Q 7(b) find eqⁿ of sphere which touches plane $3x+2y-z+2=0$, at point $(1,-2,1)$ & also cuts the sphere, $x^2+y^2+z^2-4x+6y+4=0$ Orthogonally.

Ans Let the eqⁿ of Reqᵈ Sphere Be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \quad -(1)$$

The Eqⁿ of Tangent plane to the sphere at the point $(1,-2,1)$ is

$$x(1)+y(-2)+z(1)+u(x+1)+(y-2)v+w(z+1)+d=0$$

$$(u+1)x+(v-2)y+(w+1)z+(u-2v+w+d)=0 \quad -(2)$$

→ But the given plane is

$$3x+2y-z+2=0 \quad -(3)$$

On Comparing (2) & (3)

$$\frac{u+1}{3}=\frac{v-2}{2}=\frac{w+1}{-1}=\frac{u+w+d-2v}{2}=\lambda\,(\text{say}) \quad -(4)$$

* Again the sphere cuts the given sphere orthogonally

$$2u(-2)+2v(3)+2w(0)=d+4$$

$$\boxed{6v-4u=d+4} \quad -(5)$$

from 4, $u=3\lambda-1$, $v=2\lambda+2$ → Put in (5)

$$6(2\lambda+2)-4(3\lambda-1)=d+4$$

$$16=d+4$$

$$\boxed{d=12}$$

$\therefore$ from $\underline{\underline{4}}$

$$(3k-1) - 2(2k+2) + (k-1) + d = 2\lambda$$

$$4\lambda = d - 6 \text{ as } d = 12 \Rightarrow \lambda = 3/2$$

$$u = \frac{7}{2}, \ v = 5, \ w = -\frac{5}{2}$$

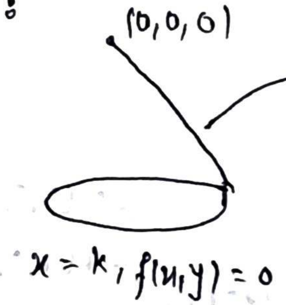
eq^n 1

Req^n Eq^n: $\boxed{x^2 + y^2 + z^2 + 7x + 10y - 5z + 12 = 0}$

Q8 (a) Prove that the eq^n of cone whose vertex is Origin & base of the curve

$z = k$, $f(x,y) = 0$ is $f\left(\frac{xk}{z}, \frac{yk}{z}\right) = 0$

Sol^n:



$$\frac{x-0}{l} = \frac{y-0}{m} = \frac{z-0}{n} \quad \text{—} \ (1)$$

$$\frac{x}{l} = \frac{y}{m} = \frac{k}{n}$$

$$P(x,y,z) = \left(\frac{kl}{n}, \frac{km}{n}, k\right)$$

Satisfies $f(x,y) = 0$

$$\therefore \ f\left(\frac{kl}{n}, \frac{km}{n}\right) = 0 \quad \text{—} \ (2)$$

from eq^n (1)

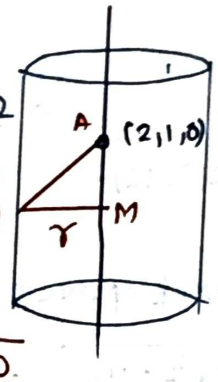
$$\frac{x}{z} = \frac{l}{n} \ \& \ \frac{y}{z} = \frac{m}{n}$$

$\therefore$ (2) becomes $f\left(\frac{xk}{z}, \frac{yk}{z}\right) = 0 \rightarrow$ Eq^n of Cone

Q8 The eq^n of right circular cylinder whose axis is $\frac{(x-2)}{2} = \frac{(y-1)}{1} = \frac{z}{3}$
(b)
and which passes through (0,0,1)

Sol^n $AP^2 = PM^2 + AM^2$

$(2-0)^2 + (1-0)^2 + (0-1)^2 = r^2 + \left[\frac{2(2-0)+1(1-0)+3(0-1)}{\sqrt{2^2+1^2+3^2}}\right]^2$



$r^2 = 6 - \frac{4}{14} \Rightarrow r^2 = \frac{40}{7} \Rightarrow r = \sqrt{\frac{40}{7}}$

$\therefore$ Now for $P(x,y,z)$; $AP^2 = PM^2 + AN^2$

$(2-x)^2 + (1-y)^2 + (0-z)^2 = \frac{40}{7} + \left[\frac{2(x-2)+1(y-1)+3(z-0)}{\sqrt{14}}\right]^2$

$\downarrow$

$(x-2)^2 + (y-1)^2 + z^2 - \left[\frac{2(x-2)+(y-1)+3z}{\sqrt{14}}\right]^2 = \frac{40}{7}$

$14[(x-2)^2 + (y-1)^2 + z^2] - [4(x-2)^2 + (y-1)^2 + 9z^2 + 4(x-2)(y-1)$
$+12z(x-2) + 6z(y-1)] = 80$

$\rightarrow 10(x-2)^2 + 13(y-1)^2 + 5z^2 - 4xy + 4x + 8y - 8 - 12zx + 24x - 6zy + 6z = 80$

$10(x^2 - 4x + 4) + 13(y^2 - 2y + 1) + 5z^2 - 4xy - 12zx - 6zy + 30z + 4x + 8y = 80$
-8

$10x^2 + x(4-40) + 40 + 13y^2 + y(8-26) + 30z + 13 - 8 - 6zy - 4xy +$
$5z^2 - 12zx = 80$

$\rightarrow 10x^2 + 13y^2 + 5z^2 + 36x - 18y + 30z + 45 - 12zx - 6zy - 4xy = 80$

Req^r eq^n of Right Circular Cone : $[10x^2 + 13y^2 + 5z^2 - 6yz - 12zx - 4xy - 36x - 18y + 30z - 35 = 0]$

2019 - 3D

Q7(a) Eqⁿ of sphere whose great circle $x^2+y^2+z^2=16;\ 2x-3y+6z=7$

1b(7)

Ans Eqⁿ of sphere through the circle $x^2+y^2+z^2=16;\ 2x-3y+6z=7$

is $(x^2+y^2+z^2-16)+\lambda(2x-3y+6z-7)=0$

Centre of sphere: $\left(-\lambda, \frac{3\lambda}{2}, -3\lambda\right)$

If the circle ~~eg~~ is great circle then $\left(-\lambda, \frac{3\lambda}{2}, -3\lambda\right)$ will lie on plane $2x-3y+6z=7$

$$-2\lambda-\frac{9\lambda}{2}-18\lambda=7 \Rightarrow \cancel{-49\lambda=14}$$

$$-49\lambda=14 \Rightarrow \boxed{\lambda=-2/7}$$

$$x^2+y^2+z^2-16-\frac{2}{7}(2x-3y+6z-7)=0$$

$$\boxed{x^2+y^2+z^2-\frac{4x}{7}+\frac{6y}{7}-\frac{12z}{7}-14=0}$$

$$\text{Centre}=\left(\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}\right)$$

$$\text{Radius}=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2+14}$$

$$r=\sqrt{15}$$

Find the locus of the vertex of cone whose guiding curve is standard ellipse in xy plane, and curve of intersection of cone and plane $x = 0$ is rectangular hyperbola. 10

Q 8(a) Eqⁿ of Right circular cone, Vertex → Origin

Axis = $x = y = z$

Generator, $2x = 3y = -5z$

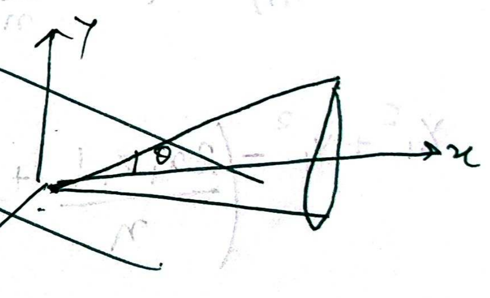
Ans If semi-vertical angle is α, Axis → $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$

$$\cos\alpha = \frac{l(x-\alpha) + m(y-\beta) + n(z-\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}, \quad (\alpha, \beta, \gamma) \to V$$

$$\cos\alpha = \frac{1(x-0) + 1(y-0) + z(1-0)}{\sqrt{3} \cdot \sqrt{x^2+y^2+z^2}}$$

* Generator ⇒ $\frac{x-0}{\cancel{30}\ 15} = \frac{y-0}{10} = \frac{z}{-6}$

$(15, 10, -6)$



* Angle b/w gen^n & Axis is also α ∴

$$\cos\alpha = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}} = \frac{1 \cdot 15 + 1 \cdot 10 + 1 \cdot -6}{\sqrt{3} \cdot \sqrt{10^2+15^2+6^2}} = \frac{\cancel{19}}{\sqrt{3}\sqrt{\cancel{19^2}}}$$

$$= \frac{1}{\sqrt{3}}$$

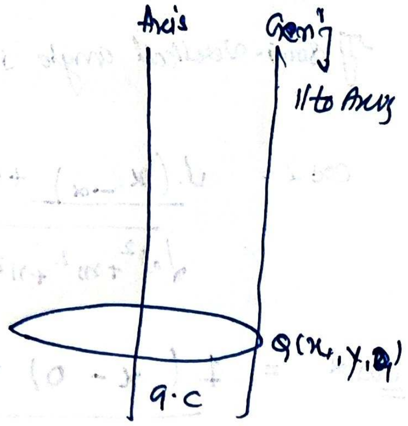
$$\therefore \frac{1}{\cancel{\sqrt{3}}} = \frac{x+y+z}{\cancel{\sqrt{3}}\sqrt{x^2+y^2+z^2}} \Rightarrow x^2+y^2+z^2 = (x+y+z)^2$$

$$\boxed{xy + yz + zx = 0}$$

Q8(b). Eq^n of cylinder, whose axis is $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

& Guiding curve $x^2+y^2+2gx+2fy+c=0 ; z=0$

Ans The eq^n of Gen^r = $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$



This meets Base at $z=0$: so

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{-z_1}{n}$$

$$Q(x,y,z) = \left(x_1 - \frac{z_1 l}{n} , y_1 - \frac{z_1 m}{n} , 0 \right)$$

Satisfies Base.

$$\left(x_1 - \frac{z_1 l}{n}\right)^2 + \left(y_1 - \frac{z_1 m}{n}\right)^2 + 2g\left(x_1 - \frac{z_1 l}{n}\right) + 2f\left(y_1 - \frac{z_1 m}{n}\right) + c = 0$$

$$x_1^2 + y_1^2 - \left(\frac{2x_1 z_1 l}{n} + \frac{2y_1 z_1 m}{n}\right) + \frac{2z_1^2 m_1^2}{n^2} + 2gx_1 - 2gz_1\frac{l}{n}$$

$$+2fy_1 - 2fz_1\frac{m}{n} \Big) = 0$$
$$+c$$

$$x_1^2 + y_1^2 - \frac{2z_1 m}{n}\left(x_1 + y_1 + g + f\right) + \frac{2z_1^2 m^2}{n^2} + 2fy_1 + 2gx_1 = 0$$
$$+c$$

Locus: $\Rightarrow \left[x^2 + y^2 + \frac{2m^2}{n^2} z^2 + 2gx_1 + 2fy_1 - \frac{2zm}{m}(x+y+g+f) + c = 0 \right]$

2018

Q7(a). A sphere of radius a passes through origin & meet the axes in A, B, C show that locus of centroid of tetrahedron OABC is $x^2+y^2+z^2 = (a/2)^2$

* Ans, Eqⁿ of sphere which passes through origin, cutting axes at $(a,0,0), (0,b,0)$ & $(0,0,c)$.

$$\rightarrow x^2+y^2+z^2-ax-by-cz=0$$

. The radius of the sphere is given to be R

$$\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2+\left(\frac{c}{2}\right)^2} = R$$

$$\left[a^2+b^2+c^2 = 4R^2\right]$$

. If the co-ordinate of the centroid of the tetrahedron OABC is (x,y,z), then

$$x = \frac{0+a+0+0}{4} = \frac{a}{4}, \quad y = \frac{b}{4}, \quad z = \frac{c}{4}$$

$$a = 4x, \quad b = 4y, \quad c = 4z$$

Eliminating a, b, c B/w ① & we get the Reciprocal locus.

$$16(x^2+y^2+z^2) = 4R^2$$

$$x^2+y^2+z^2 = \frac{a^2}{4}$$

$$\boxed{x^2+y^2+z^2 = \left(\frac{a}{2}\right)^2}$$

Q7(b) locus of vertex of cone whose G.C → standard Ellipse in xy plane & curve of intersectn of cone & plane x = 0 is a rectangular Hyperbola.

Ans Ellipse in x-y plane , ~~Rectangular Hyperbola in yz-plane~~

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \cancel{\frac{y^2}{c^2} - \frac{z^2}{d^2} = 1}$$

- ~~Now, consider the cone eqn in terms of x, y, z.~~

- Vertex (α, β, γ)

- Genr - $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \Rightarrow \frac{l}{n} = \frac{x-\alpha}{z-\gamma}, \quad \frac{m}{n} = \frac{y-\beta}{z-\gamma}$

General Point $= \left(\alpha - \frac{l\gamma}{n}, \beta - \frac{\gamma m}{m}, 0\right)$

Ellipse $\Rightarrow \frac{1}{a^2}\cdot\left(\alpha - \frac{l\gamma}{n}\right)^2 + \frac{1}{b}\left(\beta - \frac{\gamma m}{n}\right)^2 = 1$

$$\frac{1}{a^2}\left\{\alpha - \gamma\left(\frac{x-\alpha}{z-\gamma}\right)\right\}^2 + \frac{1}{b^2}\left\{\beta - \gamma\frac{(y-\beta)}{(z-\gamma)}\right\}^2$$

$$\frac{1}{a^2}\{\alpha z - x\gamma\}^2 + \frac{1}{b^2}\{\beta z - y\gamma\}^2 = \{z-\gamma\}^2$$

* it is a Rectangular hyperbola in yz plane ∴ coeff of y^2 + coeff of $z^2 = 0$

$$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{b^2} - 1 = 0$$

$$\Rightarrow \quad \alpha, \beta, \gamma \rightarrow x, y, z$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1}$$

8(a)

Ans Let $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ be the remaining gen^{s} of the cone $11yz + 6xz - 14xy = 0$

$\perp^r$ to

$\frac{x}{1} = \frac{y}{2} = z$

& lie on

$\therefore \quad l + 2m + n = 0 \quad \{ l_1 l_2 + m_1 m_2 + n_1 n_2 = \cos 90^\circ = 0 \}$

$\rightarrow l = -(2m+n)$

&

$11mn + 6 ln - 14 lm = 0$

$11mn - (2m+n)(6n - 14m) = 0$

$11mn = (2m+n)(6n-14m)$

$11mn = 6n^2 - 28m^2 - 2mn$

$13mn = 6n^2 - 28m^2$

$28m^2 + 13mn - 6n^2 = 0$

$28m^2 + 21mn - 8mn - 6n^2 = 0$

$7m(4m+3n) - 2n(4m+3n) = 0$

$(7m - 2n)(4m + 3n) = 0$

$7m - 2n = 0$	$4m + 3n = 0$
$m = \frac{2n}{7}$	$m = \frac{-3n}{4}$
$l = -\left(\frac{4n + 7n}{7}\right)$	$l = -\left(\frac{-6n + 4n}{4}\right)$
$l = -\frac{11n}{7}$	$= \frac{+2n}{4} = \frac{n}{2}$

$\cancel{l} m : n : l$

$\frac{2m}{7} : m : \frac{-11}{7}$

$2 : 7 : -11$

$l : m : n$

$\frac{n}{2} : \frac{-3n}{4} : n$

$2 : -3 : 4$

So General^{r}

$\frac{x}{-11} = \frac{y}{2} = \frac{z}{7}$ & $\frac{x}{2} = \frac{y}{-3} = \frac{z}{4}$

Q8(b)

Ans's : $\frac{x}{4} = \frac{y-2}{1} = \frac{z-0}{3}$

Gen^{l} : $\frac{x-\alpha}{4} = \frac{y-\beta}{1} = \frac{z-\gamma}{3}$; $z=0$

General point $\Rightarrow \left(\alpha - \frac{4\gamma}{3}, \beta - \frac{\gamma}{3}, 0\right)$

~~on~~ ~~base~~

$\downarrow$

$4x^2 - 2y^2 = 1$

$\downarrow$

$4\left(\alpha - \frac{4\gamma}{3}\right)^2 - 2\left(\beta - \frac{\gamma}{3}\right)^2 = 1$

$4(3\alpha - 4\gamma)^2 - 2(3\beta - \gamma)^2 = 9$

$4\{9\alpha^2 - 24\alpha\gamma + 16\gamma^2\} - 2[9\beta^2 - 6\beta\gamma + \gamma^2] = 9$

$36\alpha^2 - 18\beta^2 + 62\gamma^2 - 96\alpha\gamma + 12\beta\gamma - 9 = 0$

$\downarrow$ $\alpha \to x$, $\beta \to y$, $\gamma \to z$

$$\boxed{36x^2 - 18y^2 + 62y^2 - 96xz + 12yz - 9 = 0}$$

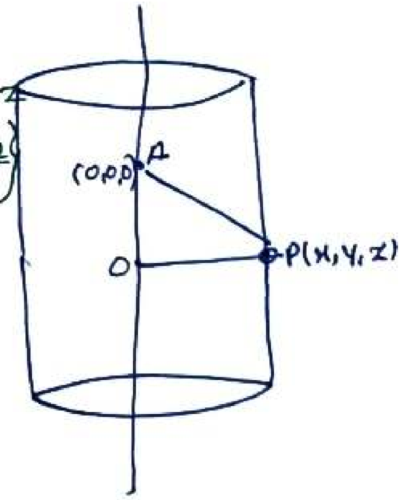
Q 8(b)

Let $P(x,y,z)$ be any point on cylinder, $OP = 4$ Given

* $OP^2 = AP^2 - OA^2$

$(x-0)^2+(y-0)^2+(z-0)^2 - \left\{\frac{2(x-0)+1(y-0)+(z-0)2}{\sqrt{4+1+4}}\right\}^2$

$= 16$



$OA = \frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}$

$x^2+y^2+z^2 - \frac{(2x+y-2z)^2}{9} = 16$

$9(x^2+y^2+z^2) - [4x^2+y^2+4z^2+4xy-4yz-8zx] = 144$

$5x^2+8y^2+5z^2-4xy+4yz+8zx-144=0$

{req^r eq^n of cylinder

eq^n of plane through origin & ⊥^r to Axis

(2) $\cancel{2x+2y-z=0}$ $\cancel{\left\{2 - \frac{1}{2}\alpha + 1\cdot\beta + \frac{1}{2}\gamma = 0\right\}}$

$2x+y-2z=0$

Area of sec^n by by this cylinder $= \pi(4)^2$

$= 16\pi$

$\cos\theta = 2/3$

$\theta \to$ Angle b/w plane & $z=0$

By $z=0$, let Area be A

Above circle is a projec^n of this sec^n on plane $2x+y-2z=0$

$A\cos\theta = 16\pi \Rightarrow A = 3\times 8$

$A = 24\pi$

Q7(b) locus of vertex of cone whose G.C → standard Ellipse in xy plane & curve of intersectⁿ of cone & plane $x=0$ is a rectangular Hyperbola.

Ans Ellipse in x-y plane, ~~Rectangular Hyperbola in yz-plane~~

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \cancel{\frac{y^2}{c^2}-\frac{z^2}{d^2}=1}$$

~~Now, consider the cone eqn in terms of x, y, z.~~

- Vertex (α,β,γ)
- Genʳ: $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n} \Rightarrow \frac{l}{n}=\frac{x-\alpha}{z-\gamma}, \ \frac{m}{n}=\frac{y-\beta}{z-\gamma}$

General Point $=\left(\alpha-\frac{l\gamma}{n}, \beta-\frac{\gamma m}{m}, 0\right)$

Ellipse ⇒ $\frac{1}{a^2}\cdot\left(\alpha-\frac{l\gamma}{n}\right)^2+\frac{1}{b}\left(\beta-\frac{\gamma m}{n}\right)^2=1$

$$\frac{1}{a^2}\left\{\alpha-\gamma\left(\frac{x-\alpha}{z-\gamma}\right)\right\}^2+\frac{1}{b^2}\left\{\beta-\gamma\frac{(y-\beta)}{(z-\gamma)}\right\}^2$$

$$\frac{1}{a^2}\{\alpha z-x\gamma\}^2+\frac{1}{b^2}\{\beta z-y\gamma\}^2=\{z-\gamma\}^2$$

* it is a Rectangular hyperbola in yz plane ∴ coeff of y^2 + coeff of $z^2=0$

$$\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}+\frac{\gamma^2}{b^2}-1=0$$

$$\Rightarrow \quad \alpha,\beta,\gamma\to x,y,z$$

$$\boxed{\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1}$$