ıs: The de Broglie Waves or Matter Waves

According to de Broglle, a matter particle having a mass in moving with a velocity v must possess a matter wavelength equivalent to

$$\lambda = \frac{h}{mv} \qquad \text{-------------------(1)}$$

where h is the universal Pianck's constant. Louis de Broglie was led to this hypothesis by considering the special theory of relativity and quantum theory. Evidence for the matter waves was found in 1927 in two different laboratories. CJ. Davisson and L.H. Germer using a metal crystal as a reflection grating and G. P. Thomson employing a metal foil as a transmission grating showed that the electrons could be diffracted and thereby established both their wave particle nature and the quantitative validity of the de Broglie's hypothesis.

Since 1927 it has been shown that material particles other than electrons have wave properties: thus diffraction effects have been observed with hydrogen and helium nuclei and also with neutrons. There is a little doubt that the wave particle duality is a property of all forms of matter. However, it can be seen from Eq. (1) that with increasing mass, the wavelengths become shorter for a given velocity and so are increasingly difficult to detect

The major advantage of diffraction of electrons and neutrons have been utilized in the study of molecular and crystal structure. Further with the electron microscope, wherein de Broglie's concept of electron waves are involved, it has been possible to resolve objects as small as 10Å in size compared with a minimtrm of about 300 nm in an ordinary microscope.

de Broglle Wavelength:

We have for electrontagnetic radiation,

$$E = mc^2 \text{ and } E = h\nu$$

$$\therefore \qquad mc^2 = h\nu = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc} \qquad \text{-------------------(2)}$$

Equation (1) gives the expression for the wavelength of a photon wave that moves through a medium when photon travels with a velocity equal to velocity of light,

Similarly, any material particle having a mass nr and moving with a velocity v must possess a de Broglie wavelength given by

$$\lambda = \frac{h}{mv}$$

de-Broglie wavelength in terms of kinetic energy

As de-Broglie waves are associated with particles that are moving with a measurable velocity v,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Characteristics of Matter Waves:

[i] From Eq. [1], $\lambda \propto \frac{1}{m}$

Thus the wavelength of matter wave is inversely proportional to the mass of the particle. The larger the mass of the particle, the shorter will be the wavelength and vice versa.

(ii) From Eq. [1], $\lambda \propto \frac{1}{v}$

Thus the matter wavelength varies inversely with the velocity of the particle. The greater the velocity of the particle, the smaller will be the matter wavelength and vice versa.

(iii) This is totally a new wave and cannot be equated to electromagnetic wave,

(iv) The velocity of matter wave depends on the velocity of matter particle, hence its velocity is not a constant whereas the velocity of electromagnetic wave is.

de Broglie's hypothesis: wave-particle duality

Light behaves as wave when it undergoes interference, diffraction etc. and is completely described by Maxwell's equations. But then, the wave nature of electromagnetic radiation is called into question when it is involved in blackbody radiation, photoelectric effect and such. Einstein forwarded his idea of photon, bundle of quantized radiant energy localized in a small volume, as a way to describe particle-like nature of light. The energy and momentum of such a photon was proposed to be,

$$E = h\nu \quad \text{and} \quad p = \frac{E}{c} = \frac{h}{\lambda}.$$

de Broglie (1924) made a great unifying, speculative hypothesis that just as radiation has particle-like properties, electrons and other material particles possess wave-like properties. For free material particles, de Broglie assumed that the associated wave also has a frequency ν and wavelength λ related to its energy E and momentum p,

$$\nu = \frac{E}{h} \quad \text{and} \quad \lambda = \frac{h}{p}. \tag{60}$$

For non-relativistic particles having mass m and moving with a velocity v and kinetic energy $E_k = mv^2/2$, the de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}} \tag{61}$$

For high energy particles, $E^2 = p^2c^2 + m_0^2c^4$, having kinetic energy $E_k = E - m_0c^2$, the momentum is $pc = \sqrt{E_k(E_k + 2m_0c^2)}$ and hence the de Broglie wavelength is,

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E_k(E_k + 2m_0c^2)}}. \tag{62}$$

For instance, the de Broglie wavelength of an object of mass $m = 1.0$kg and moving with a velocity $v = 10$m/s is ($h = 6.6 \times 10^{-34}$J-s)

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}\text{J} - \text{s}}{1.0 \times 10\text{kg} - \text{m/s}} = 6.6 \times 10^{-35}\text{m} = 6.6 \times 10^{-25}\mathring{A}.$$

The de Broglie wavelengths of an electron ($m = 9.1 \times 10^{-31}$kg) at kinetic energy $100eV$ and $0.1MeV$ are ($1eV = 1.6 \times 10^{-19}J$),

$$\begin{aligned}
\lambda &= \frac{h}{\sqrt{2mE}} = \frac{6.6 \times 10^{-34}}{(2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})^{1/2}} = 1.2\mathring{A} \\
\lambda_{\text{rel}} &= \frac{ch}{\sqrt{E(E + 2m_0c^2)}} = 0.037\mathring{A} \\
\lambda_{\text{nr}} &= \frac{h}{\sqrt{2mE}} = 0.038\mathring{A}
\end{aligned}$$

Sure enough, de Broglie's hypothesis of wave-particle duality was confirmed by Davisson and Germer (1927) and G.P. Thomson (1927) [refer Eisberg & Resnick, pg 64 – 67].

Physical significance of wave function

Wave function is the variable quantity that is associated with a moving particle at any position (x,y,z) and at any time 't', also it relates the probability of finding the particle at that point and at that time.

- The wave function Ψ is a variable quantity.
- It is associated with a moving particle i.e., $\Psi(x, y, z, t)$ where x,y,z is the position of the particle at any time t.
- It gives information about the behaviour of the particle.
- Also it gives the statistical relation between the particle and the wave.
- Ψ is complex quantity and also Ψ does not have any meaning.
- But $|\Psi|^2$ has physical meaning, where $|\Psi|^2 = \Psi^* \Psi$, Ψ^* is the complex conjugate of Ψ.
- $|\Psi|^2$ Exemplifies the probability of finding the particle per unit volume.
- $\iiint |\Psi|^2 d\tau$ represents the probability of finding the particle in a given volume $d\tau$ where $d\tau = dx.\,dy.\,dz$
- The probability has any value between 0 and 1.
- If $\iiint \Psi^* \Psi\, d\tau = 0$ there is no particle in the given range.
- If $\iiint \Psi^* \Psi\, d\tau = 1$ there is particle certainly in the given range.
- If $\iiint \Psi^* \Psi\, d\tau = 0.5$, then, there is 50% chance for finding the particle and 50% chance for not finding the particle.
- A wave function Ψ satisfying the above relation is called a normalized wave function.