"Do not write anything on question-paper except Roll Number, otherwise it shall be deemed as an act of indulging in unfair means and action shall be taken as per rules."

Roll No. 20VEIA1293

1st B.E.

3 **Maths - I**

FIRST B.E. (FIRST SEMESTER)
EXAMINATION - 2020
MA - 103 A : MATHEMATICS - I
(Common for all sections)

Time - Three Hours

Maximum Marks - 100

Note:- (1) Attempt FIVE question in all, selecting at least ONE from each section.

(2) Marks allotted to each part of the question are indicated on the right side

SECTION - A

1\. (a) If $u = \tan^{-1}\left(\dfrac{xy}{\sqrt{1+x^2+y^2}}\right)$ then prove that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{\left(1+x^2+y^2\right)^{3/2}}$$ 10

(b) If $u=\sin^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}\right)$ then prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{20}\tan u$$ 10

2. (a) Show that the minimum value of the following function is $3a^2$:

$$u=xy+a^3\left(\frac{1}{x}+\frac{1}{y}\right)$$ 10

(b) Find the asymptotes of the following curve :

$y^3-xy^2-x^2y+x^3+x^2-y^2-1=0$ 10

3. (a) Find the envelope of the straight lines

$x\cos\alpha+y\sin\alpha=l\sin\alpha\cos\alpha$

where α is a parameter. 10

(b) Trace the following curve

$x^3+y^3=3axy$ 10

SECTION - B

4. (a) Find the whole length of the asteroid

$x^{2/3}+y^{2/3}=a^{2/3}$. 10

(b) Prove that $\int_0^1\frac{dx}{\sqrt{1-x^3}}=\frac{[\Gamma(1/3)]^3}{2^{4/3}\sqrt{3}\pi}$ 10

5. (a) The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolves round its major axis. Find the surface area of the prolate sheroid generated. 10

(b) Evaluate the following integral by changing to polar coordinates:

$$\int_0^1 \int_x^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dx\, dy. \qquad 10$$

6. (a) Find the volume of the solid formed by the revolution of the loop of the curve

$y^2(a+x) = x^2(a-x)$ about the x-axis. 10

(b) Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}\, dx.$ 10

SECTION - C

7. (a) If $\vec{r}$ and r have their usual meaning, show that

$\operatorname{div} r^n \vec{r} = (n+3)r^n$

further prove that $r^n \vec{r}$ is solenoidal if $n = -3$

8+2=10

(b) Prove that 10

$$\vec{\nabla}.(\vec{a} \times \vec{b}) = \vec{b}.(\vec{\nabla} \times \vec{a}) - \vec{a}.(\vec{\nabla} \times \vec{b})$$

8. (a) Show that

$$\int_S (axi + byj + czk).\hat{n}\, dx = \frac{4\pi}{3}(a+b+c)$$

Where S is the surface of the sphere $x^2+y^2+z^2 = 1$

10

(b) Evaluate & verify Green's theorem in the plane for $\int_C (xy+y^2)\, dx + x^2\, dy$. Where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 10