M-2 2019

~~SECTION-1~~

* Second Order Differential equations - PYQ

2019

Q 3 (a) ⇒ Solve Differential eqn ⇒ 2017, 2019

$$\frac{d^2y}{dx^2} - (\cot x)\frac{dy}{dx} - (\sin^2 x)y = \cos x \sin^2 x$$

in 2018 $\cos x - \cos^3 x$ ↓ $\cos x(1-\cos^2 x)$

Ans Standard L.D.E of Second Order: $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Here $P = -\cot x$, $Q = -\sin^2 x$, $R = \cos x \sin^2 x$

* Changing ~~the~~ the Independent variable x to z

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

- where

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}, \quad Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}, \quad R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$$

So, Here $Q = -1$ {Now choose z, such that Q_1 = constant (most of the cases constant part of Q).

$$Q_1 = \frac{\cancel{-}(-\sin^2 x)}{(dz/dx)^2} = \cancel{-}1$$ Constant Part of Q

$$1 = \frac{\sin^2 x}{(dz/dx)^2} \Rightarrow \left(\frac{dz}{dx}\right)^2 = \sin^2 x \Rightarrow \frac{dz}{dx} = \sin x.$$

$dz = \sin x\, dx \longrightarrow \int \rightarrow \boxed{z = -\cos x}$

- $P_1 = \dfrac{\dfrac{d^2z}{dx^2} + P\dfrac{dz}{dx}}{\left(\dfrac{dz}{dx}\right)^2}$; $\dfrac{dz}{dx} = \sin x$, $\dfrac{d^2z}{dx^2} = \cos x$

$= \dfrac{\cos x + \sin x(-\cot x)}{\sin^2 x} = \dfrac{\cos x - \sin x \cdot \dfrac{\cos x}{\sin x}}{\sin^2 x}$

$\boxed{P_1 = 0}$

- $R_1 = \dfrac{R}{(dz/dx)^2} = \dfrac{\cos x \sin^2 x}{\sin^2 x} = \cos x = -z \Rightarrow \boxed{R_1 = -z}$

Now, Eq^n Becomes

$$\frac{d^2y}{dz^2} + 0\frac{dy}{dz} + (-1)y = -z$$

$$(D^2 - 1)y = -z$$

- Complimentary func^n, $y_c \Rightarrow$ Roots of A.E i.e $m^2 - 1 = 0$

$m^2 = 1 \Rightarrow m = \pm 1$

$$\boxed{y_c = c_1 e^{z} + c_2 e^{-z}}$$

- Particular Integral

$$y_p = \frac{-z}{D^2 - 1}$$

$$y_p = \frac{-z}{(D-1)(D+1)} = \frac{z}{(1-D)(1+D)}$$

$$y_p = z\,[1-D]^{-1}\,[1+D]^{-1}$$

$\Rightarrow$ * using Binomial expansion, $(1+x)^n = (1+nx)$, $(1-x)^n = (1-nx)$

$$y_p = z\,(1+D)(1-D)$$

$$= z\,(1-D^2) = z - \cancel{D^2 z} \rightarrow 0 \left\{ \text{as } \frac{d^2 z}{dx^2} = 0 \right\}$$

$$\boxed{y_p = z}$$

* Sol^n to D.E $y = y_c + y_p$

$$y = c_1 e^{z} + c_2 e^{-z} + z \;;\; z = -\cos x$$

$$\boxed{y = c_1 e^{-\cos x} + c_2 e^{\cos x} - \cos x}$$

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$\boxed{P_1 = 0}$

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- Particular Integral

$$y_p = \frac{-z}{D^2 - 1}$$

$$y_p = \frac{-z}{(D-1)(D+1)} = \cancel{-}\frac{z}{\cancel{-}(1-D)(1+D)}$$

$$y_p = z[1-D]^{-1}[1+D]^{-1}$$

$\Rightarrow$ * using Binomial Expansion , $(1+x)^n = (1+nx)$

$$y_p = z(1+D)(1-D)$$

$$= z(1-D^2) = z - \cancel{D^2 z}^{\,0} \quad \left\{ \text{as } \frac{d^2z}{dx^2} = \right.$$

$$\boxed{y_p = z}$$

Sol^n to D.E $y = y_c + y_p$

$$y = c_1 e^{z} + c_2 e^{-z} + z \; ; \; z = -\cos x$$

$$\boxed{y = c_1 e^{-\cos x} + c_2 e^{\cos x} - \cos x}$$

(b) Solve Using Method