

31-05-2020

TRANSFORMER

* A transformer is an electrostatic device which contain two or more than two electric circuit interlinked together by a magnetic circuit shares energy between them at a constant frequency.

* As there is no moving part in transformer all parts remain stationary it is known as the electrostatic device.

* As there is no motion hence transfer of the electrical energy takes place at constant frequency.

Principal of operation of transformer.

* The principal of operation of transformer is based on Faraday's Law of mutual induction.

* When two coils are placed near to each other and current flows in one coil then due to mutual inductance emf is induced in another coil.

* In case of D-C at there is no change in the direction of current if magnitude of D-C also is constant no magnetic field is produced hence no flux and hence no EMF will produce.
Change of

EMF equation of transformer.

When two coils are connected across the magnetic core and voltage (alternating) V_1 is applied across coil 1

then current I_1 starts flowing through the coil hence

flux ϕ is generated due to this flux through coil 1 EMF E_1 is induced through self induction.

When this flux generated flows in coil 2 then EMF E_2 is generated in coil 2 due to mutual induction. hence current I_2 starts flowing through coil 2 and we will get voltage drop V_2 across the load.

* The coil to which voltage supply is connected is known as primary coil.

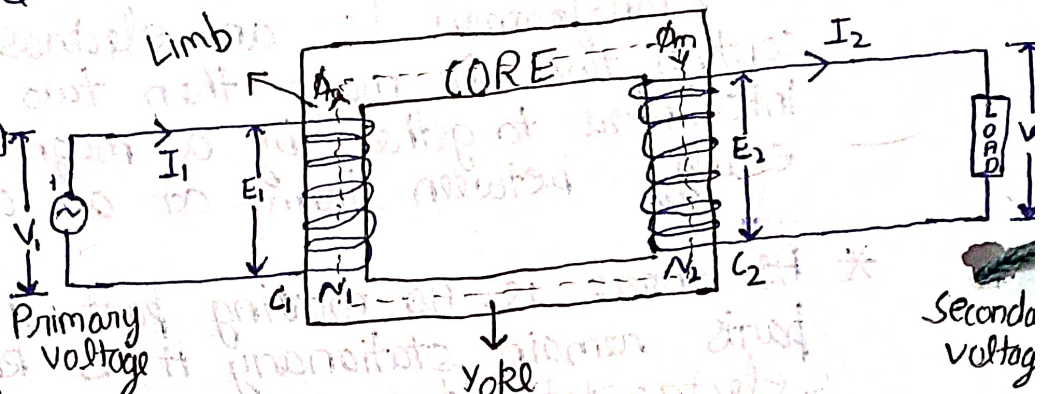
* The coil to which Load is connected is known as the secondary coil.

EMF induced in secondary coil is given by.

$$E_2 = -N_2 \frac{d\phi}{dt}$$

N_2 = Number of turns in secondary coil

ϕ = Flux linked with secondary coil



Derivation.

$$\phi = \phi_m \sin \omega t$$

$$E = -N \frac{d\phi}{dt}$$

$$E = -N \omega \phi_m \cos \omega t$$

$$E = (N \omega \phi_m) (-\cos \omega t)$$

$$[\sin(\omega t - 90^\circ) = -\cos \omega t]$$

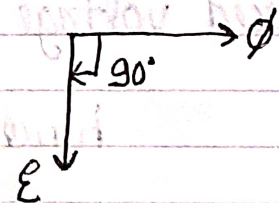
$$E = E_m \sin(\omega t - 90^\circ)$$

$$\text{Peak value} = E_m = N \omega \phi_m$$

$$\text{hence } E_{RMS} = \frac{N \omega \phi_m}{\sqrt{2}}$$

$$E_{RMS} = \frac{N \times 2\pi \times f \times \phi_m}{\sqrt{2}}$$

$$E_{RMS} = 4.44 N f \phi_m \quad \text{--- (A)}$$



* Hence induced EMF lags the flux by 90° .

* Equation (A) is known as EMF equation of transform

ϕ_m = Maximum flux in weber.

For primary.

$$E_1 = 4.44 N_1 \phi_m f$$

For secondary

$$E_2 = 4.44 N_2 \phi_m f$$

from above two we concluded that

$$\Rightarrow \frac{E_1}{N_1} = \frac{E_2}{N_2}$$

$$\Rightarrow \boxed{\frac{E_1}{N_1} = \frac{E_2}{N_2}}$$

$$\Rightarrow \boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2} = a = \text{Transformation Ratio.}}$$

* In practical transformer there is small difference between Induced voltage and a terminal voltage

$$\text{hence } \frac{V_1}{V_2} = \frac{N_1}{N_2} = a$$

* The voltage or turn Ratio is always in order of output : input or Primary : Secondary.

$$B_m = \frac{\phi_m}{A}$$

B_m = Maximum flux density in magnetic circuit (core) in tesla (T).

A = area of cross-section of core

ϕ_m = Maximum flux (Weber/Wb).

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* From transformation Ratio $a = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{V_1}{V_2}$.

$$V \propto N.$$

- The winding with high number of turns is known as high voltage winding
- The winding with less number of turns is low voltage winding

Step-UP transformer.

The voltage output or secondary voltage is greater in step up transformer.

$$V_2 > V_1 ; N_2 > N_1$$

Step down transformer.

The transformer in which output voltage is less than input is step down transformer.

$$V_1 > V_2 ; N_1 > N_2.$$

* Same transformer can be used as step up or step down transformer dependin on way it is connected in circuit.

Low voltage primary - step up

High voltage primary - Step down.

One to One transformer. (1:1).

The transformer in which output voltage is same as input voltage is called one to one

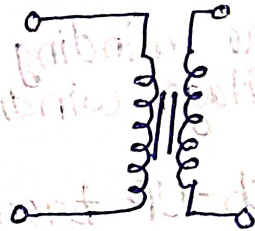
transformer.

* $V_1 = V_2 ; N_1 = N_2$.

* It is used for isolating two circuits.

Circuit symbol for two winding transformer:

Two vertical lines denote that two windings are magnetically coupled.



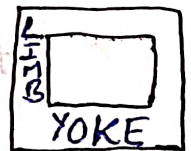
Construction of single phase transformer.

A transformer consists of two windings primary and secondary wound across a laminated core.

(i) Core.

Core is made up of thin sheets made of silicon steel. These thin sheets are called laminations.

- The lamination reduces eddy current losses.
- Silicon steel reduces the hysteresis losses.



* The laminations are staggered to minimize air gaps and are tightly clamped.

* The laminations are insulated from each other by the heat resistant coating.

* Magnetic core is used to confine flux within definite path.

* Horizontal portion of core is called 'YOKE'.

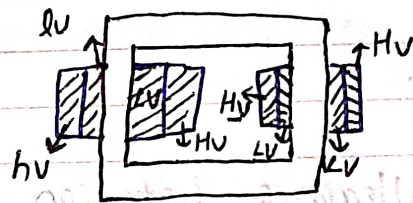
* Vertical portion of core is called 'LIMB'.

There are two basic type of construction of transformer

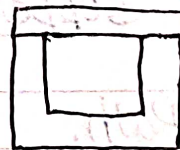
(i) Core type.

In core type transformer magnetic circuit is consist of two legs horizontal and two vertical. The low voltage winding is placed next to core then high voltage winding is placed this is done to reduce the insulating material Required.

The two windings are arranged as concentric coils Hence this winding is known as concentric or cylindrical winding.



* U shaped laminations are used in core type transformer and then they are closed by I shaped lamination at top.



Lamination (U+I shaped).

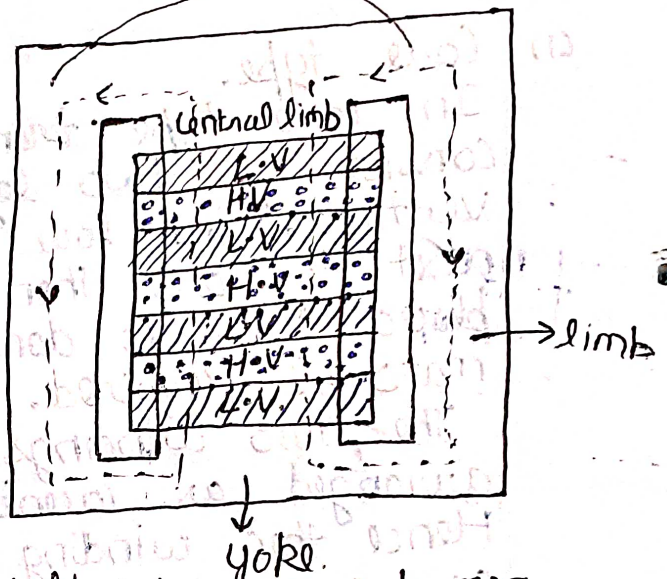
(ii) Shell type transformer.

In this type of transformer windings are put on the central limb in alternate manner (lv-hv-lv). Hence these forms sandwich hence this winding is also known as Sandwich winding or disc winding

* The outer limb provides the path for flux. $\rightarrow$ Flux Path

* The lamination shapes may be

- E+I
- U+T
- U+I



Difference between core and shell type transformer.

① Dismantle :- Core type transformer is easy to dismantle then shell type transformer, also easy to repair.

② Support against electromagnetic forces :- shell type transformer gives better support against EM forces.

③ Magnetic Path.
Shorter magnetic path in shell type hence current in shell type is lesser than core type.

④ Cooling.
Cooling is poor in shell type transformer because coils are embedded in between limbs of core as compare to core type transformer.

IDEAL TRANSFORMER.

A. Properties of ideal transformer.

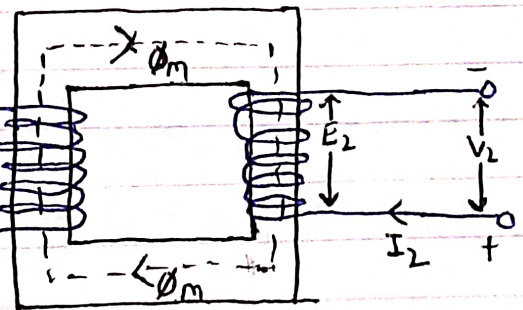
- The primary and secondary winding must have zero resistance.
- The core has infinite permeability. It means that the negligible amount of current is required to establish flux in core.
- There should be no leakage flux and leakage inductance hence all flux must be confined within core.
- There should be no hysteresis and eddy current losses. Thus efficiency of ideal transformer is 100%.

* Practical transformer behave somewhat close to ideal transformer.

Explanation.

* When V_1 applied current $I_1 = I_p$ flows which is just enough to overcome magnetomotive force (mmf) and generate EMF E_1 .

The I_p lags V_1 by 90° and by lenz law the V_1 opposes E_1 for E_1 opposes V_1 .



* Then E_2 is produced by flux ϕ_m E_2 and E_1 will be in same phase.
 As ϕ_m is produced by $I_1 = I_p$ it is in phase with it.

Here $|V_1| = |E_1|$ [For ideal transformer]
 Transformation Ratio.

$$a = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

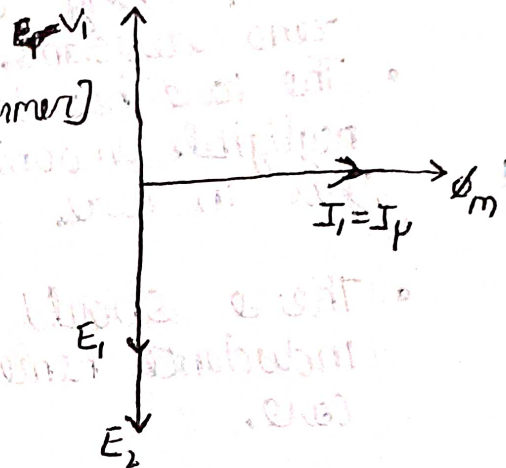
$$V_1 I_1 = V_2 I_2$$

$$S_1 = S_2$$

For ideal transformer

$$\boxed{\text{Input Kilo-volt} = \text{Output Kilo-volt}} \\ \boxed{\text{ampere} = \text{ampere}}$$

$$KVA = \frac{V_1 I_1}{1000} KVA$$



TRANSFORMER ON NO LOAD.

* When the secondary winding of transformer is open circuited then the transformer is said to be transformer on no load.

* When voltage V_1 is applied on primary winding then current I_0 starts flowing in primary circuit. This I_0 current is composed of two components I_w and I_μ .

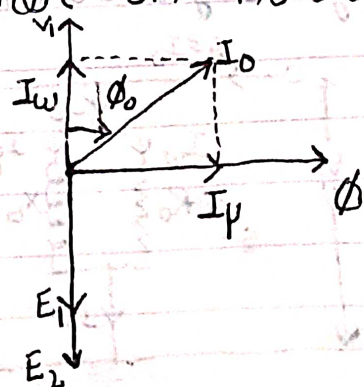
I_w :- This current is wattful component or active component of no load current (I_0). This current supplies eddy current, hysteresis and negligible I^2R loss in core and the primary winding respectively. I_w is in phase with applied voltage V_1 .

I_μ :- This current is known as wattless component or magnetising component of no load current I_0 . This component is useful in establishing the mutual flux (ϕ_m). This current remains in phase with ϕ .

Phasor diagram of transformer on no load.

ϕ_0 :- Angle ϕ_0 is known as the no load power factor angle.

$$\cos \phi_0 = \frac{I_w}{I_0}$$



hence,

$$\begin{aligned} I_W &= I_0 \cos \phi_0 \\ I_P &= I_0 \sin \phi_0 \end{aligned}$$

$$I_0 = \sqrt{I_W^2 + I_P^2}$$

* As E_1 and E_2 are produced by the same flux they must be in Phase with each other.

$$E_2 = E_1 \times \frac{T_2}{T_1}$$

$$E_2 = \frac{E_1}{a}$$

$$\phi = \phi_m \sin \omega t$$

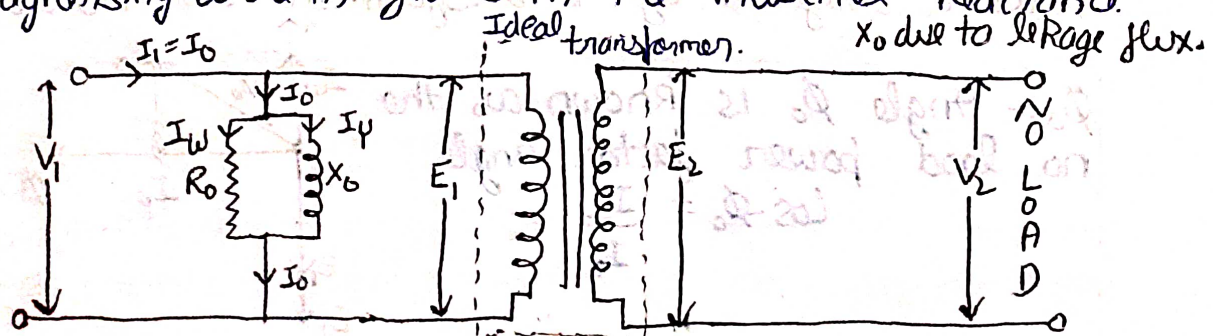
$$E_1 = E_m \sin(\omega t - 90^\circ)$$

$$E_2 = (E_2)_m \sin(\omega t - 90^\circ)$$

No Load equivalent circuit.

A Resistance R_0 is placed, which represents core, hysteresis and eddy current losses. Current I_W flows in Resistance R_0 .

The inductive Reactance X_0 placed in parallel with resistance R_0 which represents the flux loss outside the core. Current I_P (magnetising current) flows in the inductive Reactance.



From circuit

$$V_1 = I_w R_0 = I_p X_0$$

$$I_0 = I_p + I_w$$

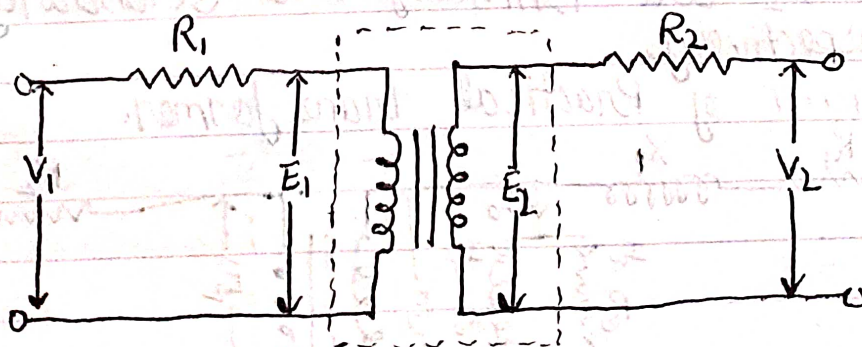
★ (Core + Eddy current + Hysteresis) losses = $I_w^2 R_0 = \frac{V_1^2}{R_0}$.

Practical transformer.

- * Both winding has Resistance
- * There are core loss, hysteresis and eddy current loss
- * Flux losses because flux is not only confined to core.
- * Permeability is finite.
- * Hence efficiency of practical transformer isn't 100%.

(ii) Winding Resistance.

Both primary and secondary winding has Resistance these resistances are represented by two Resistance (R_1 and R_2) 1 for primary and 2 for secondary.



ii) Leakage Reactance.

The leakage Reactance is due to leakage flux because flux is not at all confined within magnetic core it is diverted to non ferromagnetic surrounding (generally air or oil), because surrounding medium has finite permeability.

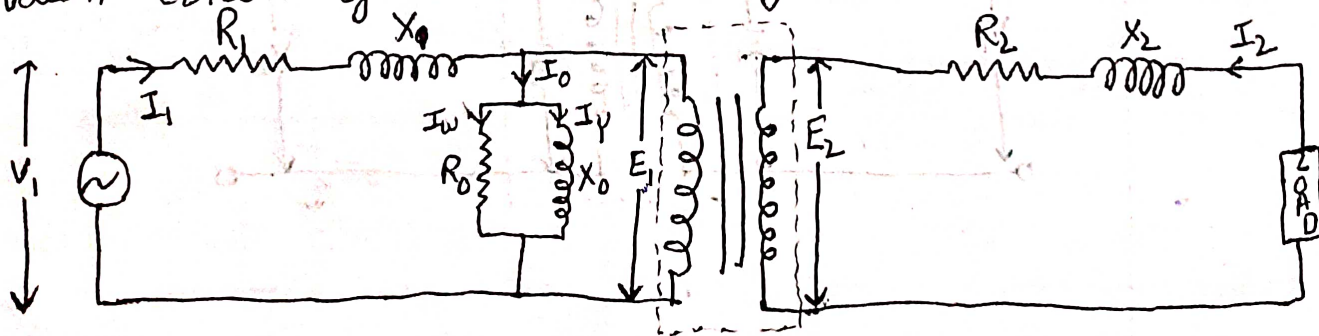
* The small portion of flux traverses different Path other than core this flux is hence known as primary leakage flux ϕ_L . When this flux ϕ_L links with primary winding it induces an EMF E_L .

Then in similar way in secondary winding ϕ_{L2} and E_{L2} are induced.

* As leakage flux linking with each winding produces a self induced EMF in that winding, hence the effect of leakage flux is equivalent to an inductance in series with each winding such that voltage drop in all series inductance is equal to that produced by leakage flux.

* The X_1 and X_2 are Primary and Secondary leakage reactance respectively.

Equivalent circuit of Practical transformer.

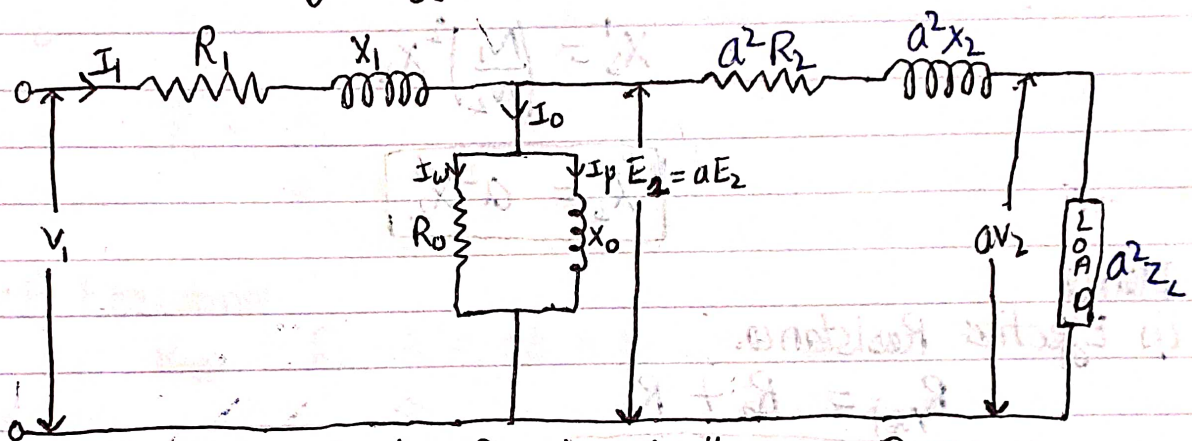


REFERENCING.

For simplifying transformer circuit Referencing is done.

* Primary circuit Referred to secondary or secondary referred to primary.

a. when secondary Referred to primary.



* Power transferred to R_2 in both case Remain same.

Power consumed by R_2 in Secondary = Power consumed by R_2 when Referred to Primary

$$I_2^2 R_2 = (I_2')^2 R_2'$$

$$R_2' = \left(\frac{I_2}{I_2'} \right)^2 \times R_2$$

By transformation Ratio $\frac{I_2}{I_2'} = \frac{N_1}{N_2}$

hence $R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2$

$$\boxed{R_2' = a^2 R_2}$$

Reactance

$$I_2^2 X_2 = (I_2')^2 X_2'$$

$$X_2' = \left(\frac{I_2}{I_2'}\right)^2 X_2$$

$$X_2' = \left(\frac{N_1}{N_2}\right)^2 X_2$$

$$\boxed{X_2' = a^2 X_2}$$

Hence

(i) Effective Resistance.

$$R_{net} = R_1 + R_2'$$

$$\boxed{R_{net} = R_1 + a^2 R_2}$$

(ii) Effective Reactance

$$X_{net} = X_1 + X_2'$$

$$\boxed{X_{net} = X_1 + a^2 X_2}$$

(iii) Effective impedance

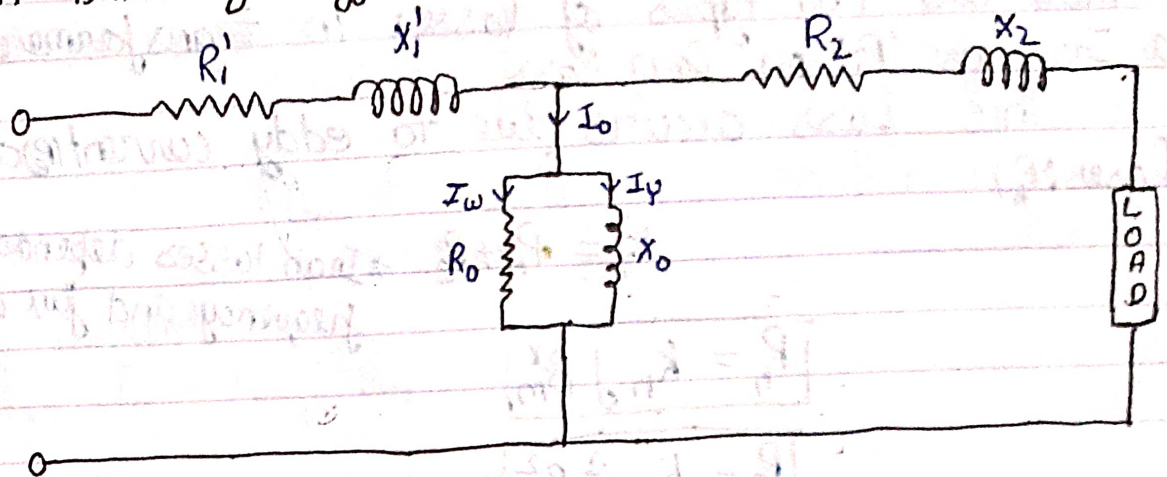
$$Z_{net} = Z_1 + Z_2'$$

$$\boxed{Z_{net} = Z_1 + a^2 Z_2}$$

$$Z_{net} = R_{net} + jX_{net}$$

$$\boxed{|Z_{net}| = \sqrt{(R_{net})^2 + (X_{net})^2}}$$

b. When Primary Referred to secondary.



$$R_1' = \frac{R_1}{a^2} ; X_1' = \frac{X_1}{a^2}$$

(i) Net Resistance

$$R_{net} = R_2 + R_1' = R_2 + \frac{R_1}{a^2}$$

(ii) Net Reactance

$$X_{net} = X_2 + X_1' = X_2 + \frac{X_1}{a^2}$$

(iii) Net impedance

$$Z_{net} = Z_2 + Z_1' = Z_2 + \frac{Z_1}{a^2}$$

$$Z_{net} = R_{net} + jX_{net}$$

$$|Z_{net}| = \sqrt{(R_{net})^2 + (X_{net})^2}$$

Losses in transformer.

There are two types of losses in transformer.

a. Iron Loss (P_i) or Core Loss

This Loss occurs due to eddy currents and hysteresis losses (P_h)

$P_i = P_h + P_e$. *Iron losses depends upon frequency and flux density in core.

$$P_h = K_h f B_m^x$$

$$P_e = K_e f^2 B_m^2$$

K_h = Proportionality constant depends upon (i) Volume (ii) Units used (iii) quality of core material.

K_e = Proportionality constant depends upon (i) Volume (ii) Units used (iii) Resistivity of core material (iv) thickness of laminations.

B_m = Maximum flux density in core.

f = Frequency of alternating flux.

* The exponent x is known as Steinmetz constant. Its value lies in between 1.5 to 2.5 and depending upon magnetic Properties.

Total Core Loss

$$P_i = P_h + P_e$$

$$P_i = K_h f B_m^x + K_e f^2 B_m^2$$

* Magnetic flux density B_m is directly proportional to voltage and inversely proportional to frequency.

* Since the value of induced voltage is approximately equal to applied voltage.

$$V_1 = E_1 = 4.44 \phi_m f T_1 = 4.44 B_m A f T_1.$$

$$B_m = \frac{V_1}{4.44 A f T_1} \Rightarrow \boxed{B_m \propto \frac{V}{f}}$$

* Eddy current

hence, $P_e = K_e f^2 B_m^2$

$$= K_e f^2 \left(\frac{V_1^2}{f^2} \right) \times \left(\frac{1}{4.44 A T_1} \right)^2$$

$$P_e = K_e V_1^2 \left(\frac{1}{4.44 A T_1} \right)^2$$

• This Relation shows that eddy current losses directly proportional to square of applied voltage and independent of frequency.

$$\boxed{P_e \propto V_1^2}$$

* Hysteresis Loss.

$$P_h = K_h f B_m^x$$

$$P_h = K_h f^{1-x} V_1^x \left(\frac{1}{4.44 A T_1} \right)^x$$

$$\boxed{P_h \propto f^{1-x} V_1^x}$$

Menu total iron Loss.

$$P_i = P_h + P_e$$

$$P_i = K_h V^x j^{1-x} + K_e V^2 \quad \star$$

Find hysteresis and eddy current Loss separately.

we know

$$P_i = K_h j B_m^x + K_e j^2 B_m^2$$

If B_m Remains constant then.

$$P_h \propto j$$

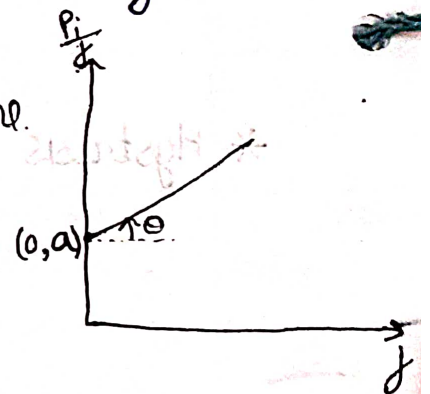
$$P_e \propto j^2$$

$$P_i = aj + bj^2$$

* For finding constants a, b no load test is performed in which secondary circuit is open circuited and on primary circuit variable voltage and frequency is applied

$$\Rightarrow \frac{P_i}{j} = a + bj \rightarrow \text{straight line}$$

$$\begin{array}{l} \text{Slope} = \tan \theta = b \\ \text{y intercept} = a \end{array}$$



$$\text{hysteresis Loss} = P_h = aj$$

$$\text{Eddy current Loss} = P_e = bj^2$$

Example.

In a transformer, the core loss is 100W at 40Hz and 72W at 30Hz. Find Hysteresis and the eddy current losses at 50Hz.

Sol

$$P_i = a + b f$$

$$P_i = 100 ; f = 40 \text{ Hz}$$

$$\frac{100}{40} = a + 40b$$

$$5 = 2a + 80b \quad \text{--- (1)}$$

$$P_i = 72 \text{ W} ; f = 30 \text{ Hz}$$

$$\frac{72}{30} = a + 30b$$

$$2.4 = a + 30b \quad \text{--- (2)}$$

$$\text{hence } a = 2.1$$

$$b = 0.01$$

$$\text{Hysteresis at } 50 \text{ Hz} = a f = 2.1 \times 50 = 105 \text{ W}$$

$$\text{Eddy at } 50 \text{ Hz} = b f^2 = 2500 \times 0.01 = 25 \text{ W}$$

Stray Loss :- Leakage flux produces eddy current and these eddy current produces losses.

Dielectric Loss :- The loss which occurs in transformer oil and solid insulation of transformer. This loss is significant in high voltage transformer. Both above losses are small and negligible.

Copper Losses or I^2R Loss (P_c).
It occurs due to winding resistances of primary and secondary winding.

Copper Loss (P_c) = Primary winding + Secondary winding copper loss

$$P_c = I_1^2 R_1 + I_2^2 R_2.$$

we know $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

$$I_1 = I_2 \times \frac{N_2}{N_1}$$

~~$$P_c = I_2^2 \frac{N_2^2}{N_1^2} R_1 + I_2^2 R_2$$~~

$$P_c = I_2^2 \frac{N_2^2}{N_1^2} R_1 + I_2^2 R_2$$

Primary Referred to Secondary = $I_2^2 \left[\frac{R_1}{a^2} + R_2 \right]$

$$P_c = I_2^2 \left[\frac{R_1}{a^2} + R_2 \right]$$

$$P_c = I_2^2 (R_{net})_1$$

Alternatively,

$$I_2 = I_1 a.$$

$$P_c = I_1^2 R_1 + I_1^2 a^2 R_2.$$

$$= I_1^2 [R_1 + a^2 R_2]$$

$$P_c = I_1^2 (R_{net})_2$$

Secondary Referred to Primary

Transformer is Rated in KVA not watt because

- Iron Loss due to voltage and copper loss due to current
- At the time of manufacturing it is not known that which type of load is connected R, L, C or RL, RC, RLC.

TRANSFORMER EFFICIENCY.

The ratio of output Power to the input Power is known as Transformer efficiency.

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} = \frac{\text{output}}{\text{output} + \text{Iron} + \text{copper}}$$

Given :- Load current :- I_2 Power factor = $\cos \phi_2$

$$\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + I_2^2 (R_{\text{net}})' + P_i}$$

At full load.

$$\eta = \frac{V_2 (I_2)_{FL} \cos \phi_2}{V_2 I_{2FL} \cos \phi_2 + (P_c)_L + P_i}$$

* As transformer is a static device there is no frictional loss, and windage loss.

* Hence transformer have efficiency as high as 99%.

Condition for Maximum efficiency.

$$\frac{d\eta}{dI_2} = 0$$

we know, $\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + I_2^2 (R_{\text{net}})' + P_i}$

Dividing by I_2 .

$$\eta = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + I_2 (R_{net})'_1 + \frac{P_i}{I_2}}$$

Here, V_2 and $\cos \phi_2$ are constant efficiency will be max when denominator minimum

$$\frac{d\eta}{dI_2} = 0$$

$$\frac{d}{dI_2} \left(V_2 \cos \phi_2 + I_2 (R_{net})'_1 + \frac{P_i}{I_2} \right) = 0$$

$$(R_{net})'_1 - \frac{P_i}{I_2^2} = 0$$

$$P_i = I_2^2 (R_{net})'_1$$

$$P_i = P_c$$

* Hence transformer have maximum efficiency when the constant iron loss is equal to variable copper loss.

current and KVA at maximum efficiency.

$$I_2^2 (R_{eq})' = P_i$$

$$I_{2M}^2 = \frac{P_i}{(R_{eq})'}$$

Multiplying and dividing by secondary current (square) at full Load I_{2fl}^2

$$I_{2M}^2 = \frac{P_i I_{2fl}^2}{I_{2fl}^2 (R_{eq})'}$$

$$I_{2M} = I_{2fl} \sqrt{\frac{P_i}{P_{cfl}}}$$

current at maximum efficiency	=	current at full Load	$\times$	$\sqrt{\frac{\text{Constant iron Loss}}{\text{full load copper Loss}}}$
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Multiplying both sides by V_2 .

$$V_2 I_{2M} = V_2 I_{2fl} \sqrt{\frac{P_i}{P_{cfl}}}$$

$S_1 = S_{fl} \sqrt{\frac{P_i}{P_{cfl}}}$

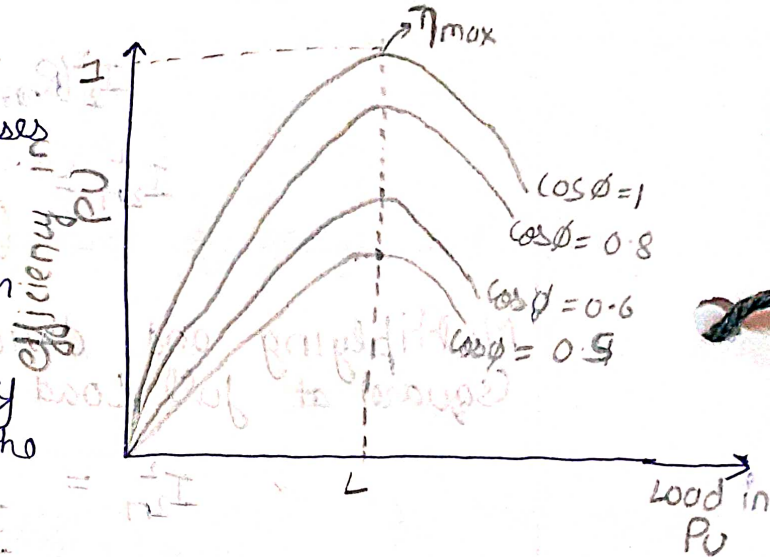
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EFFICIENCY CURVE OF TRANSFORMER

* As the Power factor increases the efficiency increases

* Highest Possible efficiency occurs at unity Power factor

* Maximum efficiency at any Power factor occurs at the same Load.



Applications of transformers.

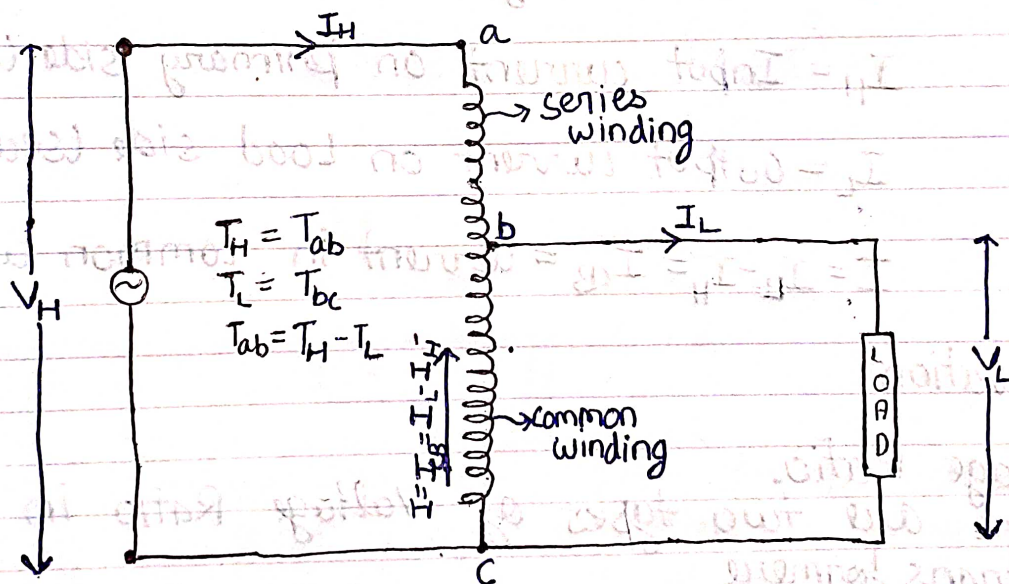
- * To change voltage and current level in an electric circuit.
- * As impedance matching for maximum Power transfer.
- * As a coupling device.
- * To isolate two electric circuit secondary and primary circuit are connected magnetically not electrically.
- * Converting high voltage A.C to high voltage D.C
- * For stepping down or stepping up the voltage.
- * Stepped down voltage is used domestically and industrially.

connection

16-09-2020

AUTOTRANSFORMER

- * A transformer in which a single winding serves as the both primary and secondary winding is known as Autotransformer.
- * The part of winding which is common to both primary as well as secondary sides is known as common winding.
- * The winding which is in series to the common winding is known as series winding.



- * Current in cb winding (I) is supposed to flow from $c \rightarrow b$ to oppose the EMF generated by the full winding (High voltage winding ac)

Important terms related to circuit

T_H = Number of turns in h.v. sides (full winding abc)

T_L = Number of turns in l.v. sides (common winding bc)

$T_H - T_L$ = Number of turns in series winding. (ab)

V_H = Input voltage on primary side.

V_L = Output voltage on Load side (secondary side)

I_H = Input current on primary side (series winding)

I_L = Output current on Load side (secondary side)

$I = I_H - I_L = I_{cb}$ = Current in common winding

Derivation

① Voltage Ratio.

There are two types of voltage Ratio in an autotransformers

② Circuit voltage Ratio / Transformation Ratio

The voltage Ratio of circuit input and output high to the low is known as circuit voltage Ratio or transformation Ratio

$$a_A = \frac{V_H}{V_L} = \frac{I_H}{I_L}$$

* a_A is called as transformation Ratio.

* a_A is always greater than 1.

* Winding voltage Ratio.

The voltage Ratio of series to the common winding is known as the winding voltage Ratio.

$$a = \frac{V_{ab}}{V_{bc}} = \frac{I_{ab}}{I_{bc}} = \frac{I_H - I_L}{I_L}$$

$$a = \frac{I_H}{I_L} - 1$$

$$a = a_A - 1$$

* If winding resistance and leakage reactance are considered negligible then winding voltage will be same as winding induced EMF

$$a = \frac{E_{ab}}{E_{bc}}$$

$$\Rightarrow a_A = a + 1$$

* This shows that transformation ratio of an autotransformer is greater if same set of winding is used as two winding transformer.

$$a_A = a + 1$$

a_A = Transformation Ratio of autotransformer.

a = winding voltage Ratio = Transformation Ratio of two winding transformer.

② Current Ratio ($\frac{I_L}{I_H} = a_A$)

By equating magnetomotive force in winding ab, bc

$$(mmf)_{ab} = (mmf)_{bc}$$

$$I_H (T_H - T_L) = I (T_L)$$

$$I_H T_H - I_H T_L = (I_L - I_H) T_L$$

$$I_H T_H - I_H T_L = I_L T_L - I_H T_L$$

$$\frac{I_H}{I_L} = \frac{T_L}{T_H}$$

$$\frac{I_L}{I_H} = \frac{T_H}{T_L} = a_A$$

$$\frac{I_L}{I_H} = a_A$$

By KCL at point B we know;

$$I = I_L - I_H$$

$$\frac{I + I_H}{I_H} = a_A$$

$$\boxed{\frac{I}{I_H} = a_A - 1}$$

* If winding ab of autotransformer converted to primary and bc converted to secondary then of two winding transformer transformation ratio is.

$$a = \frac{T_{ab}}{T_{bc}} = \frac{T_H - T_L}{T_L}$$

$$\boxed{a = a_A - 1}$$

$$a = a_A + 1$$

→ Then new transformer formed by autotransformer will have transformation ratio of $a_A - 1$.

* ③ Voltampere Relations.

* Autotransformer transfers energy from primary to the secondary side partly by magnetic link (induction) and partly by electrical link (conduction)

* Two winding transformer transfers energy only by the magnetic link (induction)

* Hence there must be two types of voltamper

- (i) Transformed voltamper
- (ii) Conducted voltamper.

(i) Transformed voltamperes
voltamperes in winding ab and bc are known as transformed voltamperes.

$$S_{\text{trans}} = I_{ab} V_{ab} = I_{bc} V_{bc}$$

$$\Rightarrow S_{\text{trans}} = I_H V_{ab} = I V_{bc}$$

(ii) Conducted Voltamper.

Power/Energy transferred directly through conduction

Input power = Transformed + Conducted

$$\text{Input Power} = V_{ac} I_{ab} = (V_{ab} + V_{bc}) I_{ab}$$

Comparing,

$$= V_{ab} I_{ab} + V_{bc} I_{ab}$$

$$\boxed{\text{Conducted VA} = V_{bc} I_{ab} = V_{bc} I_H}$$

$$S_{\text{trans}} = V_{bc} I_{bc} = V_L I$$

$$S_{\text{cond}} = V_L I_H$$

* An autotransformer is always Rated on the basis of output voltamper.

(A) Comparison between voltamper of autotrans and two winding transformer.

$$\text{Output VA of auto} = V_L I_L = V_H I_H$$

$$\text{Output of two winding} = V_L I$$

$$\frac{(VA)_{\text{auto}}}{(VA)_{\text{TW}}} = \frac{V_L I_L}{V_L I}$$

$$\frac{(VA)_{\text{auto}}}{(VA)_{\text{TW}}} = \frac{I_L}{I}$$

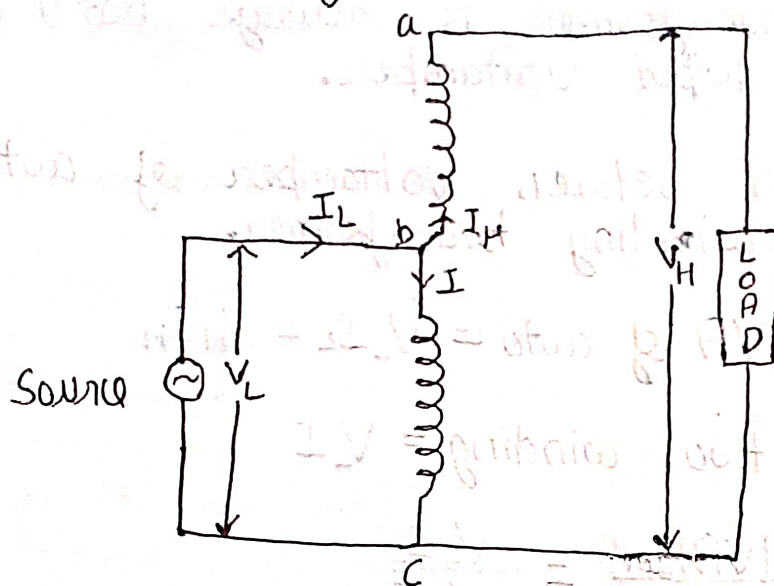
$$\frac{(VA)_{\text{auto}}}{(VA)_{\text{TW}}} = \frac{a_A}{a_A - 1}$$

$$\frac{(VA)_{\text{TW}}}{(VA)_{\text{auto}}} = \frac{a_A - 1}{a_A} = 1 - \frac{1}{a_A}$$

$$\text{Hence, } (VA)_{\text{auto}} > (VA)_{\text{TW}}$$

* Voltamper of autotransformer greater than volt-
 ampere of two winding transformer.

② Step up transformer.



By equating mmf in both windings

$$(\text{mmf})_{ab} = (\text{mmf})_{bc}$$

$$I_H N_{ab} = I N_{bc}$$

$$a_A = \frac{N_{ac}}{N_{bc}} = \frac{V_H}{V_L}$$

Here $a_A > 1$

$$\frac{I}{I_H} = \frac{N_{ab}}{N_{bc}} = \frac{N_{ac} - N_{bc}}{N_{bc}} = \frac{N_{ac}}{N_{bc}} - 1$$

$$\boxed{\frac{I}{I_H} = a_A - 1}$$

$$\frac{I + I_H}{I_H} = a_A$$

$$\frac{I_L}{I_H} = a_A$$

$$\frac{I}{I_L} = \frac{a_A - 1}{a_A}$$

By KVL in high voltage side

$$E_{ab} + E_{bc} = E_{ac}$$

$$E_{ab} = E_{ac} - E_{bc}$$

$$E_{ab} = a_A E_{bc} - E_{bc}$$

$$E_{ab} = (a_A - 1) E_L$$

③ Saving in Conducting material.

In autotransformer same winding serves as primary and secondary winding hence amount of copper required for making it is less as compared to two winding transformer.

* As less amount of copper is required so eddy current, hysteresis and resistance loss is reduced.

* Weight of copper required is directly proportional

(to the current in transformer winding and number of turns in it.

$$W_{2w} \propto I_H T_H + I_L T_L$$

$$W_{auto} \propto I_H (T_H - T_L) + (I_L - I_H) T_L$$

$$\frac{W_{auto}}{W_{2w}} = \frac{I_H (T_H - T_L) + (I_L - I_H) T_L}{I_H T_H + I_L T_L}$$

We know $I_H T_H = I_L T_L$ [equating mmf].

$$\frac{W_{auto}}{W_{2w}} = \frac{I_H T_H - I_H T_L + I_L T_L - I_H T_L}{I_H T_H + I_L T_L}$$

$$= \frac{2 I_H T_H - 2 I_H T_L}{2 I_H T_H}$$

$$= \frac{T_H - T_L}{T_H}$$

$$= 1 - \frac{T_L}{T_H} = 1 - \frac{1}{a_A}$$

$$\boxed{\frac{W_{auto}}{W_{2w}} = 1 - \frac{1}{a_A}}$$

$$\frac{1}{a_A} = 1 - \frac{W_{awto}}{W_{2w}}$$

$$\frac{1}{a_A} = \frac{W_{2w} - W_{awto}}{W_{2w}}$$

$$\text{Saving in material} = W_{2w} - W_{awto} = \frac{W_{2w}}{a_A}$$

* When a_A is unity maximum saving of copper material will be there.

* Use of auto transformer is more economical when $1/a_A$ is close to unity.

* Saving is inversely proportional to a_A .

$$\text{Saving} \propto \frac{1}{a_A}$$

④ Advantages of autotransformer

- * Autotransformer is smaller and cheaper for same output as 2-winding transformer.
- * Less winding material is required in autotransformer as compared to two winding transformer.
- * Since there is reduction in conductor and core materials ohmic and core losses are decreased hence autotransformer will have high efficiency than equivalent 2-winding transformer.
- * Saving is large if transformation ratio is small.
- * Since one winding has been completely eliminated the resistance and leakage flux of this winding is zero. Hence the voltage regulation of autotransformer is superior because of reduced voltage drop in resistance and reactance.
- * Autotransformer can have variable output voltage due to tapping point.

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⑤ Application of Autotransformer.

- * Interconnections of power systems of different voltage levels Eg 132 kV with 230 kV
- * Autotransformers with number of tapping point can be used to start induction and synchronous motors.
- * Autotransformer is used as variac (variable ac)
- * Also used when we require variable voltages over broad Ranges.