Mechenics (maons)

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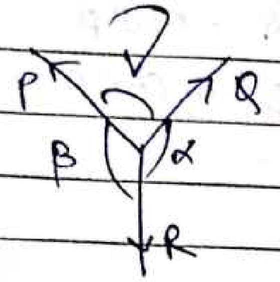
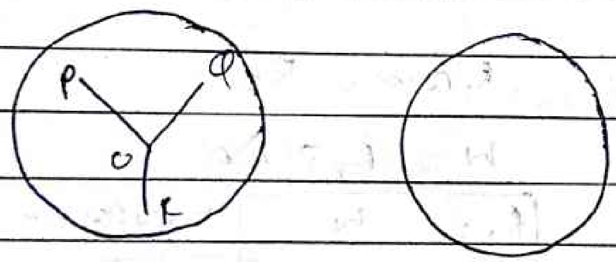
Lamis theorem :-



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

eq^n of a rigid body under the three coplanar forces



If the forces are acting a point of rigid body & they keep a body in eq^n then these 3 forces will satisfy following condition

i) All these 3 F should be in eq^n

ii) ——— ——— either meet at a pt or //.

a) point

i) If these 3 forces meet at a pt (satisfies lamis theorem)

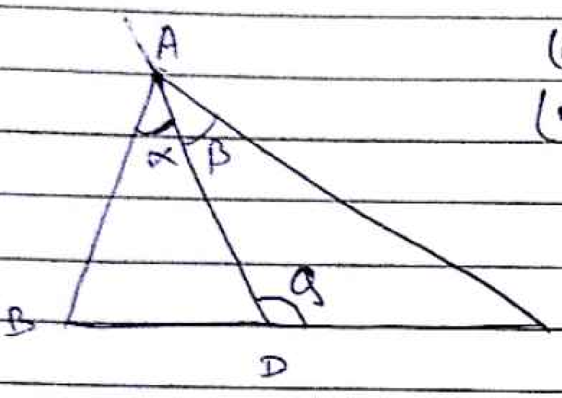
ii) should satisfies law of Δ forces

iii) Sum of resolved parts about a ⊥ dir^n should be = to '0'

b) // (if the forces are //)

i) the moment about a fixed pt in a plane should be zero.

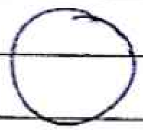
⇒ (m - n) Cot theorem

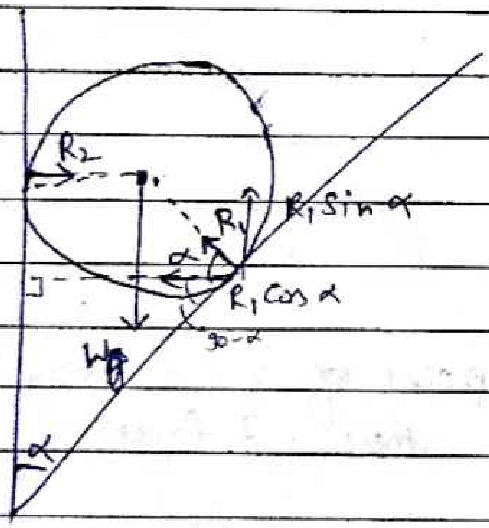


$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$(m+n)\cot\theta = n\cot B - m\cot C$

m : n

⇒  (ω) 9m find Rxn of plane on sphere



$R_1\cos\alpha = R_2$

$W = R_1\sin\alpha$

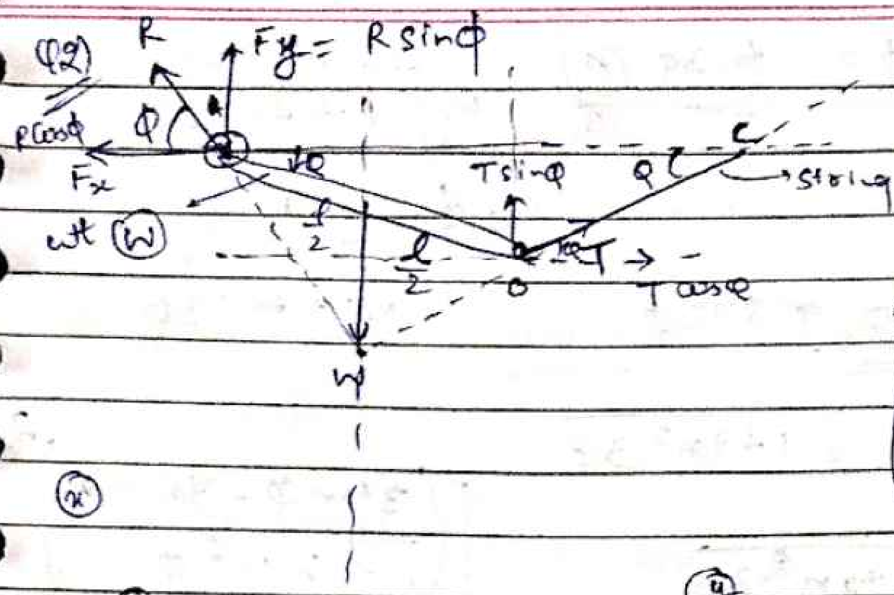
$\boxed{R_1 = \dfrac{W}{\sin\alpha} = W\operatorname{cosec}\alpha}$

$R_2 = R_1\cos\alpha$

$= \dfrac{W\cos\alpha}{\sin\alpha}$

$\boxed{R_2 = W\cot\alpha}$

(Q2)



$$\boxed{\phi = \tan^{-1}(3\tan\theta)}$$

$$\frac{W}{4}\sqrt{8+\operatorname{cosec}^2\theta}$$

or

$$\frac{W}{4}\sqrt{9+\cot^2\theta}$$

(x)

(y)

$R\cos\phi = T\cos\theta$

$$R = \frac{T\cos\theta}{\cos\phi}$$

$R\sin\phi + T\sin\theta = W$

$$T\frac{\cos\theta}{\cos\phi}\sin\phi + T\sin\theta = W$$

$$\cancel{T}\cos\theta\tan\phi + \cancel{T}\sin\theta = \frac{W}{T}$$

$$\tan\phi = \frac{\left(\frac{W}{T} - \sin\theta\right)}{\cos\theta}$$

$$\boxed{\tan\phi = \frac{W}{T\cos\theta} - \tan\theta}$$

$R =$

$\vec{M}$ about (A)

$$(W)\left(\frac{l}{2}\cos\theta\right) - T\sin\theta(l\cos\theta)$$

$$-T\cos\theta(l\sin\theta) = 0$$

$$\frac{W\cos\theta}{2} = T\sin\theta\cos\theta + T\sin\theta\cos\theta$$

$$= 2T\sin\theta\cancel{\cos\theta}$$

$$\boxed{\frac{W}{4\sin\theta} = T}$$

$$\tan\phi = \frac{\cancel{W}(4\sin\theta)}{\cos\theta\ \cancel{W}} - \tan\theta = 3\tan\theta$$

$$\boxed{\phi = \tan^{-1}(3\tan\theta)}$$

$$\boxed{\tan\phi = \frac{\tan 3\varphi}{1}}\ \frac{(P)}{(B)} \qquad h = \sqrt{1+\tan^2 3\varphi}$$

$$R = \frac{T\cos\varphi}{\cos\phi} \qquad \cos\phi = \frac{1}{\sqrt{1+\tan^2 3\varphi}}$$

$$= \cancel{\frac{\cos\varphi}{1}\frac{\sqrt{1+\tan^2 3\varphi}}{W}\ 4\sin\varphi}$$

$$= \frac{W}{4\sin\varphi}\ \frac{\cos\varphi}{1}\ \sqrt{1+\tan^2 3\varphi}$$

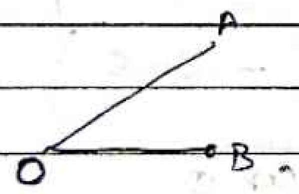
$$= \frac{W}{4\tan\varphi}\sqrt{1+\tan^2 3\varphi}$$

$$= \frac{W}{4}\sqrt{1+\frac{\tan^2 3\varphi}{\tan\varphi}}$$

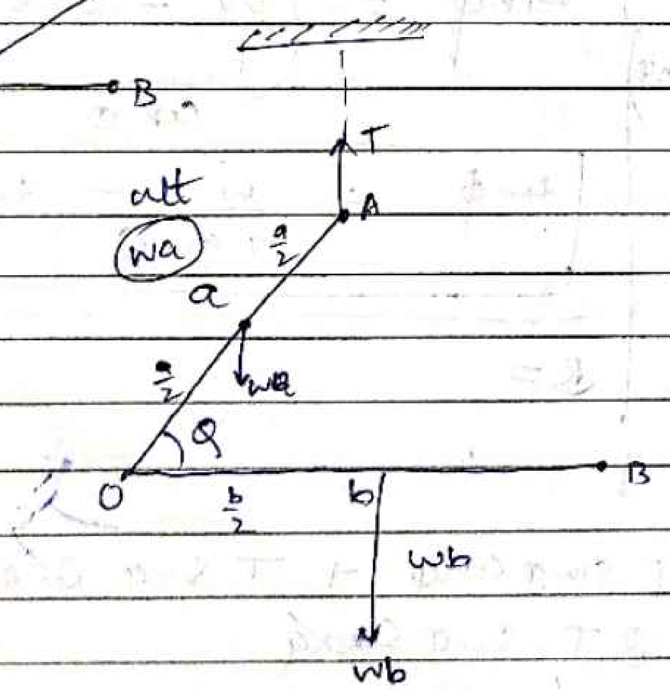
$$\left(\frac{\left(\frac{3\tan\varphi - \tan^3\varphi}{1-3\tan^2\varphi}\right)}{\cancel{\tan^{1/2}\varphi}}\right)^2$$

(Q3) wire A — B

prove $\cos\varphi = \dfrac{b^2}{a(a+2b)}$



let (w) be wt / length



$$\boxed{T = w(a+b)} \qquad M_0 = 0 \qquad T\cos\varphi - wa\left(\frac{a}{2}\right) - wb\left(\frac{b}{2}\right) = 0$$

$$\frac{\cancel{w}b^2}{\cos\varphi} = \cancel{w}(a+b) \qquad T = \frac{\frac{wa^2}{2}+\frac{wb^2}{2}}{\cos\varphi} = \frac{wb^2}{\cos\varphi}$$

$$\boxed{\cos\varphi = \frac{b^2}{(a+b)}}$$

MATHS (mechanics)

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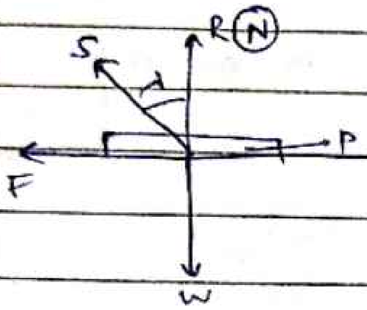
⇒ FRICTION :

STATIC — LIMITATING — KINETIC

ANGLE OF (F) :



coeff of (f) :

$\mu = \frac{F}{R}$

$F = \mu R$

$\boxed{F = \mu N}$

Relation b/w (μ) & (λ) :-

~~$S\sin\lambda = R$~~

~~$S\cos\lambda = F = \mu R$~~

~~$\tan\lambda = \frac{1}{\mu}$~~

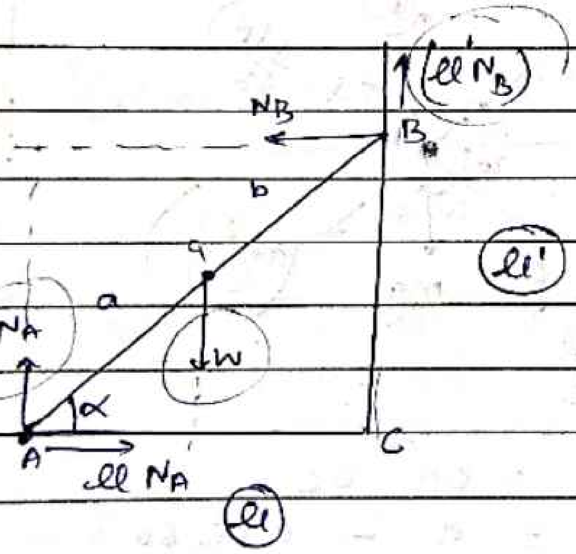
$S\cos\lambda = R$

$S\sin\lambda = F = \mu R$

$\tan\lambda = \mu$

$\boxed{\lambda = \tan^{-1}(\mu)}$

Q: Prove $\tan^{-1}\left(\frac{a - b\mu\mu'}{\mu(a+b)}\right) = \alpha$



$\vec{M}_A = 0$

$- W a\cos\alpha + N_B(a+b)\sin\alpha$

$+ \mu' N_B(a+b)\cos\alpha = 0$

$\mu' N_B + N_A = W$

$N_B = \mu N_A$

$\mu'\mu N_A + N_A = W$

$N_A(1 + \mu\mu') = W$

$\boxed{N_B = \mu\left(\frac{W}{1+\mu\mu'}\right)}$

$N_B(a+b)\left[\sin\alpha + \mu'\cos\alpha\right] = W a\cos\alpha$

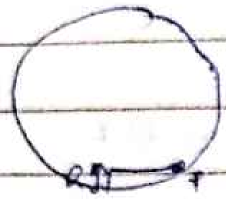
$N_B(a+b)\left[\tan\alpha + \mu'\right] = Wa$

$\tan\alpha = \left[\frac{Wa}{N_B(a+b)} - \mu'\right]$

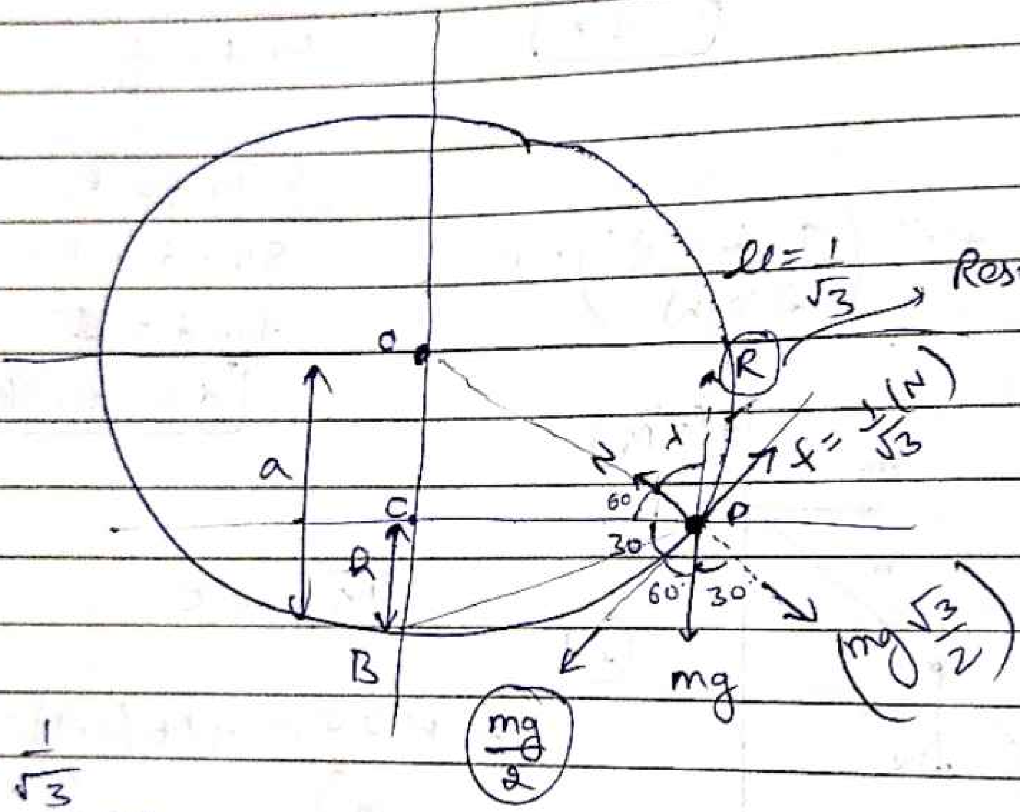
$= \left[\frac{\cancel{W}a}{\frac{\mu \cancel{W}}{1+\mu\mu'}(a+b)} - \mu'\right]$

$$\tan\alpha = \left[\frac{1+\mu\mu'}{\mu(a+b)} - \mu'\right]$$

$$= \left[\frac{1+\mu\mu' - \mu\mu'(a+b)}{\mu(a+b)}\right] = \left[\frac{1+\mu\mu'(1-(a+b))}{\mu(a+b)}\right]$$

Q)  hollow sphere $(h = ?)$ at which particle is at rest

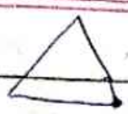
$\mu = \frac{1}{\sqrt{3}}$

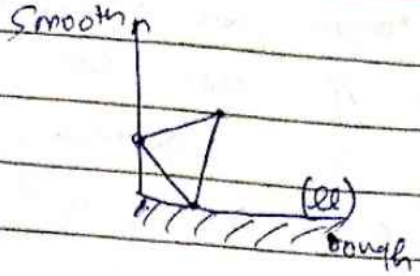


$\tan\lambda = \frac{1}{\sqrt{3}}$

$\boxed{\lambda = 30^\circ}$

$$BC = OB - OC$$
$$= a - a\sin 60$$
$$= a\left(1 - \frac{\sqrt{3}}{2}\right)$$
$$= a(1 - 0.866)$$
$$\boxed{BC = 0.134a = h}$$

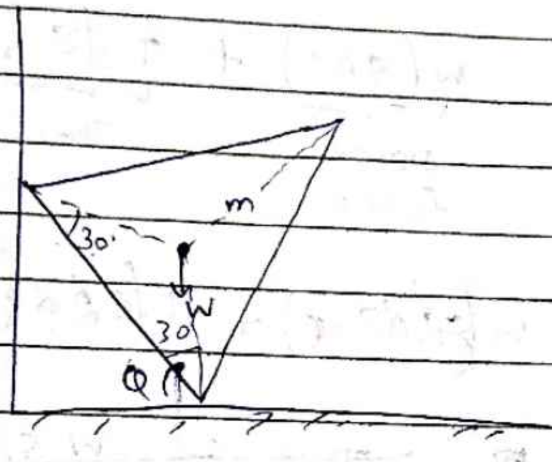
Q) 

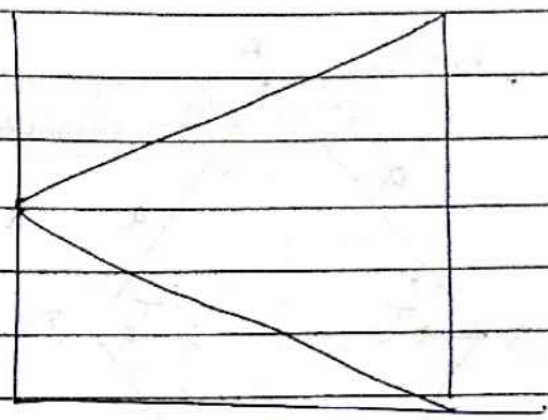


Show that the least angle its Base can make with horizontal is

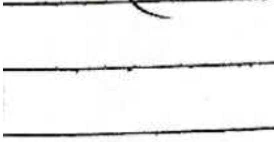
$\cot\varphi = 2\mu + \frac{1}{\sqrt{3}}$

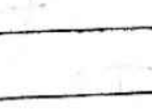
⇒





★ Virtual work $\{ w = \vec{F} \cdot \vec{\delta r} \}$



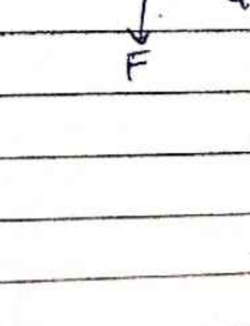


$-(w\,\delta l)$ $\quad$ $+(w\,\delta l)$

string

$-(T\,\delta l)$

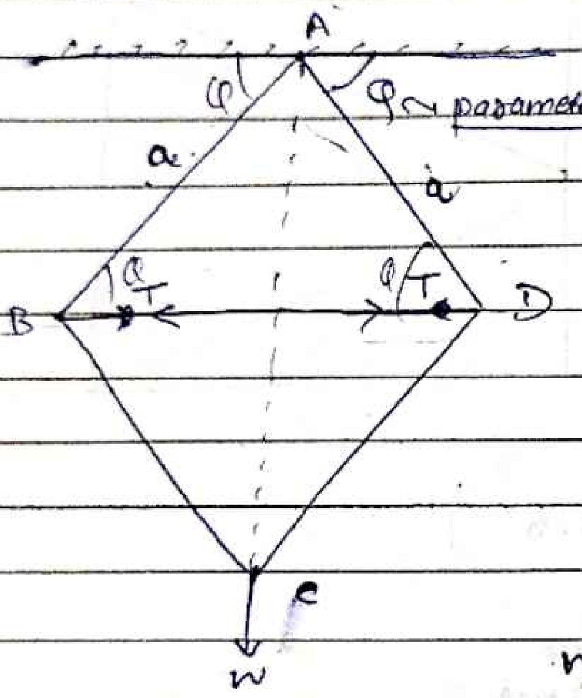
$+(T\,\delta l)$



θ → parameter

Q)

5 wt less rod of equal length are joind to form rhombus with I diagonal BD if the wt (W) is attached to pt C & system is suspended from pt A so that there is a thrust in BD $\boxed{T = \frac{W}{\sqrt{3}}}$



$$\underbrace{W(\delta AC)}_{\text{Down force}} + \underbrace{T(\delta BD)}_{\text{Thrust}} = 0$$

$$W(\delta 2a\sin\varphi) + T(\delta 2a\cos\varphi) = 0$$

$$\cancel{T = \frac{W\sin\varphi}{\cos\varphi}}$$

$$\boxed{\cancel{T = \ominus W \tan\varphi}}$$

$$W(\delta \sin\varphi) + T(\delta \cos\varphi) = 0$$

$$W\cos\varphi\, \cancel{\delta\varphi} + T(-\sin\varphi)\,\cancel{\delta\varphi} = 0$$

$$\cancel{T =} \quad W = \frac{T\sin\varphi}{\cos\varphi}$$

$$T = \frac{W}{\tan\varphi} \quad \cancel{\cot\varphi}$$

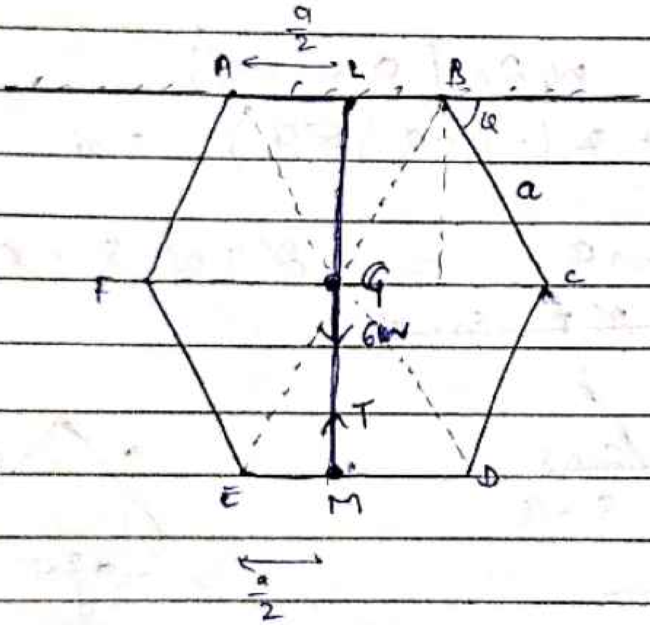
$\varphi = 60°$ $$T = \frac{W}{\sqrt{3}}$$

Q) 6 equal rods
are each of wt (W) are joind & their extremities
are made a hexagon
AB → fixed in horizontal & middle ppt of
AB & DE are joined by string.

prove $\boxed{\underset{\text{Tension String}}{(T)} = 3W}$

$6W(\delta\, LG) - T(\delta LM) = 0$

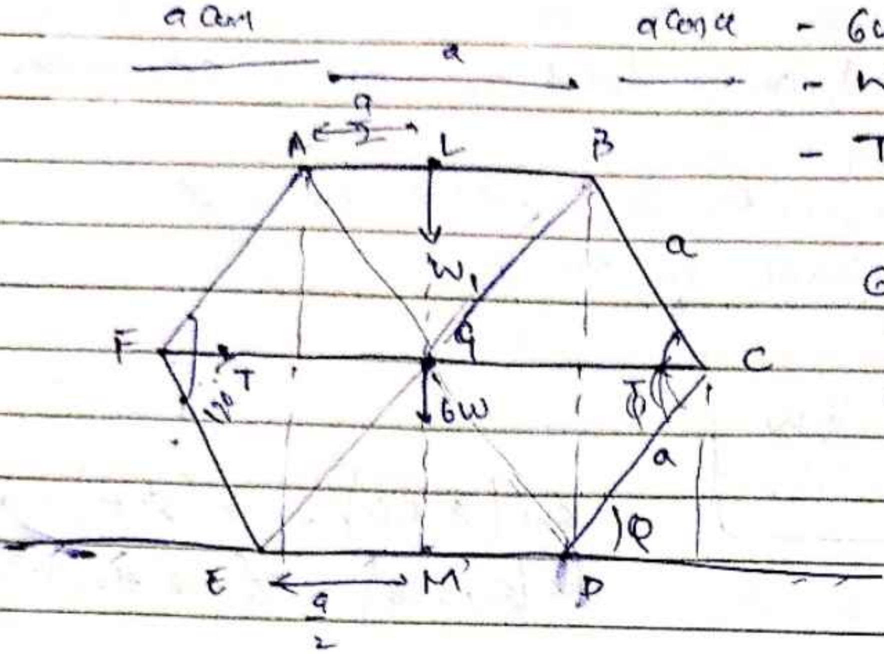
$6W(\delta a \sin\theta) - T\delta(2a \sin\theta) = 0$



$6W = 2T$

$\boxed{T = 3W}$

Q) A regular ⬡ composed of 6 rod (heavy) freely joind together & 2 opp angle are connected by string which is horizontal & rod being contact with horizontal plane & the middle pt of opp rod is placed a wt (W_1)
if (W) → wt of rod (each)
show $\underset{\text{in string}}{(T)} = \dfrac{3W + W_1}{\sqrt{3}}$



$$-6W(\delta GM) - W_1(\delta LM) - T(\delta FC) = 0$$

$$6W(\delta(a\sin\theta)) \quad W_1(\delta(2a\sin\theta)) \quad T(\delta(a+2a\cos\theta)) = 0$$

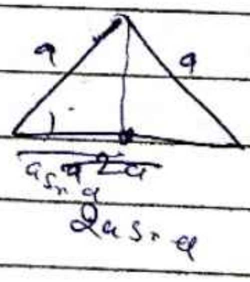
$$6W(a)(\cos\theta\,\cancel{\delta\theta}) + W_1 2a(\cos\theta\,\cancel{\delta\theta}) + Ta(0+2(-\sin\theta)\cancel{\delta\theta}) = 0$$

$$3W\cancel{a}\cos\theta + \cancel{2}W_1\cancel{a}\cos\theta = \cancel{2}T\cancel{a}\sin\theta$$

$$\cancel{4\ 8\ W\cos\theta = 2T\sin\theta}$$

$$\cancel{T = \frac{4W\cos\theta}{\sin\theta}}$$

$$= \frac{4W}{\tan\theta}$$



$$3W\cos\theta + W_1\cos\theta = T\sin\theta$$

$$T = \frac{(3W+W_1)}{\tan\theta} \qquad \theta = 60^\circ$$

$$\boxed{T = \left(\frac{3W+W_1}{\sqrt{3}}\right)}$$

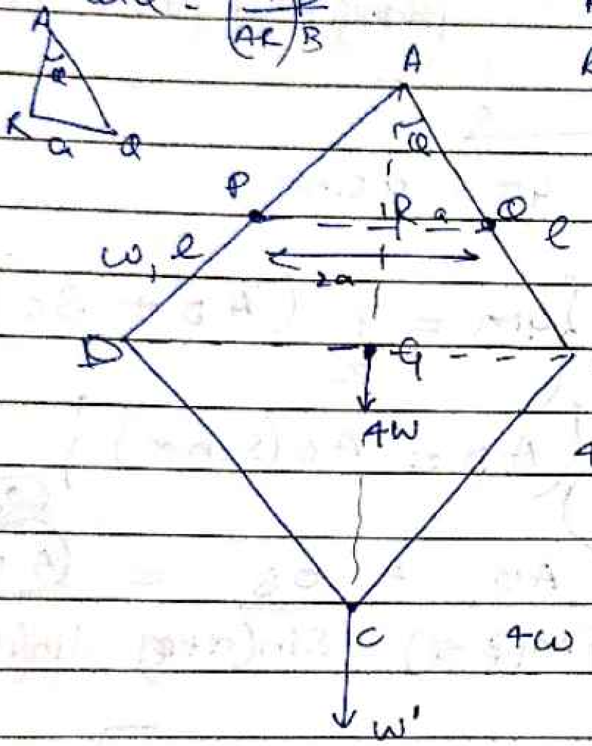
⇒

P B/P
H H/B

A rhombus is formed of 4 rod each wt (4w) length (l) with smooth join prises it rest symm. with 2 upper side at same level

prove $\sin^3\theta = \dfrac{a(4w+w')}{l(4w+2w')}$

$\cot\theta = \left(\dfrac{a}{AR}\right)\dfrac{B}{B}$



$RG = AG - AR = l\cos\theta - AR = l\cos\theta - a\cot\theta$

$RC = 2l\cos\theta - a\cot\theta$

$4w \times \delta(RG) + w'\,\delta(RC) = 0$

$4w \times \delta(l\cos\theta - a\cot\theta)$
$+ w'\delta(2l\cos\theta - a\cot\theta) = 0$

$4w\left[l(-\sin\theta)d\theta + a\,\mathrm{cosec}^2\theta\, d\theta\right]$
$+ w'\left[2l(-\sin\theta)d\theta + a\,\mathrm{cosec}^2\theta\, d\theta\right] = 0$

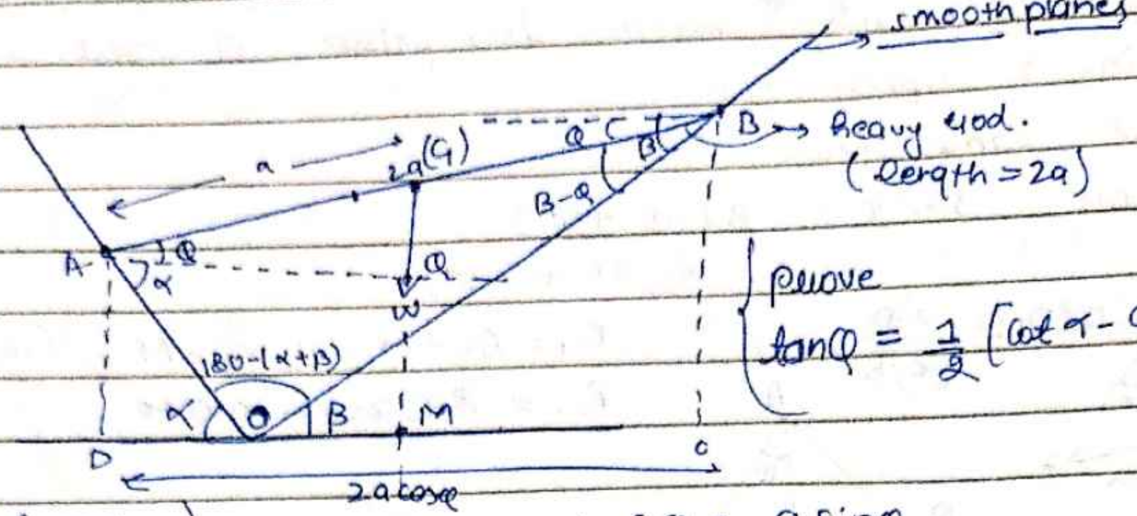
$4w\left[a\,\mathrm{cosec}^2\theta\, d\theta - l\sin\theta\, d\theta\right] + w'\left[a\,\mathrm{cosec}^2\theta\, d\theta - 2l\sin\theta\, d\theta\right] = 0$

$\dfrac{a\,\mathrm{cosec}^2\theta}{\sin^2\theta}(4w+w) - l\sin\theta(4w - 2w') = 0$

$a(4w+w') - l\sin^3\theta(4w - 2w') = 0$

$\sin^3\theta = \dfrac{a(4w+w')}{l(4w-2w')}$

eg :-



prove
$\tan Q = \frac{1}{2}\left[\cot\alpha - \cot\beta\right]$

$-w(\delta GM) = 0$

$-w\left\{\delta\, \dfrac{a\sin Q + 2a\sin\alpha \sin(\beta - Q)}{\sin(\alpha+\beta)}\right\} = 0$

$a,\ \alpha,\ \beta \to$ const

$-w\left[\dfrac{\cancel{a}\cos Q\, \cancel{dQ} + 2\cancel{a}\sin\alpha \cos(\beta - Q)(-\cancel{dQ})}{\sin(\alpha+\beta)}\right] = 0$

$+w\left[\cos Q + \dfrac{2\sin\alpha\cos(\beta-Q)}{\sin(\alpha+\beta)}\right] = 0$

$w\left[\cos Q\sin(\alpha+\beta) + 2\sin\alpha\cos(\beta-Q)\right] = 0$

$\cos Q\sin(\alpha+\beta) = -2\sin\alpha\cos(\beta-Q)$

$\dfrac{\cos(\beta-Q)}{\cos Q} = -\dfrac{\sin\alpha+\beta}{2\sin\alpha}$

$\dfrac{\cos\beta\cos Q + \sin\beta\sin Q}{\cos Q} = "$

$\cos\beta + \sin\beta\,(\tan Q) = -\dfrac{\sin(\alpha+\beta)}{2\sin\alpha}$

$\tan Q = -\dfrac{\sin(\alpha+\beta)}{\sin\beta\, 2\sin\alpha} - \dfrac{\cos\beta}{\sin\beta}$

$GQ = a\sin Q$

$\left\{GM = \dfrac{1}{2}(AD + BC)\right\}$

$\{AD = AO(\sin\alpha)\}$

$\dfrac{AO}{\sin(\beta-Q)} = \dfrac{OB}{\sin(\alpha+Q)} = \dfrac{AB\ (2a)}{\sin(180-(\alpha+\beta))}$

$AO = \dfrac{2a[\sin(\beta-Q)]}{\sin(\alpha+\beta)}$

$OB = \dfrac{2a[\sin(\alpha+Q)]}{\sin(\alpha+\beta)}$

$\boxed{AD = \dfrac{2a\sin(\beta-Q)(\sin\alpha)}{\sin(\alpha+\beta)}}$

$GM = a\sin Q + AD$

$GM = a\sin Q + \dfrac{2a\sin\alpha\sin(\beta-Q)}{\sin(\alpha+\beta)}$

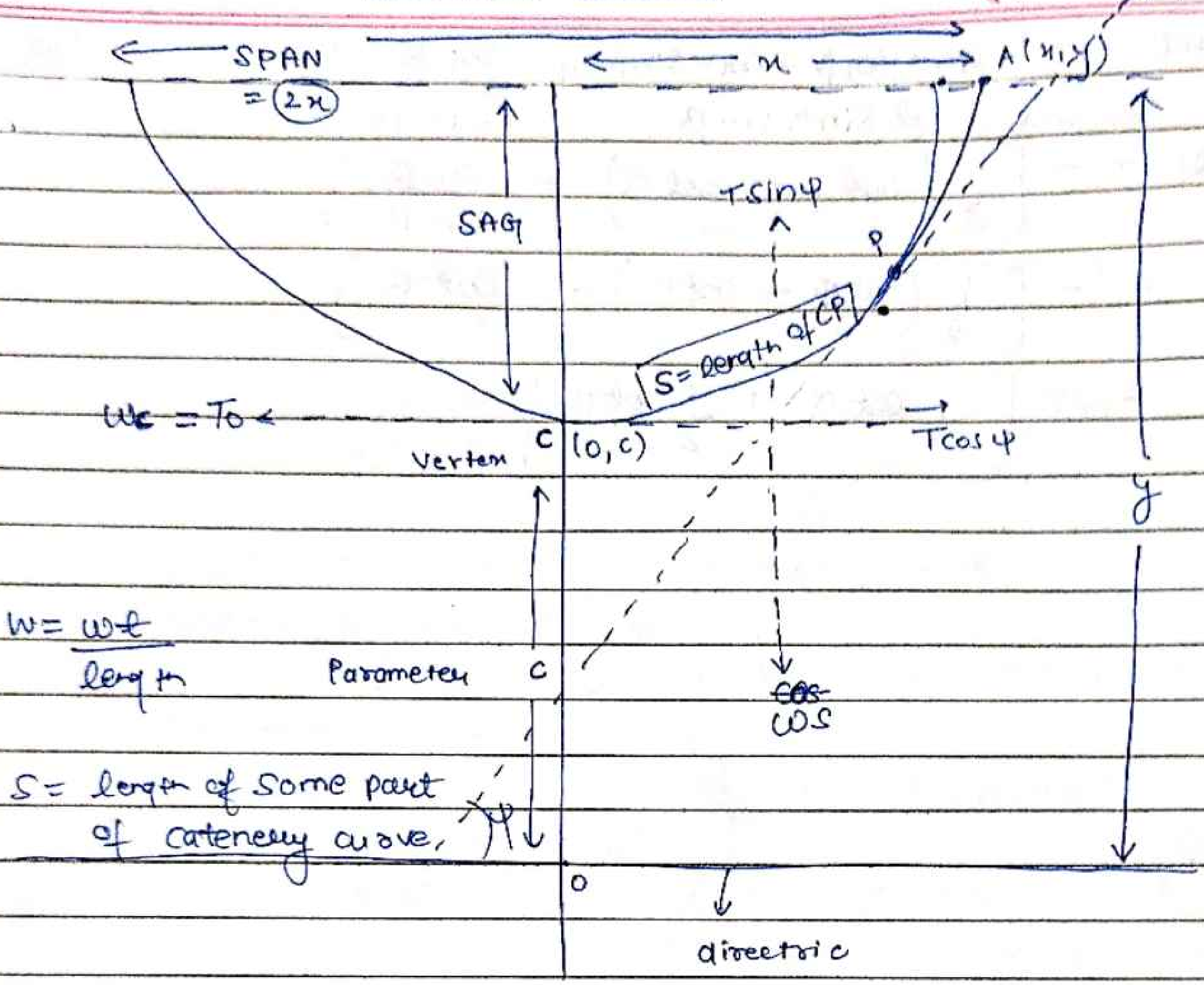
$$\tan\phi = -\left[\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{2\sin\alpha\sin\beta} + \frac{\cos\beta}{\sin\beta}\right]$$

$$\tan\phi = -\left[\frac{1}{2}\left(\cot\beta + \cot\alpha\right) + \frac{\cos\beta}{\sin\beta}\right]$$

$$= -\left[\frac{1}{2}\left(\cot\beta + \cot\alpha\right) + \cot\beta\right]$$

$$= -\left[\frac{1}{2}\cot\alpha + \frac{3}{2}\cot\beta\right]$$

Catelary Curve



Intrinsic eq^n of Catenary $\quad S = c \tan\varphi$

Cartesia " " " $\Rightarrow \quad y = c\cosh\left(\frac{x}{c}\right)$

$\boxed{OR}$

$$y = \frac{1}{2} c \left\{ e^{x/c} + e^{-x/c} \right\}$$

Some relations :-

1) $T\cos\varphi = T_0 = wc$

2) $T\sin\varphi = ws$

3) Relation b/w y & φ $\Rightarrow$ $y = c\sec\varphi$

x φ $x = c\log[\sec\varphi + \tan\varphi]$

S x $S = c\sinh(x/c)$

S y $y^2 = S^2 + c^2$ or $S = y\sin\varphi$

$$y = \frac{1}{2} c \left\{ e^{x/c} + e^{-x/c} \right\}$$

$$= \frac{1}{2} c \left\{ 1 + \cancel{x} + \frac{x^2}{c^2 2!} + \cancel{\frac{x^3}{3!}} + \dots + 1 - \cancel{x} + \frac{x^2}{2!} - \cancel{\frac{x^3}{3!}} \dots \right\}$$

$$= \frac{1}{\cancel{2}} c \left\{ \cancel{2} \left(1 + \frac{x^2}{c^2 2!} + \frac{x^4}{c^2 4!} + \frac{x^6}{6!} + \dots \right) \right\}$$

$$= c \left(1 + \frac{x^2}{c^2 2!} + \frac{x^4}{c^2 4!} + \dots \right)$$

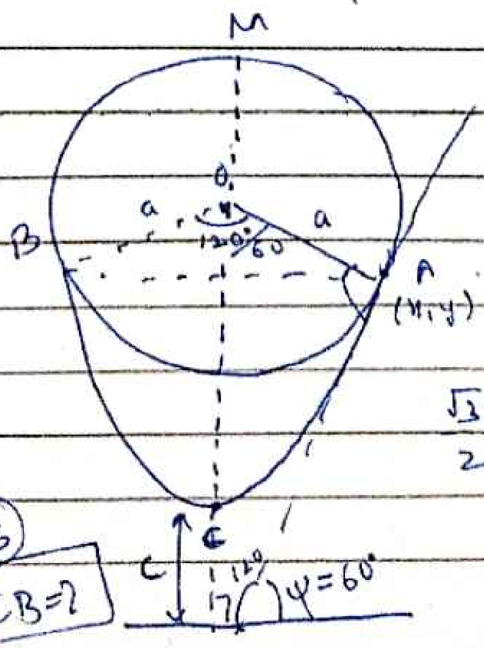
$$\boxed{x <<< c}$$

$$y = c + \frac{x^2}{2c} \Rightarrow \boxed{y - c = \frac{x^2}{2c}}$$

or $\boxed{x^2 = (2c)(y-c)}$

Hence, It is also a parabolla with vertex $(0, c)$ & whose axis is y - axis.

eg :- Show that the length of endless chain which will have a circular pulley of radius (a) so as to be contact of $2/3^{rd}$ of circumference of the pulley find the length of a chain. $= a\left[\frac{4\pi}{3} + \frac{3}{\log(2+\sqrt{3})}\right]$



$\frac{120}{360}$ 3

$AMB = \frac{2}{3}(2\pi a) = \frac{4}{3}\pi a$

$\angle AO'B = 120^\circ$ $\quad \psi = 60^\circ$

$x = c \log[\sec\psi + \tan\psi]$

$\frac{\sqrt{3}a}{2}$ $\sin 60 = c \log(2 + \sqrt{3})$

$$\boxed{c = \frac{\sqrt{3}a}{2\log(2+\sqrt{3})}}$$

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$\boxed{ACB = ?}$

$AC = \quad S = \quad c \tan 60^\circ$

$$S = \frac{\sqrt{3}\,a}{2\log(2+\sqrt{3})} \quad (\sqrt{3})$$

length of chain $= AMBC$

$$= \frac{4}{3}\pi a + 2\{S\}$$

$$= \boxed{\frac{4}{3}\pi a + \frac{\sqrt{3}\,a}{2\log(2+\sqrt{3})}}$$