

→ BASIC CONCEPTS :-

THERMODYNAMICS :

It is the science of energy transfer and its effects on properties of system.

Basic aim of thermodynamic study is to convert heat (disorganised form of energy) into work (organised form of energy) in an efficient manner.

SYSTEM :

It is a region in space upon which the study is focused or concentrated.

SURROUNDINGS :

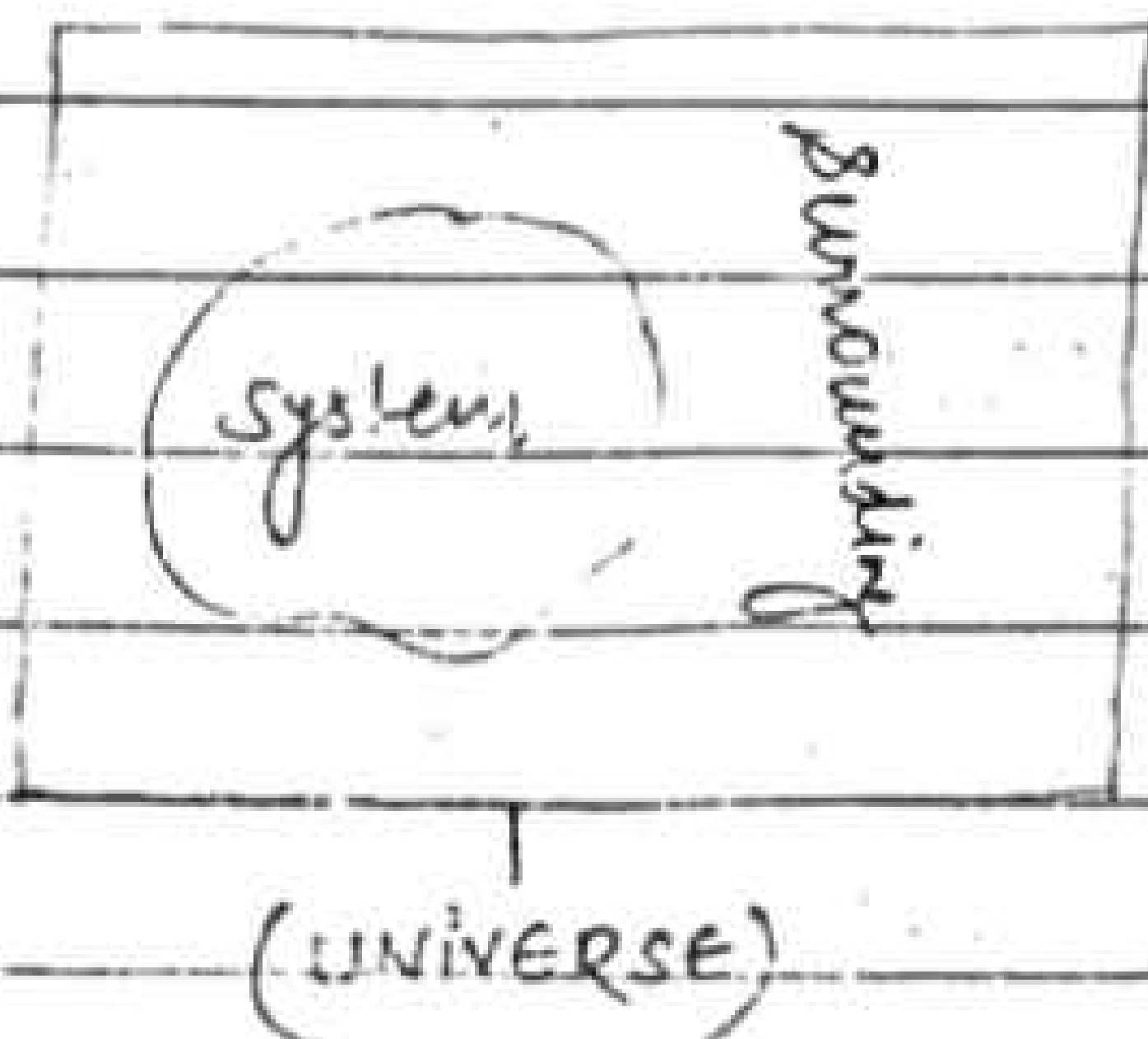
Anything external to the system is known as surroundings.

BOUNDARY :

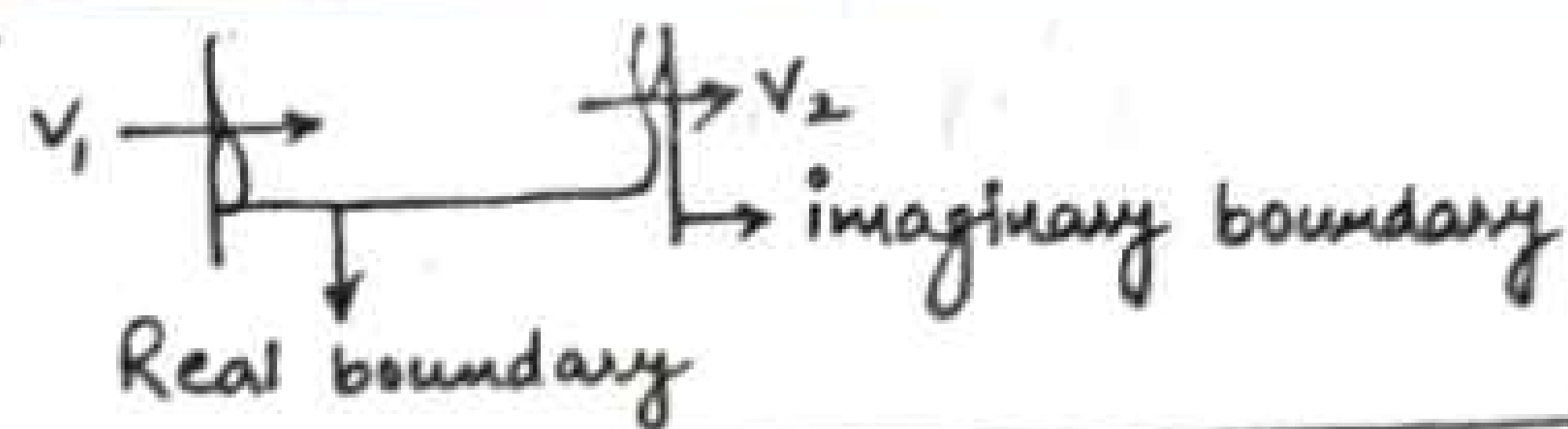
The separation between system and surroundings is known as boundary.

The boundary can be rigid or flexible, real or imaginary.

UNIVERSE = system + surroundings



> Universe is an isolated system.



→ TYPES OF SYSTEM :

SYSTEM	MASS TRANSFER	ENERGY TRANSFER	EXAMPLE
Closed	X	✓	Piston cylinder without valves
Open	✓	✓	Turbine, pump, compressor
Isolated	X	X	Universe, Hot coffee in a well insulated flask

→ MICROSCOPIC & MACROSCOPIC APPROACH OF THERMODYNAMICS :-

In microscopic approach the behaviour of individual molecule is taken into consideration and this approach is also known as statistical thermodynamics.

This approach is useful at low density (higher altitudes) because low density means low mass.

In macroscopic approach the behaviour of individual molecule is not taken into consideration but the average behaviour of molecule is taken into consideration.

This approach is also known as classical thermodynamics.

→ PURE SUBSTANCE :

A substance is said to be a pure substance if it is i) homogeneous in chemical composition
ii) homogeneous in chemical aggregation (bonding)

	2:1	2:2
(H ₂ O) Steam	H ₂ + O	H ₂ + O ₂
(H ₂ O) Water	Water	Water
	2:1	2:1
1 ✓	1 ✓	1 X
2 ✓	2 X	2 X

PROPERTY OF A SYSTEM :-

Properties are any measurable characteristics of a system. eg: P, T, V etc.

Properties are of two types -

- 1) Intensive or intrinsic
- 2) Extensive or extrinsic

Intensive properties are independent of size or mass of system. eg: P, T, ρ, μ, k etc.

Extensive properties are dependent on size or mass of system eg:- V , All forms of energy etc.

(*) Ratio of two extensive properties is an intensive property.

All specific properties are intensive properties.

eg: specific volume (v), specific internal energy (u), specific enthalpy (h), specific entropy (s)

→ STATE OF A SYSTEM :

Conditions of a system is called as state of a system. The state of a system is specified by means of its properties. As long as state is fixed, the properties are also fixed and hence properties are state functions.

PROCESS : Change of state is known as a process.

→ Important Points wrt. property :-

- 1) Properties are always point functions.
- 2) Properties are independent of past history.
- 3) Properties are exact differentials.

→ THERMODYNAMIC EQUILIBRIUM :

A system is said to be in thermodynamic equilibrium if it is in

- i) thermal equilibrium (equality of temperature)
- ii) mechanical equilibrium (equality of forces/pressures)
- iii) chemical equilibrium (equality of chemical potential)

→ REVERSIBLE & IRREVERSIBLE PROCESS :-

when A process is said to be reversible process if reversed in direction follows the same path as that of the forward path without leaving any effect on system and surrounding.

Reversible process is the most efficient process.

A process which is not a reversible process is an irreversible process. Friction is one of the reasons which makes the process irreversible.

→ QUASI-STATIC PROCESS -

A process which is carried out in a very slow manner with small potential difference is known as quasi-static process. Frictionless quasi-static process is a reversible process.

→ THERMODYNAMIC CYCLE -

A system is said to have undergone a cycle if its initial and final points are same. The change in property for a cycle is always zero. Initial and final points are same and for a cycle initial and final points are same.

→ GIBBS PHASE RULE -

		solid H_2O	ice	$P=0$
	O_2		liquid	$P=0$
		liquid	solid	$P=0$
$P=1$	$P=2$	$P=3$		
$C=1$	$C=2$	$C=3$		
$1+F=C+2 \Rightarrow F=2$	$2+F=C+2 \Rightarrow F=1$	$3+F=C+2 \Rightarrow F=0$		

According to Gibbs phase rule $P+F=C+2$
where,

P = no. of phases

F = no. of intensive variable parameters

C = no. of components

→ ZEROth LAW OF THERMODYNAMICS (CONCEPT OF TEMPERATURE):-

When a body A is in thermal equilibrium with body B and body B is in thermal equilibrium with body C, then A and C are in thermal equilibrium.

In zeroth law of thermodynamics, one body is taken as thermometer.



→ Thermometric principle :-

In this principle, the property which changes with temp. is found first and then the temperature is found. The property which helps in finding the temp. is known as thermometric property.

→ VARIOUS THERMOMETERS :-

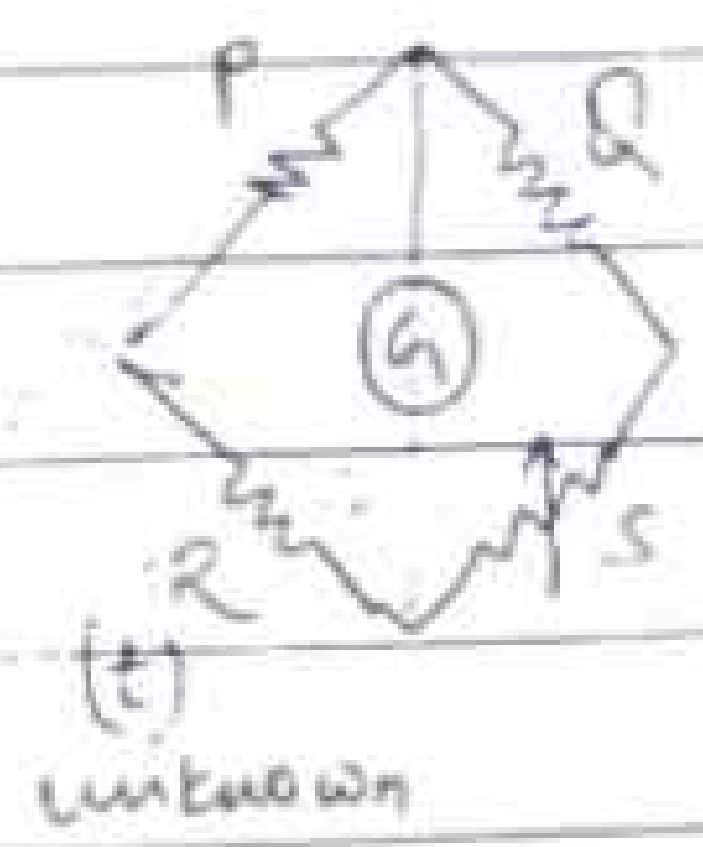
1.) Resistance Thermometer (Thermistor) :

These thermometers are based upon wheatstone Bridge principle and in this thermometer, resistance plays the role of thermometric property.

$$R = R_0(1 + \alpha t + \beta t^2)$$

for balanced W.B.

$$\frac{P}{Q} = \frac{R}{S}$$



2) THERMOCOUPLE :

Thermocouples are based upon Seebeck effect. When two dissimilar metals are joined to form two junctions and if these two junctions are maintained at different temperatures, emf or voltage is generated and this voltage is proportional to temp diff b/w two junctions, and hence in thermocouple emf plays the role of thermometric property.

3) CONSTANT VOLUME GAS THERMOMETER :

In constant gas volume thermometer pressure plays the role of thermometric property.

4) CONSTANT PRESSURE GAS THERMOMETER :

Volume plays the role of thermometric property.

→ TEMPERATURE SCALES :

Temperature scales are arbitrary.

The ice point (freezing point) and steam point (boiling point) temperature are taken as 0 and 100 on Celsius scale.

$$[K = ^\circ C + 273.15]$$

→ METHOD OF TEMPERATURE MEASUREMENT before 1954 :-

This method is based upon two reference temp. that is ice point ($0^\circ C$) and steam point ($100^\circ C$).

$$t = ap + b$$

$$t_s = ap_s + b$$

$$t_i = ap_i + b$$

$$100 = ap_s + b \quad \text{--- (3)}$$

$$0 = ap_i + b \quad \text{--- (4)}$$

> by solving for (a) & (b)

$$\Rightarrow a = \frac{100}{p_s - p_i}$$

$$b = -\frac{100}{p_s - p_i} p_i$$

$$\therefore t = \frac{100}{p_s - p_i} p - \frac{100}{p_s - p_i} p_i$$

$$\Rightarrow \boxed{t = \frac{100}{(p_s - p_i)} (p - p_i)} \quad \text{--- (A)}$$

By finding the property 'p', the corresponding temp. 't' can be found.

→ Method of finding temp. after 1954 :

It is based on a single fixed temp. i.e. tripple point of water. The tripple point of water is assigned a value of 0°C. (273.16 K).

for ideal gas

$$PV = nRT$$

if $V = \text{constant}$

$$P \propto T$$

$$= CT$$

$$\therefore C = \frac{P}{T}$$

At tripple point

$$C = \frac{p_{tr}}{T_{tr}}$$

$$P = \frac{P_{tp} \cdot T}{T_{tp}}$$

$$\Rightarrow T = \frac{P \cdot T_{tp}}{P_{tp}}$$

$$\Rightarrow T = 273.16 \left(\frac{P}{P_{tp}} \right)$$

(**) Ideal gas thermometer is independent of material of construction.

P:1 The reading t_A & t_B of two thermometers A & B agree at 0°C and 100°C and are related by $t_A = l + m t_B + n t_B^2$ b/w these temp. where l, m, n are constants. When both are immersed in oil, A reads 51 and C reads 50 determine the reading on A when C reads 25°C .

$$\text{as } t_A = l + m t_B + n t_B^2 \quad \text{--- (1)}$$

$$\text{at } 0^\circ\text{C} \Rightarrow t_A = 0 = t_B$$

$$\& \text{ at } 100^\circ\text{C} \Rightarrow t_A = 100 = t_B$$

$$\text{using } l = 0$$

$$(t_A = t_B = 100) \Rightarrow 100 = m(100) + n(100)^2$$

$$\Rightarrow L = m + 100n \quad \text{--- (2)}$$

$$\text{Now at } t_B = 50 \& t_A = 51$$

$$\Rightarrow 51 = 50m + 2500n \quad \text{--- (3)}$$

$$\text{Solving (2) \& (3)} \Rightarrow m = 1.04$$

$$n = -4 \times 10^{-4}$$

$$\therefore t_A = 0 + 1.04 t_B + (-4 \times 10^{-4} t_B)$$

$$\Rightarrow t_A = 1.04(25) + \{-4 \times 10^{-4} \times 25\}$$

$$\Rightarrow t_A = 25.75^\circ\text{C}$$

* Though the end points temp. are same, this does not mean that the intermediate temp. are also same.

2

Assertion (A): If an alcohol and mercury thermometer read exactly at 0°C and 100°C then these two thermometers will give exactly the same reading at 50°C

Reason (R): Temp scales are arbitrary

A is wrong and R is right so. Ans d

Q-3

Which of the following are intensive properties -

- i) KE
- ii) Thermal conductivity
- iii) Pressure
- iv) Volume

- a) I & II ☒ b) ii & iii c) ii, iii, iv d) 1, 3, 4

P-4

Match List I with List II

List I	List II	
1) Mercury in Glass	Pressure A)	1-C ✓
2) Thermocouple	Resistance B)	2-D ✓
3) Thermistor	Volume or length C)	3-B ✓
4) Const. v gas thermometer	emf D)	4-A ✓

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What is the?

classmate

Date

Page

Q.5 In a new temp. scale, $^{\circ}P$ the boiling and freezing point of water are $100^{\circ}P$ and $300^{\circ}P$ respectively. find the reading corresponding to $0^{\circ}C$ on Celsius scale.

Let the relation b/w both scales is linear.

$$^{\circ}P = a^{\circ}C + b$$

$$300 = a(100) + b \Rightarrow b = 300$$

$$\& \quad 100 = 100a + 300 \Rightarrow a = -2$$

$$^{\circ}P = -2^{\circ}C + 300$$

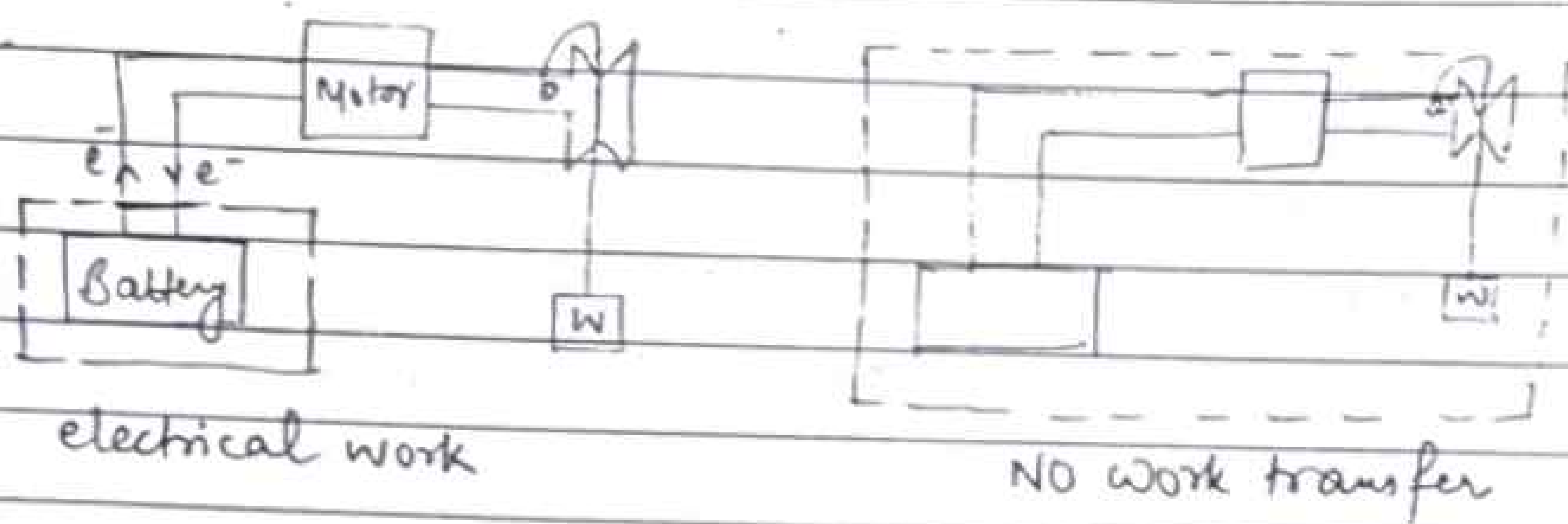
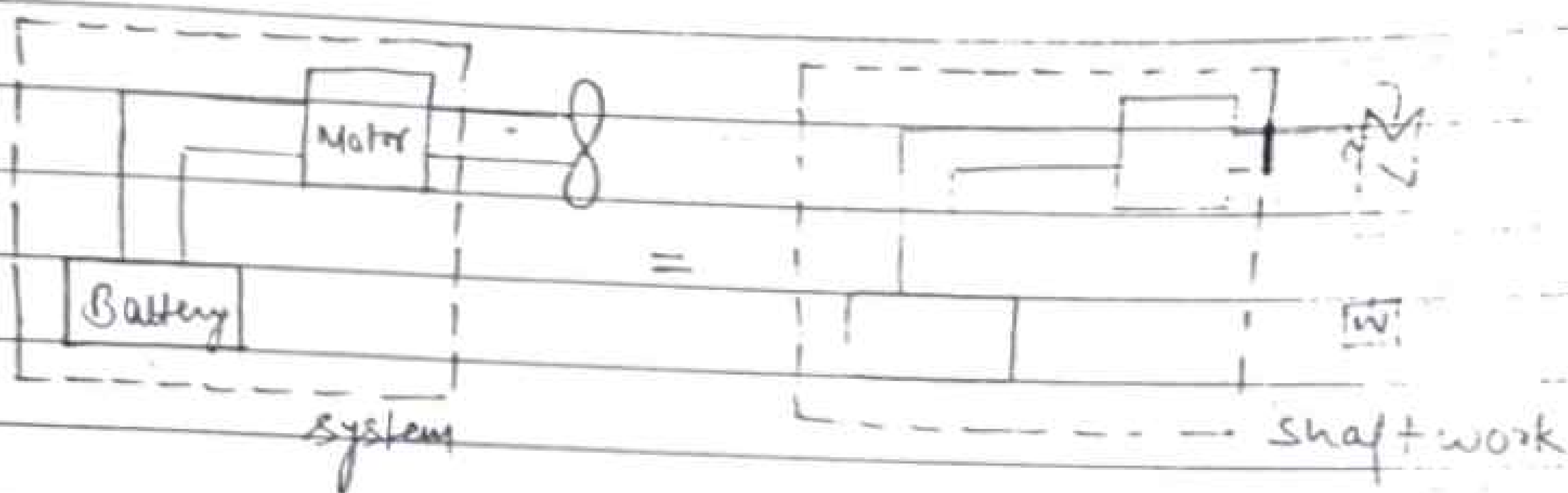
So at $^{\circ}P = 0$

$$\Rightarrow 0 = -2^{\circ}C + 300 \Rightarrow \boxed{^{\circ}C = 150^{\circ}C}$$

ENERGY INTERACTIONS (WORK & HEAT) :-

1) THERMODYNAMIC WORK :

Work is said to be done by the system if the sole effect on things external to the system can be equated to raising of weights (Weights may not be raised but the effect can be equated to raising of weights).



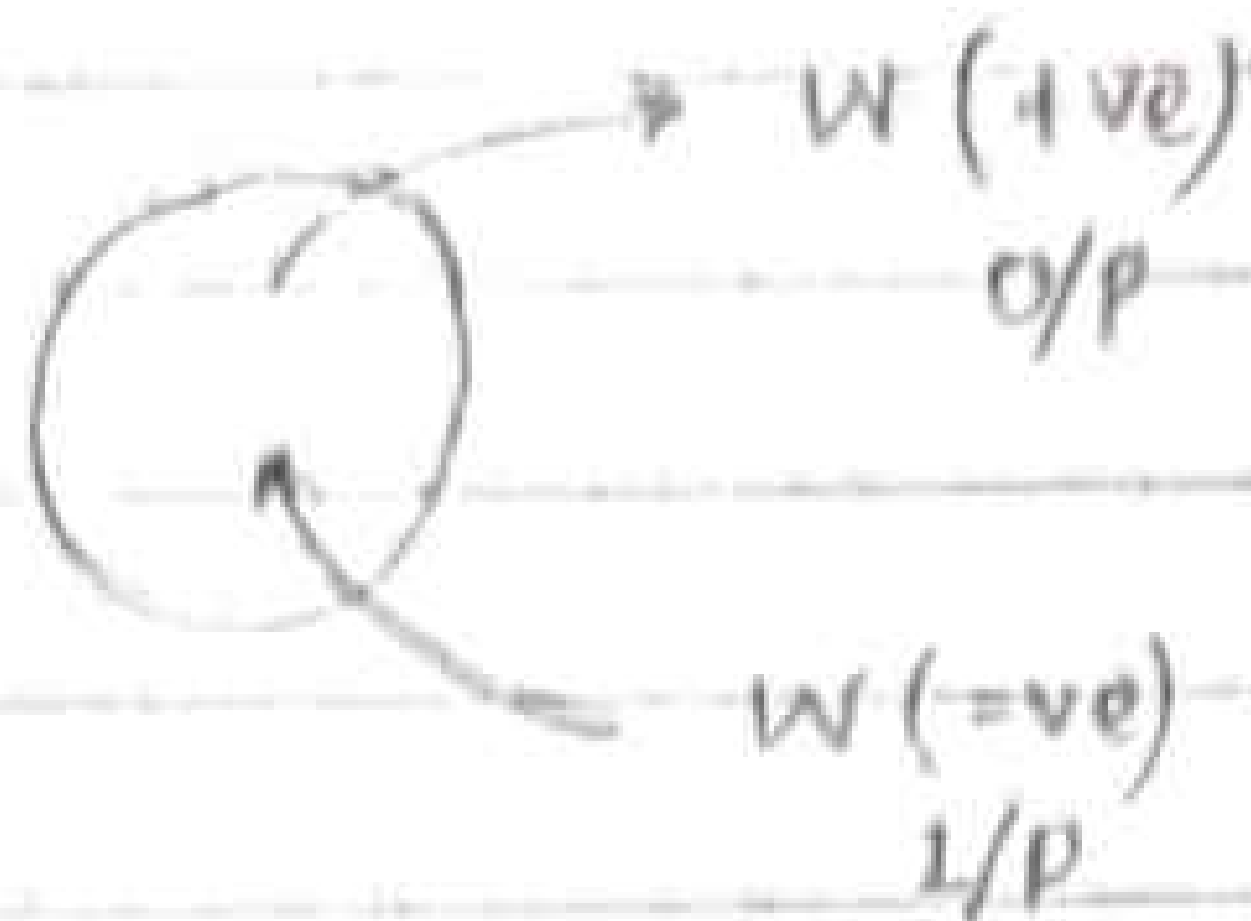
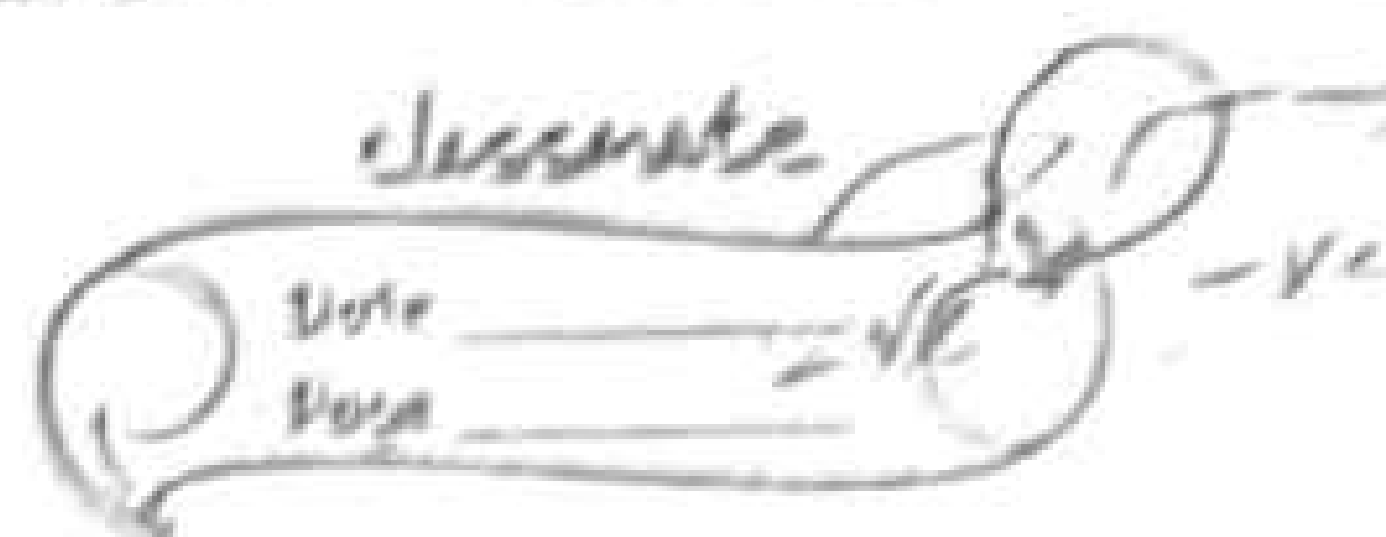
Work transfer occurs only when it crosses the boundary if it does not cross the boundary there is no work transfer and hence work transfer is a boundary phenomenon.

CONVENTION OF WORK TRANSFER :

Work done by the system is taken as +ve and work done on the system is taken as -ve.

Whenever weights are raised $\rightarrow$ work done by system

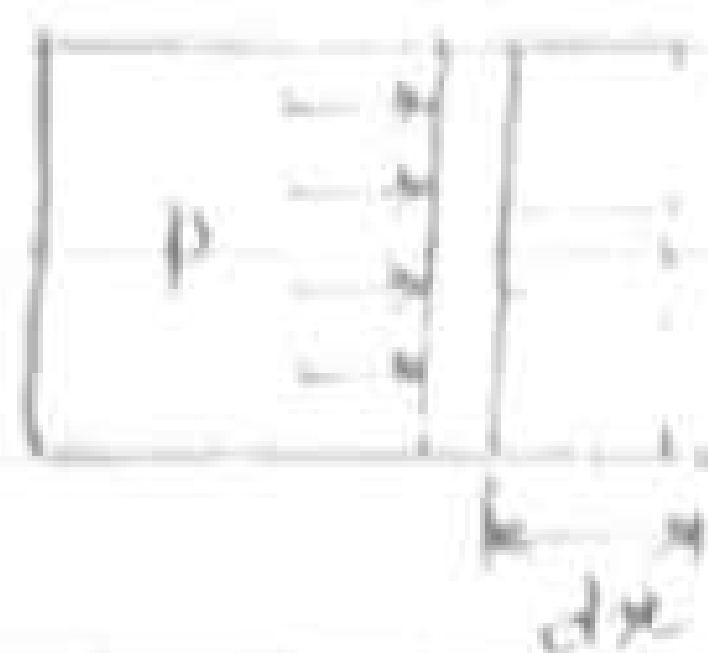
Whenever weights are lowered $\rightarrow$ work done on system



When boundary is expanding, work will be +ve

When boundary is contracting, work will be -ve

GENERALISED EQ^N FOR CLOSED SYSTEM WORK TRANSFER :
(NON FLOW WORK)



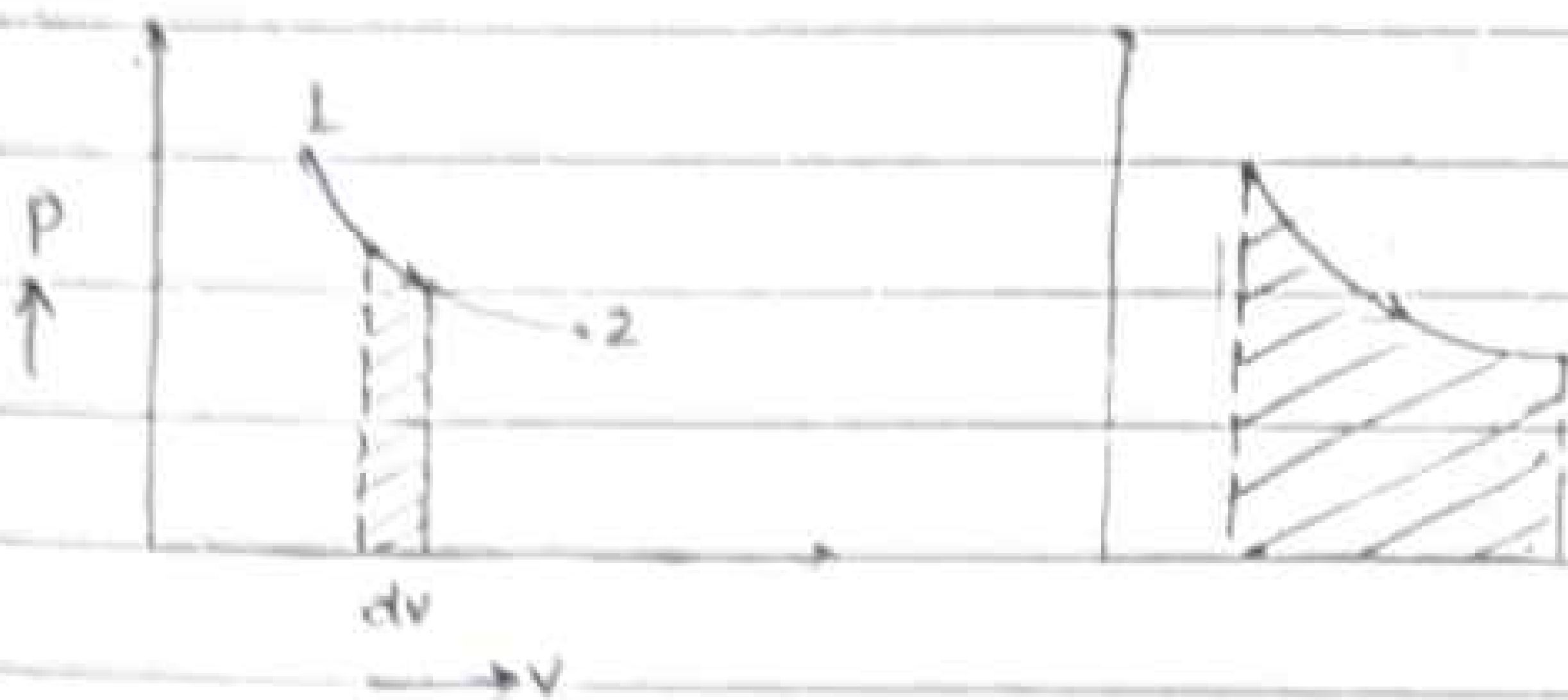
work done = force $\times$ distance

$$dW = F \times dx$$

$$dW = p A dx$$

$$dW = p dv$$

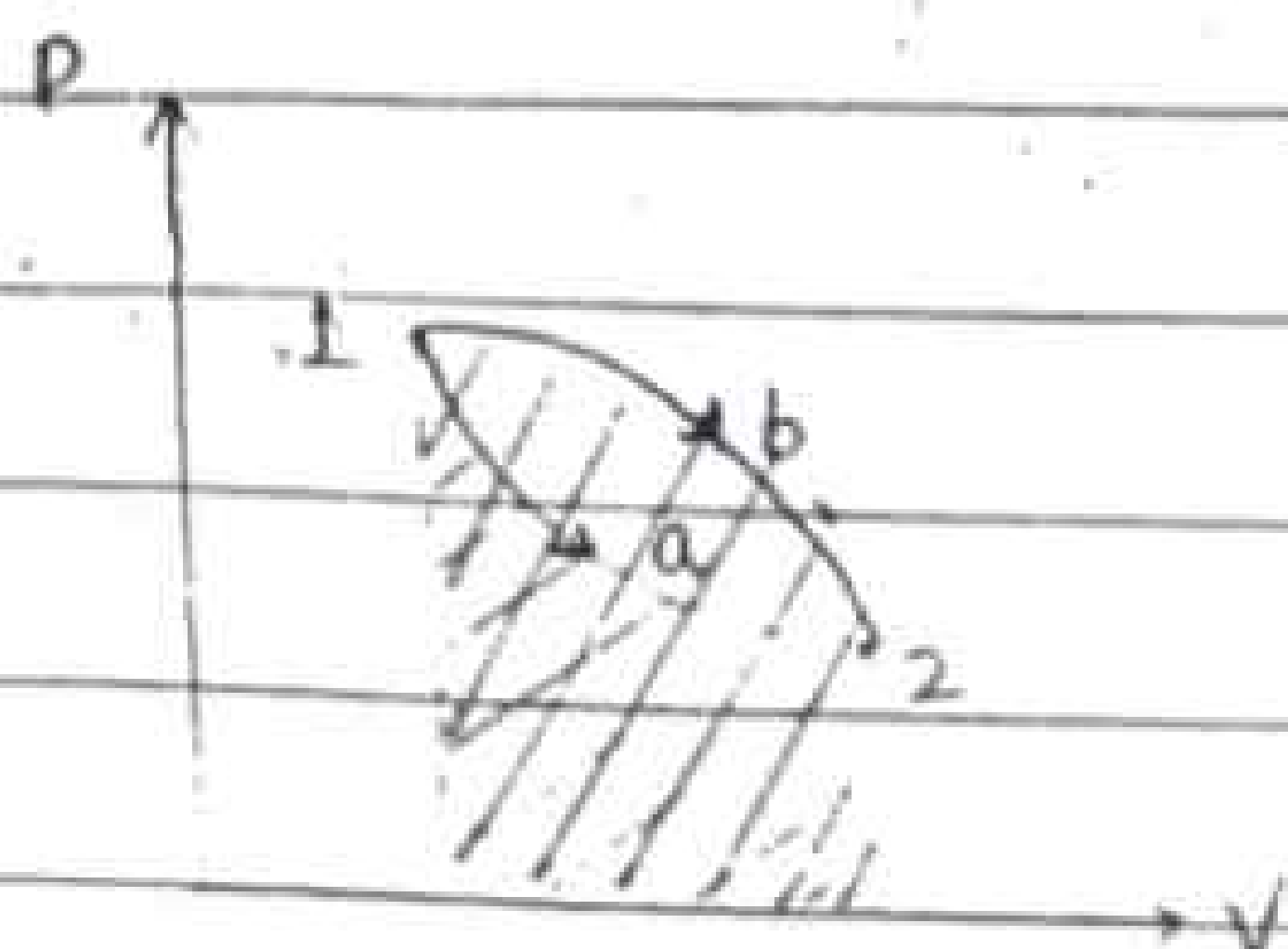
$$\& \quad W = \int p dv \quad \text{Non flow or closed system work}$$



$$\text{Area} = p dv$$

$$\text{Work} = p dv$$

(*) Area under the P-V curve when projected on volume axis gives non flow or closed system work.



$$W_A \neq W_B$$

$$\therefore \delta W \text{ or } dW$$

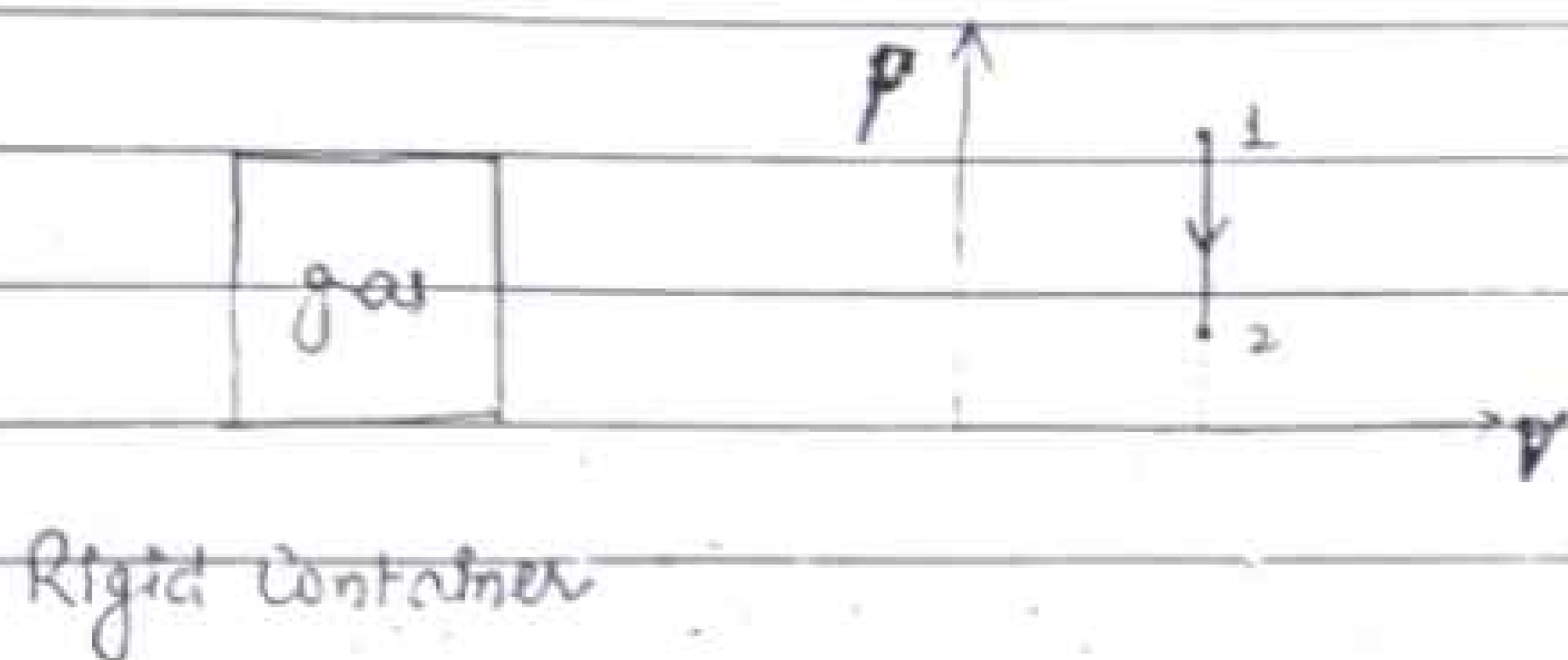
***) Though for paths A & B, end points are same, the work transfer is not same because δW is different for different paths and therefore work transfer depends upon the path. Hence the work transfer is a path function and it is not a property and it is inexact differential (dW or δW).

CONDITIONS FOR APPLYING THE EQⁿ $W = \int P dV$

- i) The system must be a closed system.
- ii) Work should cross the boundary.
- *** iii) The process must be a reversible process.

*) NON-FLOW OR CLOSED SYSTEM WORK FOR VARIOUS PROCESSES :-

i) CONSTANT VOLUME OR ISOCHORIC OR ISOMETRIC PROCESS -



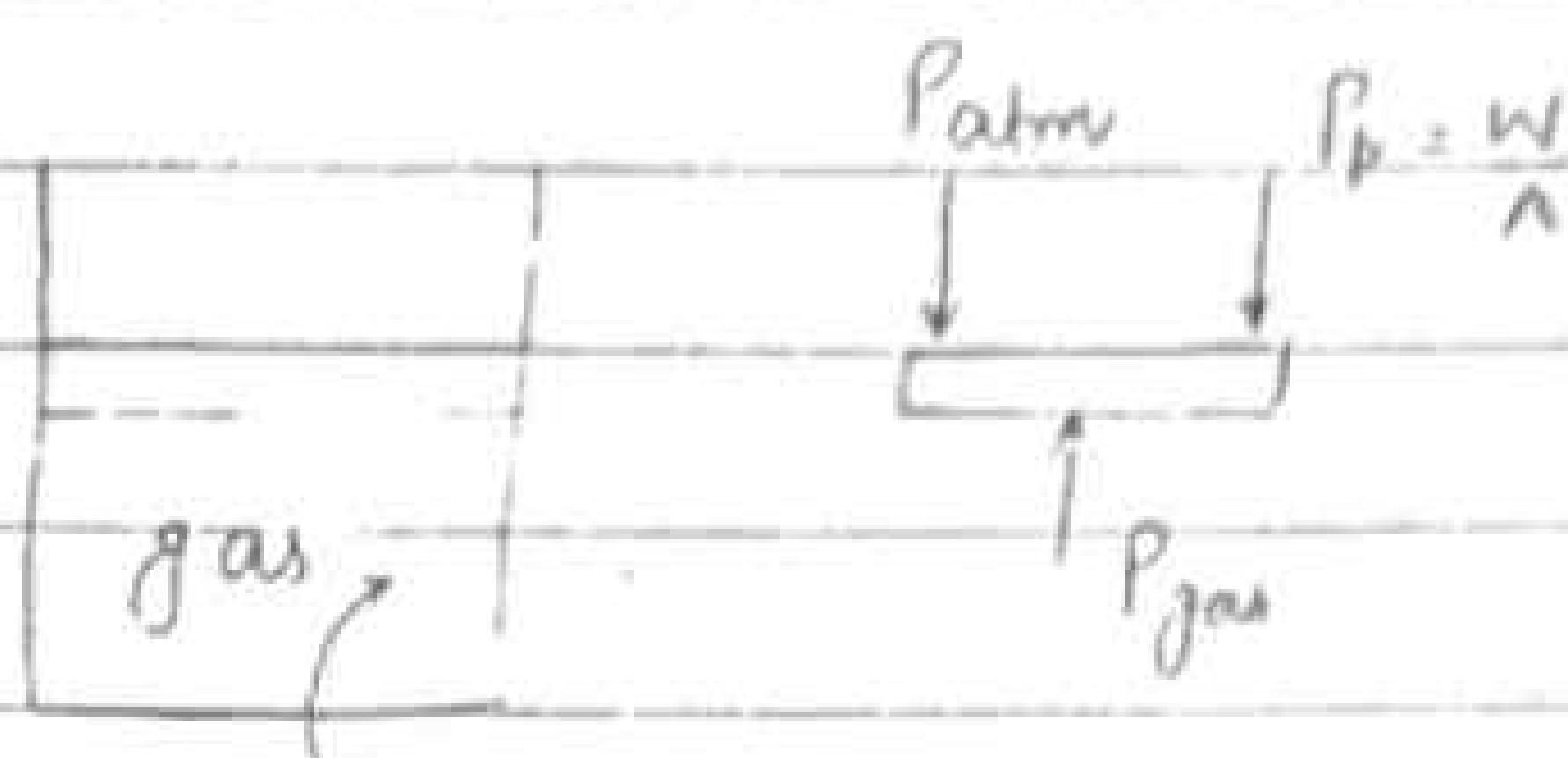
$$W = \int P dV$$

$$\text{as } V = \text{constant}$$

$$dV = 0$$

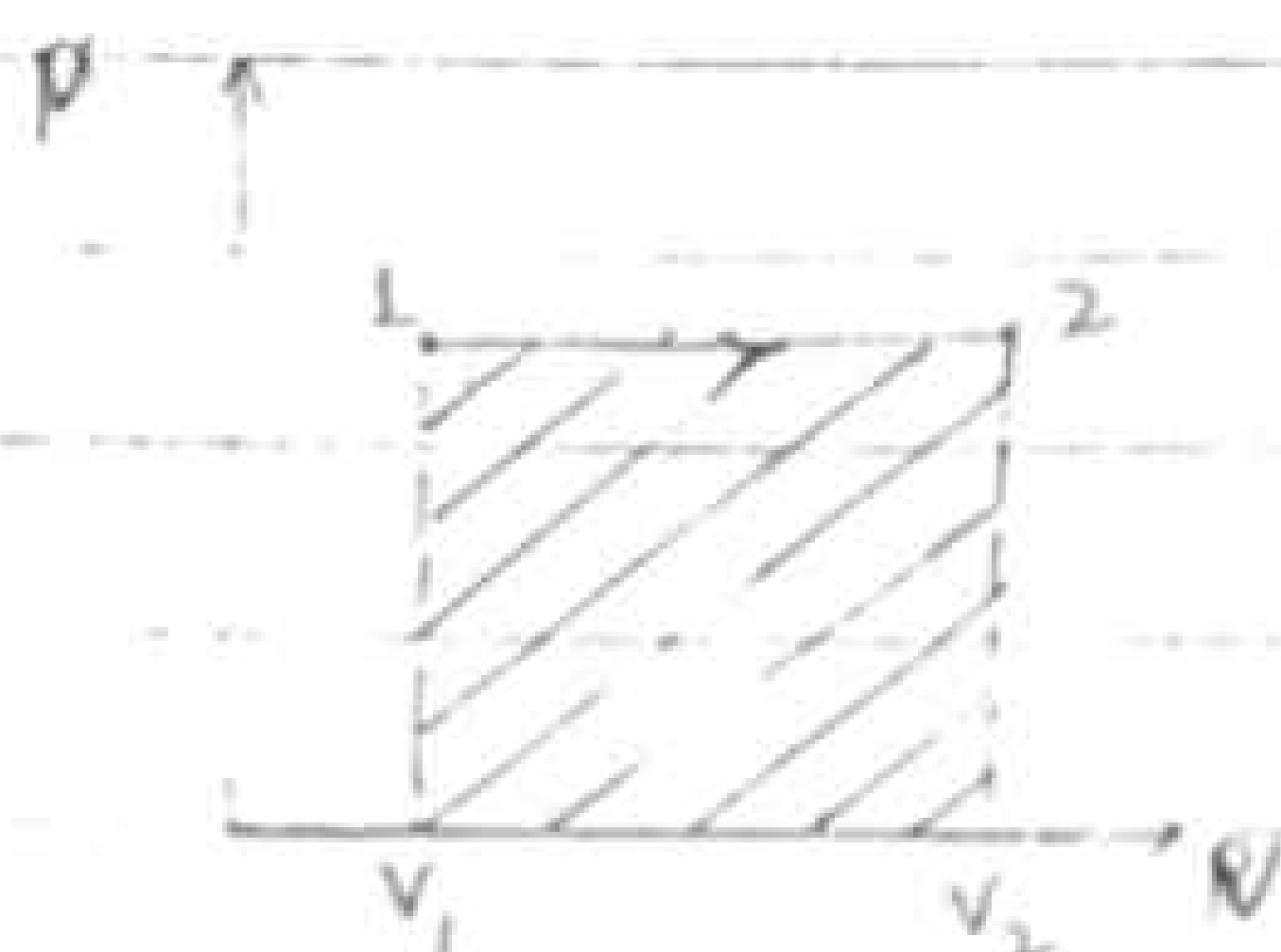
$$\therefore W = 0$$

ii) CONSTANT PRESSURE OR ISOBARIC OR ISopiestic PROCESS :-



$$P_{atm} + \frac{W}{A} = P_{gas}$$

so P_{gas} is constant.

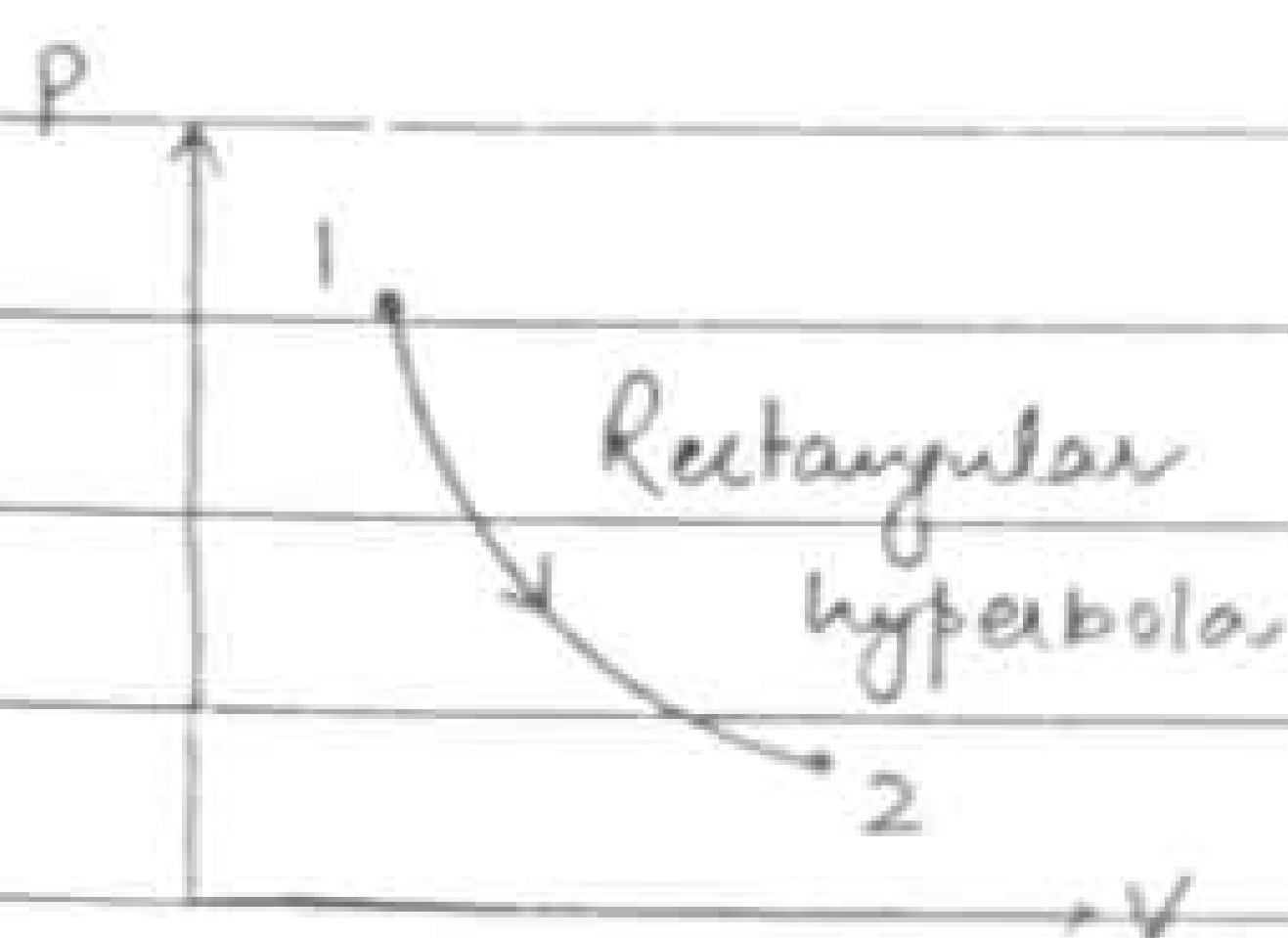


$$W = P (V_2 - V_1)$$

$$W = \int_{V_1}^{V_2} P dV$$

$$\Rightarrow [W = P(V_2 - V_1)]$$

iii) CONSTANT TEMPERATURE OR ISOTHERMAL PROCESS :-



$$PV = nRT$$

$$PV = C$$

$$P_1 V_1 = P_2 V_2 = C$$

$$W = \int_{V_1}^{V_2} P dV$$

$$\Rightarrow W = \int_{V_1}^{V_2} \frac{C}{V} dV$$

$$\Rightarrow W = C \ln \left(\frac{V_2}{V_1} \right)$$

for an ideal gas, isothermal process is a rectangular hyperbola on P-V diagram

$$\Rightarrow [W = nRT \ln \left(\frac{V_2}{V_1} \right)]$$

$$W = P_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$$

ii) ADIABATIC PROCESS :-

A process is said to be an adiabatic process if there is no heat transfer from the system or to the system. For an adiabatic process-

$$PV^\gamma = C$$

Where,

γ = Adiabatic index and greater than 1.

$$\text{So, work done} = \int_{V_1}^{V_2} P dV$$

$$= \int_{V_1}^{V_2} C V^{-\gamma} dV$$

$$= C \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$= \frac{C}{-\gamma+1} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right]$$

$$= \frac{C}{-\gamma+1} \left[V_2^{-\gamma} V_2 - V_1^{-\gamma} V_1 \right]$$

$$= \frac{1}{-\gamma+1} \left[C V_2^{-\gamma} V_2 - C V_1^{-\gamma} V_1 \right]$$

$$= \frac{1}{-\gamma+1} \left[P_2 V_2 - P_1 V_1 \right]$$

$$\Rightarrow W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

It is a closed system adiabatic work.

v.) POLYTROPIC PROCESS :-

A process is said to be a polytropic process when $PV^n = C$, in general n lies b/w 1 & γ .

In polytropic process, there is both heat and work transfer but in adiabatic process, there is only work transfer.

here n is known as polytropic index.

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

(**)

$$PV^K = C$$

$$i) \text{ for } P=C \Rightarrow K=0$$

$$\Rightarrow PV^0 = C$$

$$\Rightarrow P=C$$

$$P=C$$

$$K$$

$$0$$

$$T=C$$

$$1$$

$$ii) \text{ for } V=C \Rightarrow P^{1/K} V = C^{1/K} \quad (K=\infty)$$

$$\Rightarrow P^{1/\infty} V = C$$

$$\Rightarrow V=C$$

$$\text{Polytropic}$$

$$n$$

$$\text{Adiabatic}$$

$$\gamma$$

$$V=C$$

$$\infty$$

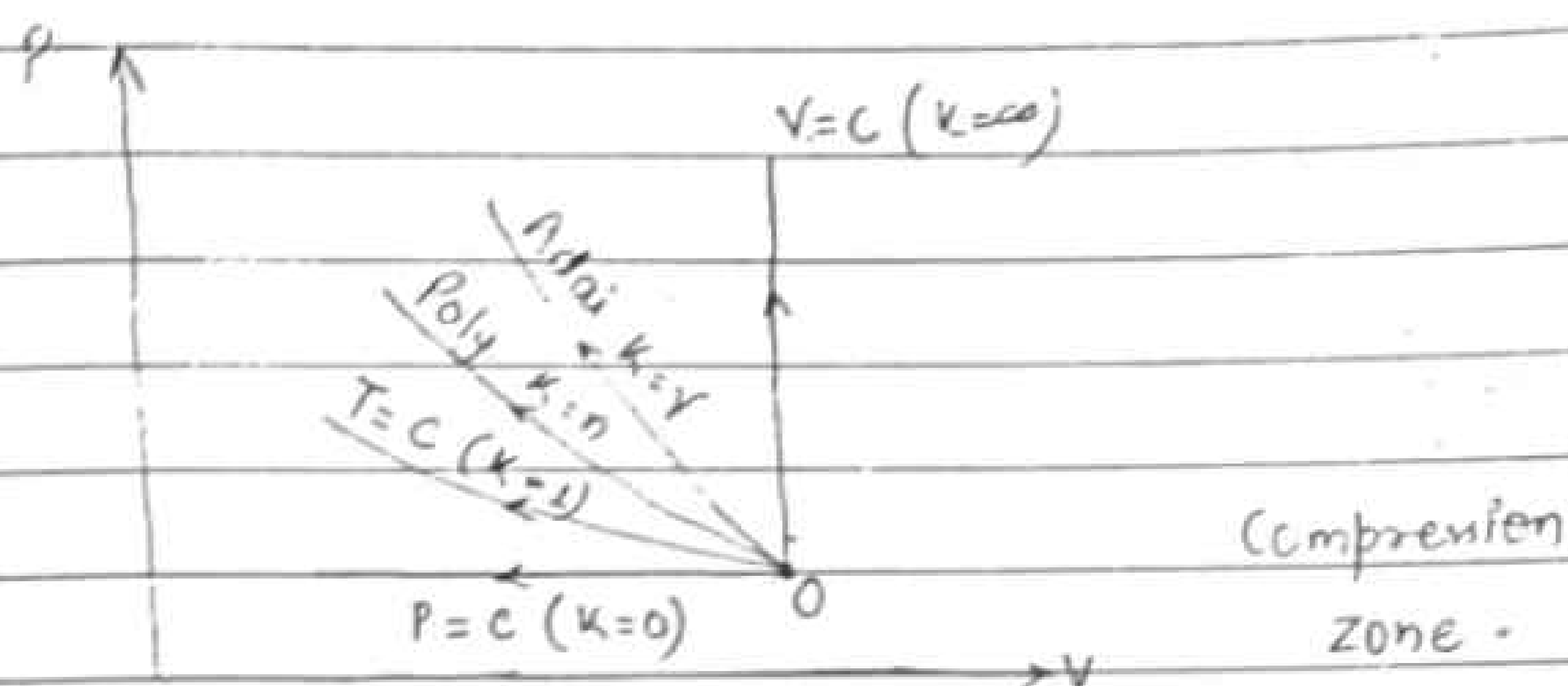
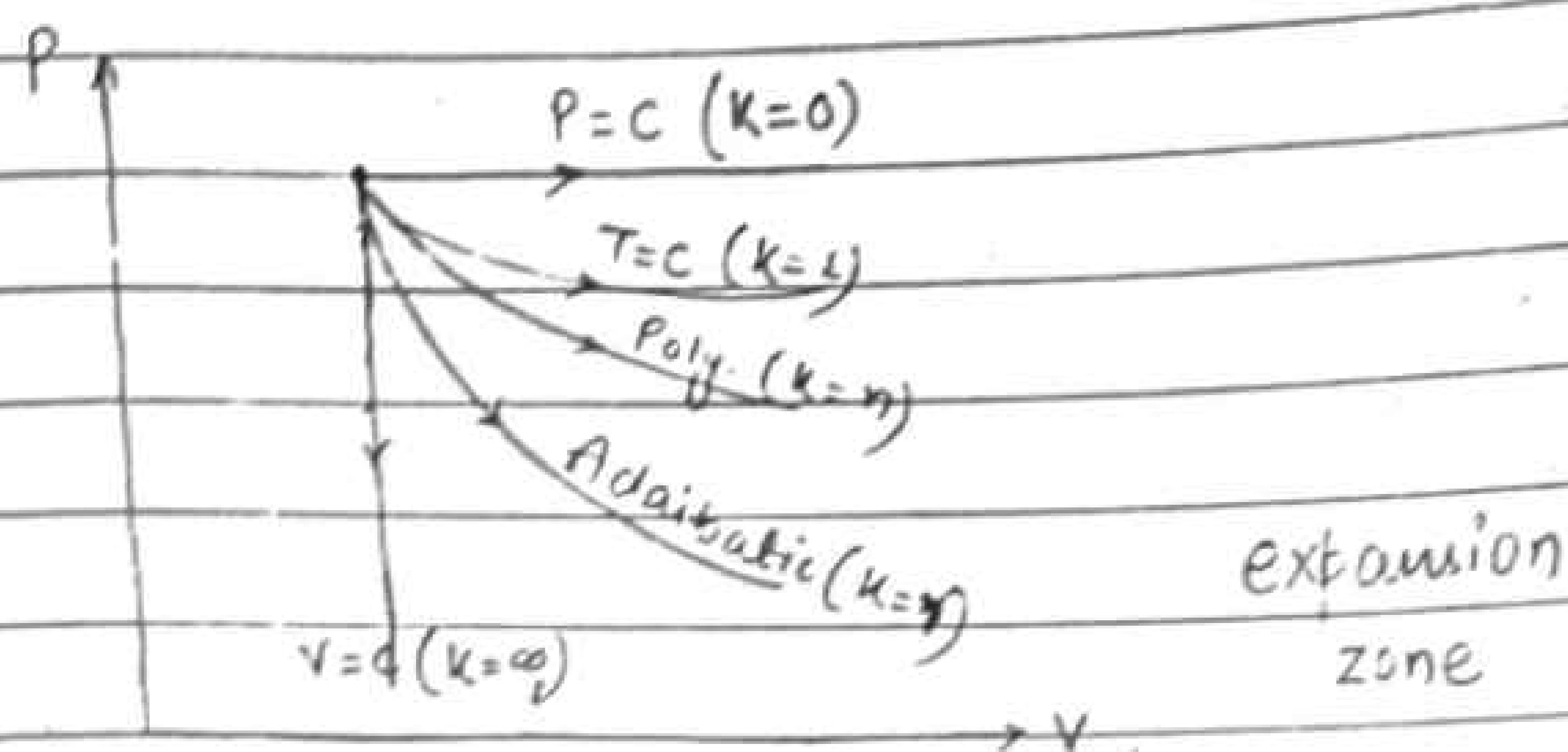
$$iii) \text{ for } T=C \Rightarrow PV^1 = C$$

$$K=1$$

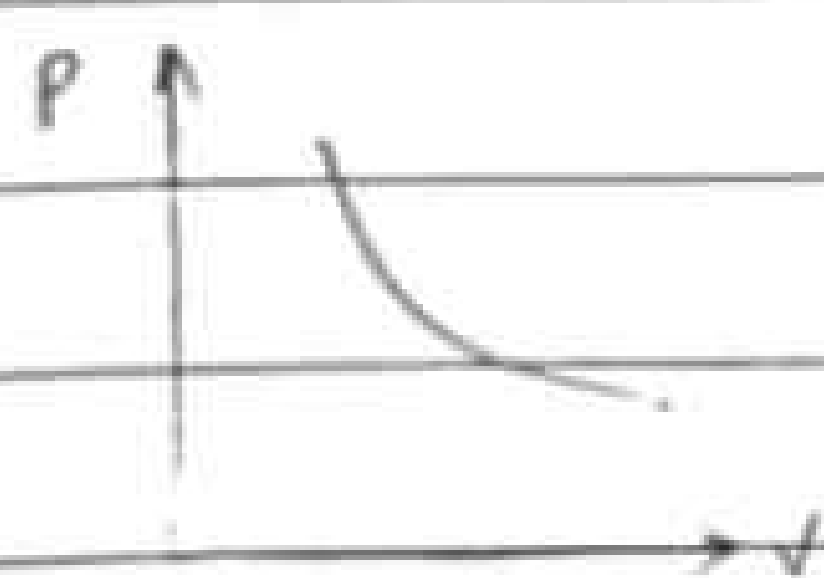
$$iv) \text{ for adiabatic } K=\gamma$$

$$v) \text{ for polytropic } K=n$$

→ Representation of various processes on P-V diagram :-



→ SLOPE OF ISOTHERMAL CURVE ON P-V Diagram -



$$PV = C$$

$$Pdv + vdp = 0$$

$$\Rightarrow Pdv = -vdp$$

$$\Rightarrow \frac{dP}{dV} = -\frac{P}{V}$$

∴ slope of isothermal curve on P-V diagram = $-\frac{P}{V}$

→ SLOPE OF ADIABATIC CURVES ON P-V Diagram :

$$PV^\gamma = C$$

$$\Rightarrow P \gamma V^{\gamma-1} dV + dP V^\gamma = 0$$

$$\Rightarrow \frac{P \gamma V^\gamma dV}{V} = -dP V^\gamma$$

$$\Rightarrow \frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$\Rightarrow \text{slope of adiabatic curve on P-V diagram} = -\frac{\gamma P}{V}$$

$$\Rightarrow \text{Adiabatic slope} = \gamma \left(-\frac{P}{V} \right)$$

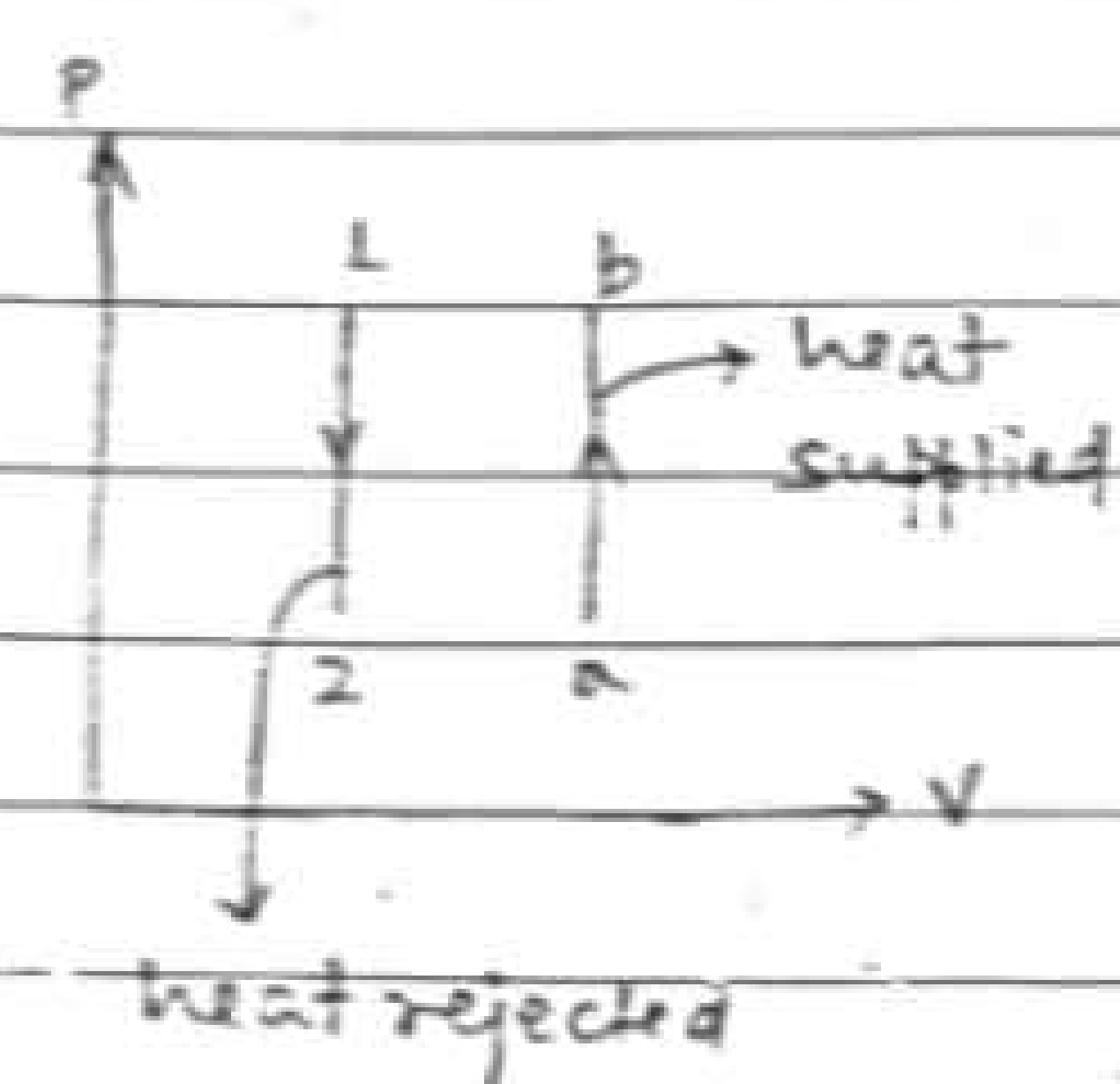
$$\Rightarrow \text{Adiabatic slope} = \gamma \left(\text{Isothermal slope} \right)$$

$$\Rightarrow \frac{\text{Adiabatic slope}}{\text{Isothermal slope}} = \gamma \quad (\text{as } \gamma > 1)$$

$$\Rightarrow \boxed{\text{Adiabatic slope} > \text{isothermal slope}}$$

→ IDEAL GAS EQUATIONS FOR VARIOUS PROCESSES :

1) CONSTANT VOLUME PROCESS :

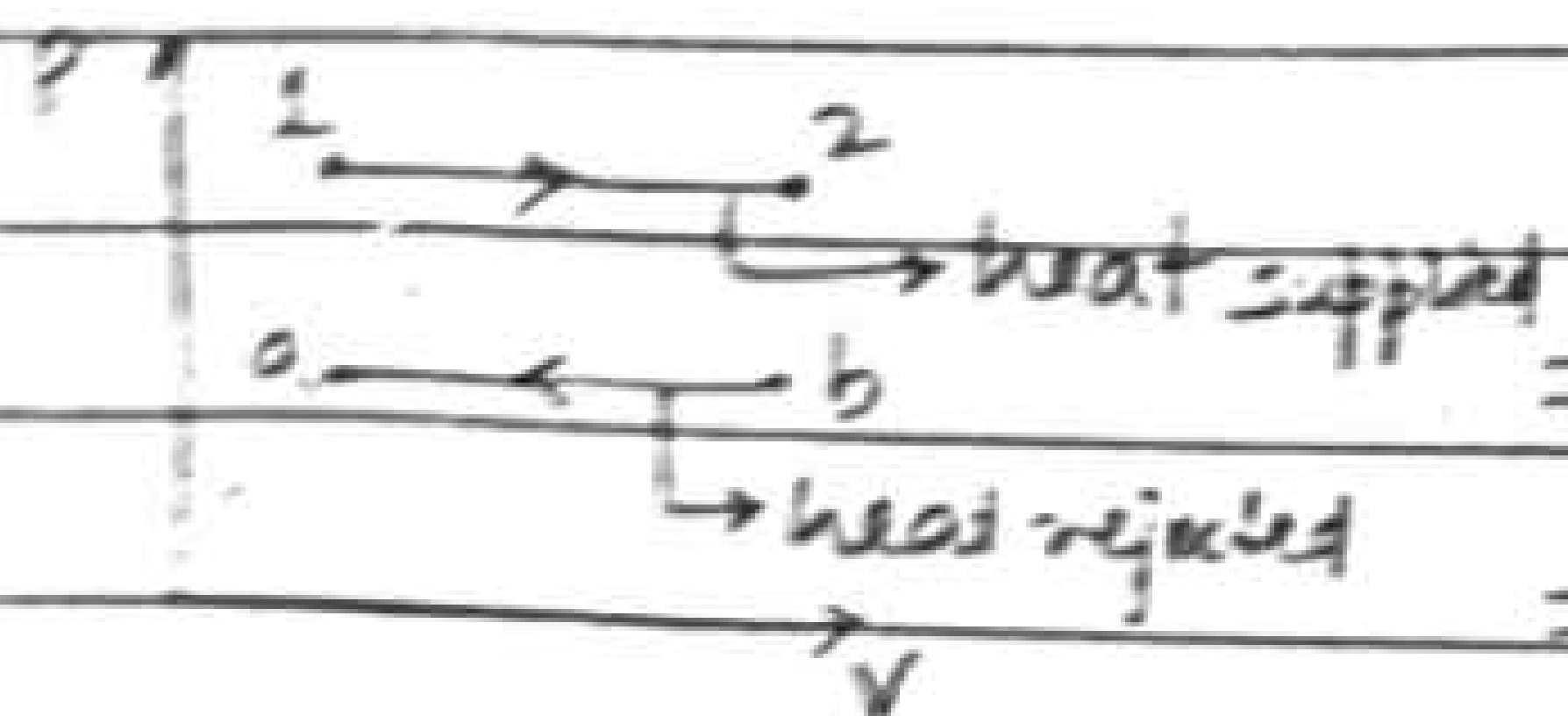


$$PV = nRT$$

$$\Rightarrow P \propto T$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

2) CONSTANT PRESSURE PROCESS :-



$$PV = mRT$$

$$V \propto T$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

3) ISOTHERMAL PROCESS :-

$$PV = mRT$$

$$PV = C$$

$$P_1 V_1 = P_2 V_2$$

4) ADIABATIC PROCESS :-

$$PV^\gamma = C$$

$$PV = mRT$$

$$\Rightarrow P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{P_2}{P_1} \right)^{1/\gamma} \quad \text{--- (1)}$$

$$\text{or } \frac{T}{V} \cdot V^\gamma = C$$

$$\Rightarrow T V^{\gamma-1} = C$$

$$\Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)} \quad \text{--- (2)}$$

From (1), (2)

$$\left(\frac{P_2}{P_1} \right)^{1/\gamma} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \quad \text{--- (3)}$$

BD

$P_1 V_1^\gamma = P_2 V_2^\gamma$	→ valid for an ideal gas undergoing adiabatic process
$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$	
$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$	

5) POLYTROPIC PROCESS :

$$PV^n = C$$

a.

$P_1 V_1^n = P_2 V_2^n$
$T_1 V_1^{n-1} = T_2 V_2^{n-1}$
$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

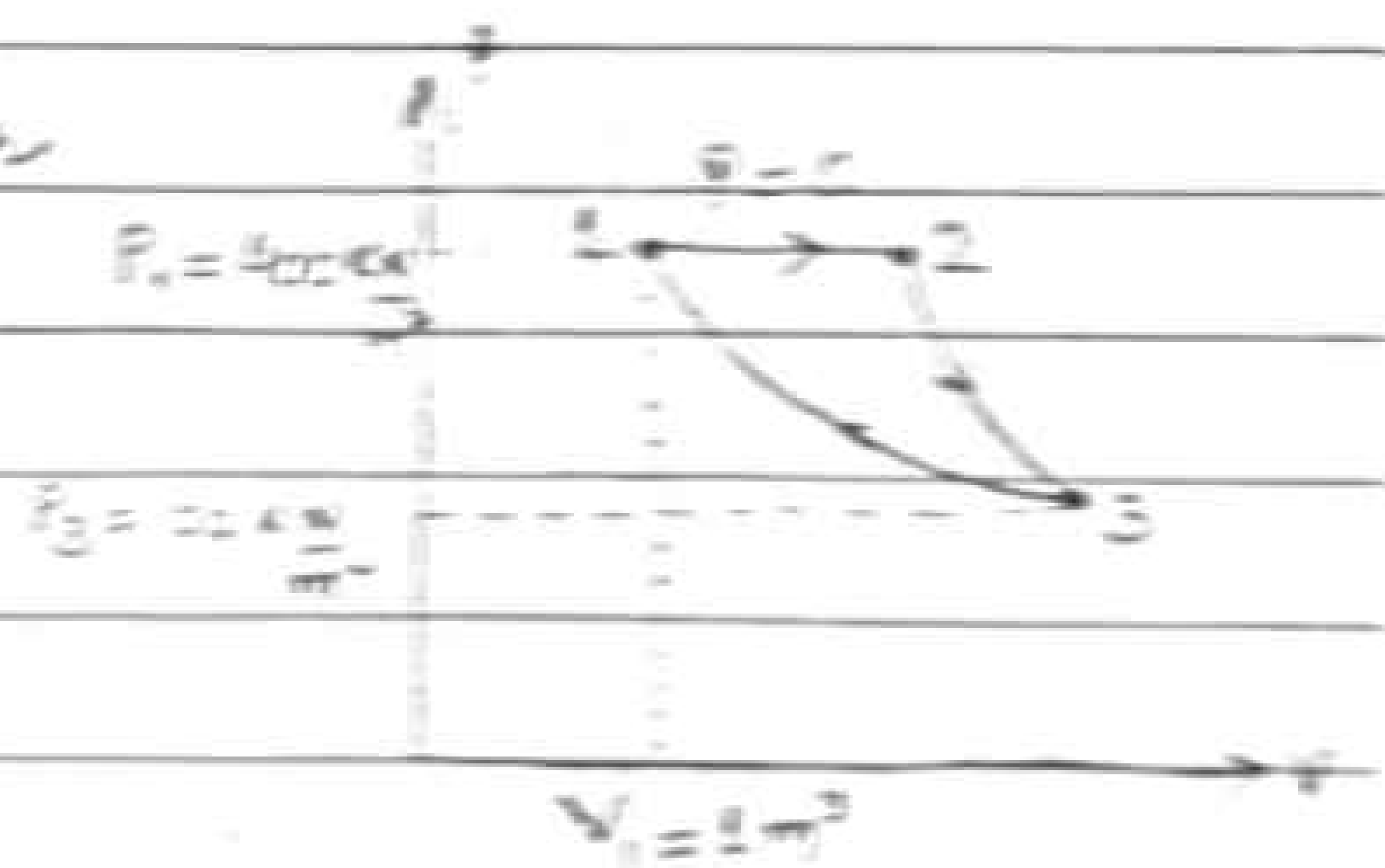
P:1 A system undergoes 3 processes as shown in fig.

1-2 isobaric process

2-3 polytropic process ($n=1.4$)

3-1 isothermal process in which $PV=C$

Find i) V_2 ii) Net work transfer



i) 3-1 $\Rightarrow T=C$
 $\Rightarrow PV=C$
 $\Rightarrow P_3 V_3 = P_1 V_1$
 $\Rightarrow V_3 = \frac{P_1 V_1}{P_3} = \frac{4 \times 1}{0.4}$
 $\Rightarrow V_3 = 10 \text{ m}^3$

And 2-3 $\Rightarrow PV^{1.4}=C \Rightarrow P_2 V_2^{1.4} = P_3 V_3^{1.4}$
 $\Rightarrow \frac{V_2}{V_3} = \left(\frac{P_3}{P_2}\right)^{1/1.4}$

$$\Rightarrow \frac{V_2}{V_3} = (4)^{1/1.4}$$

$$\Rightarrow V_2 = \underline{1.486 \text{ m}^3} \quad \text{Ans.}$$

ii) Net work done = $W_{12} + W_{23} + W_{31}$

$$\therefore W_{12} = P_1 (V_2 - V_1) = 400(1.486 - 1) = 194.4 \text{ kJ}$$

$$W_{23} = \frac{P_2 V_2 - P_3 V_3}{n-1} = \frac{400 \times 1.486 - 100 \times 4}{1.4-1} = 486 \text{ kJ}$$

$$W_{31} = P_3 V_3 \ln \left(\frac{V_1}{V_3} \right) = 100 \times 4 \times \ln \left(\frac{1}{4} \right) = -554.5 \text{ kJ}$$

$$\text{So, Net work} = 194.4 + 486 - 554.5$$

$$\Rightarrow W_{\text{net}} = \underline{125.8 \text{ kJ}} \quad \text{Ans.}$$

***) ① All Net work in a cycle is equal to area of closed region.

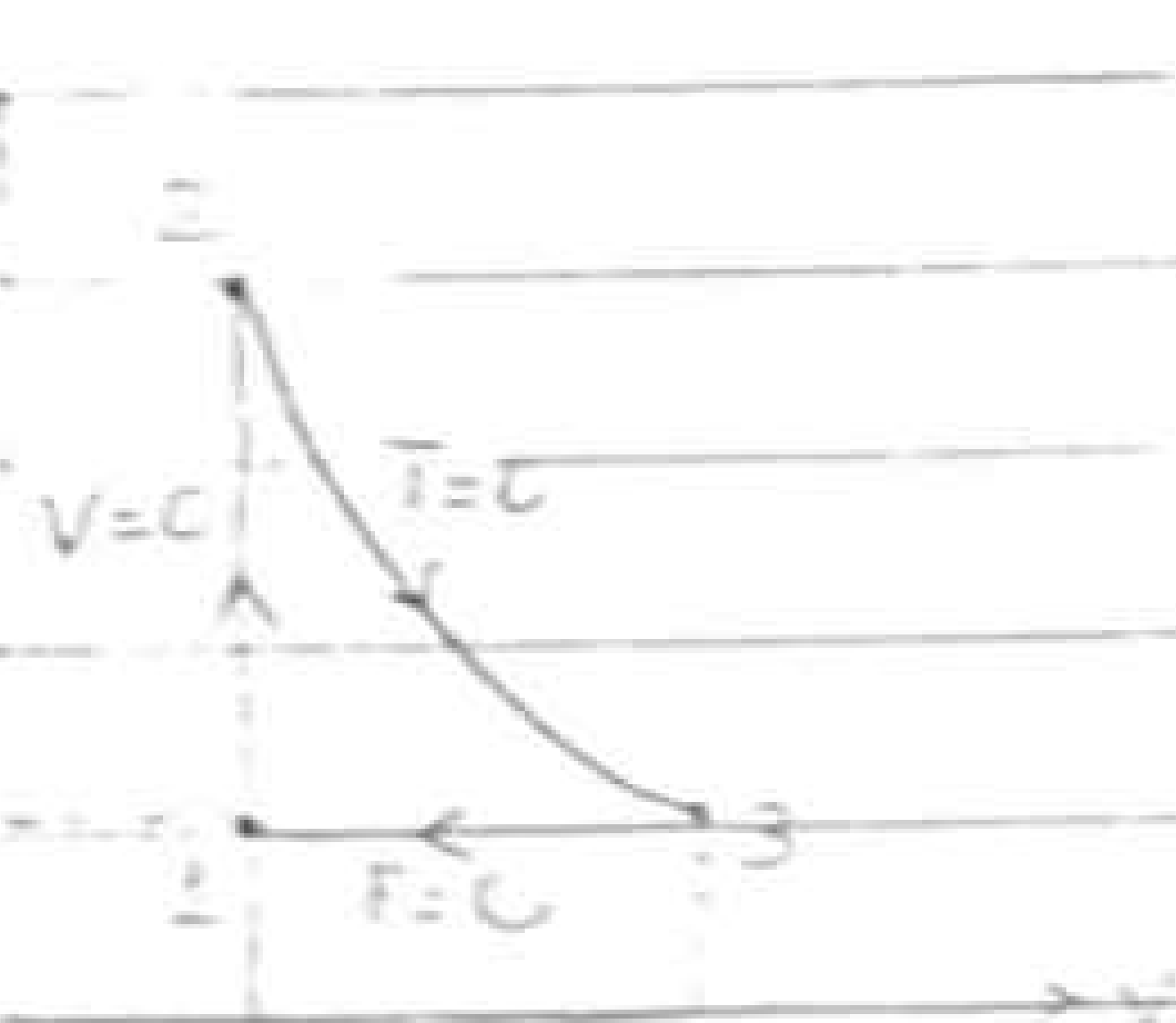
② All clockwise cycles on P-V diagram are power producing or work producing cycles.

③ All anticlockwise cycle on P-V diagram are power absorbing or work absorbing cycle.

Q.2 (2003) An ideal gas is heated at constant volume until its temp. is 3 times the original temp. It is then expanded isothermally till it reaches its original pr. The gas is then cooled at constant pr. till it is restored to the original state. Determine

i) the net work done per kg of gas

Take the initial temp as 3150K and express your answer in terms of gas constant R



for process 1-2

$$\Rightarrow W_{12} = 0$$

for (2-3) process

$$\Rightarrow W_{23} = P_2 V_2 \ln \left(\frac{V_3}{V_2} \right)$$

$$W_{23} = mRT_2 \ln(V_3/V_2) \quad \text{---(1)}$$

Now, $T_1 = 3150 \text{ K}$

$$T_2 = 3 \times T_1$$

$$T_2 = 1050 \text{ K} \quad \text{---(3)}$$

$$T_3 = 1050 \text{ K} \quad \text{---(4)}$$

for (3-1) process

$$\Rightarrow W_{31} = P_1 (V_1 - V_3) \quad \text{---(5)}$$

$$\text{As } P_2 V_2 = P_3 V_3 \Rightarrow \frac{V_3}{V_2} = \frac{P_2}{P_3} = \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{V_3}{V_2} = 3 \quad \text{---(5)}$$

Now, $W_{23} = RT_2 \ln(V_3/V_2) = 1153.5 R \quad \text{---(6)}$

$$\begin{aligned} W_{31} &= P_1 (V_1 - V_3) = P_1 V_1 \left(1 - \frac{V_3}{V_1} \right) \\ &= R(3150) \left(1 - \frac{V_3}{V_1} \right) \end{aligned}$$

$$\therefore W = 1153.5 R - 700 R$$

$$= R(3150)(1-3) = -700 R$$

$$\boxed{W_{\text{net}} = 1153.5 R} \quad \text{done}$$

09.08.2011

6.90)

3. An insulated vertical cylinder contains 0.1 kg of argon gas. In the top of a frictionless non conducting piston as shown in fig. the mass of the piston is 5 kg and it is in contact with water on the bottom of the cylinder. The cylinder is connected to a nitrogen tank at 100 bar. A pipe line fitted in a valve. The valve is opened and N_2 slowly enters into the cylinder and during this process the piston is lifted to a height of 10 cm. Initial temp and pressure of Argon are 300 K and 1 bar respectively. final temp of argon is 320 K. for argon gas take $R = 0.208 \text{ kJ/kgK}$ and $\gamma = 1.67$.
- find work done by argon
work done by nitrogen gas.

- is i) As the argon cylinder is insulated and piston is non conducting, there is no heat transfer from Argon or to Argon, therefore it undergoes adiabatic process.

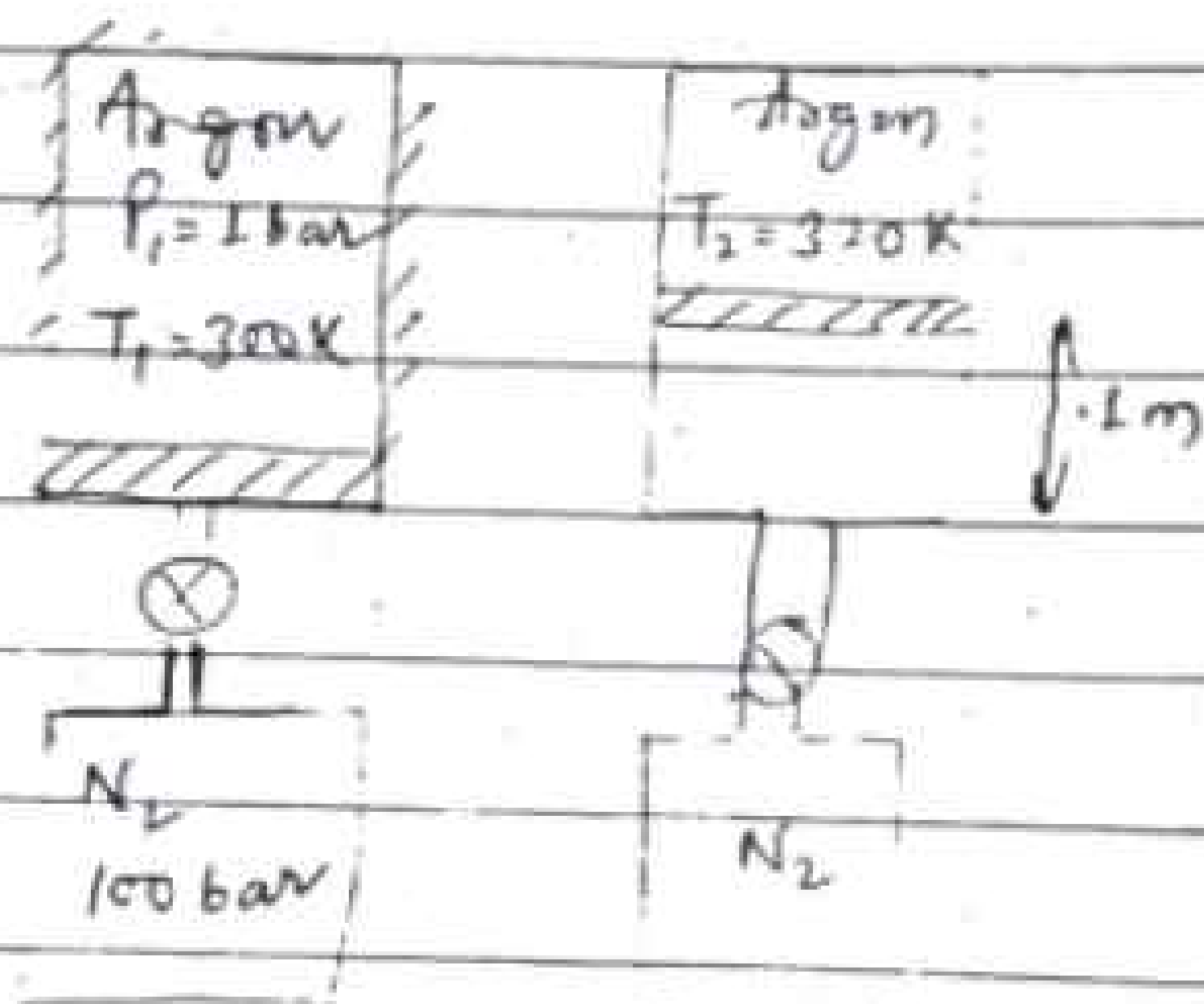
$$W_{\text{argon}} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$W_{\text{argon}} = \frac{mRT_1 - mRT_2}{\gamma - 1}$$

$$= \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$\Rightarrow W_{\text{argon}} = \frac{0.1 \times 0.208 (300 - 320)}{1.67 - 1}$$

$$\Rightarrow W_{\text{argon}} = -0.6208 \text{ kJ} \quad \text{Ans.}$$



ii) $W_{N_2} = W_{\text{argon}} + W_{\text{piston}}$

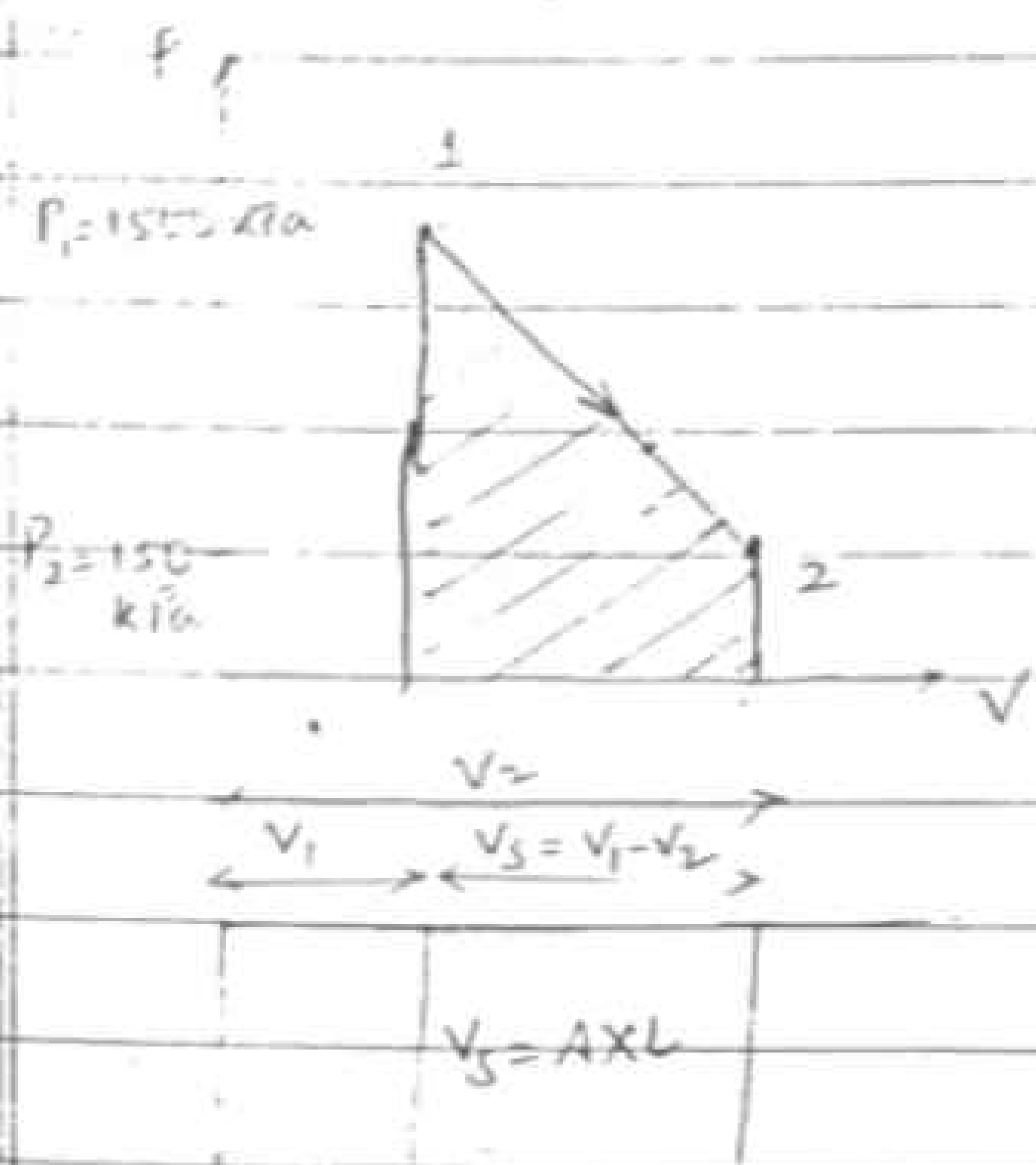
$$W_{N_2} = -0.6208 + (mgh)$$

$$W_{N_2} = -0.6208 + \frac{5 \times 9.81 \times 1}{1000}$$

$$\left(\frac{\text{kg} \times \frac{\text{m}}{\text{sec}^2} \times \text{m}}{\text{sec}^2} \right) \quad \text{N-m} = \text{J}$$

$$\Rightarrow W_{N_2} = \underline{\underline{0.6275 \text{ KJ}}} \quad \text{Ans}$$

P:4 An engine cylinder has a piston of area 0.12 m^2 and contains a gas at a pr. of 1500 kPa . The gas expands according to a process which is represented by a straight line on P-V diagram. The final pr. is 150 kPa . Calculate the work done by the gas if the stroke length is 0.3 m .



$$V_S = V_2 - V_1 = AXL$$

$$V_S = 0.036 \text{ m}^3$$

$W = \text{area under P-V curve}$

$$\Rightarrow W = \frac{1}{2} (1500 + 150) (V_2 - V_1)$$

$$\Rightarrow W = \frac{1}{2} \times 1650 \times 0.036$$

$$\Rightarrow W = 29.7 \text{ KJ}$$

P:5

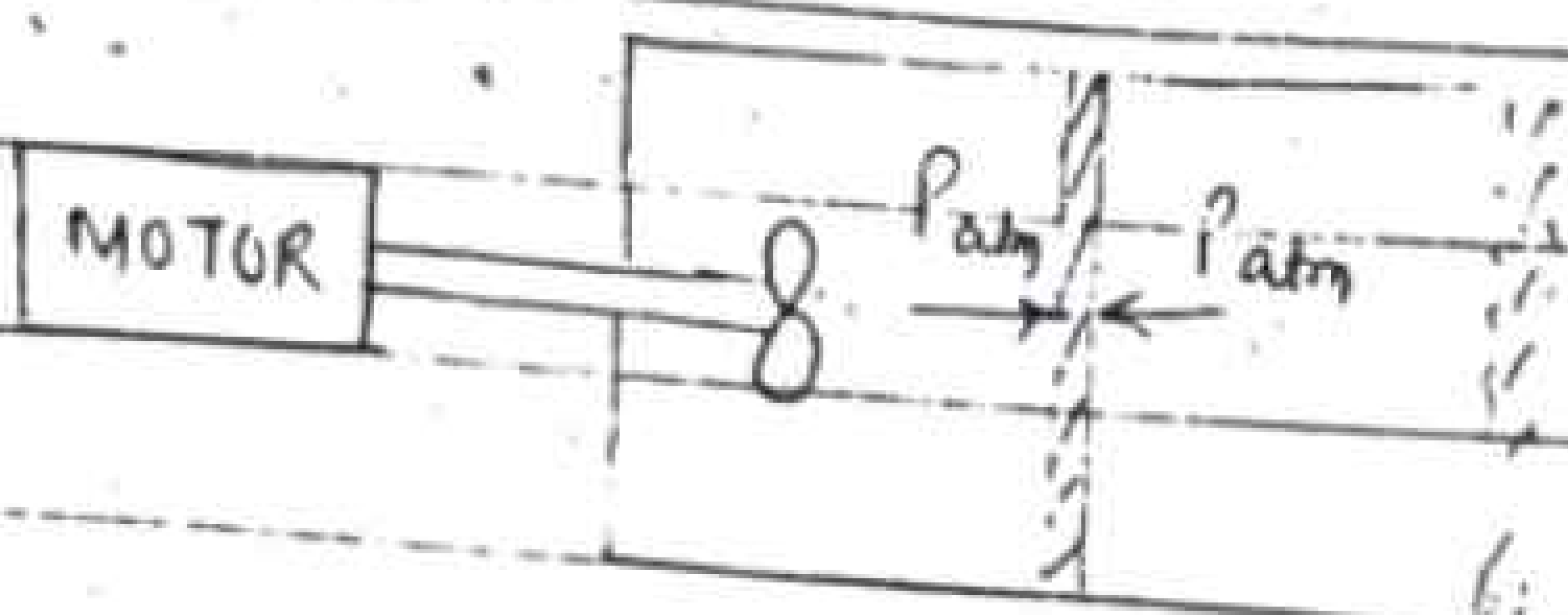
A fluid contained in a horizontal cylinder fitted with a free frictionless & leak proof piston is continuously agitated by means of a stirrer passing through the cylinder cover. The cylinder dia. is 0.4 m , during the stirring process lasting 10 minutes the piston slowly moves out a distance of 0.485 m against the atm. pressure. The net work done by the fluid is 2 KJ . The speed of the motor driving stirrer is 840 rpm . Determine the power of motor and torque in the shaft.

done on the system (-ve work)
eg: Paddling work or Stirring work

classmate

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for gas:

$$W_{\text{net}} = W_{\text{stirring}} + W_{\text{piston}} \quad \text{---(1)}$$

$$W_{\text{piston}} = P_{\text{atm}} (\Delta V)$$

$$\Rightarrow W_{\text{piston}} = 101.325 \times \left(\frac{\pi}{4} d^2 \times 0.465 \right)$$

$$\Rightarrow W_{\text{piston}} = 6.175 \text{ kJ}$$

$$\text{from (1)} \Rightarrow 2 = W_{\text{stirring}} + 6.175$$

$$\Rightarrow W_{\text{stirring}} = -4.175 \text{ kJ}$$

Now for motor $W_{\text{motor}} = 4.175 \text{ kJ}$

$$W_{\text{motor}} = 4175 \text{ J}$$

$$P_{\text{motor}} = \frac{W_{\text{motor}}}{t}$$

$$P_m = \frac{4175}{10 \times 60}$$

$$P_m = 6.95 \text{ Watt}$$

$$\therefore P_m = \frac{2\pi NT}{60}$$

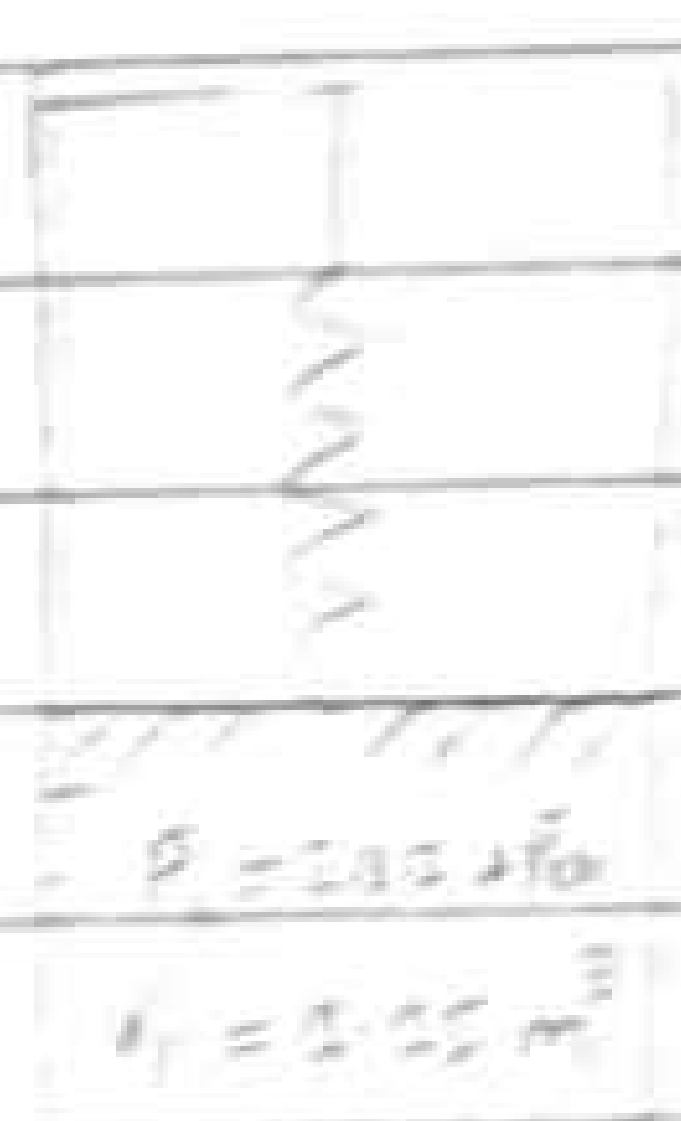
$$\Rightarrow T = 0.08 \text{ Nm}$$

Q: 6

A piston cylinder device contain 0.05 m^3 of a gas initially at 200 kPa . At this state a linear spring which has a spring constant of 150 kN/m is just touching the piston but exerting no force on it. Now heat is transferred

to find causing the piston to rise $\rightarrow$ calculate the spring work the volume under the piston increases $\frac{1}{2}$ of the x-sec area of piston 0.25 m^2 . find

- i) find to find the cylinder
- ii) work done by the gas



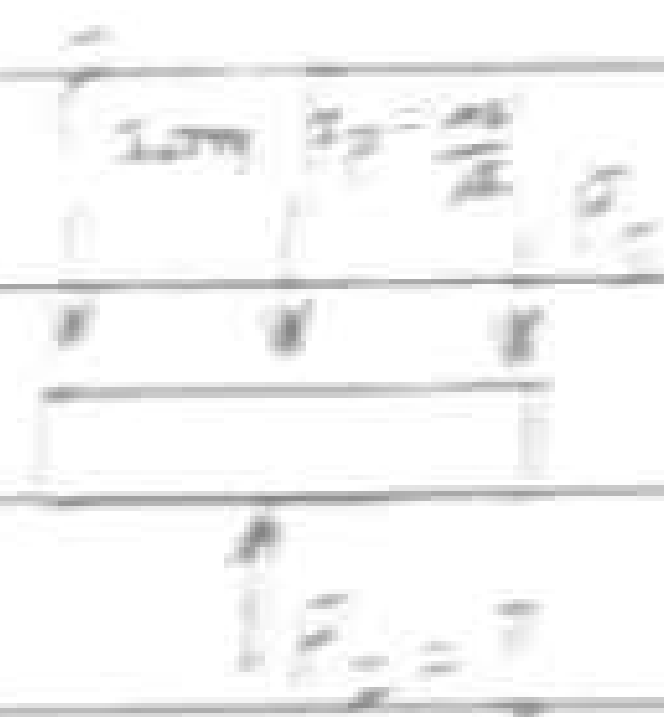
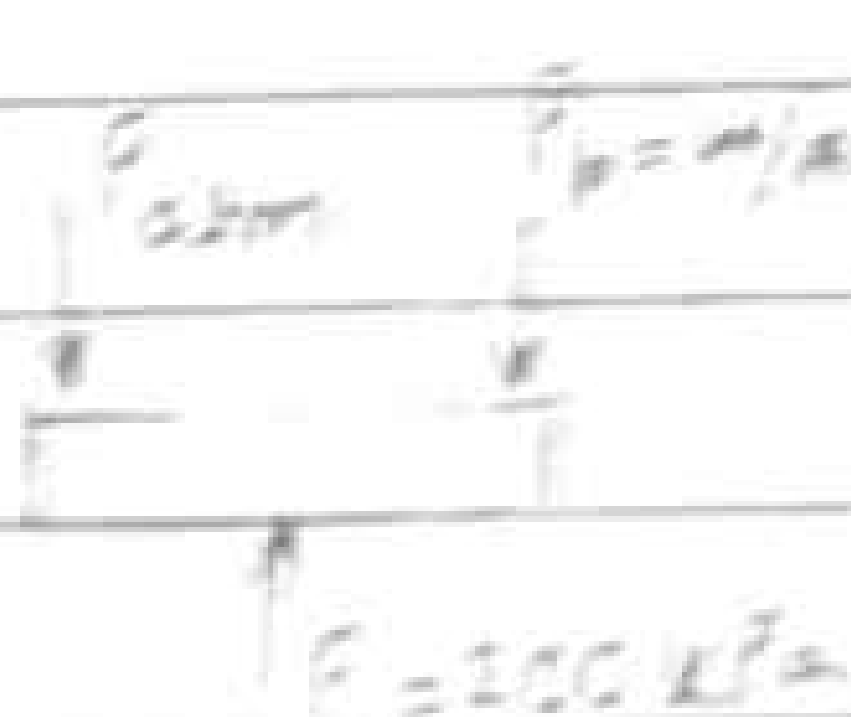
$$V_2 - V_1 = \Delta V$$

$$V_2 - V_1 = A(x)$$

$$\Rightarrow V_2 = A(x)$$

$$\Rightarrow x = V_2/A$$

$$\Rightarrow x = 0.2 \text{ m}$$



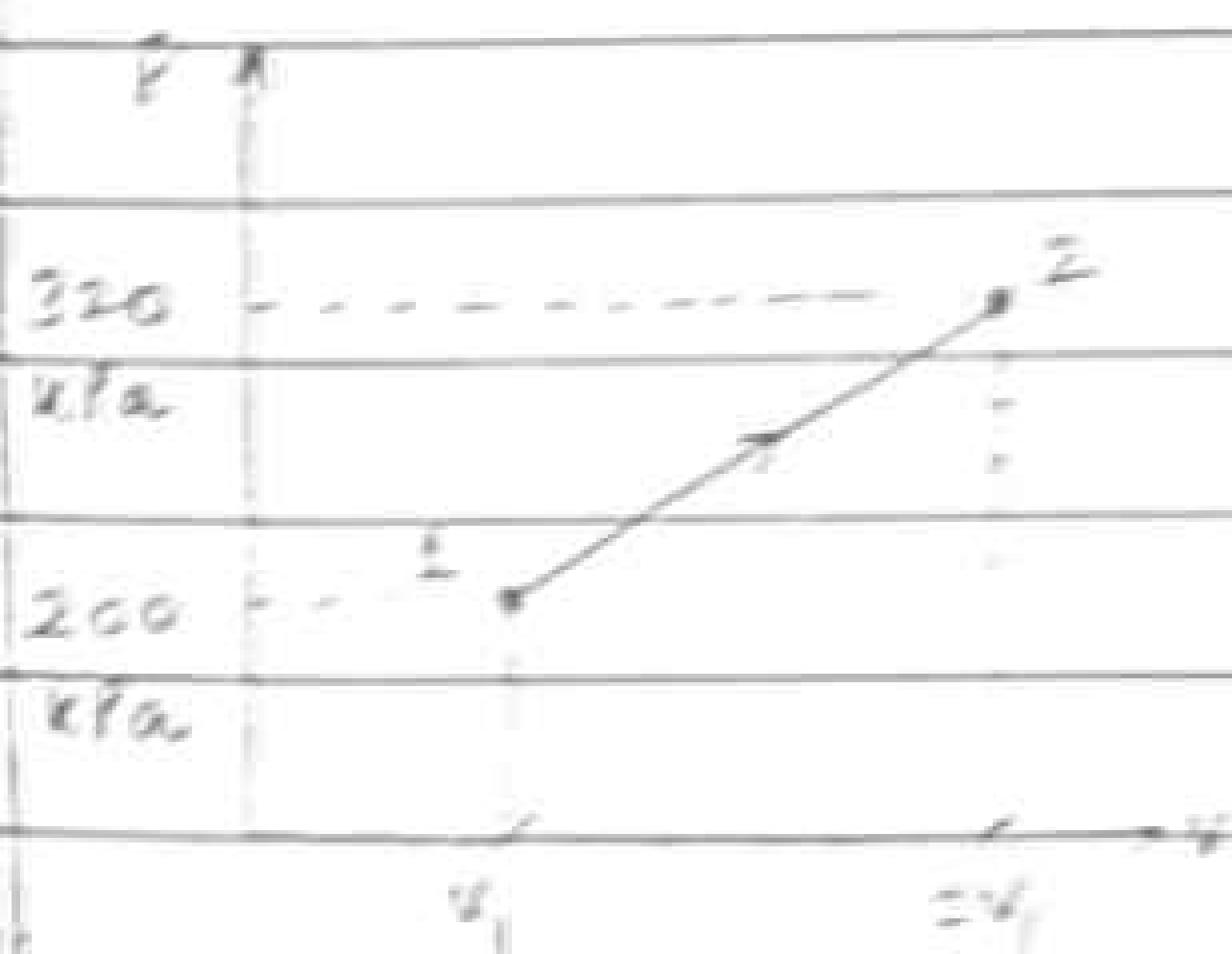
$$P_2 = \frac{Kx}{A} = \frac{150 \times 0.2}{0.25} = 120 \text{ kPa}$$

$$P_{\text{atm}} + P_p = 200 \quad \text{---(1)}$$

$$P_{\text{atm}} + P_p + P_s = P_2 \quad \text{---(2)} \Rightarrow 200 + P_s = P_2$$

$$\Rightarrow 200 + 120 = P_2$$

$$\Rightarrow P_2 = 320 \text{ kPa} \quad \text{Ans}$$



$$P_2 = 200 + P_s$$

$$P_2 = 200 + \frac{Kx}{A}$$

$$P_2 = 200 + \frac{K \times \frac{V_2 - V_1}{A}}{A}$$

$$= 320 \text{ kPa}$$

$$\text{Work done} = \frac{1}{2} (200 + 320) (V_2 - V_1)$$

$$\Rightarrow W = \frac{1}{2} \times 520 \times 0.25$$

$$\Rightarrow W = 65 \text{ kJ} \quad \text{Ans}$$

HEAT :-

The energy interactions due to temperature difference is known as heat transfer.

$$Q \propto \Delta T$$

$$Q \propto m$$

so, $Q \propto m \Delta T$

$$\Rightarrow [Q = m C \Delta T]$$

here

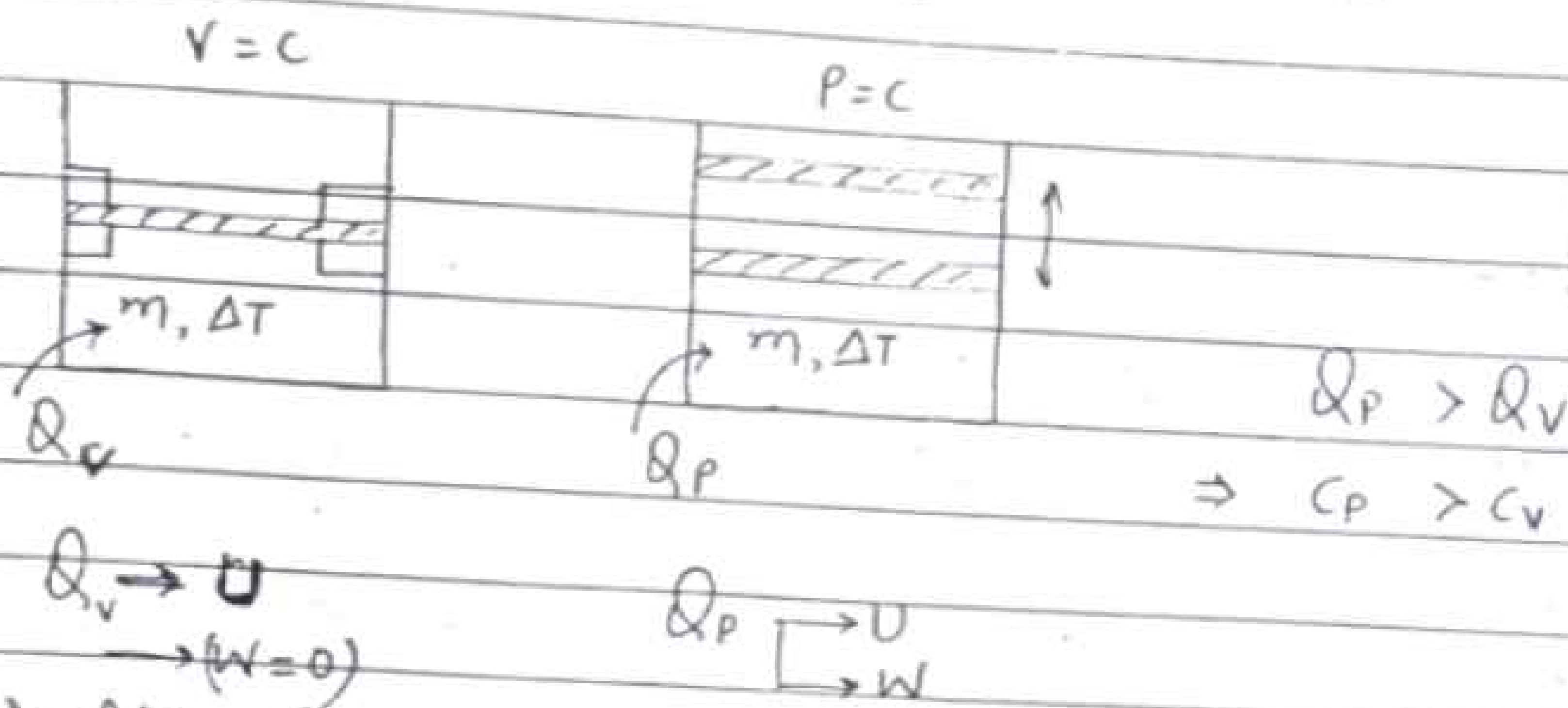
C = specific heat

if $m = 1 \text{ kg}$ then $[Q = C]$

$\Delta T = 1^\circ\text{C}$

$Q = C$ $\left(\begin{smallmatrix} 1 \text{ kg} \\ 1^\circ\text{C} \end{smallmatrix} \right)$

SPECIFIC HEAT: It is the amount of heat required to raise the temp. of unit mass substance to the unit degree temp difference.



$W = PdV$ ($V = \text{const}$) $\therefore W = 0$

Specific heat at const. pr. (C_p) is always greater than specific heat at const. volume (C_v) because C_p includes internal energy (U) and external work (W) where C_v includes only internal energy (U).

I Law of Thermodynamics (Law of conservation of energy):

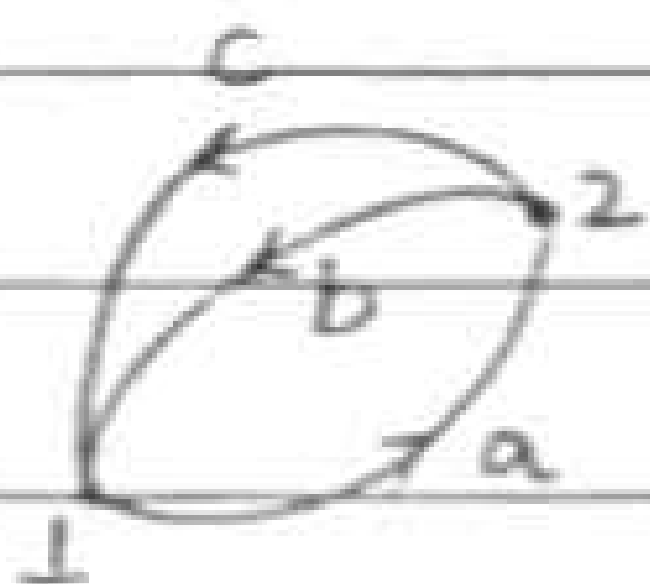
for a closed system undergoing a cycle net heat transfer is equal to net work transfer. i.e.

$$|\Sigma Q = \Sigma W| \text{ valid for a cycle (reversible or irreversible)}$$

CONSEQUENCES OF I Law of THERMODYNAMICS :

1) Heat transfer is a path function.

$P \uparrow$



$$\Sigma Q = \Sigma W$$

$$\Rightarrow (\delta Q)_{1a2} + (\delta Q)_{2b1} = (\delta W)_{1a2} + (\delta W)_{2b1} \quad \text{--- (1)}$$

$$\& (\delta Q)_{1a2} + (\delta Q)_{2c1} = (\delta W)_{1a2} + (\delta W)_{2c1} \quad \text{--- (2)}$$

Subtracting (1) & (2)

$$\Rightarrow (\delta Q)_{2b1} - (\delta Q)_{2c1} = (\delta W)_{2b1} - (\delta W)_{2c1}$$

As $(\delta W)_{2b1} \neq (\delta W)_{2c1}$ (Path function)

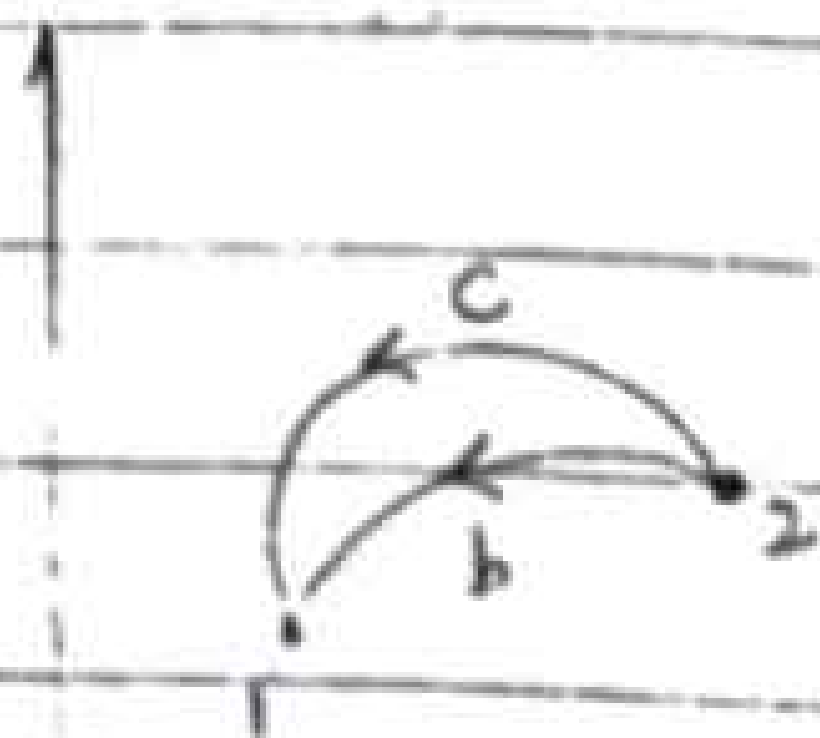
$$\Rightarrow (\delta W)_{2b1} - (\delta W)_{2c1} \neq 0$$

$$\therefore (\delta Q)_{2b1} - (\delta Q)_{2c1} \neq 0$$

$$\Rightarrow (\delta Q)_{2b1} \neq (\delta Q)_{2c1} \quad \text{--- (3)}$$

Though for paths B & C, end points are same, heat trans is not same, therefore heat transfer depends upon the path followed by process and hence it is a path function. It is inexact differential, not a property and it is a boundary phenomenon (δQ or dQ).

2) Energy is a property. but WD & heat transfer are not.



$$(\delta Q)_{2b1} - (\delta Q)_{2c1} = (\delta W)_{2b1} - (\delta W)_{2c1}$$

$$\Rightarrow (\delta Q - \delta W)_{2b1} = (\delta Q - \delta W)_{2c1}$$

The quantity $(\delta Q - \delta W)$ is same for both paths b & c and it is independent of path and it depends only on end points (2-1) therefore $(\delta Q - \delta W)$ must be a property and this property is known as energy.

$$\Rightarrow (\delta Q - \delta W)_{2b1} = (\delta Q - \delta W)_{2c1} = dE$$

$$\Rightarrow (\delta Q - \delta W)_{2b1} = dE$$

$$\Rightarrow \boxed{(\delta Q = dE + \delta W)_{2b1}} \quad \text{--- (A)}$$

> The above eqn (A) is known as I law of thermodynamics eqn for a closed system undergoing any process (reversible or irreversible process)

> The eqn $\{\delta Q = dE + PdV\}$ is the I law of thermodynamics eqn for a closed system undergoing reversible process only. because $(PdV = \delta W)$ is valid only for a reversible process.

ENERGY :

$$E = \underbrace{KE + PE}_{\text{Macroscopic}} + \underbrace{U}_{\text{Microscopic}}$$

$$dE = d(KE) + d(PE) + du$$

$$dE = du$$

So 1st law of thermodynamics for a process

$$\boxed{dQ = du + dW} \quad \text{--- (6)}$$

Eqⁿ (6) is a 1st law of TD eqⁿ for a closed stationary system undergoing any process (KE and PE changes are negligible)

INTERNAL ENERGY :

The energy associated with molecules is known as internal energy. It is an extensive property. It is generally expressed in KJ.

$$\text{Specific internal energy (u)} = \frac{U}{m}$$

$$\Rightarrow u = \frac{U}{m} \text{ (KJ/kg)} \rightarrow \text{intensive}$$

3> Energy of an isolated system is always constant.

for an isolated system : $\Delta m = 0$

$$\Delta \text{Energy} = 0$$

Work interaction = 0
Heat interaction = 0

$$\therefore dQ = du + dW$$

$$\Rightarrow Q = dU + W$$

$$\Rightarrow 0 = dU + 0$$

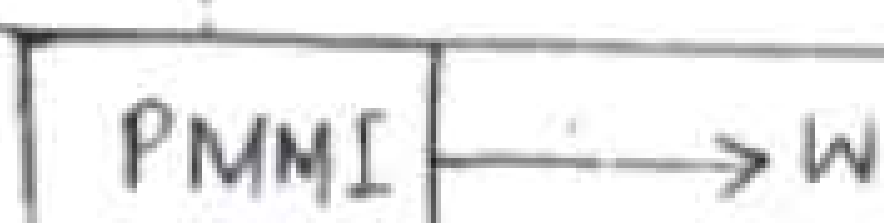
$$\Rightarrow E_2 - E_1 = 0 \quad \Rightarrow E_1 = E_2$$

4> Perpetual Motion Machine of first kind (PMM1) is impossible
continuous (based on cycle)

There can be no machine which develops work continuously without absorbing some other form of energy. If such a device is developed, then it would violate first law of

thermodynamics for a cycle. Hence PMM I is impossible.

$$\downarrow Q=0$$



$$\text{SO. } \Sigma Q \neq \Sigma W$$

ENTHALPY (H) :

In thermodynamics, the term $U+PV$ appears frequently and hence for convenience, this term is known as enthalpy.

$$H = U + PV$$

And enthalpy is an extensive property. It is generally expressed in kJ.

$$h = \frac{H}{m}$$

(specific enthalpy)

h is an intensive property.

→ For an ideal gas -

$$1) \quad U = m C_v T$$

$$\Rightarrow dU = m C_v dT$$

$$2) \quad H = m C_p T$$

$$\Rightarrow dH = m C_p dT$$

$$(20) \quad 3) \quad U = f(T) \rightarrow \text{Joule's law}$$

internal energy is a function of temp.

HEAT TRANSFER IN VARIOUS NON FLOW OR - CLOSED SYSTEM PROCESSES

1) CONSTANT VOLUME PROCESS :

$$\boxed{dQ = dU + PdV}$$

$$\text{for } V = C \Rightarrow dV = 0$$

$$\Rightarrow \boxed{dQ_v = dU} \quad \text{---(1)}$$

but for an ideal gas

$$\Rightarrow dQ_v = mC_v dT$$

Heat transfer in constant volume closed system is equal to change in internal energy.

2) CONSTANT PRESSURE PROCESS :

$$dQ = dU + PdV$$

$$dQ_p = dU + d(PV)$$

$$dQ_p = d(U + PV)$$

$$\boxed{dQ_p = dH} \quad \text{---(2)}$$

Heat transfer in constant pr. closed system process is equal to change in enthalpy.

> To show that $\boxed{C_p - C_v = R}$ for an ideal gas.

$$H = U + PV$$

$$dH = dU + d(PV) \quad \text{---(1)}$$

for an ideal gas :

$$dH = mC_p dT$$

$$dU = mC_v dT$$

$$PV = mRT$$

$$\Rightarrow m c_p dT = m c_v dT + d(mRT)$$

$$\Rightarrow m c_p dT = m c_v dT + m R dT$$

$$\Rightarrow c_p = c_v + R$$

$$\Rightarrow \boxed{c_p - c_v = R} \quad \text{--- (1)} \quad \text{Mayer's eqn}$$

$$\text{and} \quad \gamma = \frac{c_p}{c_v} \quad \text{--- (2)}$$

$$\text{Now,} \quad \gamma(c_p - c_v) = R$$

$$\boxed{c_v = \frac{R}{\gamma - 1}}$$

$$\boxed{c_p = \frac{\gamma R}{\gamma - 1}}$$

for air

$$\begin{cases} c_p = 1.005 \text{ kJ/kg}\cdot\text{K} \\ c_v = 0.718 \text{ kJ/kg}\cdot\text{K} \\ R = 0.287 \text{ kJ/kg}\cdot\text{K} \\ \gamma = 1.4 \end{cases}$$

3) ISOTHERMAL PROCESS :

$$dQ = dU + PdV$$

for an ideal gas

$$U = f(T) \quad (\text{Joule's law})$$

$$\text{as } T = C \Rightarrow U = C$$

$$dU = 0$$

$$\boxed{dQ = dW}$$

For an ideal gas undergoing isothermal process heat transfer is equal to work transfer.

4) ADIABATIC PROCESS :

There is no heat transfer in adiabatic

process ($dQ = 0$)

5) POLYTROPIC PROCESS :

$$dQ = du + dw$$

for ideal gas undergoing polytropic process

$$dQ = mC_v dT + \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$\Rightarrow dQ = mC_v (T_2 - T_1) + \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$\Rightarrow dQ = \frac{mR}{\gamma - 1} (T_2 - T_1) + \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$\Rightarrow dQ = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} + \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$\Rightarrow dQ = (P_1 V_1 - P_2 V_2) \left[\frac{1}{\gamma - 1} - \frac{1}{\gamma - 1} \right]$$

$$\Rightarrow dQ = \frac{P_1 V_1 - P_2 V_2}{(\gamma - 1)(\gamma - 1)} \left[\gamma - 1 - \gamma + 1 \right]$$

$$\Rightarrow dQ = \frac{(P_1 V_1 - P_2 V_2)}{\gamma - 1} \cdot \frac{(\gamma - \gamma)}{(\gamma - 1)}$$

$$\Rightarrow dQ_{\text{poly}} = \frac{\gamma - \gamma}{\gamma - 1} \times dw_{\text{poly}}$$

do from here

POLYTROPIC SPECIFIC HEAT (C_{poly}):

$$dQ_{poly} = \frac{\gamma - \eta}{\gamma - 1} \frac{P_1 V_1 - P_2 V_2}{\eta - 1}$$

$$= \frac{\gamma - \eta}{\gamma - 1} \frac{m R T_1 - m R T_2}{\eta - 1}$$

$$= \frac{\gamma - \eta}{\gamma - 1} \frac{m R (T_1 - T_2)}{\eta - 1}$$

$$= \frac{\gamma - \eta}{\eta - 1} m C_v (-dT)$$

$$= \frac{\eta - \gamma}{\eta - 1} m C_v dT$$

$$= m \left[\frac{\eta - \gamma}{\eta - 1} C_v \right] dT$$

$$dQ_{poly} = m C_{poly} dT$$

$$*** C_{poly} = \frac{\eta - \gamma}{\eta - 1} C_v$$

$$1 < \eta < \gamma$$

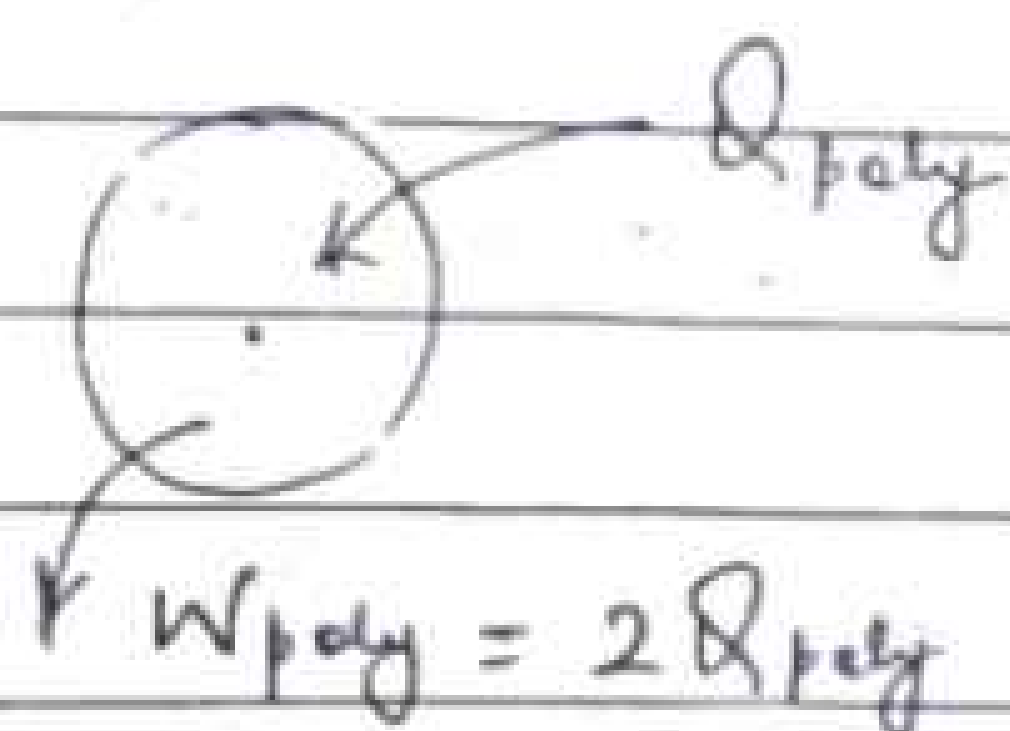
$$dQ_{poly} = \frac{\gamma - \eta}{\gamma - 1} \times W_{poly}$$

$$\begin{cases} \gamma = 1.4 \\ \eta = 1.2 \end{cases}$$

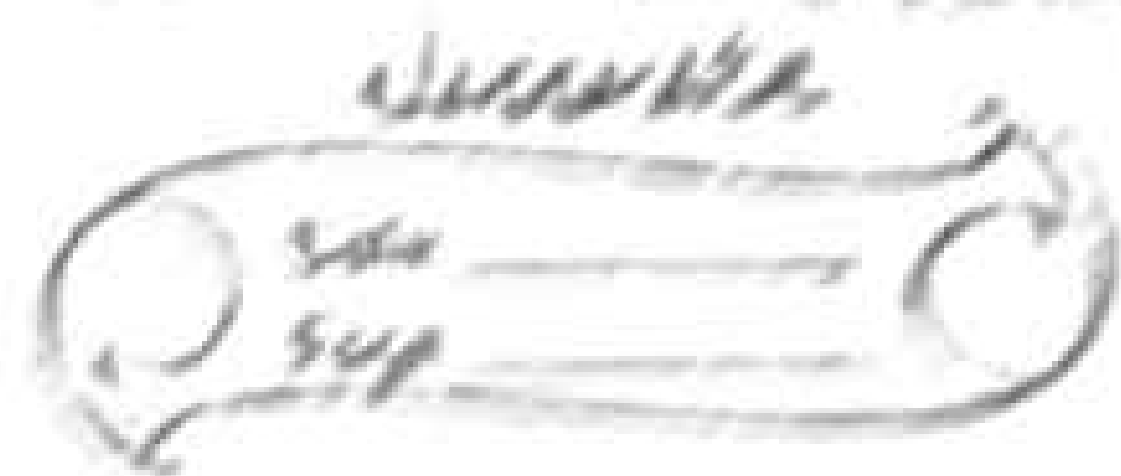
$$\Rightarrow dQ_{poly} = \frac{.2}{.4} \times W_{poly}$$

$$\Rightarrow dQ_{poly} = \frac{1}{2} W_{poly}$$

$$\Rightarrow W_{poly} = 2 dQ_{poly}$$



system is loosing its internal energy.
 $U \downarrow$ ($T \downarrow$)



for $1 < n < \gamma$. C_{poly} is negative i.e. though, heat is supplied, there is a reduction in temperature, this is because in such a polytropic process, heat transfer is more than work transfer and this extra work transfer comes from internal energy of the system. and hence there is a decrease in internal energy of the system but we know that internal energy of an ideal gas is a function of temp. and hence there is a decrease in temperature.

→ To show that $[Pv^\gamma = C]$ for an adiabatic process:

$$\bar{d}Q = du + Pdv$$

Ideal gas + Adiabatic Process + Reversible

$$\downarrow \qquad \qquad \downarrow$$

$$du = mC_v dT \qquad dv = 0$$

$$\Rightarrow du = -Pdv$$

$$\Rightarrow mC_v dT = -Pdv \quad \text{--- (1)}$$

$$\text{and } H = U + Pv$$

$$\Rightarrow dH = du + Pdv + vdp$$

$$\Rightarrow dH = dQ + vdp$$

Ideal gas + Adiabatic

$$\Rightarrow mC_p dT = vdp \quad \text{--- (2)}$$

$$\frac{2}{\gamma - 1} \ln(T) \Rightarrow \frac{mC_p dT}{mC_v dT} = \frac{vdp}{-Pdv}$$

$$\Rightarrow \gamma = \frac{C_p}{C_v} = \frac{dP}{P} \left(\frac{-v}{dv} \right)$$

$$\Rightarrow -\gamma \frac{dv}{v} = \frac{dP}{P}$$

$$\Rightarrow \ln \frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

$$\Rightarrow \ln P + \gamma \ln V = 0$$

$$\Rightarrow \ln(PV^\gamma) = \ln(C)$$

$$\Rightarrow \boxed{PV^\gamma = C} \quad \text{--- (A)}$$

*** Eqn (A) is valid for an ideal gas undergoing reversible adiabatic process (both open & closed)

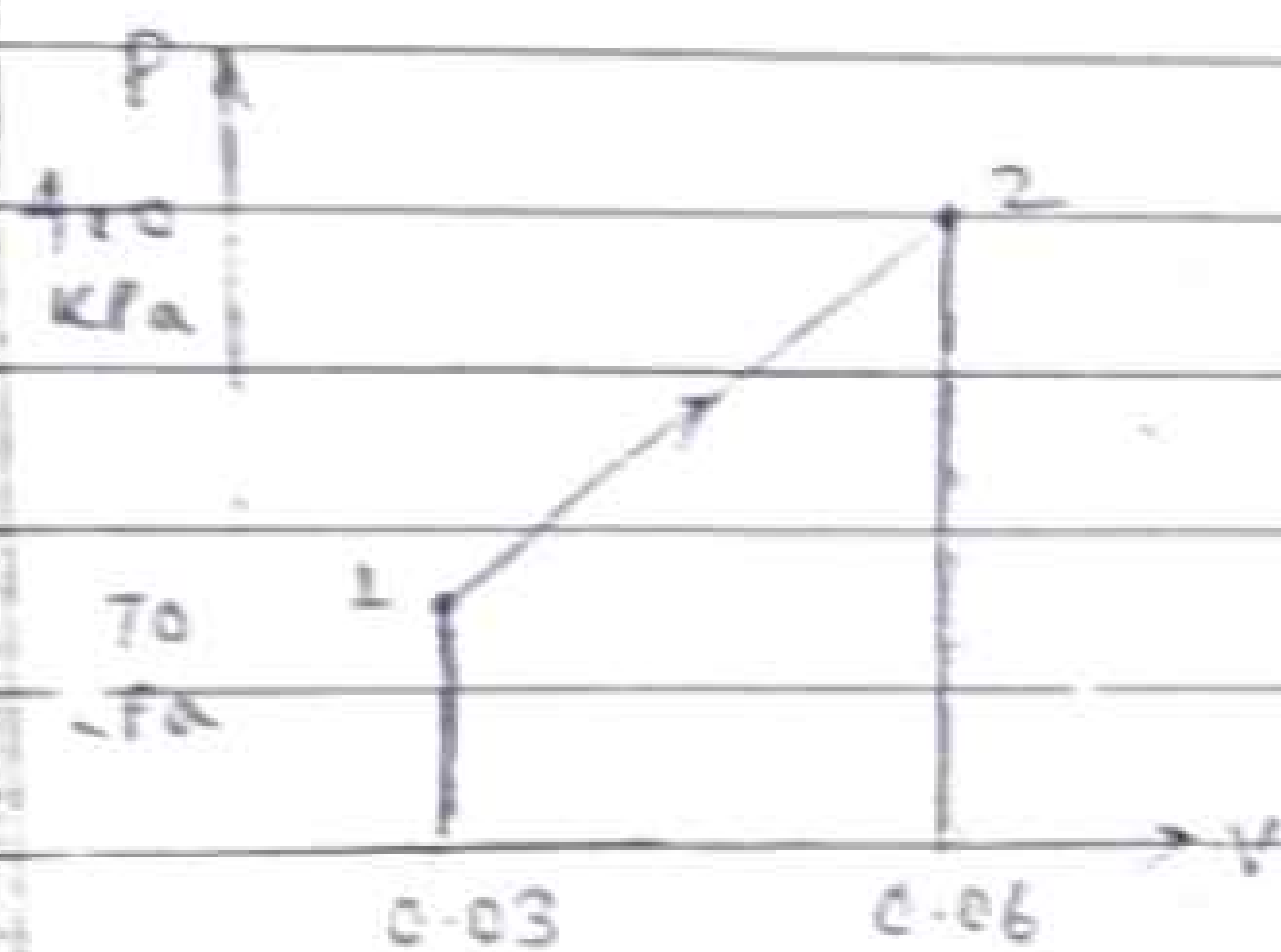
Ex 2.5 (Main)

Q. 2. A fluid is contained in a cylinder by a spring loaded frictionless piston so that p in the fluid is a linear funⁿ of volume i.e. $P = a + bV$ where a, b are constants. The internal energy of fluid is given by $U = 34 + 3.15 PV$ where U is in kJ, P in kPa, V in m^3 . If the fluid changes from initial state of

$P_1 = 170 \text{ kPa}, V_1 = 0.03 \text{ m}^3$ to a final state of

$P_2 = 400 \text{ kPa}, V_2 = 0.06 \text{ m}^3$.

Find the magnitude and direction of heat and work transfer.



Work done = Area under P-V curve

$$W = \frac{1}{2} (400 + 170) \times (0.06 - 0.03)$$

$$W = 8.55 \text{ kJ Ans.}$$

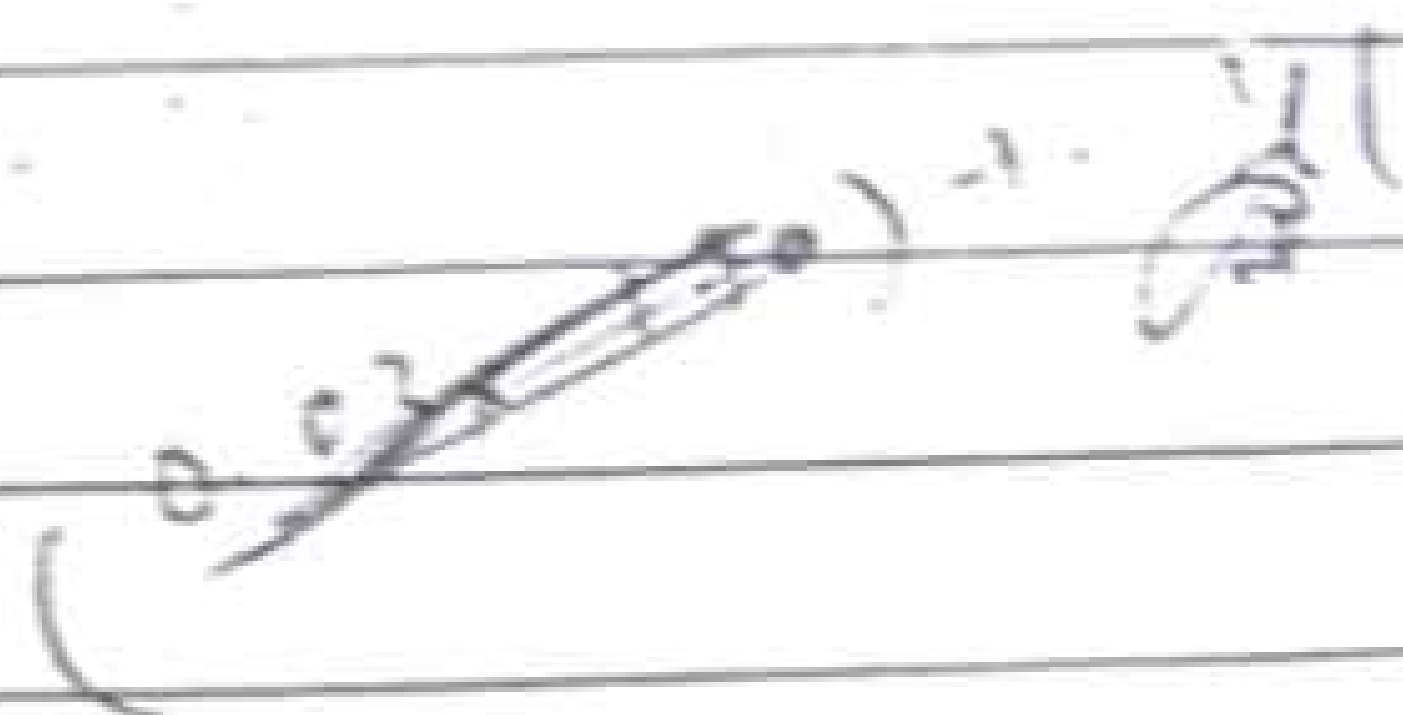
$$U_1 = 34 + 3.15 P_1 V_1$$

$$U_2 = 34 + 3.15 P_2 V_2$$

$$\Delta U = U_2 - U_1 = 3.15 (P_2 V_2 - P_1 V_1)$$

$$\Rightarrow \Delta U = 3.15 (400 \times 0.06 - 170 \times 0.03)$$

$$\Rightarrow \Delta U = 59.5 \text{ kJ}$$



$$\therefore dQ = dU + dW$$

$$\Rightarrow dQ = 54.5 + 8.55$$

$$\Rightarrow dQ = \underline{\underline{63.05 \text{ kJ}}}$$

So work is done by system.
Heat is supplied to the system.

Ex-2) An insulated rigid pr. vessel is divided into two portions by means of a partition. First part of vessel is occupied by an ideal gas at a pr. P_1 , volume V_1 and temp. T_1 . The other part is occupied by same ideal gas but at a pr. P_2 , volume V_2 and temp. T_2 . Suddenly the partition is removed and the portion mixed with each other. Show that final pr. P_3 and final temp. T_3 are given by.

$$P_3 = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2} \quad ; \quad T_3 = \frac{P_1 V_1 + P_2 V_2}{\left(\frac{P_1 V_1}{T_1}\right) + \left(\frac{P_2 V_2}{T_2}\right)}$$

P_1	P_2
V_1	V_2
T_1	T_2
m_1	m_2

Mass conservation:

$$M_3 = M_1 + M_2 \quad \quad M = \frac{PV}{RT}$$

$$\Rightarrow \frac{P_3 V_3}{RT_3} = \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}$$

P_3
T_3
$m_3 = m_1 + m_2$
$V_3 = V_1 + V_2$

$$\Rightarrow T_3 = \frac{P_3 V_3}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}} \quad \text{--- (1)}$$

Now, Energy conservation: $dQ = dU + dW$

$$\text{As } dQ = 0$$

$$dW = 0$$

$$\Rightarrow dU = 0$$

$$\Rightarrow U_f = U_i$$

$$\Rightarrow U_3 = U_1 + U_2$$

(U is an extensive property)

$$\Rightarrow m_3 C_v T_3 = m_1 C_v T_1 + m_2 C_v T_2$$

$$\Rightarrow m_3 \frac{R}{\gamma-1} T_3 = m_1 \frac{R}{\gamma-1} T_1 + m_2 \frac{R}{\gamma-1} T_2$$

$$\Rightarrow m_3 R T_3 = m_1 R T_1 + m_2 R T_2$$

$$\Rightarrow P_3 V_3 = P_1 V_1 + P_2 V_2$$

$$\Rightarrow P_3 = \frac{P_1 V_1 + P_2 V_2}{V_3}$$

$$\Rightarrow \boxed{P_3 = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}} \quad \text{--- (1)}$$

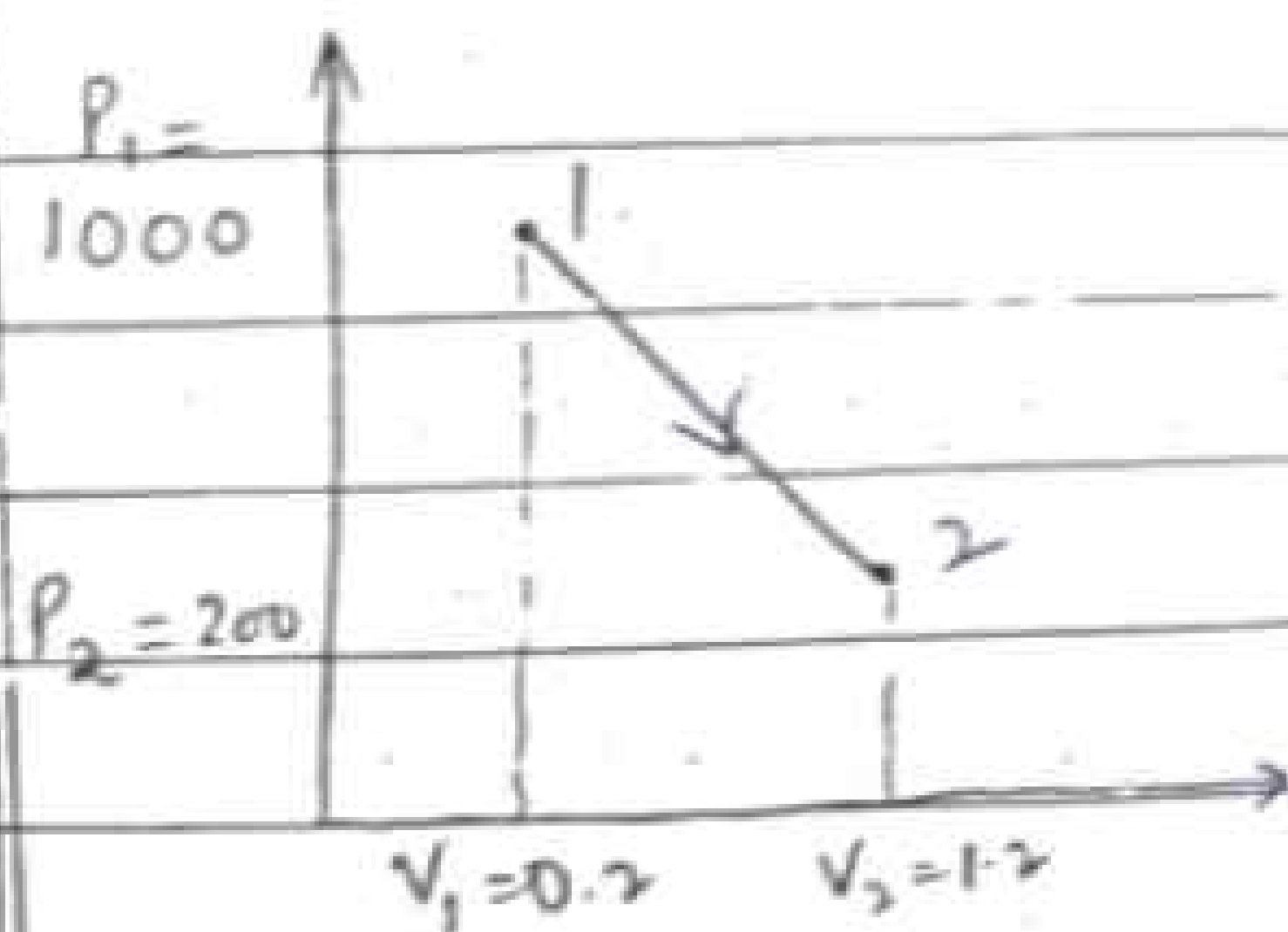
$$\hookrightarrow \boxed{T_3 = \frac{P_2 V_2 + P_1 V_1}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}} \quad \text{--- (2)}$$

= 0.5

: 3

A gas of mass 1.5 kg undergoes a reversible expansion process which follows the relationship $P = a + bV$ where a & b are constants. Initial and final pr. are 1000 kPa & 200 kPa respectively. and corresponding volumes are $V_1 = 0.2 \text{ m}^3$ and $V_2 = 1.2 \text{ m}^3$. Specific internal energy is given by $U = (1.5PV - 85) \text{ kJ/kg}$ where P is in kPa and V in m^3/kg . Calculate the heat transfer and max internal energy of gas.

Ans:



$$W = \frac{1}{2} (1000 + 200) (1.2 - 0.2)$$

$$W = 600 \text{ kJ}$$

$$U = 1.5PV - 85$$

$$U = mU$$

$$\Rightarrow U = m [1.5PV - 85]$$

$$\Rightarrow U = 1.5 \text{ PV} - 1.5 \times 85$$

$$U_1 = 1.5 P_1 V_1 = 127.5$$

$$U_2 = 1.5 P_2 V_2 = 127.5$$

$$30 \quad U_2 - U_1 = \Delta U = 1.5 (P_2 V_2 - P_1 V_1)$$

$$dU = 1.5 \times 40$$

$$dV = 60$$

$$dQ = du + dw$$

$$\Rightarrow dQ = 60 + 600$$

$$\Rightarrow dQ = 660 \text{ KJ} \quad \text{Ans. (To the system)}$$

Max^m internal energy :

$$U = 1.5PV - 127.5 \quad (\text{put } P = a + bv)$$

$$\Rightarrow U = 1.5 (a+bv)v - 127.5$$

$$\Rightarrow U = 1.5 (aV + bV^2) - 127.5$$

for $\max^m u$, $\frac{du}{dv} = 0 \Rightarrow \frac{du}{dv} = 1.5(a + 2\check{b}) = 0$

$$\Rightarrow V = -\frac{a}{2b}$$

Now, $P = a + bV \Rightarrow 000 = a + b(0.2) \quad \text{--- (A)}$

$$200 = a + 1.2b \quad \text{--- (8)}$$

Solving for (a) & b

$$\Rightarrow a = 1160$$

$$\Rightarrow b = -800$$

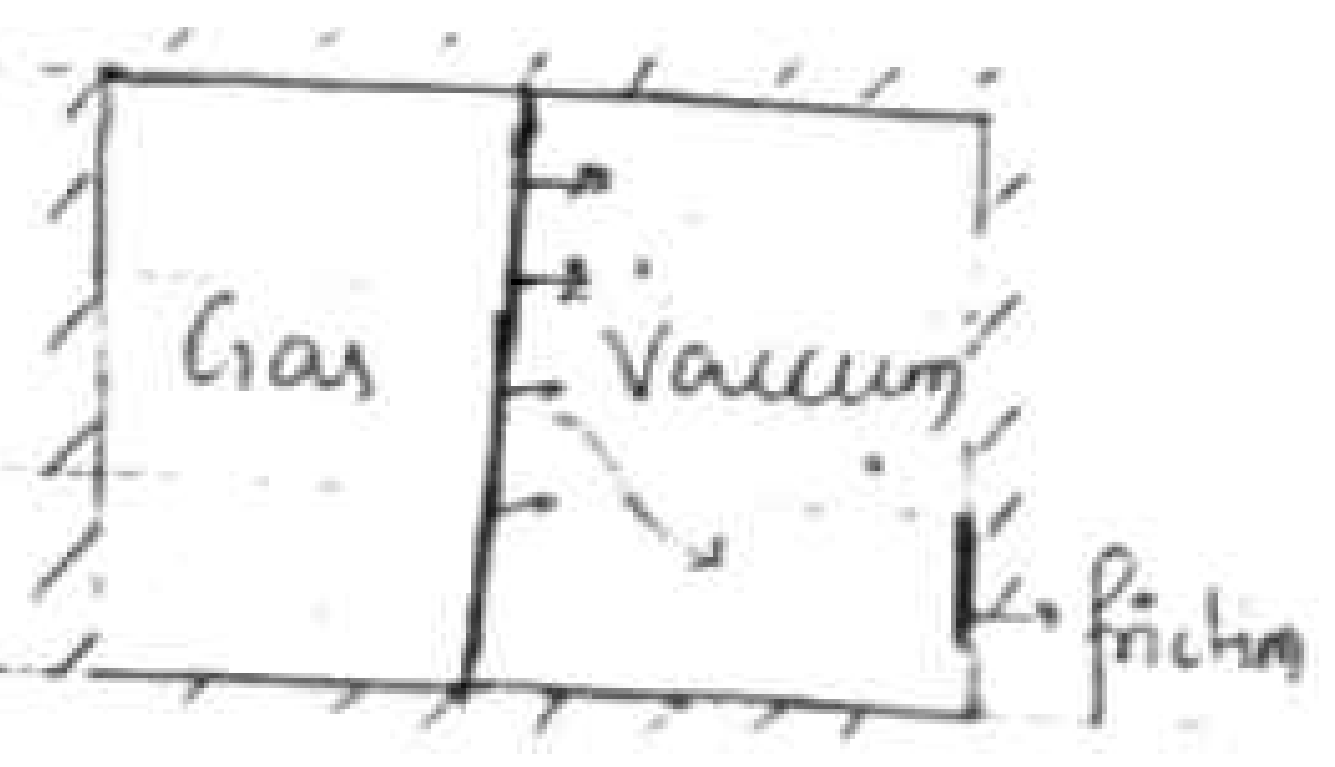
$$\therefore V = \frac{-1160}{2 \times 800} = 0.725 \text{ m}^3$$

$$\therefore U_{\max} = 1.5 \left[1160 \times 0.725 + (-800) (0.725)^2 \right] - 127.5$$

$$\Rightarrow U_{\max} = 503.25 \text{ kJ} \quad \text{Ans.}$$

→ FREE EXPANSION :-

The expansion of a gas against vacuum is known as free expansion or unrestrained expansion.



$$W_{\text{surround}} = 0$$

$$W_{\text{system}} = 0$$

(UNRESTRAINED EXPANSION)

** Free expansion work is zero.

Now using I Law :

$$\begin{aligned} dQ &= dU + dW \\ \Rightarrow 0 &= dU + 0 \\ \Rightarrow dU &= 0 \\ \Rightarrow U_2 - U_1 &= 0 \\ \Rightarrow U_2 &= U_1 \end{aligned}$$

Let ideal gas is going under free expansion -

$$U = f(T)$$

$$U_i = U_f$$

$$\Rightarrow T_i = T_f$$

$$\therefore H = U + PV$$

$$\Rightarrow H = f(T) + mRT$$

$$\Rightarrow H = \phi(T)$$

$$\text{As } T_i = T_f$$

$$\Rightarrow H_i = H_f$$

so for ideal gas (free expansion)

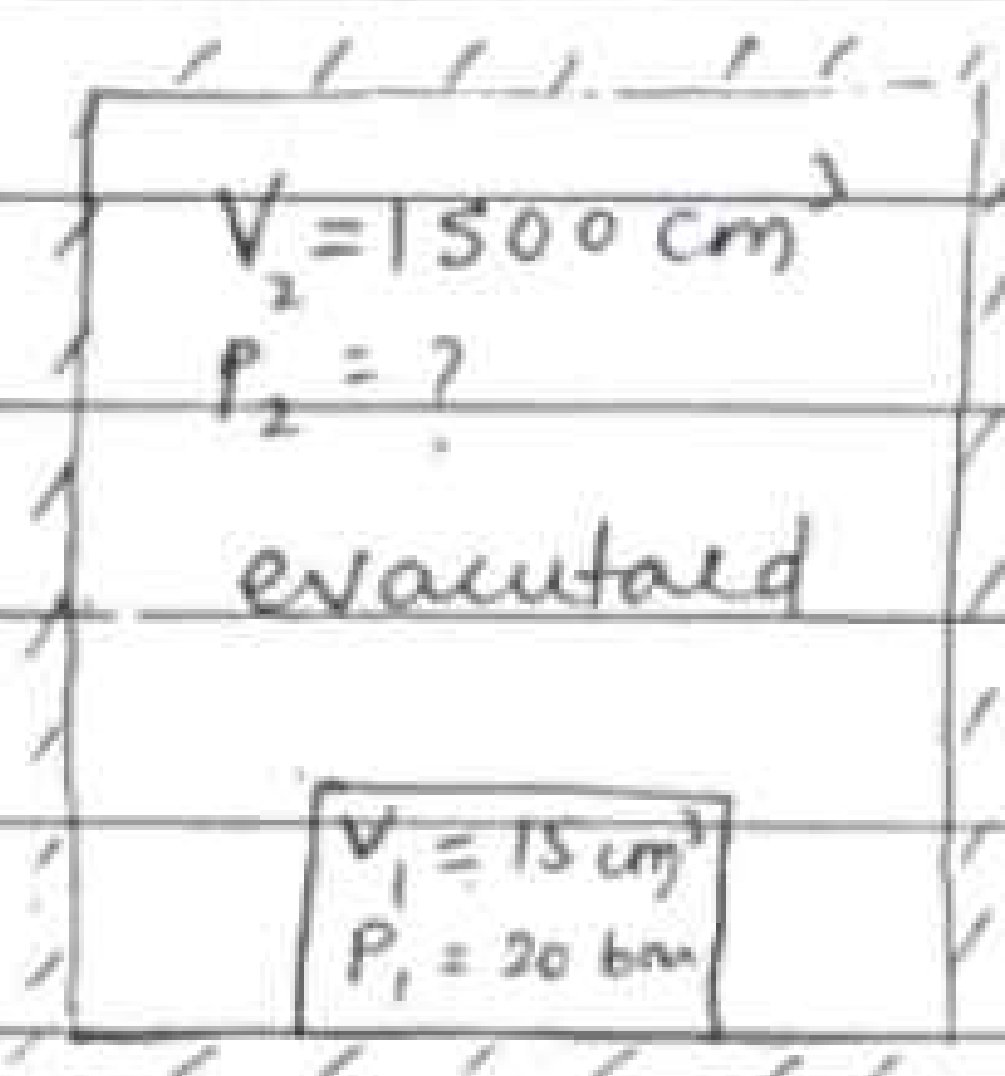
$$U_i = U_f$$

$$T_i = T_f$$

$$H_i = H_f$$

TE: Though during free expansion $Pdv \neq 0$, but work is zero because free expansion is highly irreversible whereas Pdv is valid only for a reversible process.

An ideal gas at 20 bar and 40°C is contained in a small cylinder having a volume of 15 cm^3 . This cylinder is placed inside a large container having a volume of 1500 cm^3 . The large container is perfectly evacuated and insulated. By an appropriate mean the gas is allowed to discharge and fill the large container. Find the final pr. after the entire assembly reaches equilibrium.



Mass conservation;

$$m_1 = m_2$$

$$\Rightarrow \frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2} \quad (T_1 = T_2)$$

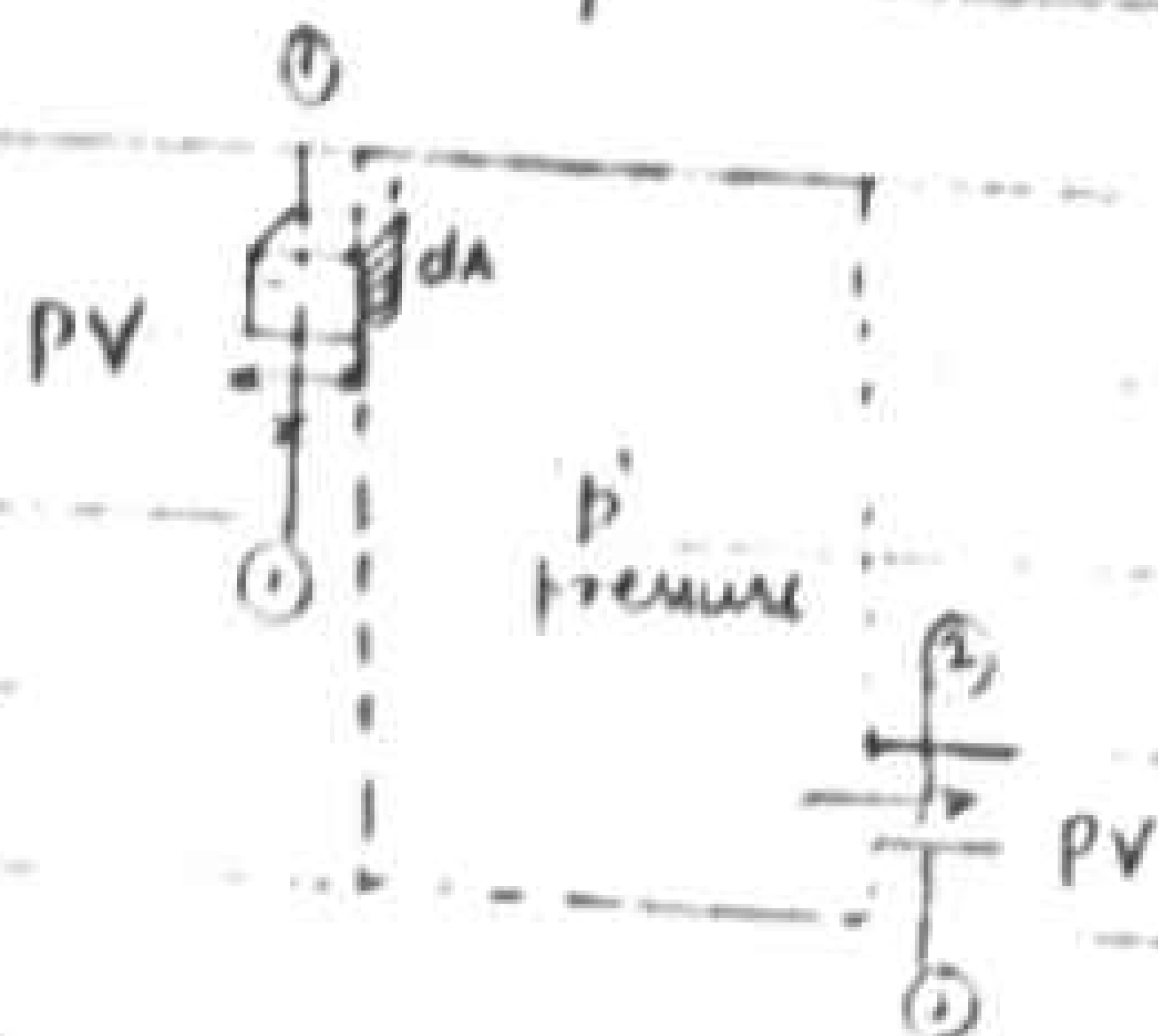
$$\Rightarrow P_1 V_1 = P_2 V_2$$

$$\Rightarrow 20 \times 15 = 1500 \times P_2$$

$$\Rightarrow P_2 = \underline{\underline{0.2 \text{ bar}}}$$

FLOW WORK :

The work transfer involved in causing the fluid element either to enter or leave the control volume is known as flow work.



$$dv = A dx \quad (1)$$

$$\text{and } v = \frac{dv}{dm} \quad (2)$$

NOW, work = force $\times$ distance

$$W = F dx$$

$$v = F dv$$

$$\text{work/mass} = P \frac{dv}{dm}$$

$$\Rightarrow W_{\text{mass}} = Pv$$

$$\Rightarrow W = Pv \times m$$

$$\Rightarrow \boxed{W = Pv} \quad (v = mv)$$

If the entry occurs at 1, then the entry flow work is $-P_1 V_1$. (work done on system)

If the exit occurs at 2, then the exit flow work is $+P_2 V_2$.

Steady flow Energy Equation (SFEE) :-

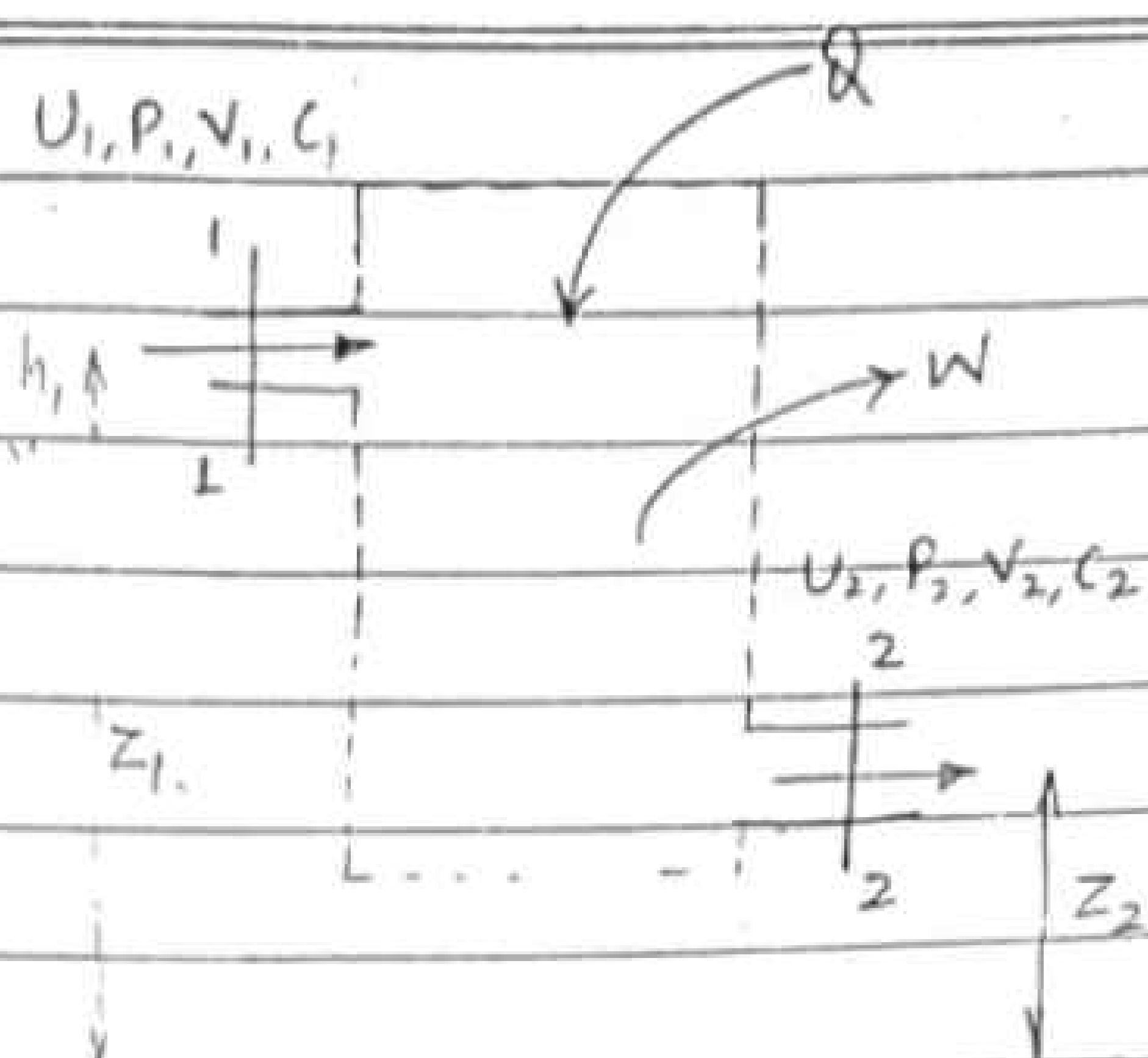
A flow is said to be steady flow if properties do not vary with respect to time at any given section. For steady flow mass entering the control volume is equal to mass leaving the control volume and energy entering into cv is equal to energy leaving cv. Therefore, for steady flow, there is no accumulation of mass and energy in control volume.

Open system $\rightarrow$ controlled volume
 closed system $\rightarrow$ controlled mass

classmate

Date _____

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$$E_{\text{entering}} = E_{\text{leaving}}$$

$$\Rightarrow \frac{1}{2} m c_1^2 + m g z_1 + U_1 + Q$$

$$= \frac{1}{2} m c_2^2 + m g z_2 + U_2 + W$$

here $W = W_{\text{shaft}} + W_{\text{cv}} + W_{\text{exit flow}}$

$$\Rightarrow W = -P_1 V_1 + W_{\text{cv}} + P_2 V_2$$

so, $\frac{1}{2} m c_1^2 + m g z_1 + U_1 + Q = \frac{1}{2} m c_2^2 + m g z_2 + U_2 - P_1 V_1 + W_{\text{cv}} + P_2 V_2$

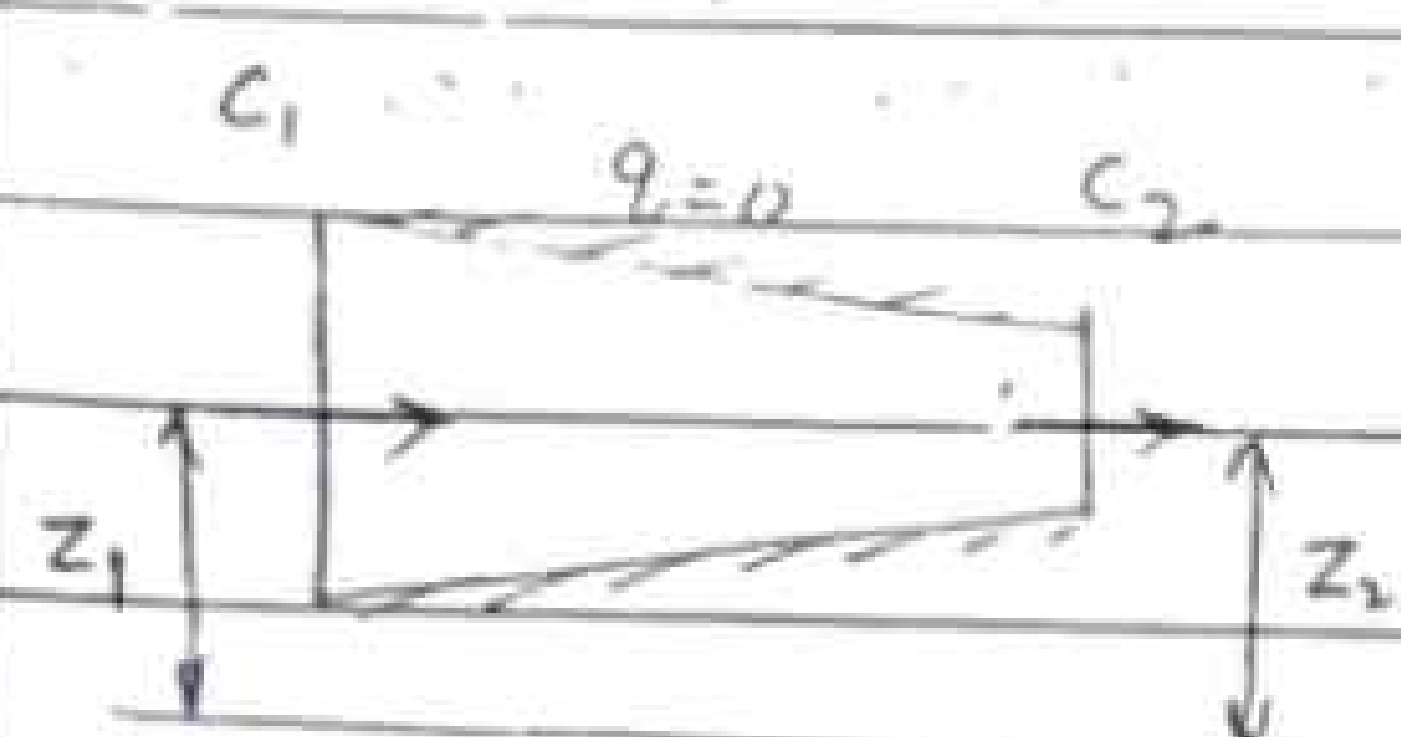
$$\Rightarrow H_1 + \frac{1}{2} m c_1^2 + m g z_1 + Q = H_2 + \frac{1}{2} m c_2^2 + m g z_2 + W_{\text{cv}}$$

$$\Rightarrow \left[h_1 + \frac{1}{2} c_1^2 + g z_1 + q = h_2 + \frac{1}{2} c_2^2 + g z_2 + w_{\text{cv}} \right] \text{ SFEE}$$

This eqn gives only control volume work. This is known as I Law of thermodynamics eqn for open system and it is valid only under steady flow condition. It can be used for reversible as well as irreversible process.

SPECIAL CASES :

1) NOZZLE :- Nozzle is a device which is used for increasing velocity at the expense of pressure energy.



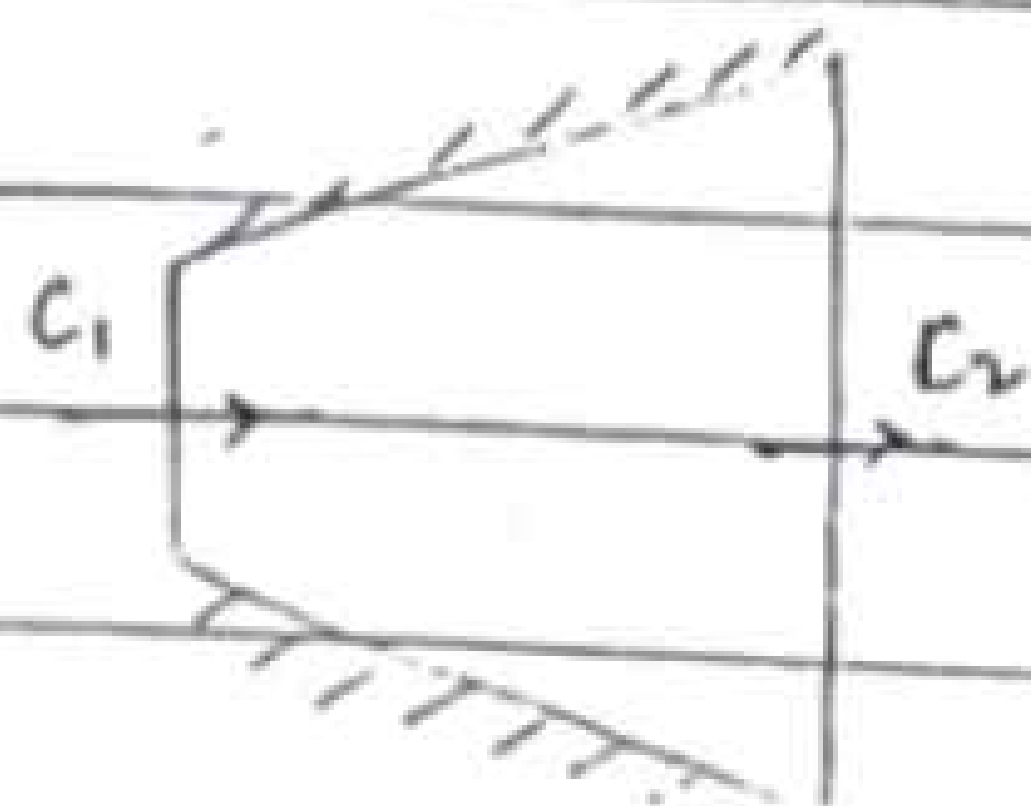
$$h_1 + \frac{C_1^2}{2} + z_1 g + q = h_2 + \frac{C_2^2}{2} + z_2 g + w_{\text{cv}}$$

$$\Rightarrow h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad C_2 \gg C_1$$

$$\Rightarrow \boxed{h_1 = h_2 + \frac{C_2^2}{2}}$$

2) TURBINE :-

Assumptions:-



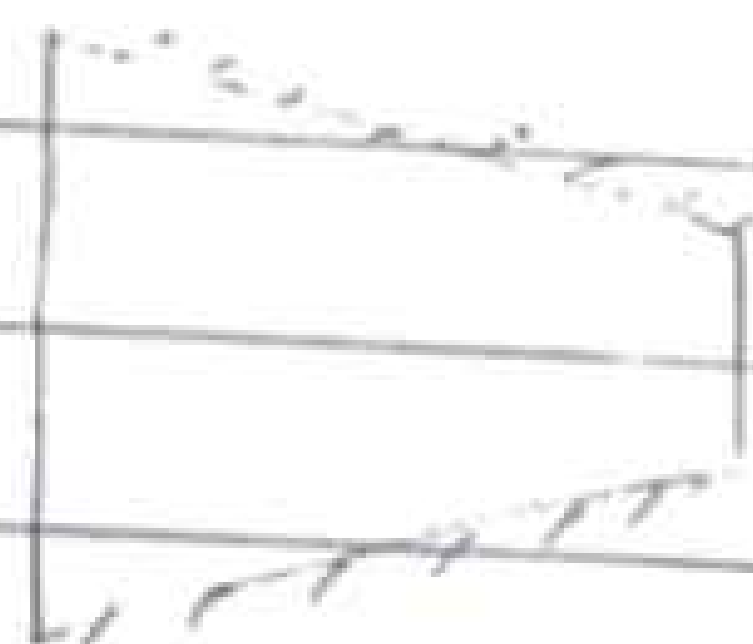
- i) steady flow
- ii) perfectly insulated ($q=0$)
- iii) P.E. changes are negligible
- iv) kinetic energy changes are negligible

$$h_1 + \frac{c_1^2}{2} + z_1 g + q = h_2 + \frac{c_2^2}{2} + z_2 g + w$$

$$\Rightarrow \boxed{w_{\text{turbine}} = h_1 - h_2}$$

3) COMPRESSOR :-

for compressor:



$$W_{\text{comp}} = -W_{\text{turbine}}$$

$$W_{\text{comp}} = -(h_1 - h_2)$$

$$\boxed{W_{\text{comp}} = h_2 - h_1}$$

Assumptions are same as above.

4) THROTTLING VALVE :-

Examples of throttling are -

- i) flow through a partially opened valve
- ii) flow through a very small opening (orifice)
- iii) flow through porous membrane.

Characteristics of throttling -

- i) No heat transfer ($dQ=0$)
- ii) No work transfer ($dW=0$)
- iii) It is irreversible process. (friction due to restriction)
- iv) Enthalpy remains constant.



$$h_1 + \frac{c_1^2}{2} + \rho_1 g z_1 + q = h_2 + \frac{c_2^2}{2} + \rho_2 g z_2 + w$$

$$c_2 = c_1$$

$$h_1 = h_2$$

isenthalpic process

UNSTEADY FLOW :-

In unsteady flow there is accumulation of mass and energy in the control volume.

Let m_i and m_e be the masses entering and leaving the control volume.

Let m_1 & m_2 be the masses in the control volume initially and finally.

i $\rightarrow$ inlete $\rightarrow$ exit

Applying conservation of mass =

$$\left(\frac{dm}{dt} \right)_{cv} = \frac{dm_i}{dt} - \frac{dm_e}{dt}$$

$$\left(\frac{dm}{dt} \right)_{cv} = m_i - m_e \quad \text{--- (1)}$$

Applying conservation of energy =

$$h_1 + \frac{c_1^2}{2} + \rho_1 g z_1 + q = h_2 + \frac{c_2^2}{2} + \rho_2 g z_2 + w$$

$$E_i = h_i + \frac{c_i^2}{2} + \rho_i g z_i + q$$

$$E_i = m_i h_i + m_i \frac{c_i^2}{2} + m_i \rho_i g z_i + Q$$

Similarly,

$$e_e = h_e + c_e^2/2 + z_e g + W_{cv}$$

$$\Rightarrow E_e = m_e h_e + m_e c_e^2/2 + m_e z_e g + W_{cv}$$

$$\text{sc.} \left(\frac{dE}{dt} \right)_{cv} = \frac{d}{dt} (m_i h_i + m_i c_i^2/2 + m_i z_i g + Q) - \frac{d}{dt} (m_e h_e + m_e c_e^2/2 + m_e z_e g + W_{cv})$$

Assuming,

✓ KE and PE changes are negligible.

$\Rightarrow$

$$E = KE + PE + U$$

$$dE = d(KE) + d(PE) + dU \Rightarrow dE = dU$$

sc.

$$\left(\frac{dU}{dt} \right)_{cv} = \frac{d}{dt} (m_i h_i + Q) - \frac{d}{dt} (m_e h_e + W_{cv})$$

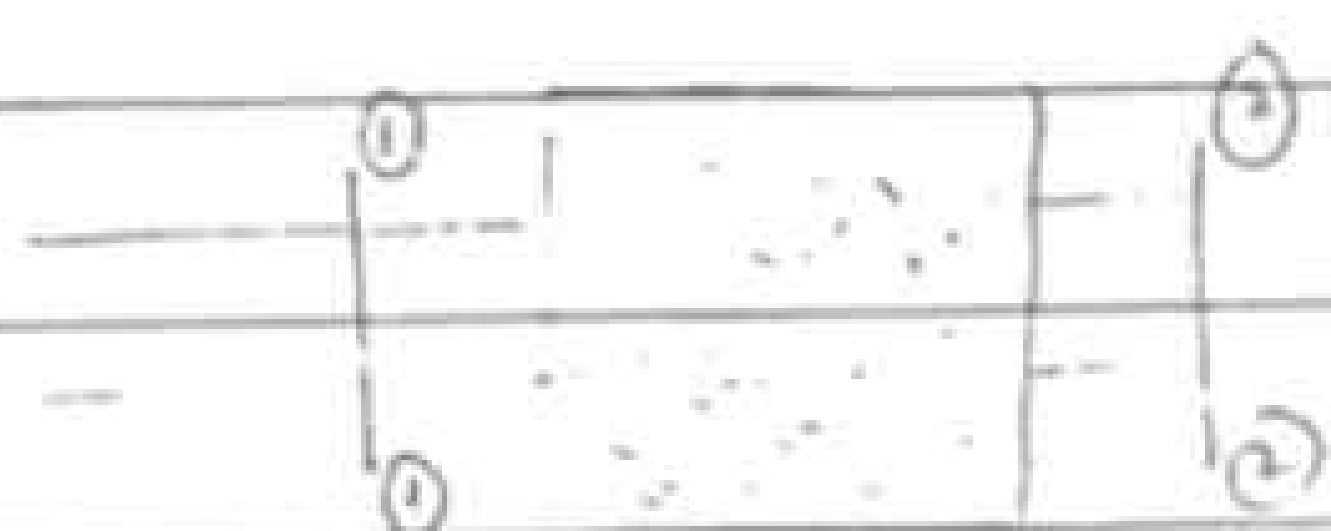
✓ Assume, h_e & h_i do not change w.r.t. time

$$\Rightarrow \left(\frac{dU}{dt} \right)_{cv} = m_i h_i \frac{d(m_i)}{dt} + \frac{dQ}{dt} - \left\{ h_e \frac{d(m_e)}{dt} + \frac{dW_{cv}}{dt} \right\}$$

$$\Rightarrow \left(\frac{dU}{dt} \right)_{cv} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv} \quad \text{--- (2)}$$

Q.:

An insulated storage tank that is initially evacuated is connected to a supply pipe line carrying a fluid at a specific internal energy u_i and specific enthalpy h_i . The valve is opened and fluid flows into the tank from the supply pipe line and reaches the pr. same as that of supply line. Show that the final specific internal energy of the fluid in the tank is equal to h_i .



$$h_1 + \frac{c_1^2}{2} + z_1 g + q = h_2 + \frac{c_2^2}{2} + z_2 g + w_{cv} \quad c_2 = c_1$$

$$h_1 = h_2$$

isoenthalpic process

UNSTEADY FLOW :-

In unsteady flow, there is accumulation of mass and energy in the control volume.

Let m_i and m_e be the masses entering and leaving the control volume.

Let m_1 & m_2 be the masses in the control volume initially and finally.

i → inlet
e → exit

Applying conservation of mass -

$$\left(\frac{dm}{dt} \right)_{cv} = \frac{dm_i}{dt} - \frac{dm_e}{dt}$$

$$\left[\left(\frac{dm}{dt} \right)_{cv} = m_i - m_e \right] \quad \text{--- (1)}$$

Applying conservation of energy -

$$\underbrace{h_1 + \frac{c_1^2}{2} + z_1 g + q}_{E_i} = \underbrace{h_2 + \frac{c_2^2}{2} + z_2 g + w}_{E_e}$$

$$E_i = h_i + \frac{c_i^2}{2} + z_i g + q$$

$$E_i = m_i h_i + m_i \frac{c_i^2}{2} + m_i z_i g + Q$$

Similarly,

$$e_e = h_e + c_e^2/2 + z_e g + W_{cv}$$

$$\Rightarrow E_e = m_e h_e + m_e c_e^2/2 + m_e z_e g + W_{cv}$$

$$\text{SC: } \left(\frac{dE}{dt} \right)_{cv} = \frac{d}{dt} \left(m_i h_i + m_i \frac{c_i^2}{2} + m_i z_i g + Q \right) - \frac{d}{dt} \left(m_e h_e + m_e \frac{c_e^2}{2} + m_e z_e g + W_{cv} \right)$$

Assuming,

✓ KE and PE changes are negligible

$$\Rightarrow E = KE + PE + U$$

$$dE = d(KE) + d(PE) + dU \Rightarrow dE = dU$$

SC:

$$\left(\frac{dU}{dt} \right)_{cv} = \frac{d}{dt} (m_i h_i + Q) - \frac{d}{dt} (m_e h_e + W_{cv})$$

✓ Assume h_e & h_i do not change w.r.t. time

$$\Rightarrow \left(\frac{dU}{dt} \right)_{cv} = m_i h_i \frac{d}{dt} + \frac{dQ}{dt} - \left\{ h_e \frac{d}{dt} (m_e) + \frac{dW_{cv}}{dt} \right\}$$

$$\Rightarrow \left(\frac{dU}{dt} \right)_{cv} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv} \quad \text{--- (2)}$$

P:

An insulated storage tank that is initially evacuated is connected to a supply pipe line carrying a fluid at a specific internal energy u_i and specific enthalpy h_i , the valve is opened and fluid flows into the tank from the supply pipe line and reaches the pr. same as that of supply line. Show that the final specific internal energy of the fluid in the tank is equal to h_i .

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e$$

as no mass is leaving the cv
 $\dot{m}_e = 0$

$$\Rightarrow \left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i \quad \text{---(1)}$$

$$\text{And } \left(\frac{du}{dt}\right)_{cv} = \dot{m}_i u_i + \dot{Q} - \dot{m}_e u_e - \dot{W}_{cv}$$

$$\Rightarrow \left(\frac{du}{dt}\right)_{cv} = \dot{m}_i h_i \quad \text{---(2)}$$

$$\Rightarrow \left(\frac{du}{dt}\right)_{cv} = \left(\frac{dm}{dt}\right)_{cv} \cdot h_i$$

$$\Rightarrow (du)_{cv} = (dm)_{cv} \cdot h_i$$

$$\Rightarrow u_2 - u_1 = (m_2 - m_1) \cdot h_i$$

$$\Rightarrow \boxed{m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_i}$$

as $m_1 = 0$

$$\Rightarrow m_2 u_2 = m_2 h_i$$

$$\Rightarrow \boxed{u_2 = h_i}$$

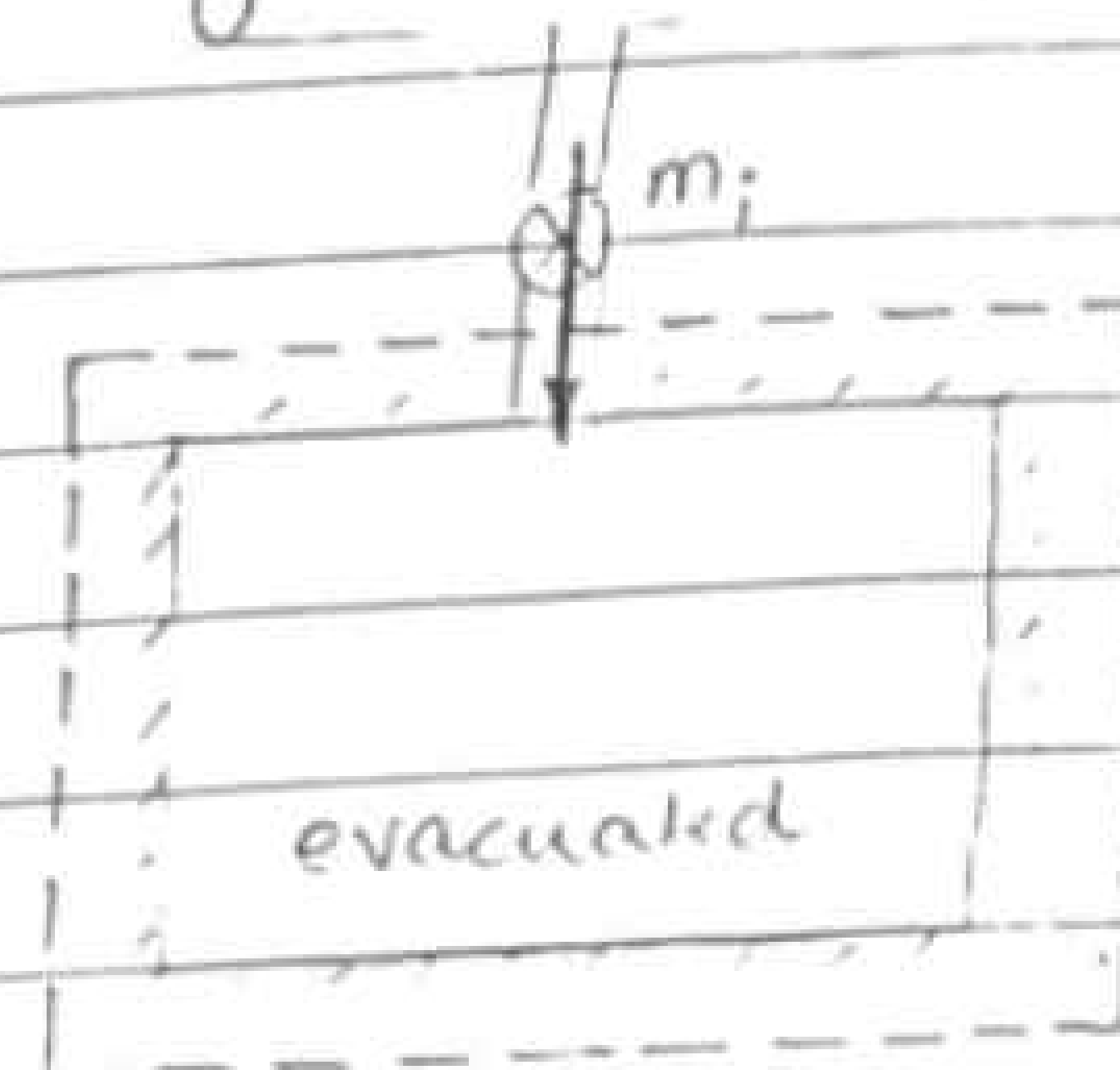
(**) for ideal gas ; $u_2 = h_i$

$$\Rightarrow C_v T_2 = C_p T_i$$

$$\Rightarrow T_2 = \frac{C_p}{C_v} T_i$$

$$\Rightarrow T_2 = \gamma T_i$$

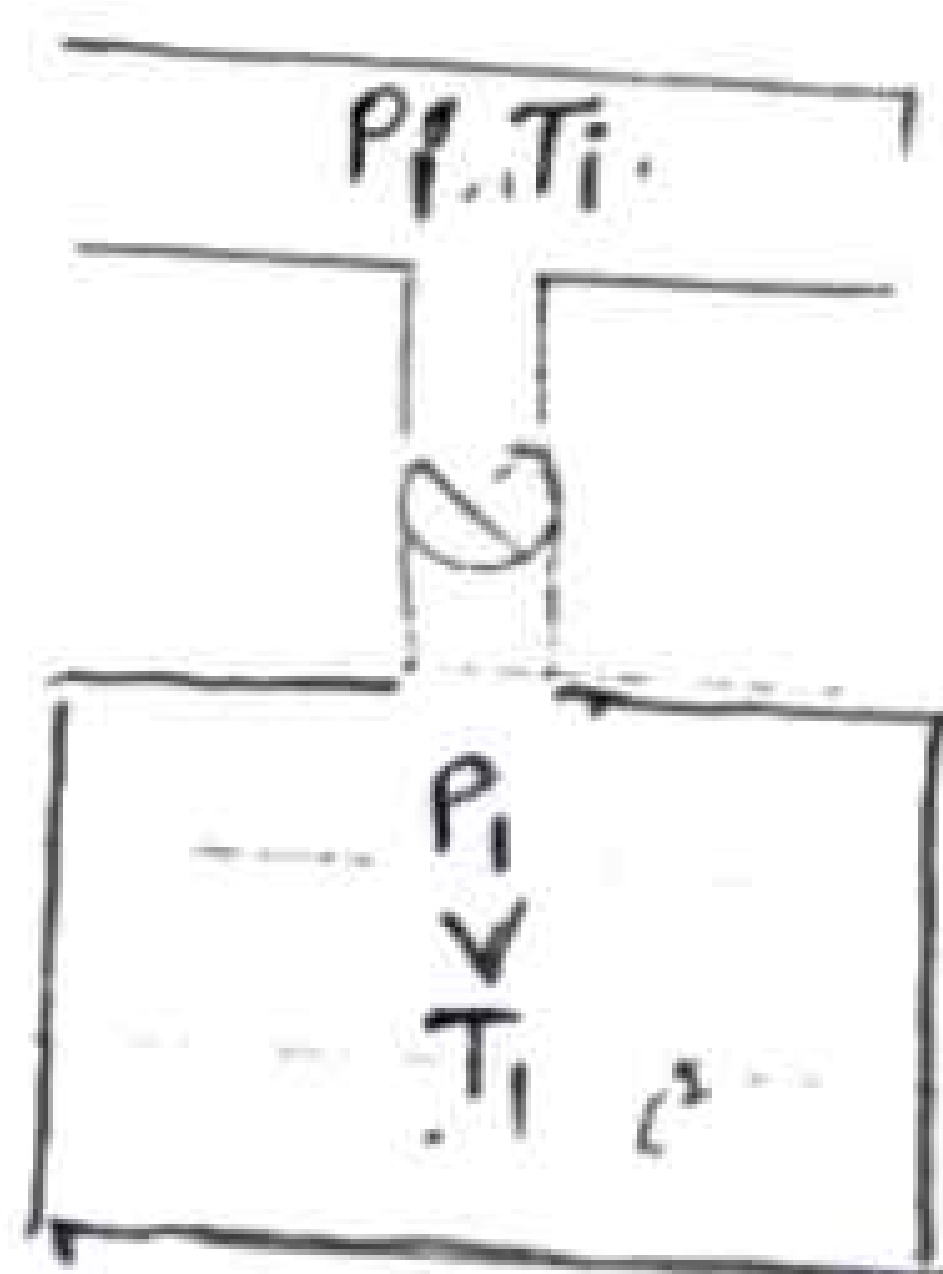
$\rightarrow u_i, h_i$



P:02 A pressure cylinder of volume V contains air at a pressure P_1 and temperature T_1 . It is to be filled from a supply air pipe line maintained at const. pr P_i & temp. T_i . Show that the final temp of air in the cylinder after it has been charged to the pressure same as the supply line pr. is given by $T_2 = \frac{\gamma T_i}{1 + \frac{P_i}{P_1} \left(\frac{\gamma T_i}{T_1} - 1 \right)}$

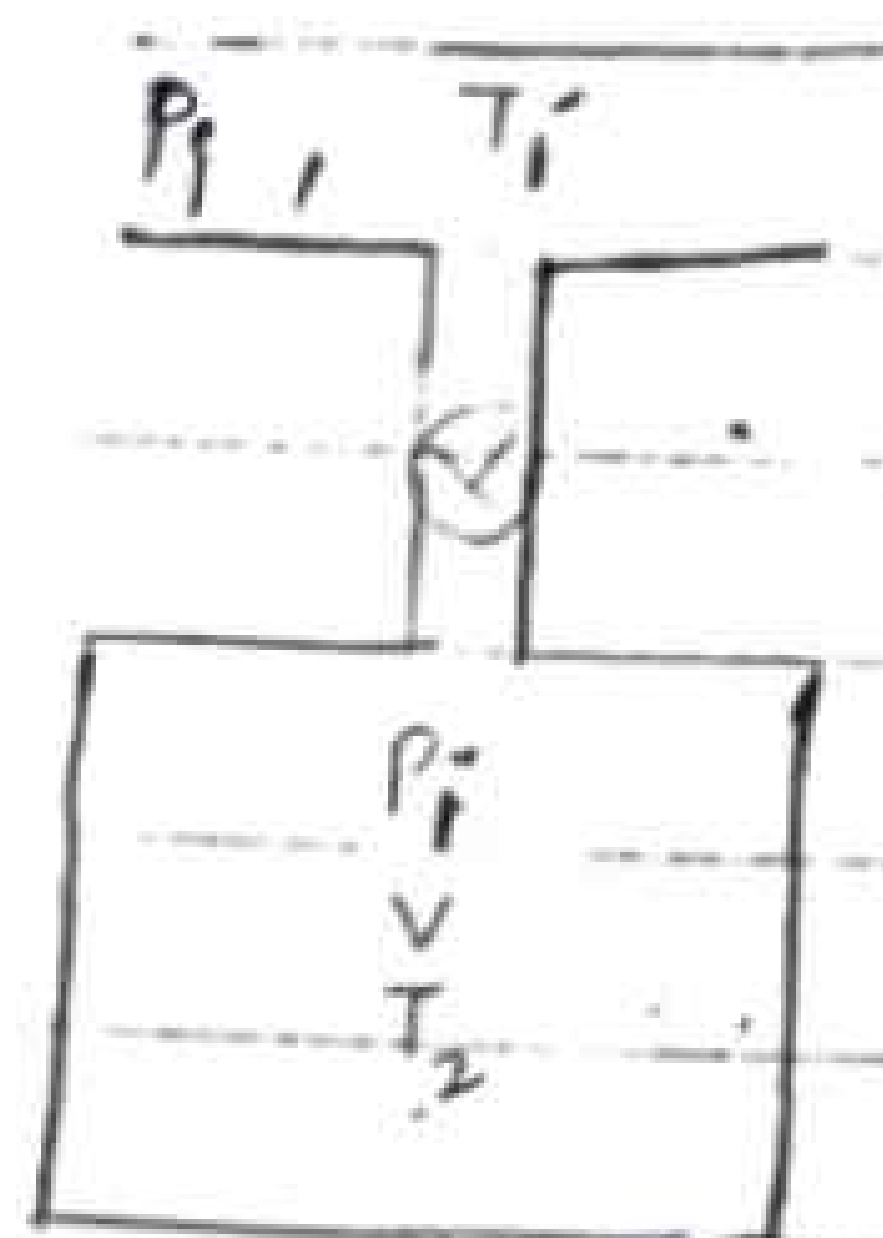
Assume the tank to be insulated.

Ans



Initial

$$(m_1 = \frac{P_1 V}{RT_1})$$



Final

$$(m_2 = \frac{P_i V}{RT_2})$$

Now, $m_2 U_2 - m_1 U_1 = (m_2 - m_1) h_i$

$$\Rightarrow m_2 C_v T_2 - m_1 C_v T_1 = (m_2 - m_1) C_p T_i$$

$$\Rightarrow m_2 T_2 - m_1 T_1 = (m_2 - m_1) \frac{C_p}{C_v} T_i$$

$$\Rightarrow \frac{P_i V}{R} - \frac{P_1 V}{R} = \left(\frac{P_i V}{RT_2} - \frac{P_1 V}{RT_1} \right) \frac{C_p}{C_v} T_i$$

$$\Rightarrow P_i - P_1 = \left(\frac{P_i}{T_2} - \frac{P_1}{T_1} \right) \gamma T_i$$

$$\Rightarrow P_i - P_1 = \gamma T_i \frac{P_i}{T_2} - \gamma T_i \frac{P_1}{T_1}$$

$$\Rightarrow P_i - P_1 + \frac{\gamma T_i P_1}{T_1} = \gamma T_i \frac{P_i}{T_2}$$

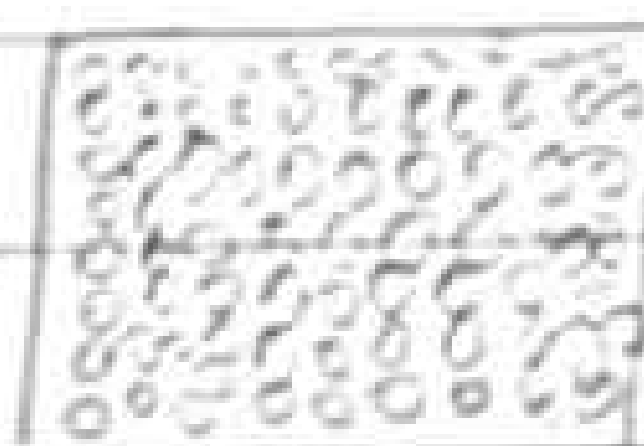
$$\Rightarrow T_2 = \frac{\gamma T_i P_i}{P_i - P_i + \gamma T_i P_i}$$

$$\Rightarrow T_2 = \frac{\gamma T_i P_i}{P_i \left(1 - \frac{P_i}{P_i} + \gamma \frac{T_i P_i}{T_i P_i}\right)}$$

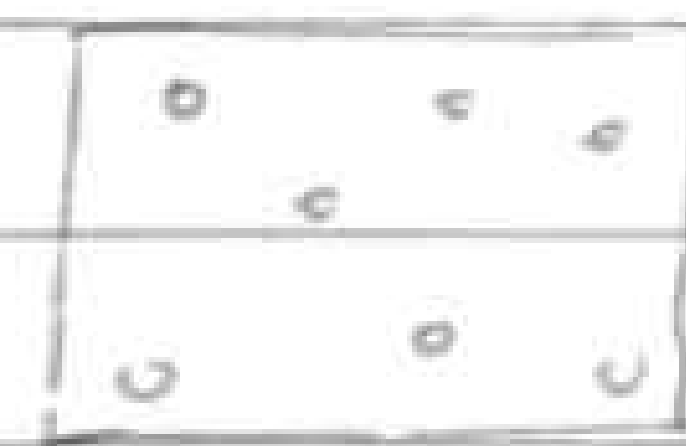
$$\Rightarrow T_2 = \frac{\gamma T_i}{1 - \frac{P_i}{P_i} + \gamma \frac{T_i}{T_i} \frac{P_i}{P_i}}$$

$$\Rightarrow T_2 = \frac{\gamma T_i}{1 + \frac{P_i}{P_i} \left(\gamma \frac{T_i}{T_i} - 1\right)}$$

*) Continuum : (continuous)



✓
continuum
applied



✗
not applied

i) Large density

ii) Less mean free path

iii) behaviour of average molecule is taken into consideration.

*** Unit of $\bar{R} = 8.314 \frac{\text{kJ}}{\text{K(mol)}}$; Molecular wt. = $\frac{\text{kg}}{\text{kmol}}$

$$\therefore \underline{R} = \frac{\bar{R}}{M} = \frac{\frac{\text{kJ}}{\text{kmol}}}{\frac{\text{kg}}{\text{kmol}}} = \frac{\text{kJ}}{\text{kg}}$$

II LAW OF THERMODYNAMICS (DIRECTIONAL LAW) :-

> I law of thermodynamics simply says that energy is conserved. It does not give any direction for a particular process. It is the second law of thermodynamics which gives direction for a particular process through the concept of entropy. and hence II law is known as directional law.

> Heat is known as low grade energy, work is known as high grade energy. Complete conversion of low grade energy (heat) into high grade energy (work) in a cycle is impossible.

> THERMAL ENERGY RESERVOIRS (TER) :-

SOURCE : It is a reservoir which can supply thermal energy without undergoing temp. change.

SINK : It is a reservoir which absorbs thermal energy without undergoing any temp. change.

> STATEMENTS :-

1.) KELVIN - PLANK STATEMENT : It is impossible to develop a device operating on a cycle which produces work continuously by exchanging heat with a single reservoir.

This device is known as PMM II and its efficiency is 100%. and therefore achieving 100% efficiency is impossible.

RESERVIOR

 $\downarrow Q$ PMMII $\rightarrow W$

$$\eta = 0/1$$

$$\eta = W/Q$$

$$\eta = 1$$

$$\eta = 100\%$$

➤ HEAT ENGINE :

It is a device which converts part of heat into work and rejects remaining to sink or surroundings.

SOURCE

 $\downarrow Q_1$ HE $\rightarrow W$ $\downarrow Q_2$

SINK

$$\text{We: } Q_1 = Q_2 + W$$

$$\Rightarrow W = Q_1 - Q_2$$

$$\text{Now, } \eta = \frac{0}{1}$$

$$\eta = \frac{W}{Q}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

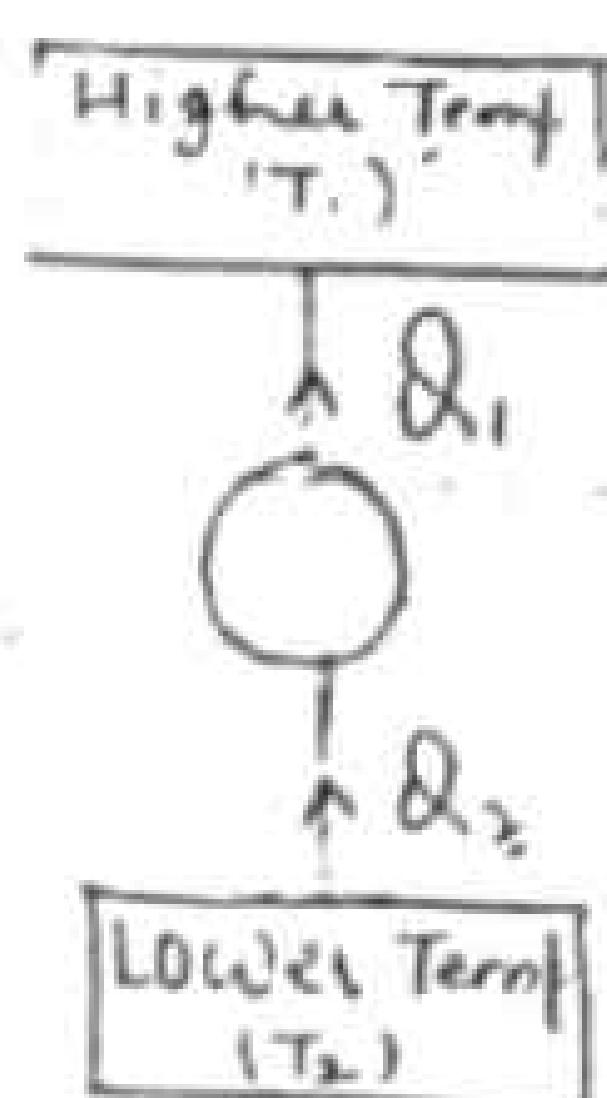
$$\boxed{\eta = 1 - \frac{Q_R}{Q_S}} \quad \text{--- (1) } \checkmark$$

Eqⁿ (1) is valid for reversible as well as irreversible heat engine.

(*) Heat engine is a logical development of kelvin-plank statement.

2.) CLAUSIUS STATEMENT :

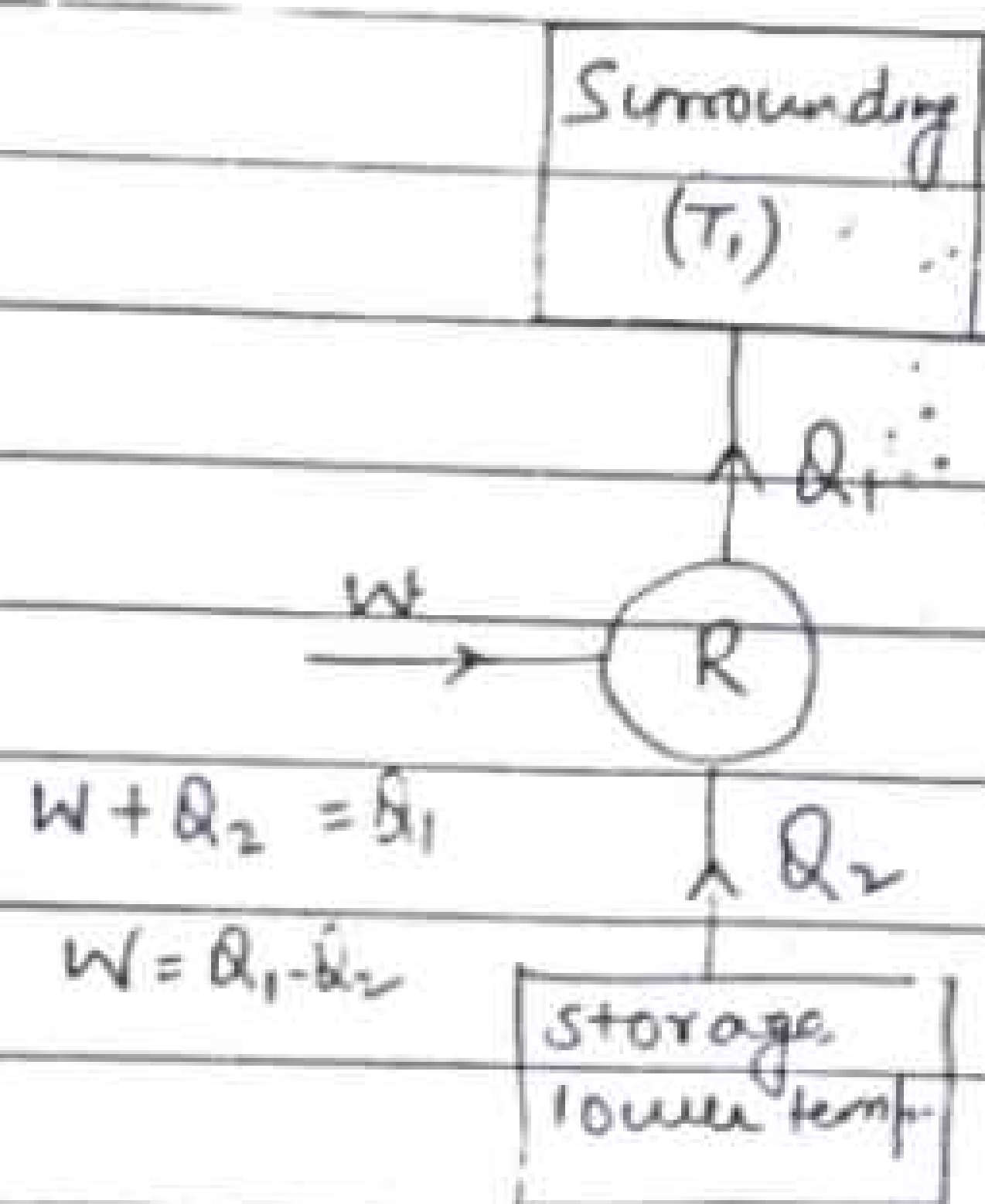
It is impossible to develop a device operating on a cycle which transfers heat from lower temp. to higher temp. without any external work input.



(Impossible)

> REFRIGERATOR :

A refrigerator is a device which maintains lower temp. compared to surroundings. As lower temp. are to be maintained continuously, therefore refrigerator operates on a cycle.



$$(COP)_R = \frac{\text{Desired effect}}{\text{Energy input}}$$

$$(COP)_R = \frac{Q_2}{W}$$

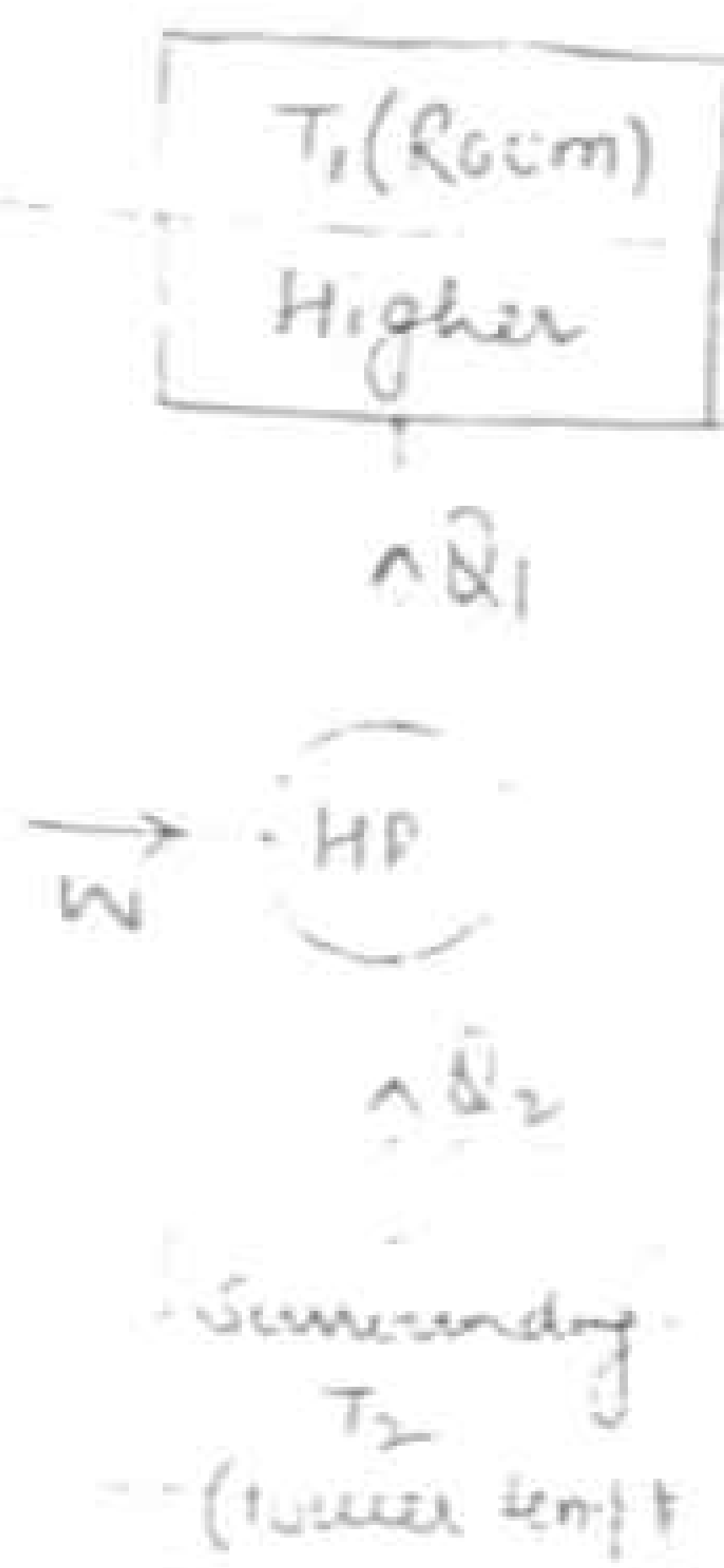
$$(COP)_R = \frac{Q_2}{Q_1 - Q_2} \quad \text{--- (2)}$$

Eqⁿ (2) is valid for reversible or irreversible process (refrigerator).

It is a logical development of Clausius statement.

➤ HEAT PUMP :

It is a device which maintains higher temp. compared to surroundings.



$$(COP)_{hp} = \frac{\text{desired effect}}{\text{Energy input}}$$

$$(COP)_{hp} = \frac{Q_1}{W}$$

$$\boxed{(COP)_{hp} = \frac{Q_1}{Q_1 - Q_2}} \quad \text{--- (3)}$$

eqn(3) is valid for both reversible and irreversible heat pump.

➤ Relationship b/w $(COP)_R$ & $(COP)_{hp}$ operating b/w same temperature limits :

$$(COP)_{hp} - (COP)_R = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2}$$

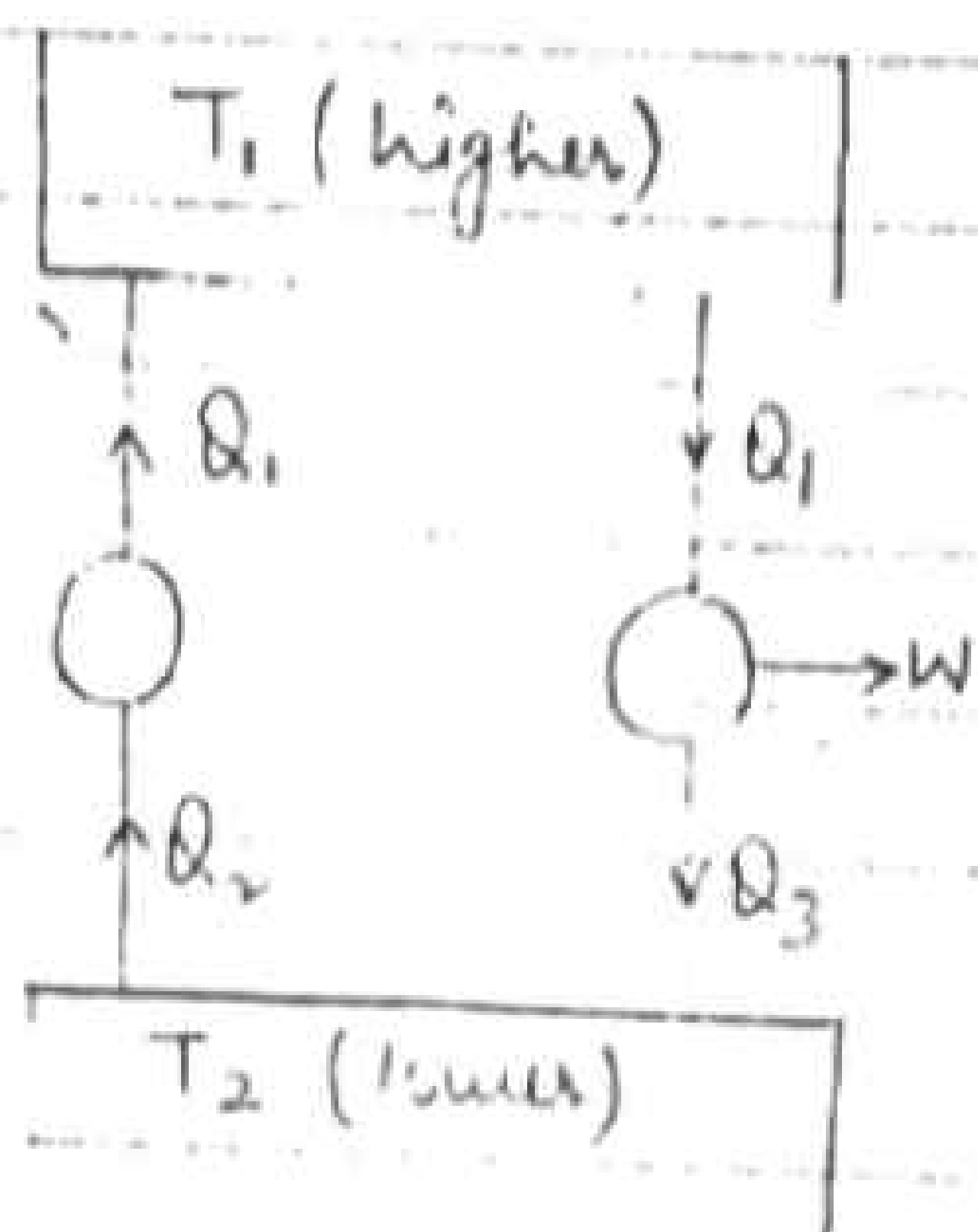
$$\Rightarrow (COP)_{hp} - (COP)_R = \frac{Q_1 - Q_2}{Q_1 - Q_2}$$

$$\Rightarrow (COP)_{hp} - (COP)_R = 1$$

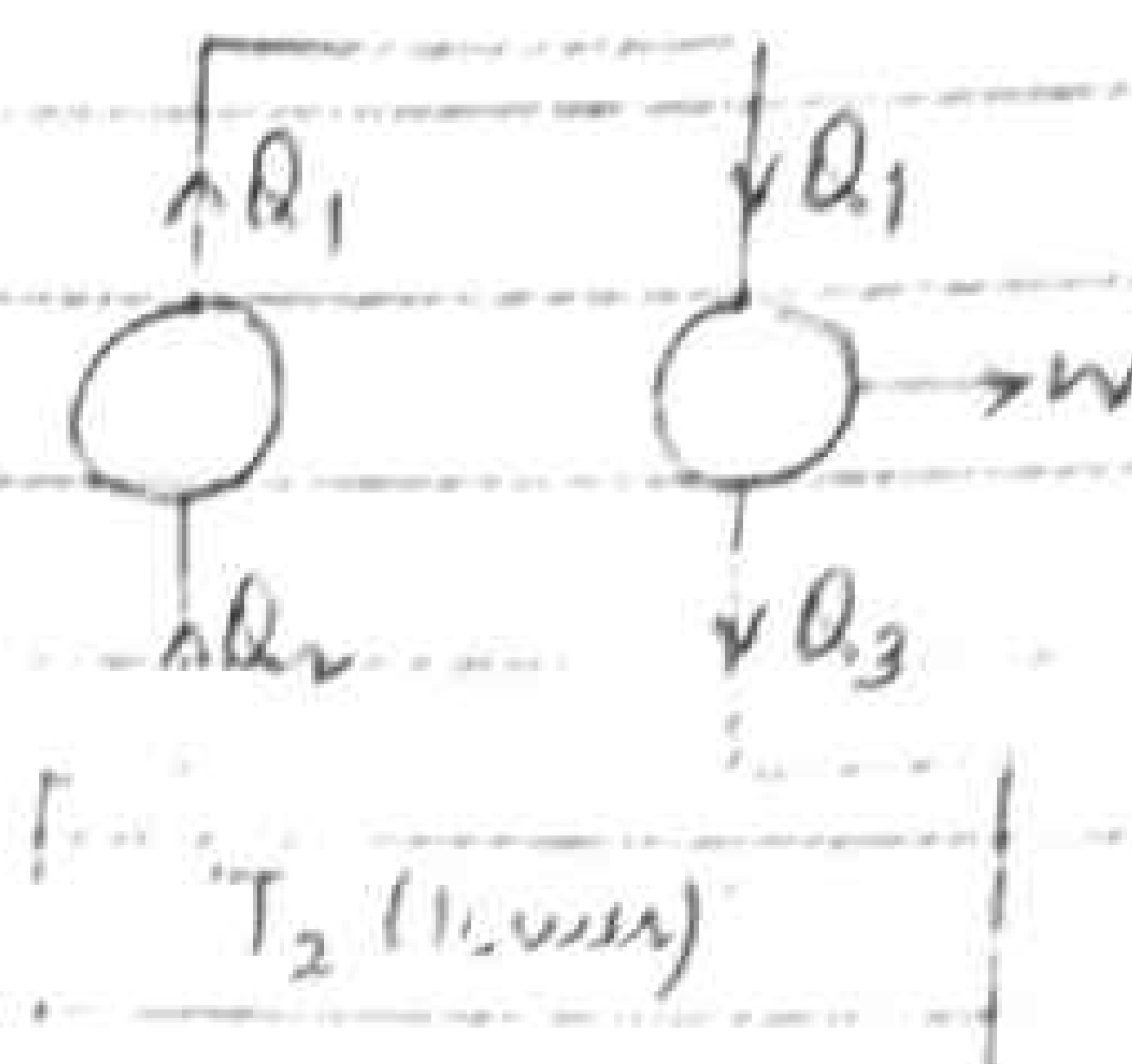
$$\Rightarrow \boxed{(COP)_{hp} = 1 + (COP)_R}$$

> To show that kelvin plank and clausius statement are equivalent statement of thermodynamics.

CASE 1 Violation of clausius statement :-



(violation of clausius)

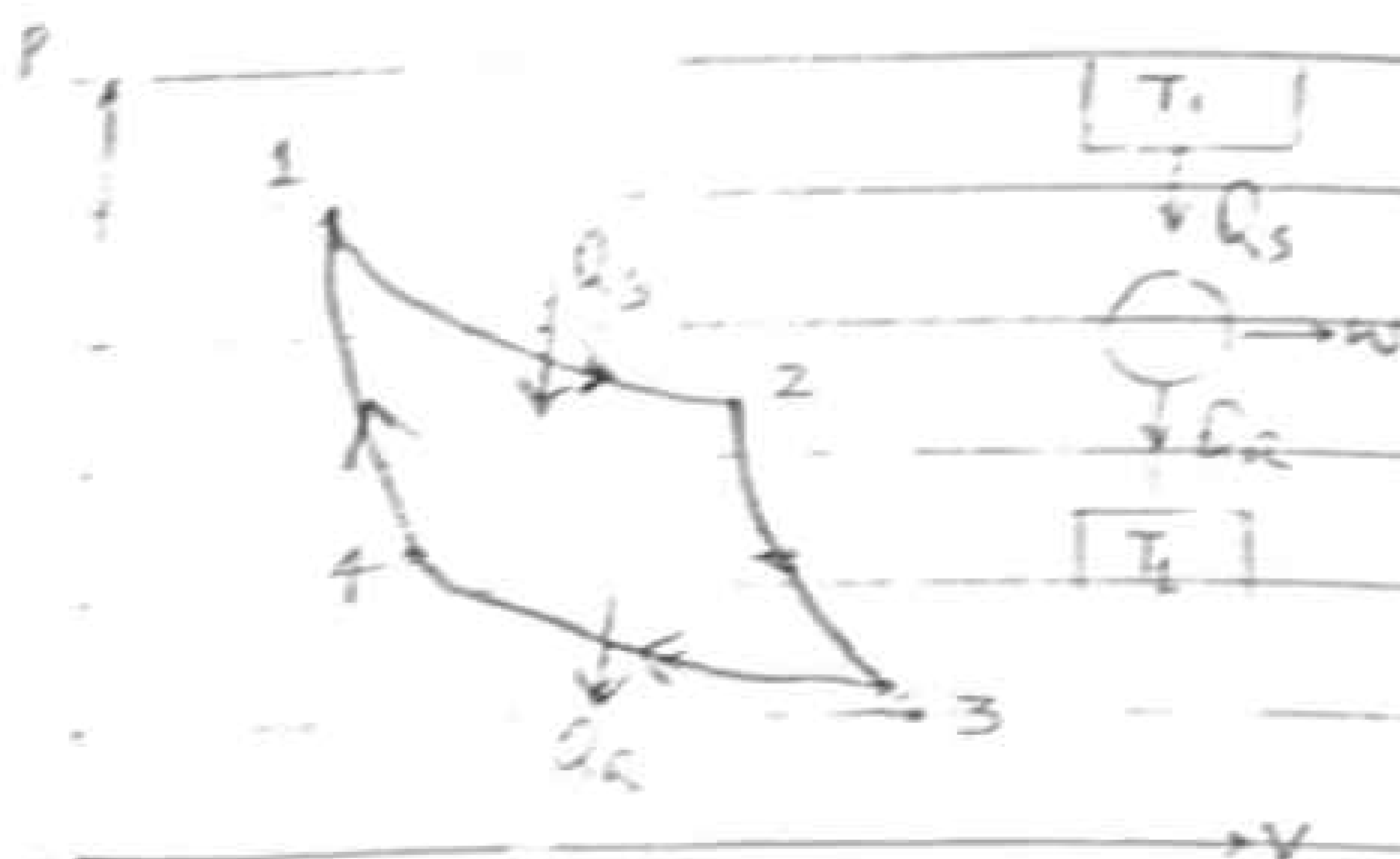


(violation of kelvin plank)

CARNOT CYCLE :

It is a reversible cycle. A cycle is said to be a reversible cycle when each process in a cycle is reversible.

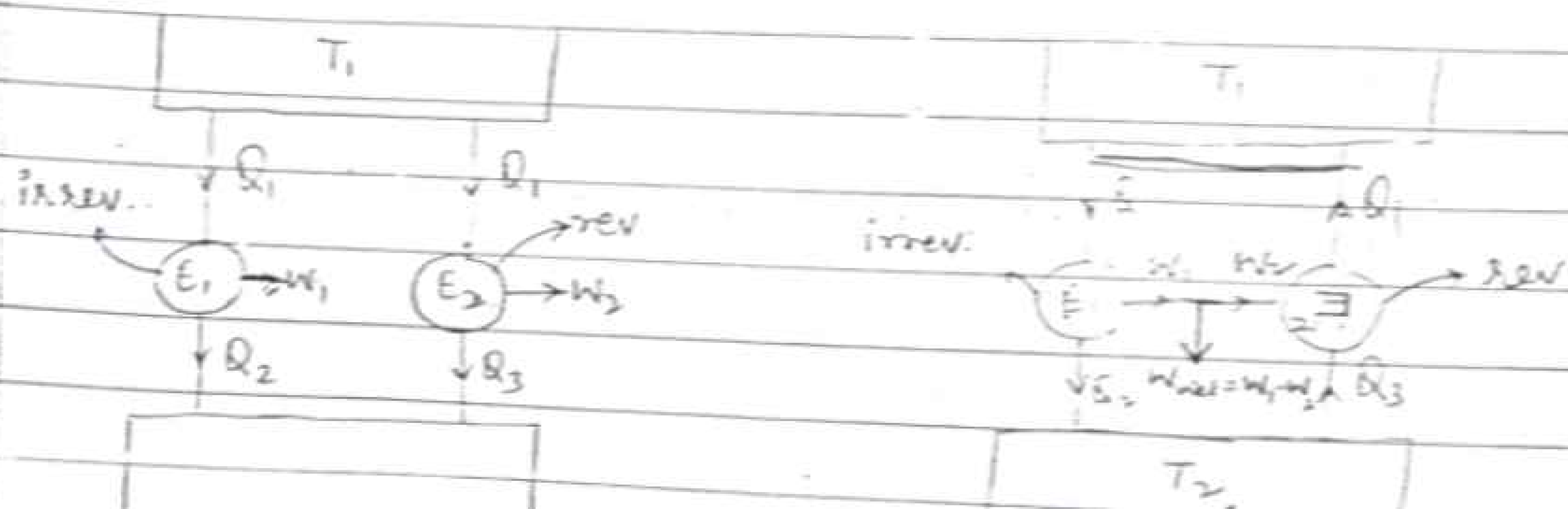
- 1-2 slow isothermal expansion (heat addition)
- 2-3 adiabatic expansion
- 3-4 isothermal compression (heat rejection)
- 4-1 fast adiabatic compression



Carnot cycle consists of two isothermal and two adiabatic process. Isothermal process is a very slow process and adiabatic process is a very fast process and hence these two combinations in a cycle are not possible. Therefore it is not a practical cycle but this cycle is used for comparing other actual cycles.

CARNOT'S THEOREM :

For various engines operating b/w same temp. limits no engine has efficiency greater than reversible cycle efficiency.



Let us assume

$$\eta_{\text{irrev.}} > \eta_{\text{revel}}$$

$$\eta_{\text{irrev.}} = \frac{W_1}{Q_1} ; \quad \eta_{\text{revel}} = \frac{W_2}{Q_2}$$

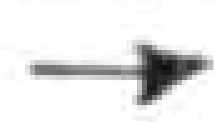
$$\frac{W_1}{Q_1} > \frac{W_2}{Q_2}$$

$$W_1 > W_2$$

With our assumption, kelvin-plank statement is violated and hence our assumption is wrong. Therefore efficiency of an irreversible engine can not be greater than the efficiency of reversible engine operating b/w same temp. limits.

Important points w.r.t. reversible cycle-

- i) Efficiency of various reversible engines working between same temp. limits is same.
- ii) η of a reversible cycle is independent of working fluid.
- iii) η of a reversible cycle depends only on temp. limits.

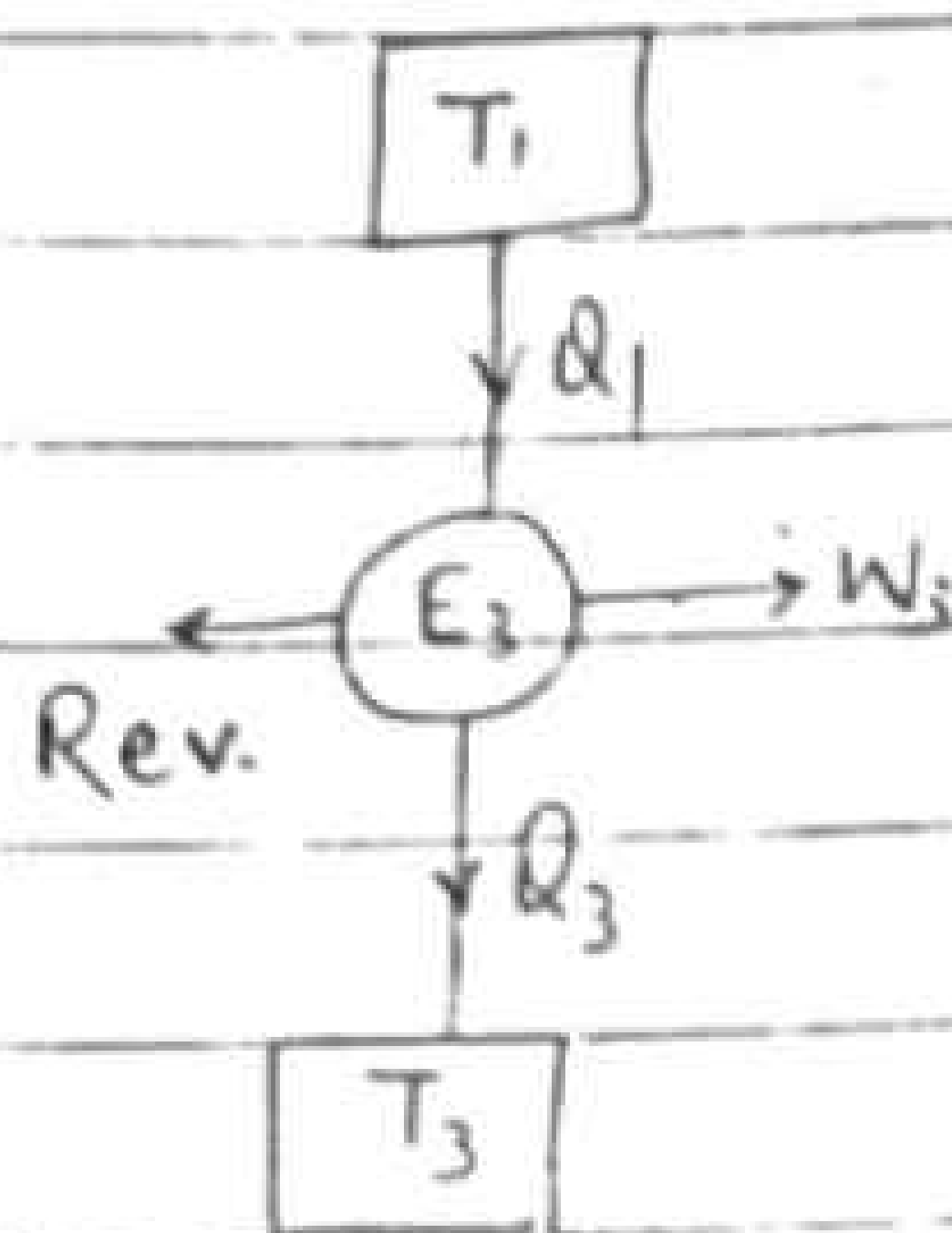
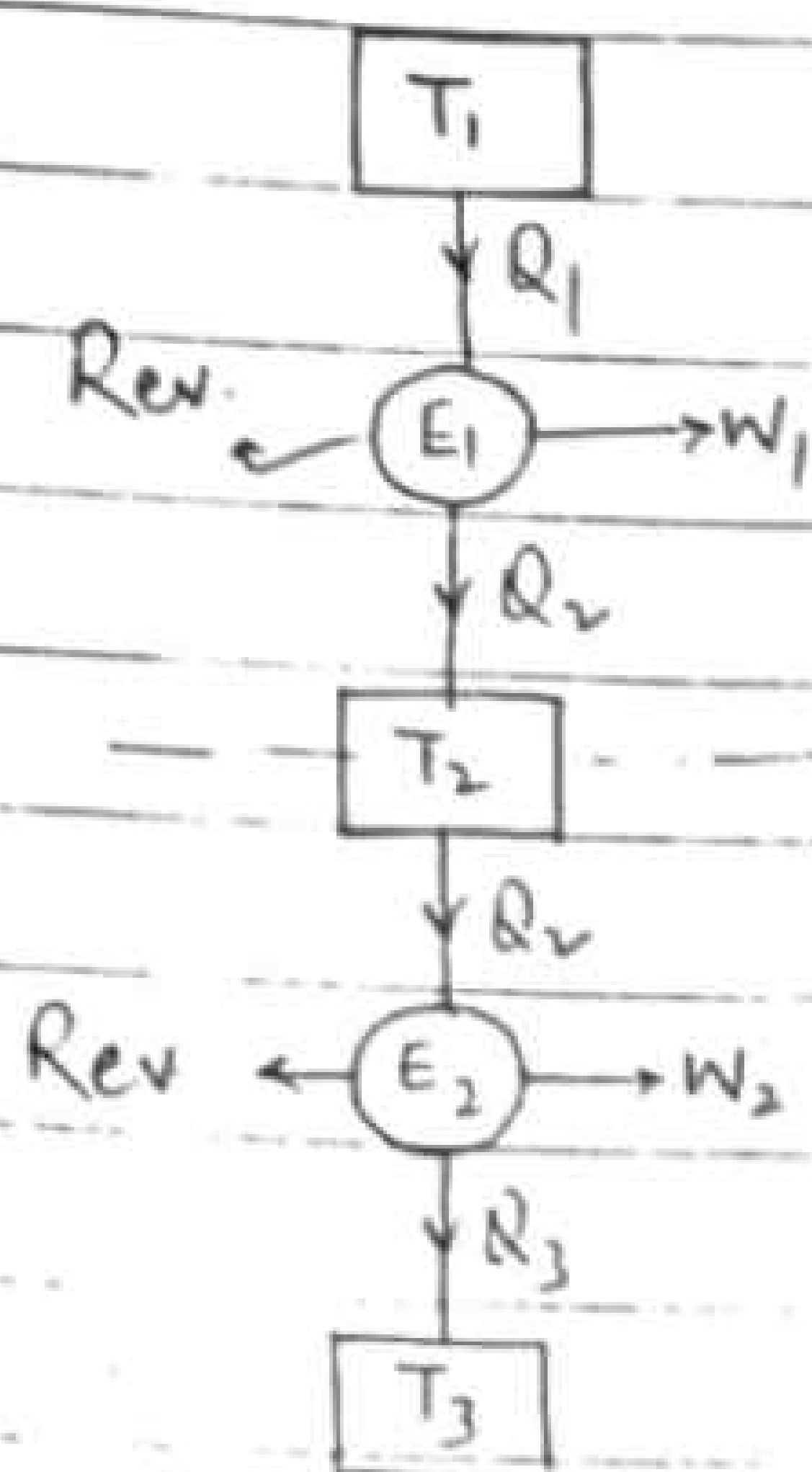


THERMODYNAMIC TEMP. SCALE :-

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\Rightarrow \eta_1 = 1 - \frac{Q_2}{Q_1}$$

$$\eta_1 = f_1(T_1, T_2)$$



$$\text{So, } 1 - \frac{Q_2}{Q_1} = f_1(T_1, T_2)$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{1}{1 - f_1(T_1, T_2)}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \phi_1(T_1, T_2)$$

$$\text{Similarly, } \frac{Q_2}{Q_3} = \phi_2(T_2, T_3)$$

$$\& \quad \frac{Q_1}{Q_3} = \phi_3(T_1, T_3)$$

$$\text{Now, } \frac{Q_1}{Q_2} = \frac{\frac{Q_1}{Q_3}}{\frac{Q_2}{Q_3}} = \frac{\phi(T_1, T_3)}{\phi(T_2, T_3)}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)}$$

$$\Rightarrow \left[\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \right]$$

valid only for reversible cycle.

(with two reversions)

→ **CLAUSIUS INEQUALITY :-**

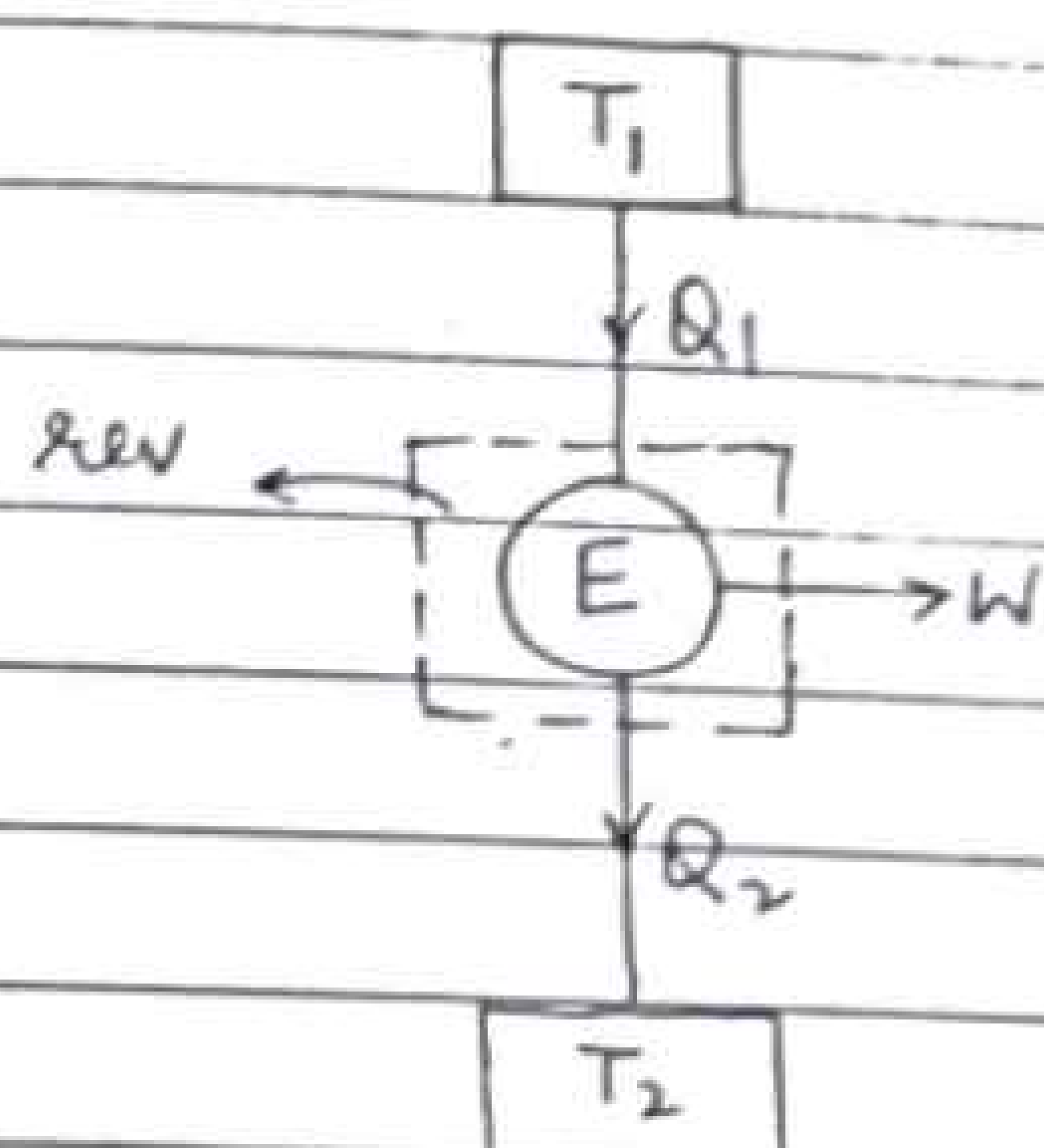
The cyclic integral of $\frac{\delta Q}{T}$ is less than or equal to zero.

$$\oint \frac{\delta Q}{T} \leq 0$$

if $\oint \frac{\delta Q}{T} < 0 \rightarrow$ irreversible cycle

if $\oint \frac{\delta Q}{T} = 0 \rightarrow$ reversible cycle

CASE 1 : REVERSIBLE CYCLE :



$$\oint_{rev} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \left(\frac{-Q_2}{T_2} \right)$$

$$\oint_{rev} \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

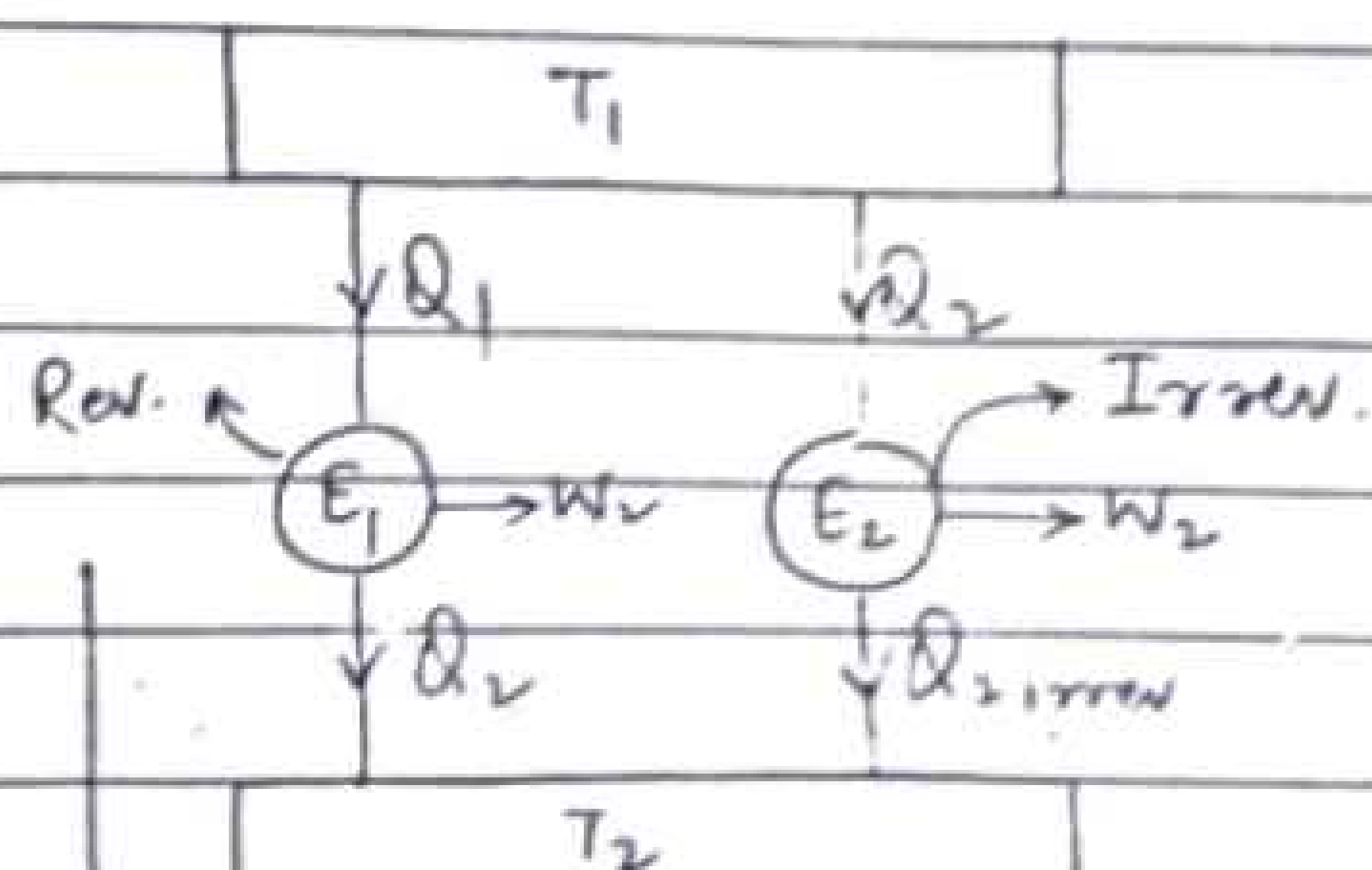
$$\Rightarrow \oint_{rev} \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$\left(\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \right)$$

$$\Rightarrow \oint_{rev} \frac{\delta Q}{T} = 0$$

(A property)

CASE 2 : IRREVERSIBLE CYCLE :



$$\eta_{rev} > \eta_{irrev}$$

$$\Rightarrow \frac{W_1}{Q_1} > \frac{W_2}{Q_1}$$

$$\Rightarrow W_1 > W_2$$

$$\Rightarrow Q_2 < Q_{2,irrev.}$$

$$\left(\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \right)$$

$$\oint_{\text{rev}} \frac{dQ}{T} = \frac{Q_1}{T_1} + \left(-\frac{Q_{2,\text{rev}}}{T_2} \right)$$

$$= \frac{Q_1}{T_1} - \frac{Q_{2,\text{rev}}}{T_2}$$

$$\oint_{\text{rev}} \frac{dQ}{T} = \frac{1}{T_2} (Q_1 - Q_{2,\text{rev}})$$

$$\Rightarrow \left[\oint_{\text{rev}} \frac{dQ}{T} < 0 \right]$$

1. A reversible heat eng.

→ EFFICIENCY OF A REVERSIBLE ENGINE :

$$\eta = 1 - \frac{Q_2}{Q_1}$$

for reversible HE $\eta_{\text{rev}} = 1 - \frac{T_2}{T_1}$

$$\left[\eta_{\text{rev}} = 1 - \frac{T_L}{T_H} = \eta_{\text{max}} \right]$$

T in 'K'

→ COP of a reversible refrigerator :

$$\eta = (\text{COP})_R = \frac{Q_2}{Q_1 - Q_2}$$

$$(\text{COP})_R = \frac{Q_2}{Q_2 \left(\frac{Q_1}{Q_2} - 1 \right)}$$

$$(\text{COP})_R = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

for reversible refrigerator : $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$(\text{COP})_{\text{HVR}} = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$(\text{COP})_{\text{HVR}} = \frac{T_2}{T_1 - T_2}$$

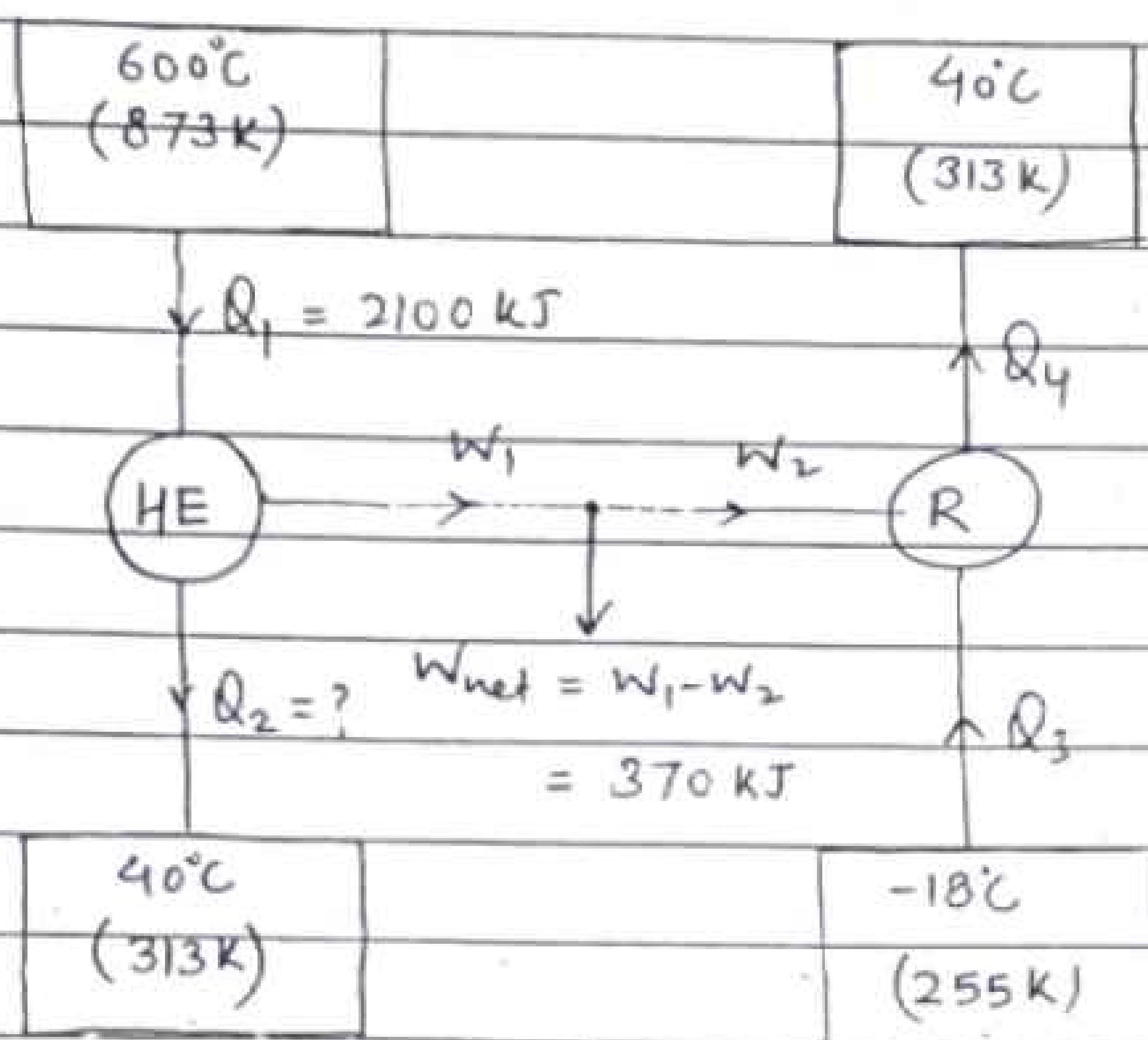
$$(\text{COP})_{\text{HVR}} = \frac{T_L}{T_H - T_L} = (\text{COP})_{\text{maxR}}$$

∴ COP of a reversible heat pump is $\frac{T_H}{T_H - T_L}$.

Ex

P: A reversible heat engine operates b/w 60°C & 40°C . This engine drives a reversible refrigerator operating b/w 40°C and -18°C . Still there is a net work output of 370 kJ , while the heat received by engine is 2100 kJ . Determine the cooling effect.

Ans: 1



for heat engine :-

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow Q_2 = \frac{Q_1 \cdot T_2}{T_1}$$

$$\Rightarrow Q_2 = 752.920$$

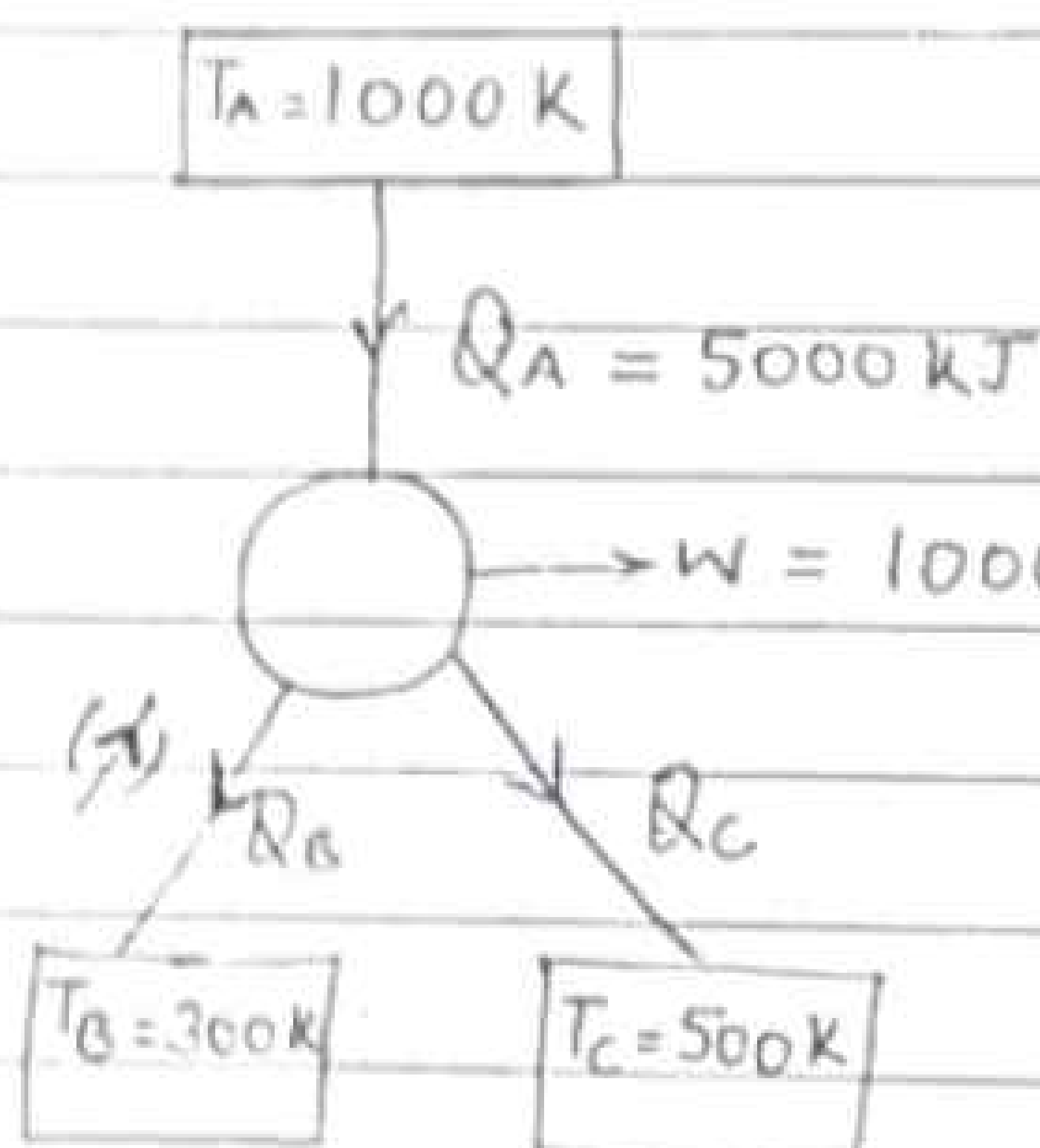
$$\begin{aligned}\therefore \text{Work } (W_1) &= Q_1 - Q_2 \\ &= 2100 - 752.92 \\ W_1 &= 1347.07 \text{ kJ}\end{aligned}$$

$$\begin{aligned}\text{So, } W_1 - W_2 &= 370 \\ \Rightarrow W_2 &= W_1 - 370 \\ \Rightarrow W_2 &= 977.07 \text{ kJ}\end{aligned}$$

$$\begin{aligned}\text{b. for refrigerator } \text{COP} &= \frac{Q_3}{W_2} = \frac{T_L}{T_H - T_L} \\ \Rightarrow Q_3 &= W_2 \cdot \left(\frac{255}{313 - 255} \right) \\ \Rightarrow Q_3 &= \underline{\underline{4295.7 \text{ kJ}}} \quad \text{Ans.}\end{aligned}$$

P: 2

fig. shows a reversible cycle during which it exchanges heat with 3 reservoirs and develops 1000 kJ of work find the magnitude and direction of Q_B & Q_C .



$$5000 = 1000 + Q_B + Q_C$$

$$\Rightarrow Q_B + Q_C = 4000$$

$$\oint \frac{dQ}{T} = 0$$

$$\Rightarrow \frac{5000}{1000} + \left(-\frac{Q_B}{300} - \frac{Q_C}{500} \right) = 0$$

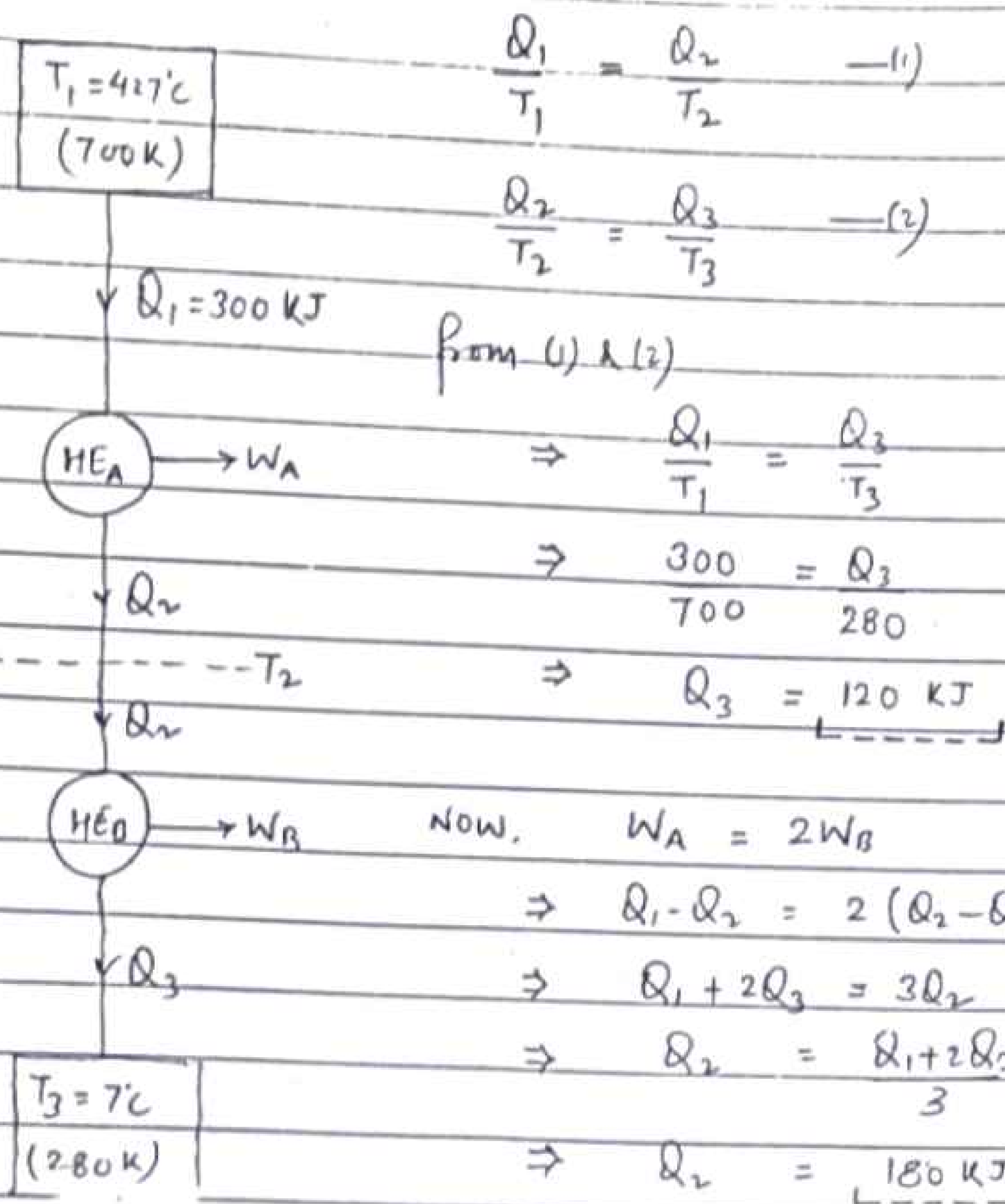
$$\Rightarrow 5 = \frac{Q_B}{300} + \frac{4000 - Q_B}{500}$$

$$\Rightarrow Q_B = \underline{\underline{-2250 \text{ kJ}}} \quad \text{Ans.}$$

$$\therefore Q_A = \underline{\underline{6250 \text{ kJ}}} \quad \text{Ans.}$$

P. 3 Two reversible HE 'A' & 'B' are arranged in series, HEA rejects heat to engine HEB. Engine A receives 300 kJ of heat at 427°C from a high temp. source while B rejects heat to a sink at 7°C . If the work output of A is 2 times that of 'B' find

- intermediate temp. b/w A & B
- efficiency of each engine
- Heat rejected by engine A & heat received by engine B
- Heat rejected to sink.



$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{--- (1)}$$

$$\frac{Q_2}{T_2} = \frac{Q_3}{T_3} \quad \text{--- (2)}$$

from (1) & (2)

$$\Rightarrow \frac{Q_1}{T_1} = \frac{Q_3}{T_3}$$

$$\Rightarrow \frac{300}{700} = \frac{Q_3}{280}$$

$$\Rightarrow Q_3 = 120 \text{ kJ}$$

$$\text{Now, } W_A = 2W_B$$

$$\Rightarrow Q_1 - Q_2 = 2(Q_2 - Q_3)$$

$$\Rightarrow Q_1 + 2Q_3 = 3Q_2$$

$$\Rightarrow Q_2 = \frac{Q_1 + 2Q_3}{3}$$

$$\Rightarrow Q_2 = 180 \text{ kJ}$$

$$\text{or } \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \frac{300}{700} = \frac{180}{T_2} \Rightarrow T_2 = 420 \text{ K}$$

$$\text{Now } \eta_{\text{HEA}} = 1 - \frac{T_2}{T_1} = 1 - \frac{420}{700} = 40\%$$

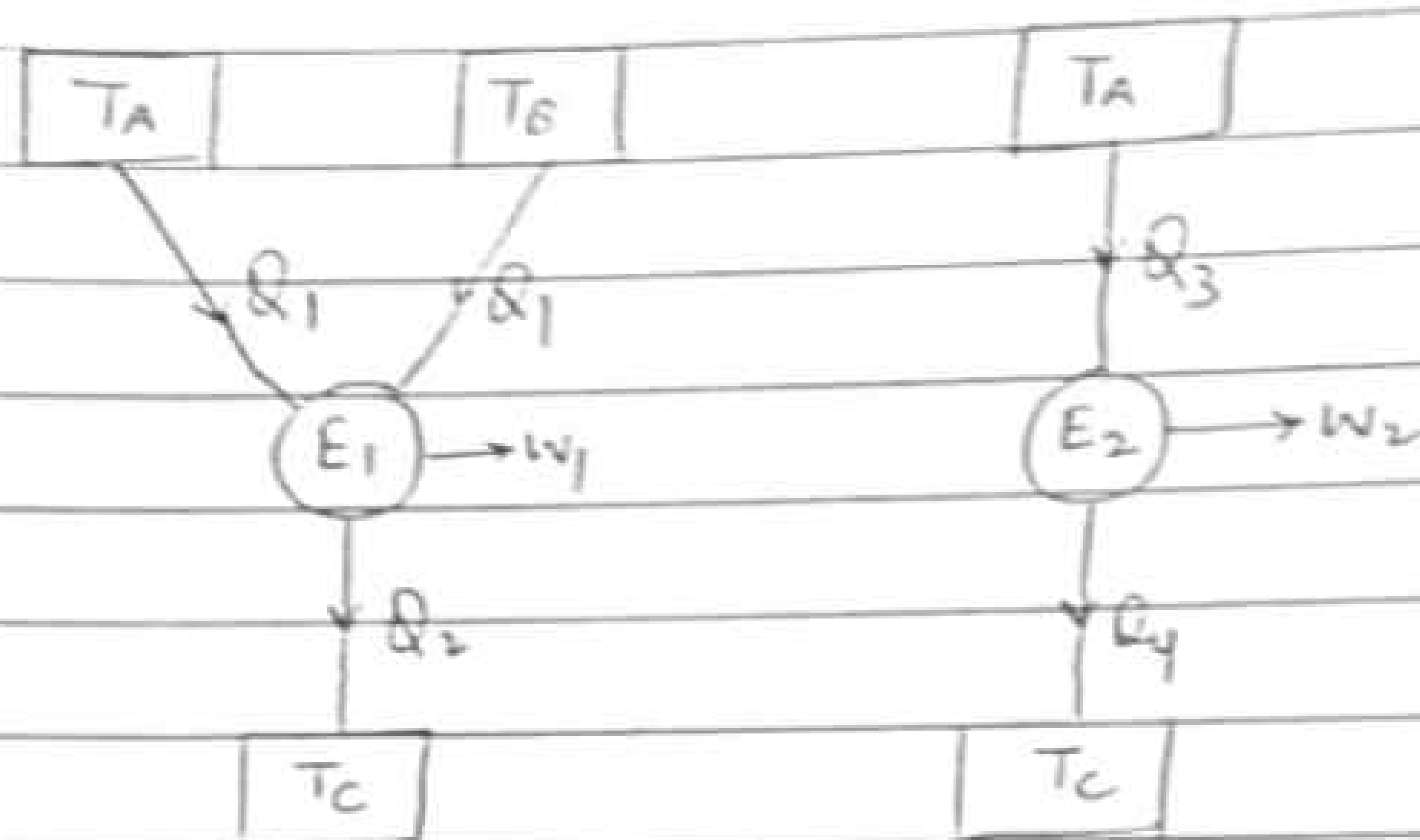
$$\eta_{\text{HEB}} = 1 - \frac{T_2}{T_3} = 1 - \frac{280}{420} = 33.33\%$$

(**)

Q:4

A reversible engine works b/w three heat reservoirs A, B, C. The engine receives equal amount of heat from reservoir A & B at temp T_A & T_B and rejects heat to a reservoir C at a temp T_C . If the efficiency of this heat engine is α times the efficiency of RHE working b/w T_A & T_C . Show that

$$\frac{T_A}{T_B} = 2(1-\alpha)T_A + (2\alpha-1)T_C$$



$$\eta = 1 - \frac{Q_2}{2Q_1}$$

$$\text{Now } \oint \frac{dQ}{T} = 0$$

$$\Rightarrow \frac{Q_1}{T_A} + \frac{Q_1}{T_B} - \frac{Q_2}{T_C} = 0$$

$$\therefore \eta = 1 - \frac{T_C}{2} \left[\frac{1}{T_A} + \frac{1}{T_B} \right] \quad (1)$$

$$\Rightarrow Q_1 \left(\frac{1}{T_A} + \frac{1}{T_B} \right) = \frac{Q_2}{T_C}$$

$$\therefore \frac{Q_2}{Q_1} = T_C \left[\frac{1}{T_A} + \frac{1}{T_B} \right]$$

$$\eta_2 = 1 - \frac{Q_4}{Q_3}$$

$$\eta_2 = 1 - \frac{T_c}{T_A} \quad \text{--- (2)}$$

$$\text{from (1) \& (2)} \Rightarrow \eta_1 = \alpha \eta_2$$

$$\Rightarrow 1 - \frac{T_c}{2} \left[\frac{1}{T_A} + \frac{1}{T_B} \right] = \alpha \left[1 - \frac{T_c}{T_A} \right]$$

$$\Rightarrow 1 - \frac{T_c}{2T_A} + \frac{T_c}{2T_B} = \alpha - \frac{\alpha T_c}{T_A}$$

$$\Rightarrow T_A - \frac{T_c}{2} + \frac{T_c T_A}{2T_B} = \alpha T_A - \alpha T_c$$

$$\Rightarrow (T_A - \alpha T_A) - \frac{T_c}{2} + \alpha T_c + \frac{T_c T_A}{2T_B} = 0$$

$$\Rightarrow \frac{T_c}{2} \frac{T_A}{T_B} = - \left\{ T_A (\alpha - 1) + T_c \left(\frac{1}{2} - \alpha \right) \right\}$$

$$\Rightarrow \frac{T_A}{T_B} = - \left\{ \frac{2T_A (\alpha - 1)}{T_c} + \frac{2T_A (1 - 2\alpha)}{2} \right\}$$

$$\Rightarrow \boxed{\frac{T_A}{T_B} = \frac{2T_A (1 - \alpha)}{T_c} + T_A (2\alpha - 1)}$$

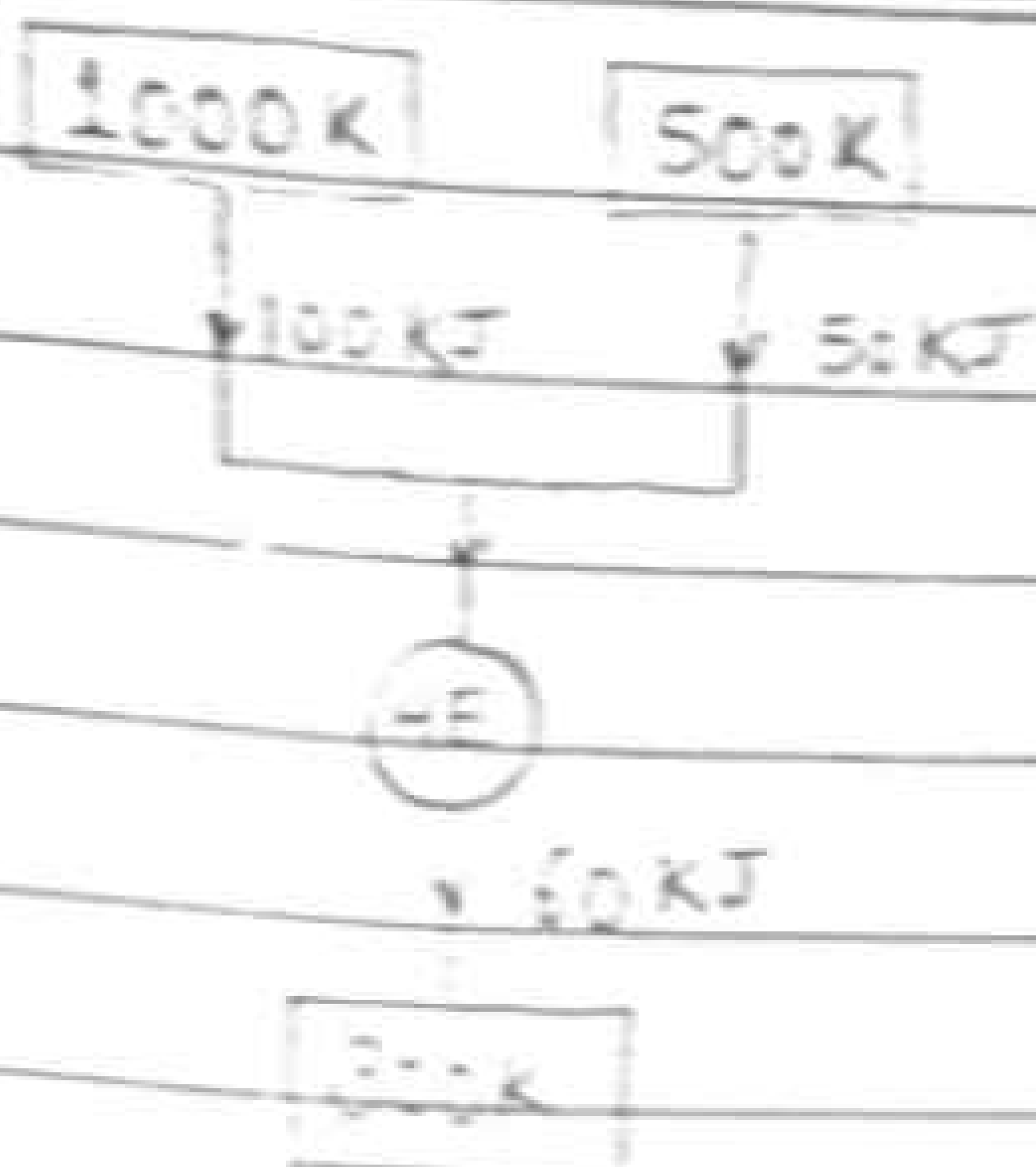
CHAPTER '4' (WORKBOOK)

DATE _____

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Ques 01

(a)



here $\oint \frac{\delta Q}{T} = 0$

$$\Rightarrow \frac{100}{1000} + \frac{50}{500} - \frac{60}{300} = 0$$

$$\Rightarrow \frac{1}{10} + \frac{1}{10} - \frac{1}{5} = 0$$

$$\Rightarrow 0 = 0$$

SO, it is a reversible HE.

Ques 02

(b)

here, $Q = 0.5 \text{ kW/m}^2$

$$\eta_{HE} = 0.2$$

$$\eta_{HE} = \frac{W}{Q}$$

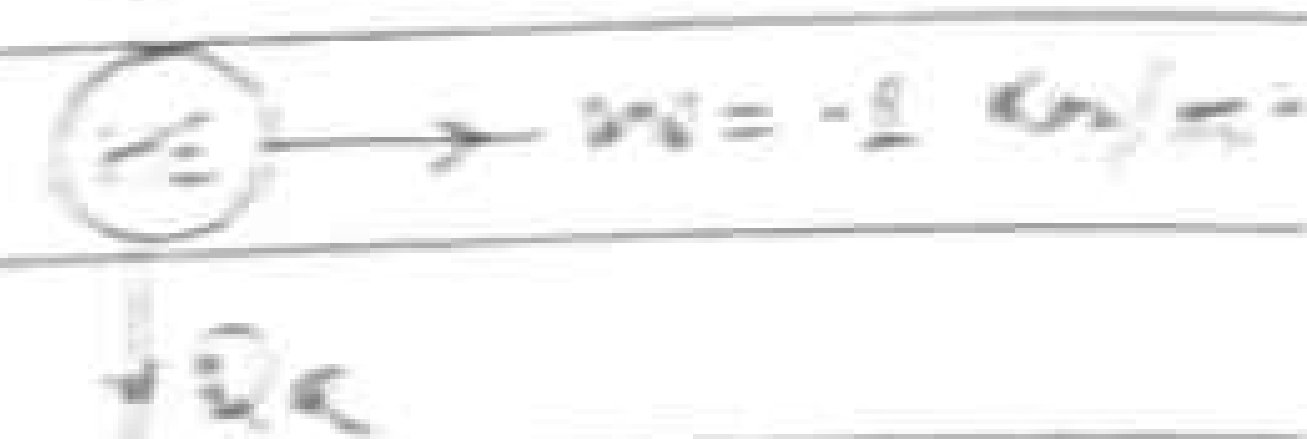
$$\Rightarrow W = 0.2 \times 0.5 = 0.1 \text{ kW/m}^2$$

SO for a power output of 1 kW, Area required will be

$$\Rightarrow 0.1 \times A = 1$$

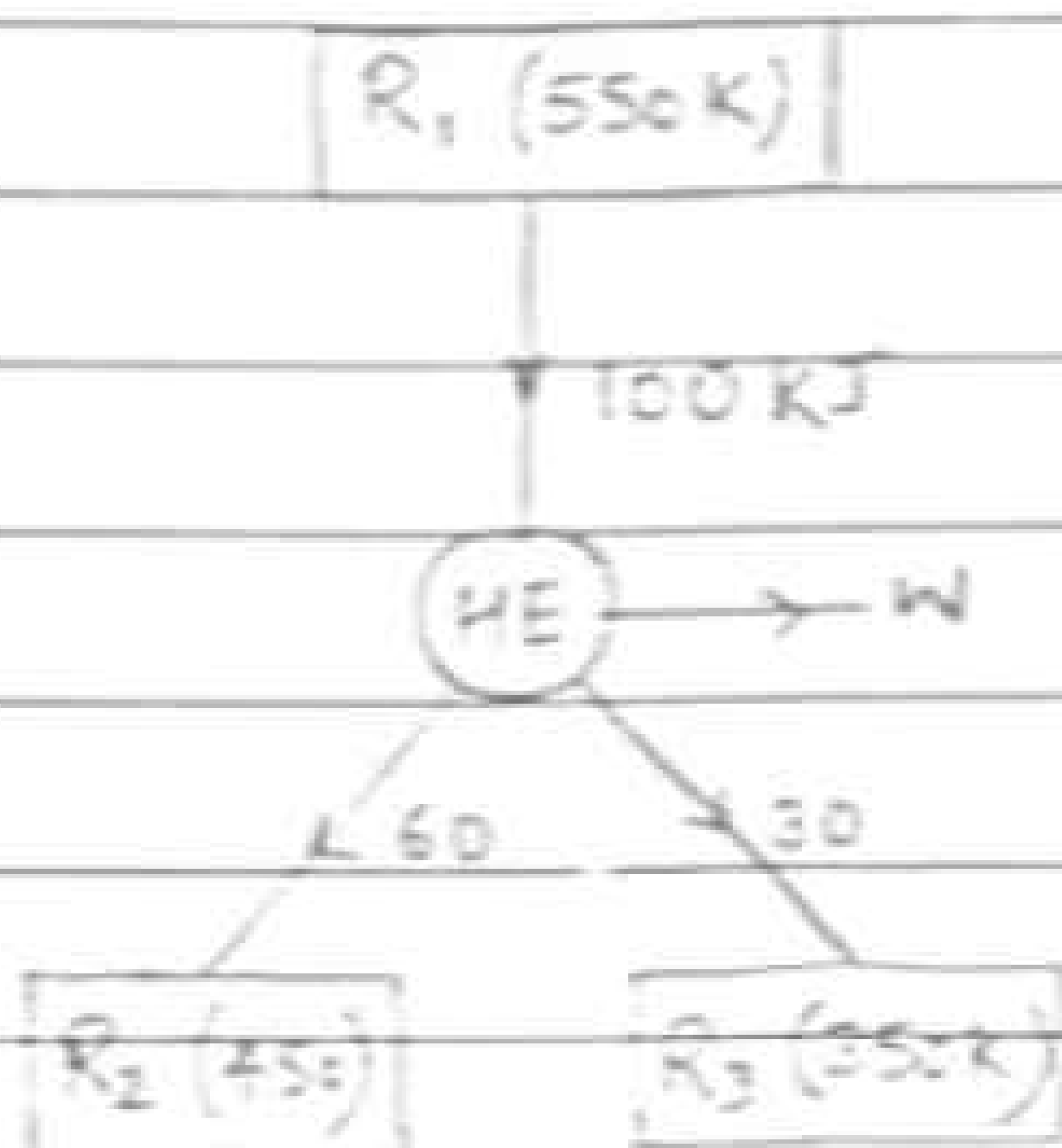
$$\Rightarrow A = 10 \text{ m}^2$$

$$Q = 0.5 \text{ kW/m}^2$$



Ques 04

(a)



from energy conservation:

$$W = 100 - (60 + 30)$$

$$W = 10 \text{ kJ}$$

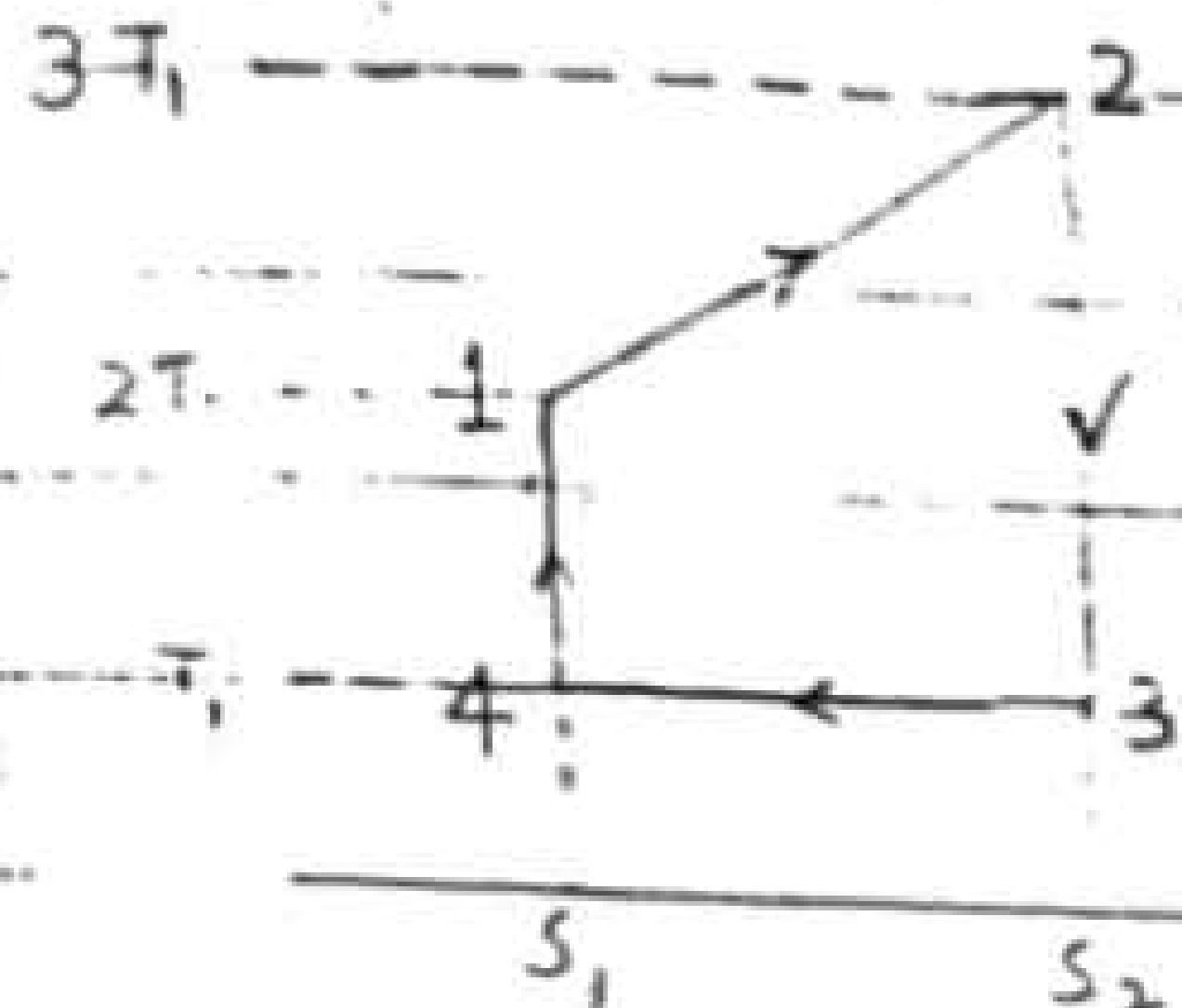
$$\therefore \eta_{HE} = \frac{W}{Q_1} = \frac{10}{100}$$

$$\eta_{HE} = 0.1$$

NS: 05

$$\eta_{HE} = \frac{W}{Q_S}$$

(a)



$$Q_S = \text{Area } 123S_2S_1$$

$$Q_S = \frac{1}{2} (2T_1 + 3T_1) (S_2 - S_1) \quad \text{--- (1)}$$

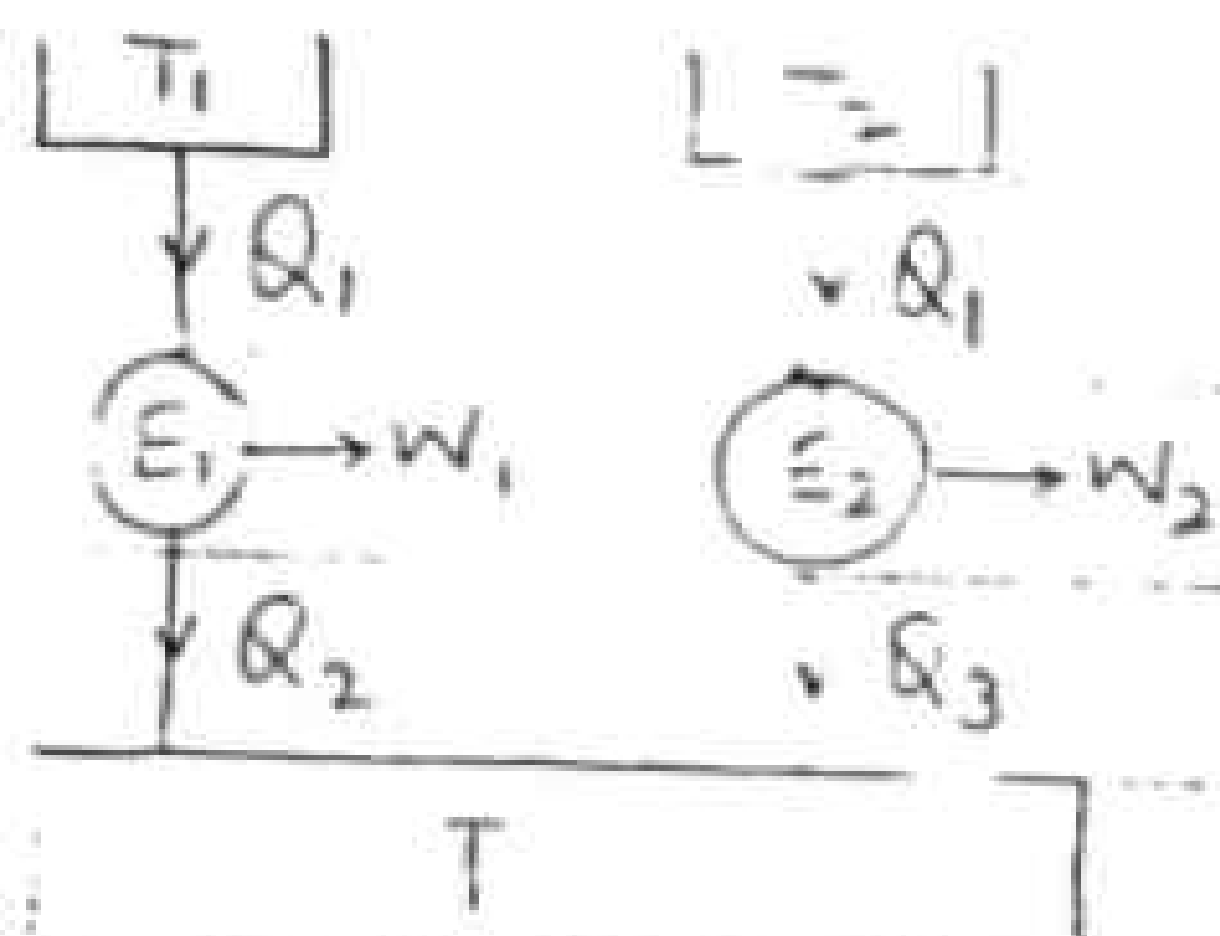
$$W = \text{Area } 12341$$

$$W = \left\{ \frac{1}{2} (2T_1 + 3T_1) - T_1 \right\} (S_2 - S_1) \quad \text{--- (2)}$$

$$\therefore \eta_{HE} = \frac{\frac{5T_1 - 2T_1}{2}}{\frac{5T_1}{2}} = \frac{3}{5} \text{ Ans.}$$

NS: 10

(b)



$$\text{As, } T_1 > T_2$$

$$\therefore \eta_{E1} = 1 - \frac{T_2}{T_1}$$

$$\text{As } \eta_{E2} = 1 - \frac{T_2}{T_2}$$

$$\Rightarrow \eta_{E1} > \eta_{E2}$$

$$\Rightarrow \frac{W_1}{Q_1} > \frac{W_2}{Q_2}$$

$$\Rightarrow \boxed{W_1 > W_2}$$

Ans: 11

(a)



According to Clausius statement, without work input, Refrigerator is not possible so

$$(COP) = \frac{Q_2}{W} \quad (W=0)$$

$$(COP)_R = \infty \quad \text{so it will never be } \infty$$

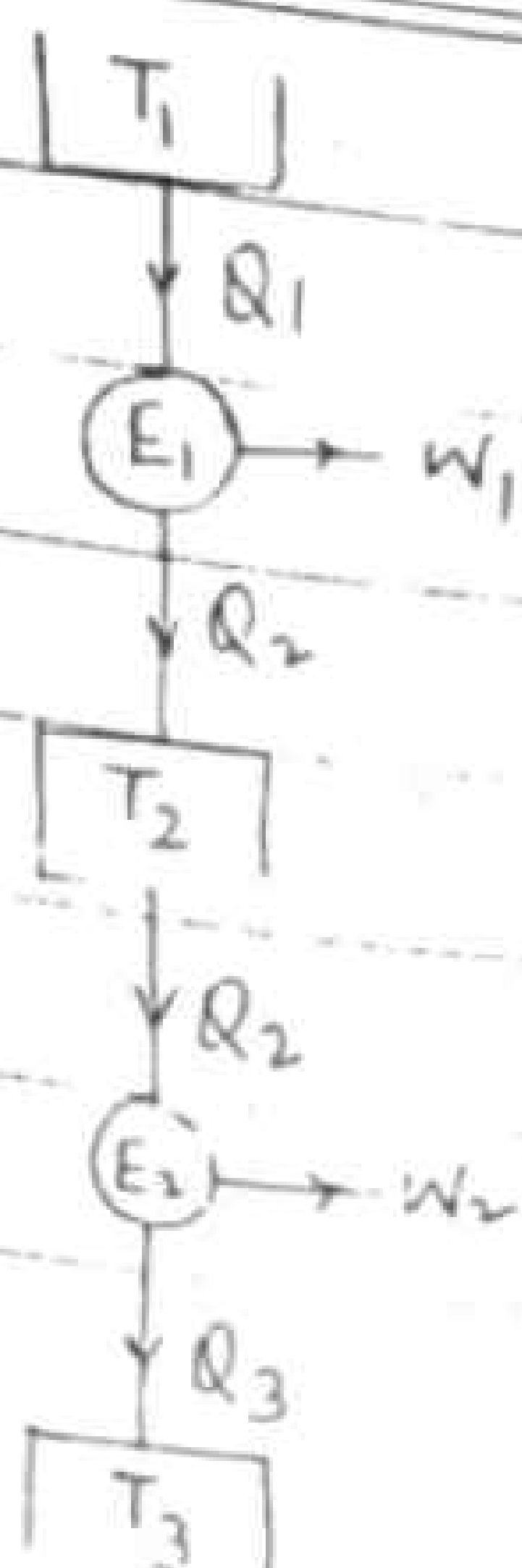
Ans: 12

(a)

$$\eta_{max} = 1 - \frac{T_2}{T_1} \quad \text{--- (1)}$$

$$(COP)_R = \frac{T_2}{T_1 - T_2} = \frac{T_2/T_1}{1 - T_2/T_1} = \frac{1 - \eta_{max}}{\eta_{max}} = \frac{1}{\eta_{max}} - 1 \text{ Ans}$$

16



i) As efficiency of both engines are same.

$$\eta_{E1} = \eta_{E2}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{Q_2}{Q_3}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{T_2}{T_3}$$

$$\Rightarrow \boxed{T_2 = \sqrt{T_1 T_3}} \quad \text{Ans.}$$

ii) As work delivered are same.

$$W_1 = W_2$$

$$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3$$

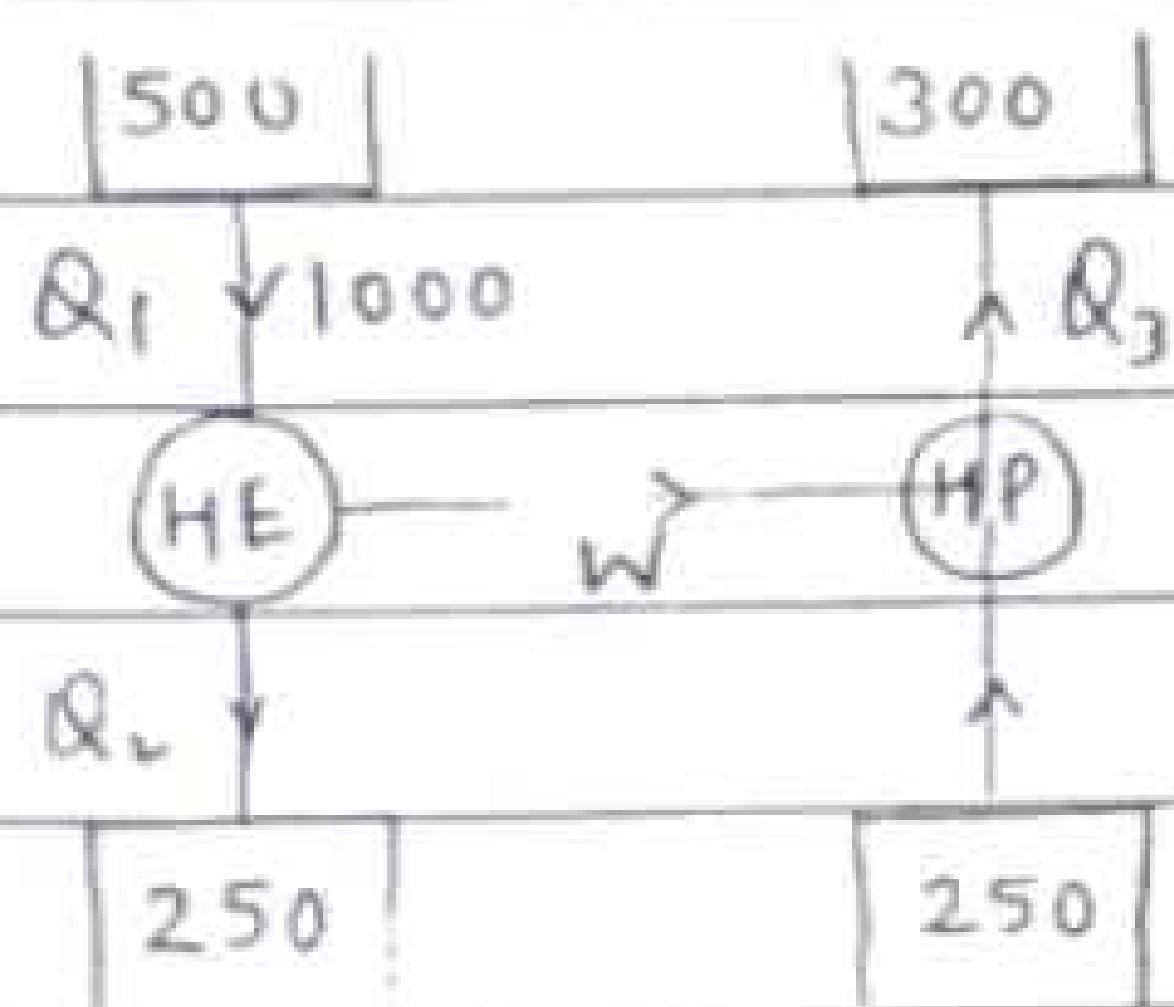
$$\Rightarrow Q_2 = \frac{Q_1 + Q_3}{2}$$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{1 + Q_3/Q_1}{2}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{1 + T_3/T_1}{2}$$

$$\Rightarrow \boxed{T_2 = \frac{T_1 + T_3}{2}} \quad \text{Ans.}$$

18



$$(COP)_{HP} = \frac{T_H}{T_H - T_L}$$

$$\Rightarrow (COP)_{HP} = 300/50$$

$$\Rightarrow (COP)_{HP} = 6 \quad \text{Ans.}$$

19

$$W = Q_1 - Q_2$$

$$\Rightarrow W = 1000 - \left(\frac{1000 \times 250}{500} \right)$$

$$\Rightarrow W = 500 \text{ kJ}$$

$$\text{So, } Q_3 = 500 \times 6$$

CHAPTER - 03 (WORKBOOK)

Ans: 08

As compression is adiabatic,

(C) $P_1 = 0.1 \text{ MPa}$
 $T_1 = 300 \text{ K}$

$P_2 = 1 \text{ MPa}$
 $T_2 = ?$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow T_2 = 300 \times 1.9306$$

$$\Rightarrow T_2 = 579.209 \text{ K}$$

Applying SFEE

$$h_1 + \cancel{0} = h_2 + W$$

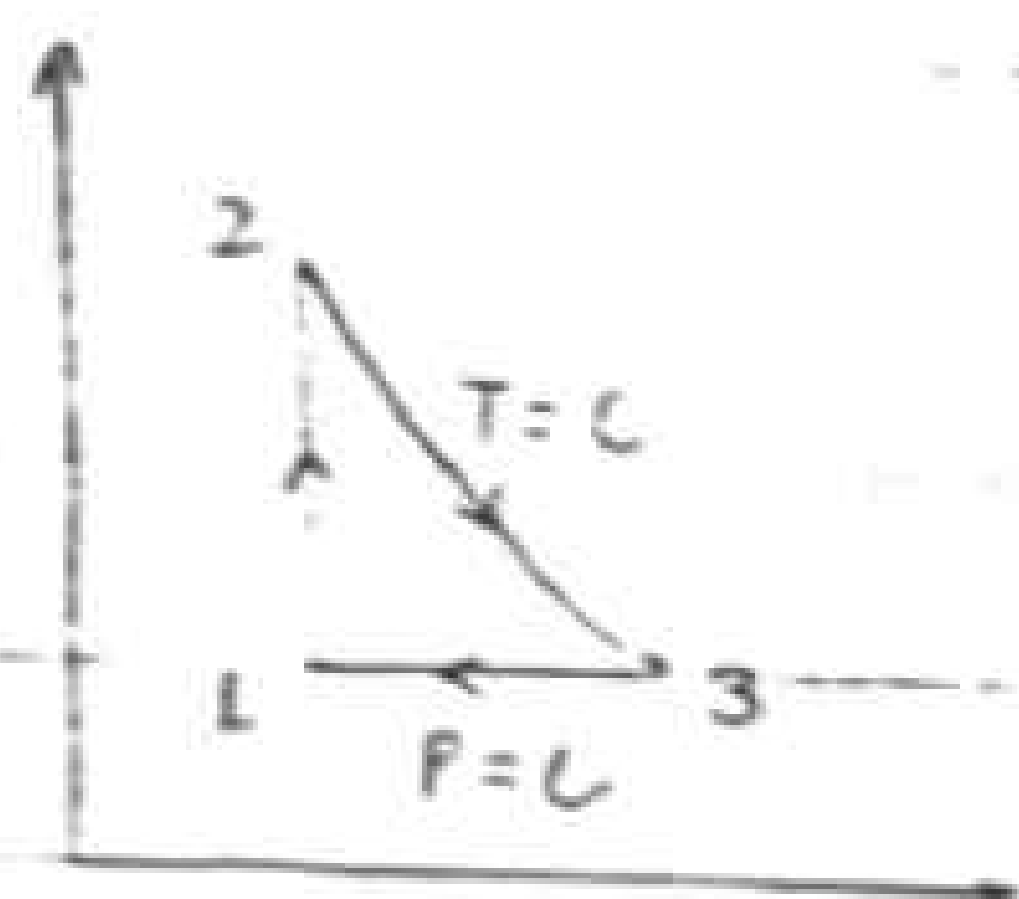
$$\Rightarrow W = h_1 - h_2$$

$$\Rightarrow W = C_p (T_1 - T_2)$$

$$\Rightarrow W = -28.605 \text{ kW} \quad (\text{sign shows comp. work})$$

Ans: 11

(d)



for 1-2 : $\dot{Q}_{12} = 50 \text{ kJ}$

(V=C) $\dot{W}_{12} = 0$

for 2-3 : $\dot{W}_{23} = 500 \text{ kJ}$

T=C $\dot{Q}_{23} = 500 \text{ kJ}$

for 3-1 : $\dot{W}_{31} = -200 \text{ kJ}$

P=C $\dot{Q}_{31} = ?$

Now, $\Sigma \dot{Q} = \Sigma \dot{W}$

$$\Rightarrow 50 + 500 + \dot{Q}_{31} = 0 + 500 - 200$$

$$\Rightarrow \dot{Q}_{31} = -250 \text{ kJ} \quad \text{Ans.}$$

ΔU for 3-1

$$\Rightarrow dQ = dU + dW$$

$$\Rightarrow -250 = dU - 200$$

$$\Rightarrow dU = -50 \text{ kJ} \quad \text{Ans.}$$

Ans: 16

$P_1 = 10 \text{ bar}$

$$\Rightarrow h_1 = h_2$$

(d)

$P_2 = 1 \text{ bar}$

$$\Rightarrow U_1 + P_1 V_1 = U_2 + P_2 V_2$$

$V_1 = 0.5 \text{ m}^3/\text{kg}$

$$\Rightarrow U_2 - U_1 = P_1 V_1 - P_2 V_2$$

$V_2 = 2 \text{ m}^3/\text{kg}$

$$\Rightarrow dU = (10^3 \times 0.5 - 10^3 \times 2) \text{ kJ/kg}$$

for throttling process ($h_1 = h_2$)

$$\Rightarrow dU = (500 - 200)$$

$$\Rightarrow \boxed{dU = 300 \text{ kJ/kg}} \quad \text{Ans.}$$

Ans: 17

(d)

When heat is supplied at constant volume, all the heat $\uparrow$ the internal energy (Δu) and given by $Q_v = m C_v T$

When heat is supplied at const pr., some part of heat $\uparrow$ the du and some do work so $Q_p = m C_p dT$

NOW $\frac{Q_v}{Q_p} = \frac{m C_v dT}{m C_p dT},$

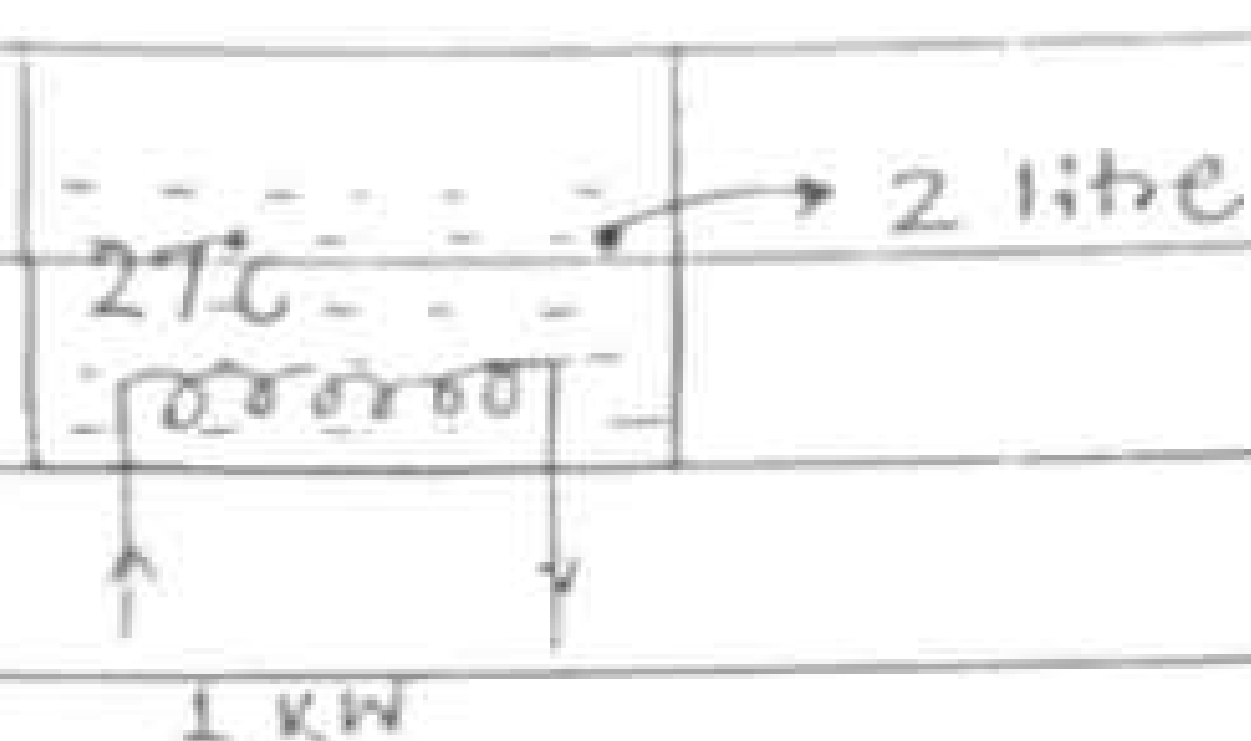
$$\Rightarrow \frac{du}{Q_p} = \frac{C_v}{C_p} = \frac{1}{\gamma}$$

$$\Rightarrow \frac{du}{Q_p} = \frac{5}{7}$$

$$\Rightarrow \boxed{du = \frac{5}{7} (Q_p)}$$

Ans: 23

(a)



$$Q_{\text{dissipate}} = 160 \text{ J/s}$$

Applying energy balance eqⁿ:

$$m C_p (\Delta T) = (1000 - 160) \times t \text{ (J)}$$

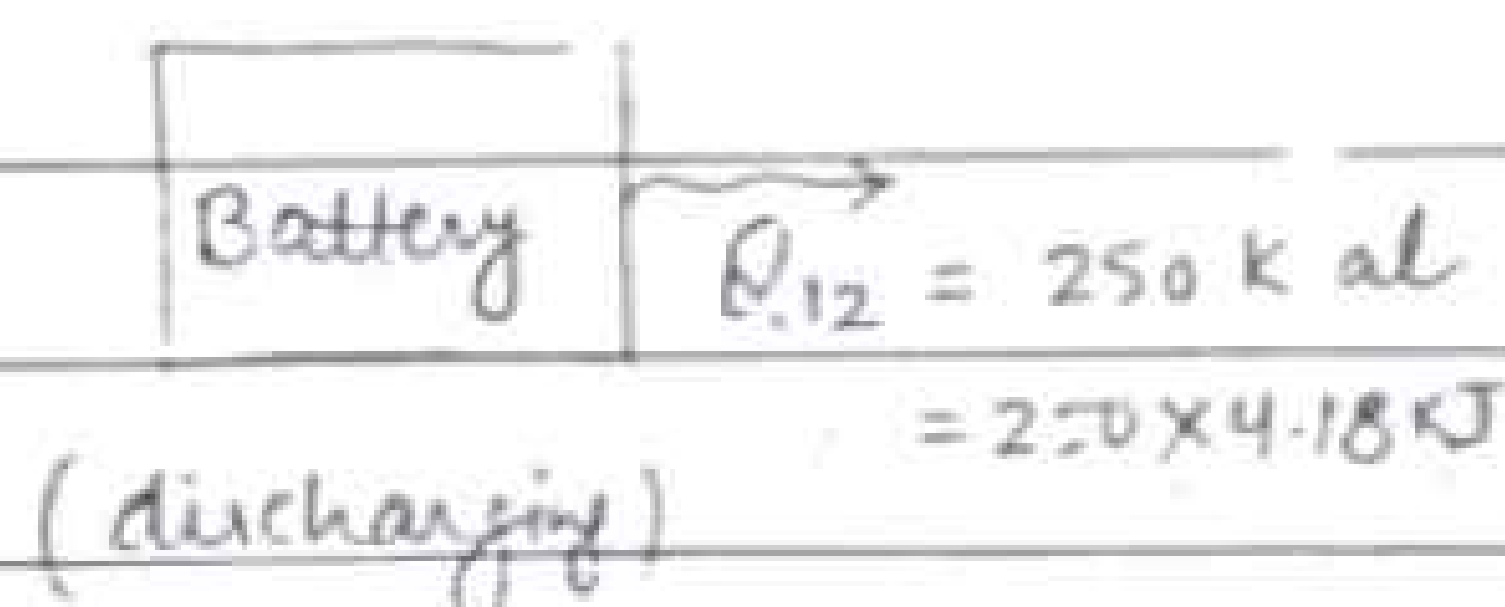
$$\Rightarrow \frac{1000 \times 2 \times 4.2 \times 10^3 \times 50}{1000} = 840 \times t$$

$$\Rightarrow t = 500 \text{ sec.}$$

$$\Rightarrow t = 8 \text{ minute } 20 \text{ sec. Ans.}$$

Ans: 24

(c)



Applying $\Sigma W = \Sigma Q$

$$\Rightarrow +Q_{12} + Q_{21} = W_{12} + W_{21}$$

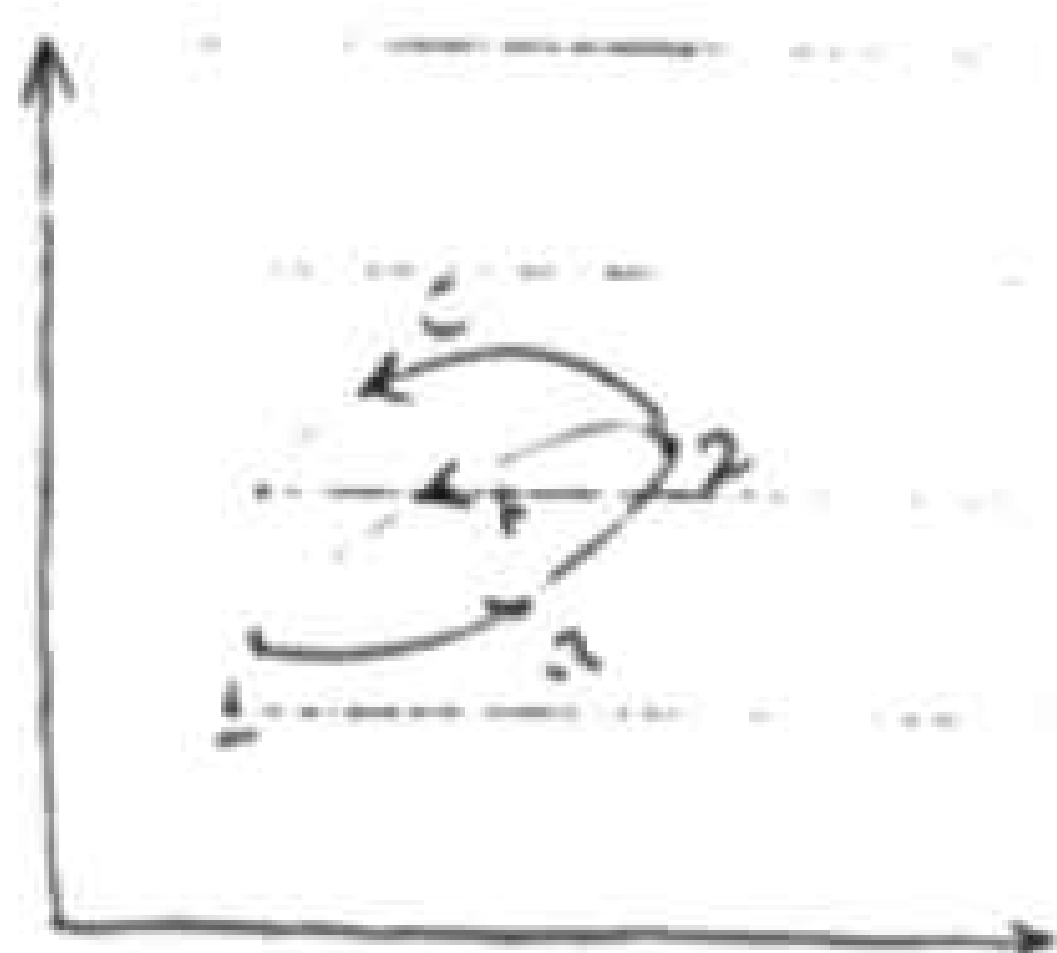
$$\Rightarrow -1045 + Q_{21} = -1908$$

$$\Rightarrow Q_{21} = -861.3 \text{ kJ Ans.}$$

ENTROPY :-

CASE 1 REVERSIBLE CYCLE :

for reversible process



$$\oint_{\text{rev}} \frac{dQ}{T} = 0$$

$$\Rightarrow \left(\frac{dQ}{T} \right)_{1 \rightarrow 2} + \left(\frac{dQ}{T} \right)_{2 \rightarrow 3} + \left(\frac{dQ}{T} \right)_{3 \rightarrow 4} + \left(\frac{dQ}{T} \right)_{4 \rightarrow 1} = 0$$

$$\Rightarrow \left(\frac{dQ}{T} \right)_{1 \rightarrow 2} + \left(\frac{dQ}{T} \right)_{2 \rightarrow 1} = 0$$

1 to 2 : $\rightarrow$ rev. cycle1 to 2 : $\rightarrow$ rev. cycle

$$\Rightarrow \left(\frac{dQ}{T} \right)_{2 \rightarrow 1} = - \left(\frac{dQ}{T} \right)_{1 \rightarrow 2}$$



$$\Rightarrow \left(\frac{dQ}{T} \right)_{2 \rightarrow 1}^{\text{rev}} = - \left(\frac{dQ}{T} \right)_{1 \rightarrow 2}^{\text{rev}}$$

(**) The quantity $\left(\frac{dQ}{T} \right)_{\text{rev}}$ is same for both paths b & c, and it does not depend upon path and it depends only on end points, therefore $\left(\frac{dQ}{T} \right)_{\text{rev}}$ must be a property and this property is known as entropy.

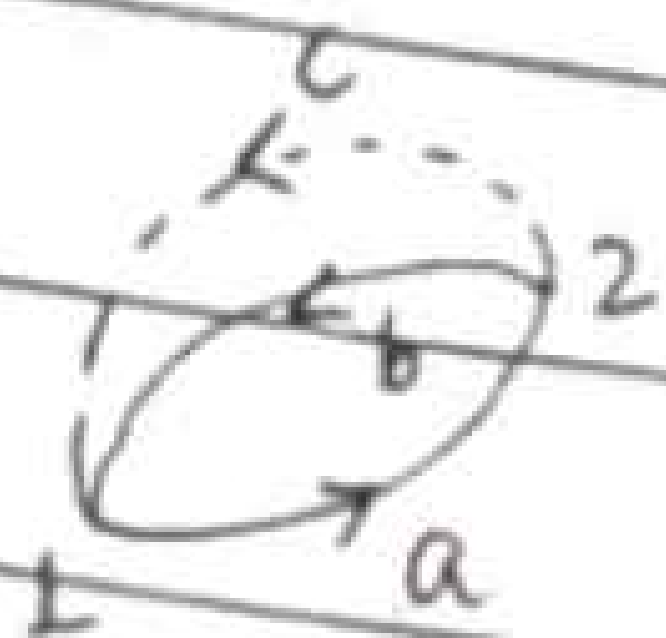
$$\left(\frac{dS}{T} \right)_{2 \rightarrow 1} = \left(\frac{dQ}{T} \right)_{2 \rightarrow 1} = dS$$

$$\Rightarrow \boxed{dS = \left(\frac{dQ}{T} \right)_{\text{rev}}} \quad \text{***} \quad (1)$$

Eqn (1) is 2nd Law of thermodynamics for a process.

CASE 2 IRREVERSIBLE CYCLE :

$$\oint_{\text{irrev}} \frac{dQ}{T} < 0$$



$$\oint_{\text{rev}} \frac{\delta Q}{T} = 0 \Rightarrow \left(\frac{\delta Q}{T} \right)_{a \rightarrow b} + \left(\frac{\delta Q}{T} \right)_{b \rightarrow c} = 0$$

$$\Rightarrow \left(\frac{\delta Q}{T} \right)_{a \rightarrow b} = - \left(\frac{\delta Q}{T} \right)_{b \rightarrow c}$$

$$\text{Now } \oint_{\text{irrev}} \frac{\delta Q}{T} < 0$$

$a \rightarrow b \rightarrow c \rightarrow a$ → rev cycle

$a \rightarrow b \rightarrow c \rightarrow a$ → irrev cycle

$$\Rightarrow \left(\frac{\delta Q}{T} \right)_{a \rightarrow b} + \left(\frac{\delta Q}{T} \right)_{b \rightarrow c} < 0$$

$$\Rightarrow - \left(\frac{\delta Q}{T} \right)_{b \rightarrow c} + \left(\frac{\delta Q}{T} \right)_{b \rightarrow c} < 0$$

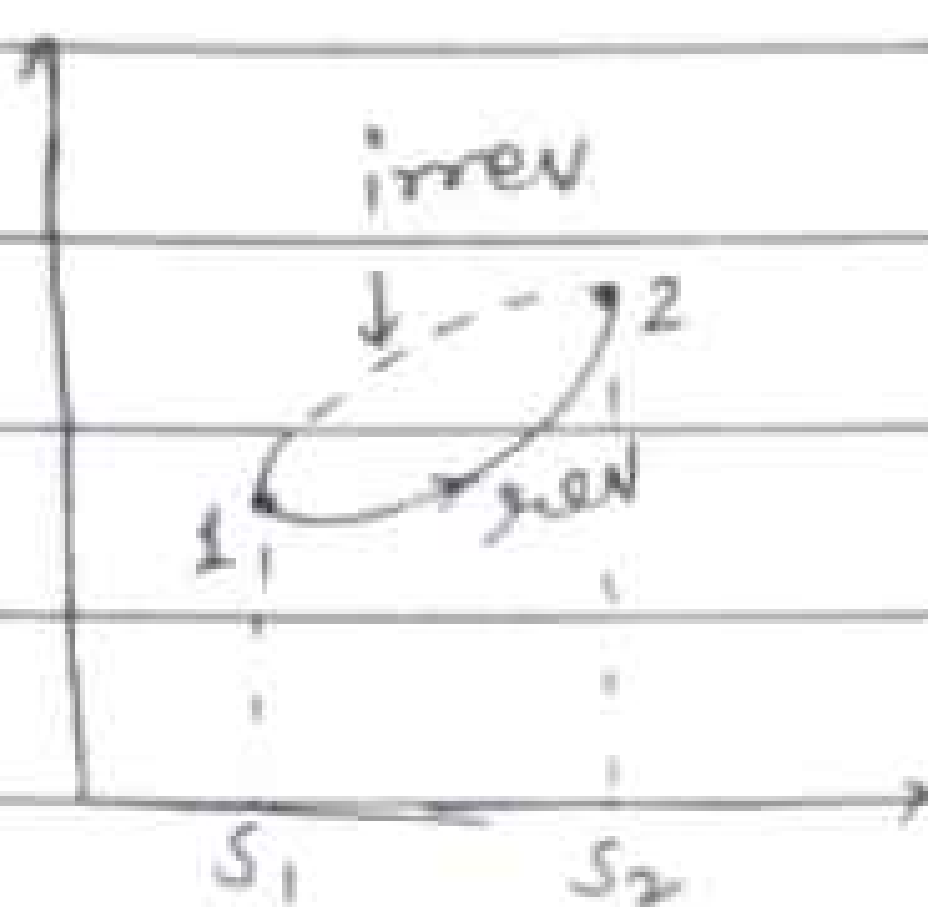
$$\Rightarrow \left(\frac{\delta Q}{T} \right)_{\text{irrev}} < \left(\frac{\delta Q}{T} \right)_{\text{rev}}$$

$$\Rightarrow \left(\frac{\delta Q}{T} \right)_{\text{irrev}} < ds$$

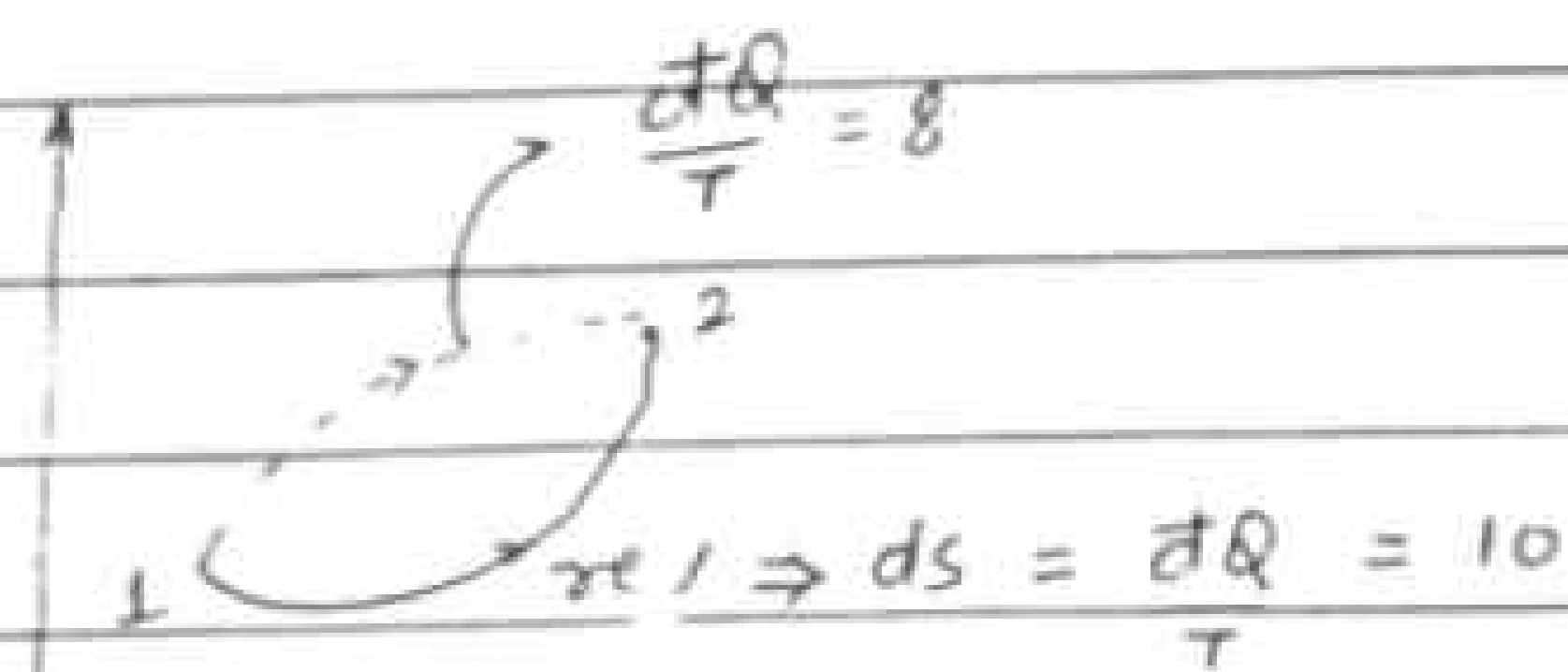
$$\text{So, } ds > \left(\frac{\delta Q}{T} \right)_{\text{irrev}} \quad (2)$$

from eqⁿ (1) & (2)

$$ds > \frac{\delta Q}{T} \quad ***$$



$$(ds)_{\text{rev}} = s_2 - s_1$$



$$\text{So, } (ds)_{\text{rev}} = (ds)_{\text{irrev}}$$

2. As entropy is a property, it depends only on end points and hence if end points are same, entropy change for reversible and irreversible process must be same.

1. To find net entropy change for irreversible process, replace it with a reversible process with same end points and then find the entropy change for this reversible path and the entropy change for irreversible process will be same as obtained for reversible process but this value will be greater than $\left(\frac{dQ}{T}\right)_{\text{irrev}}$

2. OK is not possible b/c at OK it violates second law statement.

→ ENTROPY CHANGE OF A SYSTEM IN REVERSIBLE PROCESS:

CASE 1: When heat is supplied to the system, =

$$dS = \left(\frac{dQ}{T}\right)_{\text{rev}} \rightarrow +ve$$

$$dS = +ve$$

$$dS > 0$$

$$Q_2 > Q_1$$

$$\boxed{Q_2 > Q_1} \quad \text{--- (A)}$$

When heat is added to the system in a reversible process, the system entropy will increase.

CASE 2: When heat is rejected from system =

$$dS = \left(\frac{dQ}{T}\right)_{\text{rev}} \rightarrow -ve$$

$$dS = -ve$$

$$S_2 - S_1 < 0$$

$$\Rightarrow \boxed{S_2 < S_1} \quad \text{--- (0)}$$

When heat is rejected from a system in a reversible manner, system entropy will decrease.

3 Reversible adiabatic process :

$$ds = \left(\frac{dQ}{T} \right)_{\text{rev}} \rightarrow 0$$

$$\Rightarrow ds = 0$$

$$\Rightarrow S_2 - S_1 = 0$$

$$\Rightarrow \boxed{S_2 = S_1 = \text{const}} \rightarrow \text{isentropic} \begin{cases} \rightarrow \text{Reversible} \\ \rightarrow \text{Adiabatic} \end{cases}$$

A reversible adiabatic process is always isentropic process

E: In a reversible process, system entropy can $\uparrow$, can $\downarrow$ or can remain constant depending on heat transfer.

ENTROPY CHANGE FOR IRREVERSIBLE PROCESS (FOR SYSTEM) :-

$$ds > \left(\frac{dQ}{T} \right)_{\text{irrev}}$$

$$\Rightarrow ds = \left(\frac{dQ}{T} \right)_{\text{irrev}} + (\delta S)_{\text{generation}}$$

entropy generation

+ve quantity

$$ds = \left(\frac{dQ}{T} \right)_{\text{rev}} + (\delta S)_{\text{gen}}$$

$$\Rightarrow ds = -3 + 3 \quad \text{irreversible}$$

> Though a reversible adiabatic process is always isentropic and isentropic process need not be reversible adiabatic.

> Adiabatic Process ($dQ=0$)

(Reversible)

$$ds = \left(\frac{dQ}{T} \right)_{rev}$$

$$ds = 0$$

$$S_2 = S_1 = \text{isentropic}$$

(Irreversible)

$$ds = \left(\frac{dQ}{T} \right)_{irr} + (ds)_{gen}$$

$$ds = (ds)_{gen}$$

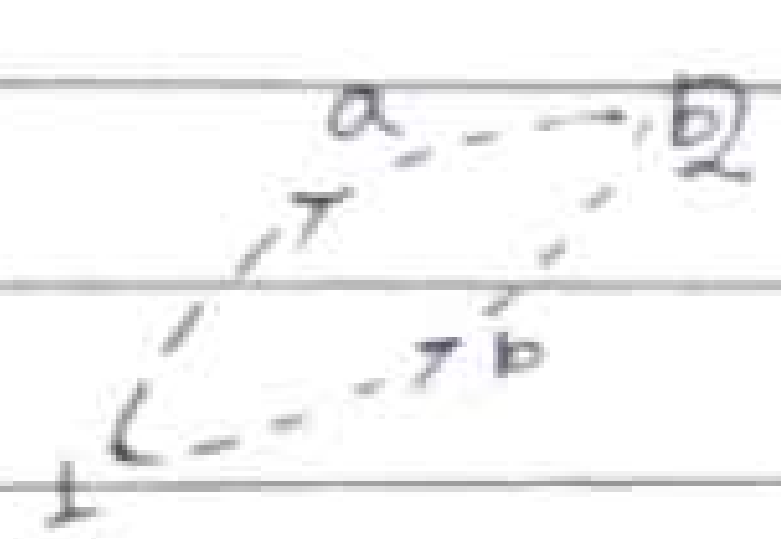
$$ds > 0$$

$$S_2 > S_1 \quad \text{Non isentropic}$$

> An adiabatic process is isentropic only if it is reversible. If the adiabatic process is irreversible, it must be nonisentropic.

** In an adiabatic process, the system entropy will never decrease.

ENTROPY GENERATION :



Let process 'a' be more irreversible than process 'b'

$$irr \rightarrow ds = \left(\frac{dQ}{T} \right)_{irr} + (ds)_{gen}$$

↑

$$rev \rightarrow ds = \left(\frac{dQ}{T} \right)_{rev} + 0$$

As irreversibility ↑, $(ds)_{gen}$ ↑.

Entropy $\rightarrow$ extensive property

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Now for process 'a' to 'b'

$$(ds)_a = \left(\frac{\delta Q}{T}\right)_{\text{rev a}} + (\delta s)_{\text{gen a}}$$

$$(ds)_b = \left(\frac{\delta Q}{T}\right)_{\text{rev b}} + (\delta s)_{\text{gen b}}$$

Now $(ds)_a = (ds)_b$

but $(\delta s)_{\text{gen a}} \neq (\delta s)_{\text{gen b}}$

so $(\delta s)_{\text{gen a}} \neq (\delta s)_{\text{gen b}}$

Though entropy is a property, entropy generation is not a property, it depends upon the extent of irreversibilities. Greater the irreversibility, greater is the entropy generation. Hence entropy generation is a path function and it is not a property.

PHYSICAL MEANING OF ENTROPY :

Entropy is a measure of molecular disorder. Greater the disorder, greater is the entropy and lower is the efficiency.

Entropy is an extensive property.

ENTROPY OF UNIVERSE :

sim

SYS

so, universe is an isolated system.

so for universe $\rightarrow \delta S = 0$

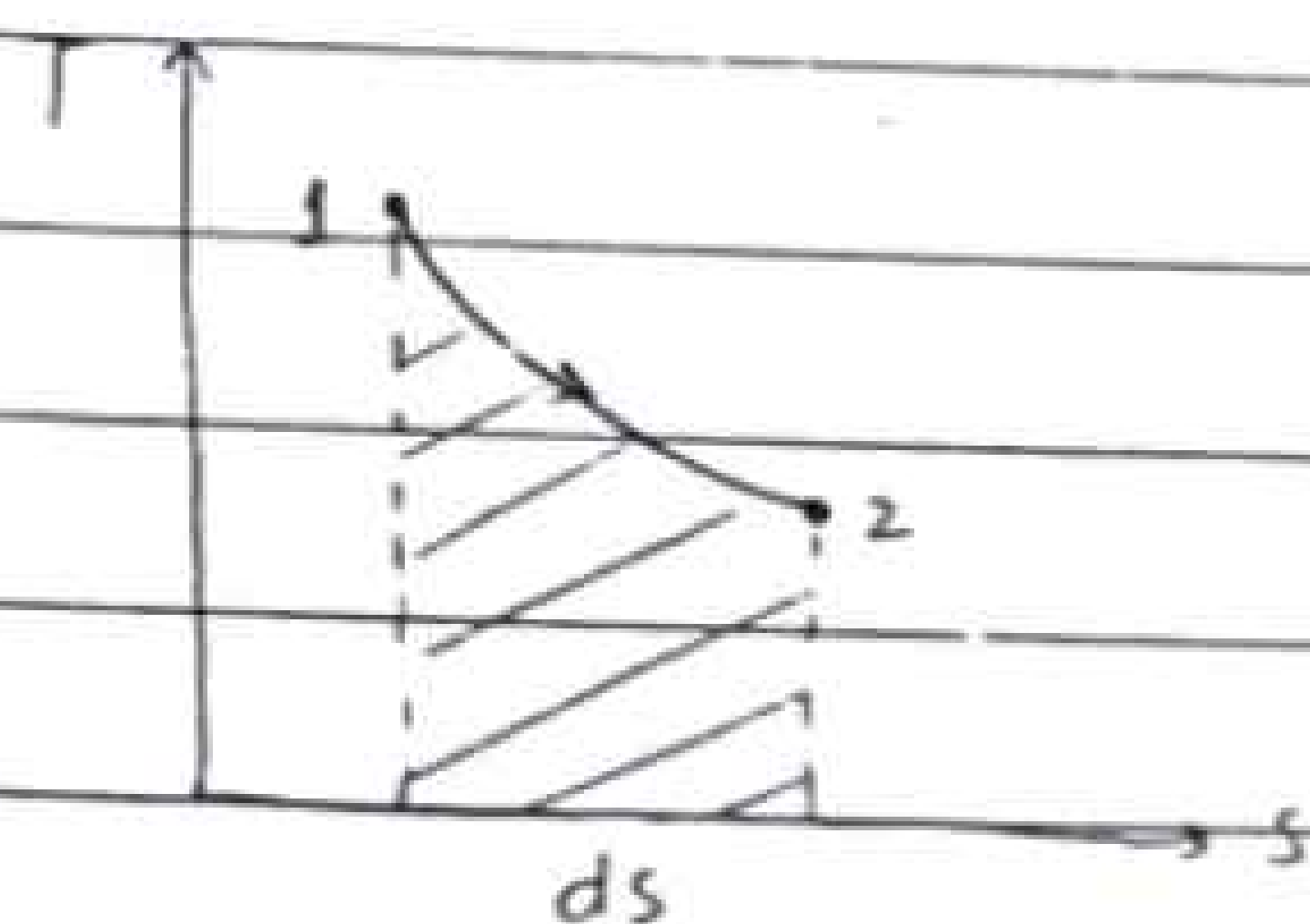
$$\Rightarrow (ds)_{\text{universe}} \geq \left(\frac{0}{T}\right)$$

$$\Rightarrow \boxed{(ds)_{\text{univ}} \geq 0} \quad \text{****} \quad \begin{array}{l} = \text{for reversible} \\ > \text{for irreversible} \end{array}$$

$$\text{SO} \quad \boxed{(ds)_{\text{system}} + (ds)_{\text{sur}} \geq 0}$$

System entropy can $\uparrow$, can $\downarrow$. Similarly surrounding entropy can $\uparrow$, can $\downarrow$ but the entropy change for universe can never decrease, this is known as principle of increase of entropy.

→ T-S Diagram :



$$\text{SO. Area} = T ds \quad \text{---(1)}$$

but for reversible process

$$dQ = T ds \quad \text{---(2)}$$

from (1), (2)

$$\boxed{\text{Area} = dQ_{\text{rev}}}$$

Area under the curve when projected on entropy axis gives reversible heat transfer.

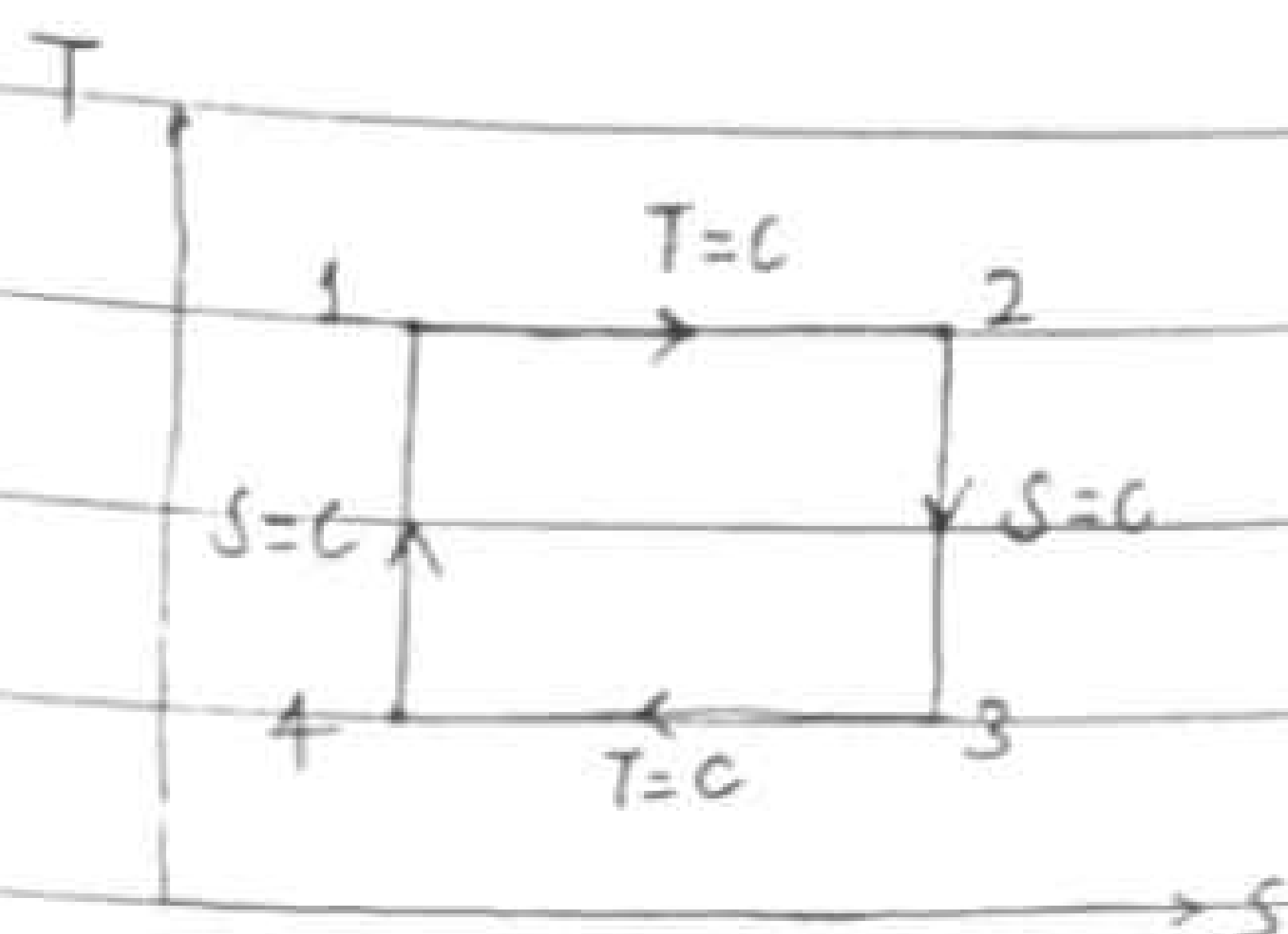
→ Representation of Carnot cycle on T-S diagram :-

1-2 rev isothermal heat addition (expansion)

2-3 rev adiabatic expansion

3-4 rev isothermal heat rejection (compression)

4-1 rev adiabatic compression



COMBINED I Law & II Law of THERMODYNAMICS :-

$$\begin{aligned} dQ &= du + PdV && \text{from I Law} && \text{rev. path} \\ dQ &= T ds && \text{from II Law} && \text{rev. path} \end{aligned}$$

$$\Rightarrow \boxed{Tds = du + PdV} \text{ --- (1) --- for any process}$$

This eqⁿ is valid for reversible as well as irreversible process. (for any process)

$$H = U + PV$$

$$dH = du + PdV + v dP$$

$$dH = dQ + v dP$$

$$\Rightarrow dQ = dH - v dP$$

$$\Rightarrow \boxed{Tds = dH - v dP} \text{ --- (2) --- for all processes}$$

eqⁿ(2) is valid for any process (reversible or irreversible because it connects various properties).

Representation of constant pr. and const. volume line on T.S diagram :-

$$Tds = du$$

$$\Rightarrow Tds = mc_v dT$$

$$\Rightarrow \left(\frac{dT}{ds}\right)_{v=c} = \frac{T}{c_v}$$

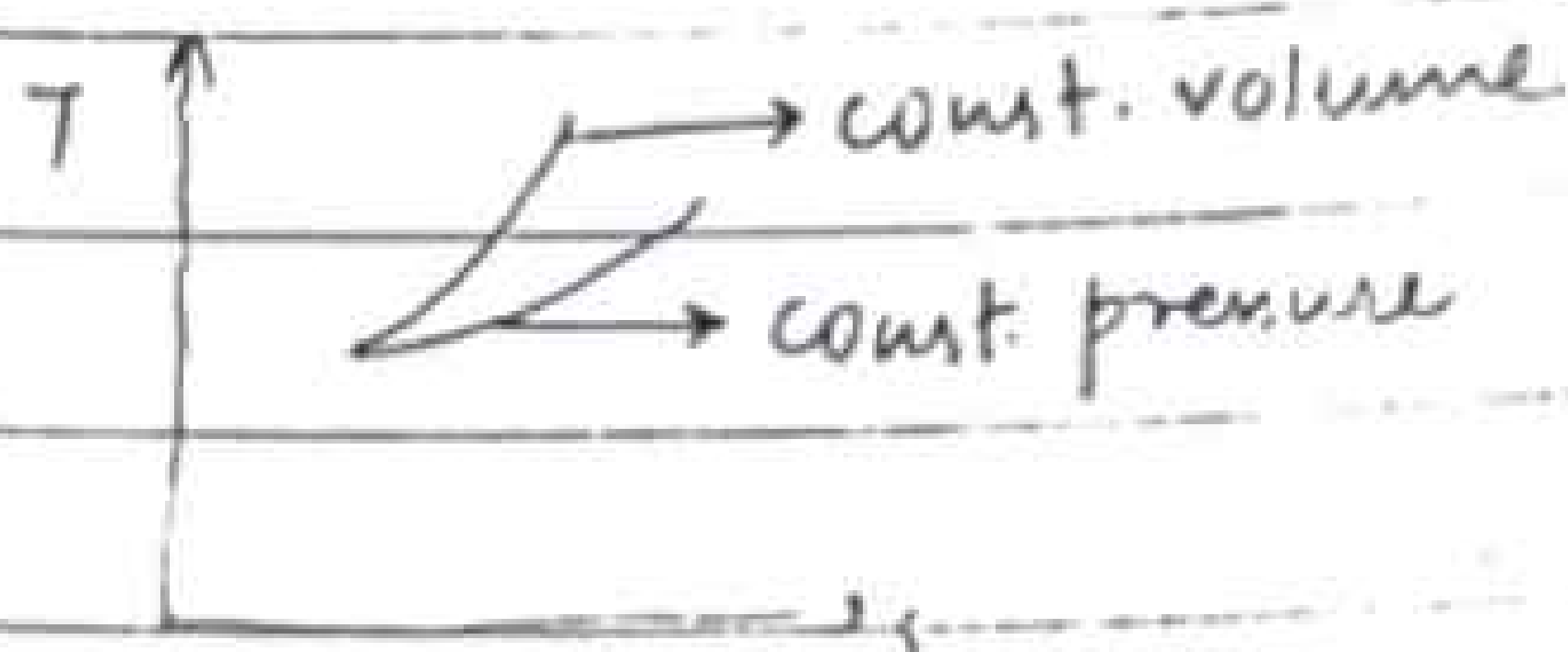
$$Tds = dh$$

$$\Rightarrow Tds = mc_p dT$$

$$\Rightarrow \left(\frac{dT}{ds}\right)_{p=c} = \frac{T}{c_p}$$

$$\text{As } c_p > c_v$$

$$\Rightarrow \left[\left(\frac{dT}{ds}\right)_{p=c} < \left(\frac{dT}{ds}\right)_{v=c}\right]$$



As $c_p > c_v$, slope of constant volume lines is greater than slope of constant pressure lines

ENTROPY CHANGE FOR AN IDEAL GAS :

$$\text{As } Tds = du + Pdv$$

$$\text{for ideal gas : } du = mc_v dT$$

$$PV = mRT$$

$$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} mc_v \frac{dT}{T} + \int_{V_1}^{V_2} \frac{mR}{V} dV$$

$$\Rightarrow s_2 - s_1 = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow \boxed{s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)} \quad \text{--- (1)}$$

$$\text{Also, } Tds = dh - vdp$$

$$\Rightarrow ds = \frac{dh}{T} - \frac{vdp}{T}$$

$$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} mc_p \frac{dT}{T} - \int_{P_1}^{P_2} \frac{mR}{P} dP$$

$$C_p - C_v = R$$

$$\Rightarrow C_p = R + C_v$$

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$$\Rightarrow S_2 - S_1 = m C_p \ln\left(\frac{T_2}{T_1}\right) - m R \ln\left(\frac{P_2}{P_1}\right)$$

$$\Rightarrow \boxed{S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)} \quad \text{--- (2)}$$

from eqn (1) $\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} \quad (\text{as } PV = mRT)$

$$\Rightarrow S_2 - S_1 = C_v \ln\left(\frac{P_2}{P_1} \cdot \frac{V_2}{V_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_v \ln \frac{V_2}{V_1} + R \ln \frac{V_2}{V_1}$$

$$\Rightarrow S_2 - S_1 = C_v \ln\left(\frac{P_2}{P_1}\right) + (C_v + R) \left[\ln\left(\frac{V_2}{V_1}\right) \right]$$

$$\Rightarrow \boxed{S_2 - S_1 = C_v \ln\left(\frac{P_2}{P_1}\right) + C_p \ln\left(\frac{V_2}{V_1}\right)} \quad \text{--- (3)}$$

Ques)

1

A closed system executes a reversible cycle 1234561 consisting of 6 processes. During process 1-2 the system receives 1000 kJ of heat at a const. temp. of 500 K. 2-3 is adiabatic expansion in which system temp. decreases from 500 K to 400 K. During 3-4 process 800 kJ of heat is added at a const. temp. of 400 K. 4-5 is adiabatic expansion in which system temp. ↓ from 400 K to 300 K. 5-6 is const. temp. heat rejection at 300 K. 6-1 is adiabatic comp. Represent the cycle on T-S diagram and find net work and efficiency of the cycle.

Ans: 1



$$\text{for 1-2. } ds = \frac{dQ}{T} = \frac{1000}{500} = 2 \text{ kJ/K}$$

Net work done = enclosed area by cycle 1234561

$$\Rightarrow W = 100 \times 4 + 100 \times 2$$

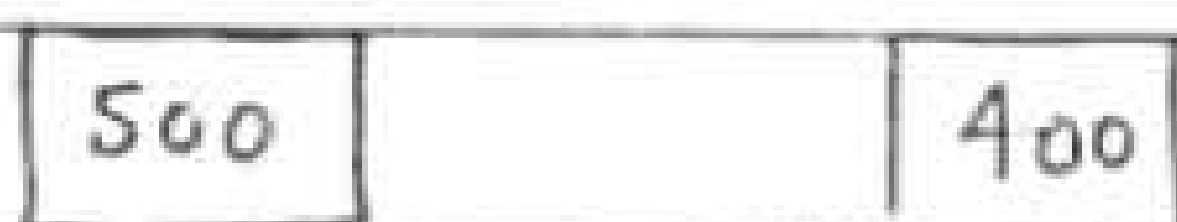
$$\Rightarrow \boxed{W = 600 \text{ KJ}} \quad \text{--- Ans.}$$

A.d. efficiency = $\frac{W}{Q_s}$

$$\eta = \frac{600}{1800}$$

$$\boxed{\eta = 33.33 \%} \quad \text{--- Ans.}$$

Alternate :-



As reversible cycle

$$\Rightarrow \oint \frac{dQ}{T} = 0$$

$$\Rightarrow \frac{1000}{500} + \frac{800}{400} - \frac{Q_R}{300} = 0$$

$$\Rightarrow \underline{\underline{Q_R = 1200 \text{ KJ}}} \quad \checkmark$$

→

$$ds = \left(\frac{dQ}{T} \right)_{\text{irrev}} + (ds)_{\text{gen}}$$

external
interactions

internal
irreversibilities

- Entropy changes occur due to external interactions and internal irreversibilities. In case of reversible process as there are no internal irreversibilities, the entropy change occurs only due to external interactions.

In an adiabatic process, as system does not react with surroundings in the form of heat transfer, therefore entropy change for surroundings is zero in an adiabatic process.

Air is flowing steadily in an insulated pipe. The pr. and temp. at two stations 1 & 2 are given below. Establish the direction of flow assuming air as an ideal gas.

Take $C_p = 1.005 \text{ kJ/kgK}$ and $R = 0.287 \text{ kJ/kgK}$.

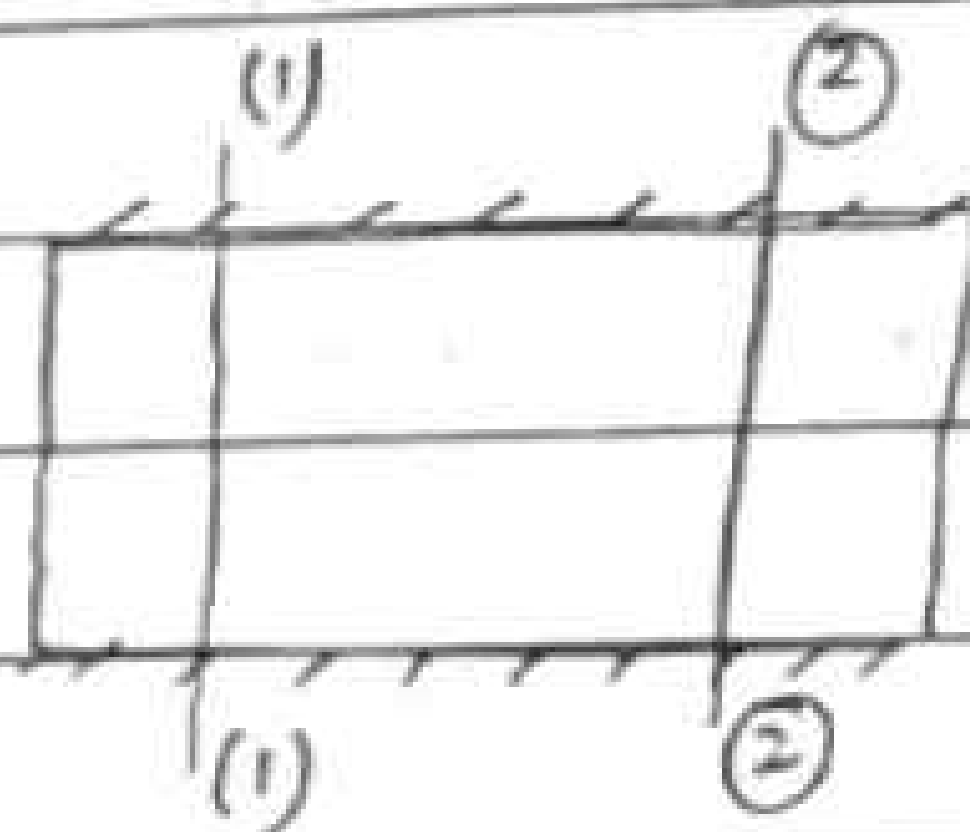
	Station 1	Station 2
$P \rightarrow$	130 kPa	100 kPa
$T \rightarrow$	50°C	13°C

A flow is possible if and only if $(\Delta S)_{univ} \geq 0$

$$\Rightarrow (\Delta S)_{sys} + (\Delta S)_{sur} \geq 0 \quad (\text{As insulated})$$

$$\Rightarrow (\Delta S)_{sys} \geq 0$$

Assume the flow from (1) to (2)

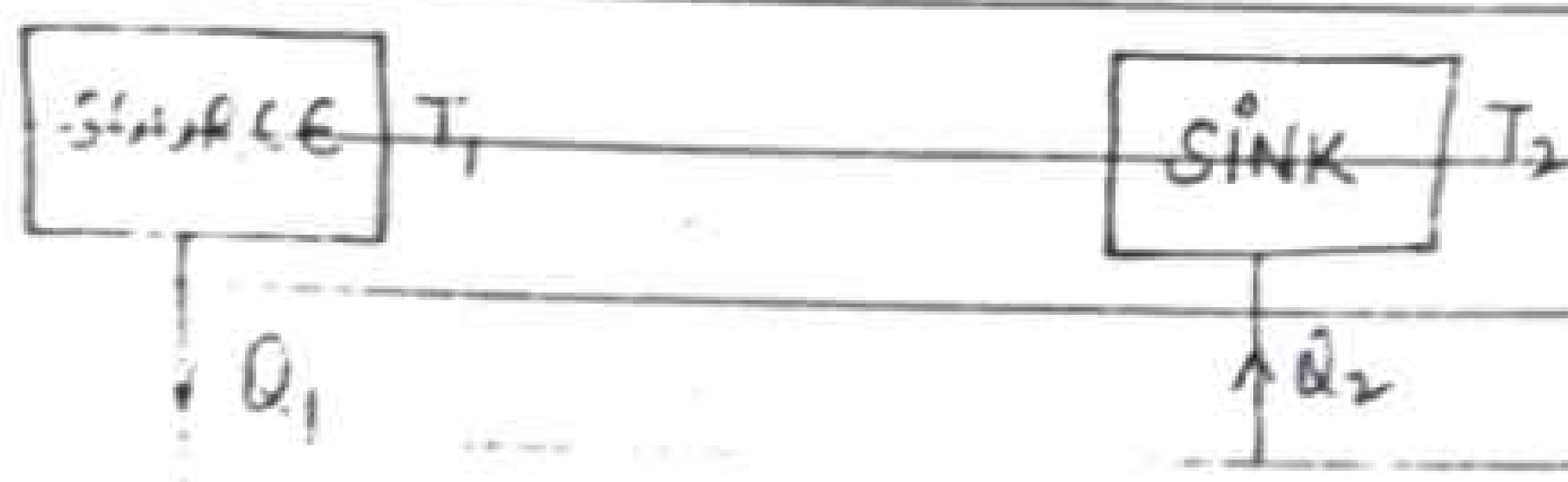


$$\text{Then } (S_2 - S_1) = C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$(\Delta S)_{sys} = 1.005 \ln \left(\frac{273+13}{273+50} \right) - 0.287 \ln \left(\frac{100}{130} \right)$$

$$(\Delta S)_{sys} = -0.0469 \text{ kJ/kgK}$$

ENTROPY CHANGE FOR A RESERVIOR :-



$$ds = \frac{dQ}{T}$$

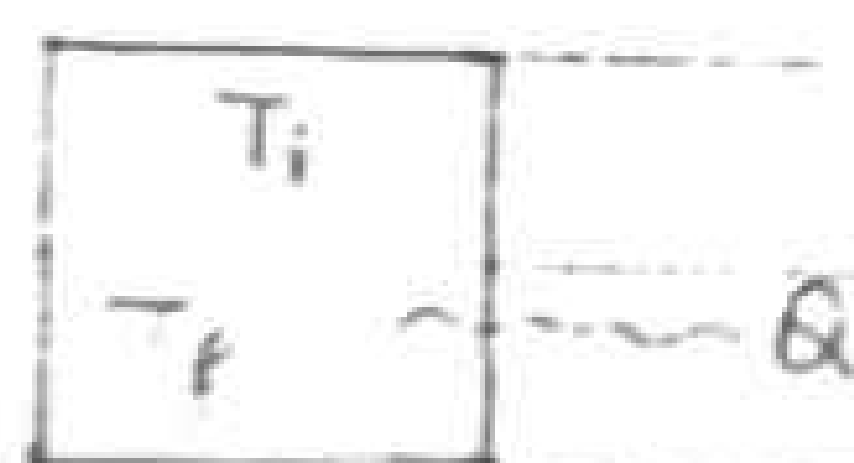
$$ds = \frac{dQ}{T}$$

$$ds = -\frac{Q_1}{T_1}$$

$$ds = \frac{Q_2}{T_2}$$

T_1, T_2 in K.

ENTROPY CHANGE FOR A FINITE BODY :-



$$dQ = mc dT$$

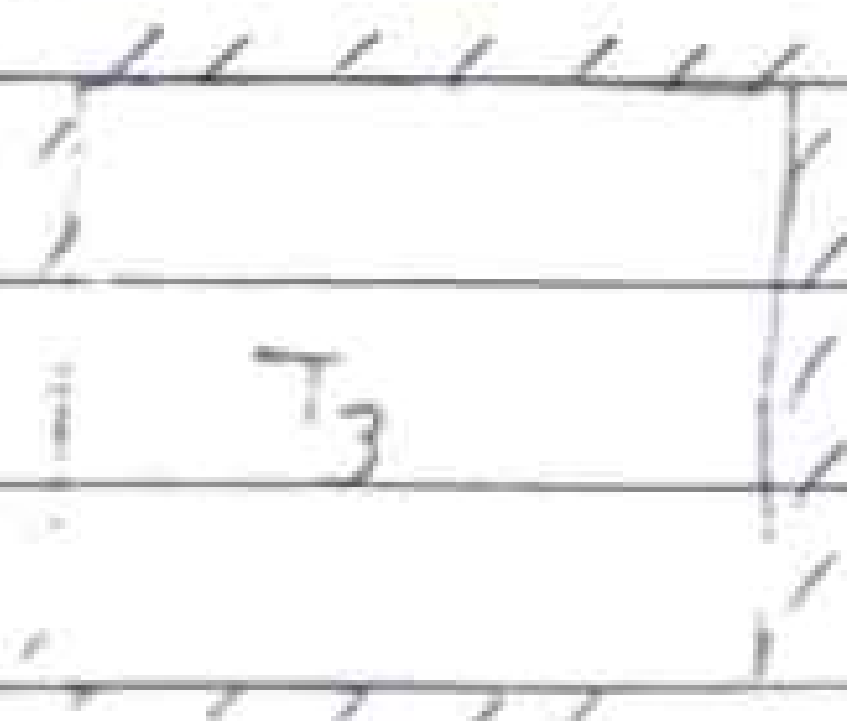
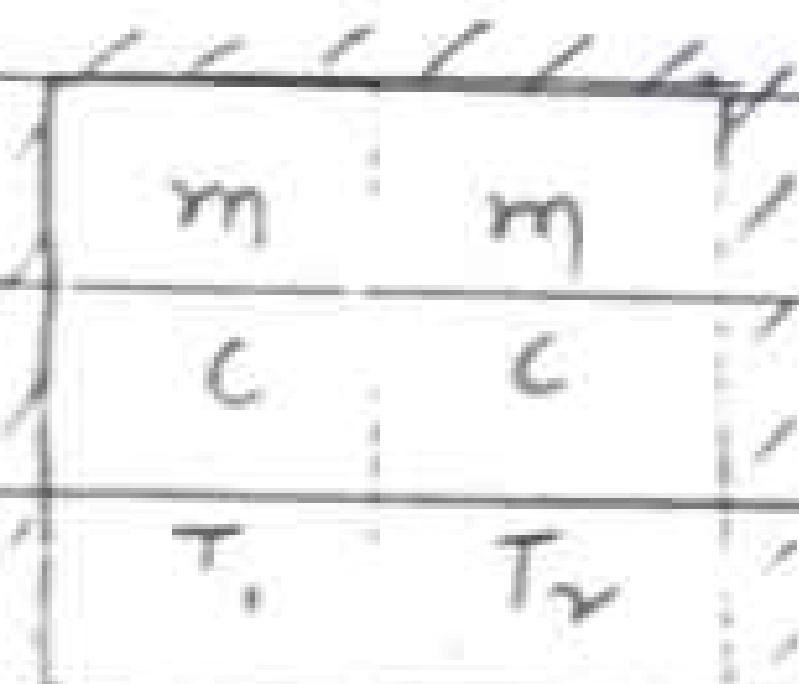
$$ds = \frac{dQ}{T}$$

$$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_i}^{T_f} \frac{mc dT}{T}$$

$$\Rightarrow s_2 - s_1 = mc \ln \left(\frac{T_f}{T_i} \right)$$

*****)
(24)

Q:8(wb) Show that mixing is an irreversible process.



$$\Delta S = \Delta S_1 + \Delta S_2$$

$$\Rightarrow \Delta S = mc \ln \left(\frac{T_3}{T_1} \right) + mc \ln \left(\frac{T_3}{T_2} \right)$$

$$\Rightarrow \Delta S = mc \ln \left(\frac{T_3^2}{T_1 T_2} \right)$$

$$\Rightarrow \Delta S = mc \left\{ \ln \left(\frac{T_3}{\sqrt{T_1 T_2}} \right) \right\}$$

$$\Rightarrow \Delta S = 2mc \ln \left(\frac{T_3}{\sqrt{T_1 T_2}} \right) \quad \text{--- (1)}$$

Now, assume $T_1 > T_2$

so, heat lost by 1 = heat gain by 2

$$\Rightarrow mc(T_1 - T_3) = mc(T_3 - T_2)$$

$$\Rightarrow T_3 = \frac{T_1 + T_2}{2} \quad \text{--- (2)}$$

from (1), (2) $\Delta S = 2mc \ln \left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right] \quad \text{--- (A)}$

As we know

$$AM > GM$$

$$\frac{T_1 + T_2}{2} > \sqrt{T_1 T_2}$$

so, from eqn (A), $\frac{T_1 + T_2}{2(\sqrt{T_1 T_2})}$ will be greater than 1

$$\text{so } (\Delta S)_{\text{system}} > 0$$

And for adiabatic, $(\Delta S)_{\text{sur}} = 0$

$$(\Delta S)_{\text{universe}} = (\Delta S)_{\text{system}}$$

$$\Rightarrow [(\Delta S)_{\text{univ}} > 0] \quad \text{--- (B)}$$

$\Rightarrow$ Mixing process is irreversible.

Q: 9 (WB)

$$\begin{array}{|c|c|} \hline m & T_H \\ \hline \end{array} \quad T_F$$

$$Q_1 = mc(T_H - T_F)$$



$$W = Q_1 - Q_2$$

$$\Rightarrow W = mc[T_H - T_F - T_F + T_L]$$

$$\Rightarrow W = mc[T_H + T_L - 2T_F] \quad \text{--- (1)}$$

$$(\Delta S)_{\text{universe}} \geq 0 \quad (\text{reversible})$$

$$\Rightarrow (\Delta S)_{\text{un}} = 0$$

$$\Rightarrow (\Delta S)_{\text{sys}} + (\Delta S)_{\text{sur}} = 0 \quad \text{--- (i)}$$

Now, system undergoes cycle and for a cycle entropy is properly so $(\Delta S)_{\text{sys}} = 0$

$$\Rightarrow (\Delta S)_{\text{sur}} = 0$$

$$\Rightarrow \Delta S_1 + \Delta S_2 = 0$$

$$\Rightarrow mc \ln\left(\frac{T_f}{T_H}\right) + mc \ln\left(\frac{T_H}{T_f}\right) = 0$$

$$\Rightarrow \ln\left(\frac{T_f^2}{T_L T_H}\right) = 0$$

$$\Rightarrow T_f^2 = T_L T_H$$

$$\Rightarrow T_f = \sqrt{T_L T_H} \quad \text{--- (2)}$$

$$\text{So, } W = mc (T_f + T_H - 2\sqrt{T_L T_H})$$

(25)

Q: 19 (wb) As adiabatic $\Rightarrow (\Delta S)_{\text{sur}} = 0$

$$\therefore (\Delta S)_{\text{system}} = \text{Total } (\Delta S)$$

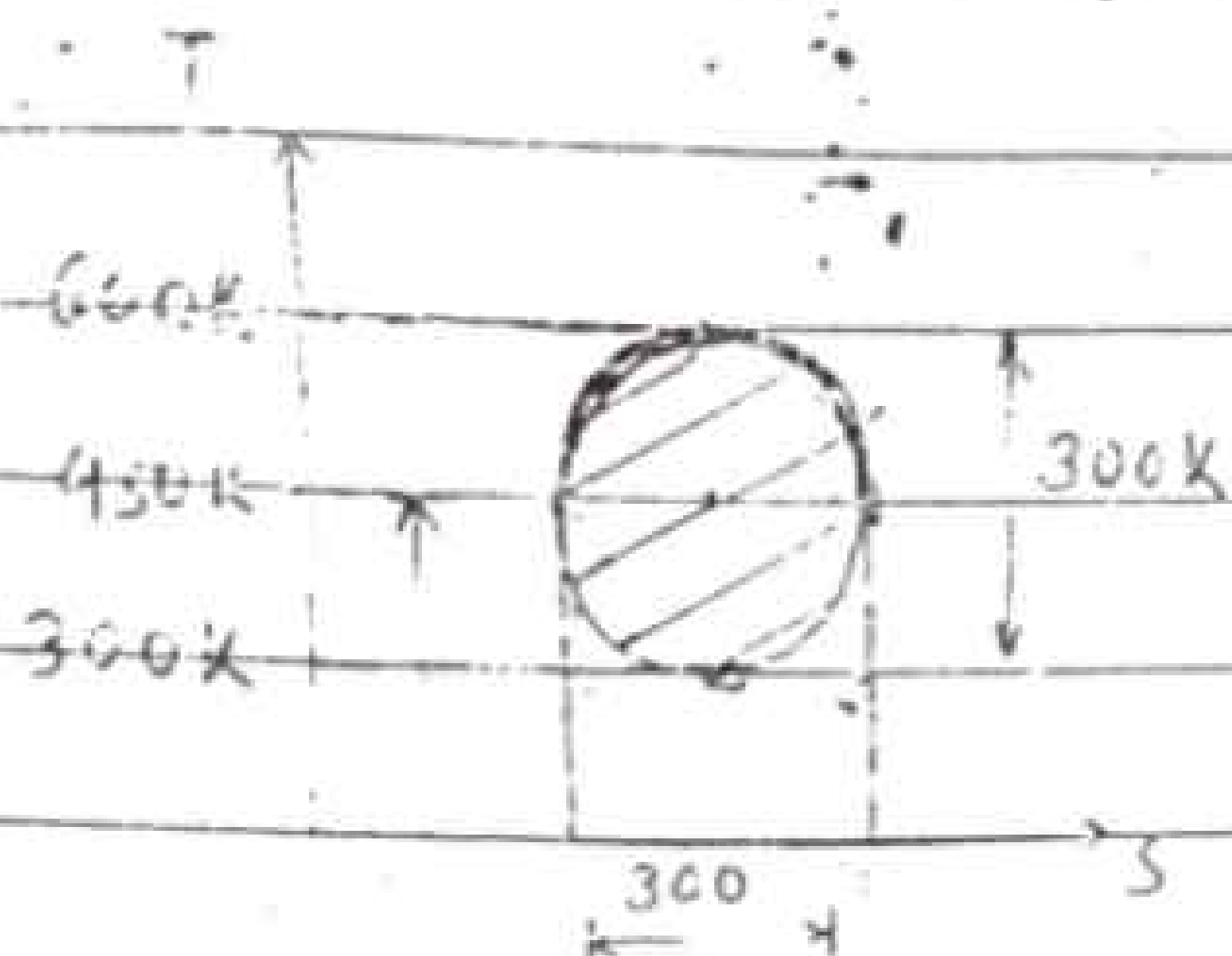
for free expansion : $(T_1 = T_2)$
+ ideal gas

$$(\Delta S)_{\text{system}} = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$= C_v \ln(1) + R \ln(2)$$

$$\Rightarrow (\Delta S)_{\text{system}} = \underline{R \ln 2}$$

Q: 21



$$W_{DCK} = \frac{\pi}{4} (300)^2 = 70685.83$$

$$Q_{\text{supplied}} = 450 \times 300 + \frac{\pi}{4} (300)^2$$

$$Q_{\text{supplied}} = 135000 + 35342.91$$

$$Q_{\text{supplied}} = 170342.91 \text{ KJ}$$

$$\text{So, } \eta = \frac{70685.83}{170342.91} = 0.4149 = 41.5\% \quad \text{Ans}$$

Q: 26

Rigid tank

$$\Delta V = 0 \Rightarrow V_1 = V_2$$

= 0

$$S_2 - S_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow S_2 - S_1 = C_v \ln\left(\frac{T_2}{T_1}\right) \quad T_2 < T_1$$

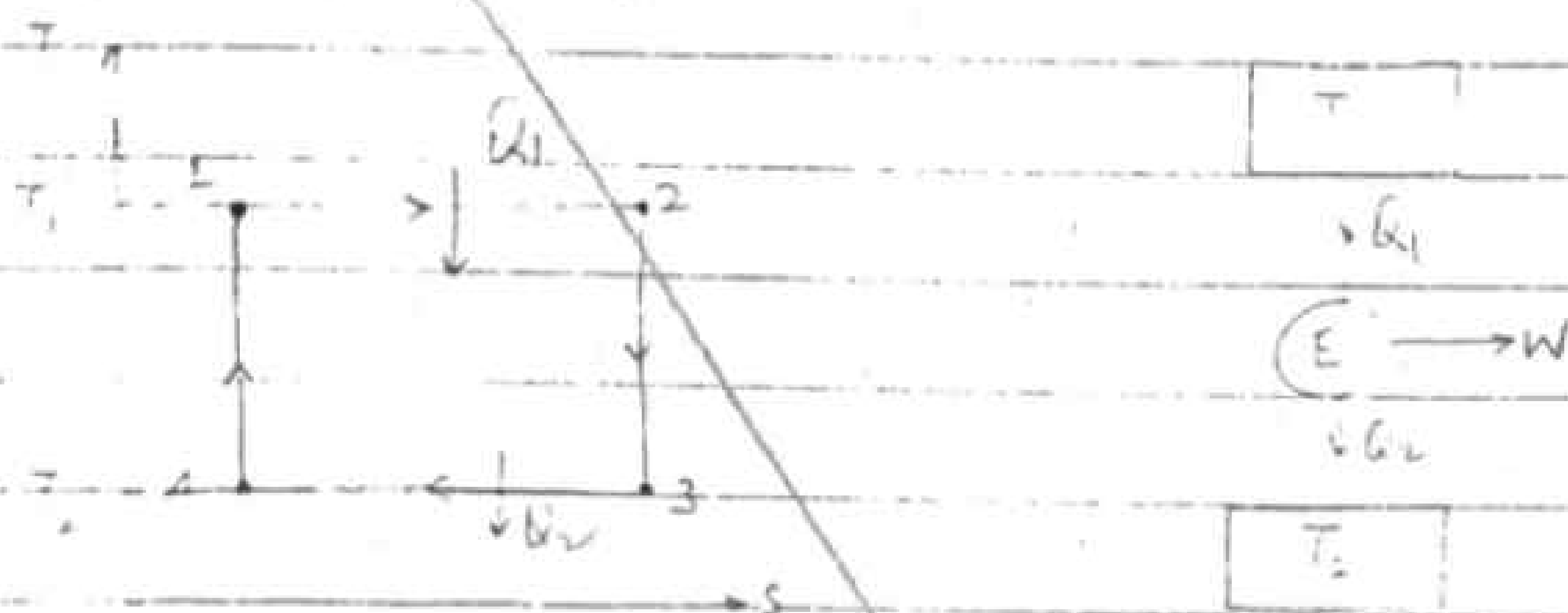
$$\Rightarrow S_2 - S_1 = -C_v \ln\left(\frac{T_1}{T_2}\right)$$

$$\Rightarrow \Delta S < 0$$

→ AVAILABLE ENERGY, AVAILABILITY & IRREVERSIBILITY :-

1) AVAILABLE ENERGY :

The maximum possible amount of work that can be obtained in a cycle is known as available energy.



$$\eta = \frac{W}{Q_1}$$

$$\eta_{\max} = \frac{W_{\max}}{Q_1} \quad \text{---(1)} \quad$$

$$\eta_{\max} = 1 - \frac{T_2}{T_1} \quad (\text{for reversible cycle}) \quad \text{---(2)} \quad$$

$$1 - \frac{T_2}{T_1} = \frac{W_{\max}}{Q_1}$$

$$\Rightarrow \boxed{W_{\max} = Q_1 \left(1 - \frac{T_2}{T_1} \right)}$$

This work is further maximum for a given value of Q_1 & T_1 when the temp of heat rejection (T_2) is minimum. And the lowest possible temp of heat rejection is i.e. of surroundings (T_0), therefore

$$\boxed{AE = Q_1 \left(1 - \frac{T_0}{T_1} \right)} \quad \text{---(3)}$$