined will also have an a^2 and NOT a^6.

In sum, all entries in a column 'look alike.'

How Is A Complicated Expression Simplified?

When you need to simplify a complicated expression think of your college experience. When you first start college and hear that you need to accumulate 64 or 128 credits it sounds like a huge challenge.

In reality, you only take about 16 credits each term.

Your focus should be only on one term at a time.

As you finish each term your credits are combined in a new transcript.

Simplifying a complex expression should be done the same way. Remember "SSC"

Step #1 **S**eparate the expression into terms.

Step #2 **S**implify each term.

Step #3 **C**ombine the results.

Example #1 Simplify:

$5x - (7xy - 2y) - 2(3x + 5yx + y)$

$5x \mid - (7xy - 2y) \mid -2(3x + 5yx + y)$ Step #1 Separate (terms)

$+5x \mid - 7xy + 2y) \mid - 6x - 10yx - 2y$ Step #2 Simplify (each term)

$+5x \quad -7xy \quad +2y$ Step #3 Combine (in columns)

$-6x \quad -10xy \quad -2y$ Note: $-10yx$ rewritten $-10xy$

Answer: $-1x - 10xy$ Note: $+2y - 2y = 0$

Fund firms lower bar for younger investors

SATURDAY, JULY 21, 2007

They'll slash the minimum investment below cost of an iPod

By ELEANOR LAISE
THE WALL STREET JOURNAL

Mutual-fund firms have a new message for young investors: Stop whining and give us your money.

In an attempt to attract the next generation of customers, Wall Street is rolling out a host of new offerings designed to squelch the excuses that keep many 20- and 30-somethings from investing outside of their 401(k) plans.

Can't afford the hefty four- and five-figure minimums required by many mutual funds? They'll slash the minimum investment below the cost of an iPod. Not sure which funds to buy, and in what amounts? They're launching all-in-one products that take much of that work off your hands. Don't know how to save for a car, a house and that far-off mirage called retirement? They'll deluge you with online tools and easy-to-enroll individual retirement accounts.

ASSOCIATED PRESS

Charles Schwab Corp. this year cut the minimum investment on its Schwab funds to $100, from a typical range of $2,000 to $2,500, and created a streamlined IRA with low investment minimums aimed at younger people.

Charles Schwab Corp. this year cut the minimum investment on its Schwab funds to $100, from a typical range of $2,000 to $2,500, and created a streamlined IRA with low investment minimums aimed at younger people. Late last year, American Century Investments rolled out an investment program for younger clients that includes $500 minimums on certain funds, while Fidelity Investments launched an online tool that helps new investors develop a simple financial plan based on their answers to five questions.

START EARLY

The most important thing, though, is simply to start early. An investor who starts saving at age 32 and contributes $100 a month to a tax-deferred account will have about $195,000 at 65, assuming an 8 percent annual return, according to T. Rowe Price. But an investor who starts saving at 22 and contributes $100 a month for 10 years — and then stops saving — will have more than $255,000 by 65.

Choosing the right vehicle. Some of the new offerings try to steer young people into tax-deferred accounts, such as IRAs. Schwab's new 15-Minute IRA, for example, is specifically designed to appeal to time-strapped young investors, offering a simple application that helps clients open an account and select investments in a single step. Fidelity, meanwhile, offers a SimpleStart IRA that helps young investors start saving with as little as $200 a month.

Why Does It Pay To Start Investing Early In Life?

Two examples of a young investor are given.

The 32 year old invests $100 a month until age 65.

1. How much did he invest each year?

2. For how many years did he invest that amount?

3. What was the total that was invested?

4. What was the values of the investment at age 65?

The 22 year old also contributed to his retirement fun d $100 per month but for only 10 years.

5. How much did he contribute each year?

6. How much was contributed after 10 years when he stopped?

7. What was the total contributed.

8. What was his retirement fund worth at age 65?

1. The 32 year old invested $1200 each year.

2. He invested for 65-32=33 years.

3. The Total invested was $39,600

4. The value of the account was $195,000 (see article)

5. The 22 year old also put $1200 into his account each year.

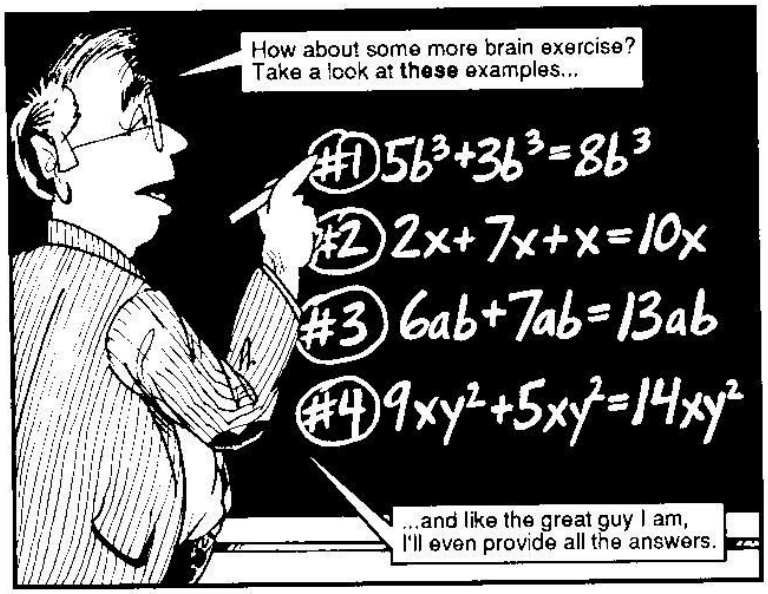
6. After 10 years $12,000 was contributed.

7. According to the article, his retirement fund was worth more than $255,000.

Note that even though the 22 year old only contributed $100 a month for 10 years, his fund was worth more than the 32 year old who contributed $100 a month for 33 years.

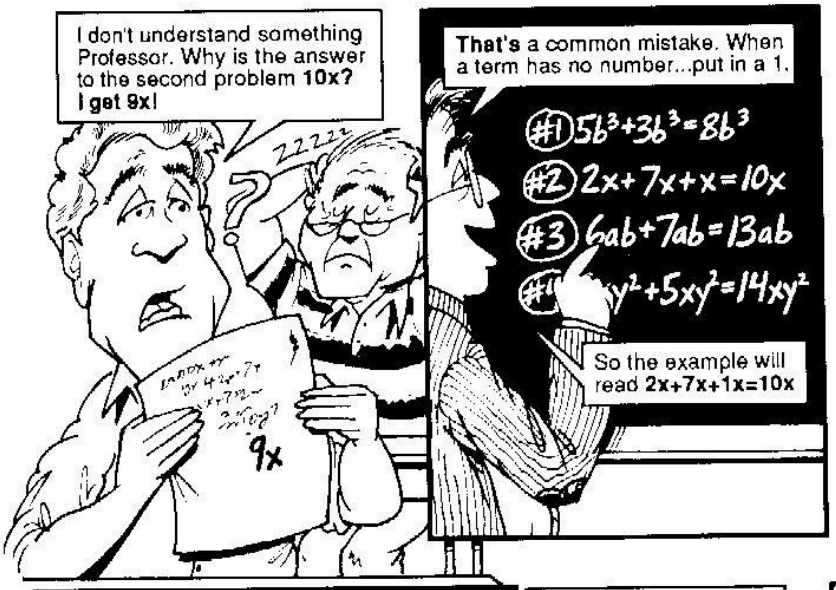
The two examples prove that it is best to start investing for your retirement as early as possible.

Ok class, here's an easy problem. What do you get when you add 3 apples and 2 apples? Anyone?
3a+2a=
Hmm, ... you get a very, very tough Algebra problem !
It's time for those old VISUAL AIDS!
That's sticking to your diet Professor!
Never mind that! Just solve this problem.
3a+2a=
That's easy...5a².
Are you sure? Here...watch!
WHOA! Here are 3 apples or as we'll call them...3a.
Now...if I pick up 2 more apples or another 2a...
BONK
...I'll ask again! What's 3a + 2a? And please, don't anybody say "It's Professor Weissman about to make applesauce!"
I got it... the answer is 5 apples, er...I mean 5a.
Right...just 5a. NOT SQUARED! Besides, apples are round anyway (he he).
3a+2a=
So when do we get a² Professor?
When we multiply! But we'll do that later...right now we're combining. We say that 3a and 2a are LIKE TERMS! They have the same letters. Just combine the numbers and keep the letter.
COEFFICIENTS
3a+2a=5a
LITERAL FACTORS
The numbers are called "COEFFICIENTS" and the letters are called the "LITERAL FACTORS!"


How about some more brain exercise? Take a look at **these** examples...
#1 $5b^3+3b^3=8b^3$
#2 $2x+7x+x=10x$
#3 $6ab+7ab=13ab$
#4 $9xy^2+5xy^2=14xy^2$
...and like the great guy I am, I'll even provide all the answers.

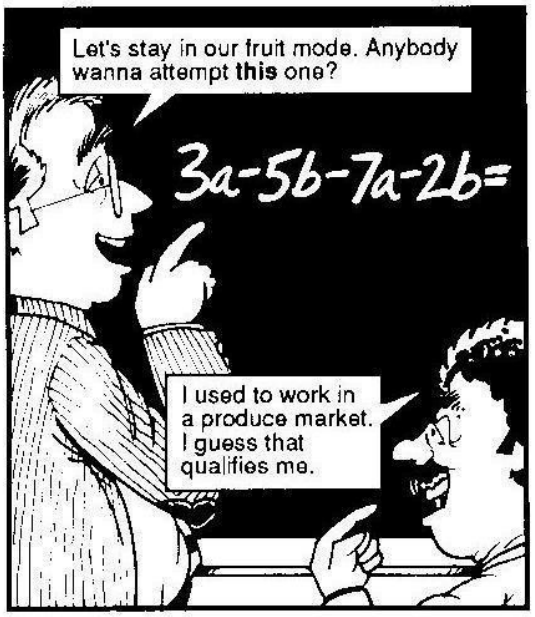
Hey Professor... where **were** you when I was taking the SAT?

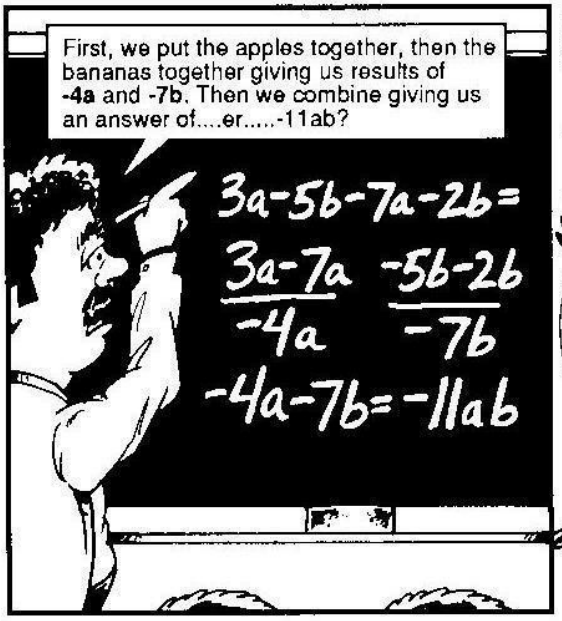
Or when my father caught me sneaking home 3 o'clock in the morning?

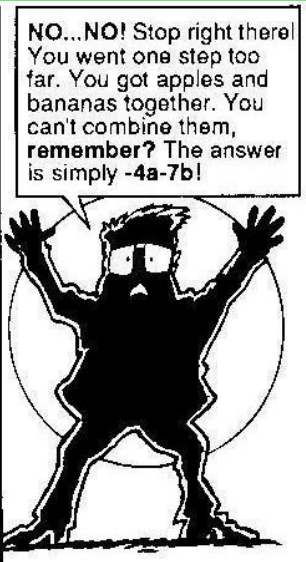

I don't understand something Professor. Why is the answer to the second problem **10x? I get 9x!**
That's a common mistake. When a term has no number...put in a 1.
#1 $5b^3+3b^3=8b^3$
#2 $2x+7x+x=10x$
#3 $6ab+7ab=13ab$
So the example will read **2x+7x+1x=10x**
9x

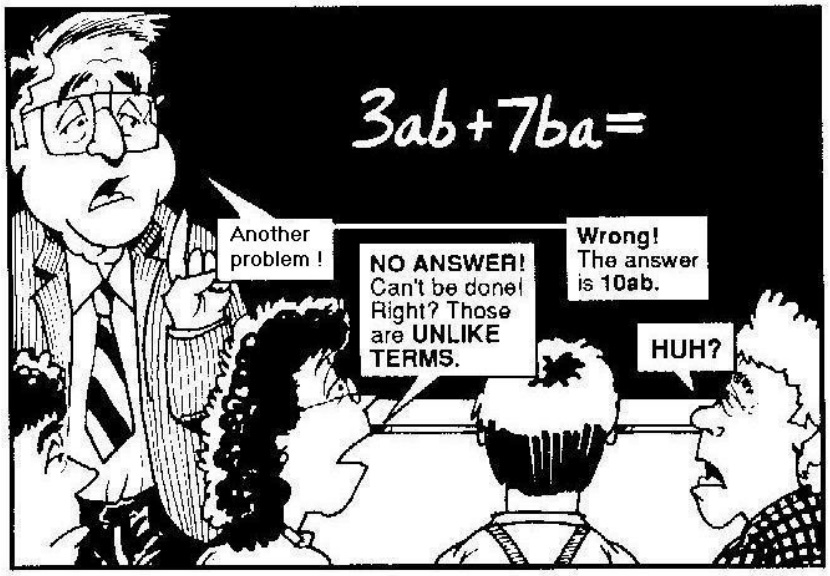
What about the first example? Shouldn't the answer be **8b⁶?**
Again...that's what we do when we multiply. Remember, we're combining now!
#1 $5b^3+3b^3=8b^3$
#2 $2x+7x+x=10x$

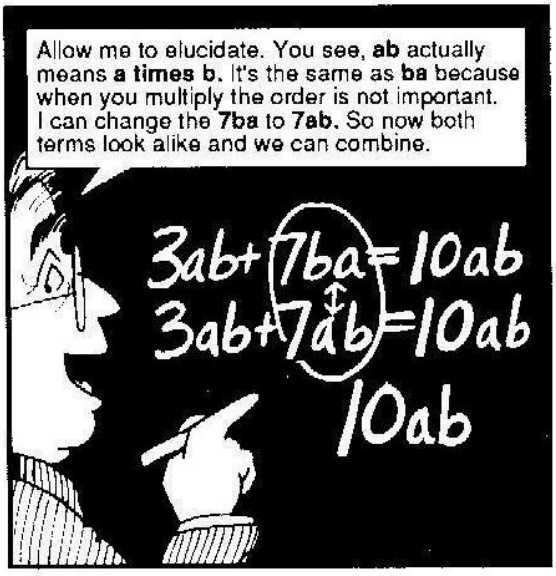
Now...back to my fruit again. I've got **3 apples** and 2 **bananas.** What do I get when I combine them?
3a+2b=
5ab?
I know how we can combine those apples and bananas Professor!
I won't even **ask** what you're doing with a blender. The two terms, **3a** and **2b** are **UNLIKE!** Unlike terms **can't** be combined. You don't have 5 apples and you don't have 5 bananas and you certainly don't have 5 applebananas! Ain't no such kind of fruit!
SO WHAT'S THE ANSWER?
There **is** no answer! You can't add apples and bananas... that is unless you want to make fruit salad.


Let's stay in our fruit mode. Anybody wanna attempt **this** one?
3a-5b-7a-2b=
I used to work in a produce market. I guess that qualifies me.

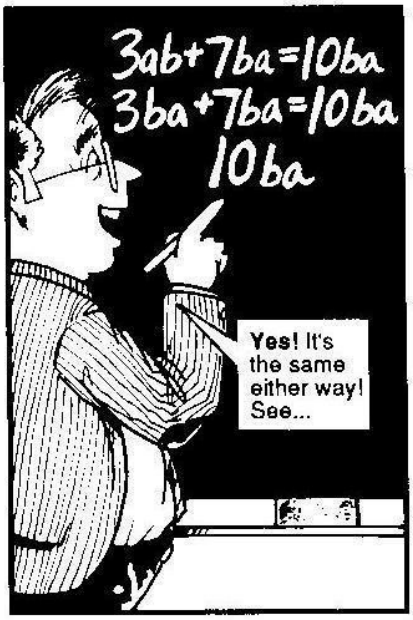

First, we put the apples together, then the bananas together giving us results of **-4a** and **-7b**. Then we combine giving us an answer of....er.....-11ab?
3a-5b-7a-2b=
3a-7a -5b-2b
-4a -7b
-4a-7b=-11ab

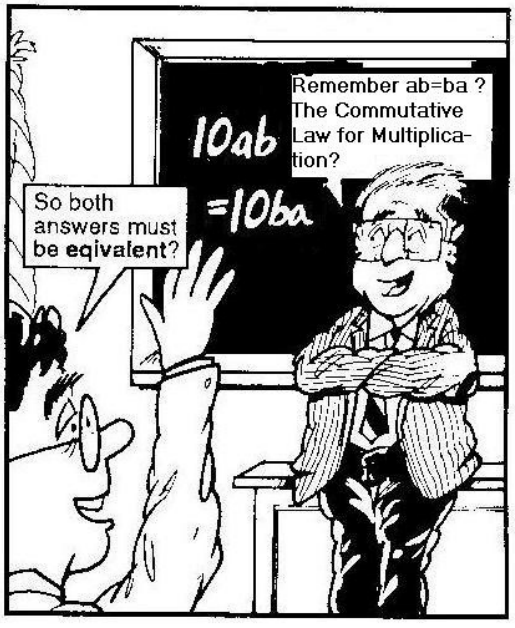

NO...NO! Stop right there! You went one step too far. You got apples and bananas together. You can't combine them, **remember?** The answer is simply **-4a-7b**!

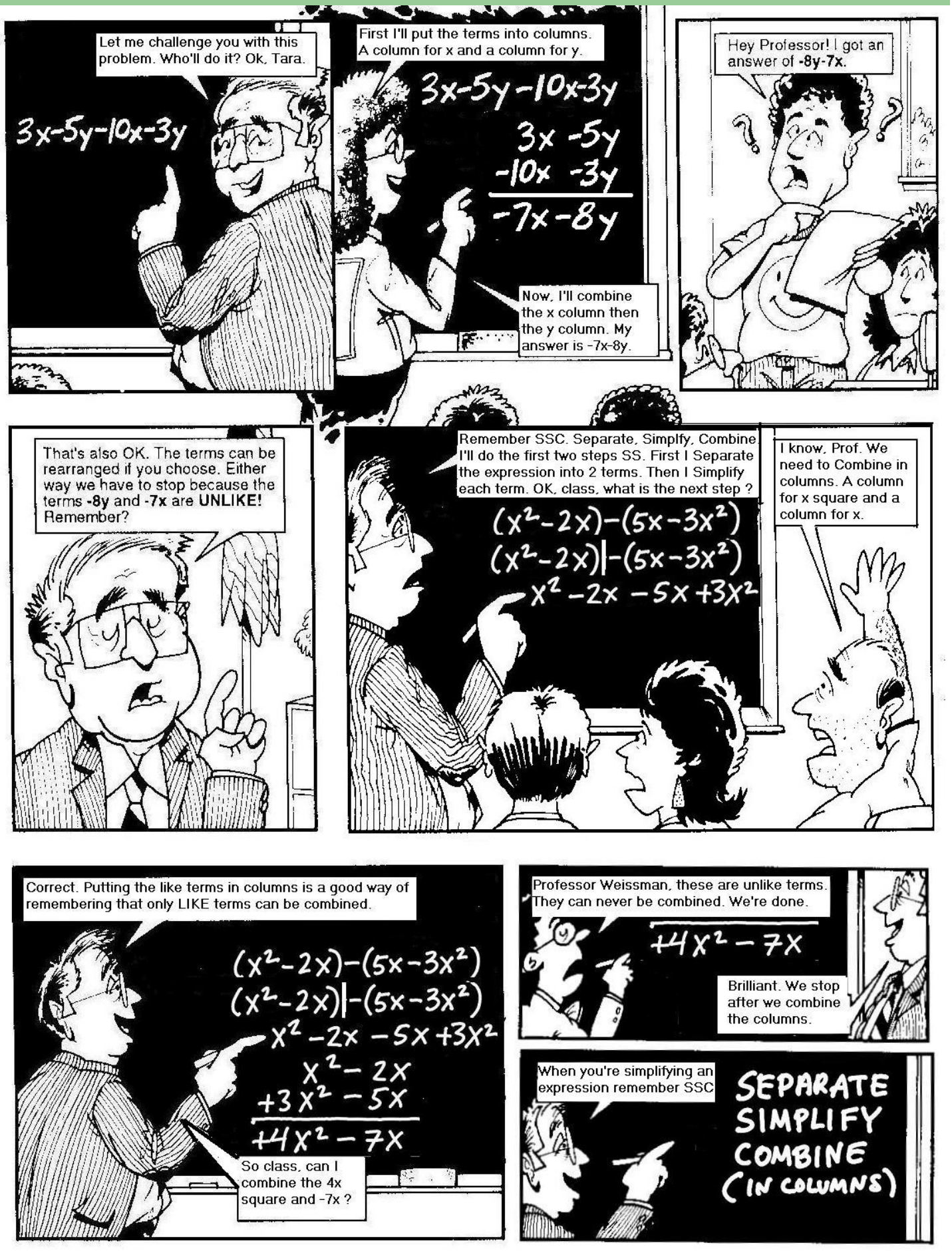

3ab+7ba=
Another problem !
NO ANSWER! Can't be done! Right? Those are **UNLIKE TERMS.**
Wrong! The answer is **10ab.**
HUH?


Allow me to elucidate. You see, **ab** actually means **a times b.** It's the same as **ba** because when you multiply the order is not important. I can change the **7ba** to **7ab.** So now both terms look alike and we can combine.
3ab+7ba=10ab
3ab+7ab=10ab
10ab

I guess that's what they mean by being fruitful and multiplying. So, Professor, wouldn't **10ba** also be correct?


3ab+7ba=10ba
3ba+7ba=10ba
10ba
Yes! It's the same either way! See...


10ab
=10ba
Remember ab=ba ? The Commutative Law for Multiplication?
So both answers must be **eqivalent?**


Let me challenge you with this problem. Who'll do it? Ok, Tara.
3x-5y-10x-3y
First I'll put the terms into columns. A column for x and a column for y.
3x-5y-10x-3y
3x -5y
-10x -3y
-7x-8y
Now, I'll combine the x column then the y column. My answer is -7x-8y.
Hey Professor! I got an answer of -8y-7x.
That's also OK. The terms can be rearranged if you choose. Either way we have to stop because the terms -8y and -7x are UNLIKE! Remember?
Remember SSC. Separate, Simplfy, Combine I'll do the first two steps SS. First I Separate the expression into 2 terms. Then I Simplify each term. OK, class, what is the next step ?
(x²-2x)-(5x-3x²)
(x²-2x)|-(5x-3x²)
x² -2x -5x +3x²
I know, Prof. We need to Combine in columns. A column for x square and a column for x.
Correct. Putting the like terms in columns is a good way of remembering that only LIKE terms can be combined.
(x²-2x)-(5x-3x²)
(x²-2x)|-(5x-3x²)
x² -2x -5x +3x²
x² -2x
+3x² -5x
+4x² -7x
So class, can I combine the 4x square and -7x ?
Professor Weissman, these are unlike terms. They can never be combined. We're done.
+4x² -7x
Brilliant. We stop after we combine the columns.
When you're simplifying an expression remember SSC
SEPARATE
SIMPLIFY
COMBINE
(IN COLUMNS)

Exercise Set 14

1. Combine Like Terms

   a) 5d + 3d

   b) 5m + 7m +m

   c) -5m –7m –m

   d) 7n—10n

   e) 3m+2n+7m+n

   f) 3m-2n-7m-n

   g) 5x + 2y

   h) $3c^2 + 7c^2$

   i) 4mn + 7mn

   j) 20ab –ab

   k) 5cd + 6dc

   l) 3a-5b-7b-8a+5ab

   m) $7xy^2 - 10xy^2$

   n) $4ab^2 + 6a^2b$

   o) -7am +7am

   p) 5x-7y+2y-9x+5y

   q) 5-3x-9-2x

   r) 10m-3-2-10m+5

2. Simplify

   a) - (-5a)

   b) - (2b-1)

   c) - (-3e-7e)

   d) 2x - (-5x)

   e) b - (-b)

   f) 2a-7 - (5+6a)

   g) 3m-7 + (2m+1)

   h) (2a-4b)

   i) (2a-4b) + (6a-7b)

   j) (2a-4b) - (6a-7b)

   k) Subtract –3x from 7x

   l) Subtract 2a+5b from 7a-2b

   m) Find the sum of –3x and –2x

   n) Find the difference of –3x and –2x

   o) What terms when combined with 7h—8g will equal zero?

   p) 2.3x+5x+0.16x

   q) 40x-3.4x

   r) 23y-1.71y

   s) 3p-7p

   t) 7p-3p

   u) (1/2)a+2.5a

   v) (2/3)x + (4/5)x

3. Fill in the blanks.

   a) Only _____ terms can be combined.

   b) To combine like terms, combine the _____ and keep the _____ _____.

   c) Unlike terms can not be __.

   d) Two other ways to write +7xy are __ and ___

   e) The numerical coefficient of –xy is _

   f) In –xy, x and y are __ __.

4. True or False? Why?

   a) 2a+3b=5ab

   b) $5b^3+7b^3=12b^6$

   c) There is no numerical coefficient in the term: xy

   d) 12mn and -5nm are like terms.

   e) 2x and x^2 are equivalent.

5. Find the perimeter of a square with side

   a) 8 mm

   a) 3 1/2 inches.

   b) 7.3 cm.

6. Find the side of a square whose perimeter is:

   a) 100 feet

   b) 2 2/3 yards

7. $2b - b^4 + 3 - 6b^2$

   a) How many terms are there?

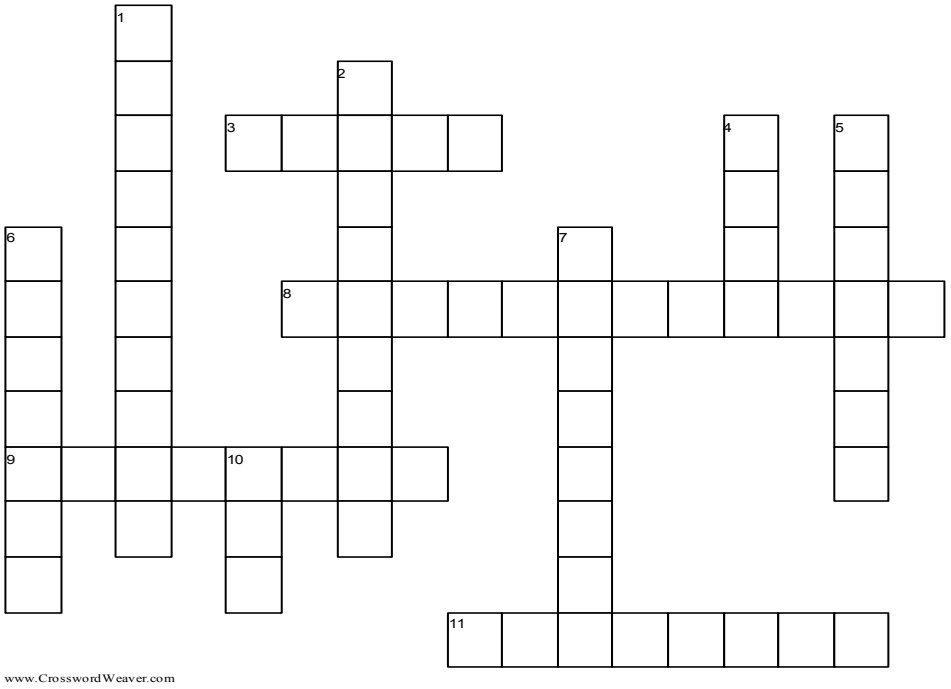
   b) What is the coefficient of the 1st term?

   c) How many factors are there in the 4th term?

   d) Arrange the terms in descending powers.

   e) Arrange the terms in descending powers and ac count for missing terms.

Crossword #14



ACROSS

3 what we separate an expression into

8 combine these

9 one term

11 many wives

DOWN

1 start with highest exponent

2 three terms

4 you can only combine these terms

5 parts of a term

6 put like terms in these

7 two terms

10 water and oil don't

Jokes Set #14

The math professor's six-year-old son knocks at the door of his father's study.
"Daddy", he says. "I need help with a math problem I couldn't do at school."

"Sure", the father says and smiles. "Just tell me what's bothering you."

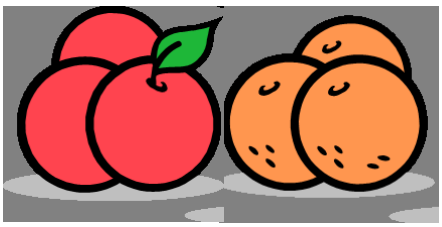
"Well, it's a really hard problem: *There are four ducks swimming in a pond, when two more ducks come and join them. How many ducks are now swimming in the pond?"*

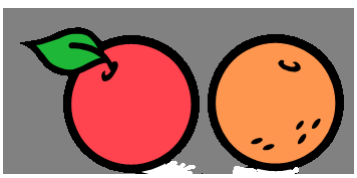
The Professor stares at his son with disbelief: "You couldn't do that?! All you need to know is that 4 + 2 = 6"

"Do you think, I'm stupid?! Of course, I know that 4 + 2 = 6. But what does this have to do with ducks!?"

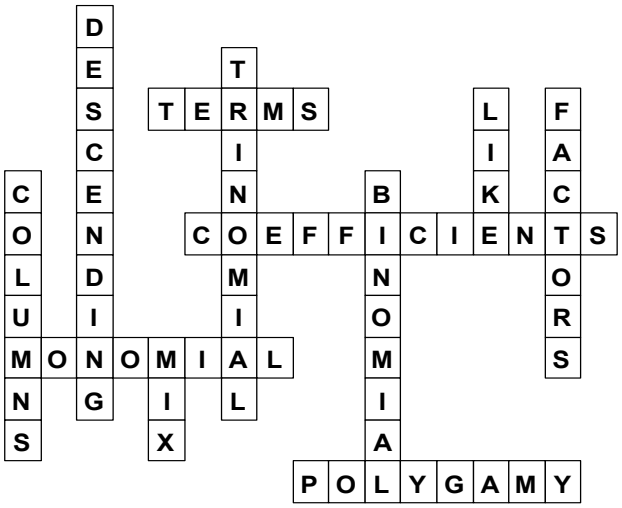
Brain Teasers Set #14

Apples and Oranges: In the garage behind your house, you have three big boxes. One of the boxes is labeled "Apples", another one "Oranges" and the last one "Apples and Oranges". The only thing you know is that none of boxes is labeled correct. How can you correctly re-label all the boxes, if you are only allowed to take out ONE fruit from ONE of the boxes?





Crossword #14 Solution



a) +5a
b) -2b+1
c) +10e
d) +7x
e) +2b
f) -1a –12
g) +5m+6
h) +2a –4b
i) +8a-11b
k) +10x
l) +5a-7b
m) -5x
n) -1x
o) -7h+8g
p) 12.46x
q) 36.6x
r) 21.29y
s) -4p
t) +4p
u) +3a
v) (22/15)x

3.

a) like
b) coefficients, literal factors
c) combined
d) 7xy, 7yx
e) -1
f) literal factors

4.

a) False
b) False
c) False
d) True
e) False

5.

a) 32 mm
b) 12 1/4 inches
c) 59.29 cm

Answers to Exercise Set 14

1.

a) +8d
b) +13m
c) -13m
d) -3n
e) +10m+3n
f) -4m –3n
g) 5x+2y
h) +10c^2
i) +11mn
j) +19ab
k) +11cd
l) -5a-12b+5ab
m) -3xy^2
n) 4ab^2+6a^2b
o) 0
p) -4x
q) -5x-4
r) 0

2.

d) 2/3 yd

6.

a) 25 ft.
b) 2/3 yd

7..

a) 4
b) 2
c) 3
d) – b^4 — 6b^2 +2b + 3
e)– b^4 +0b^3— 6b^2 +2b + 3

Brain Teaser #14 Answer

Answer: You take a fruit from the box labeled "apples and oranges". If you pull out an apple, then you should put the "apple" label on that box. If you pull out an orange, then put the "orange" label on that box. Then since you knew all three boxes had incorrect labels, you would switch the remaining two labels as well. If you pulled out an apple, You now know that box should be labeled “Apples.”You know the box labeled “Oranges” isn’t correct, so the only other thing it can be is “Apples/Oranges. Leaving the last label Apples,” which should be changed to “Oranges.” Follow the 3 steps [1],[2], and [3] in the table below.

Apples/Oranges [will be renamed Apples]	Oranges [Will be renamed Apple/Oranges]	Apples [Will be renamed Oranges]
[1] You pick an apple. Rename this box apples.	[2] This can’t be oranges since all boxes are labeled wrong. It can’t be apples since first box is apples. Label this box Apple/ Oranges.	[3] This can only be oranges. Label this oranges.