25th february.

Electricity And Magnetism

L-1

***Field:** A field is a region of space in which a physical quantity is well defined at all points at a given instant of time.

- Well defined means the physical Quantity must be
  - Single valued
  - Continuous
  - Definite
- Any field can be expressed as a mathematical funcn of space & time. → $f(x, y, z, t)$ OR $f(\vec{r}, t)$

* Gravitational field of Earth (Time independent) • Magnetic field of Earth (Time Dependent)

→ On the Nature of Physical Qty. the field is of two types -
1) Scalar field
2) Vector field.

L-2

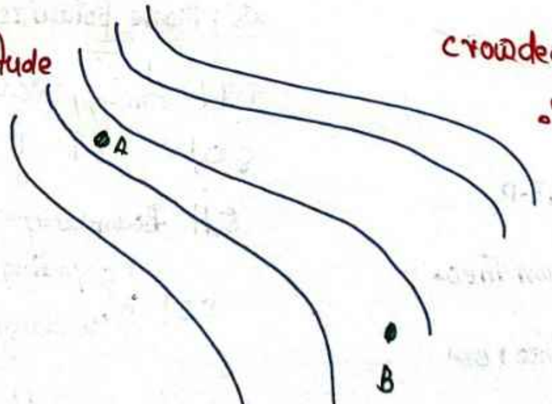
*** Scalar field:** A region of space in which a scalar Qty. is well defined at every point & at every instant of time is called a scalar field.

- The field at a given point can be completely expressed just by a number
  eg - Density of earth this can be expressed as $\rho(x, y, z, t)$
  - A scalar field (Density) is not defined in the location of black hole.
  - The variation in tempr in a room can be expressed as $T(x, y, z, t)$.

* A scalar field is expressed by imaginary contours. which can be obtained By connecting all those points which have same values for a physical Quantity

eg - Equipotential Surface. * The Magnitude of the scalar field is expressed By the crowdedness of the contours

* More is the Crowdedness more will be the Magnitude of the field.



crowdedness : $A > B$

$\therefore$ Magnitude of field

$A > B$

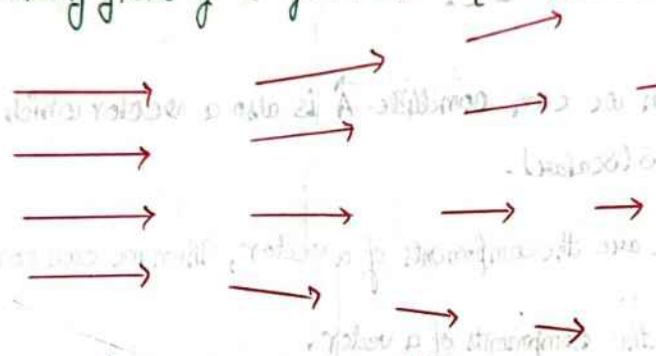
* Vector field. – A region of space in which a vector qty is well defined with a unique direction at every point & at every instant of time is called a vector field.

L-3

eg. • force on a compass needle in Earth's Magnetic field
• Electric field
• Velocity field of a fluid in Motion. → $\vec{V}(x,y,z,t)$

Vector fields are represented with the help of vectors.

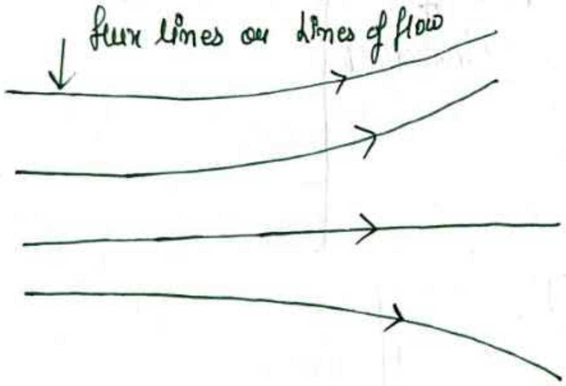
a)



a) Length of Arrow → strength of field.
• Arrowhead → direcn of field

b)

flux lines or lines of flow



b) Tangent at curve → direcn of field

No. of field line → strength of field. per unit Area, which must be $\perp^r$ to flow of field lines

* We adopt the convn of that the no. of lines per unit area at right angle to the lines is proportional to the field strength.

L-4

* The Del Operator.

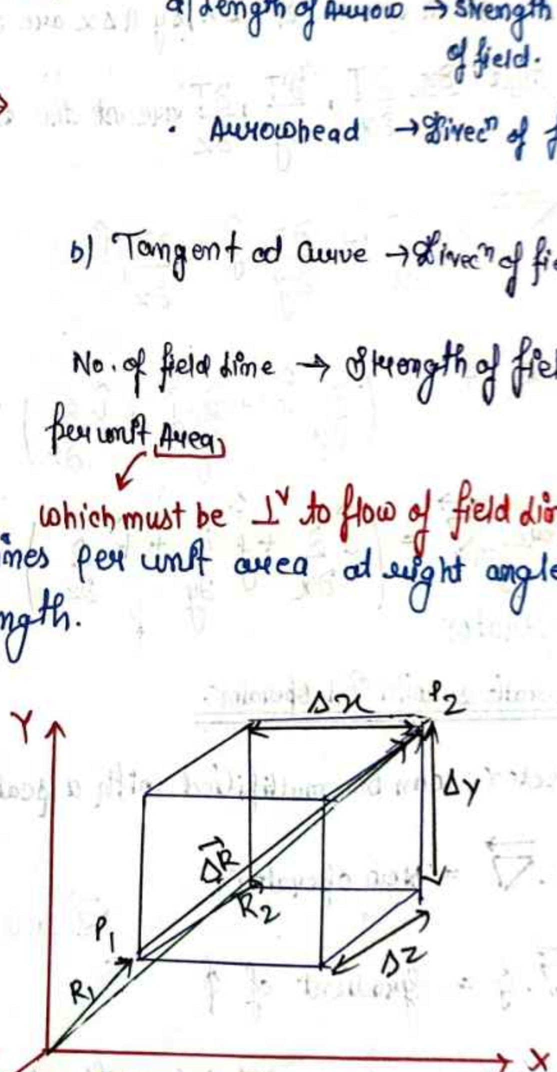
• A particle is moving along x axis with a unifor speed of 5 m/s, find distance covered by it in 3sec.

Ans $\Delta x = 5 \times 3 = 15\,m$

$\Delta x = \frac{dx}{dt} \times \Delta t$ → change in time

↓ change in x ; ↓ Rate of change of x w.r.t time

• As x is a fn of time one variable ∴ it can be written as $x(t)$



• Consider distribution of tempr in a room & consider arbitrarily a point $P_1(x,y,z)$ repr by a vector $\vec{R_1} = x\hat{i} + y\hat{j} + z\hat{k}$ having temp T_1.

T_2 is the tempr at P_2 & $P_2 = (x+\Delta x,\ y+\Delta y,\ z+\Delta z)$

$\vec{R_2} = (x+\Delta x)\hat{i} + (y+\Delta y)\hat{j} + (z+\Delta z)\hat{k}$.

• Change in tempʳ b/w P_1 & P_2 = $\Delta T = T_2 - T_1$

$$\Delta T = \left(\frac{\partial T}{\partial x}\right)\Delta x + \left(\frac{\partial T}{\partial y}\right)\Delta y + \left(\frac{\partial T}{\partial z}\right)\Delta z$$

$$\overrightarrow{\Delta R} = \vec{R_2} - R_1 \Rightarrow \Delta \vec{R} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

• $\Delta x, \Delta y$ & Δz are the components of a vector $\overrightarrow{\Delta R}$.

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
$$S = \vec{A}\cdot\vec{B}$$
$$\therefore S = A_xB_x + A_yB_y + A_zB_z$$

As S is a scalar, $\vec{B}$ is a vector then we can conclude $\vec{A}$ is also a vector which is in dot product with $\vec{B}$ to give us S (Scalar).

• As ΔT is a scalar & $\Delta x, \Delta y$ & Δz are the components of a vector, Then we can conclude that ~~∂x~~ $\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}$ are ~~at~~ the components of a vector.

$$\therefore \vec{\nabla} T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

$$\vec{\nabla}.T = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \hat{k}\frac{\partial}{\partial z}\right)T$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$ is called Del operator & , Thus. $\Delta T = \vec{\nabla}T.\Delta\vec{R}$

Del operator

1.5 Operations with Del operator.

• A vector can be multiplied with a scalar ; $k\vec{A}$ = a new vector ~~whose k times~~ , ϕ = scalar funcⁿ.

$$\phi.\vec{\nabla} = \text{New operator};$$

$$\vec{\nabla}.\phi = \text{gradient of } \phi$$

; $\vec{\nabla}$ acts on a scalar fn & result will be a vector function.

• A vector can be multiplied with another Vector: $\vec{A}\cdot\vec{B}$ = Scalar Qty

$\vec{A}\times\vec{B}$ = Vector Qty.

$\vec{A}$ = vector function

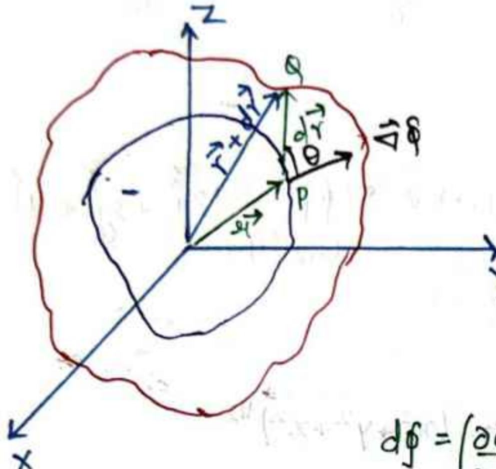
$$\vec{\nabla}.\vec{A} = \text{Divergence of } \vec{A}$$

• Divergence [. operation] of $\vec{\nabla}$ with a vector funcⁿ

→ Resultant will be a scalar funcⁿ.

- $\vec{\nabla} \times \vec{A}$ = Curl of $\vec{A}$; Operates only on a vector func^n, Resultant will be a vector func^n.

1.6 Gradient of a Scalar function.



$\phi(r)$ = Scalar function at point P

$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$

$\phi(r+dr) \rightarrow$ Scalar fn at Q.

$\vec{r} + d\vec{r} = \hat{i}(x+dx) + \hat{j}(y+dy) + \hat{k}(z+dz)$

→ The change in ϕ from P to Q from Taylor's approximation:

$$d\phi = \left(\frac{\partial \phi}{\partial x}\right) \cdot dx + \left(\frac{\partial \phi}{\partial y}\right) dy + \left(\frac{\partial \phi}{\partial z}\right) dz$$

$$d\phi = \left(\hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}\right) \cdot \left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$d\phi = \vec{\nabla}\phi \cdot d\vec{r} \rightarrow d\phi = \vec{\nabla}\phi \, dr \cos\theta$$

$$\frac{d\phi}{dr} = \vec{\nabla}\phi \cos\theta \quad \Bigg| \quad \frac{d\phi}{dr} = \text{Space rate of change of } \phi$$

↳ Direction Dependent.

If $\theta = 0^\circ$, then $\cos\theta = 1$

⇓

$\frac{d\phi}{dr}$ is maximum $\therefore \left(\frac{d\phi}{dr}\right)_{Max^m} = \vec{\nabla}\phi$

→ Hence, The Magnitude of the gradient of a scalar function is equal to the Maximum space rate of change of the scalar function

→ The direction of the gradient is the direction in which $\frac{d\phi}{dr}$ is Maximum.

- If we move in such a way that $\theta = 90^\circ$, in that case, $\frac{d\phi}{dr} = \nabla\phi \cos 90^\circ \rightarrow \frac{d\phi}{dr} = 0$

⇒ ϕ = constant func^n [We are moving along a level surface].

↓

Defines a level surface

* We can conclude that direction of gradient is always perpendicular to the level surface.

L-7

Q find $\vec{\nabla}\phi$ at $(1,2,-1)$ if $\phi(x,y,z) = 3x^2y - yz^2$

Sol^n $\vec{\nabla}\phi = \hat{i}\frac{\partial}{\partial x}\phi + \hat{j}\frac{\partial}{\partial y}\phi + \hat{k}\frac{\partial}{\partial z}\phi = \hat{i}\frac{\partial}{\partial x}(3x^2y - yz^2) + \hat{j}\frac{\partial}{\partial y}(3x^2y - yz^2) + \hat{k}\frac{\partial}{\partial z}(3x^2y - yz^2)$

$$\vec{\nabla}\phi = \hat{i}(6xy) + \hat{j}(3x^2 - z^2) + \hat{k}(-2yz)$$

$$\vec{\nabla}\phi\Big|_{(1,2,-1)} = \hat{i}(6\times1\times2) + \hat{j}(3(1)^2 - (-1)^2) + \hat{k}(-2\cdot2\cdot(-1)) \Rightarrow \nabla\phi\Big|_{(1,2,-1)} = 12\hat{i} + 2\hat{j} + 4\hat{k}$$

Q find gradient of r

Sol^n $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ $\therefore$ $r = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2}$

$$\vec{\nabla}.r = \hat{i}\frac{\partial r}{\partial x} + \hat{j}\frac{\partial r}{\partial y} + \hat{k}\frac{\partial r}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{1/2} + \hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{1/2} + \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{1/2}$$

$$= \hat{i}\cdot\frac{1}{2}(x^2+y^2+z^2)^{-1/2}\cdot 2x + \hat{j}\,\frac{1}{2}(x^2+y^2+z^2)^{-1/2}\cdot 2y + \hat{k}\,\frac{1}{2}(x^2+y^2+z^2)^{-1/2}\cdot 2z$$

$$\vec{\nabla}.r = \frac{\hat{i}x + \hat{j}y + \hat{k}z}{(x^2+y^2+z^2)^{1/2}} \Rightarrow \vec{\nabla}.r = \frac{\vec{r}}{r}$$

$$\therefore \boxed{\vec{\nabla}r = \hat{r}}$$

L-8

Q Show that $\nabla r^n = n r^{n-2}\hat{r}$.

Sol^n $\vec{r} = \hat{i}x + y\hat{j} + \hat{k}z$; $r = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2}$ OR $r^2 = x^2+y^2+z^2$

$$r^n = (x^2+y^2+z^2)^{n/2}$$

$$\vec{\nabla}.r^n = \hat{i}\frac{\partial r^n}{\partial x} + \hat{j}\frac{\partial r^n}{\partial y} + \hat{k}\frac{\partial r^n}{\partial z}$$

$$\nabla r^n = \sum \hat{i}.\frac{\partial}{\partial x}(x^2+y^2+z^2)^{n/2}$$

$$\nabla r^n = \sum \hat{i}\frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}.2x$$

$$\nabla r^n = n(x^2+y^2+z^2)^{\frac{n}{2}-1}\left[\hat{i}x + \hat{j}y + \hat{k}z\right]$$

$$\nabla r^n = n(r^2)^{n/2-1}.\vec{r}$$

$$\nabla r^n = n r^{n-2}.\vec{r}$$

$$\nabla r^n = n r^{n-2}.r\hat{r}$$

$$\boxed{\therefore \nabla r^n = n r^{n-1}\hat{r}}$$

Q. If $\phi = \log_e |\vec{r}|$, find $\vec{\nabla}\cdot\phi$. $\rightarrow$ $\vec{\nabla}\phi = \frac{1}{r}\cdot\hat{r}$

Sol^n $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$, $|\vec{r}| = (x^2+y^2+z^2)^{1/2}$

$$\phi = \ln (x^2+y^2+z^2)^{1/2} = \frac{1}{2}\ln|x^2+y^2+z^2|$$

$$\nabla\phi = \frac{1}{2}\nabla \ln(x^2+y^2+z^2) = \frac{1}{2}\sum \hat{i}\frac{\partial \ln(x^2+y^2+z^2)}{\partial x}$$

$$= \frac{1}{2}\sum \hat{i}\frac{2x}{x^2+y^2+z^2} = \frac{1}{x^2+y^2+z^2}(\hat{i}x+\hat{j}y+\hat{k}z)$$

$$\boxed{\nabla\phi = \frac{\vec{r}}{r^2} = \frac{\hat{r}}{r}}$$

L-10

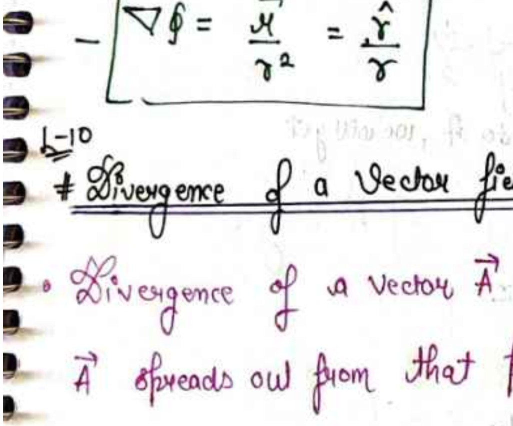
Divergence of a Vector field.

- Divergence of a vector $\vec{A}$ i.e $\vec{\nabla}\cdot\vec{A}$ is a measure of how much that vector $\vec{A}$ spreads out from that point.

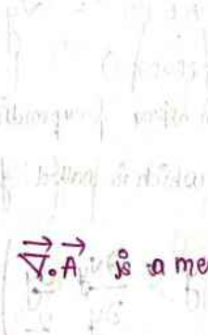
$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

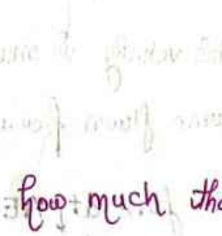
* Field whose divergence is always zero is called solenoidal fields.



+ve Divergence

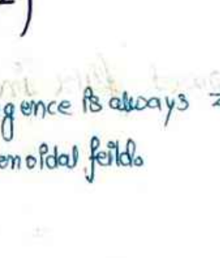


+ve Divergence.



zero divergence.

Divergence of Electric field is zero for a point charge.

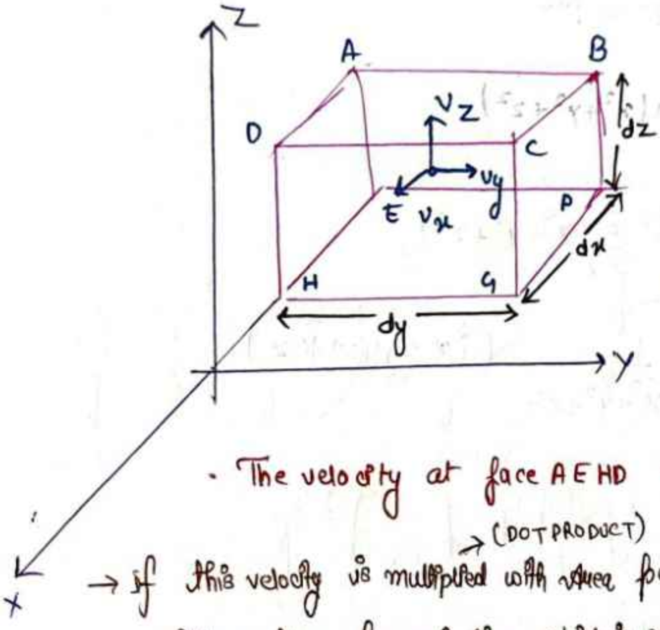


Also true for dipole

$\vec{\nabla}\cdot\vec{E} = 0$ Except at the location of point.

> Gauss Theorem : $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$, $\vec{\nabla}\cdot\vec{B} = 0$ (Always)

L-4 # Physical Significance of Divergence.



- $\frac{\partial v_x}{\partial x}, \frac{\partial v_y}{\partial y}, \frac{\partial v_z}{\partial z}$ = velocity gradient along x, y & z direction.

- Let us consider that at the centre of the parallelopiped, velocity is $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

- The velocity at face AEHD $= v_y - \frac{\partial v_y}{\partial y} \cdot \frac{dy}{2}$

→ If this velocity is multiplied (DOT PRODUCT) with area perpendicular to it, we will get volume flow per unit time, which is called flux.

- The inward flux at face AEHD $= \left(v_y - \frac{\partial v_y}{\partial y}\frac{dy}{2}\right) dx\,dz$

- The outward flux of at face BCGF $= \left(v_y + \frac{\partial v_y}{\partial y}\frac{dy}{2}\right) dx\,dz$

→ Net outward flux in Y direction $= \left(\cancel{v_y} + \frac{\partial v_y}{\partial y}\frac{dy}{2} - \left(\cancel{v_y} - \frac{\partial v_y}{\partial y}\frac{dy}{2}\right)\right) dx\,dz$

$$= \frac{\partial v_y}{\partial y} \cdot dx\,dy\,dz$$

- Similarly for X-Direction & Z-Direction

Net outward flux in X dirn $= \frac{\partial v_x}{\partial x} \cdot dx\,dy\,dz$

Net outward flux in Z dirn $= \frac{\partial v_z}{\partial z} \cdot dx\,dy\,dz$

- Net outward flux through the parallelopiped $= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) \cdot dx\,dy\,dz$

→ Net outward flux through the parallelopiped per unit volume $= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

- Net Outward flux per unit ~~time~~ volume $= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$

$= \vec{\nabla}.\vec{v}$ = Divergence of velocity

∴ Divergence of a vector field will give us the Net outward flux per unit volume

#L-12 Q 1) find the Divergence of $\vec{r}$.

Ans $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$; $div\,\vec{r} = \vec{\nabla}.\vec{r} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(\hat{i}x + \hat{j}y + \hat{k}z)$

$\therefore\ div\,\vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} \Rightarrow \boxed{div\,\vec{r} = 3}$

Q 2) $\vec{A} = x^2z\,\hat{i} - 2y^2z^2\,\hat{j} + x^2y^2z\,\hat{k}$, find div.$\vec{A}$ at point (1,2,1)

Ans $\vec{\nabla}.\vec{A} = div\,\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$

$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{\partial}{\partial x}(x^2z) + \frac{\partial}{\partial y}(-2y^2z^2) + \frac{\partial}{\partial z}(x^2y^2z)$

$\vec{\nabla}.\vec{A} = 2xz - 4yz^2 + x^2y^2$

$div\,\vec{A}\Big|_{(1,2,1)} = 2(1)(1) - 4(2)(1)^2 + (1)^2(2)^2 \rightarrow \left[div\,\vec{A}\Big|_{(1,2,1)} = -2\right]$

L-13

Q1) Show that $\vec{\nabla}.(r^n\vec{r}) = (n+3)\,r^n$

Ans $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$, $r^n = (x^2+y^2+z^2)^{n/2}$

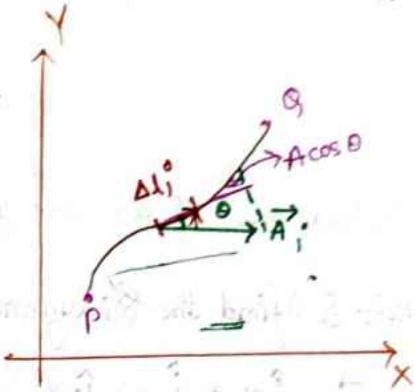
$\vec{\nabla}.(r^n\vec{r}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(r^n(\hat{i}x + \hat{j}y + \hat{k}z)\right) = \frac{\partial}{\partial x}(r^n.x) + \frac{\partial}{\partial y}(r^n y) + \frac{\partial}{\partial z}(r^n.z)$

$= \sum\frac{\partial}{\partial x}\left[(x^2+y^2+z^2)^{n/2}.x\right] = \sum (x^2+y^2+z^2)^{n/2}.1 + x.\frac{n}{2}(x^2+y^2+z^2)^{\frac{n-2}{2}}.2x$

$= 3(x^2+y^2+z^2)^{n/2} + n(x^2+y^2+z^2)^{\frac{n}{2}-1}.(x^2+y^2+z^2) = 3(x^2+y^2+z^2)^{n/2} + n(x^2+y^2+z^2)^{n/2}$

$= (n+3)\underbrace{(x^2+y^2+z^2)^{n/2}}_{r^n} \Longrightarrow \boxed{\vec{\nabla}.(r^n.\vec{r}) = (n+3)\,r^n}$

L-14 # Line Integral (Path Integral)



- In line integral some quantity related to the field is to be integrated between two given points in space. The curve b/w P & Q ie C may be open or closed.

* In line integral there is only one independent variable

→ $\int_C \phi\, d\vec{l}$, $\int_C \vec{A}\cdot d\vec{l}$, $\int_C \vec{A}\times d\vec{l}$ — ϕ → Scalar field, $\vec{A}$ → vector field.

Vector, Scalar, Vector

- for closed curve : $\oint_C$, for example $\int_C F\cdot dl$ = Work done , $\int_C E\cdot dl$ = Potential Diff^r

→ $\int_C \vec{A}\cdot d\vec{l} = \int_C A\cos\theta\, dl$ = The integral of the Tangential Component of $\vec{A}$ along C.

- If it is along a closed curve, Then it is also called circulation of $\vec{A}$ around C.

* $\int_C \vec{F}\cdot d\vec{l}$ = Path Dependent ; if F is frictional force

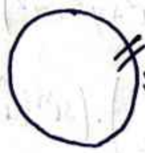
$\int \vec{F}\cdot d\vec{l}$ = Path independent ; If F is electrostatic force on a charge particle in an EF

∴ Nature of $\int \vec{F}\cdot d\vec{l}$ depends upon the nature of $\vec{F}$.

L-15 Surface Integral.

- Associated with scalar as well as with vector fields.

→ Simplest form: $\int_C \phi\, dS$ → SCALAR ; ϕ → SCALAR , eg



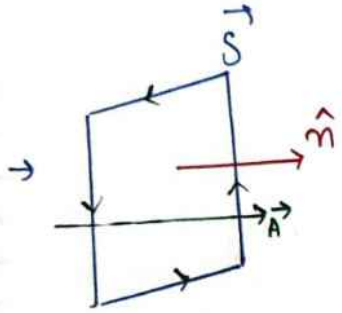
$\oint_S \rho(r)\cdot dS$ = Mass of the shell ; ρ → Mass/Area

Imp. to us → FLUX - $\int_S \vec{E}\cdot d\vec{S}$, $\int_S \vec{B}\cdot d\vec{S}$

∴ $\underbrace{\int_S \phi\, d\vec{S}}_{\text{vector}}$, $\boxed{\underbrace{\int_S \vec{A}\cdot d\vec{S}}_{\text{scalar}}}$, $\underbrace{\int_S \vec{A}\times d\vec{S}}_{\text{vector}}$; for closed surface : $\oint_S \vec{A}\cdot d\vec{S}$

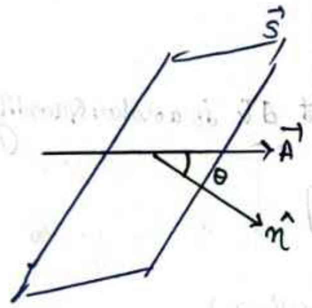
$\vec{dS} = \hat{n}\, dS$ = Vector Area Element

→ $\hat{n}$ is outward drawn normal for a closed area



• $\vec{A} \perp$ to the plane , $\vec{A} \parallel$ to $\vec{S} = \hat{n} S$

Here Surface Integral $= AS$



Here

Surface integral $= AS\cos\theta = \vec{A}\cdot\vec{S}$

$= A(S\cos\theta)$

$= A$ (component of Area whose plane is $\perp$ to A)

• for uneven shape, Surface integral $= \sum_{i=1}^{n} \vec{A}_i \cdot \Delta\vec{S}_i$

If $n \to \infty$, $\Delta S_i \to 0$ $\quad S\cdot I = \sum_{i=1}^{n} \vec{A}_i \cdot \Delta\vec{S}_i$

$\therefore$ Surface integral $= \int_S \vec{A}\cdot d\vec{s}$, if the surface is closed, Surface integral $= \oint_S \vec{A}\cdot d\vec{s}$.

L-16 Evaluation of Surface Integral. [JYADA KAAM KA NAHI HAI]

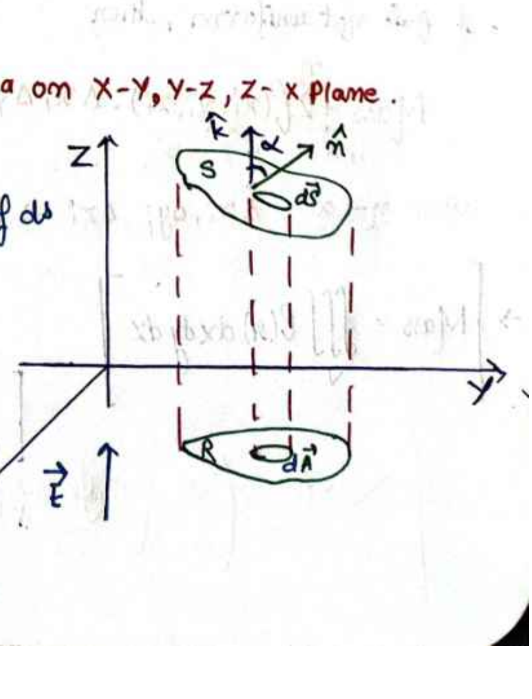
$\int_S \vec{A}\cdot d\vec{s}$, WE have to translate ds in terms of Co-ordinate differential of our choosen Co-ordinate System

eg- in case of sphere • $dS = a\sin^2\theta\, d\theta\, d\phi$

* for a general surface we take the projecⁿ of area on X-Y, Y-Z, Z-X plane.

$d\vec{s} = \hat{n}\, ds$; $\hat{n}$ = unit vector $\perp$ to the plane of ds

$d\vec{A}$ = Projection of $d\vec{s}$ on XY plane



* from fig ; $dA = ds\cos\alpha$; $ds = \dfrac{dA}{\cos\alpha}$

$ds = \dfrac{dA}{\hat{n}\cdot\hat{k}}$

• Let S is given by $f(x,y,z) = 0$

$\cdot \dfrac{\vec{\nabla}.f}{|\vec{\nabla} f|}$ = unit vector $\perp^r$ to S

$\therefore dS = \dfrac{dA\,|\vec{\nabla}.f|}{\vec{\nabla}.f.\hat{k}}$

$$\hat{k}.\vec{\nabla}f = \left(\hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}\right).\hat{k} = \frac{\partial f}{\partial z}$$

$$\rightarrow \boxed{dS = \frac{dA\,|\vec{\nabla}f|}{\partial f/\partial z}}$$

1.14 Volume Integral : It is simpler because the volume element $d\tau$ is a scalar Quantity.

$d\tau = dx\,dy\,dz$ [In Cartesian Co-ordinate]

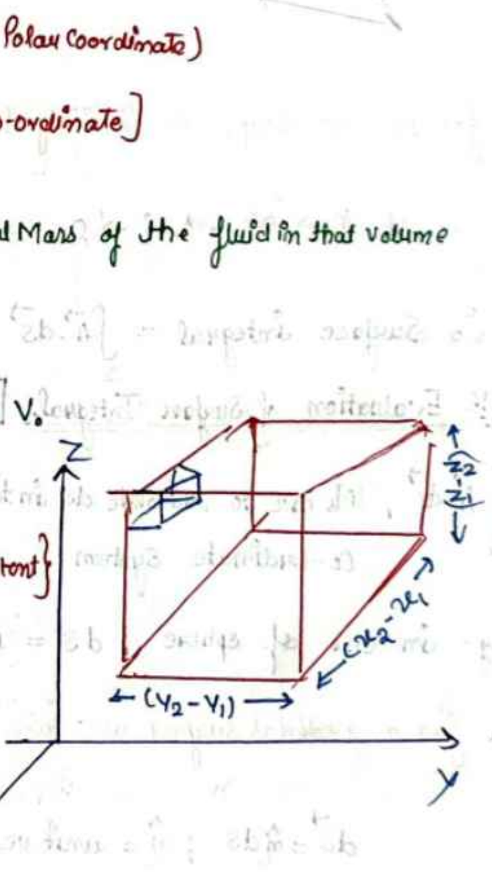
$d\tau = r^2 dr\, \sin\theta\, d\phi$ [In spherical Polar coordinate)

$d\tau = \rho\, d\rho\, d\phi\, dz$ [In cylindrical Co-ordinate]

$\rightarrow \overbrace{\int_V \phi\, d\tau}^{\text{SCALAR}}\ ,\ \overbrace{\int_V \vec{A}.d\tau}^{\text{VECTOR}}$

eg, ① $\int_V \underset{\text{Density}}{\rho(r)}\, d\tau$ = Total Mass of the fluid in that volume

② $\int_V \rho(r)\vec{v}\, d\tau$ = Total linear Momentum of fluid in volume V.



* Mass of cuboid = $\rho\,(x_2-x_1)(y_2-y_1)(z_2-z_1)$ {ρ = constant}

· If ρ is not uniform, then

$$\text{Mass} = \sum_{i=1}^{n} \rho(x_i, y_i, z_i).\Delta x_i \Delta y_i \Delta z_i$$

when $n \rightarrow \infty$, $\Delta x_i, \Delta y_i, \Delta z_i \rightarrow 0$, then

$$\rightarrow \left[\text{Mass} = \iiint_V \rho(r)\, dx\, dy\, dz\right]$$

$$\left[\int_V \vec{A}.d\tau = \hat{i}\int_V A_x d\tau + \hat{j}\int_V A_y d\tau + k\int_V A_z d\tau\right]$$

L:18

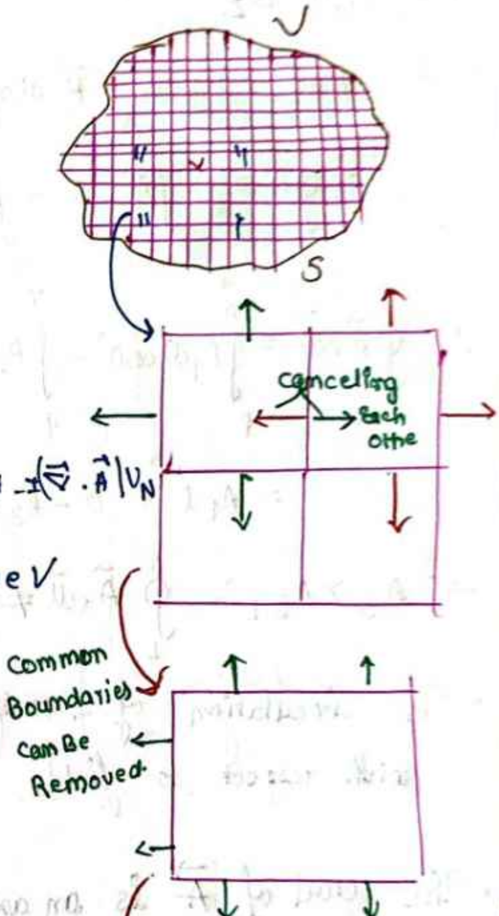
Gauss's Divergence Theorem

→ This theorem relates the total flux of a vector field out of a closed surface to the Integral of the divergence of the vector field over the enclosed volume V.

• If a vector $\vec{A}$ & its first derivative are continuous in a region then Gauss' Divergence theorem states that:

$$\oint_S \vec{A}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{A})\,dV$$

" The surface integral over a surface S of any vector $\vec{A}$ is equal to the volume integral of Divergence of $\vec{A}$ over the volume enclosed by S".



* PROOF : Consider a volume V in which a vector field is continuous & Differentiable.

→ Let us divide V into ~~continuously~~ arbitrarily small N volumes ΔV_i

• $\vec{\nabla}\cdot\vec{A}$ = Net Outward flow of A per unit volume

$(\vec{\nabla}\cdot\vec{A})\Delta V_i$ = Net outward of A in $\Delta V_i = (\vec{\nabla}\cdot\vec{A})\Delta V_1 + \dots + (\vec{\nabla}\cdot\vec{A})V_N$

→ $\sum_{i=1}^{N}(\vec{\nabla}\cdot\vec{A})\Delta V_i$ = Net outward of A through volume V

∴ $\sum_{i=1}^{N}(\vec{\nabla}\cdot\vec{A})\Delta V_i$ = The outward flow of A through the surface.

→ if $N\to\infty$ then $\Delta V_i \to 0$, then LHS will be

$$\boxed{\iiint_V (\vec{\nabla}\cdot\vec{A})\,dA = \oint_S \vec{A}\cdot d\vec{S}}$$

→ The RHS Becomes the total outflow to the exterior while LHS is the sum of the outflows of Differential Elements.

• Physical meaning of Divergence : $\vec{\nabla}\cdot\vec{A} = \int_{\Delta S}\vec{A}\cdot d\vec{S} \,/\, \Delta V$

$$(\vec{\nabla}\cdot\vec{A})\Delta V = \int_{\Delta S}\vec{A}\cdot d\vec{S}$$

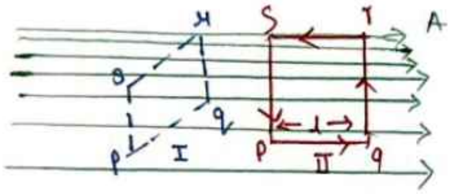
• Hence, The sum of all the outflow ~~of~~ in the Vol V will just be the sum of through the exterior boundary. In the limit of ∞ subdivis^n these sums become integral.

L-19

Physical Significance of Curl.

- Consider a vector field $\vec{A}$ for which $\oint \vec{A}\cdot\vec{dl} \neq 0$

- The line integral of $\vec{A}$ along pqrs in situation I is



$$\oint_L A\cdot dl = \int_p^q \vec{A_1}\cdot\vec{dl} + \int_q^r \vec{A_2}\cdot\vec{dl} + \int_r^s \vec{A_3}\cdot\vec{dl} + \int_s^p \vec{A_4}\cdot\vec{dl}$$

- Lameller field

$$\oint \vec{E}\cdot\vec{dl} = 0$$

But for magnetic field

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{in}$$

Non lamellar vector ~~field~~ field.

zero circulation: $\left[\oint_I \vec{A}\cdot dl = 0\right] \longleftarrow$ In situation I, Angle B/W A's & dl is always 90°

- The line integral of $\vec{A}$ along PQRS in situation II is -

$$\oint_L \vec{A}\cdot dl = \int_p^q \vec{A_1}\cdot\vec{dl} + \int_q^r \vec{A_2}\cdot\vec{dl} + \int_r^s \vec{A_3}\cdot\vec{dl} + \int_s^p \vec{A_4}\cdot\vec{dl}$$

$$\therefore \oint_L \vec{A}\cdot\vec{dl} = \int_p^q A_1 dl \cos 0^\circ + \int_q^r A_2 dl \cos 90^\circ + \int_r^s A_3 dl \cos 180^\circ + \int_S^P A_4 dl \cos 270^\circ$$

$$= A_1 l + 0 - A_3 l + 0 = (A_1 - A_3)\, l \longleftarrow \text{Max}^m \text{ circulat}^n \text{ cond}^n$$

$\rightarrow A_3 > A_1 \therefore \oint_L \vec{A}\cdot\vec{dl} \neq 0$

- The circulation of a vector depends upon the orientation of the loop with respect to field.

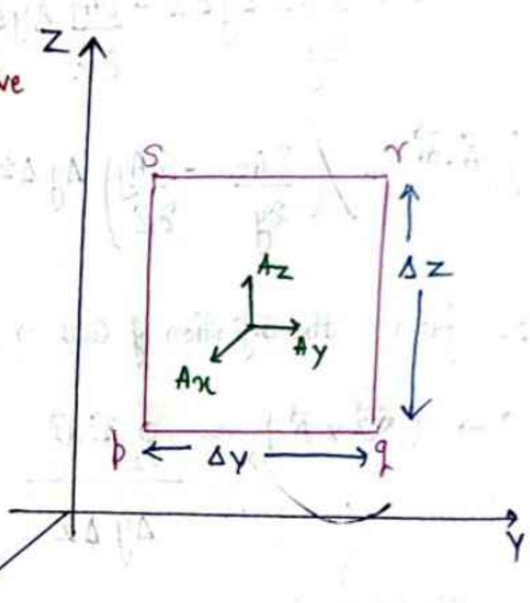
- The curl of $\vec{A}$ is an axial vector whose magnitude is the maximum circulation of A per unit area, as the area tends to zero & whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

$$\left[\vec{\nabla}\times\vec{A} = \left(\lim_{\Delta S\to 0} \frac{\oint \vec{A}\cdot\vec{dl}}{\Delta S}\right)_{MAX^M}\cdot\hat{n}\right]$$

← Mathematical representat^n of Physical significance of Curl.

L-20

Curl in Cartesian Co-ordinate



- Consider a vector funcⁿ $\vec{A}$ s.t $\vec{A}$ & its first derivative are continuous in a given region of space
- At the centre of pqrs vector $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

→ Assume that there is some positive gradient along positive direction of X, Y & Z axes.

- To do line Integral along pqrs, we need the value of pq, qr, rs & sp.
- Value of A_y at pq (midpoint)
- Along PQ, we need not to find the value of A_x & A_z because these are normal to pq ($\vec{dl}$) so their line integral vanish.

$$A_y' = A_y - \frac{\partial A_y}{\partial z}\left(\frac{\Delta z}{2}\right)$$

- Along qr ; $A_z' = A_z + \frac{\partial A_z}{\partial y}\left(\frac{\Delta y}{2}\right)$
- Along rs ; $A_y'' = A_y + \frac{\partial A_y}{\partial z}\left(\frac{\Delta z}{2}\right)$
- Along sp : $A_z'' = A_z - \frac{\partial A_z}{\partial y}\left(\frac{\Delta y}{2}\right)$

$$\oint_L \vec{A}\cdot\vec{dl} = \int_p^q \vec{A_y}'\cdot dl + \int_q^r \vec{A_z}'\cdot\vec{dl} + \int_r^s \vec{A_y}''\cdot\vec{dl} + \int_s^p \vec{A_z}''\cdot\vec{dl}$$

$$= \left(A_y - \frac{\partial A_y}{\partial z}\cdot\frac{\Delta z}{2}\right)\Delta y + \left(A_z + \frac{\partial A_z}{\partial y}\frac{\Delta y}{2}\right)\Delta z - \left(A_y + \frac{\partial A_y}{\partial z}\cdot\frac{\Delta z}{2}\right)\Delta y - \left(A_z - \frac{\partial A_z}{\partial y}\frac{\Delta y}{2}\right)\Delta z$$

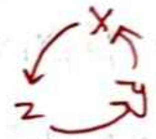
$$\oint_L \vec{A}\cdot\vec{dl} = \cancel{A_y\Delta y} - \frac{\partial A_y}{\partial z}\frac{\Delta z}{2}\cdot\Delta y + \cancel{A_z}\Delta z + \frac{\partial A_z}{\partial y}\cdot\frac{\Delta y}{2}\cdot\Delta z - \cancel{A_y}\Delta y - \frac{\partial A_y}{\partial z}\frac{\Delta z}{2}\Delta y - \cancel{A_z} + \frac{\partial A_z}{\partial y}\frac{\Delta y}{2}\cdot\Delta z$$

$$\oint_L \vec{A}\cdot d\vec{l} = \frac{\partial A_z}{\partial y}\Delta y\Delta z - \frac{\partial A_y}{\partial z}\Delta y\Delta z$$

$$\oint_L \vec{A}\cdot d\vec{l} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\Delta y\,\Delta z$$

$\therefore$ from the definition of Curl $\rightarrow$ Curl is line Integral per unit area

$$\rightarrow (\vec{\nabla}\times\vec{A})_x = \frac{\oint_L \vec{A}\cdot d\vec{l}}{\Delta y\,\Delta z} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i}$$



Similarly

$$(\vec{\nabla}\times\vec{A})_y = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$$

$$(\vec{\nabla}\times\vec{A})_z = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\therefore \vec{\nabla}\times\vec{A} = (\vec{\nabla}\times\vec{A})_x\hat{i} + (\vec{\nabla}\times\vec{A})_y\hat{j} + (\vec{\nabla}\times\vec{A})_z\hat{k}$$

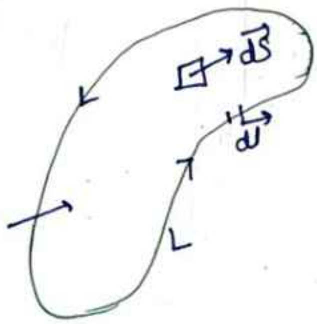
$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

$$\therefore \boxed{\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}}$$

L-21
Stoke's Theorem : This theorem relates an integral over an open surface to the line Integral around the curve Bounding the surface.

- Stoke's theorem states that the circulation (line Integral around a closed path) of a Vector field $\vec{A}$ around a path L is equal to the surface integral of the curl $\vec{A}$ over the open surface S Bounded By L provided $\vec{A}$ & $\vec{\nabla}\times\vec{A}$ are continuous on $\vec{S}$.

* $\oint_L \vec{A}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{A})\,d\vec{s}$

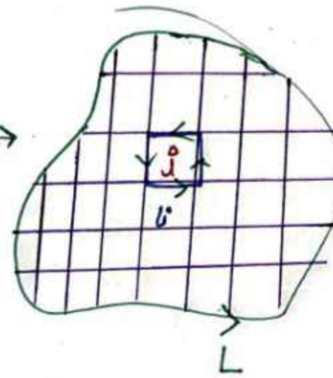


- for a closed surface there is not any perimeter $\therefore \int_S (\vec{\nabla}\times\vec{A})d\vec{s} = 0$

→ The surface over which stoke's theorem is applied must have two surfaces. It can't be applied to Moebius strip Because it only has one surface & Direction for outward Normal is not defined.

L-22
Proof of Stoke's Theorem

- Let us assume that Surface S is made up of a large no. (countable) of small elements



- Consider One Such element . Δs_i

$\vec{\nabla}\times\vec{A}$ = line Integral per unit ~~...~~ Area.

$(\vec{\nabla}\times\vec{A})\cdot\Delta\vec{s}_i$ = line integral about Δs_i

$(\vec{\nabla}\times\vec{A})\cdot\Delta\vec{s}_i = \oint_{l_i} \vec{A}\cdot\vec{dl}$

for the whole surface : $\sum(\vec{\nabla}\times\vec{A})\cdot\Delta\vec{s}_i = \oint_{l_1}\vec{A}\cdot\vec{dl} + \oint_{l_2}\vec{A}\cdot\vec{dl} + \dots + \oint_{l_n}\vec{A}\cdot\vec{dl}$

if $N\to\infty$ then $\Delta s_i \to 0$

$$\left[\int_S (\vec{\nabla}\times\vec{A})\,d\vec{s} = \oint_L \vec{A}\cdot\vec{dl}\right]$$

The curl of Electric field.

- We know that any stationary field of central forces is conservative i.e the work done by the forces of this field is independent of the path.

$(W_{12})_{\text{conservative field}}$ = same for any path

$W_{12} = -W_{21} \Rightarrow [W_{12} + W_{21} = 0] \Rightarrow$ work done over a closed path for conservative field is zero

$\oint_L \vec{f}\cdot\vec{dl} = 0$; In case of stationary Electric field

$\oint_L q\vec{E}\cdot\vec{dl} = 0$; if $q = 1C$ or unit

$\left[\oint_L \vec{E}\cdot\vec{dl} = 0\right]$

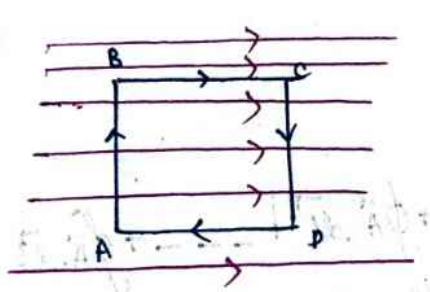
→ Applying stoke's Theorem :- $\oint_L \vec{E}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{E})\cdot\vec{ds} = 0$

$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = 0}$ - This eqn implies that Electric vector field has special characteristics. it is not like other vector functions.

* Any vector field which satisfy the above condn is called Irrotational field.

- from above condn we can draw some conclusions -

1) The field lines of Electrostatic field cannot be closed. They start from positive charge & end at negative charge or Infinity.

2) 

$\oint_L \vec{E}\cdot\vec{dl} = \cancel{\int_A^B \vec{E}_1\cdot\vec{dl}}^{\,0} + \int_B^C \vec{E}_2\cdot\vec{dl} + \cancel{\int_C^D \vec{E}_3\cdot\vec{dl}}^{\,0} + \int_D^A \vec{E}_4\cdot\vec{dl}$

$E_2 \neq E_4 \Rightarrow \oint_L \vec{E}\cdot\vec{dl} \neq 0$

$\therefore$ Given Representation doesn't Represent EF lines.

→ Any line can't represent electric field.

3) from Vector calculus, we know that curl of gradient of any scalar funcn is zero

- $\vec{\nabla}\times\vec{E} = 0 \Longrightarrow \vec{E}$ can be expressed as gradient of some scalar funcn i.e electric Potential.

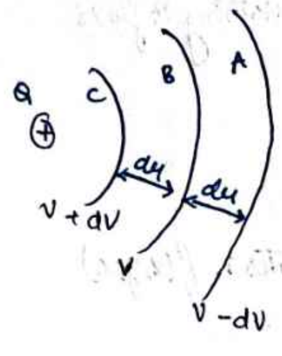
1.24 * Relationship B/W Electric field & Electric Potential.

· We know that $\vec{\nabla}\times\vec{E}=0$ & also curl of a scalar funcⁿ is zero

$$\therefore \vec{\nabla}\times\vec{E} = \vec{\nabla}\times(\vec{\nabla}V) = 0 \quad \text{OR} \quad \vec{\nabla}\times(-\vec{\nabla}V) = 0$$

Thus $\boxed{\vec{E} = -\vec{\nabla}.V}$

BY EXTERNAL AGENT



· dV = amount of work done in moving a unit positive charge (+1C) from one point to another without acceleration.

$$\rightarrow dV = \vec{F}.\vec{dr} = -q\vec{E}\cdot\vec{dr} \quad ; \quad q = +1C$$

$$dV = -\vec{E}.\vec{dr} \quad \text{---(1)}$$

$$\int dV = V = -\int_{D}^{A}\vec{E}.\vec{dr} \quad ; \quad V = -\int\vec{E}.\vec{dl}$$

* dV = Negative of the work done by the Electrostatic force in bringing a unit +ve charge from one point to another point without acceleration

Also $$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$dV = \left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)\cdot(\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$dV = (\vec{\nabla}V).\vec{dr} \quad \text{---(2)}$$

* from (1) & (2)

$$-\vec{E}.\vec{dr} = \vec{\nabla}V.\vec{dr}$$

$$\left[\vec{E} = -\vec{\nabla}V\right]$$

· Electric field $\vec{E}$ is always directed in direction of decreasing potential.

* The direction of gradient is along the direction in which the potential, thus the negative sign is introduced in formula.

L-25 # Gauss' law in Electrostatics.

- The flux of Electric field through a closed surface is equal to the algebraic Sum of the charges enclosed by this surface, divided by ε_0

$$\left[\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0} = \phi\right] \leftarrow \text{Integral form}$$

* A charge lying outside the flux contributes Nothing to flux.

- ϕ doesn't depend upon size of surface.

$$\oint \vec{E}\cdot d\vec{s} = \int_V \rho\, d\tau \Big/ \varepsilon_0 \quad -① \qquad ; \ \rho \rightarrow \text{function of Co-ordinates i.e } \rho(x,y,z)$$

$V \rightarrow$ Volume of charged Body lying inside the surface.

* Using Divergence Theorem

$$\oint \vec{E}\cdot d\vec{s} = \int_V (\vec{\nabla}\cdot\vec{E})\, d\tau \quad -②$$

from ① & ②

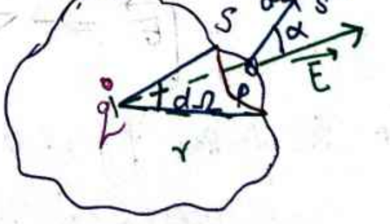
$$\int_V (\vec{\nabla}\cdot\vec{E})\, d\tau = \frac{\int_V \rho\, d\tau}{\varepsilon_0}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}} \rightarrow \text{Differential form of Gauss' law}$$

L-26
Proof of Gauss' Theorem

- Consider a point charge q enclosed in a closed surface S. Take a point P on the surface

→ Consider an area element at P such that the value of $\vec{E}$ at every point of which is same & direction of area element is uniquely defined.



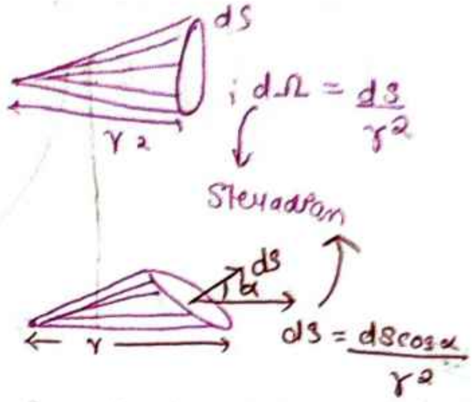
- from the Def^n of Electric flux; The flux associated with $d\vec{s}$:

$$d\phi = \vec{E}\cdot d\vec{s} = E\, ds \cos\alpha$$

- let Point P is at a distance r from charge q.

$\cdot\ E = \dfrac{q}{4\pi\varepsilon_0 r^2} \Rightarrow d\phi = \dfrac{q}{4\pi\varepsilon_0 r^2}\, ds\cos\alpha$; $\dfrac{ds\cos\alpha}{r^2} \rightarrow$ Solid angle $\rightarrow d\Omega$

$$d\phi = \frac{q}{4\pi\varepsilon_0}\, d\Omega$$



· Total flux associated with S is

$$\phi = \oint_S \frac{q}{4\pi\varepsilon_0}\, d\Omega = \frac{q}{4\pi\varepsilon_0}\oint_S d\Omega$$

$$\phi = \frac{q}{\cancel{4\pi}\varepsilon_0}\cdot \cancel{4\pi} \Rightarrow \left[\phi = \frac{q}{\varepsilon_0}\right]$$

* Because any close surface makes an angle of 4π steradian.

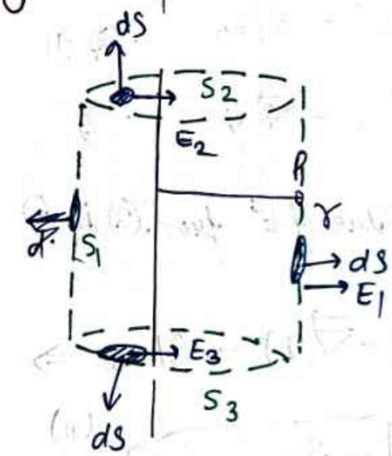
$\rightarrow\ \phi = \oint_S \vec{E}\cdot\vec{ds} = q/\varepsilon_0$

$\vec{E}$ due to a line charge

* Consider a uniformly charged wire of Infinite length

$\rightarrow$ We have to find $\vec{E}$ at a point P which is at a distance r from the wire

· Consider a Gaussian Surface in the form of cylinder of Radius r & length l.



· Applying Gauss Theorem $\oint_{\text{CYLINDER}} \vec{E}\cdot\vec{ds} = q_{in}/\varepsilon_0$

$$\int_{S_1}\vec{E}_1\cdot\vec{ds} + \int_{S_2}\vec{E}_2\cdot\vec{ds} + \int_{S_3}\vec{E}_3\cdot\vec{ds} = \frac{\lambda l}{\varepsilon_0}$$

; $\lambda \rightarrow$ Charge per unit length $\rightarrow$ linear charge Density

$$\oint_{S_1} E_1\, ds\cos 0 + \cancelto{0}{\oint_{S_2} E_2\cos 90\, ds} + \cancelto{0}{\oint_{S_3} E_3\, ds\cos 90} = \frac{\lambda l}{\varepsilon_0}$$

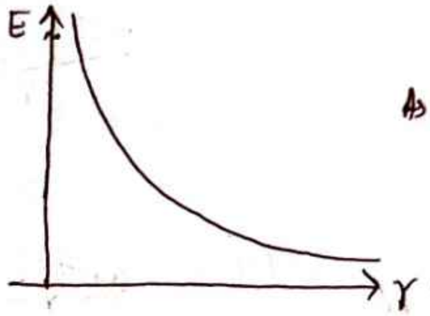
; Assumption $\lambda > 0$

$$\oint_{S_1} E_1\cdot ds = \frac{\lambda l}{\varepsilon_0} \Rightarrow E_1\oint_{S_1} ds = \frac{\lambda l}{\varepsilon_0}$$

$\Rightarrow$ E_1 is same on the surface of cylinder & Also the Resultant E.F ie $E_1 = E$

$\cdot\ E \cdot 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}}$; $\vec{E}$ is radially outwards at every point ; $\hat{r}$ → unit vector pointing radially outwards.

$\boxed{E \propto \frac{1}{r}}$



As $r \to \infty$, $E \to 0$; if the wire is surrounded by any medium, then

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_m r}\hat{r}}$$

L-28

Laplace & Poisson's Equation

- In cases of conductor if ρ is not known in advance, since charge is moving

* We have two scalar functions associated with $\vec{E}$, one is the potential $V(x,y,z)$ & second is divergence of $\vec{E}$ $(\vec{\nabla}\cdot\vec{E})$

Acc. to Gauss Theorem (in free space)

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad -①$$

$$\vec{E} = -\vec{\nabla}V \quad -②$$

- Putting value of $\vec{E}$ from ② in ①

$$\vec{\nabla}(-\vec{\nabla}V) = \rho/\varepsilon_0 \Rightarrow \vec{\nabla}\cdot\vec{\nabla}V = -\rho/\varepsilon_0$$

$$\boxed{\nabla^2 V = -\rho/\varepsilon_0} \quad -(III) \leftarrow \text{Poisson's Eq}^n \text{ for free space}$$

- ∇^2 is called Laplacian operator

∇^2 → divergence of gradient of ---

* In terms of Cartesian co-ordinates ⇒ $$\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho}{\varepsilon_0}\right]$$ Poisson's Eq^n in Cartesian Coordinates.

* Whenever $\rho = 0$, $\therefore$ Eq^n Reduce to $\left[\nabla^2 V = 0\right] \rightarrow$ Laplace Equation

for In-Homogeneous Medium

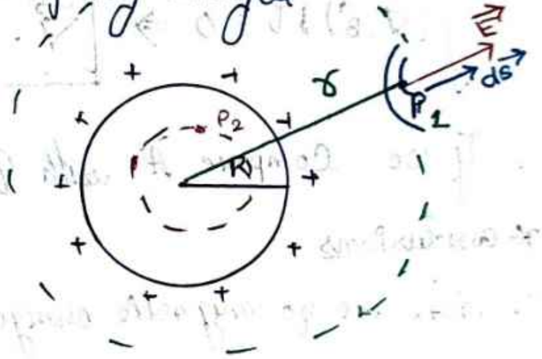
· Eq^n (iii) can be written as : $\vec{\nabla} \cdot (-\varepsilon \vec{\nabla} V) = \rho$

→ for Homogeneous Medium, $\varepsilon \rightarrow$ constant $\therefore$

$$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon}} \quad , \quad \boxed{\nabla^2 V = 0}$$

· Any func^n which satisfy Laplace Eq^n is called Harmonic func^n.

L-29' # $\vec{E}$ due to a Spherical Shell. → Thin & Uniformly Charged



· R = Radius , q = charge on shell

1. When point is lying outside shell.

· Applying Gauss' theorem

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\varepsilon_0} \Rightarrow \oint_S E\, ds \cos\theta = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E \oint_S ds = \frac{q}{\varepsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow \boxed{E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}} \rightarrow \boxed{E \propto \frac{1}{r^2}}$$

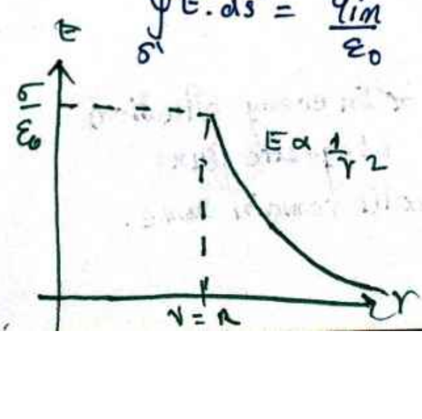
2. When point is on the Surface of shell $\Rightarrow r = R$

$$\therefore \boxed{E_{surface} = \frac{q}{4\pi\varepsilon_0 R^2}} \longrightarrow E_{surface} = \frac{1}{\varepsilon_0} \frac{q}{4\pi R^2} \rightarrow \boxed{E_{surface} = \frac{\sigma}{\varepsilon_0}}$$

· $\sigma = q/4\pi r^2$ = Charge/Area = Surface charge Density

3. When point lies inside the shell.

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\varepsilon_0}$$, as there is not any charge lying inside Gaussian Surface i.e $q_{in} = 0 \Rightarrow \boxed{E_{in} = 0}$



L-30 Divergence of $\vec{B}$.

- Acc. to Gauss' law of Magnetostatics, the flux of $\vec{B}$ associated with any closed surface is always Zero. i.e $\oint_S \vec{B}.\vec{da} = 0$ —(1)

→ Using Gauss Divergence Theorem

$$\oint_S \vec{B}.\vec{ds} = \int_V (\vec{\nabla}.\vec{B})\, d\tau \quad \text{—(11)}$$

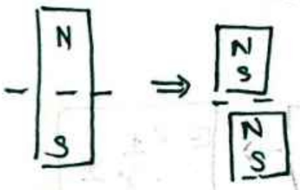
- from (1) & (11)

$$\int_V (\vec{\nabla}.\vec{B})\, d\tau = 0 \Rightarrow \boxed{\vec{\nabla}.\vec{B} = 0}$$

- If we Compare A with Gauss' Law of electrostatics i.e $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$

*** Conclusions**

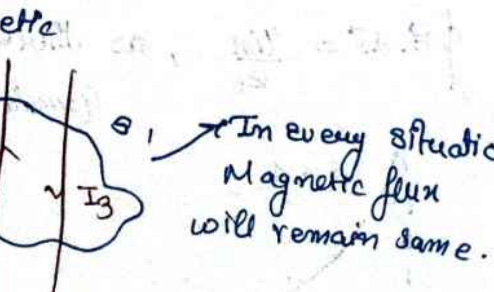
1. There are no magnetic charges i.e independent Magnetic Monopoles doesn't exist.



2. The lines of Vector $\vec{B}$ have neither begining nor ending, they are continuous.

3. The no. of field lines of $\vec{B}$ emerging from any volume is equal to the no. of field lines entering volume.

4. The flux of $\vec{B}$, Through a close surface Doesn't depends upon the shape & location of the closed Surface. This is reffered to as Law of conservation of Magnetic flux.

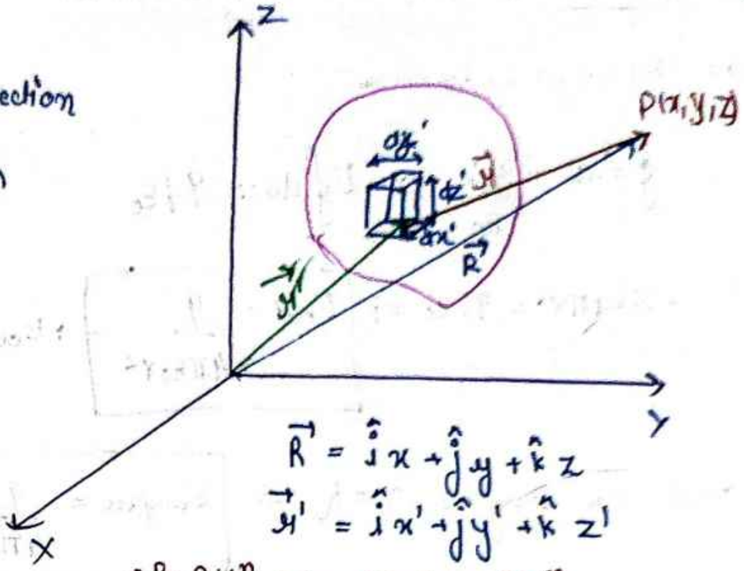


- Magnetic field is not a conservative field.

→ In every situation Magnetic flux will remain same.

[L-31]

Divergence of $\vec{B}$



- Consider a conductor of arbitrary cross section containing current I through it. Then according to Biot Savart's law

- $$\vec{B}(r) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(r') \times \hat{r}}{r^2}\, d\tau' \quad -(i)$$

- $\vec{B}$ is a func^n of (x, y, z)
- $\vec{J}$ is a func^n of (x', y', z')

; $d\tau' = dx'dy'dz'$

$\vec{R} = \hat{i}x + \hat{j}y + \hat{k}z$

$\vec{r}' = \hat{i}x' + \hat{j}y' + \hat{k}z'$

- By Add^n Δ law of vector Add^n

$\vec{R} = \vec{r}' + \vec{r} \Rightarrow \left[\vec{r} = \vec{R} - \vec{r}'\right]$

$\therefore \vec{r} = (x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}$

- Taking divergence of eq^n ①:

$$\vec{\nabla}\cdot\vec{B} = \frac{\mu_0}{4\pi}\int \vec{\nabla}\cdot\left(\vec{J} \times \frac{\hat{r}}{r^2}\right) d\tau'$$

(x', y', z') (x, y, z)

; The integration is over the primed co-ordinates & the divergence is to be taken w.r.t non primed Coordinates.

* Using $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

$$\therefore \vec{\nabla}\cdot\left(\vec{J}\times\frac{\hat{r}}{r^2}\right) = \frac{\hat{r}}{r^2}\cdot(\vec{\nabla}\times\vec{J}) - \vec{J}\cdot\left(\vec{\nabla}\times\frac{\hat{r}}{r^2}\right)$$

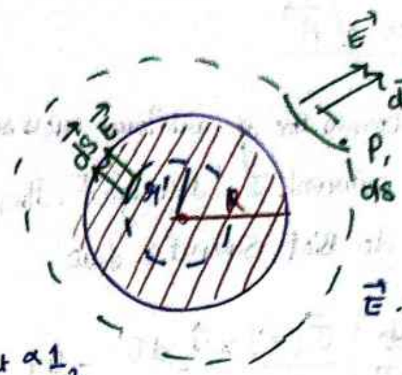
- $\vec{\nabla}\times\vec{J} = \vec{0}$, $\vec{\nabla}\times\frac{\hat{r}}{r^2} = \vec{0}$, using $\vec{\nabla}\times r^n\hat{r} = 0$

$$\therefore \vec{\nabla}\cdot(\vec{J}\times\hat{r}/r^2) = 0;$$

The above Results imply that

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$\vec{E}$ due to uniformly Charged Sphere.



- q = total charge in the sphere

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\oint ds = q/\varepsilon_0$$

$\vec{E}\cdot\vec{ds} = E\,ds\cos 0 = E\,ds$

- $E \cdot 4\pi r^2 = q/\varepsilon_0 \Rightarrow \boxed{E_{out} = \dfrac{q}{4\pi\varepsilon_0 r^2}} \rightarrow E_{out} \propto \dfrac{1}{r^2}$

- At the Surface, $r = R \Rightarrow \boxed{E_{surface} = \dfrac{q}{4\pi\varepsilon_0 R^2}} \rightarrow$ Max^m

* If the point is lying inside the sphere then:

$$\oint \vec{E}_{in}\cdot\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

, $\frac{4}{3}\pi R^3$ Volume contains q charge

unit volume contains $\dfrac{q}{\frac{4}{3}\pi R^3}$

$$\oint E_{in}\,ds\cos 0 = \frac{q r'^3}{R^3\varepsilon_0}$$

$\frac{4}{3}\pi r_1'^3$ Volume contains $\dfrac{\frac{4}{3}\pi r'^3 q}{\frac{4}{3}\pi R^3} = \dfrac{q r_1'^3}{R^3}$

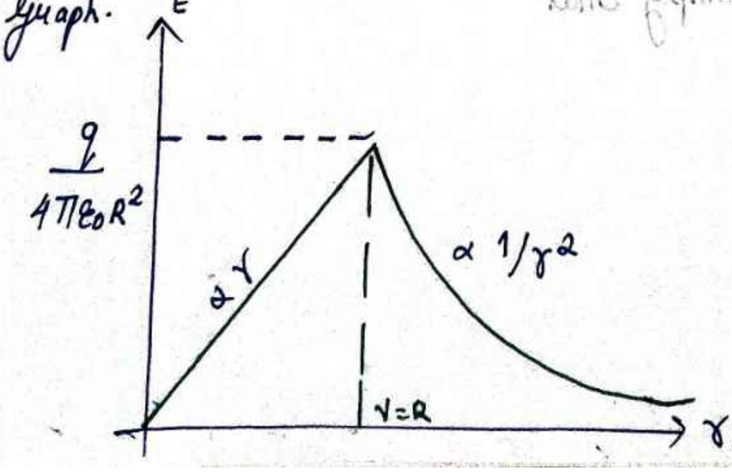
$$E_{in}\oint ds = \frac{q r'^3}{R^3\varepsilon_0}$$

$$E_{in}\cdot 4\pi r^2 = \frac{q r'^3}{R^3\varepsilon_0} \rightarrow \boxed{E_{in} = \frac{q r'}{4\pi R^3\varepsilon_0}} \rightarrow E_{in} \propto r$$

$$E_{in} = \frac{q r'}{3\times\frac{4}{3}\pi R^3\varepsilon_0} \Rightarrow \boxed{E_{in} = \frac{\rho r'}{3\varepsilon_0}}$$

; $\rho = \dfrac{q}{\frac{4}{3}\pi R^3}$

E v/s r graph.

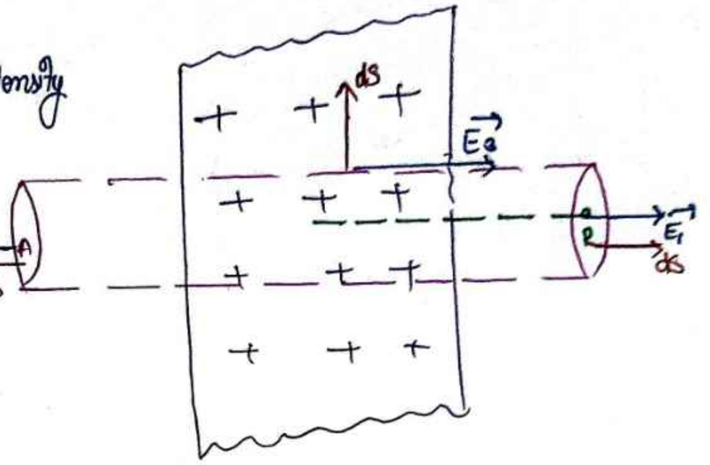


L-83 # $\vec{E}$ due to thin uniform sheet of Infinite length

- σ = Surface charge Density

→ We have to calculate electric field intensity at a distance r from the sheet.

- Consider a cylinderical gaussian surface.
The length of cylinderical surface is $2r$
- Point P is at one of its top.
- By Gauss' Theorem, Total flux = $\frac{q_{in}}{\varepsilon_0}$



→ flux through top + flux through Bottom + flux through curved surface $= \frac{q_{in}}{\varepsilon_0}$

$$\int \vec{E_1}\cdot\vec{ds} + \int \vec{E_2}\cdot\vec{ds} + \int \vec{E_3}\cdot\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

$$\int E_1 ds + \int E_2 ds + \cancel{\int E_3 ds \cos 90}^{\,0} = \frac{q_{in}}{\varepsilon_0}$$

$$E_1 A + E_2 A = \frac{q_{in}}{\varepsilon_0}$$

Here $E_1 = E_2 = E$

$$\therefore 2EA = \frac{q_{in}}{\varepsilon_0}$$

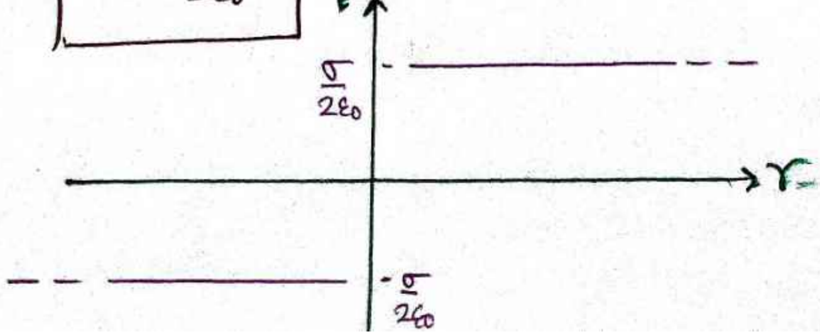
; If $\sigma > 0$, then E will be directed outwards

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

If $\sigma < 0$, then E will be directed inwards.

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

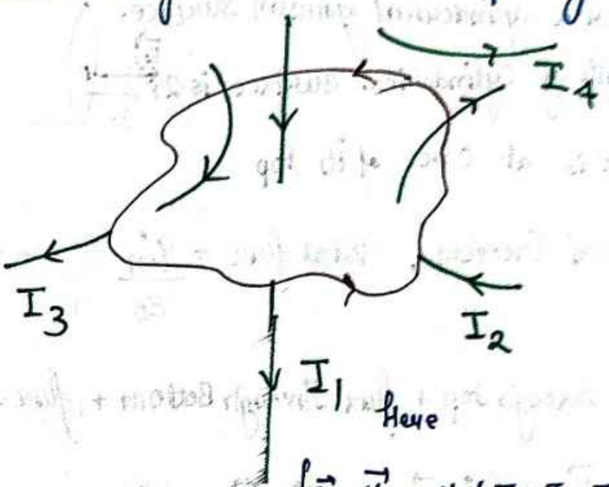
; E v/s r, graph.



1.34. Curl of $\vec{B}$ $(\vec{\nabla}\times\vec{B})$

- Ampere's circuital law states that for the magnetic field of a direct current in a vaccumm, the line integral of vector $\vec{B}$ around an arbitrary contour C is equal to the product of μ_0 by the algebraic sum of the currents enveloped by the contour C i.e

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I \quad , \quad I = \sum_{i=1}^{k} I_i$$



Here

$$\oint \vec{B}\cdot\vec{dl} = \mu_0(I_2 - I_1 - I_3)$$

- If the current is distributed over the volume, then

$$I = \int \vec{J}\cdot\vec{ds}$$

Here $\vec{J}$ is current density at the point where ds is taken

$$\therefore \oint_C \vec{B}\cdot\vec{dl} = \mu_0\int_S \vec{J}\cdot\vec{ds} \quad -(1)$$

- By using Stokes Theorem; $\oint_C \vec{B}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{B})\cdot\vec{ds} \quad -(11)$

- from (1) & (2)

$$\int_S (\vec{\nabla}\times\vec{B})\, ds = \mu_0\int_S \vec{J}\cdot\vec{ds}$$

1) Diff^r wrt x

2) Diff^r wrt y

$$\boxed{(\vec{\nabla}\times\vec{B}) = \mu_0\vec{J}}$$

Ampere circuital form law

Work Done over a closed Path is not zero.

; $\left[\vec{\nabla}\times\vec{E} = 0\right]$ Work Done over a closed Path is zero

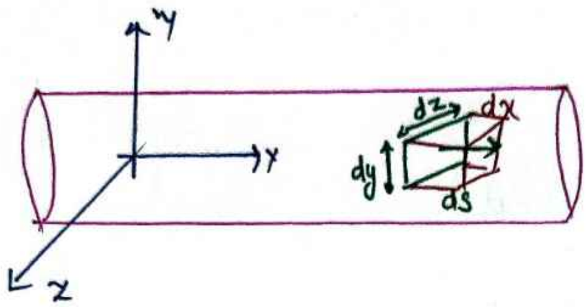
$\therefore \vec{B}$ can't be written as negative gradient of some scalar func^n.

- $\vec{B}$ is not a potential field but it is a solenoidal field.

↳ a.k.a Vortex field

L-35 # The Continuity Equation

• $dS = dy\,dz$, $\vec{v}$ = velocity of flow of +ve charge



• The volume current density J is defined as the amount of charge passing per unit ~~time~~ Area per unit time through a surface element held normal to the flow of positive charge

$$\therefore J = \frac{dq}{dt\,dy\,dz} \quad , \quad v = \frac{dx}{dt} \rightarrow dt = \frac{dx}{v}$$

$$\rightarrow J = \frac{\vec{v}\,dq}{dx\,dy\,dz} \Rightarrow \left[\vec{J} = \rho\vec{v}\right] \text{---(1)}$$

OR

$$\left[J = \frac{dI}{dS}\right] \longrightarrow \vec{J}\cdot\vec{dS} = dI$$

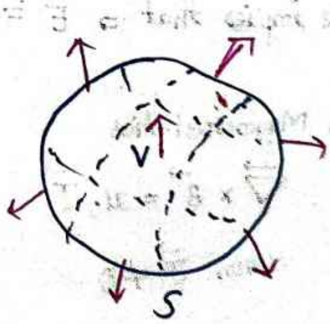
$$\left[I = \int \vec{J}\cdot\vec{dS}\right]$$

Current flowing through the surface S. OR ~~normal~~ Charge flowing per unit time through the surface S.

Now $\oint \vec{J}\cdot\vec{dS}$ = Current ~~(Charge~~ leaving through close surface S OR

Charge leaving per unit time through the close surface S.

* from law of conservation of Charge → $\left[\oint \vec{J}\cdot\vec{dS} = -\frac{dq_{in}}{dt}\right]$



We know, $\rho = \frac{dq_{in}}{dV} \Rightarrow dq_{in} = \rho\, dV$

$$q_{in} = \int_V \rho\, dV \qquad \cancel{\oint \vec{J}\cdot\vec{dS} =}$$

$$\therefore \oint \vec{J}\cdot\vec{dS} = -\frac{d}{dt}\int_V \rho\, dV$$

$$\oint \vec{J}\cdot d\vec{S} = -\int_V \frac{\partial \rho}{\partial t} dV \quad - (*)$$; We use partial derivative because $\rho(x,y,z,t)$

* Using Gauss' Divergence Theorem

$$\oint_S \vec{J}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{J})\, dV$$

Therefore eq^n (*) Becomes $\int_V (\vec{\nabla}\cdot\vec{J}) dV = -\int \frac{\partial \rho}{\partial t} dV$

1) diff^r wrt x
2) diff wrt y
3) diff^r wrt z

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}}$$

- Mathematical statement - The time rate of decrease of charge within a given volume must be equal to the Net Outward flow through the surface.

* In Steady state (Magnetostatics) ; $\boxed{\vec{\nabla}\cdot\vec{J} = 0}$ → continuity Equation

L-36

Magnetic Scalar Potential.

- In electrostatics $\vec{\nabla}\times\vec{E} = 0$ (Always) & The curl of gradient fn of a scalar fn is also zero
- This implies that ⇒ $\vec{E} = -\vec{\nabla}V$ [V = Electrostatic

* In Magnetostatics

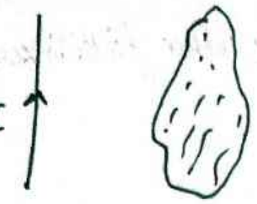
$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ when $\vec{J} \neq 0$, $\vec{\nabla}\times\vec{B} = 0$ when $\vec{J} = 0$; $\vec{B}$ - Magnetic flux Intensity, $\vec{H}$ - Magnetic field Intensity.

- We know $B = \mu_0 H$

$\vec{\nabla}\times\vec{H} = \vec{J}$ $\qquad$ $\vec{\nabla}\times\vec{H} = 0$

• $\vec{J} \neq 0 \Rightarrow$

Examples $\Rightarrow$ I



| $\vec{J} = 0$

examples $\Rightarrow$ 1) Permanent Magnets

2) Some Region of CoAxial Cable

• When $\vec{J}=0 \Rightarrow \vec{\nabla} \times \vec{H} = 0$

$$\vec{H} = -\vec{\nabla} V_m$$

• V_m = Magnetic Scalar Potential, $\left[\nabla^2 V_m = 0\right] \Rightarrow$ Laplace Eq^n

↳ In free Space & in Homogeneous Magnetic Material

• In the Region where $\vec{J} \neq 0$, there $\nabla^2 V_m$ is not defined.

• Dimension of V_m is Ampere

→ V has single value at a given point But V_m can have more than one value at a given point.

L-37 # Magnetic Vector Potential.

• In Electrostatics: $\vec{\nabla} \times \vec{E} = 0$. Also the curl of gradient of any fn is zero $\Rightarrow \vec{E} = -\vec{\nabla} V$

→ We can add any constant to V, Because gradient of any constant is always zero

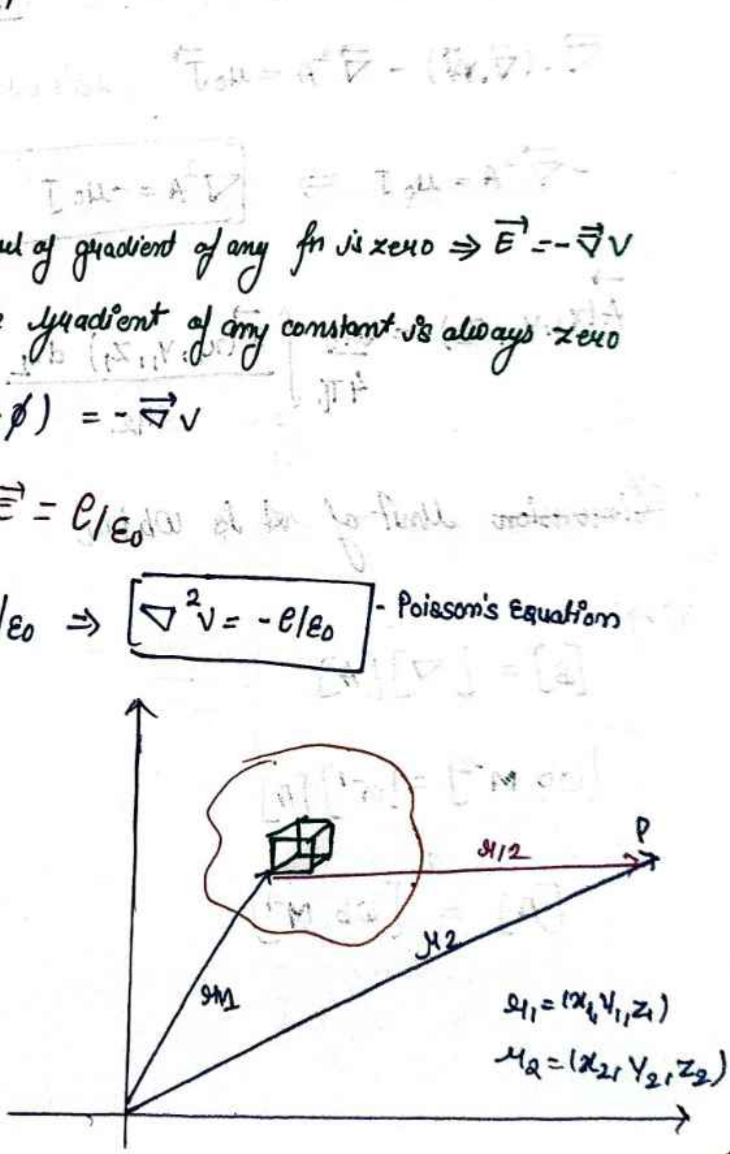
$$V' = V + \phi \;;\; \vec{E} = -\vec{\nabla}(V + \phi) = -\vec{\nabla} V$$

* from Differential form of gauss law: $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$

$\therefore \vec{\nabla}(-\vec{\nabla} V) = \rho/\epsilon_0 \Rightarrow -\vec{\nabla}^2 V = \rho/\epsilon_0 \Rightarrow \boxed{\nabla^2 V = -\rho/\epsilon_0}$ - Poisson's Equation

$$V(x_2, y_2, z_2) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(x_1, y_1, z_1)\, dv_1}{r_{12}}$$

Based on Assumption, as $r_{12} \to \infty$, $V \to 0$



• In Magnetostatics, $\vec{\nabla}\cdot\vec{B} = 0$ & The divergence of curl of any vector field is zero

$\therefore \left[\vec{B} = \vec{\nabla}\times\vec{A}\right]$

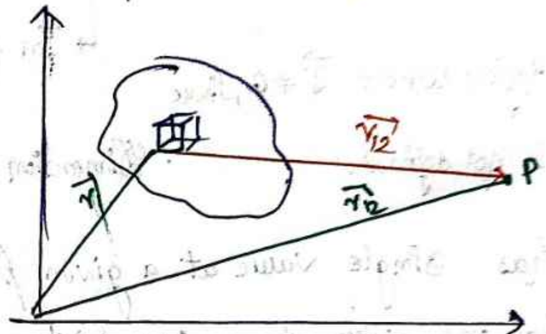
• Can be applied when 1) $\vec{J} = 0$ & 2) $\vec{J} \neq 0$

* We can even extend (OR we) $\vec{A}$ in Time Varying Case

• gradient of any scalar field can be added to $\vec{A}$ to give same $\vec{B}$.

$$A' = (\vec{A} + \vec{\nabla}\psi) \quad , \quad \vec{\nabla}\times A' = \vec{\nabla}\times(\vec{A} + \vec{\nabla}\psi) = \vec{\nabla}\times\vec{A} = \vec{B}$$

• from Ampere's Circuital law



$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0\vec{J}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2 A = \mu_0\vec{J}$$; let's select / choose $\vec{A}$ such that $\vec{\nabla}\cdot\vec{A} = 0$

$$\therefore \quad -\vec{\nabla}^2 A = \mu_0 J \Rightarrow \boxed{\nabla^2 A = -\mu_0 J} \rightarrow$$ Poisson's Eq^n in Magnetostatics

$$\vec{A}(x_2, y_2, z_2) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(x_1, y_1, z_1)\, dv_1}{r_{12}}$$

• Dimension Unit of A is Wb/m

Rough

$$[B] = [\nabla][A]$$

$$[\text{Wb M}^{-2}] = [\text{m}^{-1}][A]$$

$$[A] = [\text{Wb M}^{-1}]$$

L-8c

Displacement Current [Differential form]

- $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ [Ampere Circuital law]

- Divergence of curl is always zero

* from above

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\mu_0\vec{J} = \mu_0(\vec{\nabla}\cdot\vec{J}) = 0$$

$$\Rightarrow \vec{\nabla}\cdot\vec{J} = 0 \text{ [Always]}$$

- from Continuity Eq^n; $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$

* $-\frac{\partial \rho}{\partial t} = 0$; In steady state [ρ = constant]

$-\frac{\partial \rho}{\partial t} \neq 0$; when we are dealing with Time varying fields.

- Maxwell resolved this problem, By -

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} \text{ (continuity eq}^n\text{)} \quad , \quad \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \text{ [Differential form of Gauss law]}$$

$$\rho = \varepsilon_0(\vec{\nabla}\cdot\vec{E})$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial}{\partial t}(\varepsilon_0\vec{\nabla}\cdot\vec{E})$$

$$\vec{\nabla}\cdot\vec{J} = -\varepsilon_0\vec{\nabla}\cdot\frac{\partial \vec{E}}{\partial t} = \vec{\nabla}\cdot\frac{\partial}{\partial t}(\varepsilon_0\vec{E})$$

- The problem can be solved By Adding $\frac{\partial}{\partial t}(\varepsilon_0 E)$ to $\vec{J}$

$$\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\left(\vec{J} + \frac{\partial \varepsilon_0\vec{E}}{\partial t}\right) \qquad \text{let } \vec{D} = \varepsilon_0\vec{E}$$

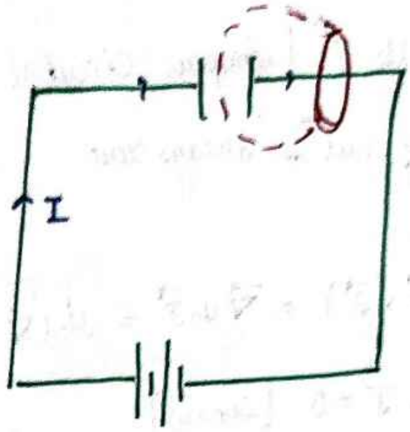
Electric Displacement Vector

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = \mu_0(\vec{J} + \vec{J}_D)} \quad ; \ B = \mu_0 H$$

$$\rightarrow \mu_0(\vec{\nabla}\times\vec{H}) = \mu_0(\vec{J} + \vec{J}_D) \Rightarrow \boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

- $I = \int_S \vec{J}\cdot d\vec{S}$ & $\boxed{I_D = \int_S \vec{J}_D\cdot d\vec{S}}$

L-39 Displacement Current



- Ampere's Circuital law - $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$

[for plane surface for the loop C]

→ Maxwell modified this law By Adding I_D = displacement current in the conduction current

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0(I + I_D) \quad ; \quad I_D = \int_S \vec{J_D}\cdot\vec{ds} \ , \ \vec{J_D} = \varepsilon_0 \frac{\partial E}{\partial t}$$

. E.F inside a condenser ; $E = q/A\varepsilon_0$

$$\frac{\partial E}{\partial t} = \frac{1}{A\varepsilon_0}\frac{\partial q}{\partial t} = \frac{I}{A\varepsilon_0}$$

$$\therefore \oint_C \vec{B}\cdot\vec{dl} = \mu_0\left(I + \int_S \varepsilon_0 \cdot \frac{I}{A\varepsilon_0}\, ds\right)$$

* for plane surface, $E = 0$ $\therefore \oint_C \vec{B}\cdot\vec{dl} = \mu_0 I$

for curved surface, $I = 0$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\left(\int_S \frac{I}{A}\, ds\right)$$

$$= \mu_0 \cdot \frac{I}{A}\int_S ds$$

$$= \mu_0 \frac{I}{\cancel{A}}\cancel{A}$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

∴ Modified Ampere Circuital law : $\left[\oint_C \vec{B}\cdot\vec{dl} = \mu_0(I + I_D)\right]$

L-40

Maxwell Equations

1) Gauss law of Electrostatics

$$\oint \vec{E}\cdot\vec{ds} = q/\varepsilon_0$$

OR

$$\oint \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\int_v \rho dV \rightarrow \oint(\varepsilon_0\vec{E})\cdot\vec{ds} = \int_v \rho dV$$

$$\therefore \oint_s \vec{D}\cdot\vec{ds} = \int_v \rho dV$$

2) Gauss law of Magnetostatics.

$$\oint_S \vec{B}\cdot\vec{dS} = 0 \quad ; \vec{B} = \mu_0\vec{H}$$

$$\oint_S(\mu_0\vec{H})\cdot\vec{ds} = 0 \Rightarrow \oint_s \vec{H}\cdot\vec{ds} = 0$$

③ Faraday's law

$$\oint \vec{E}\cdot\vec{dl} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

$$\vec{B} = f(x,y,z,t)$$

$$e = -\frac{d\phi}{dt}$$

$$e = \oint \vec{F}\cdot\vec{dl} = \oint q\vec{E}\cdot\vec{dl} = \oint \vec{E}\cdot\vec{dl}$$

$$-\frac{d\phi}{dt} = -\frac{d}{dt}\left(\int_S \vec{B}\cdot\vec{ds}\right) = -\int \frac{\partial}{\partial t}\vec{B}\cdot\vec{dS}$$

④ Modified Ampere Circuital law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0(I + I_D)$$

$$I = \int_S \vec{J}\cdot\vec{dS} \quad \& \quad I_D = \int_S \vec{J}_D\cdot\vec{dS}$$

$$\oint \vec{B}\cdot dl = \mu_0\int_S(\vec{J}+\vec{J}_D)dS = \mu_0\int_S\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)dS$$

$$J_D = \frac{\partial D}{\partial t} = \varepsilon_0\frac{\partial E}{\partial t}$$, using $\vec{B} = \mu_0\vec{H}$ $\therefore \mu_0\oint \vec{H}\cdot\vec{dl} = \mu_0\int_S\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)ds$

$$\oint \vec{H}\cdot\vec{dl} = \int_S\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)\vec{dS}$$

L-41

Physical Significance of Maxwell's equation

$$\left[\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right) \quad \text{OR} \quad \vec{\nabla}\times\vec{H} = \left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)\right]$$

1) Modified Ampere circuital law in Differential form

2) Space Variation of $\vec{B}$ with its sources.

3) $\vec{B}$ can be produced By Both $\vec{J}$ or By Time Varying $\vec{E}$

4) In steady state, $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ OR $\vec{\nabla}\times\vec{H} = \vec{J}$

5) Integral form: $\oint\vec{B}\cdot\vec{dl} = \mu_0(I+I_D) = \mu_0\int_S\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)d\vec{S}$

~~B) Go~~ $\left[\vec{\nabla}\cdot\vec{B} = 0\right]$

1) Gauss law of Magnetostatics in Differential form

2) Non Existence of Magnetic charges monopoles.

3) Magnetic flux lines forms a closed loop. They never converge or diverge from a point

4) Integral form $\oint\vec{B}\cdot\vec{dl} = 0$

5) True for stationary as well as time varying fields.

L-42

$$\left[\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad \text{or} \quad \vec{\nabla}\cdot\vec{D} = \rho\right]$$

1) Coulomb's law ~~in~~ OR Gauss' law of Electrostatics in Differential form

2) This is true for Both steady & Time Varying fields.

3) Relation B/w space variation of $\vec{E}$ with its source ~~or~~ (sink) ρ

4) In source free space, $\vec{\nabla}\cdot\vec{E} = 0$

5) Integral form: $\oint\vec{E}\cdot\vec{dS} = \frac{1}{\varepsilon_0}\int_V \rho\, dV$

L-44

$$\left[\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}\right]$$

1) Differential form of faraday's law of Electromagnetic Induction.

2) A time varying MF can also be a source of Electric field.

3) Relation B/w space dependence of Electric field with time dependence of Magnetic field.

4) The Non-Zero Curl of $\vec{E}$ shows that ~~Electrof~~ Electric flux lines can Also form a closed loop

5) for steady state, $\vec{\nabla}\times\vec{E} = 0$

6) Integral form : $\oint_l \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$

7) The negative sign in integral form is due to lenz law.

L-44

Derivation of Maxwell's Equations in Differential form

1) $\vec{\nabla}\cdot\vec{B} = 0$ [Gauss's law of Magnetostatics]

$\oint \vec{B}\cdot d\vec{S} = 0$ — (1)

→ from Gauss' Divergence Theorem $\oint \vec{B}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{B})\, dV$ — (2)

from (1) & (2) ⇒ $\int_V (\vec{\nabla}\cdot\vec{B})\, dV = 0$

diff^r w.r.t V

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

2) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ or $\vec{\nabla}\cdot\vec{D} = \rho$ [Gauss' law of Electrostatics]

$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho\, dV$ — (1)

→ using Gauss' Divergence Theorem $\oint \vec{E}\cdot d\vec{s} = \int_V (\vec{\nabla}\cdot\vec{E})\, dV$ — (2)

from (1) & (2) $\int_V (\vec{\nabla}\cdot\vec{E})\, dV = \frac{1}{\varepsilon_0}\int \rho\, dV \xrightarrow[\text{w.r.t } V]{\text{diff}^r} \left(\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\varepsilon_0}\right) = 0 \rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0}$

$\int_V (\vec{\nabla}\cdot\vec{E} - \rho/\varepsilon_0)\, dV = 0$

$\vec{\nabla}\cdot\varepsilon_0\vec{E} = \rho$

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

1-45

- $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ [Faraday's law of EMI]

- $e = -\frac{d\phi}{dt}$ -(i) ; e = W.D by over a closed path in bringing a unit +ve charge once around a close ckt

$$e = \oint_c \vec{F}\cdot\vec{dl} = \oint_c q\vec{E}\cdot\vec{dl} \; ; q = 1C \; \therefore \; e = \oint_c \vec{E}\cdot\vec{dl}$$

- Using Stoke's Theorem

$$e = \oint_c \vec{E}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{E})\, dS \quad -(ii) - 1$$

$$\therefore \text{ from (i) \& (ii)} \Rightarrow \int_S (\vec{\nabla}\times\vec{E})\, d\vec{S} = \frac{-\partial\phi}{dt} \quad - (iii)$$

- Now $\frac{d\phi}{dt} = \frac{d}{dt}\int_S \vec{B}\cdot\vec{dl} = \int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$ —(iv)

$$\text{from (iii) \& (iv)} \Rightarrow \int_S (\vec{\nabla}\times\vec{E})\, d\vec{S} = -\int_S \frac{\partial B}{\partial t}\cdot d\vec{S}$$

$$\therefore \int_S \left[(\vec{\nabla}\times\vec{E}) + \frac{\partial \vec{B}}{\partial t} \right] . d\vec{S} = 0$$

Diff^{r} wrt x then Y [OR SO]

$$(\vec{\nabla}\times\vec{E}) + \frac{\partial \vec{B}}{\partial t} = 0$$

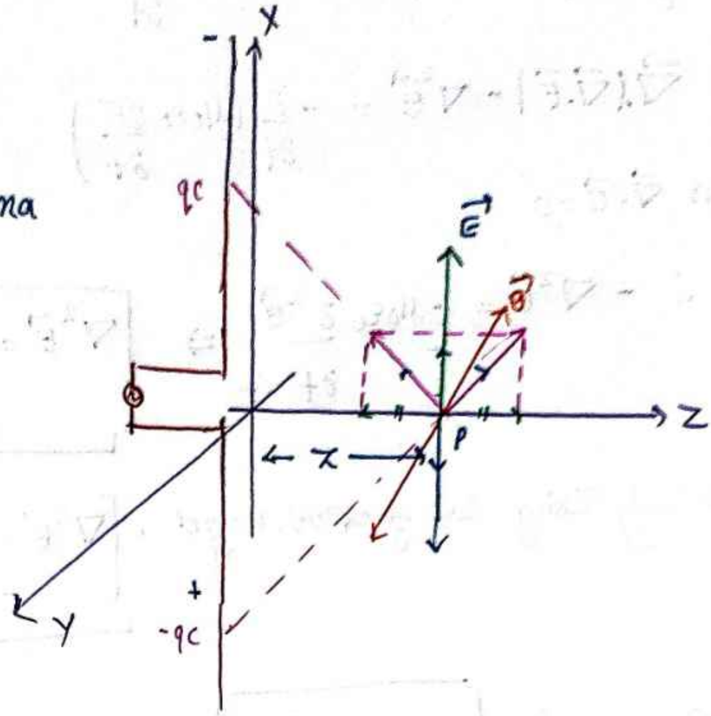
$$\therefore \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}}$$

- We start from a scalar eq^n & ends up getting a vector eq^n in final Result.

L-46

Electromagnetic Waves.

- $e = e_0 \sin\omega t$ [Alternating emf source]
- If Point P lies very close to dipole antenna then $E_x = E_0 \sin\omega t$



- The electric field at P is

$$E_x = E_0 \sin\omega\left(t - \frac{Z}{\omega}\right)$$

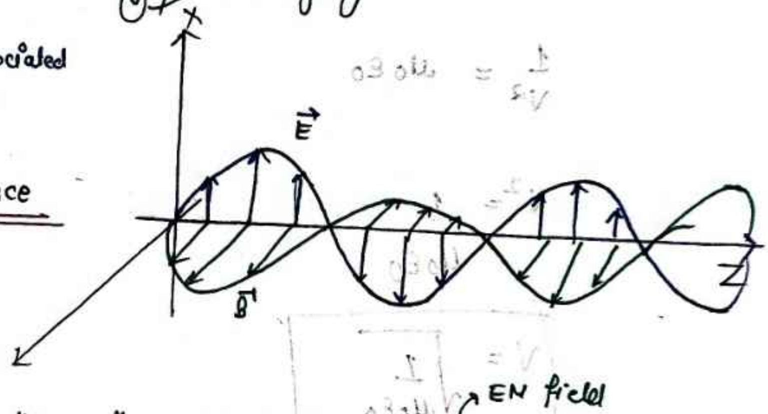
Periodic in Both space & Time

- $\vec{B}_{wire} = \frac{\mu_0}{2\pi}\frac{2I}{Z}$; $I \to$ sinusoidal fn

$\therefore \vec{B}_{wire} \propto \sin\omega t$

- Magnetic field : $B_Y = B_0 \sin\omega t$
- Magnetic field at $P \to B_y = B_0 \sin\omega\left(t - \frac{Z}{\textcircled{c}}\right)$ → speed of light
- EM wave also have energy associated with it

L-47 Eqⁿ of EM wave in free space



- Maxwell's Eqⁿ

(I) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

(II) $\vec{\nabla}\cdot\vec{B} = 0$

(III) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu_0\frac{\partial\vec{H}}{\partial t}$

(IV) $\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

OR

$\vec{\nabla}\times\vec{H} = \vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}$

- from these eqⁿ maxwell concluded EM waves can leave source & travel through space — EM field
- In Source free Space Eqⁿ Becomes → $\rho = 0$ & $\vec{J} = 0$

(I) $\vec{\nabla}\cdot\vec{E} = 0$

(II) $\vec{\nabla}\cdot\vec{B} = 0$

(III) $\vec{\nabla}\times\vec{E} = \cancel{-\mu_0\varepsilon_0\frac{\partial E}{\partial t}}\ -\frac{\partial\vec{B}}{\partial t}$

(IV) $\vec{\nabla}\times\vec{H} = \varepsilon_0\frac{\partial E}{\partial t}$

- Taking curl of (iii) : $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

as $\vec{\nabla}\cdot\vec{E} = 0$

$$\therefore\ -\nabla^2\vec{E} = -\mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \Rightarrow \boxed{\nabla^2\vec{E} = \left(\frac{\partial^2\vec{E}}{\partial t^2}\right)\mu_0\varepsilon_0} \quad \text{--- (5)}$$

- Similarly Taking Curl of eq^n(4), we get, $\boxed{\nabla^2\vec{B} = \mu_0\varepsilon_0 \frac{\partial^2\vec{B}}{\partial t^2}}$ — (6)

- 3-D wave eq^n - $\boxed{\partial^2 g = \frac{1}{v^2}\frac{\partial^2 g^2}{\partial t^2}}$

- Comparing eq^n (5) & (6) with 3D wave eq^n $\Rightarrow$ $\vec{E}$ & $\vec{B}$ are 3D wave eq^n &

$$\frac{1}{v^2} = \mu_0\varepsilon_0$$

$$v^2 = \frac{1}{\mu_0\varepsilon_0}$$

$$\boxed{v = \sqrt{\frac{1}{\mu_0\varepsilon_0}}}$$

$$v = 3 \times 10^8\ m/s = c$$

Both $\vec{E}$ & $\vec{B}$ travels in Vaccum & with speed of light.

* for dielectric Medium

$$\boxed{v = \sqrt{\frac{1}{\mu\cdot\varepsilon}}}$$

L-48

Wave Equation for Plane Polarized EM Wave

- Transverse Electromagnetic Wave (TEM)
- Transverse Plane : Normal of Plane is in same direc^n as EM wave & it contains $\vec{E}$ & $\vec{B}$

* Consider an EM wave in which $\vec{E}$ is polarised in X Direction ie $E_y = E_z = 0$ and EM wave is propogating in z direction.

$\therefore \vec{E} = E_x \hat{i}$ ↳ E_x changes with x & not y. $\frac{\partial E_x}{\partial y} = 0$

- from Maxwell Eq^n in Source free space - $\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$ -(I) & $\vec{\nabla} \cdot \vec{E} = 0$ -(II)

$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial E}{\partial t}$ -(III) $\quad \vec{\nabla} \cdot \vec{H} = 0$ -(IV)

Now $\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = -\hat{j}\left(-\frac{\partial E_x}{\partial z}\right) + k\left(\cdot\frac{\partial E_x}{\partial y}\right)$

$$\vec{\nabla} \times \vec{E} = \hat{j}\frac{\partial E_x}{\partial z} - \hat{k}\cancel{\frac{\partial E_x}{\partial y}}^{0} = 0 \text{ as } \frac{\partial E_x}{\partial y} = 0 \text{ as E is a func}^n \text{ of z only}$$

$$\therefore \vec{\nabla} \times \vec{E} = \hat{j}\frac{\partial E_x}{\partial z} \quad -(5)$$

- Direction of $\vec{\nabla} \times \vec{E}$ will give us the direction of $\vec{H}$ i.e $\vec{H}$ has only y component

$\therefore \boxed{\vec{H} = \hat{j} H_y}$; $H_x = H_z = 0$

- from eq^n (1) & (5) $\Rightarrow -\mu_0 \frac{\partial \vec{H}}{\partial t} = \frac{\partial E_x}{\partial z}$ --(6)

- Now $\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_y & 0 \end{vmatrix} \Rightarrow \vec{\nabla} \times \vec{H} = \hat{i}\left(-\frac{\partial}{\partial z}H_y\right) + \hat{k}\cancel{\left(\frac{\partial H_y}{\partial x}\right)}$ 0 ; as $\vec{H}$ is fn of z only.

$$\vec{\nabla} \times \vec{H} = -\hat{i}\frac{\partial H_y}{\partial z} \quad -(7)$$

· from eq^n 3 & 7 $\Rightarrow$ $-\dfrac{\partial H_y}{\partial z} = \varepsilon_0 \dfrac{\partial E_x}{\partial t} \Rightarrow \left[\dfrac{\partial H_y}{\partial z} = -\varepsilon_0 \dfrac{\partial E_x}{\partial t}\right]$ -- 8

* Diff^n 6 wrt z & 8 wrt t

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \frac{\partial^2 H_y}{\partial z \partial t} \quad - (9)$$

$$\frac{\partial^2 H_y}{\partial t \partial z} = -\varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad - (10)$$

* from 9 & 10 $\Rightarrow$ $\dfrac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \dfrac{\partial^2 E_x}{\partial t^2}$; $\dfrac{\partial^2 E_x}{\partial z^2}$

Dir^n of Polarisation

dir^n of Propogat^n

; similarly $\dfrac{\partial^2 H_y}{\partial z^2} = \mu_0 \varepsilon_0 \dfrac{\partial^2 H_y}{\partial t^2}$

· $E_x(z,t) = E_0 \cos(\omega t - kz)$

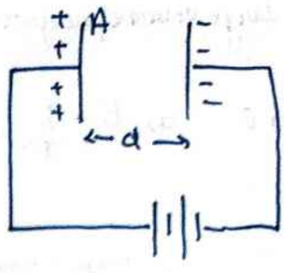
$H_y(z,t) = H_0 \cos(\omega t - kz)$

· $\omega = \dfrac{2\pi}{T} = 2\pi\nu$ = angular frequency , $k = \dfrac{2\pi}{\lambda}$ = ~~ratio~~ Wave No.

· $\dfrac{E_x}{H_y} = \dfrac{E_0}{H_0} = \dfrac{E_0}{B_0}\mu_0 = c\mu_0 = 377\,\Omega$ #

Characteristic Impedence of free space.

L-49 Energy Density of Electric field.



- Volume $= Ad$
- P.E energy due to two point charges $= \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

$$= q_1 V_1$$

- for n point charges P.E : $U = \frac{1}{2}\sum_{i=1}^{N} q_i V(r_i)$

- Distribution of charge over a volume, then $P.E = \frac{1}{2}\int_{\tau} (\rho\, d\tau) . V \quad \text{--(1)}$

- from Gauss law $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0 \Rightarrow \rho = \varepsilon_0(\vec{\nabla}.\vec{E})$

Put in eq^n (1)

$$U = \frac{1}{2}\int \varepsilon_0\, \vec{\nabla}.\vec{E}\; V\, d\tau \quad \text{--(2)}$$

Now, $\vec{\nabla}(V\vec{E}) = V(\vec{\nabla}.\vec{E}) + (\vec{\nabla}V).\vec{E}$

$$V(\vec{\nabla}.\vec{E}) = \vec{\nabla}(V\vec{E}) - (\vec{\nabla}V)\vec{E}$$

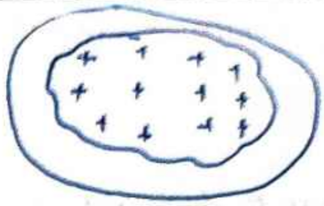
Put in eq^n (2) $\Rightarrow U = \frac{\varepsilon_0}{2}\int_{\tau} \vec{\nabla}.(V\vec{E})\,d\tau - \frac{\varepsilon_0}{2}\int_{\tau} (\vec{\nabla}V).\vec{E}\,d\tau$

Using Gauss Divergence Theorem: $\int_{\tau} \vec{\nabla}.\vec{A}\,d\tau = \oint_{S} \vec{A}.\vec{ds}$

$$\therefore U = \frac{\varepsilon_0}{2}\int_{S} V\vec{E}.\vec{ds} - \frac{\varepsilon_0}{2}\int_{\tau}(\vec{\nabla}V).\vec{E}\,d\tau \qquad ; \text{using } E = -\vec{\nabla}V$$

$$U = \frac{\varepsilon_0}{2}\int_{S} V\vec{E}.\vec{ds} + \frac{\varepsilon_0}{2}\int_{\tau} E^2\,d\tau$$

· for very large volume & surface , $r \to \infty$



$$\oint_S V\vec{E}\cdot d\vec{s} \to 0 \quad \text{as } E \propto \frac{1}{r^2},\ V \propto \frac{1}{r},\ d\vec{s} \propto r^2$$

∴ for a Very Very large volume

$$U = \frac{\varepsilon_0}{2}\int_\tau E^2 d\tau$$

diff^r wrt τ

$$\frac{dU}{d\tau} = \frac{\varepsilon_0 E^2}{2}$$

$$\boxed{U_E = \frac{\varepsilon_0 E^2}{2}}$$

Energy Density → Energy per unit volume

L50 # Energy Density of Magnetic field.

· $U = \frac{1}{2}LI^2$ (I) [Energy stored in Inductor]

$$\phi = LI \quad -(II) \qquad , \phi = \int_S \vec{B}\cdot d\vec{S} = \int_S (\vec{\nabla}\times\vec{A})\cdot d\vec{s}$$

Magnetic Vector Potential

- Using Stoke's Theorem

$$\int_S (\vec{\nabla}\times\vec{A})\cdot d\vec{s} = \oint_L \vec{A}\cdot d\vec{l} = \phi \quad -(III)$$

$$\text{from (2) \& (3)} \Rightarrow \oint_L \vec{A}\cdot d\vec{l} = LI \quad -(IV)$$

· from (I) & (IV)

$$U = \frac{1}{2} I \oint_L \vec{A}\cdot d\vec{l} = \frac{1}{2}\oint_L \vec{A}\, I\cdot d\vec{l}$$

· for a Volume Current Distribution

$$\mathcal{U} = \frac{1}{2}\int_V \vec{A}\cdot\vec{J}\, d\tau \quad -(*) \quad \text{as} \quad I = \int_S \vec{J}\cdot\vec{ds}$$

· using Ampere's Circuital law in Differential form

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\therefore \vec{J} = \frac{\vec{\nabla}\times\vec{B}}{\mu_0}$$

Put in $(*)$ → $\mathcal{U} = \frac{1}{2}\int_V \vec{A}\cdot\left(\frac{\vec{\nabla}\times\vec{B}}{\mu_0}\right) d\tau = \frac{1}{2\mu_0}\int_V \vec{A}\cdot(\vec{\nabla}\times\vec{B})\, d\tau \rightarrow (\#)$

Now $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}(\vec{\nabla}\times\vec{B})$ & $\vec{\nabla}\times\vec{A} = B$

$$\vec{A}\cdot(\vec{\nabla}\times\vec{B}) = \vec{B}\cdot\vec{B} - \vec{\nabla}\cdot(\vec{A}\times\vec{B})$$

Put in eq^n $(\#)$:

$$\mathcal{U} = \frac{1}{2\mu_0}\int_V \vec{B}^2 d\tau - \frac{1}{2\mu_0}\int_V \vec{\nabla}\cdot(\vec{A}\times\vec{B})\, d\tau$$

; using Gauss' Divergence Theorem

$$\int_V \vec{\nabla}\cdot\vec{A}\, d\tau = \int_S \vec{A}\cdot d\vec{S}$$

$$\therefore \int_V \vec{\nabla}\cdot(\vec{A}\times\vec{B})\, d\tau = \int_S (\vec{A}\times\vec{B})\, d\vec{S}$$

$$\mathcal{U} = \frac{1}{2\mu_0}\int_V \vec{B}^2 d\tau - \frac{1}{2\mu_0}\int_S (\vec{A}\times\vec{B})\, ds$$

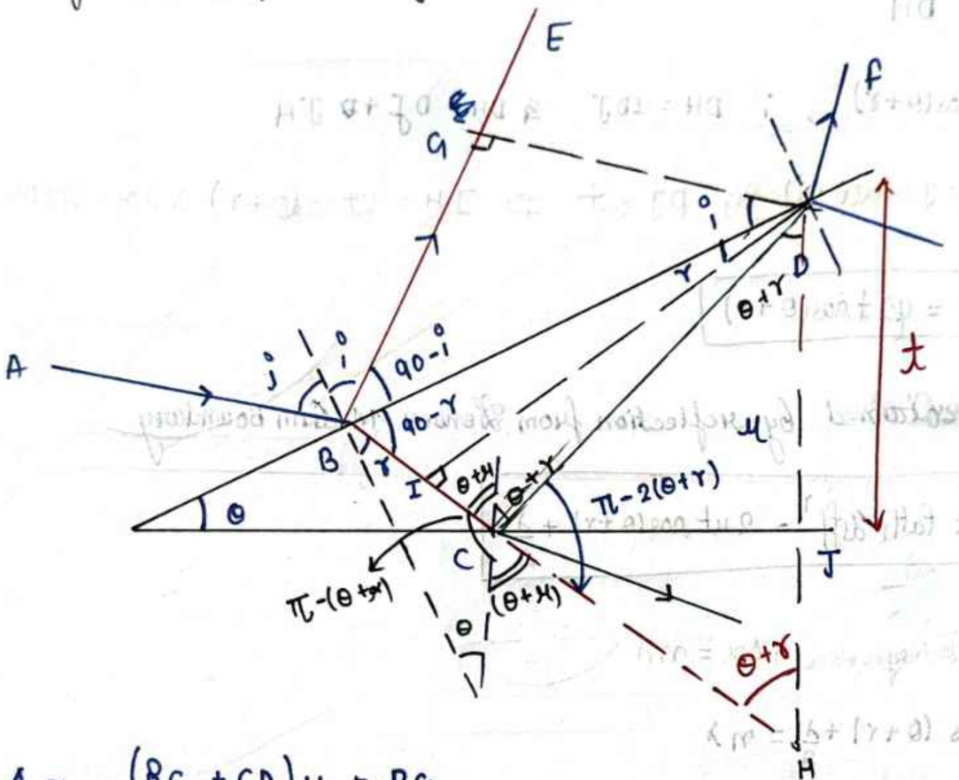
Integrating Over Whole Space ∴ Surface integral Becomes zero

$$\mathcal{U} = \frac{1}{2\mu_0}\int_V B^2 d\tau \rightarrow \text{diff}^r \text{ wrt } V \Rightarrow \frac{d\mathcal{U}}{dV} = \frac{B^2}{2\mu_0}$$

$$\therefore \boxed{\mathcal{U}_B = B^2/2\mu_0}$$

Interference

Wedge shaped film Interference



- $\Delta x = (BC + CD)\mu - BG$

$BC = BI + IC$

- In $\Delta\, CDJ$ & $\Delta\, CJH$ are congruent $\therefore CD = CH$

$\rightarrow \Delta x = \mu(BI + IC + CH) - BG$

$= \mu(BI + IH) - BG = \mu BI + \mu IH - BG \quad —(1)$

$\mu = \dfrac{\sin i}{\sin r}$, In ΔBDG , $\sin i = \dfrac{BG}{BD}$

$\therefore \mu = \dfrac{BG}{BI}$ In $\Delta\, BID$, $\sin r = \dfrac{BI}{BD}$

$[BG = \mu BI]$

SUBSTITUTING in(1), we get

$\Delta x = \cancel{\mu BI} + \mu IH - \cancel{\mu BI} \rightarrow \boxed{\Delta x = \mu IH}$

- In Δ ~~ICD~~ IHD

$$\cos(\theta+r) = \frac{IH}{DH}$$

$IH = DH\cos(\theta+r)$; $DH = 2DJ$ & $DH = DJ + JH$

$IH = 2DJ\cos(\theta+r)$; $DJ = t \Rightarrow IH = 2t\cos(\theta+r)$ & $\Delta x = 2IH$

$$\boxed{\Delta x = \mu 2t\cos(\theta+r)}$$

- Since BE is obtained by reflection from Denser Medium Boundary

$$\boxed{\text{Net Path diff}^r = 2\mu t\cos(\theta+r) + \frac{\lambda}{2}}$$

→ for constructive Interference : $\Delta x = n\lambda$

$$2\mu t\cos(\theta+r) + \frac{\lambda}{2} = n\lambda$$

$$\therefore \boxed{2\mu t\cos(\theta+r) = (2n-1)\frac{\lambda}{2}}$$

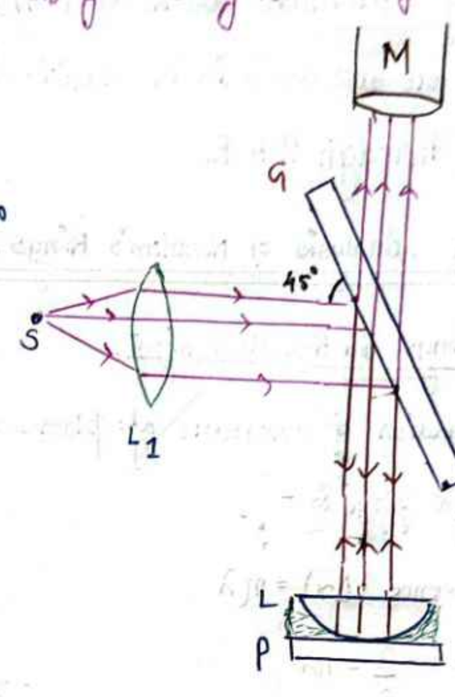
→ for Destructive Interference, $\Delta x = \frac{\lambda}{2}(2n+1)$

$$\frac{\lambda}{2} + 2\mu t\cos(\theta+r) = (2n+1)\lambda/2$$

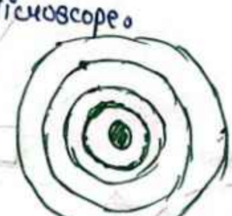
$$\therefore \boxed{2\mu t\cos(\theta+r) = n\lambda}$$

Newton's Ring Experiment. → Based on "Interference By Division of Amplitude".

- S = Source of Monochromatic light.
- L_1 = Convex lens
- G = Plane glass plate inclined at angle of 45°
- L = Plano convex lens
- P = Plane glass plate of same material as of G.
- M = Travelling Microscope.



- fringe pattern →



- When a convex surface of a plano convex lens of large radius of curvature is placed on plane glass plate air film of increasing thickness is formed B/w the lens & the plate.
- The thickness of the film is zero at the point of contact & gradually increases as one proceeds outwards.
- When a parallel beam of Monochromatic light is made to fall on such a film as shown in fig. a pattern of bright & dark, concentric rings are observed round the point of contact. These rings are called Newton Rings.
- formed due to interference of light waves reflected from the upper & lower surface of film of air enclosed b/w lens & plate.
- The Experimental set up for producing Newton's Rings is as shown in fig. S is source of monochromatic light placed at the focus of upper lens L_1.

→ A parallel beam of light rays fall on Glass plate G at angle of 45°.

- Plate G reflects part of light towards air film enclosed B/w ~~the~~ lens L & Glass Plate P.
- light transmitted through lens L on reflection from surface of glass plate P interfers with light reflected from lower surface of L.

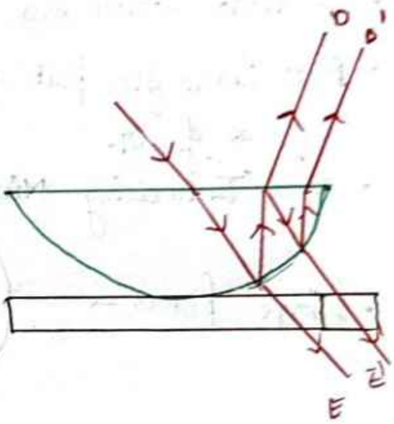
→ These 2 reflected beams travels upwards & enters observer's eye through a travelling microscope M when microscope is focused at the point of contact of the lens & plate.

- A pattern of Alternate Dark & Bright Rings with central ring dark is seen.

→ The rings are also seen in transmitted light i.e the light passing Downward through Plate P.

Mathematical Analysis of Newton's Rings.

(I) Newton's Rings in Reflected Light.



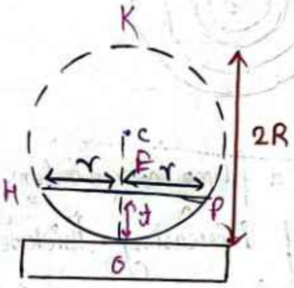
→ Let R be radius of Curvature of plane convex lens L.

- Condn for Bright fringe is -

Path difference $(\Delta x) = n\lambda$

$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$



- for Normal incidence, $\angle i = \angle r = 0$

for air film, $\mu = 1$

So, $2 \cdot 1 t \cos 0 = (2n+1)\frac{\lambda}{2}$

Ring Pattern.

$\therefore \boxed{2t = (2n+1)\frac{\lambda}{2}}$ — ① ; $n = 0, 1, 2, \ldots$

* from Geometry

$HE \times EP = KE \times EO \Rightarrow r \times r = (KO - EO) \times EO$

$\therefore r^2 = (2R - t) \times t$; $2R >>>> t \therefore 2R - t = 2R$

$r^2 = 2Rt \rightarrow \left[t = \frac{r^2}{2R}\right]$ — ②

- Using ② in ①

$\frac{2r^2}{2R} = (2n+1)\frac{\lambda}{2} \Rightarrow r_n^2 = (2n+1)\lambda R/2 \Rightarrow r_n = \sqrt{(2n+1)\frac{\lambda R}{2}}$

$\therefore$ Diameter of n^{th} Bright fringe. $D_n = 2r_n$

$\boxed{D_n = 2\sqrt{(2n+1)\frac{\lambda R}{2}}} \rightarrow \left[D_{BRIGHT} \propto \sqrt{(2n+1)}\right]$ Hence, dia. of Bright fringes are proportional to sq. root of odd natural numbers.

- The condⁿ for Dark fringe: Path Difference $(\Delta x) = (2l+1)\lambda/2$

$$2\mu t\cos r - \frac{\lambda}{2} = l\lambda + \frac{\lambda}{2} \rightarrow 2\mu t\cos r = (l+1)\lambda$$

So $2\mu t\cos r = n\lambda$; $n = l+1 \rightarrow$ Integer

for normal incidence, $r=0$ & for Air film $\mu = 1$

$$2t(1)\cos 0 = n\lambda \rightarrow \boxed{2t = n\lambda} \text{ --- (3)}$$

using (2) in (3)

$$\frac{2r^2}{2R} = n\lambda \Rightarrow r_n^2 = n\lambda R \Rightarrow r_n = \sqrt{n\lambda R}$$

- for $n=0$; $\boxed{r_n = 0}$; Hence Central Ring at the point of contact of lens & the Plate is a Dark Ring in Reflected light.

- Diameter of n^{th} Dark Ring is $D_n = 2r_n$

$$\boxed{D_n = 2\sqrt{n\lambda R}} \rightarrow \left[(D_n)_{DARK} \propto \sqrt{n}\right]$$

Hence, dia. of dark rings are proportional to square root of natural numbers.

- $(D_2)_{Dark} - (D_1)_{dark} = 2\sqrt{2\lambda R} - 2\sqrt{1\lambda R} = 2\sqrt{\lambda R}(\sqrt{2} - 1)$

$$= 0.414 \cdot 2\sqrt{\lambda R}$$

- $(D_3)_{Dark} - (D_2)_{Dark} = 2\sqrt{3\lambda R} - 2\sqrt{2\lambda R} = 2\sqrt{\lambda R}(\sqrt{3} - \sqrt{2})$

$$= 0.318 \cdot 2\sqrt{\lambda R}$$

- Thus with increase in order Dark Rings become more & more narrower

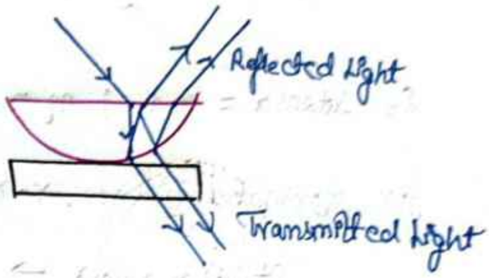
* Similarly, with increase in order Bright Rings also becomes more & more narrower.

* Newton's Rings in Transmitted lights

- The cond^n for Bright Rings in transmitted light is given by

Path diff^r $(\Delta x) = n\lambda$, $n = 0, 1, 2, \ldots$

$2\mu t \cos r = n\lambda$



- for normal incidence $\angle i = \angle r = 0$; for air film $\mu = 1$

$\therefore\ 2(1)t.\cos 0 = n\lambda \rightarrow \boxed{2t = n\lambda}$ — (1)

&

$t = r^2/2R$ — (2)

using (2) in (1)

$$\frac{2r_n^2}{2R} = n\lambda \rightarrow r_n^2 = n\lambda R \longrightarrow \boxed{r_n = \sqrt{n\lambda R}}$$

- Diameter of n^{th} Bright Ring is given by : $D_n = 2r_n$

$\boxed{(D_n)_{BRIGHT} = 2\sqrt{n\lambda R}}$; for $n = 0$, $r_n = 0$ Hence Central Ring in transmitted light is Bright Ring.

- The cond^n for ~~Dark~~ Dark Rings in transmitted lights

Path diff^r $(\Delta x) = (2n+1)\lambda/2$; $n = 0, 1, 2 \ldots$

$2\mu t \cos r = (2n+1)\lambda/2$

Ring Pattern →

∴ for normal incidence $\angle i = \angle r = 0$; for air film $\mu = 1$

$2.1 t \cos 0 = (2n+1)\lambda/2 \Rightarrow 2t = (2n+1)\lambda/2$ — (3)

using (2) in (3) $\quad \frac{2r_n^2}{2R} = (2n+1)\lambda/2 \rightarrow r_n^2 = (2n+1)\lambda R/2$

$$r_n = \sqrt{(2n+1)\frac{\lambda R}{2}}$$

- Diameter of n^{th} ~~bright fringe~~ dark ring is given by $D_n = 2r_n$

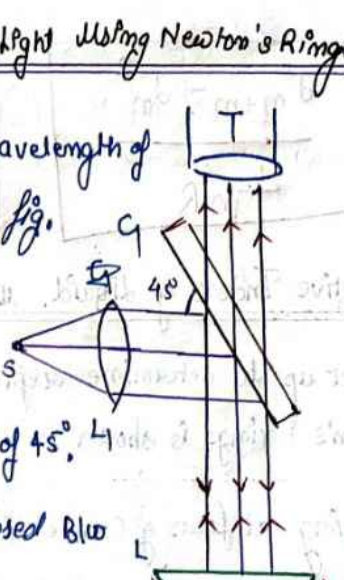
$$\boxed{(D_n)_{DARK} = 2\sqrt{(2n+1)\frac{\lambda R}{2}}}$$

Difference B/w in Newton's Rings b/w Transmitted Light & Reflected Light

Newton's Rings in Reflected Light	Newton's Ring in Transmitted Light
1) Ring Pattern 	① Ring Pattern 
2) $(D_m)_{BRIGHT} = 2\sqrt{(2m+1)\dfrac{\lambda R}{2}}$	② $(D_n)_{BRIGHT} = 2\sqrt{n\lambda R}$
3) $(D_N)_{DARK} = 2\sqrt{n\lambda R}$	③ $(D_m)_{DARK} = 2\sqrt{(2n+1)\dfrac{\lambda R}{2}}$
4) Contrast is good.	④ Contrast is Poor.

- Hence, The Ring Patterns in Reflected & Transmitted light are complimentary to each other.

Determination of the Wavelength of Sodium Light Using Newton's Rings.

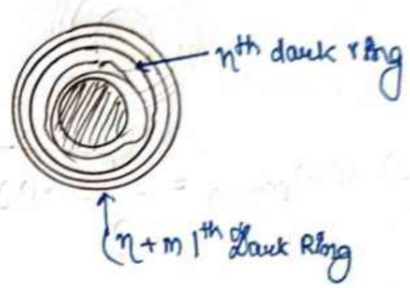


- The experimental set up to determine wavelength of Sodium light using Newton's Rings is shown in fig.
- S is sodium lamp laying at focus of Convex lens L_1. A parallel beam of light rays fall on the glass plate G at angle of 45°.
- Plate G reflects lights towards air film enclosed B/w the lens L & plane Glass Plate P.
- Light transmitted through lens L on reflection from glass plate P interfere with light reflected from lower surface of L.
- Thus two reflected beams travel upwards & enter Observer's Eye through travelling Microscope T.

- When travelling microscope T is focused at point of contact of lens L & Plate P, A pattern of alternate dark & Bright Rings (Newton's Rings) with central Ring dark is seen.

- The diameter of n^{th} dark Ring is $d_n = 2\sqrt{n\lambda R}$

$$d_n^2 = 4(n\lambda R) \quad -(1)$$



- The diameter of $(n+m)^{th}$ dark Ring is -

$$d_{n+m} = 2\sqrt{(n+m)\lambda R}$$

$$d^2_{n+m} = 4(n+m)\lambda R \quad -(2)$$

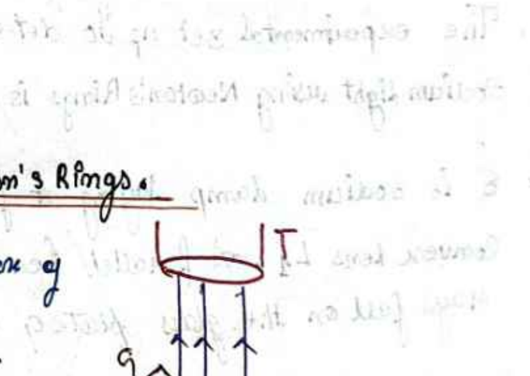
- (2) - (1)

$$d^2_{n+m} - d^2_n = 4(n+m)\lambda R - 4(n\lambda R) = 4m\lambda R$$

$$\boxed{\lambda = \frac{d^2_{n+m} - d_n^2}{4mR}}$$

Determination of Refractive Index of Liquid using Newton's Rings.

- The experimental set up to determine refractive index of liquid using Newton's rings is shown.

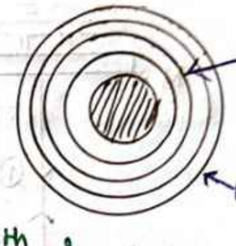


- S is sodium lamp lying at focus of Convex lens L_1.
- A parallel beam of light falls on glass plate G at an angle of 45°.
- Plate (G) reflects light towards liquid film enclosed b/w plano-convex lens (L) & plane glass plate (P).
- Light transmitted through lens (L) on reflection from glass plate P interferes with light reflected from lower surface of L.

· These 2 reflected beams travel upwards & enter observer's eye through travelling microsoft when microscope is focused at point of contact of lens & plate, a pattern of alternate dark & bright rings (Newton's Rings) with central dark Ring is seen.

· for Dark fringe Rings

$2\mu t = n\lambda$, $t = r^2/2R$



$$\therefore r_n = \sqrt{\frac{n\lambda R}{\mu}} \Rightarrow (D_n)_{liq} = 2\sqrt{\frac{n\lambda R}{\mu}}$$

squaring

$$D_n^2 = \frac{4n\lambda R}{\mu} \text{ -(1)}$$

similarly. The dia. of $(n+m)^{th}$ ring is case of liquid film of refractive index (μ) is

$$D_{m+n} = \sqrt{\frac{2(m+n)\lambda R}{\mu}}$$

squaring

$$(D^2_{m+n})_{liq} = \frac{4(n+m)\lambda R}{\mu} \text{ (2)}$$

→ (2) − (1)

$$D_{m+n}^2 - D_n^2 = \frac{4(m+n)\lambda R}{\mu} - \frac{4n\lambda R}{\mu}$$

$$D^2_{m+n} - D_n^2 = \frac{4m\lambda R}{\mu}$$

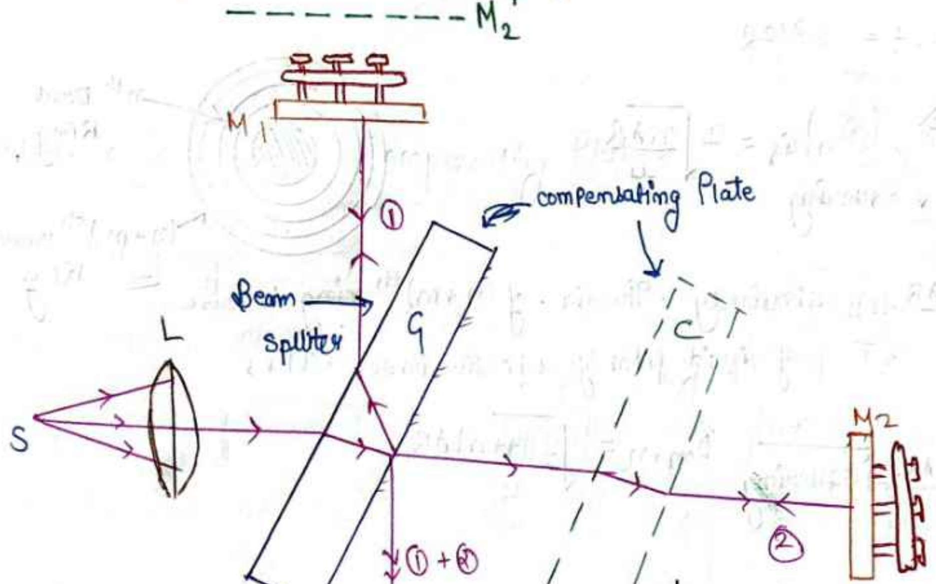
$$\boxed{\mu = \frac{4m\lambda R}{(D^2_{n+m})_l - (D^2_n)_l}}$$

· Where R is radius of curvature of plano convex lens which can be determined using spectrometer.

Michelson Interferometer.

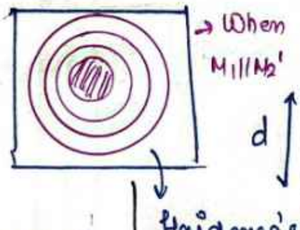
· Instruments which works on the principle of interference are known as Interferometers.

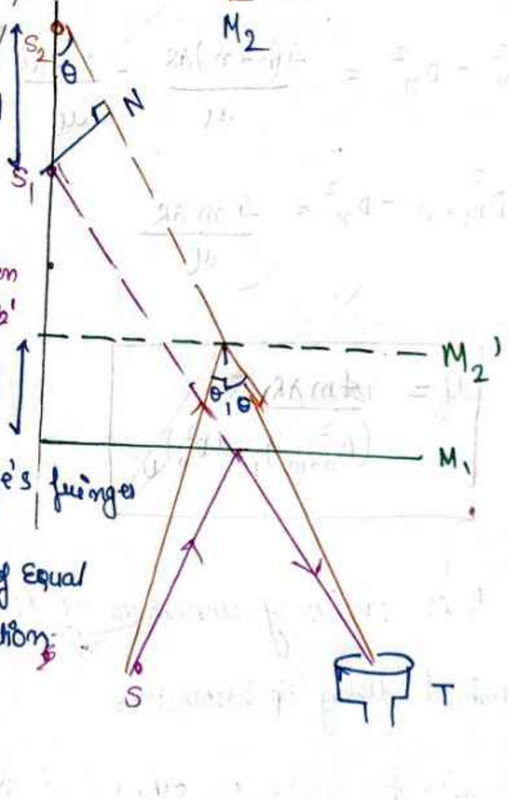
→ Michelson designed one such interferometer to determine the Wavelength of light, thickness of strips & standardization of Meter.



· Types of fringes

① Circular fringes →





Haidenge's fringes OR fringes of Equal inclination

$S_1N = 2d\cos\theta$

· Effective Path diff^n $= 2d\cos\theta - \frac{\lambda}{2}$

* for Bright fringe, $\Delta x = n\lambda$

$2d\cos\theta - \lambda/2 = n\lambda$

$\therefore \boxed{2d\cos\theta = (2n+1)\lambda/2}$

* for Dark fringe, $\Delta x = (2l+1)\lambda/2$

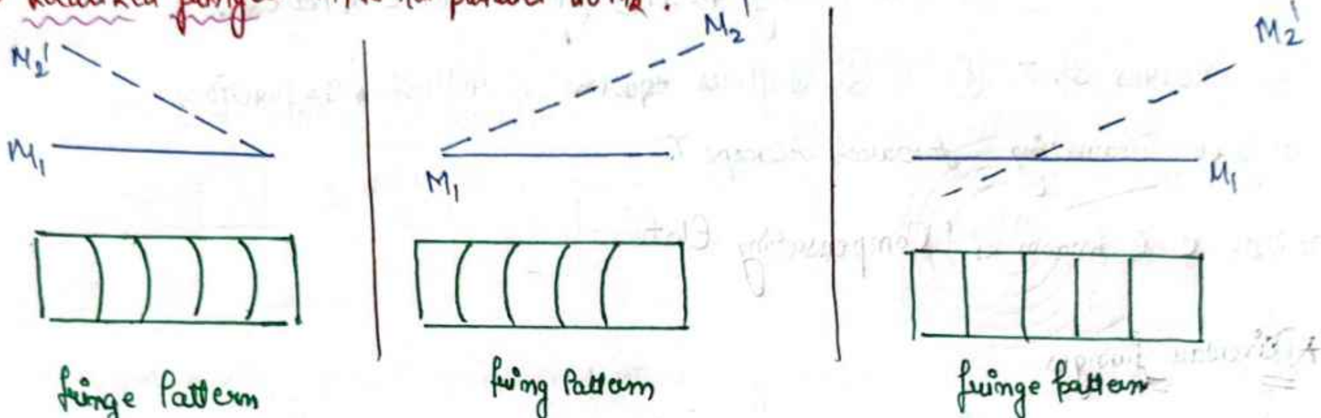
$2d\cos\theta - \lambda/2 = (2l+1)\lambda/2 \rightarrow 2d\cos\theta = 2l\lambda + \lambda = (l+1)\lambda = n\lambda \; ; n = l+1$

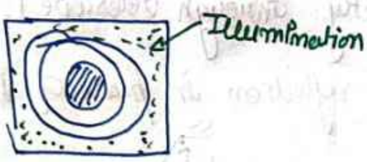
$\therefore \boxed{2d\cos\theta = n\lambda}$

• If M_1 & M_2' coincide, then eff. Path diff$^r = \frac{\lambda}{2}$ → Destructive Interference

• Observat^n in telescope :  ← completely dark field of view.

② Localized fringes. M_1 is not parallel to M_2'.



③ White light fringes 

• Central Dark Ring is Obtaine

• Some Colourful Rings are obtained then Uniform Illumination will be Observed.

* Construction & working

• It consists of a source (S) of Monochromatic light placed at focus of lens L.

• Lens L converts light coming from source into a parallel beam of light which is then allowed to fall on glass Plate G (Partially Silvered at its Back surface) which is inclined at an angle of 45°

• Half of light energy falling on plate G is reflected towards M_1 in the form of Beam (1)

• And other Half is transmitted towards Mirror M_2 as Beam (2)

• The Beam (1) after reflection from highly Polished Plane Mirror M_1 gets transmitted through plate G towards telescope T.

• The Beam (2) after reflection from highly Polished Mirror M_2 gets reflected towards telescope T.

- These 2 rays travelling towards telescope T interfere with each other & fringes are observed by the eye looking into mirror M_1 through telescope T.

→ Another glass plate C of same material as that of G parallel to plate G is introduced in the path of beam (2) B/W Mirror 2 & Plate G.

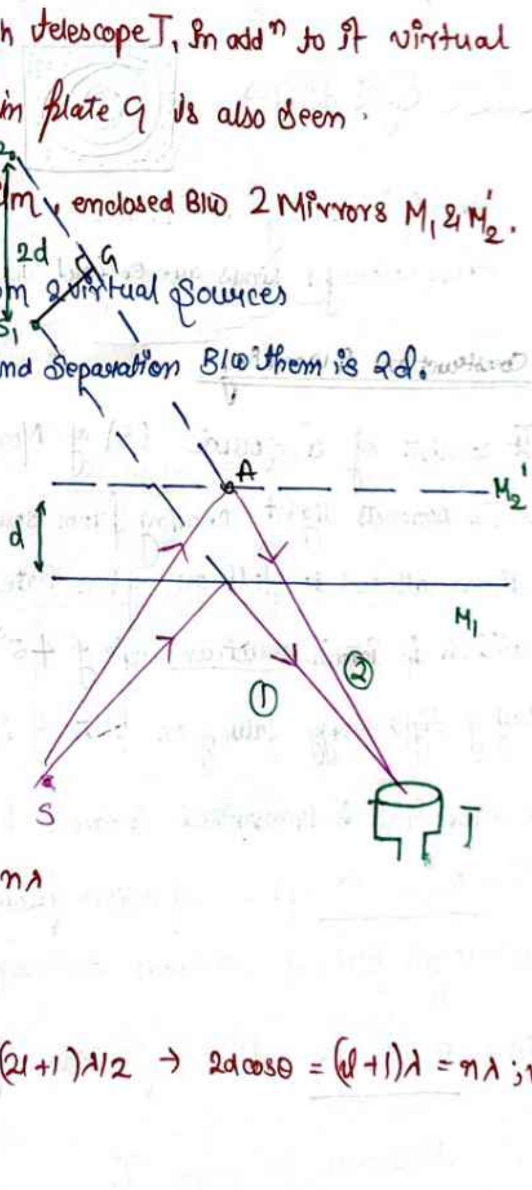
- It ensures that (1) & (2) suffers equal no. of reflections & refractions before travelling towards telescope T.

* Plate C is known as "Compensating Plate".

(i) Circular fringes

- When Mirror M_1 is seen directly through telescope T, In addⁿ to it virtual image M_2' of mirror M_2 formed by reflection in plate G is also seen.
- This forms a system equivalent to air film enclosed B/W 2 Mirrors M_1 & M_2'.

→ The 2 interfering beams appears to come from 2 virtual sources S_1 & S_2 derived from single source S and separation B/W them is 2d.



- Draw $S_1G \perp S_2A$, In Right Angled ΔS_1S_2G

$$\frac{S_1G}{S_1S_2} = \frac{Base}{Hypotenuse} = \cos\theta \Rightarrow \frac{S_1G}{2d} = \cos\theta$$

$$\rightarrow \boxed{S_1G = 2d\cos\theta}$$

- Eff. Path difference $(\Delta x) = 2d\cos\theta - \lambda/2$

- for Bright fringe, $\Delta x = n\lambda$ ∴ $2d\cos\theta - \lambda/2 = n\lambda$

$$\boxed{2d\cos\theta = (2n+1)\lambda/2} \rightarrow \text{Maxima}$$

for Dark fringe ; $\Delta x = (2l+1)\lambda/2$ ∴ $2d\cos\theta - \lambda/2 = (2l+1)\lambda/2 \rightarrow 2d\cos\theta = (l+1)\lambda = n\lambda$; $n = l+1$

$$\boxed{2d\cos\theta = n\lambda} \rightarrow \text{Minima}$$

- When M_1 & M_2 are exactly perpendicular to each other. Then M_1 & M_2' will be exactly ~~perpendicular~~ parallel to each other.

→ In that case we get circular fringes also known as Haidenger's fringes or fringes of equal inclination.

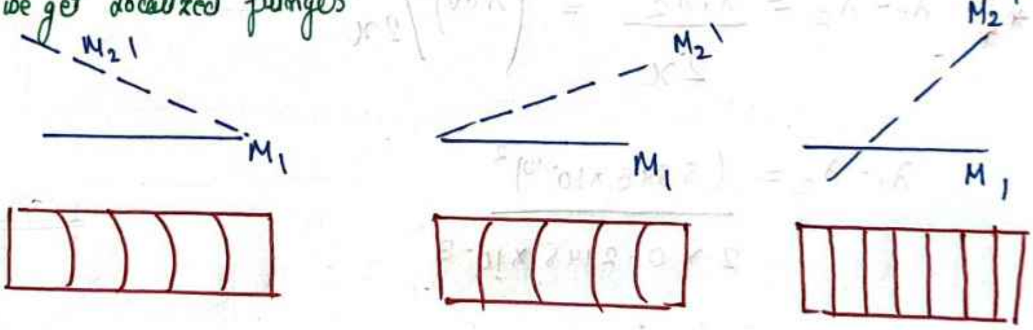
- fringe Pattern →

When $d \neq 0$ When $d = 0$

② Localised fringes

- When M_1 & M_2 are not exactly perpendicular to each other then M_1 & M_2' will not be parallel
- In this we get at air film of varying thickness enclosed B/W M_1 & M_2 (wedge shaped film).
- In such a film we get localized fringes.

- Orientation of → M_1 & M_2?
- fringe Pattern



③ fringes in white light

- In case of white light source, the centre ring is found to be dark. & all around it are 8 to 10 colourful fringes & Beyond that white light is seen Due to Overlapping of Different Colours.

2024, Ist sem

Q In Michelson's experiment the Mirror is displaced by 0.6939 mm to 0.9884 mm for two consecutive maximum indistinctness positions. If the Mean wavelength of D line is 5893 Å, Deduce the difference B/w two spectral lines.

Ans As we know that Maximum distinctness were found to be
0.6939 mm & 0.9884 mm i.e $x = 0.9884 - 0.6939$
$= 0.2945$ mm

Mean Wavelength of the doublet used = 5893 Å

* A waveform signal's wavelength is defined as the separation B/w two identical points (Adjacent crests) in adjacent cycles as the signal travels through space or over a wire

** $$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2x} = (\lambda_{av})^2 / 2x$$

$$\lambda_1 - \lambda_2 = \frac{(5893 \times 10^{-10})^2}{2 \times 0.2945 \times 10^{-3}}$$

$$= 5.89 \times 10^{-10}$$

$\therefore \lambda_1 - \lambda_2 = 5.89$ Å → Difference B/w two spectral lines is 5.89 Å

Determination of Difference of two nearly Equal wavelengths, Using Michelson Inferometer

- Michelson is set for circular fringes. Let the source has two wavelengths λ_1 & λ_2 ($\lambda_1 > \lambda_2$) which are very close to eachother

- The two wavelengths form their separate fringe patterns, But Because of Very Small change change in wavelength, the two patterns overlap.

- As the mirror M_1 is moved slowly the two patterns separate out slowly & when path difference is such that dark fringe of λ_1 falls on bright fringe of λ_2, the Result is Max^m indistinctness.

- When the path difference is such that bringh fringe of λ_1 falls on bright fringe or λ_2 {or same for dark fringes}, the result is Max^m Distinctness.

- Let the Mirror M_1 is to be moved through a distance x B/w two sucessive positions of Max^m Distinctness ~~is posn~~

- In this position n^{th} fringe of λ_1 must coincide with $(n+1)^{th}$ fringe of λ_2

$$\therefore \quad x = \frac{n\lambda_1}{2} = \frac{(n+1)\lambda_2}{2}$$

$$\Rightarrow \quad n = \frac{2x}{\lambda_1} \quad \& \quad (n+1) = \frac{2x}{\lambda_2}$$

$\therefore$ Subtracting we get

$$1 = \frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} = 2x\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right) \Rightarrow \lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2}$$

~~$\lambda_1 \lambda_2 = \lambda \Rightarrow \lambda$~~ $\quad \lambda_1 \approx \lambda_2 \approx \lambda \Rightarrow \lambda_1\lambda_2 \approx \lambda^2$ where $\lambda = \frac{\lambda_1 + \lambda_2}{2}$

$$\boxed{\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}}$$

, Thus by measuring distance x moved By Mirror M_1 the difference B/w wavelengths can be Determined.

* Determination /Measurement of Wavelength of Monochromatic light using Michelson inferometer.

- Monochromatic light from source S is allowed to fall on plate G_1.
- Then inferometer is adjusted for circular fringes.
- The movable mirror M_1 is displaced such that $G_1M_1 = G_1M_2$
- The mirrors M_1 & M_2 are made perfectly perpendicular to each other with the help of levelling screws at the Back of mirrors.
- Consequently, concentric Circular fringes are observed when viewed through telescope.
- Suppose the separatn B/w Real mirror M_1 & Virtual Mirror M_2 is such that a bright fringe of n^{th} order is formed at the centre of the field of view
- This fringe must satisfy the condition

$$\Delta = 2e = (n+1)\lambda$$

It is seen that when the value of e becomes $\left(e+\frac{\lambda}{2}\right)$, the n^{th} fringe at centre is replaced by $(n+1)^{th}$ fringe.

In other words on displacing the mirror M_1 by a distance of $\frac{\lambda}{2}$. One fringe is displaced in the telescope. is counted.

- Now the Mirror is gradually ~~displaced~~ moved & the number of fringes displaced in the telescope is counted.

- Let on moving M_1 through distance x, the no. of fringes displaced is N,

- On moving the mirror by $\lambda/2$, the no. of fringes displaced is 1

$\therefore$ On moving the mirror by x, the no. of fringes displaced will be $N = 2x/\lambda$

$\therefore \boxed{\lambda = \frac{2x}{N}}$; knowing the value of x & N experimentally, the wavelength of light used can be determined.

Effect on Newton Rings of

Q 3.11.a (i) Oil of lens < μ < glass plate is introduced.

Sol^n When a little oil is introduced B/w the lens ($\mu = 1.5$) & glass plate ($\mu = 1.75$) ($\mu = 1.65$)

then in this situat^n Both the Boundaries i.e lens to oil & oil to glass plate

Both are rarer to denser type.

· So a phase change of π will occur at both the Boundaries. In result

Dark & Bright fringes will be interchanged within the area of liquid.

* Plano-Convex lens of Small Radius is used

b) As we know that diameter of Bright & dark ring is directly proportional to the square root of the Radius of curvature of lens.

· Hence, if we use a lens of small radius of curvature, the diameter of the rings will get closer.. ~~Thi~~

· This will effect on percentage error in measurement of Diameter.

* Plano Convex lens is Raised By Height ΔH

d) We know that air film thickness is almost zero at Point of contact & gradually increases towards the periphery of the lens.

· $\Delta = 2t + \frac{\lambda}{2}$, for Max^m intensity, for Min^m Intensity

$$2\cancel{\Delta} = \cancel{(2n+1)\lambda/2} \qquad \cancel{2t}\ \cancel{\Delta} = \cancel{2t}\ n\lambda + \cancel{\tfrac{\lambda}{2}}$$

$$(2n+1)\lambda/2 + \cancel{\tfrac{\lambda}{2}}$$

. So, at point of contact, $t = 0$

$$\Delta = 2 \times 0 + \frac{\lambda}{2} = \frac{\lambda}{2}$$ ⇒ Path diff^r corresponds to Dark fringe at centre

If plano convex lens is lifted by Δh, then

$$t = \Delta h$$

$$\Delta = 2 \cdot \Delta h + \frac{\lambda}{2}$$ ⇒ Rings will shift ~~as~~ in accordance to the value of Δh.

Q What will happen if white light is used in Newton's Ring? 4 M

- When white light is used instead of monochromatic light. the interference pattern will not be as clear & distinct.
- This is because white light consists of a range of ~~wavelengt~~ wavelengths (colours); each will produce its own set of interference fringes.
- As a result, you will see a mixture of colour in the rings making it more difficult to observe & analyze the interference pattern.
- In summary, using white light in Newton's Ring Experiment will result in a more complex & colourful pattern compared to using monochromatic light, making it harder to study & analyze the interference fringes.

Q8.3

Sol^n

$D_n = 2\sqrt{n\lambda R}$ ⇒ Dia. of n^{th} dark ring

$D_{n+m} = 2\sqrt{(n+m)\lambda R}$

$* \; D_{n+m}^2 - D_n^2 = 4m\lambda R$

Case

$n = 4$; $D_n = 0.4 cm$

$n + m = 10$; $D_{n+m} = 6.2 cm$

$m = 6$

$D_{n+m}^2 - D_n^2 = 4m\lambda R$

$\cancel{(0.4)^2 - (} \quad (6.2)^2 - (0.4)^2 = 4 \times 6 \times \lambda R$

$\lambda R = 1.595$

Case 2

$n = 10$

$n + m = 18$

$m = 8$

$D_{18}^2 - (6.2)^2 = 8 \times \lambda R$

$D_{18} = \sqrt{(6.2)^2 + 8 \times 1.595}$

$$\boxed{D_{18} = 7.15 \, cm}$$

* Difference B/w Michelson Rings & Newton's Rings.

Michelson fringes	Newton Rings
1) They're formed due to Virtual air film	· They're formed due to real air film.
2) The air film is of equal thickness.	· The air film is of wedge shap 
3) They're also known as Haidenger fringes, { fringes of equal inclinatⁿ }	
4) Their order of Central fringe is of Max^m Range	· Their order of central fringe is zero (ie we start from $n = 0$)
5) They're a result of locus ~~film~~ of air film of equal thickness	· They are result of locus of air film of Varie thickness
6)	· They're localised in the plane of wedge shape air film
7) They're viewed with the help of telescope	· They're viewed with help of travelling microscope.

2023, IInd sem → 2018, IInd sem

Q3

(III) In michelson inferometer, $\lambda = \frac{2x}{n}$

Ans

Here $x = 0.5893\,mm = 0.05893 \times 10^{-3}\,m$

& $n = 200$

$$\lambda = \frac{2 \times 0.05896 \times 10^{-3}}{200} = 5896 \times 10^{-10}\,m$$

$$\lambda = 5896\,\mathring{A}$$

Q 4(a) 2014, IInd Sem

- Radius of n^{th} dark ring due to $\lambda_1 = \sqrt{n\lambda_1 R}$

- Radius of $(n+1)^{th}$ dark ring due to $\lambda_2 = \sqrt{(n+1)\lambda_2 R}$

- acc to Que.

$$\sqrt{n\lambda_1 R} = \sqrt{(n+1)\lambda_2 R}$$

$$n\lambda_1 R = (n+1)\lambda_2 R$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$n(\lambda_1 - \lambda_2) = \lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

[- Radius of n^{th} dark ring due to $\lambda_1 = \sqrt{\frac{\lambda_1 \lambda_2 R}{\lambda_1 - \lambda_2}}$]

Q 5(b)

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

: Here $D_{15} = 0.590$ cm, $p = 15 - 5 = 10$

$D_5 = 0.336$ cm

$R = 100$ cm ie 1m

$$\lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100} = \frac{0.3481 - 0.112896}{4000}$$

$$\boxed{\lambda = 5.8801 \times 10^{-5}\,\text{cm} = 5880.1\,\text{Å}}$$

Q 2013 IIIrd sem

- dia N^{th} ring $= 4 \cdot 8mm$, dia of $(n+8)^{th} = 7mm$, $R = 2m$, $\lambda = ?$

Solⁿ $\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4 \times R \times P}$, $P = 8$

$D_n = 4.8 \text{ mm} = 4.8 \times 10^{-3}$

$D_{n+8} = 7.0 = 7 \times 10^{-3}$

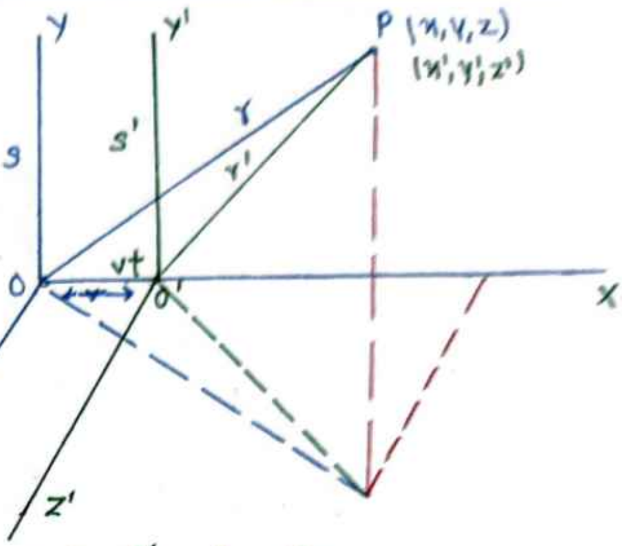
$$\lambda = \frac{(7^2 - 4.8^2) \times 10^{-6}}{4 \times 2 \times 8} = 0.405.625 \times 10^{-6} \times 10^4 \times 10^{-4}$$

$$\boxed{\lambda = 4056.25 \mathring{A}}$$

Relativistic Mechanics

L-1

* Galilean Transformation



- A particle in space can be located from a knowledge of its co-ordinates with its reference to a particular frame of reference. These coordinates will be diff^r in diff^r frame of references.
- Sometimes it is necessary to transform the co-ordinates from one reference to the other. This transformation of coordinates from one inertial frame of reference ~~is called to~~ another ~~to~~ inertial frame of reference is called Galilean Transformation.

→ Such sets of equations are called Transformation Equations.

- Now, consider two inertial frames of reference $S(x,y,z)$ & $S'(x',y',z')$ as shown in the fig. The Observer O is in reference frame S & O' in reference frame S'. The Observer O' moves with uniform velocity 'v' relative to O along the x axis. At time $t=0$, O & O' are coincident.
- After time t, $OO' = vt$

* Transformation of Position

- Consider a particle at P as shown in the fig.

Here $\vec{r} = \vec{r'} + vt$ or $\vec{r'} = \vec{r} - vt$ — ①

* As v is parallel to x axis ∴ separating the vector eq^n into three components

$$\left.\begin{array}{l} x' = x - vt \\ y' = y,\ z' = z\ \&\ t' = t \end{array}\right\} \text{— (II)}$$

- It is assumed that the two observers ~~,~~ are using the same time. It means that time measurement is independent of the mot^n of observers.

* The set of eq^n (II) are called ~~Galilean~~ Galilean transformations, because by using these eq^n, the results observed in one Reference frame can be transformed into the other reference frame.

Transformation of Velocity

- The velocity $\vec{v}$, of Particle P relative to Observer O is given by ; $\vec{v} = \frac{d\vec{r}}{dt}$

$$\vec{v} = \hat{i}\frac{d\vec{x}}{dt} + \hat{j}\frac{dy}{dt} + \hat{k}\frac{dz}{dt}$$

- The velocity $\vec{v}'$ of Particle P wrt Observer O' is given by, $\vec{v}' = \frac{d\vec{r}'}{dt}$

$$\vec{v}' = \hat{i}\frac{dx'}{dt} + \hat{j}\frac{dy'}{dt} + \hat{k}\frac{dz'}{dt}$$

* Differentiating eq^n (1) wrt t, we get

$$\frac{dr'}{dt} = \frac{dr}{dt} - v \Rightarrow \vec{V}' = \vec{V} - \vec{v} \quad \text{(III)}$$

- Diff^r eq^n (2) wrt t, we get

$$\frac{dx'}{dt} = \frac{dx}{dt} - v \Rightarrow \vec{v}_x' = \vec{v}_x - v$$

$$\frac{dy'}{dt} = \frac{dy}{dt} \;\&\; \frac{dz'}{dt} = \frac{dz}{dt} \Rightarrow \vec{v}_y' = \vec{v}_y \;\&\; \vec{v}_z' = \vec{v}_z$$

- Therefore, we can write : $\vec{v}_x' = \vec{v}_x - v$, $\vec{v}_y' = \vec{v}_y$ and $\vec{v}_z' = \vec{v}_z$ — (IV)

Transformation of Accelaration

- Accelaration of P relative to O is given by ; $\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2}$

- Accelaration of P relative to O' is given by ; $\vec{a}' = \frac{d\vec{V}'}{dt} = \frac{d^2\vec{r}'}{dt^2}$

* Diff^r eq^n (III) wrt t $\Rightarrow$ $\frac{d\vec{V}'}{dt} = \frac{d\vec{V}}{dt}$ $\left[\because v \text{ is constant} \therefore \frac{dv}{dt} = 0\right]$

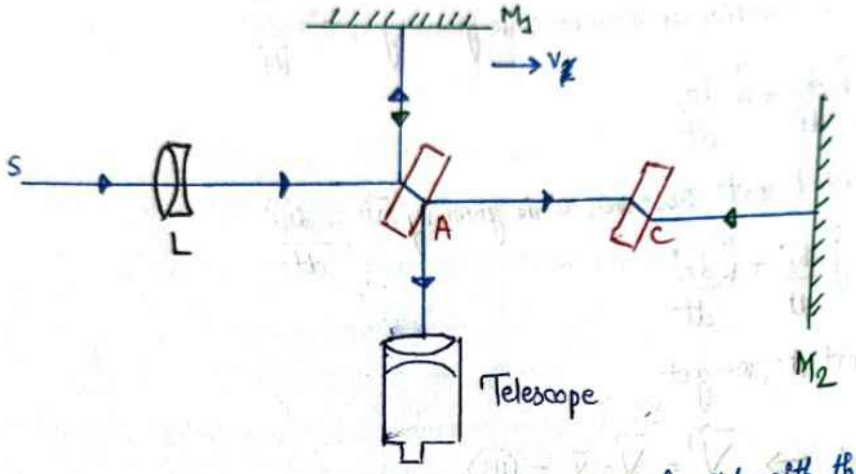
$$\Rightarrow \vec{a}' = \vec{a} \quad \text{(V)}$$

- Now Diff^r eq^n (IV), we get

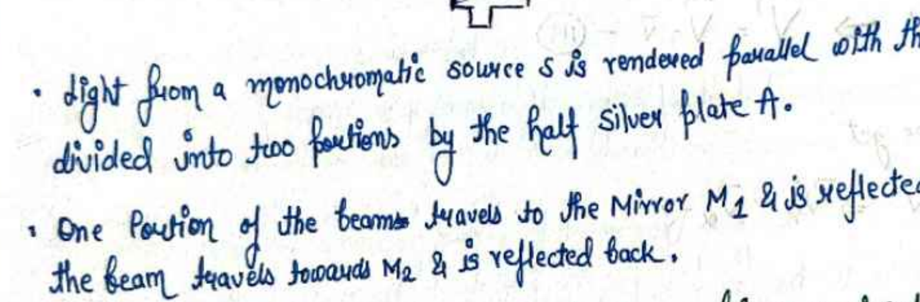
$$a_x' = a_x, \; a_y' = a_y, \; a_z' = a_z \quad \text{(VI)}$$

→ It means both the Observers O & O' observe the same accelaration, It also means accelarati remains invariant or unchanged when transformed from One frame of Reference to the other frame of Reference moving with uniform relative velocity.

Michelson Morley Experiment



- Light from a monochromatic source S is rendered parallel with the lens L & it is divided into two portions by the half silver plate A.
- One Portion of the beams travels to the Mirror M_1 & is reflected back. The other portion of the beam travels towards M_2 & is reflected back.
- The two reflected beams interfere & interference fringes are viewed with the help of telescope.
- The apparatus is arranged to move along the direction of Earth's Orbit round the sun. The speed of movement of the apparatus is equal to v i.e, speed of earth in its orbit.



- Acc. to Galilean frame of Reference, F is a fixed frame & F' is a frame of Reference moving with velocity v in the direction of the movement of apparatus. The velocity of light in the direcn of Movement of frame F' = c-v & in the opposite direcn equals to c+v
- Suppose T_1 is the time taken by light to travel from A to M_2' & T_2 be the time taken from M_2' to A as shown in fig. , $D = AM_1 = AM_2$

- Total time taken by light (to & fro) $\therefore T = T_1 + T_2 = \frac{2Dc}{c^2 - v^2}$ i.e. $\frac{D}{c+v} + \frac{D}{c-v}$

- Total distance travelled by light, $X_1 = T \times C = \frac{2Dc^2}{c^2 - v^2}$

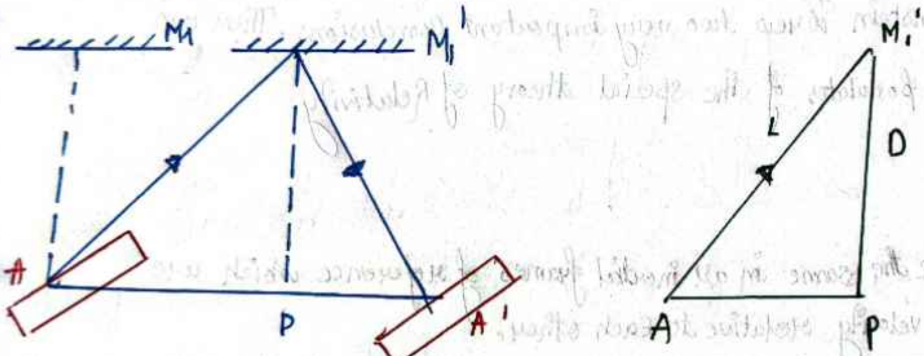
OR

$$X_1 = 2D\left[\frac{1}{\frac{c^2 - v^2}{c^2}}\right] \Rightarrow \cancel{X_1 = 2D\left[\frac{1}{1 - \frac{c^2}{v^2}}\right]}$$

OR

$$X_1 = 2D\left[\frac{1}{1 - \frac{v^2}{c^2}}\right] = 2D\left[1 - \frac{v^2}{c^2}\right]^{-1} = 2D\left[1 + \frac{v^2}{c^2} + \cdots\right]$$ Neglecting the higher powers

$$X_1 = 2D\left[1 + \frac{v^2}{c^2}\right] \quad \text{--- (I)}$$



- Let the time taken by light to travel from A to M_1' be T'. Here, M_1 is shifted to M_1' in this time A is shifted to P as shown in fig.

- Then, distance $AP = vT'$

But $T' = L/c \therefore AP = \frac{vL}{c}$

→ Applying Pythagoras theorem in $\Delta APM_1'$ $\therefore L^2 = D^2 + \frac{v^2 L^2}{c^2}$

$$L^2\left[1 - \frac{v^2}{c^2}\right] = D^2 \Rightarrow L^2 = D^2/1 - v^2/c^2 \Rightarrow L = D\left[1 - \frac{v^2}{c^2}\right]^{-1/2}$$

$$L = D\left[1 + 2\frac{v^2}{c^2}\right]$$ $\therefore$ Total distance travelled by light in time T in going from A to M_1' and back to $A' = 2L = 2D\left[1 + \frac{2v^2}{c^2}\right]$

$$X_2 = 2D\left[1 + \frac{v^2}{2c^2}\right] \quad \text{--- (II)}$$

• from eq^n ① & ②, the path difference is $(x_2 - x_1)(\Delta x)$

$$(x_2 - x_1) = \left(2D\left[1+\frac{v^2}{2c^2}\right]\right) - \left(2D\left[1+\frac{v^2}{c^2}\right]\right) = \frac{Dv^2}{c^2} \quad - ③$$

$$x_2 - x_1 = \frac{Dv^2}{c^2} \quad - (iii)$$

• if the apparatus is turned through 90°, the diff^n will be equal to $-\frac{Dv^2}{c^2}$.
Thus the displacement in the interference fringe in $\frac{8v^2}{c^2}$

* But in this experiment no displacement of fringes was observed. This shows that this experiment is a negative experiment.

L-3 · Postulates of the Special Theory of Relativity

• In 1950, Albert Einstein drew two very important conclusions. These are known as fundamental postulates of the special theory of Relativity

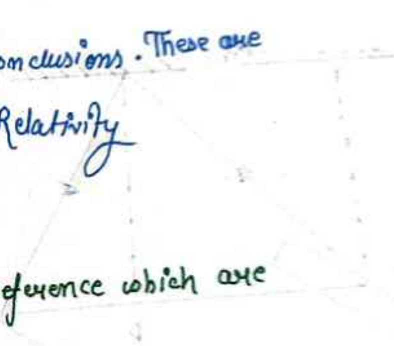
• These Postulates are

1) All physical laws are the same in all inertial frames of reference which are moving with constant velocity relative to each other.

2) The speed of light in vacuum is the same in every inertial frame.

• The theory based on these two postulates applies to all inertial frames and is called "Special Theory of Relativity".

L-4 Lorentz Transformation of Space & time.



• Consider two frame of reference A & B, along the direction of the X Axis, A is fixed & B is moving with constant speed v.

• Initially both the frames have the same origin of co-ordinates. After a time t frame of reference B has moved distance $OO' = vt$.

• for the point P in space the co-ordinates are (x, y, z) with reference to the frame A & (x', y', z') with reference to the frame B.

• Acc to Galilean frame of Reference

$$\left.\begin{array}{l} x' = x - vt \\ y' = y,\ z' = z,\ t' = t \end{array}\right\} - ①$$

→ Diff^r w.r.t t ∴ $\frac{dx'}{dt} = \frac{dx}{dt} - v$ — (II) OR $c' = c - v$ — (III)

• Acc to the second postulate of the special theory of Relativity the velocity of light remains constant in free space.

• The eq^n : $x' = x - vt$ is in accordance with the law of Mechanics.

• The new transformation of x-co-ordinate must be similar to this eq^n, when the value of v is extremely small as compared to the velocity of light c.

→ The simplest form possible form of the eq^n can be , $x' = k(x - vt)$ — (IV)

Here k depends upon only on the value of v & doesn't depend upon value of x & t.

→ Acc. to the first postulate of special theory of relativity, observations made in frame of reference B must be identical to those made in A, except for a change in the sign of v & having the same value for the constant of proportionality.

$$\therefore\ x = k(x' + vt') \ — (V)$$

* As the relative motion is confined to x only of A & B ∴ $y' = y$ — (VI)
$z' = z$ — (VII)

* Eq^n (IV) & (V) will not held if $t' = t$

• The value of x' is substituten in eq^n (5) from eq^n (4) this gives

$$x = k\,[k(x - vt) + vt']$$

$$= k^2(x - vt) + kvt' \ — (8)$$

$$kvt' = x - k^2(x - vt)$$

$$t' = \frac{x - k^2(x - vt)}{kv} = \frac{x - k^2 x + k^2 vt}{kv}$$

$$t' = kt + \left(\frac{1-k^2}{kv}\right)x \quad -(9)$$

- To find the value of k, consider two reference frames A & B.

* The spaceship in reference frame A measures the time t & spaceship in reference frame B measures the time t'.

* The x-coordinates for both of the ship will be -

$$x = ct \quad -(10)$$

$$x' = ct' \quad -(11)$$

- The value of c must remain constant in both the frames of reference acc. to second postulate of special theory relativity.

- substituting value of x' & t' in eq^n (11)

$$k(x-vt) = c\left[kt + \left(\frac{1-k^2}{kv}\right)x\right] \quad -(12)$$

$$kx - kvt = ckt + cx\left(\frac{1-k^2}{kv}\right)$$

$$kx - cx\left(\frac{1-k^2}{kv}\right) = kct + kvt$$

$$x\left(k - c\left(\frac{1-k^2}{kv}\right)\right) = ckt\left(1 + \frac{v}{c}\right)$$

$$x = \frac{ckt(1+v/c)}{k\left[1 - c\left(\frac{1-k^2}{k^2 v}\right)\right]}$$

$$x = ct\left[\frac{\left(1+\frac{v}{c}\right)}{1 - \left(\frac{c}{v}\right)\left(\frac{1}{k^2} - 1\right)}\right]$$

* This value of equal must be equal to value of x in eq^n x

$$ct = ct\left[\frac{(1+v/c)}{1 - \left(\frac{c}{v}\right)\left(\frac{1}{k^2} - 1\right)}\right]$$

$$\therefore 1 - \left(\frac{c}{v}\right)\left(\frac{1}{k^2} - 1\right) = 1 + \frac{cv}{vc} \Rightarrow -\frac{c}{v}\left(\frac{1}{k^2} - 1\right) = \frac{v}{c}$$

$$1 - \frac{1}{k^2} = \frac{v^2}{c^2} \Rightarrow \frac{1}{k^2} = 1 - \frac{v^2}{c^2} \Rightarrow k^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\left[k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right] - (12)$$

- When the value of v is extremely small in comparison to c, the value of k is practically equal to 1
- Now, substituting value of k in eq (4), (6), (7) & (9), we get

$$x' = k(x - vt) \Rightarrow x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - (13)$$

$$y' = y - (14)$$

$$z' = z - (15)$$

$$t' = kt + \left(\frac{1 - k^2}{kv}\right)x \therefore t' = kt + \frac{x}{kv} - k\left(\frac{x}{v}\right) = kt + \frac{x}{v}\left(\frac{1}{k} - k\right)$$

OR

$$\cancel{t' = t + \frac{x}{v}\left[1 - v^2\right.} \quad t' = kt + kx/v\left(\frac{1}{k^2} - 1\right)$$

OR

$$t' = t + \frac{x}{v}\left[1 - \frac{v^2}{c^2} - 1\right] \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - (16)$$

* Eq^n (13), (14), (15) & (16) are called Lorentz transformations
- These eq^n gives the conversion for the measurements of time, space made in stationary frame A to their counterparts in the moving frame B

• The inverse Lorentz transformation for measurements of time & space made in the frame B to their counterparts in frame A can be written as follows

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y = y', \quad z = z' \quad \& \quad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

* Velocity Addition Theorem

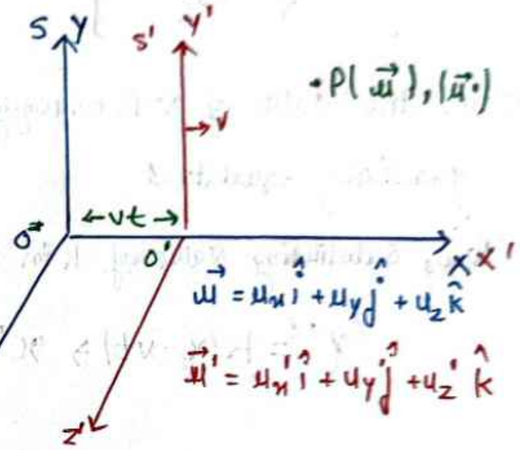
• Galilean Transformation

$x' = x + vt$, $u_x' = u_x - v$

$y' = y$, $u'_y = u_y$

$z' = z$, $u'_z = u_z$

$t' = t$



• Lorentz transformations

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z \quad \& \quad t' = \frac{t - vx}{\sqrt{1 - \frac{v^2}{c^2}}}$$

* Consider two frames S & S', where S' is moving towards +ve x direction relative to S with velocity v.

• Suppose a particle moves with $\vec{u}$ whose components are u_x, u_y, u_z relative to frame S & with velocity $\vec{u}'$ whose components are u_x', u_y' & u_z' relative to frame S'.

→ $u_x = \frac{dx}{dt}$ — (1) & $u_x' = \frac{dx'}{dt'}$ — (4)

$u_y = \frac{dy}{dt}$ — (2) $\quad u_y' = \frac{dy'}{dt'}$ — (5)

$u_z = \frac{dz}{dt}$ — (3) $\quad u_z' = \frac{dz'}{dt'}$ — (6)

Acc to Lorentz transform^n

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad — (7)$$

$$t' = \frac{t - vx}{\sqrt{1 - \frac{v^2}{c^2}}} \quad — (8)$$

* Differentiating eq^n (7), we get ,

$$dx' = \frac{dx - vdt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad — (9)$$

Differentiating (8), on Both sides

$$dt' = \frac{dt - vdx}{\sqrt{1 - \frac{v^2}{c^2}}} \quad — (10)$$

Using (9) & (10) in (4), we get

$$u'_x = \frac{dx'}{dt'} = \frac{dx - v\,dt}{\sqrt{1-\frac{v^2}{c^2}}} \Big/ \frac{dt - \frac{v}{c^2}dx}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$u'_x = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx} \times \frac{1/dt}{1/dt} = \frac{\frac{dx}{dt} - v\frac{dt}{dt}}{\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}}$$

$$\therefore \boxed{u'_x = \frac{u_x - v}{1 - u_x \cdot \frac{v}{c^2}}}$$

* Acc. to Lorentz Transformation

$$z' = z$$

$$dz' = dz \quad \text{—(11)}$$

$\times \frac{1}{dt'}$

$$\frac{dz'}{dt'} = \frac{dz}{dt'}$$

$$u_z' = \frac{dz}{dt - \frac{v}{c^2}dx} \times \sqrt{1-\frac{v^2}{c^2}}$$

$$u_z' = \frac{\frac{dz}{dt} \times \sqrt{1-\frac{v^2}{c^2}}}{\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}} \quad \text{—(13)}$$

$$\boxed{u'_z = \frac{u_z \cdot \sqrt{1-\frac{v^2}{c^2}}}{1 - u_x\frac{v}{c^2}}}$$

$$y' = y$$

$$dy' = dy \quad \text{—(12)}$$

Dividing by dt'

$$\frac{dy'}{dt'} = \frac{dy}{dt'}$$

$$u_y = \frac{dy}{dt - \frac{v}{c^2}dx} \cdot \sqrt{1-\frac{v^2}{c^2}}$$

$$u_y' = \frac{\frac{dy}{dt} \cdot \sqrt{1-\frac{v^2}{c^2}}}{\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}} \quad \text{—(14)}$$

$$\boxed{u_y' = \frac{u_y \cdot \sqrt{1-\frac{v^2}{c^2}}}{1 - u_x\frac{v}{c^2}}}$$

* if $v <<<< c$ then

$$u'_x = u_x - v,\ u_y' = u_y \ \&\ u_z' = u_z$$

* Proof of Second Postulate of Theory of Relativity

- $u'_x = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}}$

Let $u_x = c$

$$u_x' = \frac{c - v}{1 - \frac{cv}{c^2}}$$

$$u'_x = \frac{c - v}{1 - \frac{v}{c}}$$

$$u'_x = \frac{c - v}{(c - v)} \cdot c \Rightarrow \boxed{u'_x = c}$$

velocity with reference to frame s' is also c.

$\therefore$ It is not possible for a particle to move with a velocity greater than light.

L-5 · Time Dilation

- The word Time Dilation mean to lengthen an interval of time.
- Consider two systems S & S'. Let S' be moving with velocity v wrt S in +ve x direc^n.
- Let a clock situated in the system S, at posit^n "x" & give signals at interval, Δt

$$\Delta t = t_2 - t_1 \quad -(1)$$

- If this interval is observed by an observer in system S', then Δt is given by

$$\Delta t' = t_2' - t_1' \quad -(11) \rightarrow$$

- from Lorentz transformations, we have

$$t_1' = \frac{t_1 - \left(\frac{vx}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \quad -(3)$$

&

$$t_2' = \frac{t_2 - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad -(4)$$

• Putting the value of t_1' & t_2' from eq^n ③ & ④ in eq^n ②, we get

$$\Delta t' = t_2 - \frac{vx}{c^2} - t_1 + \frac{vx}{c^2} \Big/ \sqrt{1-\frac{v^2}{c^2}}$$

$$\left\{ \Delta t' = \frac{t_2 - t_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} \right\}$$

• This eq^n shows that $\Delta t' > \Delta t$ i.e., the time interval in system S' is greater than time interval in system S.

L-6

Q1 A certain process requires 10^{-6} sec to occur in an atom at rest in laboratory. How much time will this process require to an observer in the lab. when atom is moving with speed 5×10^{7} m/s.

Ans. Given: $\Delta t = 10^{-6}$ sec ; $v = 5\times10^{7}$ m/s $= 5\times10^{9}$ cm/s

we know that, $\Delta t' = \dfrac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\Delta t' = \frac{10^{-6}}{\left[1-\left(\frac{5\times10^{9}}{3\times10^{8}}\right)^2\right]^{1/2}} = \frac{10^{-6}}{\left[1-\frac{25}{900}\right]^{1/2}}$$

$$\Delta t' = 1.028\times10^{-6}\ \text{sec}$$

Q2 A certain particle called μ-mesons has a life time of 2×10^{-6} sec, what is the mean life time when the particle is travelling with speed of 2.994×10^{10} cm/sec. How far does it go during one ~~life time~~, mean life.

Ans Here, given that, $\Delta t = 2\times10^{-6}$ s , $v = 2.994\times10^{10}$ cm/s

we know that, $\Delta t' = \dfrac{\Delta t}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \dfrac{2\times10^{-6}}{\left[\left(1-\left(\frac{2.994\times10^{10}}{3\times10^{10}}\right)^2\right)\right]^{1/2}} = 31.7\times10^{-6}$ sec

Distance travelled by the particle in one mean life $S = v\times t'$

$$S = 2.994\times10^{10}\times31.7\times10^{-6}$$

$$[S = 9510\text{m}]$$

Length Contraction

- Let us consider a rigid rod at rest in a moving frame S', moving along X-axis, with a speed v.
- The length of rod is equal to the difference B/w coordinates of its two end points
- Thus an observer in the spaceship measures the length of the Rod to be

$$L_0 = x_2' - x_1' \quad \text{— (1)}$$

→ L_0 is said to be the proper length of the Rod.

- Now, an observer on the earth frame S will have to determine the length of Rod by marking forward end position & the rearward posit^n^ of the rod at the same instant of time. Thus he will mark the coordinates of Rod as x_1 & x_2 in his frame.

- Acc. to Lorentz transformation eq^n^

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ we have}$$

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \& \quad x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

So,

$$x_2' - x_1' = \frac{x_2 - vt - x_1 + vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— (11)}$$

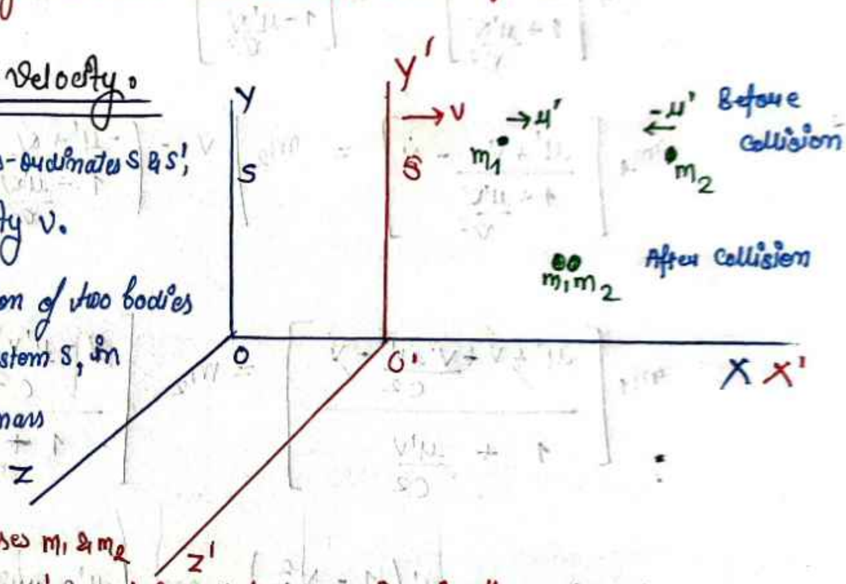
- where, $L = x_2 - x_1$ is the length of the rod as measured by an observer in stationary frame.
- It follows that, $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$

- Since $\sqrt{1-\frac{v^2}{c^2}}$ is always less than 1, so L is less than L_0.
- It means that if an observer at rest w.r.t a body measures its length to be L, an observer moving with a relative speed v w.r.t will find it shorter than its proper length by a factor of $\sqrt{1-\frac{v^2}{c^2}}$
- Hence, the Rod in the spaceship is contracted as measured from earth.
- This effect is symm^r; a rod on earth will undergo contract^n when measured from space ship

* This effect is known as length contract^n; Occurs in direc^n of mot^n

L-8 Variation of Mass with Velocity.

- Consider two systems of co-ordinates S & S', the latter moving with velocity v.
- Now, Consider ~~of the~~ a collision of two bodies in system S' & view it from system S, in order to consider variation of mass with velocity.



* Suppose two bodies of masses m_1 & m_2 are travelling with velocities u' & -u' parallel to X Axis in the system S'

- Suppose two bodies collide & after collision combine into one body

* So, acc. to conservation of mass, the mass of combined bodies after collision is equal to 2m'.

- As two bodies were moving with same velocity in opposite direc^n, Hence after collision they are at rest in system S'

→ Using the law of Add^n of velocities, the velocities u_1 & u_2 is system S corresponding to u_1' & $-u_2'$ are given by

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad \& \quad u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \quad \text{—} (1)$$

· Let the mass of the body traveling with velocity u_1 be m_1 & that of the Body moving with velocity u_2 be m_2.

* Applying the principle of conservation of Momentum, we have

$$m_1 u_1 + m_2 u_2 = (m_1+m_2) v \quad - (2)$$

· As after collision two bodies are combined into one body & moving with velocity in frame S.

* from Eqⁿ (1), putting Values of u_1 & u_2 in eqⁿ (2), we get

$$m_1\left[\frac{u'+v}{1+\frac{u'v}{c^2}}\right] + m_2\left[\frac{-u'+v}{1-\frac{u'v}{c^2}}\right] = (m_1+m_2)v$$

$$m_1\left[\frac{u'+v}{1+\frac{u'v}{c^2}} - v\right] = m_2\left[v - \left(\frac{-u'+v}{1-\frac{u'v}{c^2}}\right)\right]$$

$$m_1\left[\frac{u'+\cancel{v}+\frac{v^2u'}{c^2}-\cancel{v}}{1+\frac{u'v}{c^2}}\right] = m_2\left[\frac{\cancel{v}-\frac{u'v^2}{c^2}+u'-\cancel{v}}{1+\frac{u'v}{c^2}}\right]$$

$$\cancel{\frac{m_1}{m_2} = \frac{u'\left(1-\frac{v^2}{c^2}\right)}{u'\left(1+\frac{v^2}{c^2}\right)}} \qquad m_1\left[\frac{u'-\frac{u v^2}{c^2}}{1+\frac{u'v}{c^2}}\right] = m_2\left[\frac{u'-\frac{u'v^2}{c^2}}{1-\frac{u'v}{c^2}}\right]$$

$$* \quad \frac{m_1}{m_2} = \left(1+\frac{u'v}{c^2}\right)\Big/\left(1-\frac{u'v}{c^2}\right) \quad -(3)$$

· from Eqⁿ (1), It can be proved that : $\frac{u'v}{c^2} = \frac{2c^2 - u_1^2 - u_2^2 - 2\sqrt{(c^2-u_1^2)^{\frac{?}{2}}(c^2-u_2^2)}}{u_1^2 - u_2^2}$ (4)

· Putting Value of $\frac{u'v}{c^2}$ from eqⁿ (4) in eqⁿ (3), we get:

$$\frac{m_1}{m_2} = \frac{\left(1-\frac{u_2^2}{c^2}\right)^{1/2}}{\left(1+\frac{u_1^2}{c_1^2}\right)^{1/2}}$$

· Let the body of mass m_2 be moving with zero velocity in system S, before collision i.e $u_2 = 0$, then

$$\frac{m_1}{m_2} = \frac{1}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

· In commonly used notation ; $m_1 = m$, & $m_2 = m_0$ & $u_1 = v$

$$\therefore \frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

OR

$$\left[m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \right]$$

* This is the Relative formula for the variation of Mass with velocity, Here m_0 is said to be resting mass & m is the effective mass.

→ Now consider the following situations:-

(I) When v is very small compared to c, $\frac{v^2}{c^2}$ will be negligible in comparison to unity. ∴ $m = m_0$

i.e - At velocities which are much smaller than velocity of light, the mass of moving object is the same as its rest mass.

(II) When v is comparable to c, then $\sqrt{1-\frac{v^2}{c^2}}$ less than unity i.e $m > m_0$

i.e at velocities which are comparable to velocity of light, the mass of moving object appears to be greater than at rest.

(III) When $v = c$ or greater than c, then $\frac{v^2}{c^2}$ will become equal to or greater than unity, so $m \to \infty$

i.e at velocities which are equal to greater than c, the mass of object becomes infinite or imaginary.

L-9

Q Deduce the fractional inc. of mass of particle with for velocity 0.1 c.

Ans Since, we know that, $m = m_0 / \sqrt{1-\frac{v^2}{c^2}}$

$$\Rightarrow \frac{m}{m_0} = 1/\sqrt{1-\left(\frac{0.1c}{c}\right)^2} = \frac{1}{(1-0.01)^{1/2}} = (1-0.01)^{-1/2}$$

$$\frac{m}{m_0} = 1+0.005 \Rightarrow \frac{m}{m_0} - 1 = 0.005$$

$$\frac{m-m_0}{m_0} = \text{fractional inc. in mass} = 0.005$$

L-10 Relation Between Energy & Momentum.

- Since, Both the energy E & momentum p of a particle have diff^r values in diff^r reference frames.

* However the Quantity $(E^2 - p^2c^2)$ is invariant & has same value in diff^r reference frame.

$$\therefore E = mc^2$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{--- (1)}$$

* Squaring Both sides of eq^n (1), we get

$$E^2 = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}} \Rightarrow E^2\left(1 - \frac{v^2}{c^2}\right) = m_0^2 c^4$$

$$E^2 - \frac{E^2 v^2}{c^2} = m_0^2 c^4 \Rightarrow E^2 = \frac{\underbrace{E^2}_{mc^2} v^2}{c^2} + m_0^2 c^4$$

$$E^2 = m^2c^4\left(\frac{v^2}{c^2}\right) + m_0^2 c^4$$

$$\therefore E^2 = m_0^2 c^4 + c^2 (mv)^2$$

- Denoting $mv = p$, we write

$$E^2 = p^2c^2 + m_0^2 c^4 \quad \text{--- (ii)} \quad ; \quad (m_0c^2)^2 = E_0^2$$

$$\downarrow$$

$$E = c\sqrt{p + m_0^2 c^2} \quad \text{--- (iii)}$$

- we can also write eq^n (2) as

$$E^2 = E_0^2 + p^2c^2$$

$$\boxed{E^2 - p^2c^2 = E_0^2}$$

The above eq^n doesn't contain v, which implies that the quantity $(E^2 - p^2c^2)$ is independent of the velocity of particle.

- Alternatively, we can express momentum p in terms of Energy E as

$$\boxed{p = \frac{\sqrt{E^2 - E_0^2}}{c}}$$

L-11 Problem on Equivalence of Mass & Momentum

Q How much electric energy can theoretically be obtained by anihilation of 1g of matter?

Ans We know that, $E = mc^2$; Here $m = 1g$, $c = 3\times10^{10}$ cm/s

$\therefore E = 1\times(3\times10^{10})^2 = 9\times10^{20}$ ergs

further $1ev = 1.602\times10^{-12}$ ergs

or $E = \dfrac{9\times10^{20}}{1.602\times10^{-12}}$ eV $\therefore \boxed{E = 5.618\times10^{32}\,eV}$

L-12 An e^- is moving with speed 0.99c, what is its Total energy? find Ratio of Newtonian K.E to the relativistic K.E.

Ans Given, $v = 0.99c$

Since we know that, $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{9.1\times10^{-31}}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}} = \dfrac{9.1\times10^{-31}}{\sqrt{1-0.98}} = \dfrac{9.1\times10^{-31}}{0.141}$

$$m = 64.5\times10^{-31}\,kg$$

Now, total energy ; $E = mc^2$

$= (64.5\times10^{-31}\,kg)\times(3\times10^{8}\,m/s)^2$

$= 5.8\times10^{-13}$ Joule

- Newtonian kinetic Energy $= \frac{1}{2}m_0v^2$

Relativistic K.E = Total energy - Rest Energy

$= mc^2 - m_0c^2$

* $\dfrac{\text{Newtonian K.E}}{\text{Relativistic K.E}} = \dfrac{\frac{1}{2}m_0v^2}{mc^2 - m_0c^2} = \dfrac{1}{2}\dfrac{m_0}{(m-m_0)}\times\left(\dfrac{v}{c}\right)^2 = \dfrac{1}{2}\dfrac{9.1\times10^{-31}}{55.4\times10^{-31}}\times\left(\dfrac{0.99c}{c}\right)^2$

$$\boxed{\dfrac{\text{Newtonian KE}}{\text{Relativistic K.E}} = 0.08}$$

Quantum Mechanics.

* Planck's Quantum Hypothesis

- According to Classical theory, the energy changes of radiators take place, continuously But according to Planck's idea the energy changes takes place Discontinuously & Discretely as an integral Multiple of a small unit of Energy which is called a Quantum.
- He put forward the Hypothesis that the Radiation field in a uniform temperature enclosure ~~is~~ remains in a state of Dynamic Equilibrium, by way of emission & absorption processes.,
- for this to take place he assumed the presence of Harmonic Oscillators of Molecular Dimensions lined By Along the walls of Enclosure
- They absorb energy from ~~Oscillator~~ radiation field & Give it Back to field in characteristic way as proposed By Planck.
- Any such ~~Radiator~~ Oscillator Does not Radiate until energy reaches an integral Multiple of a Certain Minimum, Called a Quantum of Energy, And it is taken to be proportional to the frequency of the oscillator.

* The Main pts that Establish the Radiatn Law are—

1. An oscillator absorbs energy from the Radiation field & Delivers it Back to the field in Quanta of $0, E, 2E, \ldots$ etc, where E is a Quantum of Energy proportional to frequency ν of oscillator. $E = h\nu$, $h \to$ Planck's Constant.

- The number of Oscillators emitting particular energy is given By the statistical Distributn Law of Boltzman.

* Planck's Radiation law

- Acc. to Bose-Einstein Distribution law, the No. of photons having Energy in the Range u & $u+du$ are given By

$$\eta(u)\,du = \frac{g(u)\,du}{e^{\alpha+\beta u}-1} \quad -(1) \qquad , \text{ as } \eta_i = \frac{g_i}{e^{\alpha+\beta u_i}-1}$$

- As photons ~~as~~ are continuously absorbed OR emitted by the walls of Black Body, so the no. of photons may not be constant i.e

$$\eta = \eta_1 + \eta_2 + \dots + \eta_k$$

$$\eta = \sum_{i=1}^{k} \eta_i \neq \text{constant}$$

$$\sum_{i=1}^{k} dn_i \neq 0 \quad -(2)$$

- But in the derivation of eq^n (B) we used $\sum_{i=1}^{k} \alpha\, dn_i = 0 \quad -(3)$

→ from eq^n (2) & (3), we have

$$\boxed{\alpha = 0} \; -(4) \quad \text{Also } \boxed{\beta = \frac{1}{kT}} \; -(5) \quad , k \rightarrow \text{Boltzmann Constant}$$

* using (4) & (5) in eq^n (1), we get

$$\boxed{\eta(u)\,du = \frac{g(u)\,du}{e^{u/kT}-1}}$$

- As $u = h\nu$, so no. of photons having frequency in Range of ν & $\nu + d\nu$ are

$$\boxed{\eta(\nu)\,d\nu = \frac{g(\nu)\,d\nu}{e^{h\nu/kT}-1}} \quad -(6)$$

* The no of phase space cells corresponding to the Momentum p & $p+dp$ are given by

$$g(p)\,dp = \frac{4\pi V p^2}{h^3}\,dp \quad -(7)$$

$V \to$ Volume of System
$h^3 \to$ Volume of space cell
$p \to$ Momentum

• Due to Left Handed & Right Handed Polarisation of photons, the system may be considered ~~of~~ to be Made up of 2 subsystems each Having Volume (V) & Total Volume is taken as $2V$, so from eq^n (7)

$$g(p)\,dp = \frac{4\pi(2V)p^2\,dp}{h^3}$$

$$g(p)\,dp = \frac{8\pi V p^2\,dp}{h^3}$$

• As Momentum of Photon $p = \frac{h\nu}{c} \Rightarrow dp = \frac{h}{c}d\nu$

So we have

$$\cancel{g(p)\,dp} = \frac{8\pi V\left(\frac{h\nu}{c}\right)^2\left(\frac{h\,d\nu}{c}\right)}{h^3} = g(\nu)\,d\nu$$

$$g(\nu)\,d\nu = \frac{8\pi V \nu^2\,d\nu}{c^3} \quad -(8)$$

No. of phase space cells correspond^n to frequency ν & $\nu + d\nu$

Using (8) in (6)

$$\cancel{g(\nu)\,d\nu}\ \eta(\nu)\,d\nu = \frac{8\pi V \nu^2\,d\nu}{c^3\left(e^{h\nu/kT} - 1\right)} \quad -(9)$$

- The Energy of Radiations in frequency range ν to $\nu + d\nu$ is given by-

$$\eta(\nu)\, d\nu \cdot h\nu = h\nu \times \frac{8\pi V \nu^2 d\nu}{c^3(e^{h\nu/kT}-1)} = \frac{8\pi V h\nu^3 d\nu}{c^3(e^{h\nu/kT}-1)}$$

- Energy Density (which is amount of energy / unit Volume) is given by

$$E_\nu\, d\nu = \frac{8\pi \cancel{V} h\nu^3 d\nu}{c^3(e^{h\nu/kT}-1)} \Big/ \cancel{V}$$

$$\therefore \boxed{E_\nu\, d\nu = \frac{8\pi h\nu^3 d\nu}{c^3(e^{h\nu/kT}-1)}} \Rightarrow$$ Planck's Radiation law in terms of frequency

- As $\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$, so

- $$E_\lambda\, d\lambda = \frac{8\pi h \left(\frac{c}{\lambda}\right)^3 \cdot \left(-\frac{c}{\lambda^2} d\lambda\right)}{c^3\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

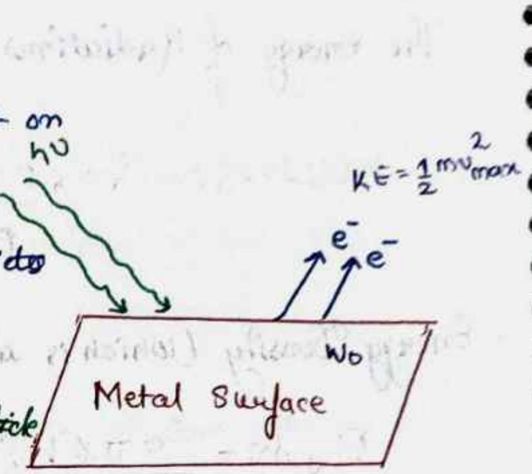
Neglecting -ve sign

- $$\boxed{E_\lambda\, d\lambda = \frac{8\pi h c\, d\lambda}{\lambda\left(e^{\frac{hc}{\lambda kT}} - 1\right)}}$$

Planck's Radiation law in terms of Wavelength in the Wavelength Range λ to $\lambda + d\lambda$.

Einstein Photoelectric Effect Equation

- In 1905, Einstein explained photoelectric effect on the Basis of ~~that~~ Planck's Quantum Theory acc. to which light Radiations consist of tiny particles packets of Energy called photons.



- Einstein Suggested that when a photon of particle energy $h\nu$ falls on Metal Surface, its energy is used in 2 ways. A part of it equal to work function W_0 is used in ejecting the e^- out of the surface.
- Remaining part appears as Max^m kinetic energy of ejected e^-.

$$\left[h\nu = W_0 + \frac{1}{2}mv_{max}^2\right] - (1)$$

→ Now, $W_0 = h\nu_0$ — (2) ; where ν_0 is threshold frequency

- Using (2) in (1), we get

$$h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2$$

~~$h(\nu - \nu_0)$~~

$$\boxed{h(\nu-\nu_0) = \frac{1}{2}mv_{max}^2}$$ → Einstein's Photoelectric equation.

* Explanation of Laws of Photoelectric Emission By Einstein's Equation.

- Einstein's Photoelectric Eq^n is : $h(\nu-\nu_0) = \frac{1}{2}mv_{max}^2$ — (1)

(a) If $\nu < \nu_0$, then from eq^n (1) - $\frac{1}{2}mv_{max}^2$ Becomes -ve which is not possible. Hence, No photoelectron will be ejected for $\nu < \nu_0$

(b) from eqⁿ ①, it is clear that maximum velocity V_{max} of photo e^- depends on frequency of ~~indirect~~ incident radiation. It is also clear that V_{max} increases with increase of ν_0.

c) When the intensity of incident light is doubled, the no. of incident photons are doubled which leads to double the no. of photo e^- emitted But Max^m K.E is still equal ~~to~~ to $h(\nu-\nu_0)$.

- It depends upon only ν of radiations & not on intensity of light. In this way independence of Max^m K.E with intensity of incident light is explained.

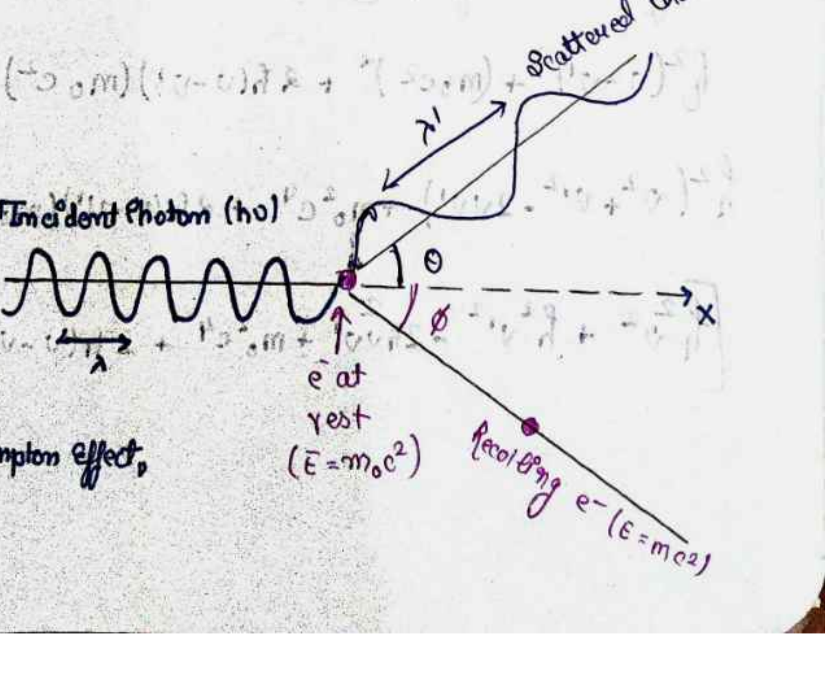
d) As the emission of a ~~photoelectric~~ photoelectron involves a collision B/W incident photon & an e^- of Metal, so photoelectric emission is an instantaneous process & there is no time lag B/W incidence of photon & emission of e^-

- Thus ~~f~~ Einstein successfully explained photoelectric effect. On the Basis of Planck's Quantum Theory.

- It also proves that Electromagnetic Radiations consists of tiny packets of Energy called photons which Behaves like particles.

Compton Effect.

- When a high frequency radiations are scattered by e^- of the scatterer. The frequency of the scattered wave is smaller than the frequency of incident Radiation. This Phenomena is called Compton effect.



- A·H Compton in 1923 explained this effect By considering the particle nature of Electromagnetic wave.
- The change in wavelength of High Energy Radiation During scattering process is called Compton shift & is given by

$$\boxed{\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)}$$

* Proof

Before Collision	After Collision
1) Energy of incident Radiation $= h\nu$	1) Energy of scattered photon $= h\nu'$
2) Momentum of incident photon $= \frac{h\nu}{c}$	2) Momentum of scattered photon $= \frac{h\nu'}{c}$
3) Rest Mass energy of $e^- = m_0c^2$	3) Energy of $e^- = mc^2$, where $m = m_0/\sqrt{1-v^2/c^2}$
4) Momentum of $e^- = 0$	4) Momentum of Recoiling $e^- = mv$

- Using law of Conservation of Energy

$$\rightarrow h\nu + m_0c^2 = h\nu' + mc^2$$

$$h\nu - h\nu' + m_0c^2 = mc^2 \implies \left[h(\nu-\nu') + m_0c^2 = mc^2\right] \quad \text{--- (1)}$$

* Squaring Both sides of Eqn (1)

$$h^2(\nu-\nu')^2 + (m_0c^2)^2 + 2h(\nu-\nu')(m_0c^2) = m^2c^4$$

$$h^2(\nu^2+\nu'^2-2\nu\nu') + m_0^2c^4 + 2h(\nu-\nu')(m_0c^2) = m^2c^4$$

$$\left[h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + m_0^2c^4 + 2h(\nu-\nu')m_0c^2 = m^2c^4\right] \quad \text{--- (2)}$$

* Applying law of Conservation of Momentum along X Axis

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$h\nu = h\nu'\cos\theta + mvc\cos\phi \rightarrow \left[mvc\cos\phi = h\nu - h\nu'\cos\theta\right] - (3)$$

Squaring Both sides

$$m^2v^2c^2\cos^2\phi = (h\nu - h\nu'\cos\theta)^2 \Rightarrow \left[m^2v^2c^2\cos^2\phi = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta\right] \rightarrow (4)$$

* Applying law of Conservatⁿ of Momentum along Y Axis

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$$

$$mv\sin\phi = \frac{h\nu'}{c}\sin\theta$$

Squaring Both sides $\rightarrow \left[m^2v^2c^2\sin^2\phi = h^2\nu'^2\sin^2\theta\right] - (5)$

- Adding eqⁿ (4) & (5)

$$m^2v^2c^2(\sin^2\phi + \cos^2\phi) = h^2\nu^2 + h^2\nu'^2(\sin^2\theta + \cos^2\theta) - 2h^2\nu\nu'\cos\theta$$

$$\therefore \left[m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta\right] - (6)$$

* Subtracting eq (6) from eqⁿ (2)

$$m^2c^2(c^2 - v^2) = -2h^2\nu\nu' + 2h^2\nu\nu'\cos\theta + m_0c^4 + 2hm_0c^2(\nu - \nu')$$

$$\left[m^2c^2(c^2 - v^2) = -2h^2\nu\nu'(1 - \cos\theta) + m_0c^4 + 2h\,m_0c^2(\nu - \nu')\right] - (7)$$

- As $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow$ Squaring $\rightarrow m^2 = \dfrac{m_0^2}{1 - \frac{v^2}{c^2}}$

$$\left[m^2 = \frac{c^2m_0^2}{c^2 - v^2}\right] - (8)$$

• Using eqⁿ ⑧ in ⑦

$$\frac{c^2 m_0^2}{c^2-v^2}\cdot c^2(c^2-v^2) = -2h^2 \nu\nu'(1-\cos\theta) + m_0^2 c^4 + 2hm_0c^2(\nu-\nu')$$

$$\cancel{m_0^2 c^4} = -2h^2\nu\nu'(1-\cos\theta) + \cancel{m_0^2 c^4} + 2hm_0c^2(\nu-\nu')$$

$$\rightarrow \cancel{2}h^{\cancel{2}}\nu\nu'(1-\cos\theta) = \cancel{2h}m_0c^2(\nu-\nu')$$

$$\frac{h}{m_0c}(1-\cos\theta) = \frac{c(\nu-\nu')}{\nu\nu'}$$

$$\frac{h}{m_0c}(1-\cos\theta) = c\left(\frac{\nu}{\nu\nu'} - \frac{\nu'}{\nu\nu'}\right)$$

$$\frac{h}{m_0c}(1-\cos\theta) = c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right)$$

$$\frac{h}{m_0c}(1-\cos\theta) = \frac{c}{\nu'} - \frac{c}{\nu}$$

$$\therefore \frac{h}{m_0c}(1-\cos\theta) = \lambda' - \lambda$$

$$\rightarrow \boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

$h \rightarrow$ Planck's constant

$m_0 \rightarrow$ Rest mass

$\Delta\lambda \rightarrow$ Change in wavelength of photon.

$\theta \rightarrow$ Scattering angle

Angle ~~at~~ which scattered photon makes with X axis

$c \rightarrow$ speed of light

Heisenberg Uncertainty Principle

- In classical Mechanics, One can find position & momentum of moving objects easily, if you are provided some Initial conditions.

- In Quantum Mechanics, the particle is described in terms of wave packet.

→ The particle is likely to be found anywhere within the wave packet moving with group velocity. Hence, it is impossible to know where the particle exactly is & what exactly is its momentum at any instant.

* Heisenberg uncertainty principle states that "It is impossible to measure accurately & simultaneously both the position of a particle along a particular Direction {Say X-Axis} & Momentum of the particle in same Direction (p_x)

- If Δx = uncertainty in the measurement of position along X-Direction.

  Δp_x = uncertainty in the measurement of momentum in same Direction.

- Then product of these two uncertainties is of the order of $\hbar$ ie $\Delta x \cdot \Delta p_x \approx \hbar$

OR $$\boxed{\Delta x \cdot \Delta p_x \approx \frac{h}{2\pi}} \quad -(1)$$

Similarly $$\left[\Delta y \cdot \Delta p_y \approx \hbar \quad \& \quad \Delta z \cdot \Delta p_z \approx \hbar\right] - (2)$$

- The Relatⁿ given by eqⁿ (1) & (2) are known as "position-momentum" uncertainty relatⁿ.

- Heisenberg Uncertainty Relatⁿ is applicable to all those pairs whose products has Dimensions of $M^1L^2T^{-1}$

ex- $\Delta E \cdot \Delta t \approx \hbar$

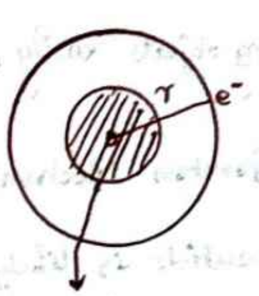
$\Delta J \cdot \Delta\theta \approx \hbar$

Angular momentum   Angle

Size & Energy of Hydrogen in Ground State.

Size of Hydrogen Atom

· Uncertainty in the position of electron (Δx) is $= r$



Nucleus

· We know that acc. to Heisenberg's uncertainty principle

$$\Delta x \cdot \Delta p \geq \hbar \Rightarrow \text{for derivat}^n \ \Delta x \cdot \Delta p = \hbar$$

$$\Delta p = \frac{\hbar}{(\Delta x)} = \frac{\hbar}{r}$$

$$PE = -\frac{e^2}{4\pi\epsilon_0 r}$$

$$KE = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m}$$

$$= \frac{p^2}{2m}$$

· The Minimum momentum of the e^- is $p = \Delta p = \frac{\hbar}{r}$

, Total Energy (E) $=$ K.E + P.E

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\therefore E = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} \quad , \ p = \Delta p = \frac{\hbar}{r}$$

$$\rightarrow E = \frac{\hbar^2}{2mr^2} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

; To find out the value of r at which the energy is Min^m : $\frac{dE}{dr} = 0$

diff^r wrt r

$$\frac{dE}{dr} = \frac{d}{dr}\left(\frac{\hbar^2}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r}\right) = 0$$

$$\frac{dE}{dr} = \frac{\hbar^2}{2m}\frac{d}{dr}(r^{-2}) - \frac{e^2}{4\pi\epsilon_0}\frac{d}{dr}\left(\frac{1}{r}\right) = 0$$

$$\therefore \frac{\hbar^2}{2m}(-2)r^{-3} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{-1}{r^2}\right) = 0$$

$$\frac{-\hbar^2}{mr^3} + \frac{e^2}{4\pi\varepsilon_0 r^2} = 0$$

$$\frac{\hbar^2}{m r^{\cancel{3}}} = \frac{e^2}{4\pi\varepsilon_0 \cancel{r^2}} \Rightarrow \boxed{r_0 = \frac{4\pi\hbar^2\varepsilon_0}{me^2}}$$

$\Rightarrow$ Radius at which Energy is Minm ie Ground St

* Now, we calculate the Minm Energy of e^-

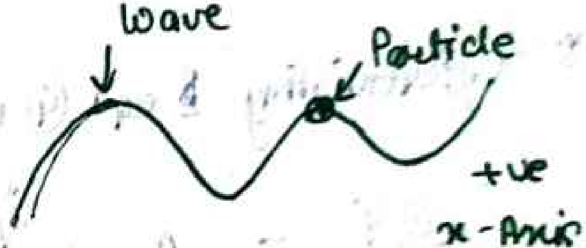
$$E_0 = \frac{\hbar^2}{2m} \cdot \frac{me^4}{(4\pi\varepsilon_0)^2\hbar^{4}} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2 \cdot me^2}{(4\pi\varepsilon_0)\hbar^2}$$

$$E_0 = \frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2} - \frac{me^4}{(4\pi\varepsilon_0)^2\hbar^2}$$

$$\boxed{E_0 = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}}$$

$\Rightarrow$ Energy of Hydrogen atom in ground State

~~Time~~ Schrodinger Time-Dependent Wave Eq^n.



- Consider a particle of mass m moving in +ve direction of X Axis with velocity v. Acc. to de Broglie hypothesis every moving particle has a wave associated with it which is called Matterwave whose wavelength is given by

Propogat^n Constant

$$\lambda = \frac{h}{mv} = \frac{h}{p_x} \Rightarrow p_x = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k \Rightarrow \boxed{k = p_x/\hbar} \quad -(1)$$

- Acc. to Quantum Physics, a wave consists of tiny particles of energy called photons. The Energy of each photon is given by $E = h\nu = \frac{h}{2\pi} \cdot 2\pi\nu = \hbar\omega$

$$\boxed{\omega = \frac{E}{\hbar}} \quad -(2)$$

- The wave associated with moving particle can be represented by a wave funcⁿ $\psi(x,t)$ as – $\left[\psi(x,t) = a e^{-i(\omega t - kx)}\right]$ – (3)

Using (1) & (2) in (3)

$$\psi(x,t) = a e^{-i\left(\frac{E}{\hbar}t - \frac{p_x x}{\hbar}\right)}$$

$$\left[\psi = a e^{-i/\hbar\,(p_x x - Et)}\right] \quad -(4)$$

- Now, Total energy (E) of the Particle, E = KE + PE is given by

$$E = \frac{p_x^2}{2m} + V$$

Multiplying Both sides by ψ

$$\left[E.\psi = \frac{p_x^2\psi}{2m} + V\psi\right] \quad -(5)$$

* Differentiating eqⁿ (4) w.r.t time

$$\frac{\partial\psi}{\partial t} = a . e^{i/\hbar\,(p_x x - Et)} . \frac{i}{\hbar}(-E)$$

$$= \psi . \frac{i.i}{i\hbar}(-E)$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \psi E} \quad -(6)$$

• Differentiating (4) w.r.t x

$$\left[\frac{\partial \psi}{\partial x} = a\, e^{i/\hbar (p_x x - Et)} \cdot \frac{i}{\hbar}(p_x) = \psi \cdot \frac{i}{\hbar}(p_x)\right] - 7$$

• Diff^{n} (7) w.r.t x

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{i}{\hbar} p_x \frac{\partial \psi}{\partial x} = \frac{i}{\hbar} \times p_x \cdot \left(\psi \frac{i}{\hbar} p_x\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{i^2}{\hbar^2} \psi p_x^2 \Rightarrow \left[\psi p_x^2 = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2}\right] - (9)$$

• using (6) & (9) in (5)

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}\left[-\hbar^2 \frac{\partial^2 \psi}{\partial x^2}\right] + V\psi}$$

$\Rightarrow$ This is Schrodinger time dependent wave eq^n in one Dimension.

~~In 1-D~~ in 3-D $\Rightarrow$ $\left[i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi\right]$

$$i\hbar \frac{\partial \psi}{\partial t} = \psi\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$$

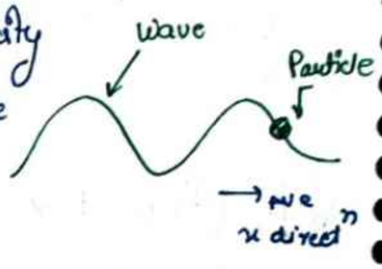
$$\boxed{E\psi = \psi H}$$

where $i\hbar \frac{\partial}{\partial t}$ = Energy Eigen Value, E

$-\frac{\hbar^2}{2m}\nabla^2 + V^2$ = Hamiltonian Operator, H

Schrodinger Time Independent Equation

- Consider a particle of mass 'm' moving with velocity 'v' along +ve direction of x-direction let P & E be the momentum & Energy of the particle respectively.



- According to De-Brogli hypothesis every moving particle is associated with a wave called matterwave.

- The wave funcⁿ describing the behaviour of wave associated with moving particle can be expressed as – $\psi(x,t) = a e^{\frac{i}{\hbar}(Px - Et)}$, a → amplitude of wave

$$\left[\psi = a e^{\frac{i}{\hbar}(Px - Et)}\right] \quad -(1)$$

→ We know that Schrodinger time dependent wave eqⁿ is given by

$$\left[i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi\right] \quad -(2)$$

∴ Diffⁿ eqⁿ (1) w.r.t t

$$\frac{\partial \psi}{\partial t} = a e^{i/\hbar(Px - Et)} \cdot \frac{i}{\hbar} \cdot (-E)$$

$$\frac{\partial \psi}{\partial t} = -\frac{iE\psi}{\hbar} \times \frac{i}{i} = \frac{E\psi}{i\hbar}$$

$$\left[i\hbar \frac{\partial \psi}{\partial t} = E\psi\right] \quad -(3)$$

- Let potential energy (V) of the particle depends only on position x & doesn't depend on time 't'. In this case wave func^n $\Psi(x,t)$ can be written as:-

- $$\Psi(x,t) = \phi(x).\mu(t)$$

$$\rightarrow \left[\Psi = \phi.\mu\right] - (4)$$

- Differentiating (4) w.r.t x

$$\frac{\partial\Psi}{\partial x} = \mu\frac{d\phi}{\partial x}$$

Diff^r w.r.t x

$$\left[\frac{\partial^2\Psi}{\partial x^2} = \mu\frac{\partial^2\phi}{\partial x^2}\right] - (5)$$

* Using (3), (4), & (5) in (2)

$$E\Psi = -\frac{\hbar^2}{2m}\left(\mu\frac{\partial^2\phi}{\partial x^2}\right) + V\phi\mu$$

$$E\phi\mu = -\frac{\hbar^2}{2m}\left(\mu\frac{\partial^2\phi}{\partial x^2}\right) + V\phi\mu$$

→ Dividing Both sides by μ

$$E\phi = -\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + V\phi \Rightarrow E\phi + \frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} - V\phi = 0$$

- $$\boxed{\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + (E-V)\phi = 0}$$

⇒ This is schrodinger time independent wave eq^n in 1D

- In 3D ⇒ $\frac{\hbar^2}{2m}\nabla^2\phi + (E-V) = 0$

Interpretation of Wave function.

- Wave function is an essential element of a Quantum Mechanical System.
- Any meaningful can be about Quantum Mechanical System can be obtained from wave func^n
- It is denoted by $\Psi(\vec{r},t)$ for a 3-Dimensional Motion of System & By $\Psi(x,t)$ for a 1-Dimensional motion of System along x axis.
- We get Ψ by solving a second order Differential eq^n called Schrodinger eq^n

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

- Schrodinger Eq^n in Quantum Mechanics plays the same role as is played by Newton's Second Law of Motion ($F=ma$) in classical mechanics.
- In Quantum Mechanics a particle is expressed by group of waves called wave packets
- Every wave is associated with a quantity which varies periodically with space & time
- for eg in light wave, electric & Magnetic field varies periodically with space & time.
- Similarly, in a wave packet, it is the func^n Ψ which varies periodically with space & time.
- Ψ being a complex func^n has no any physical significance it is not an observable Quantity.

→ When Ψ is multiplied By its complex Conjugate Ψ^*, we get a real Quantity which has a definite Physical Significance.

$\Psi = A + iB$

$\Psi^* = A - iB$

$\Psi \times \Psi^* = |\Psi|^2 = (A+iB)(A-iB)$

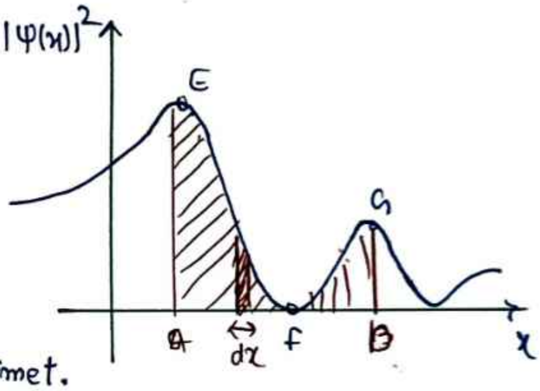
$\Psi\Psi^* = A^2 - (iB)^2$

$$\boxed{\Psi\Psi^* = |\Psi|^2 = A^2 + B^2}$$

* Physical Significance of Wave function.

* $|\psi|^2 = A^2 + B^2$; $|\psi|^2$ is known as probability density.

- In 1926, Max Born provided the statistical interpretation of $|\psi|^2$
- According to him the probability of finding a particle at any point at a given time is proportional to $|\psi|^2$
- Large value of $|\psi|^2$ at a ~~poit~~ point means high probability of finding the particle at that point & vice versa at given time

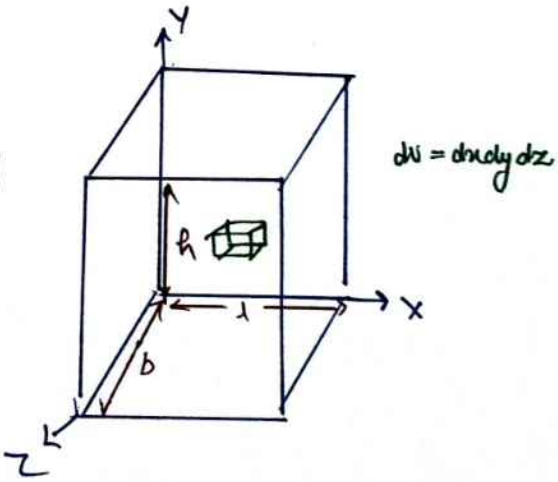


* for 1-Dimensional Motion

→ $\int_a^b |\psi(x,t)|^2 dx$ gives the probability of finding the particle B/w a & b at any time t.

* for 3-Dimensional Motion

→ $\iiint_{0\,0\,0}^{l\,b\,h} |\psi(\vec{r},t)|^2 dv$ → Probability of finding the particle in a cuboid having dimensions l, b & h at any time t_0



Normalization Condition for Wave function.

· Schrodinger wave eqⁿ (1D) - $-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = E\psi$ — (1)

Solⁿ

$\psi(x)$

$c_1\psi(x)$

$c_2\psi(x)$

$c_3\psi(x)$

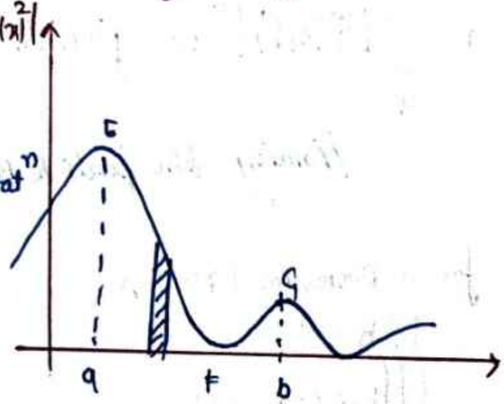
$c_4\psi(x)$

* Points to Be noted

1) Not all of these equations solutions of eqⁿ (1), can be used for a real particle.
2) These are mathematical functions / solutions
3) Only one of them can be used for real particle
4) Now, what is it that Determines which one of these Solⁿ can actually provide Quantum Mechanical Description of given Particle.

· $|\psi|^2 = A^2 + B^2$ → Real Quantity & $|\psi|^2$ is known as probability Density ; if $\psi = A + iB$

$\int_a^b |\psi(x)|^2 dx = P_{ab}$; $\boxed{\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1}$ — (*) ← Max Born Interpretatⁿ

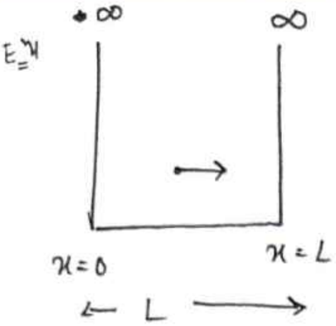


→ $\int_{-\infty}^{\infty} |\psi(x)|^2 dx \neq 1$

· Now for $N.\psi(x)$, $\int_{-\infty}^{\infty} |N\psi(x)|^2 dx = 1$

$$N^2 \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$$\boxed{N^2 = \frac{1}{\int_{-\infty}^{\infty} |\psi(x)|^2 dx}}$$



$$-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = E\psi \quad -(1)$$

$$\psi(x) = \sin\left(\frac{n\pi}{Lx}\right)$$

- $N\psi(x) = N\sin\left(\frac{n\pi}{Lx}\right)$; $\int_{-\infty}^{\infty} |N\psi(x)|^2 dx = 1$

$$N^2\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \rightarrow N^2\int_{-\infty}^{\infty} \sin^2\left(\frac{n\pi}{Lx}\right) dx = 1$$

$$N^2 \cdot \frac{L}{2} = 1 \Rightarrow \boxed{N = \sqrt{\frac{2}{L}}}$$

Probability Current Density

- We know that motion of a material particle is associated with ~~wave function~~ the motion of wave fn.
- ~~If the probability of finding outside this region of space decreases with time the~~
- If the probability of finding a particle in some bounded region of space decreases with time then the probability of it finding outside this region must in by same amount.
- The time dependent Schrodinger's wave eqn

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad -(1)$$

- Multiplying both sides with ψ^*, we get

$$i\hbar\psi^*\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\psi^*\nabla^2\psi + \psi^* V\psi \quad -(2)$$

- Taking complex conjugate of eq^n (1), we get

$$-j\hbar \frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^* \quad \text{--- (3)}$$

- Multiplying Both sides of eq^n (3) By ψ

$$\left[-j\hbar\, \psi \frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m}\psi\nabla^2\psi^* + \psi V\psi^*\right] \text{- (4)}$$

Subtracting (4) from (2)

$$j\frac{\hbar}{} \psi^* \frac{\partial \psi}{\partial t} - \left(-j\hbar\, \psi\frac{\partial\psi^*}{\partial t}\right) = -\frac{\hbar^2}{2m}\psi^*\nabla^2\psi + \psi^* V\psi - \left(-\frac{\hbar^2}{2m}\psi\nabla^2\psi^* + \psi V\psi^*\right)$$

$$j\hbar\left(\psi^*\frac{\partial\psi}{\partial t} + \psi\frac{\partial\psi^*}{\partial t}\right) = -\frac{\hbar^2}{2m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$$

$$\frac{\partial}{\partial t}\left(\psi^*\psi\right) = \frac{-\hbar}{2mi}\nabla\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right)$$

$$\left[\frac{\partial}{\partial t}\left(\psi^*\psi\right) + \nabla\cdot\frac{\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = 0\right] \text{--- (5)}$$

- We know that eq^n of Continuity in Hydrodynamics associated with Conservation of fluid of density ρ & current density $\vec{J}$ is given by

$$\left[\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0\right] \text{--- 6}$$

• On comparing 5 & 6, we get

$$\rho = \psi^* \psi = |\psi|^2 = \text{~~Prob~~ Position probability Density}$$

&

$$\left[\vec{J} = \frac{\hbar}{2mi} \left(\psi^* \nabla \psi - \psi \nabla \psi^* \right) \right]$$ ⇒ known as probability Current Density.

• On comparing ⑤ & ⑥, we get

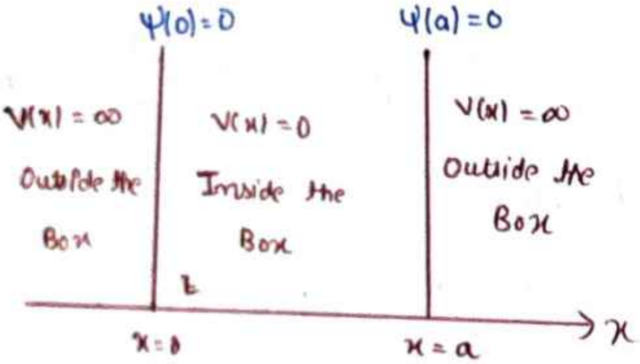
$$\rho = \psi^* \psi = |\psi|^2 = \text{Position probability Density}$$

&

$$\left[\vec{J} = \frac{\hbar}{2mi} \left(\psi^* \nabla \psi - \psi \nabla \psi^* \right) \right] \Rightarrow \text{known as probability Current Density.}$$

~~Particle~~ free Particle in One Dimensional Box

• In Quantum Mechanics, a box means a region of space in which particle experiences no force i.e potential energy of this particle is zero in this Region & infinite outside this Region.



• 1-D Box means that particle is allowed to move along a straight line say along x-axis

• Consider a particle of mass m confined to move in 1-D Box of length 'a' along X-Axis under following Conditions -

(1) The walls of the Box are ~~elastic~~ rigid & perfectly elastic so that particle rebounds with same K.E after making elastic collision with the wall & total Energy (E) of the particle remains constant.

(2) The walls of the Box are non-penetrable, so that there is no probability of finding the particle outside the Box.

Potential energy funcⁿ V(x) for 1-D Box

$$V(x) = \begin{cases} 0 & , 0 < x < a \\ \infty & , x < 0 \ \& \ x > a \end{cases}$$

- On comparing (5) & (6), we get

$$\rho = \psi^* \psi = |\psi|^2 = \text{Position probability Density}$$

&

$$\left[\vec{J} = \frac{\hbar}{2mi} \left(\psi^* \nabla \psi - \psi \nabla \psi^* \right) \right] \Rightarrow \text{known as Probability Current Density.}$$

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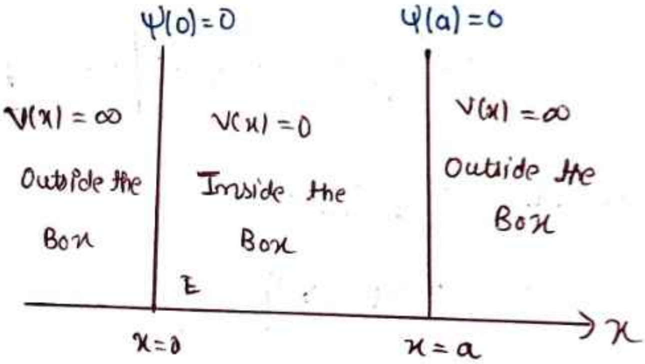

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- Potential energy funcⁿ V(x) for 1-D Box

$$V(x) = \begin{cases} 0 & , 0 < x < a \\ \infty & , x < 0 \ \& \ x > a \end{cases}$$

- 1-D Schrodinger wave eq^n is given by

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

- for $0<x<a$, $V=0$, $\therefore$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0$$

③

$$② \leftarrow \left[\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0\right] \left[k = \sqrt{2mE}/\hbar\right]$$

$$E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \Rightarrow 2mE = p^2$$

$$p = \sqrt{2mE}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{2\pi\sqrt{2mE}}{h}$$

$k = \frac{\sqrt{2mE}}{\hbar}$ ⇒ SQUARING BOTH SIDES

$$\therefore k^2 = 2mE/\hbar^2$$

- General sol^n of eq^n ② is $\rightarrow \left[\psi(x) = A\sin kx + B\cos kx\right]$ — ④

At $x=0$, $\psi(x)=0$

$$0 = 0 + B \Rightarrow [B=0] \text{ — ⑤}$$

- Using ⑤ in eq^n ④ $\Rightarrow \left[\psi(x) = A\sin kx\right]$ — ⑥

- At $x=a$, $\psi(x)=0$

$\therefore$ $0 = A\sin ka \Rightarrow \sin ka = 0$ as $A \neq 0$

$\therefore$ $ka = 0, \pm\pi, \pm 2\pi \pm 3\pi$; $ka = 0$; Not possible as Both k & a can't be zero

$ka = -n\pi$ as $\sin(-x) = -\sin x$ & A can absorb -ve sign as it's constant

- In General

$$ka = n\pi \quad ; n = 1,2,3 \text{ --}$$

$$\left[k = \frac{n\pi}{a}\right] \text{ - ⑦}$$

; Comparing ③ & 7

$\frac{n\pi}{a} = \frac{\sqrt{2mE}}{\hbar^2}$ ⇒ Squaring

$$\frac{n^2\pi^2}{a^2} = \frac{2mE}{\hbar^2}$$

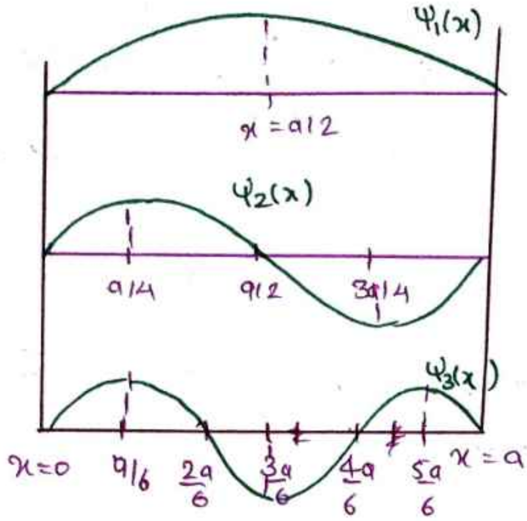
$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

- Eigen functions : $\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$

$\psi_1(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$

$\psi_2(x) = \sqrt{\frac{2}{a}} \sin \frac{2\pi x}{a}$ ... & so on

$\psi_3(x) = \sqrt{\frac{3}{a}} \sin \frac{3\pi x}{a}$



* NODES & POSTⁿ OF NODES

- Within the Box there are certain points at which $\psi(x) = 0$, such points are called nodes

- At nodes, $\psi_n(x) = 0 \Rightarrow \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a} = 0 \Rightarrow \sin\frac{n\pi x}{a} = 0 = \sin P\pi$; $P = 0, 1, 2,$

$$\frac{n\pi x}{a} = P\pi \Rightarrow x = \frac{Pa}{n} \Rightarrow x = 0, \frac{a}{n}, \frac{2a}{n}, \frac{3a}{n} ----$$

Post^n of Node

for $n = 1$, $x = 0, a$

for $n = 2$, $x = 0, a/2, a$

for $n = 3$, $x = 0, \frac{a}{3}, \frac{2a}{3}, a$

* ANTINODE & POSTⁿ OF ANTINODES

- Within the Box there are certain points at which $\psi(x) = \max^m$, such points are called antinodes

- At antinodes, $\psi_n(x) = \text{Max}$ i.e when $\sin\frac{n\pi}{a}x = 1 \Rightarrow \sin\frac{n\pi x}{a} = \sin(2p+1)\frac{\pi}{2}$; $P = 0, 1, 2 --$

$$\frac{n\pi x}{a} = (2P+1)\frac{\pi}{2} \Rightarrow x = \frac{(2P+1)a}{2n}, \quad x = \frac{a}{2n}, \frac{3a}{2n}, \frac{5a}{2n}$$

Post^n of antinode

for, $n = 1$, $x = a/2$

for $n = 2$, $x = a/4, 3a/4$

for $n = 3$, $x = a/6, 3a/6, 5a/6$