

① Fundamentals Of Surveying

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Surveying is art of determining relative position of points on, above and below the earth surface presenting it graphically or numerically.

⇒ Objectives of Surveying

- ① To determine relative position of points.
- ② To Layout or markout proposed structure on the ground.
- ③ To determine relative quantities like Area and volume.

→ Methods of Representation :-

① Numerical Representation :- $AB = 10\text{KM}$

② Graphical Representation :- $A \xrightarrow{10\text{KM}} B$

Graphical representation is done in form of Plan or Map.

PLAN :- Large Scale

MAP :- Small Scale

Note :-

Vertical distances can be presented with help of contour or spot level. Contour is a imaginary line joining the points of same level (elevation) on earth surface.

- Spot levels are heights of individual points.
- Contour gives better visualization in comparison to spot level.

Basic Definitions:-

① Shape of Earth

(a) Oblate Spheroid:-> Slightly flat at poles. Polar axis is 43.5 km smaller than equatorial axis.

(b) Ellipsoide:->

Equatorial Section is elliptical in nature.

(c) Ovaloid:->

Southern hemisphere is slightly larger than northern hemisphere.

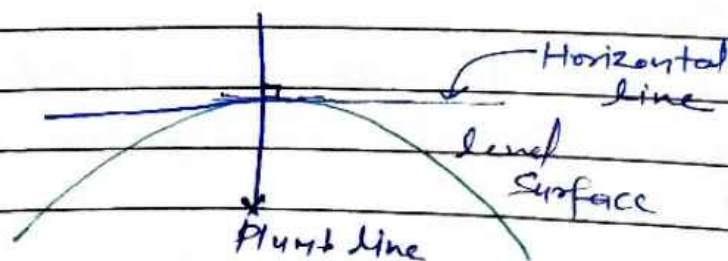
⇒ We can observe that no geometrical figure completely defines shape of earth. Hence, a new name is been given to shape of earth that is "Geoid".

Note:- For calculation purpose we'll assume earth to be spherical in nature.

② Level Surface

It is a curved surface parallel to spherical surface of earth and hence every point on it is equidistant from the centre of earth.

Every moment on level surface is perpendicular to plumb line.



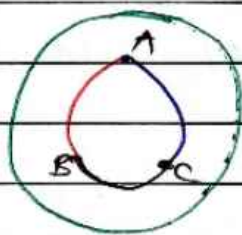
③ Horizontal Plane :-

It is a tangible plane to the earth surface which is also perpendicular to plumb line.

④ Great Circle :- It is a imaginary circle passing through centre of the earth.
A Great circle $\neq$ divides earth into two equal parts.

⑤ Spherical Triangle :-

Spherical triangle is a triangle which is formed on the surface of sphere by intersecting by 3 arcs of different Great Circle.



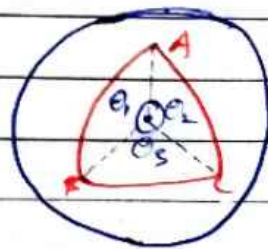
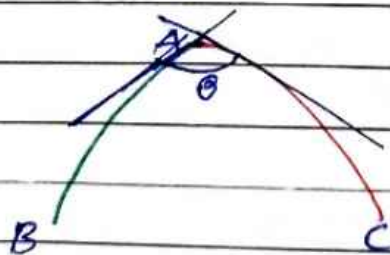
The arc enclosing spherical triangle and called as it's side and the angles in which these arc intersects are called as Spherical angles.

Spherical Angles is defined as angle btw tangent to great circles and the points of intersection.

$$\widehat{AB} = \angle O_1$$

$$\widehat{BC} = \angle O_3$$

$$\widehat{AC} = \angle O_2$$



length of a side of a spherical triangle is defined as angle subtended by that side at centre of the surface.

Properties of Spherical Triangle

- ① Each side of a this must be less than equal to π .
- ② Each angle must be less than 180°
- ③ Sum of 3 sides must be btw "0-360"
- ④ Sum of 3 angles of a spherical Triangle must be btw $[180^\circ - 540^\circ]$
 ↓ inclusive ↓ Exclusive.

Note :- Amount by which sum of angles of spherical triangle ~~must be~~ exceed by 180° is called as spherical excess.

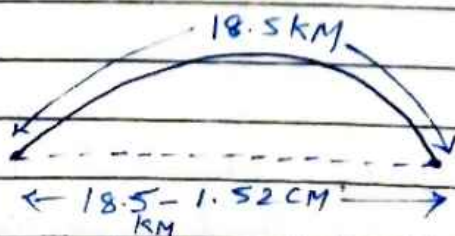
- Surface area of a spherical Triangle must be less than $2\pi r^2$.
- Greater angle is opposite to greater side

Classification of Surveying

Surveying can be classified into various means.
Ex:- Instrument used, Purpose, Place of survey etc.

But mainly Surveying is classified into two types:-

- 1) Geodetic Surveying
- 2) Plane Surveying



Geodetic

→ It is done for large area and we consider earth surface as curved.

→ Area $\geq 195.5 \text{ km}^2$

→ It is done by survey of India to establish control points through out the country.

→ It uses spherical Trigonometry.

Plane Survey

- Plane surveying is done for small area and we consider earth surface as flat surface. that is curvature of earth is neglected.

- Area $< 195.5 \text{ km}^2$

- It is done for local survey.

- It uses plane Trigonometry.

Note :- In plane surveying plotted measurements are projected on a horizontal plane.

For area $= 195.5 \text{ km}^2$ Spherical excess $= 1 \text{ Second}$.

Classification based on Purpose

① Topographical Survey :-

These survey are used to obtain maps which show details of Natural and manmade features on earth surface including elevation information.

Scale :- $1:25000$ to $1:1000000$

② Engineering Survey :-

These survey are used for Engineering works.

Ex:- Dam, Highways, Canal, Railways, etc.

Building Work $\rightarrow 1:50$ to $1:200$

Bridges & Other Civil Engineering work $\rightarrow 1:500$ to $1:2500$

Highways $\rightarrow 1:1250$ to $1:50000$

③ Cadastral Surveying :-

These are used to represent Property boundaries.

Scale - $1:1000$ to $1:5000$

④ Hydrographic Surveying :-

These survey are done on near the water bodies such as lake, river, bay etc.

⑤ Astronomic Survey :-

With help of astronomic survey we can determine latitude, longitude and local mean time of any place.

⑥ Geological Survey :-

It is used to obtain info. about different strata of earth surface.

Classification based on Instrument.

① Chain Survey:-

It is simplest type of survey in which only linear measurements are done with help of chain or tape.

→ No Angular measurements is done.

② Compass Survey:-

In compass surveying direction and horizontal angles are measured with help of compass and distances are measured with help of chain or tape.

③ Levelling:-

In this type of survey vertical distances are measured w.r.t. a fixed frame of reference which is generally called 'Datum'.

④ Plane-Table Surveying:-

In this type a plan or Map is prepared by determining direction of various points and taking their linear measurements with chain or tape.

⑤ Theodolite Surveying:-

In this type Horizontal and vertical angles are measured with help of theodolite & distances are measured with chain or tape.

⑥ Tachometric Surveying:- We measure horizontal & vertical distance optically with the help of instrument called an instrument. Tachometer.

① Photogrammetric Surveying :-

In this kind of surveying we survey a Area with help of photographs.

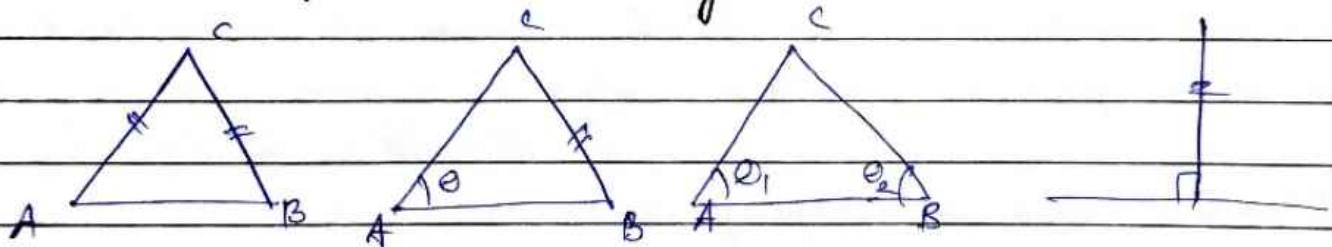
Principles of Surveying :-

① Working from Whole to Part :-

Main objective of working from whole to part is to localise the errors & prevent their accumulation.

If we work from part to whole errors gets accumulation or it gets maximised to greater extent.

② Locate a point atleast by two measurements :-



Scale :-

$$\text{Scale} = \frac{\text{Length of a line on Plan / Map}}{\text{Length of a line on Ground}}$$

Numerical Scale

Graphical Scale

Engineer Scale

Representative

1 cm of Plan or
Map distance

Fraction Scale (R_f)

= Ground
distance

It is Fractional

Form of Scale

$[1 : 10^6]$

Large Scale & Small Scale :-

SCALE-A

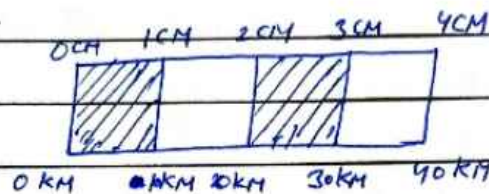
$$1\text{cm} = 10\text{m}$$

Larger the denominator smaller will be scale.

SCALE-B

$$1\text{cm} = 100\text{m}$$

Graphical Scale:-



Graphical scale is line drawn on map & marking ground distance directly on it.

⇒ Graphical scale has advantage over numerical scale b'coz distance on map can be determined by actual scaling even though map has shrunk.

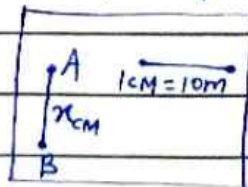
In case of shrinkage of map graphical scale also changes with Map. hence ratio is unaffected.

i) Before Shrink:-

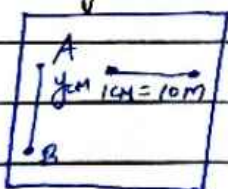
Let distance btw two points on Plan or Map = x cm

Let Scale is 1:1000

Therefore ' x ' cm on plan or Map equals to a ground distance of $1000x$ cm.



ii) After Shrinkage:-



Now distance btw same points on plan or Map = y cm

y cm on plan / Map = $1000y$ cm at ground distance.

$$\text{Shrunk Scale} = \frac{Y \times 1}{x : 1000} \rightarrow \text{Original Scale}$$

Shrinkage Factor or Shrinkage Ratio equals to :-

$$\frac{\text{Shrink Length}(y)}{\text{Original Length}(x)} = \frac{\text{Shrink Scale}}{\text{Original Scale}} = \frac{\text{Shrink R.F.}}{\text{Original R.F.}}$$

- * Shrinkage Factor < 1 (always)
- * Shrink Scale - Small Scale
- Original Scale - Large Scale

Q Plan of an Area has shrunk such that a line originally 10 cm long now measures as 9.5 cm only if original scale of plan was 1 cm = 10 m. R.F. = 1:1000 then determine

- Shrinkage Factor
- Shrink Scale
- Correct length if measured length of line was 100 m and correct area if measured Area was 100 m².

Solⁿ:- @ S.F. = $\frac{9.5}{10} = 0.95$

① S.S. = Original Scale $\times$ S.F.

$$= \frac{1}{1000} \times 0.95$$

S.S. = $\frac{1}{1052.63}$

(C) S.F. = $\frac{y \times 1000}{x \times 1000} = \frac{1000y}{1000x}$

$$S.F. = \frac{\text{Measured distance}}{\text{Correct distance}} = \frac{M.D.}{C.D.}$$

$$C.D. = \frac{100}{0.95} = 105.263 \text{ m}$$

$$C.A. = \frac{100}{(0.95)^2} = 110.803 \text{ m}^2$$

Error due to Wrong Measuring Scale:-

$$C.L. = \frac{\text{R.F. of Wrong Scale}}{\text{R.F. of Correct Scale}} \times M.L.$$

Q- A Surveyor Measures distance btw two points marked on the plan drawn to a scale of $1\text{cm} = 1\text{m}$, and found it to be 50m . Later he discovered that he ~~can~~ has used a wrong scale of $1\text{cm} = 50\text{cm}$ for the measurement. Determine correct length. What should be correct area if measured area is 100m^2 .

$$\text{Sol}^n:- C.L. = \frac{1/50}{1/100} \times 50 = 100 \text{ m}$$

$$C.A. = \left(\frac{1/50}{1/100} \right)^2 \times 100 = 400 \text{ m}^2$$

Use of Vernier in Scale:-

Vernier is a device used for measuring readings which are fractional part of smallest division to main scale.

The reading is given by:-

Reading before index on main scale + $N \times LC$

$N \rightarrow$ No. of vernier division which perfectly co-incides with main scale division.

L.C. $\rightarrow$ One division at main scale - One division of vernier scale.

Types of Vernier:-

- ① Direct Vernier:- 'n' divisions of vernier scale are equal to (n-1) division of main scale. One vernier division is slightly smaller than one division of main scale.

$$n(V) = (n-1)S$$

$$L.S. = S - V$$

$$L.S. = S - \left(\frac{n-1}{n}\right)S$$

$$L.C. = \frac{S}{n}$$

- ② Retrograde Vernier:-

'n' divisions on vernier scale are equal to (n+1) division on main scale.

- One vernier division is slightly larger than one division on main scale.
- Vernier division and main scale division ($\uparrow$)es in opposite dirxⁿ whereas in direct vernier both ($\uparrow$)es in same dirxⁿ.

$$n(V) = (n+1)S$$

$$L.C. = V - S$$

$$L.C. = \left(\frac{n+1}{n}\right)S - S$$

$$L.C. = \frac{S}{n}$$

Q The main plate of Theodolite is divided into 1080 parts & 60th division of Vernier scale consider with 59th division of main scale, determine least count of Theodolite.

Solⁿ:-
$$L.C. = \frac{S}{N} = \frac{\left(\frac{360}{1080}\right)}{60} = 20''$$

Special Type of Vernier:-

(i) Double Vernier:-

If graduation of main scale ($\uparrow$ less in one dirⁿ only we have a single vernier only whereas Main scale is graduated in both the dirⁿ a double vernier is used.

Ex:- Theodolite

→ Double vernier consist of two direct verniers Extending in both the direction with index mark in Centre.

(ii) Double Folded Vernier:-

A double folded vernier is a special type of double vernier whose total length is exactly half of combined length of corresponding double vernier.

→ It is used when required to reduce length of vernier scale.

Ex:- Sextant.

→ In double folded vernier, the vernier reads from Index mark (0-mark) towards either of extrem division and then from other extrem division towards centre in the same direction.

3) Extended Vernier :-

When divisions at main scale are very close & it becomes very difficult to determine exact graduation where coincidence occurs if vernier of normal length is used.

Hence Extended Vernier is very helpful when available length of V.S. is small and required to have small least count without making vernier division very close.

In Extended vernier 'n' division of vernier scale are equal to '2n-1' division of main scale.

$$nV = (2n-1)S$$
$$L.C. = 2S - \left(\frac{2n-1}{n} \right) S$$

$$L.C. = \frac{S}{n}$$

Precision & Accuracy :-

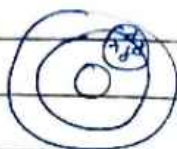
① Precision :- Degree of fineness and care with which any physical Measurement is done.

② Accuracy :- Degree of ~~fineness~~ Perfection.
→ A value is considered to be accurate when it is close to true value.

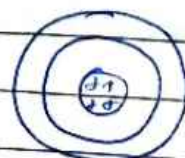
→ Precise value represents set of observations that are closely grouped & have small deviation from true value.



A.V.



P.V.



AV & PV

Note:- To plot or draw a plan accuracy required is also defined by scale of plan. A Good draftsman can plot smallest length of 0.25 mm. Now for a scale of 1:1000 (that is 1mm on plan = 1m on ground) smallest plotable distance is 0.25m. Hence for survey at scale of 1:1000 accuracy desired in field should be 0.25m.

Mistake:-

These are discrepancies cause due to carelessness, misunderstanding & poor judgement.

Mistakes can be corrected if only discovered, mistakes can be avoided by adopting careful procedure.

Error:- There are discrepancies other than mistake.

$$\boxed{\text{Error} = \text{M.V.} - \text{T.V.}} = -(\text{Correction})$$

$$\therefore \text{Correction} = \text{T.V.} - \text{M.V.}$$

① Systematic Error:- (Cumulative Error)

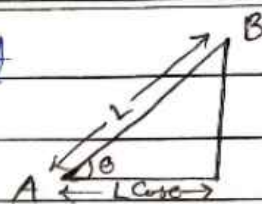
- These errors follows definite Mathematical or physical law.
- These are cumulative in nature.

② Random Error:- (Accidental Error/Compensating Error)

- These are errors other than systematic errors.
- These are compensating in nature.

2] Linear Measurement

Main purpose of Surveying is to plot Map of an area. As the Map is plotted on a horizontal plane. The distances shown are horizontal projection on this plane.



In surveying distance btw 2-points is Horizontal distance. When slope distance are measured in field these are always reduced to equivalent horizontal distance for the preparation of Map.

⇒ Methods of Linear/Horizontal Measurement :-

- ① Direct-Method :- Chain or Tape
- ② Optical-Method :- Tacheometry
- ③ EDMI :- Electronic distance measuring instrument.
Geodimeter, Distomat
Mekometer, Tellurometer
Total-Station

Chain-Surveying :-

In this type of surveying the distance are measured with chain or Tape. Chain is used for smaller area.

- In process of joining chaining the survey team consist of follower (0m end) and leader (20 or 30m end).
- Leader puts the arrow in ground and follower collects it.
- Number of Arrows with the follower signifies chain length measured.

Procedure in chain Surveying :-

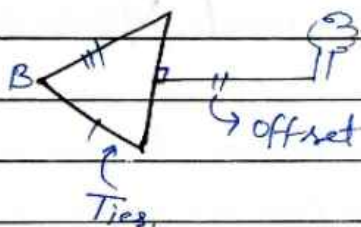
For small Area details of various points can be collected with help of ties or offsets.

⇒ Offset :-

Offsets are lateral distances measured from the survey line to the point whose information to be collected.

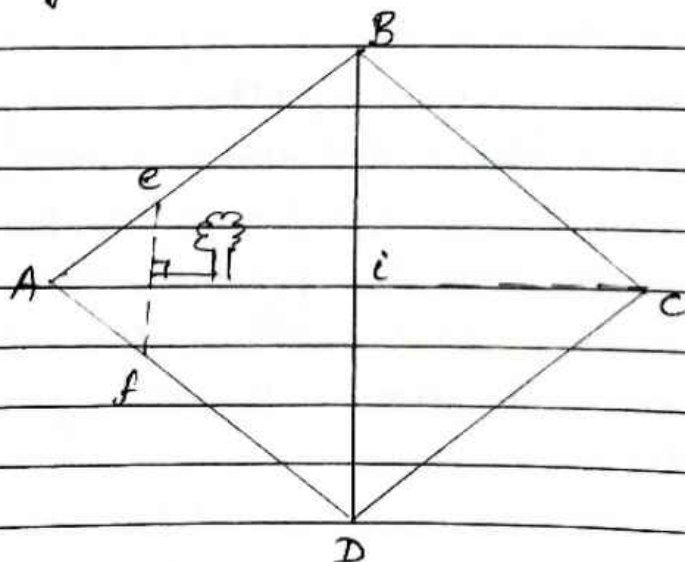
Perpendicular Offset → Simply Offset

Oblique Offset → Tie



When area is large Error introduced due to large offset will be more. Therefore to overcome this difficulty large area is first of all converted into a Network of triangle and then details are collected with respect to sides of these triangle using Ties and offsets.

Basic Definations :-



Main Station:-

It is a point where two sides of a Triangle meet and these station decides boundary of survey area.

Ex:- A, B, C & D.

Main Survey Line:-

These are the lines joining main station. Ex- AB, BC, CD, AD, BD.

Tie Station / Subsidiary Station / Auxiliary Station:-

These are the stations on main survey line. Ex:- e & f.

Tie Line / Auxiliary Line / Subsidiary Line:-

These are the lines joining Tie stations. These are helpful in locating inner details of the area which is to be surveyed. Ex:- ef

Base line:-

It is longest main survey line which generally passes through the central part of area.

Ex.- BD.

Check - Line:-

It is provided to check the accuracy of field work. It is not for plotting of any point.

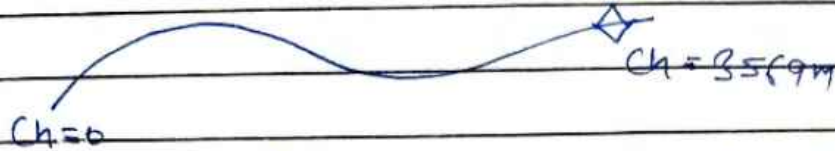
Ex - C_i.

Measured length of check-line should be equal to length of check-line scaled from the plan.

Chainage:-

Distance ✓

Not displacement ✗



Chainage is the distance measured along the dirⁿ of the progress of survey. It is distance of a well defined point from the starting point.

The term chainage is generally used in linear projects. ex:- Highway, Railways, Canal etc.

They are used to represent important features in work.

Well Conditioned Triangle:-

In chain Surveying attempt should be made to form a triangle such that intersections of lines should be clear for plotting purpose, hence for obtaining clear intersection angles should be in range of 30° - 120° .

Best possible well conditioned triangle. is equilateral Triangle.

Equipments used in chain Surveying

① Surveying Chain:-



Types of chain :-

- 1) Revenue Chain $\rightarrow$ 33ft 16 Links
- 2) Gunter's Chain $\rightarrow$ 66 ft 100 Links
- 3) Engineer's Chain $\rightarrow$ 100 ft 100 Links
- 4) Metric Chain $\rightarrow$ a) 20m $\rightarrow$ 100 Link
b) 30m $\rightarrow$ 150 Link

Note:-

1 Mile = 1.6093 km = 80 Gunter's Chain

1 Acre = 4046.856 m² = 10 $\times$ (Gunter's chain)²

1 Yard = 3ft = 0.9144m

② TAPE:-

1) Cloth or Linen Tape:-

It is made up of closely woven linen or synthetic material and varnished to resist moisture.

This tape doesn't remain straight during strong winds.

2) Metallic Tape:-

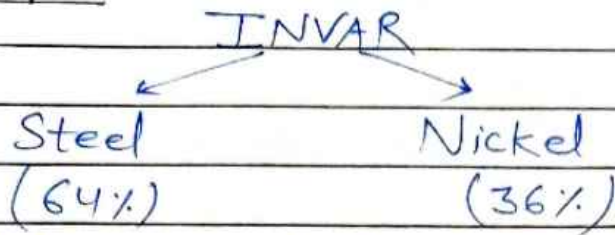
It is made up of linen with brass or copper wire woven into it and varnished.

This tape gives better result than in comparison to cloth tape because stretching is reduced.

3) Stainless-Steel Tape:-

Made up of Stainless steel.

4) Invar Tape



These tapes are highly accurated and have small value of thermal coefficient of Linear Expansion (α).

→ This tape is very soft and hence required special attention on field.

Accessories Used in Surveying

① Pegs:- These are used to mark definate point on ground semi permanently or temporary



② Arrow:- It is used by leader to mark end point of chain length which is later collected by follower.



③ Ranging Rod:- It is used to locate intermediate points along a st. line and this process is called as Ranging.

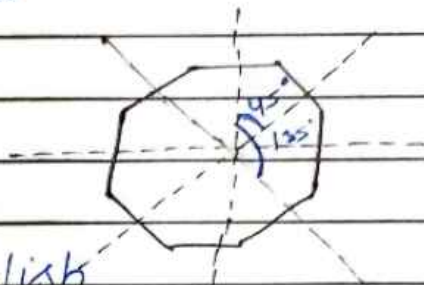
④ Offset Rod:-

It is a ranging rod with slots made at right angle. It can be used to establish perpendicular offset with respect to survey line.



⑤ Cross-Staff / Open Cross Staff :-

It is used for establishing perpendicular offsets w.r.t. survey line at any point.

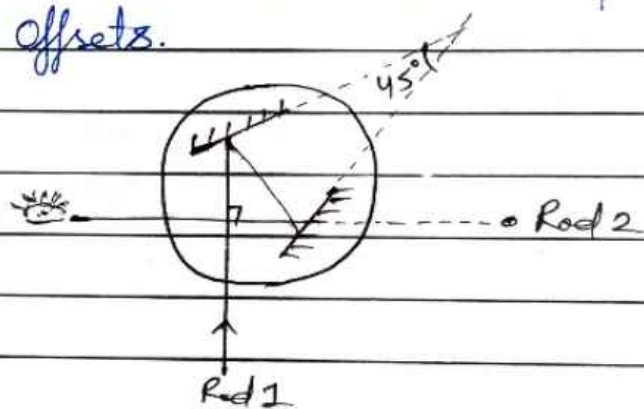


⑥ French Cross staff :-

With help of this we can establish 45°, 90°, 135° offsets.

⑦ Optical Square :- It is used to set out perpendicular offsets.

- It used principle of double reflection.
- Index mirror fully silvered and other half not silvered.



⑧ Prism Square :-

It has similar arrangement as compare to optical square with mirror replaced by prism.

⑨ Simple Clinometer :- It is used to measure slope of ground or vertical angle.



⑩ Ranging :- It is process of establishing intermediate points along a st. line.

i) Direct Ranging :-

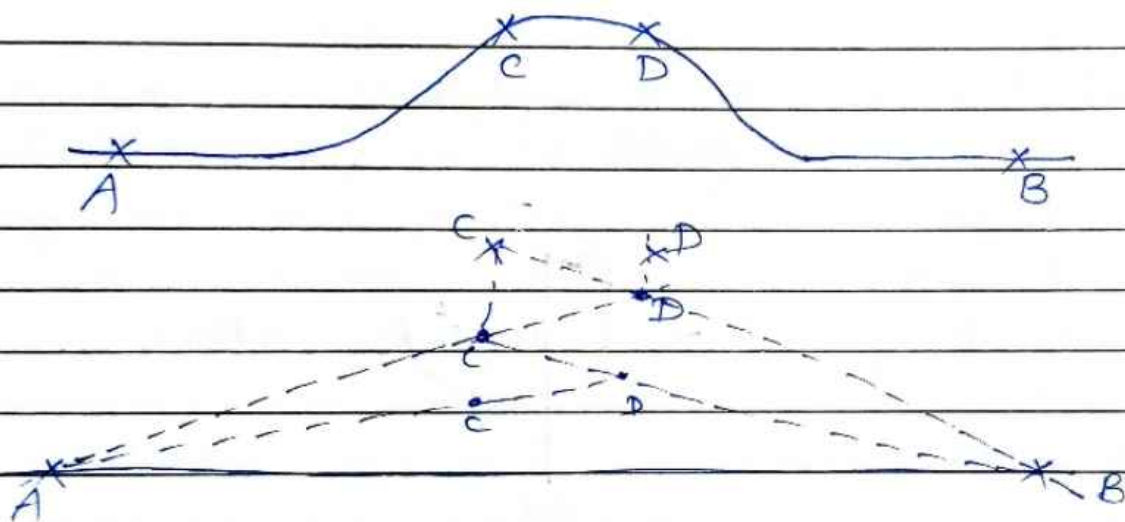
When intermediate points are established by direct observation from end stations.

2) Indirect Ranging / Reciprocal / Repeated Alignment Method :-

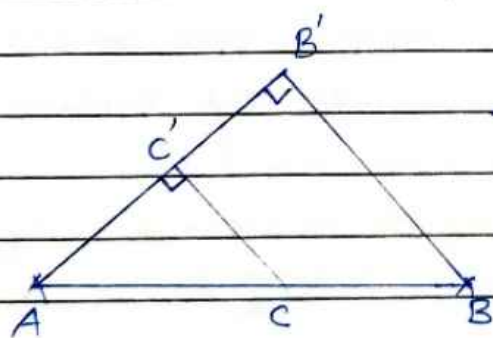
This method is used when end stations are not intervisible due to rising ground b/w them or due to long distance b/w end stations.

During this process we select two intermediate stations (C & D) such that :-

From station C $\rightarrow$ B & D are visible } Mandatory
From station D $\rightarrow$ A & C are visible } Condition.



③ Random Line Method $\rightarrow$



$$\frac{AB'}{BB'} = \frac{AC'}{CC'} (P)$$

- AB is original Survey line to be established
- B' is the point from where station B is visible and line AB' is clear.
- By determining AB', BB' & AC' we can determine CC' with the help of similar triangle.

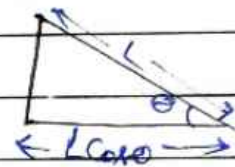
Linear Measurement with chain :-

(A) On smooth level surface :- If the distance to be measured is longer than chain length then by establishing intermediate points on the st. line we can determine the distance.

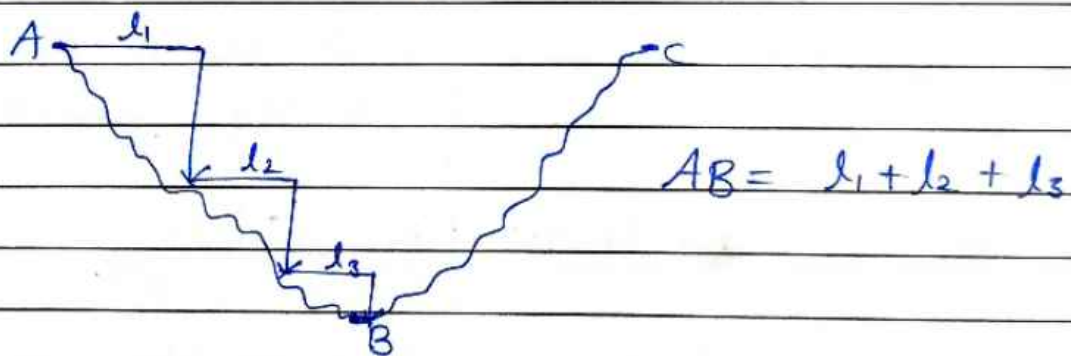
(B) On Sloping Ground :-

On sloping ground the horizontal distance can be measured by :-

- ① Stepping
- ② Measured along the slope



if ground slope is regular or uniform the measuring along the slope is possible but if variable slope is observed then stepping method is used.



While stepping we follow stepping down the hill.

"Errors in Chaining" :-

① Systematic Error :- Directly proportional length of line.

$$e \propto L$$

② Random Error :- Directly proportional to $\sqrt{\text{length}}$ of line

$$e \propto \sqrt{L}$$

Correction

Chain

- ① Correction for Standardization
- ② Correction for Slope

Tape

- ① Correction for Standardization
- ② Slope Correction
- ③ Pull Correction
- ④ Temperature Correction
- ⑤ Sag Correction
- ⑥ M.S.L. Correction
- ⑦ Mis-Alignment Correction

① Correction for Standardization (C_s) $\Rightarrow$

It is also called as "Correction for absolute length".

- Nominal length / Designated length $\rightarrow L'$ (20m or 30m)
- Actual length / Absolute length $\rightarrow L$
- Measured length $\rightarrow L'$
- Correct length $\rightarrow L$

if actual length of tape doesn't matches with nominal length of tape then correction for standardization is need to be done.

→ If tape is actually shorter than nominal length of the tape, then measured length is greater than correct length.

→ Therefore error is positive & correction is Negative.

→ If Tape is actually [Shorter] then correction will be [subtractive].

→ Error Per Chain length: $\rightarrow \frac{l'-l}{l'}$
No. of chain length = $\frac{L'}{l'}$

$$\text{Error in M.L. (L')} = \frac{(l'-l) \times L'}{l'}$$

$$\text{Correction in M.L. (L') i.e. } Ca = \frac{(l-l') \times L'}{l'}$$

$$\text{Correct length } L = L' + \frac{(l-l') \times L'}{l'}$$

$$\text{Correct length } L = L' \times \frac{l}{l'}$$

$$C.L. = \left(\frac{A.L.}{N.L.} \right) \times M.L.$$

$$C.A. = \left(\frac{A.L.}{N.L.} \right)^2 \times M.L.$$

$$C.V. = \left(\frac{A.L.}{N.L.} \right)^3 \times M.L.$$

Q. A line of true length 500m when measured by 20m tape is reported to be 502m then correct length of the tape is.

Solⁿ:-

$$C.L. = \frac{A.L. \times M.L.}{N.L.}$$

$$\Rightarrow 500 = \frac{502 \times A.L.}{20}, \quad A.L. = 19.92m.$$

Q Area of plan of an old survey was plotted to a scale of $1\text{cm} = 10\text{m}$. Now measures as 90.5cm^2 as determined by planimeter. Plan is found to have shrunk such that a line originally 10cm long now measures as 9.5cm only. A Note on the plan states that a chain used was 9cm too short determine True area of survey if ~~Normal~~ Nominal length of the chain was 20m .
Solⁿ:-

$$\text{Area before Shrinkage} = \frac{90.5}{(0.95)^2} = 100.27\text{m}^2$$

$$\text{Ground Area} = 10027.7\text{m}^2 \quad (1\text{cm} = 10\text{m})$$

$$\begin{aligned} \text{Correct area} &= \left(\frac{19.91}{20}\right)^2 \times 10027.7 \\ &= 9937.65\text{m}^2 \end{aligned}$$

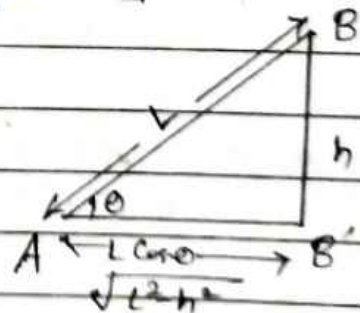
② Correction For Slope :- $C_g \Rightarrow$

We know that slope distance is always greater than horizontal distance hence correction for slope is always Negative.

Case ① Angle or Slope ' θ ' is Measured

$$C_g = L \cos \theta - L$$

$$\boxed{C_g = -L(1 - \cos \theta)}$$



Case ② Height difference ' h ' is measured

$$C_g = \sqrt{L^2 - h^2} - L$$

$$C_g = \left[L \left(\sqrt{1 - \frac{h^2}{L^2}} \right) - L \right]$$

$$C_g = L \left[\left(1 - \frac{h^2}{2L^2} - \frac{h^4}{8L^4} + \dots \right) - 1 \right]$$

$$C_g = -\frac{h^2}{2L} - \frac{h^4}{8L^3}$$

Neglecting higher order term.

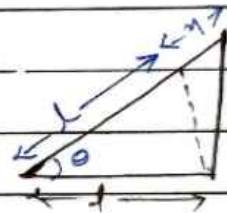
$$C_g = -\frac{h^2}{2L}$$

Note:- If higher order terms are not neglected then magnitude of correction for slope will increase by $\frac{h^4}{8L^3}$.

Hypotenious Allowance:-

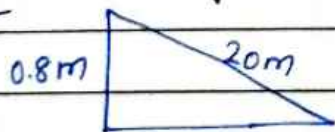
$$n = L \sec \theta - L$$

$$n = L (\sec \theta - 1)$$



② If downhill end of a 20m slope is held 80cm too low then correction for slope will be.

Solⁿ:-



$$C_g = -\frac{h^2}{2l} = \frac{-0.8^2}{2 \times 20} = -0.016m$$

③ Correction for Pull:- (C_p) :-

$$C_p = \frac{(P - P_0) L}{AE}$$

P = Pull applied in the field.

P_0 = Standard pull (60-80 N)

A = Cross-sectional Area of Tape

E = Young's Modulus of Elasticity

L = Nominal length of tape or measured length of line.

Q A Steel tape 30m long was standardized with a pull of 65N. If pull at the time of measurement was 80N. Determine correction for tape length. If Wt. of tape is 10N. $E = 2 \times 10^5 \text{ N/mm}^2$, $\gamma_{\text{steel}} = 77.10 \text{ kN/m}^3$.

Solⁿ:-

$$W = \gamma A L$$

$$10 = \frac{77.10 \times 10^3}{10^9} \times A \times 30 \times 10^3$$

$$A = 4.32 \text{ mm}^2$$

$$C_p = \frac{(P - P_0)l}{AE} = \frac{(80 - 65)30}{4.32 \times 2 \times 10^5} = +0.52 \times 10^{-3} \text{ mm}$$

④ Correction for Temperature :- (C_t)

$$C_t = l \alpha (t - t_0)$$

α = Thermal Coefficient of linear Expansion

t = Field Temp.

t_0 = Standard Temp.

l = Nominal length of tape or measured length of line.

→ if $t > t_0$ then $C_t = (+)ve$

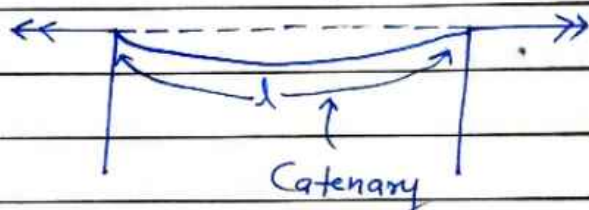
Otherwise $C_t = (-)ve$

Q If measured length of line was 1000m by a 30m Tape when field temp. were 30°C and standard temp was 25°C determine correct length of line ($\alpha = 10^{-5}/^\circ\text{C}$)

$$\begin{aligned} \text{Sol}^n:- \quad C_t &= l \alpha (t - t_0) \\ &= 10^3 \times 10^{-5} \times (30 - 25) \\ &= +0.05 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{C.L} &= 1000 + 0.05 \text{ m} \\ &= 1000.05 \text{ m} \end{aligned}$$

⑤ Correction for Sag:->



When tape is supported btw two ends, it takes the shape of a Catenary.

→ If tape is flat standardise and used in catenary then correction will be "**Negative**". b'coz horizontal chord length is always shorter than curved length.

For determination of correction for sag the curve is assumed to be parabola insted of catenary.

$$C_s = \frac{-w^2 l^3}{24P^2} = \frac{-(wl)^2 l}{24P^2} = \frac{-W^2 l}{24P^2}$$

w = Wt. per unit length

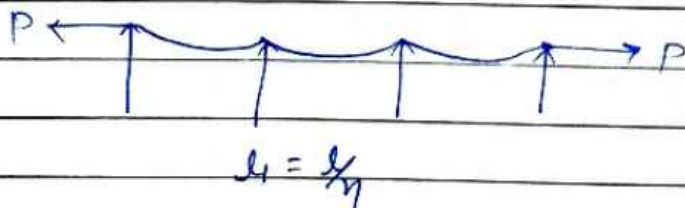
l = Nominal length (Suspended length)

P = Force applied

W = Total weight of Tape

→ If l is total length of tape which is suspended in ' n ' number of bays.

$$\therefore 1 \text{ bay} \cdot l_1 = \frac{l}{n}$$



$$C_s = \frac{-n \times w^2 \left(\frac{l}{n}\right)^3}{24P^2} = \frac{-w^2 l^3}{24n^2 P^2}$$

$$C_s = -\frac{W^2 l}{24 P_n^2}$$

Note:- If tape is standardise in catenary and used for flat then correction for sag will be (+ve)

Q Normal Tension P_n is a theoretical pull at which correction for pull is numerically equal to correction for Sag.

$$\frac{W^2 l}{24 P_n^2} = \frac{(P_n - P_0) l}{AE}$$

Q Determine Sag correction for 30m steel tape under a pull of 80N in 3 days of 10m each. c/s area of Tape is 8 mm^2 and unit wt. of steel may be taken 77 kN/m^3 .

Solⁿ:- $W = \rho A L = \frac{77 \times 10^3}{10^9} \times 8 \times 30 \times 10^3 = 18.48 \text{ N}$

$$C_s = -\frac{(18.48)^2 \times 30}{24 \times 8^2 \times 80^2} = -7.41 \times 10^{-3} \text{ m}$$

Q Determine Normal Tension for a steel tape supported b/w two supports 10m apart if standard tension is 65N and weight of tape per meter is 0.62N.

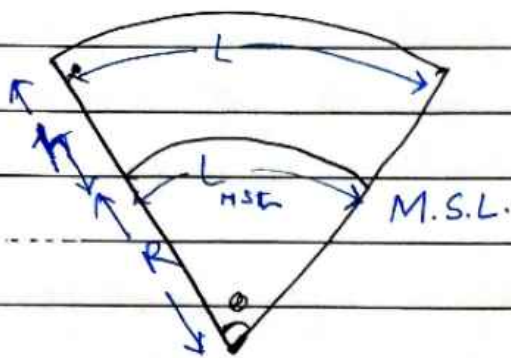
$E = 2 \times 10^5 \text{ N/mm}^2$, c/s Area of tape is 8 mm^2 .

Solⁿ:- $\frac{W^2 l}{24 P_n^2} = \frac{(P_n - P_0) l}{AE}$

$$P_n = 162.29 \text{ N}$$

$$\frac{(0.62 \times 10)^2}{24 \times P_n^2} = \frac{(P_n - 65)}{8 \times 2 \times 10^5}$$

⑥ Correction for MSL :-



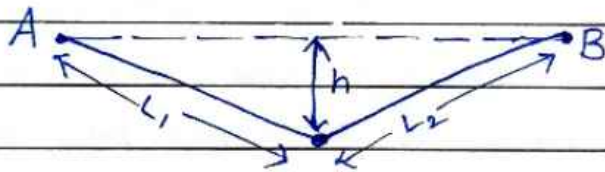
$$\begin{aligned} L_{MSL} &= L_{MSL} - L \\ &= R\theta - (R+h)\theta \\ &= -h\theta \end{aligned}$$

$$C_{MSL} = \frac{-hL}{R+h}$$

$$h \ll R$$

$$C_{MSL} = \frac{-hL}{R}$$

⑦ Correction for Mis-Alignment :-



$$C_M = (\sqrt{L_1^2 - h^2} + \sqrt{L_2^2 - h^2}) - (L_1 + L_2)$$

"Field Work in chain Surveying"

① Reconnassiance Survey:- It is preliminary inspection of the area to be surveyed to get some idea of terrain and to identify principle features of the ground.

It helps the surveyor to decide upon best possible arrangement of triangles.

② Marking of Stations:- After completion of reconnassiance survey best suitable position of main survey station are marked on the ground such that they can be easily identified afterward if required.

③ Running of Survey Lines:- Survey lines are run to measure distance btw main station and to locate adjacent details by taking offsets.

④ Taking Offsets:-

Note:- ① Max. length of offset depends on accuracy desired, Nature of ground, Scale of plotting.

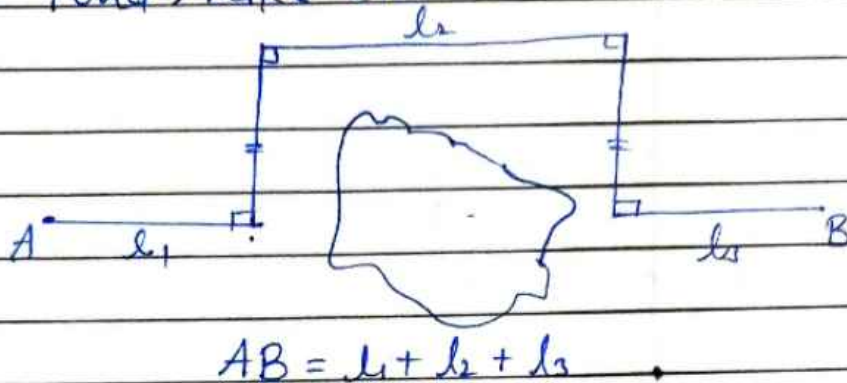
② Degree of accuracy of offset depends on scale of plotting, length of offset, importance of detail whose information need to be collected.

① Reconnaissance → ② Preliminary → ③ Detail Survey

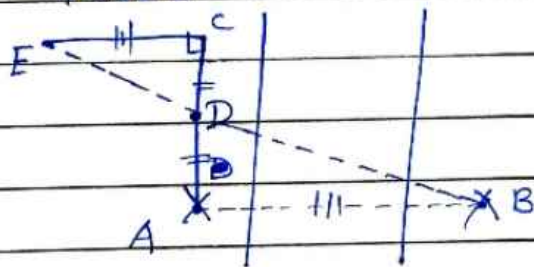
"Obstacle in Chaining"

Case I :-

Chaining round of obstacle is possible
Ex:- Pond, Lake etc.

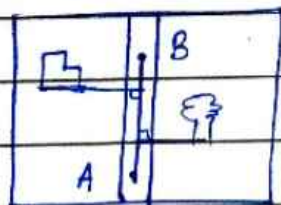


Case II :- Chaining round of obstacle is not possible
Ex:- River

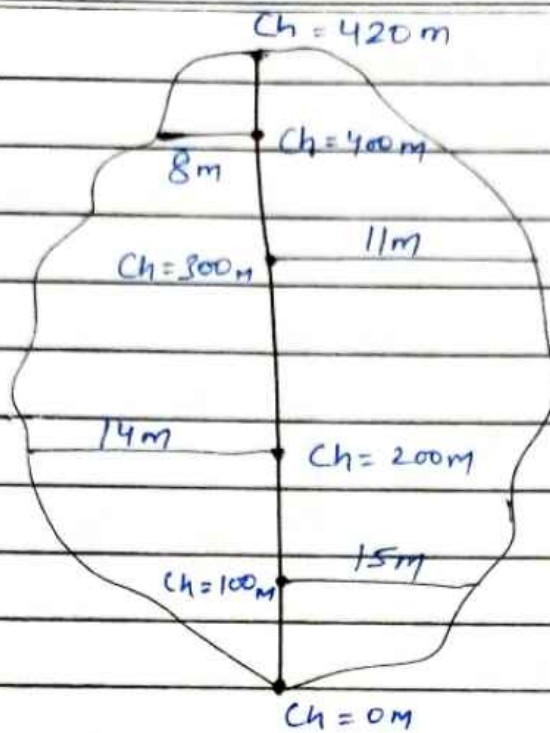


Case III :- Recording of Measurement

Double line field book is used to record chain survey and recording starts from bottom toward Top.



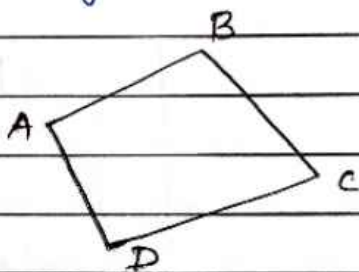
Cross-Staff Survey $\Rightarrow$ It is a special type of survey (chain) which is used to locate boundary of the field for calculating approx Area of field.



During this survey Area is plotted on a suitable Scale

3 COMPASS Surveying

- Compass Surveying is branch of Surveying in which dirⁿ of survey lines are established with the help of compass and length is measured with the help of chain or Tape.
- Compass are generally used to run a traverse in the field.
- In Surveying traverse is a frame work consisting of series of straight lines forming a open or closed polygon.

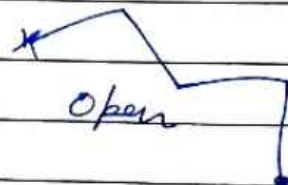
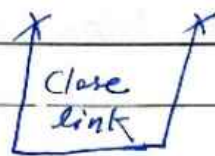
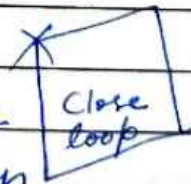


Points such as A & B are called as Traverse Station or Traverse Point. At each traverse station where sides meet angles are measured.

- Depending on instrument used for the measurement of angles traverse can be classified as compass traverse, plane table traverse, Theodolite traverse.

Type of Traverse:-

- ① Closed Traverse:- It is a traverse which starts from a point of known location and closes either at same point or at another point of known location.



② Open Traverse :-

It starts from a point of known location and closes at another point of unknown location.

- Open traverse is used for linear projects.
- An open traverse can be checked by Method of chords or by Astronomical Observations.

"Difference"

Chain Surveying

- ① Only Linear measurements are done.
- ② Framework consist of Network of Triangle
- ③ Check lines or proof lines are required.
- ④ Used for smaller Area and accuracy desired is low.

Traverse Surveying

- Linear & Angular Measurements are done.
- Framework consist of Open or Closed Traverse.
- Not required, as accuracy is checked by method of adjustment.
- Used for larger area Accuracy desired is high

Measurements of Angles :- Direction of survey line can be defined in two ways

- ① Relative to each other
- ② Relative to some fixed reference direction.

This fixed reference direction in language of Surveying is called as Meridiane.

"There are 4 types of Meridians"

① True Meridian :-

True Meridian at a point is the line joining True North and true South of the earth surface through that point

- True Meridian is determined with help of Astronomical observation of Sun & Star.
- True Meridian at a place doesn't ~~to~~ changes with the passage of time.

② Magnetic Meridian :-

Magnetic meridian at a point is direction indicated by a freely suspended Magnetic bar or needle. Provided that it is not affected by local Magnetic field.

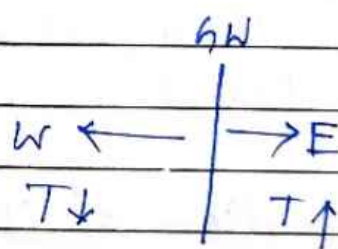
Magnetic meridian at a place is not fix, it changes with the passage of time.

③ Grid Meridian :-

$$1 \text{ hr} = \frac{360}{24} = 15^\circ$$

$$30 \text{ min} \rightarrow 7.5^\circ$$

$$\frac{82.5^\circ \text{E}}{7.5} = 11$$



For survey of a country true meridian passing through central place taken as reference meridian for the whole country and such a reference meridian is called as Grid Meridian.
Ex :- 82.5°E Allahbad

① Arbitrary Meridiane:-

It is the meridian taken in arbitrary direction. Generally it is taken in the direction from a traverse station to a well defined point such as top of chimney, Tower etc.

Sometimes it is also direction of first traverse line.

BEARING:-

Bearing is the horizontal angle b/w a survey line and reference Meridian.

Bearing can be True Bearing, Magnetic Bearing, Grid Bearing or Arbitrary bearing.

True bearing of a survey line does not change with the passage of time. Whereas Magnetic bearing changes with passage of time b'coz magnetic meridian changes.

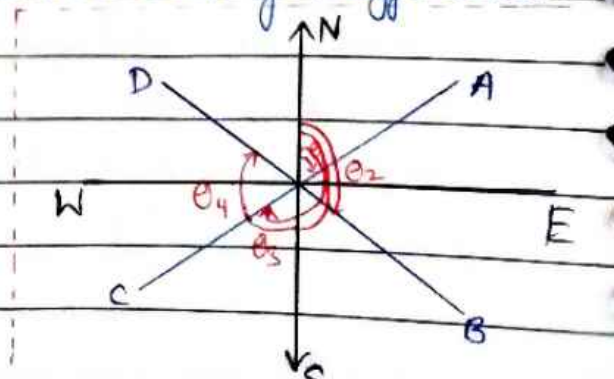
Note:- For all important surveys True bearing is preferred over magnetic Bearing.

⇒ Designation of Bearing:->

★ ★ ① Whole Circle Bearing System (WCBS) :->

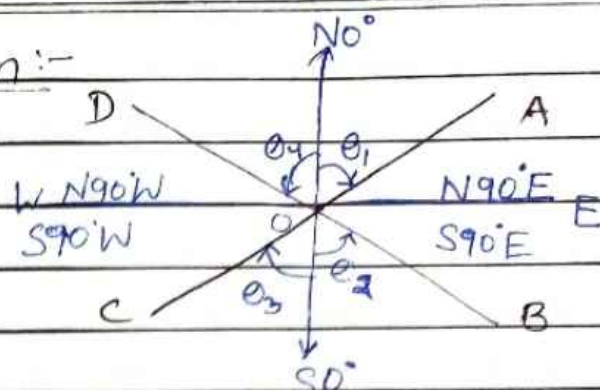
It is horizontal clock-wise angle measured from North end of reference Meridian towards survey line.

- It is also called as AZIMUTH
- It ranges b/w ~~0-180~~ 0-360°
- ★ Prismatic Compass.



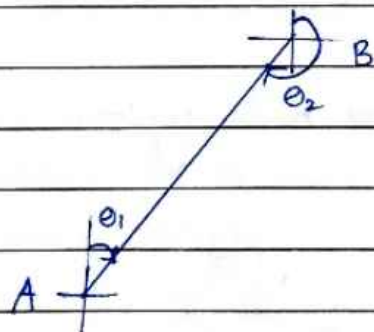
② Quadrantal Bearing System:- (Reduce Bearing)

Line	Q.B.
OA	N θ_1 E
OB	S θ_2 E
OC	S θ_3 W
OD	N θ_4 W



- It is the acute angle b/w Survey line and reference Meridian.
- It can be measured from North or South end in clockwise or Anti-clockwise direction. It's value ranges b/w 0-90°
- Surveyor's Compass.

Fore Bearing & Back Bearing:-



Line	F.B.	B.B.
AB	θ_1	θ_2
BA	θ_2	θ_1

Fore Bearing is the bearing of line measured in dirⁿ of progress of survey.

Back Bearing is bearing of a line measured in dirⁿ opposite to the progress of survey.

Check:-

$$B.B. = F.B. \pm 180^\circ$$

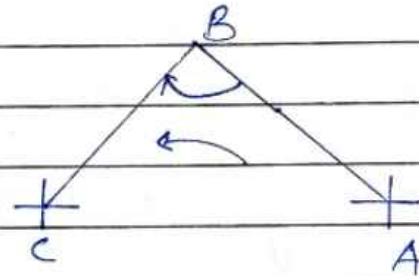
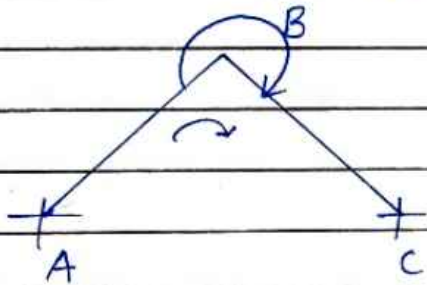
$$+Ve \rightarrow F.B. < 180^\circ$$

$$-Ve \rightarrow F.B. \geq 180^\circ$$

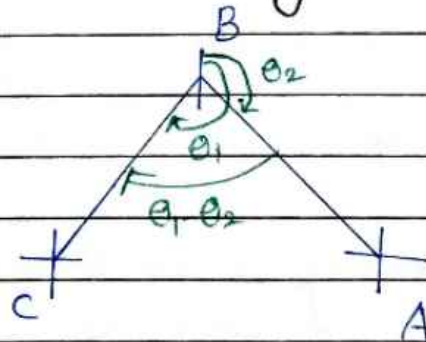
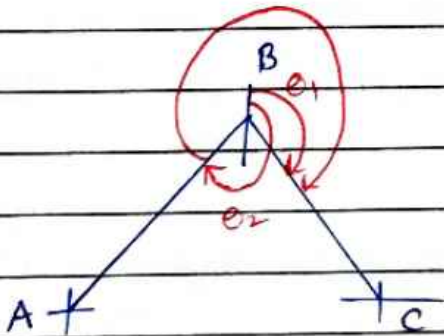
If Fore Bearing of a line is given in Quadrantal Bearing system then Back bearing can be obtained by replacing North by South, South by North, East by West and West by East, also keeping the same value of 'θ'.

Ex:- $S 50^\circ E \rightarrow N 50^\circ W$
 $N 40^\circ W \rightarrow S 40^\circ E$

Included Angles:- It is angle measured in clockwise direction from previous line to the next line.



→ Calculation of included angle from bearing:-

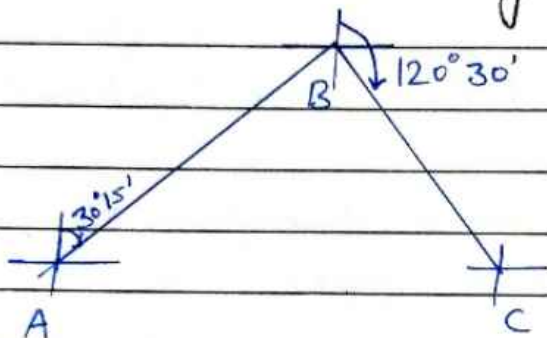


Included Angle = F.B. of Next line - B.B. of Previous line

Included Angle = F.B. of Next line - B.B. of Previous line + 360°

If a Negative value is obtained then simply add 360° to get required value of included angle.

Q WBC of line AB & BC are $30^\circ 15'$ & $120^\circ 30'$ respectively. What is included angle, $\angle B$ or angle ABC?



$$\angle B = 89^\circ 45'$$

$$\text{included Angle} = 270^\circ 15'$$

$$= 59^\circ 45' + 90^\circ + 120^\circ 30' = 270^\circ 15'$$

$$= 120^\circ 30' - (130^\circ 15' + 180^\circ) + 360 = 270^\circ 15'$$

Q Determine the value of included angle in a close compass traverse.

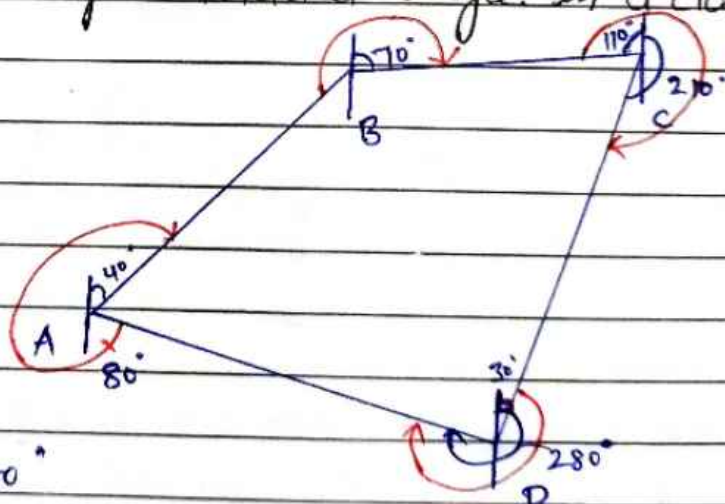
Line F.B.

AB 40°

BC 70°

CD 210°

DA 280°



$$\angle A = 40^\circ + 180^\circ + 80^\circ = 300^\circ$$

$$\angle B = 180^\circ - 40^\circ + 70^\circ = 210^\circ$$

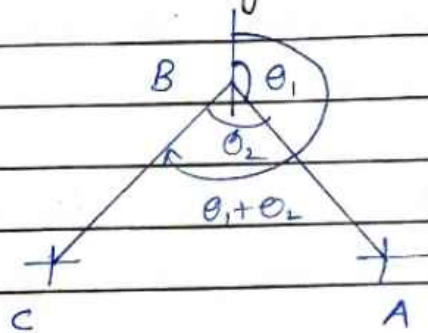
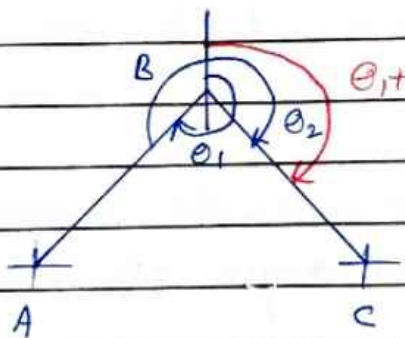
$$\angle C = 110^\circ + 210^\circ = 320^\circ$$

$$\angle D = 280^\circ - 30^\circ = 250^\circ$$

$$\text{Sum of Exterior Angles} = (2n+4)90^\circ$$

$$\text{Sum of Interior Angles} = (2n-4)90^\circ$$

Calculation of Bearing From Included Angle:-



$$\text{F.B. of Next line} = \text{B.B. of Previous line} + \text{Included Angle}$$

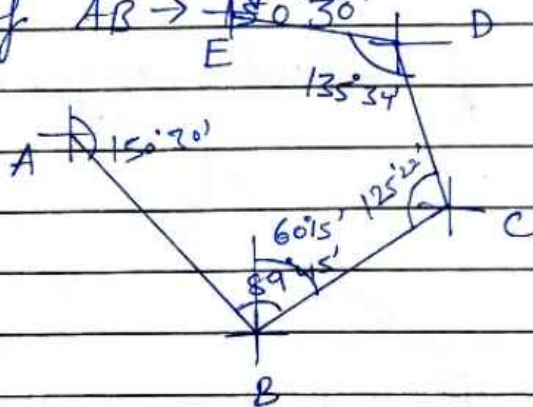
If the value obtained is more than 360° then subtract 360° to get the required value.

Q- Following included angles were observed in an open traverse angle $\angle ABC = 89^\circ 45'$, $\angle BCD = 125^\circ 22'$, $\angle CDE = 135^\circ 34'$

Calculate bearing of all the lines if

F.B. of AB $\rightarrow 150^\circ 30'$

Soln:-



$$\text{FB of BC} = (150^\circ 30' + 180^\circ) + 89^\circ 45' - 360^\circ = 60^\circ 15'$$

$$\text{FB of CD} = (60^\circ 15' + 180^\circ) + 125^\circ 22' - 360^\circ = 5^\circ 37'$$

$$\text{FB of DE} = (5^\circ 37' + 180^\circ) + 135^\circ 34' = 321^\circ 11'$$

Magnetic Field Of Earth $\Rightarrow$

Earth act as a powerful Magnet with its magnetic lines of forces running from magnetic south towards magnetic North.

- A magnetic Needle aligns itself in the direction parallel to earth Magnetic field.

Magnetic lines of forces are vertical at poles and they become parallel to earth surface at equator.

Angle of dip $\therefore$ It is vertical angle btw earth surface and Magnetic lines of forces is called as dip angle.

At Poles Dip = 90°

At Equator Dip = 0°

Btw Equator & poles dip = $0-90^\circ$

~~Lines join~~

Isoclinic Line $\Rightarrow$ Lines joining points of equal angle of Dip on earth surface is known as Isoclinic line. Ex:- Latitude.

Aclinic Line $\Rightarrow$

A special case of isoclinic line when angle of Dip is zero is called as Aclinic line.
Ex:- Equator.

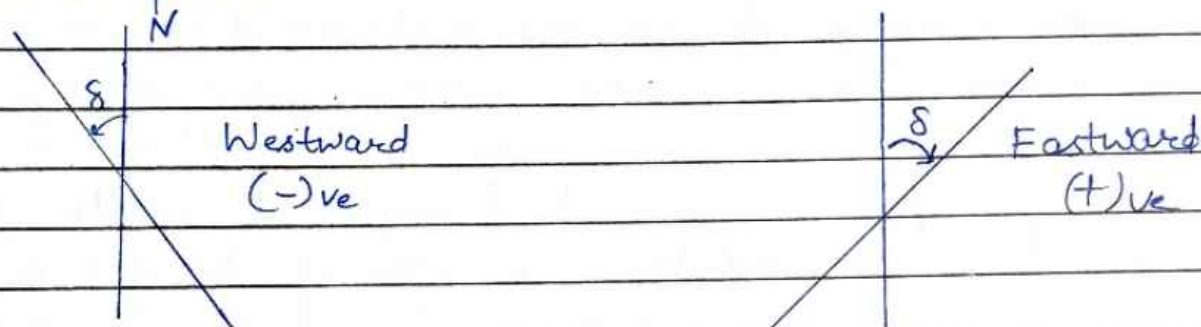
Magnetic Declination $\therefore$

Generally Magnetic Meridian and True meridian don't coincide with each other. Horizontal angle btw True meridian & Magnetic meridian at the time of observation is called as Magnetic Declination.

→ Magnetic Declination in the field can be determined by observing Magnetic Bearing & True Bearing of a Survey line.

→ Declination at a place changes from time to time.

→ Declination at a ~~place~~ time changes from ~~change~~ place to place.



→ Variation of Declination is shown by Isogonic Line.

⇒ Isogonic Line :-

These are lines joining points of same declination on earth surface at a given point of time.

⇒ Agonic Line :-

These are special Isogonic line which pass through points having zero declination at all the points on Agonic line magnetic meridian coincides with True meridian.

Variation of Magnetic Declination :- These are 4 types of variation which occurs in magnetic declination of a place.

① Secular Variation :- Secular variation of declination occurs continuously over a long period of time.

This variation approximately follows Sine curve over a period of 300 years. There is no reliable method of predicting secular variation.

2) Angular Variation $\rightarrow$

Angular variation is variation of declination in a year. It causes due to revolution of earth around sun.

Annual variation is not same at annual rate of change of secular variation.

3) Diurnal Variation :-

Diurnal variation is variation of declination in one day. It causes due to rotation of earth about its own axis.

$\rightarrow$ It is observed that diurnal variation is

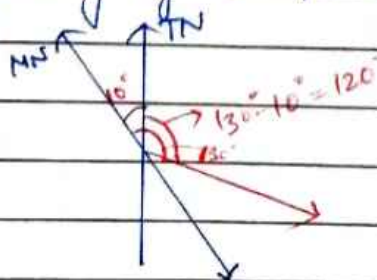
- i) Greater in day than in night
- ii) Greater in Summer than in night
- iii) Greater in poles than at equator
- iv) It changes from year to year

4) Irregular Variation $\rightarrow$

It caused due to volcanic eruption earthquake or storm.

① Magnetic Bearing of a Survey line AB is 130° determine True bearing of same line if magnetic declination is $10^\circ W$

Solⁿ:-



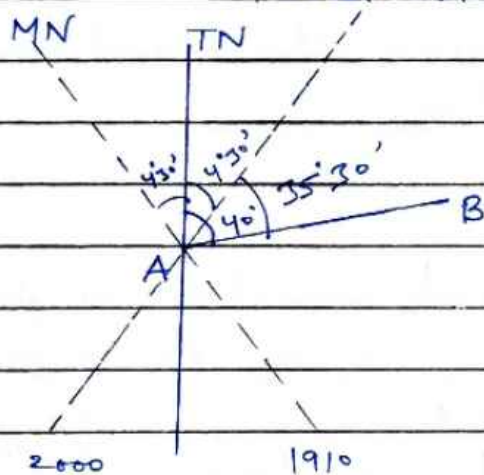
Q If a line AB had magnetic bearing of $44^{\circ}30'$ in year 1910 when declination was $4^{\circ}30'$ west determine Magnetic bearing of same line in year 2000 when annual rate of change of declination is $6'E$.

Solⁿ:- 1910 $\rightarrow$ 2000
90 years

1 year change = $6'$

90 $\rightarrow 90 \times 6' = 540' \Rightarrow \underline{9^{\circ} E}$

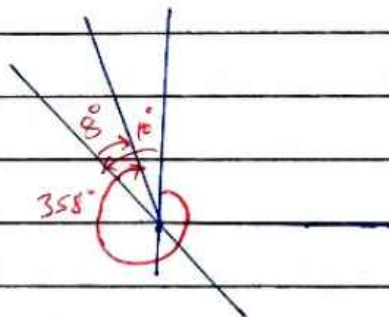
Total change in declination = $9^{\circ} E$



Magnetic bearing
 $44^{\circ}30' - 9^{\circ}$
 $= 35^{\circ}30'$

Q If true bearing of a survey station A = 358° and Magnetic bearing of the same was 8° fore bearing of AB when measured by prismatic compass = 300° . Determine true fore bearing of AB.

Solⁿ:-



True bearing = $300^{\circ} - 10^{\circ}$
 $= 290^{\circ}$

Local Attraction $\Rightarrow$

It is the attraction magnetic needle to a local Magnetic field other than earth Magnetic field.

It is caused by any iron material nearby Compass Station ex:- Iron pipe, Iron fence, Steel vehicle, Iron deposit, Transmission line etc.

- $\rightarrow$ To determine local attraction it is mandatory to take fore bearing and back bearing of each survey line.
- $\rightarrow$ The difference btw F.B. & B.B. should be equal to
- $\rightarrow$ If $F.B. - B.B. \neq 180^\circ$ then local attraction is expected at
 - 1) Either of the station A or B.
 - 2) At both stations A and B.
- $\rightarrow$ If $F.B. - B.B. = 180^\circ$ then both the stations are free from local attraction.

Correction for Local Attraction $\Rightarrow$

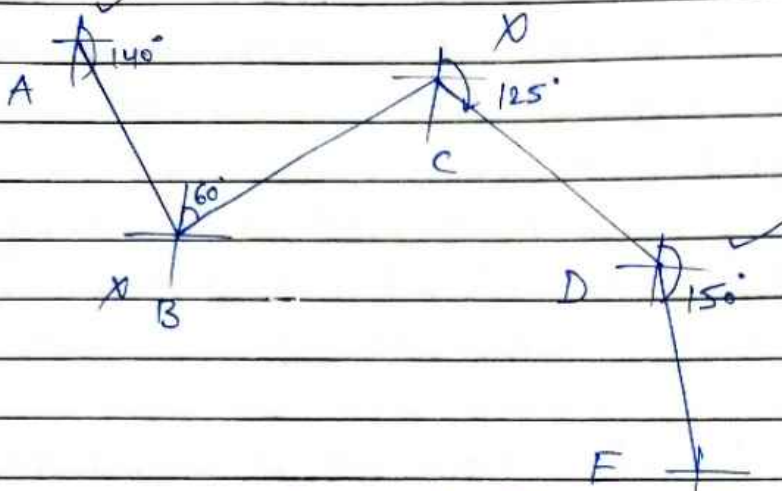
METHOD:-1

• By calculating local attraction @ each station
In this method local attraction @ each station is determined and then required correction is applied to observed bearings.

Example

Line	FB	BB
AB	140°	318°
BC	60°	240°
CD	125°	302°
DE	150°	

Solⁿ:-



Correct B.B. of AB = 318°

Correct F.B. of AB = $318^\circ - 180^\circ = 138^\circ$

Obs. F.B. of AB = 140°

Correction at Station A = -2°

$\therefore$ Correct FB of CD = 125°

Correct BB of CD = $125 + 180 = 305^\circ$

Observed B.B. of CD = 302°

So Correction at D = $+3^\circ$

Since Observed FB of DE = 150°

$\therefore$ Correct FB of DE = $150 + 3$
= 153°

Steps:-

- ① Observe a line whose FB & BB differ by 180°
- ② End stations of such line are free from local attraction
- ③ With the help of correct bearing and correction for local attraction we can correct the bearing at all the other stations.

Observations:-

Corrections for all the bearings are taken constant at each and every station.

- This method is suitable for open traverse.
- This method fails in case of close traverse that is when instrumental errors are present or theoretical sum doesn't match with actual sum.

② METHOD-2:- (By Included Angles)

- This method is most suitable for closed traverse.
- In this method included angles are used to determine correct bearings.
- Since all the bearings taken at a station are equally affected in same direction hence included angles are free from local attraction.

STEPS:-

- ① Determine included Angles.
- ② Observe a line whose F.B. & B.B. differ exactly by 180° .
- ③ With the help of correct bearing and included angles we can determine correct bearing at all the other stations.

Q- If Observed bearing are as given below then determine included angles.

Line	FB	BB
AB	$150^{\circ}30'$	$329^{\circ}45'$
BC	$78^{\circ}0'$	$256^{\circ}30'$
CD	$42^{\circ}30'$	$223^{\circ}45'$
DE	$315^{\circ}45'$	$134^{\circ}15'$
EA	$220^{\circ}15'$	$40^{\circ}15'$

Solⁿ:-

FB - BB

$$\angle A = 150^{\circ}30' - 40^{\circ}15' = 110^{\circ}15'$$

$$\angle B = 78^{\circ}0' - 329^{\circ}45' + 360^{\circ} = 108^{\circ}15'$$

$$\angle C = 42^{\circ}30' - 256^{\circ}30' + 360^{\circ} = 146^{\circ}0'$$

$$\angle D = 315^{\circ}45' - 223^{\circ}45' = 92^{\circ}0'$$

$$\angle E = 220^{\circ}15' - 134^{\circ}15' = 86^{\circ}0'$$

$$\text{Sum of all angles} = 110^{\circ}15' + 108^{\circ}15' + 146^{\circ} + 92^{\circ} + 86^{\circ} = 542^{\circ}30'$$

$$\text{Theoretical Sum} = (2n-4)90^{\circ} = 540^{\circ}$$

- We can observe that theoretical sum and actual sum don't match with each other and this discrepancy is due to instrumental error.

$$\text{Error} = \text{M.V.} - \text{T.V.} = +2^{\circ}30'$$

$$\text{Correction per angle} = -30'$$

Corrected Angle

$$\angle A = 109^{\circ}45'$$

$$\angle B = 107^{\circ}45'$$

$$\angle C = 145^{\circ}30'$$

$$\angle D = 97^{\circ}30'$$

$$\angle E = 85^{\circ}30'$$

Q A Compass traverse ABCD was Run and following bearings were observed determine stations affected by local attraction & correct bearing by both methods.

Line	FB.	B.B.
AB	$80^{\circ}45'$	260°
BC	$130^{\circ}30'$	$311^{\circ}35'$
CD	$240^{\circ}15'$	$60^{\circ}15'$
DA	$290^{\circ}30'$	$110^{\circ}10'$

Solⁿ:- We can observe that FB & BB of line CD are differing by 180° so the stations C & D are free from local attraction.

Method-1:-

We can observe all the bearings taken from C & D would be correct.

Correction at station A = $+20^{\circ}$

$\therefore$ Observed FB of AB = $80^{\circ}45'$

$\therefore$ Correct FB of AB = $81^{\circ}5' + 180^{\circ}$
= $261^{\circ}5'$

Observed BB of AB = 260°

So correction @ B = $261^{\circ}5' - 260^{\circ}$
= $+1^{\circ}5'$

$\therefore$ Observed B.B. of BC = $130^{\circ}30'$

Correct FB of BC = $131^{\circ}35'$

Method 2:- LA = $330^{\circ}35'$ LC = $288^{\circ}40'$

LB = $230^{\circ}30'$ LD = $236^{\circ}15'$

Actual Sum = LA + LB + LC + LD = 1080°

Theoretical Sum = $(2n+4)90^{\circ} = 1080^{\circ}$

Correct FB of DA = $290^{\circ}30'$, BB = $110^{\circ}30'$

Correct FB of AB = $110^{\circ}30' + 330^{\circ}35' - 360^{\circ} = 815^{\circ}$, BB = $261^{\circ}5'$

Correct FB of BC = $261^{\circ}5' + 230^{\circ}30' - 360^{\circ} = 131^{\circ}35'$

Correct BB of BC = $311^{\circ}35'$ (OK)

4 TRAVERSING

Traverse is a frame work consisting of straight line, forming a closed and open polygon.

Types :)

- ① Closed Traverse
- ② Open Traverse

Measurements in Traversing:-

Linear Measurement

- ① Chain or Tape
- ② Techeometer
- ③ EDM I

Angular Measurement

- (i) Loose Needle Method
- (ii) Fast Needle Method
- (iii) Included angle Method
- (iv) Deflection angle Method
- (v) Direct angle Method

In traversing linear measurement equipment should be selected such that degree of accuracy is of the same order of angular Measurement.

If very precise equipments are used for angular measurement then equally precise equipment shall be used for distance measurement.

⇒ If 'θ' is least count of angular measurement equipment then linear measurement equipment should have error of the order of

$$\tan \theta = \frac{\delta l}{l}$$

Q If least count of compass is $30'$. Find order of error permissible in linear measurement used in compass survey.

Solⁿ:-

$$\tan 30' = \frac{SL}{L} = 0.00873$$

($30'$ - Minutes not degree)

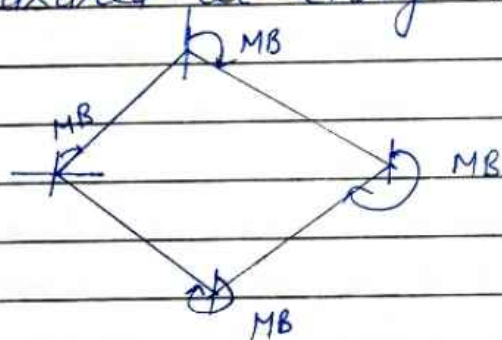
$$\text{if } L = 1m = 1000mm$$

$$\therefore SL = 8.73mm$$

Angular Measurement

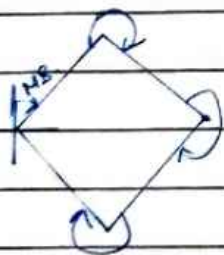
① Loose/Free Needle Method:-

In this method Magnetic Bearing of each Traverse Line is measured at every station.



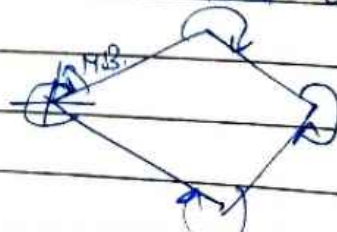
② Fast Needle Method:-

In this method Magnetic bearing is determined for one traverse line which is generally the first traverse line and included angles are determined for other traverse line.



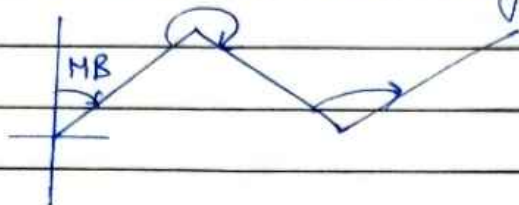
This is more accurate than previous method.

③ Included Angle Method:- In this method magnetic Bearing of any one line is measured which is generally first traverse line and included angles are measured at all the stations.

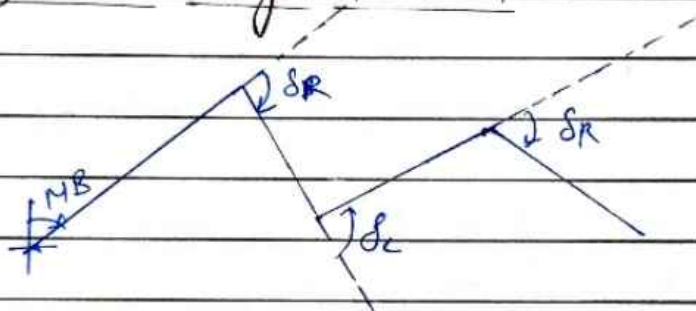


④ Direct Angle Method:-

This method is generally used in Open Traverse.
In this method Direct angles towards are measured.



⑤ Deflection Angle Method:-



It is used for Open Traverse in which the value of deflection angle is small like canal, pipeline, Sewer etc.

ANGLE OF MISCLOSURE $\Rightarrow$

In loop traverse the difference btw actual sum of measured angles and theoretical sum is called as Angle of Misclosure.

It's permissible Limit is

$$E = K\sqrt{N}$$

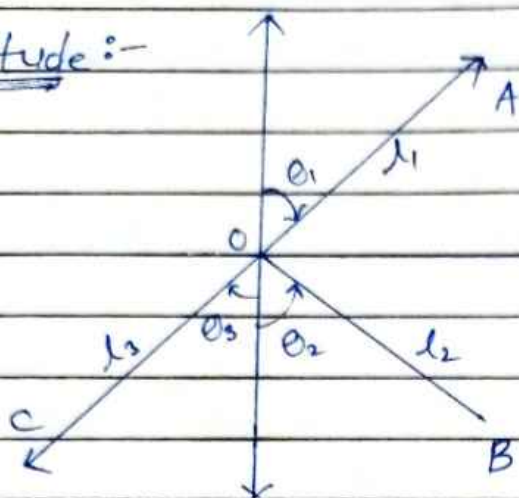
N = No. of sides / Angles

K = Constant whose value depends on least count of Theodolite no. of repetition & accuracy desired.

Generally for ordinary theodolite survey $K = 20''$

LATITUDE & DEPARTURE :->

Latitude :-



$$LoA = l_1 \cos \theta_1$$

$$DoA = l_1 \sin \theta_1$$

$$LoB = -l_2 \cos \theta_2$$

$$DoB = +l_2 \sin \theta_2$$

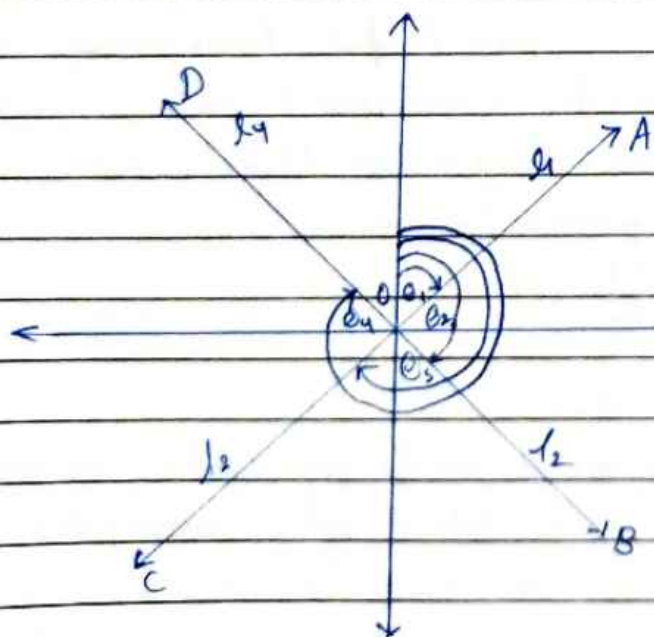
$$LoC = -l_3 \cos \theta_3$$

$$DoC = l_3 \sin \theta_3$$

Orthographic Projection of a line on N-S Meridian
Positive $\rightarrow$ Northing, Negative $\rightarrow$ Southing

Departure :-

Orthographic projection of a line on E-W Meridian
(+)ve $\rightarrow$ Easting, (-)ve $\rightarrow$ Westing



$$LoA = l_1 \cos \theta_1$$

$$DoA = l_1 \sin \theta_1$$

$$LoB = l_2 \cos \theta_2$$

$$DoB = l_2 \sin \theta_2$$

$$LoC = l_3 \cos \theta_3$$

$$DoC = l_3 \sin \theta_3$$

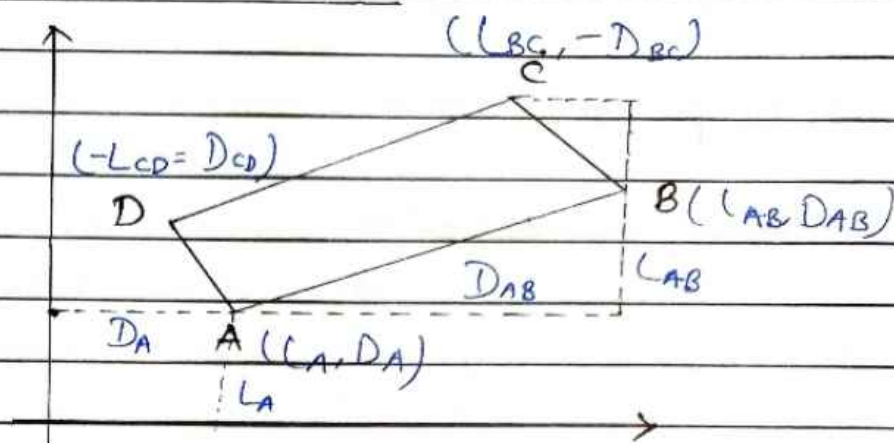
$$LoD = l_4 \cos \theta_4$$

$$DoD = l_4 \sin \theta_4$$

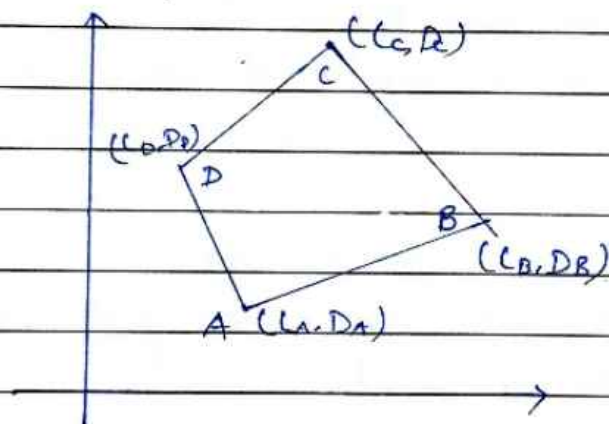
Note In case of Web Algebraic sign of Latitude and departure is given by Sine & Cos itself.

Consecutive / Dependent Coordinates $\Rightarrow$

In consecutive coordinate system coordinate of Next point is defined w.r.t. previous point.



Independent Coordinate:-



$$L_C = L_A + L_{AB} + L_{BC}$$

$$D_C = D_A + D_{AB} + D_{BC}$$

If coordinates of any point of traverse is defined w.r.t. common origin then it is called as Independent coordinate system or Total latitude or Total departure.

$\Rightarrow$ Total latitude or departure of any point = Latitude or departure of initial point + Algebraic sum of Latitude and departure of all the line upto that point.

Note :- In Independent coordinate error is localised whereas in consecutive coordinate system error gets maximised

Checks in Closed Traverse :-

① Loop Traverse :- $\sum L = 0$
 $\sum D = 0$

② Link Traverse :-

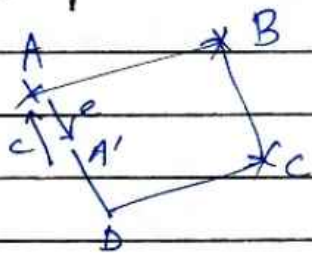
$\sum L$ = Latitude of last Point - Latitude of First point

$\sum D$ = Departure of last point - Departure of First point

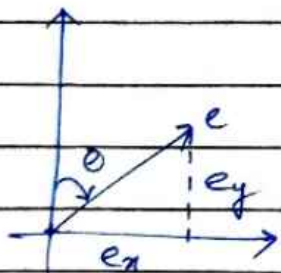
⇒ Closing Error :-

If a traverse has error of closer. It will not close while plotted on paper or sheet.

Case-1 (Loop Traverse) :-



Closing error = AA'
that is coordinate of A' w.r.t. Starting point.



$$e_x = \sum D$$

$$e_y = \sum L$$

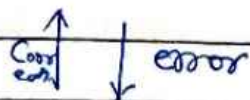
$$e = \sqrt{(e_x)^2 + (e_y)^2}$$

$$\tan \theta = \frac{e_x}{e_y}$$

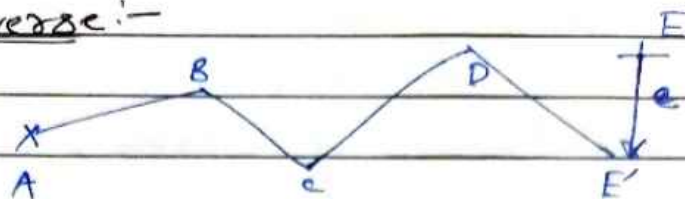
$$\theta = \tan^{-1} \left(\frac{|e_x|}{|e_y|} \right)$$

The quadrant of closing error will be decided with the help of algebraic sign of e_x and e_y .

Magnitude of correction is same as magnitude of closing error but direction is opposite.



Link Traverse:-



Quadrant of closing error will be decided with the help of algebraic sign of e_x and e_y .

$$e = \sqrt{(e_x)^2 + (e_y)^2} \quad \theta = \tan^{-1} \frac{|e_x|}{|e_y|}$$

Q- Coordinate of E is (25, 5) & E' is (20, 1) Determine Magnitude & direction of Closing Error.

Solⁿ:- $E = (25, 5)$

$E' = (20, 1)$

$$e_x = 1 - 5 = -4$$

$$e_y = 20 - 25 = -5$$

$$e = \sqrt{(e_x)^2 + (e_y)^2}$$

$$= \sqrt{(-4)^2 + (-5)^2} = \sqrt{41} = 6.403$$

$$\theta = \tan^{-1} \left(\frac{|e_x|}{|e_y|} \right) = \tan^{-1} \left(\frac{-4}{-5} \right) = 38.65^\circ$$

Degree of Accuracy :-

Also known as Error of Closure / Relative Accuracy.

$$\text{Relative Error of Closure} = \frac{e}{P} = \frac{1}{P/e}$$

e = Magnitude of Closing error

P = Perimeter of Traverse.

Adjustment Of Traverse :-

If there is error of closure then it's has to be adjusted such that error of closure becomes 'zero' and Traverse is closed Geometrically.

① Arbitrary Method :-

In this method closing error is distributed Arbitrary according to experience of surveyor based on field condition.

② Bowditch Rule / Compass Rule :-

It is generally used for adjusting a Traverse in which angles and distances are measured with same precision. Bowditch rule also assumes that error is random in nature.

Therefore probable error in a traverse line is directly proportional to square root of length of line.

$$e_p = \text{Probable Error} \propto \sqrt{L}$$

$$\frac{\text{Error in Latitude or Departure of a line}}{\text{Total Error in Lat or Dep.}} = \frac{\text{Length of line}}{\text{Perimeter of Traverse}}$$

$$e_{L-AB} = e_y \times \frac{AB}{P}$$

When Traverse is adjusted by this Rule both length & Bearings of lines are changed.

→ Compare to Transit Rule lengths are changed less and angles are changed more.

③ Graphical Method / Proportionate Method:-

This method is based on Bowditch Rule, Hence It is used for compass Traverse.

④ Transit Rule :-

This is used to balance a Traverse in which angular measurements are more accurate than linear measurements.

Ex:- Theodolite Traverse.

$$\frac{\text{Error in Lat./Dep. of a line}}{\text{Total error in Lat. or Dep.}} = \frac{\text{Numerical Value of Lat. or Dep. of line}}{\text{Arithmetic Sum of Lat. or Departure}}$$

In Transit Rule angles are changed less but length are changed more.

- * In Transit Rule if a line is having zero latitude or zero departure then there will not be any correction to latitude or departure respectively.

Axis - Method :-

In Axis-Method corrections are only applied to length.

- ⇒ Angles are measured precisely and Distances are not measured precisely.
- ⇒ Since direction of length lines are not affected general shape of traverse is not affected.

Gales Traverse Table :-

Traverse Calculations are generally done in tabular form which is known as Gales Traverse Table.

In this case we ultimately determine independent coordinates of various points after applying necessary correction.

It is better to select Independent coordinate such that whole traverse lies in first quadrant and independent coordinate of these points become positive.

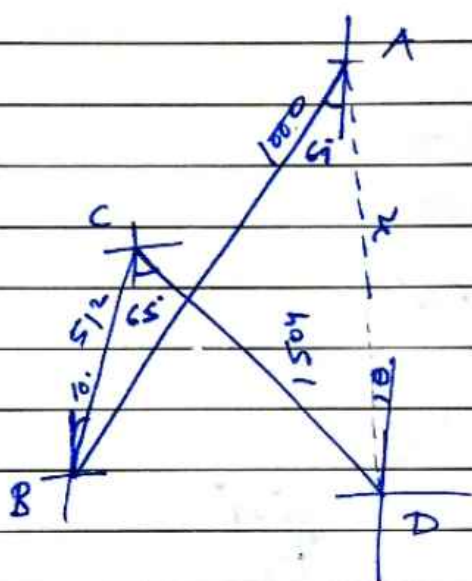
Inversing :-

It is calculation of length and bearings of line from end station coordinates.

Q A Surveyor started from point A and travelled 1000 steps in $S67^\circ W$ direction & reached a point B then he changed his dirⁿ and travelled 512 steps in $N10^\circ E$ & reached a point C. then again he changed his direction and travelled 1504 steps in $S65^\circ E$ and reached D.

Determine how many steps in which dirⁿ he shall move to reach A.

Solⁿ:-



Quardental bearing at
DA = $N45.5^\circ W$

$$\sum L = -1000 \cos 67^\circ + 512 \cos 10^\circ - 1504 \cos 65^\circ + x \cos \theta = 0$$

$$\sum D = -1000 \sin 67^\circ + 512 \sin 10^\circ + 1504 \sin 65^\circ - x \sin \theta = 0$$

$$x \cos \theta = 522.127$$

$$x \sin \theta = 531.489$$

$$\Rightarrow x = 745 \text{ Steps}$$

$$\theta = \cancel{45.5^\circ} 45.5^\circ$$

Q Determine Missing Readings

Line	Length	Bearing
AB	200	Roughly East
BC	98	178°
CD	x	270°
DA	86.4	1°

$$\sum L = 200 \cos \theta + 98 \cos 178^\circ + n \cos 270^\circ + 86.4 \cos 1^\circ = 0$$

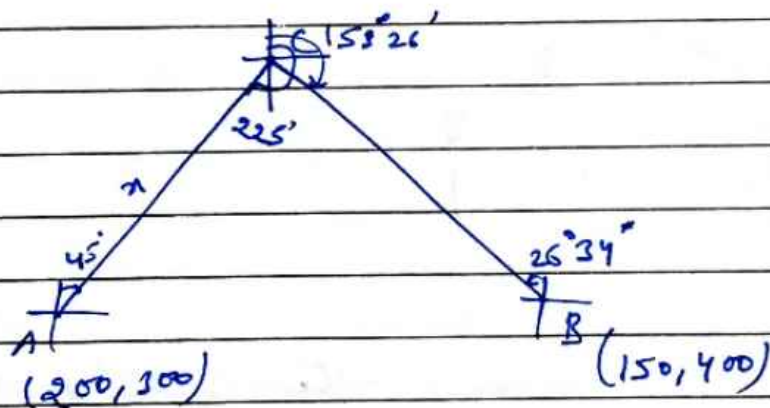
$$\theta = 86.68^\circ$$

$$\sum D = 200 \sin 86.68^\circ + 98 \sin 178^\circ + n \sin 270^\circ + 86.4 \sin 1^\circ = 0$$

$$n = \textcircled{204.59 \text{ m}}$$

Q- Bearing of two Inaccessible stations A & B taken from station C were 225° & $153^\circ 26'$ respectively. Find C.

Station	N	E
A	200	300
B	150	400



$$200 + x \cos 45^\circ = 150 + y \cos 26^\circ 34'$$

$$300 + x \sin 45^\circ = 400 - y \sin 26^\circ 34'$$

$$x = 70.707, y = 111.802$$

$$L_c = 200 + 70.707 \cos 45^\circ = 250$$

$$D_c = 300 + 70.707 \sin 45^\circ = 350$$

* 5 LEVELLING

Levelling is the operation of determining height difference of various point.

In Topographical work heights are measured w.r.t. a common frame of reference which is called as "Datum".

→ Generally Datum is Mean Sea level. Mean level of sea is avg. height of high and low tides over a long period of time. That is 19 years.

Terminology:-

① Elevation (RL):- It is vertical distance of a Point above of below datum.

Elevation is also called as RL, Height of a point etc. Vertical distances are always measured along the dirⁿ of Gravity which is plumb line.

② Altitude:-> Altitude is vertical distance above datum. Above the datum Elevation and altitude are same.

③ Axis OF Telescope:- It is the line joining Optical centre of objective to centre of eyepies

④ Line Of Sight:- It is the line joining intersection of cross-hair to optical centre of Objective.

④ Line Of Collimation :-

In levelling line of Sight has to be horizontal while taking the readings. When line of sight becomes horizontal that is perpendicular to plumb line is called as line of collimation.

⑤ Height OF Instrument :- It is elevation of plane of collimation.

⑥ Station :-

It is a point of unknown elevation.

Back-Sight :-

Back-Sight is the first Staff reading taken after instrument is set-up on the point of known elevation.

Height of Instrument	=	R.L. Of Benchmark	+	Back Sight
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Back-Sight is taken on Bench-mark for first setting of instrument and there after on change point for next setting of instrument.

Fore Sight :-

It is the last staff reading taken from a perpendicular setting of instrument. Fore Sight is taken at the last point of Survey or at change point.

Elevation of Staff Station	=	H.I. - Fore Sight
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Turning Point / Change Point :-

It is the point denoting shifting of level.

There are two readings taken at change point one is fore sight from previous setting of instrument and second is back sight from next setting of instrument.

Intermediate Point / Sight :-

It is the staff reading taken b/w back sight and fore sight from a particular setting of instrument.

Both Intermediate & Fore Sight are taken at a point of unknown elevation.

Fore Sight is taken either at change point or last point of survey, whereas I.S. is taken at any other points.

Parallax :-

It is apparent movement at any of image relative to cross-hair.

Bench-Mark :-

It is a fix reference point of known elevation.

⇒ Depending upon permanency and Precision bench marks are classified as.

① GTS Bench Mark :- (Great Trigonometrical Survey)

These are established by Survey of India throughout the country very accurately with respect to Bombay height.

2) Permanent B.M. :-

These are established at closer interval btw widely spaced GTS B.M. State PWD and Survey of India.

③ Temporary Bench Mark :-

These are the temporarily established whenever required. Generally these are points at which work is closed for a day and from which work would be started for Next day.

④ Arbitrary Bench Mark :-

These are the B.M. whose elevation doesn't refer to any fixed datum, such as MSL.

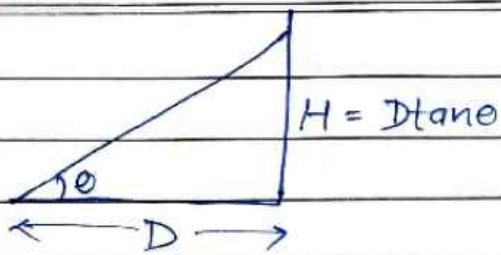
Elevation of such B.M. are arbitrary assumed for levelling of a small Area.

Different Methods of Levelling :-

① Direct Levelling :- (SPIRIT Levelling)

- ⇒ It is most common method of levelling.
- ⇒ A Spirit level fixed to the telescope of a levelling instrument is used to make line of sight horizontal.
- ⇒ Vertical distances are measured with respect to horizontal line of sight.

② Trigonometric Levelling :- It is an indirect method of levelling in which difference of elevation is determined indirectly from horizontal distance & vertical angle.



③ BAROMETRIC Levelling $\Rightarrow$

It is another type of indirect levelling in which elevations are determined indirectly from change in atmospheric pressure.

- Atmospheric Pressure decreases with the increase in elevation.
- Altimeter $\Rightarrow$ (Determines Altitude of Aeroplane)
Aneroid barometer and Mercury barometer are used for determining changes in atmospheric pressure.

④ Hypsometric Levelling $\Rightarrow$ In this process of levelling elevation is determined by observing the temp. at which boiling of water starts.
As Altitude increases boiling point of water decreases.

CLASSIFICATION OF Direct Levelling $\Rightarrow$

① SIMPLE Levelling $\Rightarrow$ In this method only one setting of instrument is done to determine height difference b/w two points.

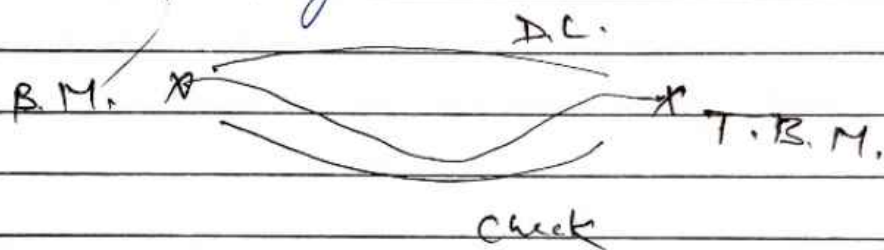
② Differential / Compound Levelling $\Rightarrow$ In this method more than one setting of instrument is required to determine height difference. It is used when:-

- Two points are at larger distance.
- Difference of elevation btw two points is larger.
- Observations btw two points.

③ Check Levelling:-

It is a type of differential levelling done for the purpose of checking elevation of different points which are already obtained.

Generally check levelling is done at the end of days work from last station to the starting station of that day.



④ FLY LEVELLING:-

It is a type of differential levelling used for determination of approx elevation of different points.

Only back Sight and Fore sights are taken and line of sight kept as long as possible.

It is used when accuracy desired is low and time available is less.

⑤ Profile Levelling / Longitudinal Levelling:- It is a type of differential levelling done for the purpose of determine elevation of ground surface along a fix line.

- ⇒ Fin line is generally centre line of the project.
- ⇒ It is used for finding gradient along longitudinal section.

Cross-Sectional Levelling:-

It is a type of differential levelling done to determine difference of elevation of ground surface along the lines perpendicular to centre line.

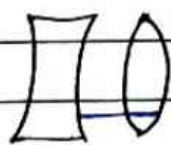
Generally profile levelling and cross levelling are done simultaneously for estimation of earthwork.

⇒ "Optical Defect In Lense of Telescope":-

① Chromatic Abberation:- It occurs in telescope due to dispersion of white light.

⇒ White light splits into component of colours which results in formation of rainbow near image due to which focusing become difficult.

This defect is removed by using combination of lences as discussed below.



(Canada Balsam)

→ These two lences are joined together by a chemical as Canada Balsam.

② Spherical Abberation:- It occurs due to defect in spherical surface of lence.

→ It is also removed by same combination of lences as discussed above.

Note:-

Achromatism :- Absence of chromatic Abberation.

Aplanation :- Absence of Spherical Abberation.

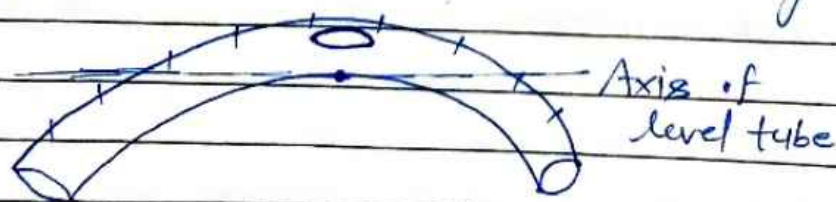
Level Tube :- Level tube or bubble tube is an essential part of most of the surveying instruments as it is used for making line of sight horizontal and vertical axis co-inciding with plumb line.

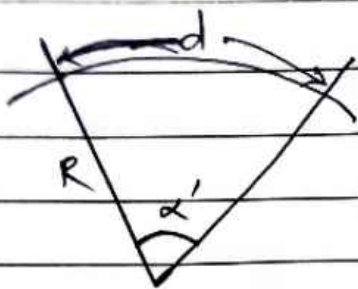
Level tube is a glass tube sealed at both ends, which is partially filled with a sensitive liquid leaving enough space for the formation of air bubble.

- ⇒ Liquid should be (a) Stable & Non-freezing at Ordinary Temperature.
(b) Low Viscosity.

Previously Alcohol, Ether & chloroform were used but these days purified synthetic Alcohol is used.

- C/s of Level-tube is circular in nature.
- Upper surface of level tube of graduation which are symmetrical about centre and equally-spaced.
- Generally Graduations are 2mm long.





$$\alpha' = \frac{d}{R} \times 206265$$

Conversion from Radian to Seconds

- The longitudinal line segment Tangent to mid-point of inner curved surface is called as Axis of level Tube
- During the construction of equipment line of sight is made parallel to axis of level tube.
- When air bubble comes in centre Axis of level tube becomes horizontal & thereby line of sight becomes Line of Collimation.

Sensitivity of Level Tube $\Rightarrow$

Sensitivity of level tube is expressed in terms of angle in seconds that subtended at the centre by an arc of level tube = 1 division.

- It is expressed in Second per mm.
- Radius of curvature of upper surface of tube is also a Measure of Sensitivity.
- If the radius of level tube is larger is more sensitive.

Q:- Bubble Tube A

$$\alpha'_A = \frac{20'}{2 \text{ mm}}$$

$$\alpha'_A = \frac{d}{R} \times 206265$$

$$R = \frac{2 \times 10^{-3}}{20} \times 206265$$

$$R_A = 20.63 \text{ m}$$

Bubble Tube B

$$\alpha'_B = \frac{10'}{2} \text{ mm}$$

$$R_B = \frac{2 \times 10^{-3}}{10} \times 206265$$

$$R_B = 41.25 \text{ m}$$

"B is More Sensitive"

Note :- Bubble tube 'B' is more sensitive because for 10 seconds of deflection air bubble moves for 2 mm.

Whereas in A 20 sec. deflection is required for movement of 2 mm.

Sensitivity of level Tube can be increased by changing following Parameter.

- ① Increasing Radius of curvature of tube
- ② By decreasing Roughness of inner walls of Tube.
- ③ By reducing viscosity of liquid.
- ④ By Increasing diameter of tube
- ⑤ By Increasing length of air bubble.
- ⑥ By Reducing Temperature.

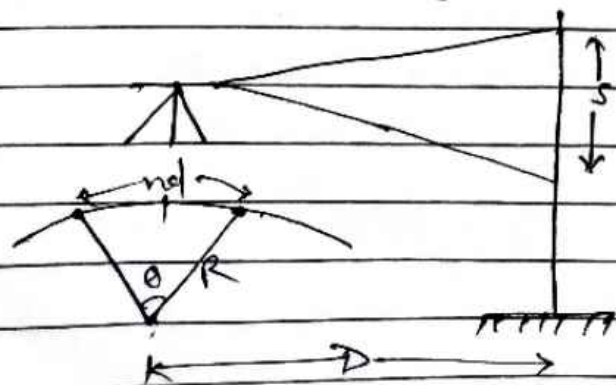
Note:-

Sensitivity Expressed as sec/division is not a definite quantity unless length of one division is also specified.

→ If Not specified then length of one division can be taken as 2 mm.

→ Length of air bubble changes with temp. and also under the action of gravity.

Determination of Sensitivity of Bubble Tube



$$\theta = \frac{nd}{R} = \frac{s}{D}$$

$$\alpha' = \frac{d}{R} = \frac{s}{nd} \times 206265$$

S = Staff intercept

n = No. of division moved by Air Bubbles.

D = Horizontal distance btw vertical axis of instrument & staff station.

d = Length of one division of level tube.

Q If bubble have sensitivity of $20''/2\text{mm}$ determine error in staff reading on a vertically held staff at a distance of 200m . Caused by 3 divisions out of centre.

Solⁿ:-
$$\alpha' = \frac{S}{nD} \times 206265$$

$$20'' = \frac{S}{3 \times 200} \times 206265$$

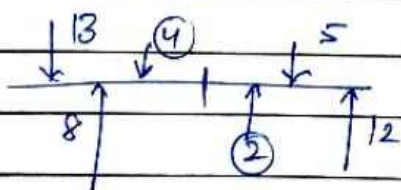
$$S = \frac{20 \times 3 \times 200}{206265}$$

$$S = 0.58\text{m}$$

Q The following Observations were made to determine sensitivity. The distance btw vertical instrument & Staff station is 100m . length of one division is 2mm .

Bubble Tube	Case	Bubble Reading LHS	Bubble Reading RHS	Staff- Reading
A	①	13	5	1.681
	②	8	12	1.767
B	①	15	3	1.635
	②	6	14	1.788

Solⁿ:- Bubble Tube A



$$n = 4 + 2 = 6$$

$$\alpha'_A = \frac{S}{nD} = \frac{1.767 - 1.681}{6 \times 100} \times 206265$$

$$= 29.56''$$

Bubble Tube B

$$\alpha'_B = \frac{1.788 - 1.635}{10 \times 100} \times 206265$$

$$\alpha'_B = 31.55''$$

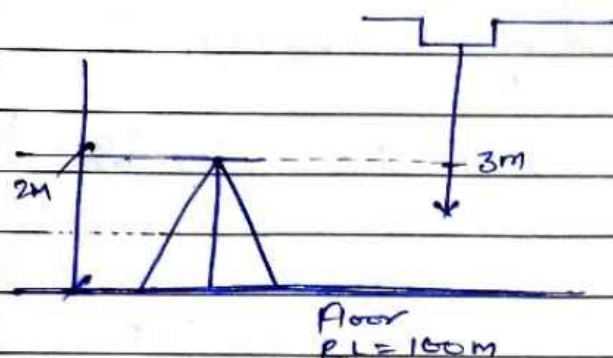
$\therefore$ A is more sensitive

Theory Of Simple Levelling:-

level is set-up at a suitable place btw two stations.

- Instrument need not be in line with end stations.
 - R.L. of Ground point of Instrument station is not determined.
 - Staff should be vertical & line of sight should be horizontal.
 - To ensure verticality of staff, staff is wavered slightly towards instrument & then away from instrument. This process is called as "Rocking" of staff.
- During this process instrument man should observe smallest reading.
- Very intense sunshine causes Air close to earth to shimmer. Hence, avoid staff reading less than 0.5m.

USE Of Inverted Staff



R.L. of Soffit of beam

$$(100 + 2) - (-3)$$

$$\Rightarrow \underline{105m}$$

When the point whose elevation has to be determined is much above the line of sight. Example $\Rightarrow$ Soffit of beam or Soffit of Slab, then to determine level of that point, staff is kept inverted with zero end touching that point.

Such readings are recorded with a (-)ve sign in level field book.

Level Field Books :- A level field book is used to record reduce level of various points. There are two methods of reducing the levels.

$$R.L. = H.I. - \text{Readings.}$$

$$H.I. = R.L. + B.S.$$

- ✓ (i) Collimation Method / Height of Instrument Method.
- ✓ (ii) Rise and Fall Method.

Check ① $\Sigma B.S. - \Sigma F.S. = \text{Last R.L.} - \text{First R.L.}$

② $\Sigma B.S. - \Sigma F.S. = \Sigma \text{Rise} - \Sigma \text{Fall} = \text{Last R.L.} - \text{First R.L.}$

Q. Instrument having been moved after third, 6th & 8th readings 2.328, 1.666, 0.988, 2.090, 2.864, 1.262, 0.602, 1.982, 1.044, 2.684 meters.

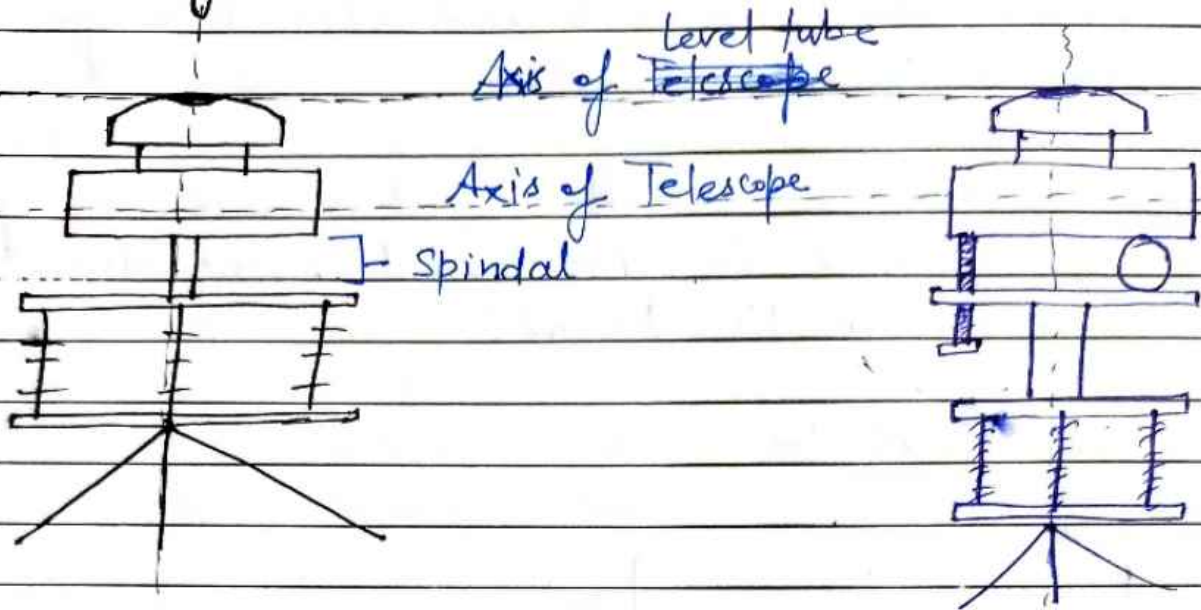
Enter the above readings in a page & calculate RL of points if first reading was taken with a staff held on bench mark of 432.384 m.

Solⁿ:-

Station	B.S.	IS	FS	Rise	Fall	RL	Remarks
1	2.228					432.384	BM
2		1.606		0.622		433.006	
3	2.090		0.988	0.618		433.624	TP1
4		2.864			0.774	432.850	
5	0.602		1.262	1.602		434.452	TP2
6	1.044		1.982		1.380	433.072	TP3
7			2.684		1.640	431.432	

Checks 5.964 Fall 0.952 Fall 0.952 0.952 Checked

Dumpy Level :-



- In dumpy level Telescope and spindal are cast as one piece.
- levelling is done using levelling screw.
- Under proper adjustment axis of level tube should be perpendicular to vertical axis of Instrument.

Note :- In Tilting level this adjustment is not required.

Permanent Adjustment of Dumpy Level

- ① Axis of level tube should be parallel to line of Sight.
- ② Axis of level tube should be perpendicular to vertical axis of Instrument.

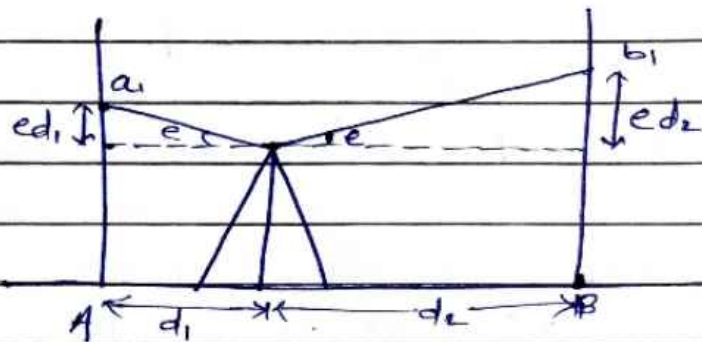
This adjustment is not required in tilting level.

- ③ Horizontal cross-hair should be in plane perpendicular to vertical axis of Instrument.

When air Bubble is in the centre Axis of level tube should be Horizontal and also line of Sight should be Horizontal.

If Air bubble is in the centre and line of Sight is inclined to horizontal plane then it is called as "Collimation Error".

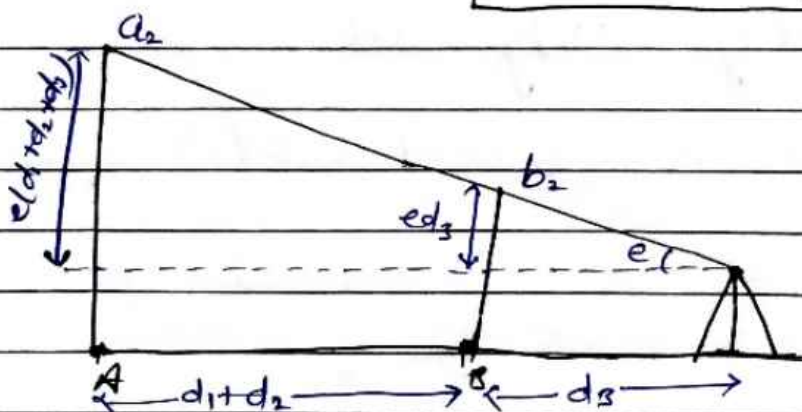
Two Peg Test :-



$$\Delta h_{AB} = (a_1 - ed_1) - (b_1 - ed_2)$$

if $d_1 = d_2$

$$\Delta h_{AB} = a_1 - b_1$$



$$\Delta h_{AB} = (a_2 - e(d_1 + d_2 + d_s)) - (b_2 - ed_3)$$

$$\Delta h_{AB} = (a_2 - b_2) - e(d_1 + d_2)$$

$$(a_1 - b_1) = (a_2 - b_2) - e(d_1 + d_2)$$

$$e = \frac{(a_2 - b_2) - (a_1 - b_1)}{d_1 + d_2}$$

Q- In Order to check adjustment of a dumpy level the following procedure was followed A, B, C & D were selected in a st. line such that $AB = BC = CD = 30\text{m}$.

Determine whether the instrument has collimation error or not if yes then how much.

Solⁿ:-

Inst.	Staff	Reading
St ⁿ	A	C
B	1.926	1.462
D	2.445	1.945

Instrument Station B

$$\Delta h_{AC} = 1.926 - 1.462 = 0.464$$

Instrument Station D

$$\Delta h_{AC} = 2.445 - 1.945 = 0.5$$

Since Δh_{AC} is not equal in both the cases, hence there is collimation error.

$$e = \frac{0.5 - 0.464}{60} = 0.0006 \text{ m/m}$$

EFFECT of Curvature :-

It occurs because difference btw level line and horizontal line increases as the distance of staff station from instrument station increases.

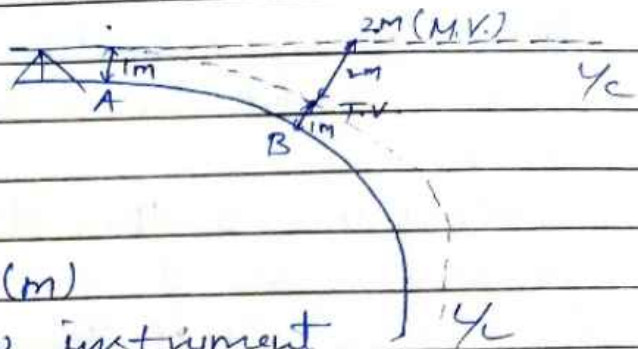
Level line is a curved line whereas line of collimation is a horizontal line which is tangential to level line.

Therefore points on earth's surface appears to be much lower than actually they are.

$$M.V. > T.V.$$

$$F = +Ve$$

$$C_c = (-) Ve$$



$$C_c = -0.0785d^2$$

C_c = Curvature Correction in (m)

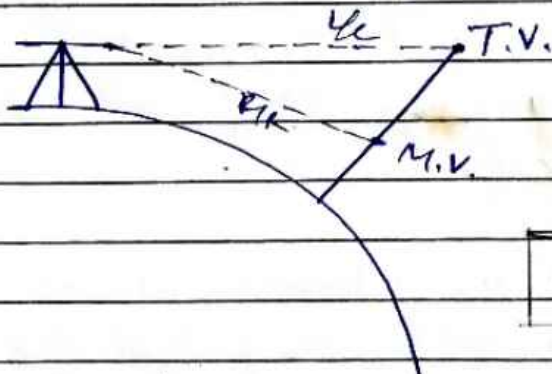
d = Horizontal distance btw instrument and Staff Station. in km.

Refraction Correction :-

Density of air in Atmosphere decreases with increase in altitude. Air is denser near the earth.

A ray of light travels from thinner Medium to denser medium. So it bends towards the Normal.

Hence line of Sight doesn't remain Horizontal but it bends downwards towards the centre of the earth.



$$M.V. < T.V.$$

$$F = (-) Ve$$

$$C_r = +Ve$$

$$C_r = \frac{1}{7} |C_c| = +0.0112d^2$$

Combined Correction

$$C = C_c + C_r$$

$$= -0.0785d^2 + 0.0112d^2$$

$$C = -0.0673d^2$$

d - km

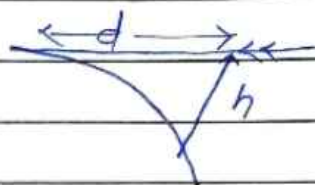
C - m

In ordinary survey effect of curvature & refraction are neglected for a distance less than 250 m.

Distance of Visible Horizon :-

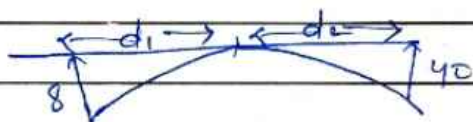
$$h = 0.0673d^2$$

$$d = 3.855\sqrt{h}$$



Q. An observer standing on deck of a ship just sees the light house which is 40m above the sea level. If height of observer's eye is 8m above the sea level then determine the distance btw Observer & light house.

Solⁿ:-



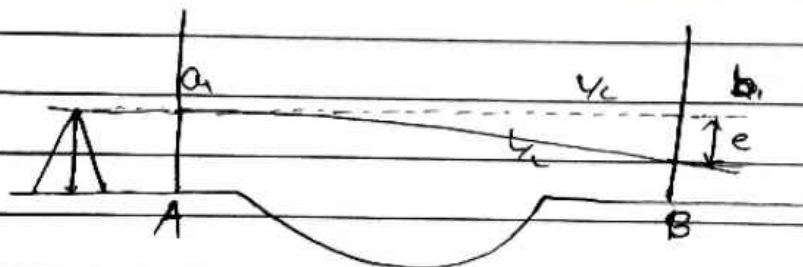
$$d_1 + d_2 = 3.855(\sqrt{8} + \sqrt{40})$$
$$= 35.28 \text{ km}$$

Reciprocal Levelling :->

It is used to determine correct difference in elevation of two points which are quite a larger distance apart and it is not possible to set up the instrument midway btw these points.

Ex:- River Opposite banks
Opposite Sides of valley.

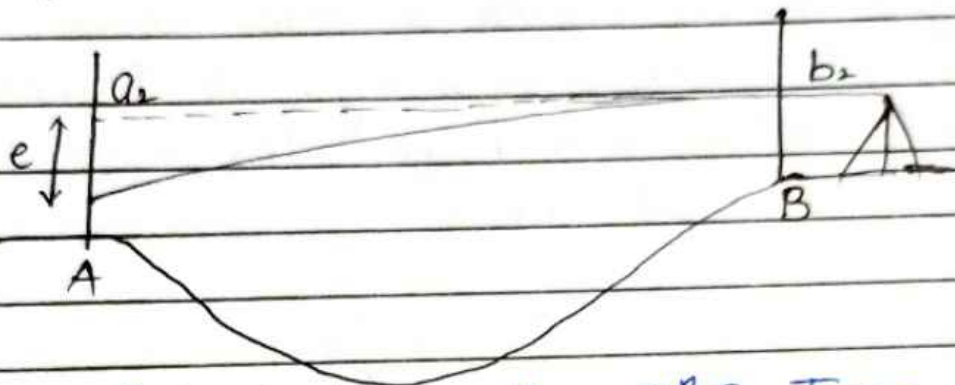
- Reciprocal levelling eliminates error due to
i) Curvature ii) Refraction iii) Collimation



'e' is total error which is due to collimation refraction and curvature.

$$e = e_c + e_r + e_k$$

e_c = Collimation error assumed as inclined upward and positive.



Assuming $St^n A$ is lower than $St^n B$ ~~True~~

$$\text{True } \Delta h_{AB} = a_1 - (b_1 - e) \quad \text{--- (1)}$$

$$\Delta h_{AB} = (a_2 - e) - b_2 \quad \text{--- (2)}$$

$$\text{(1) + (2)} \quad 2\Delta h_{AB} = (a_1 - b_1) + (a_2 - b_2)$$

$$\Delta h_{AB} = \frac{(a_1 - b_1) + (a_2 - b_2)}{2}$$

Calculation for Collimation Error

$$e = e_c + e_r + e_k$$

$$e_c = e - e_r - e_k$$

$$\Delta h_{AB} = a_1 - (b_1 - e)$$