Mossyrock Dam Analysis

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Advanced Systems Engineering Final Exam
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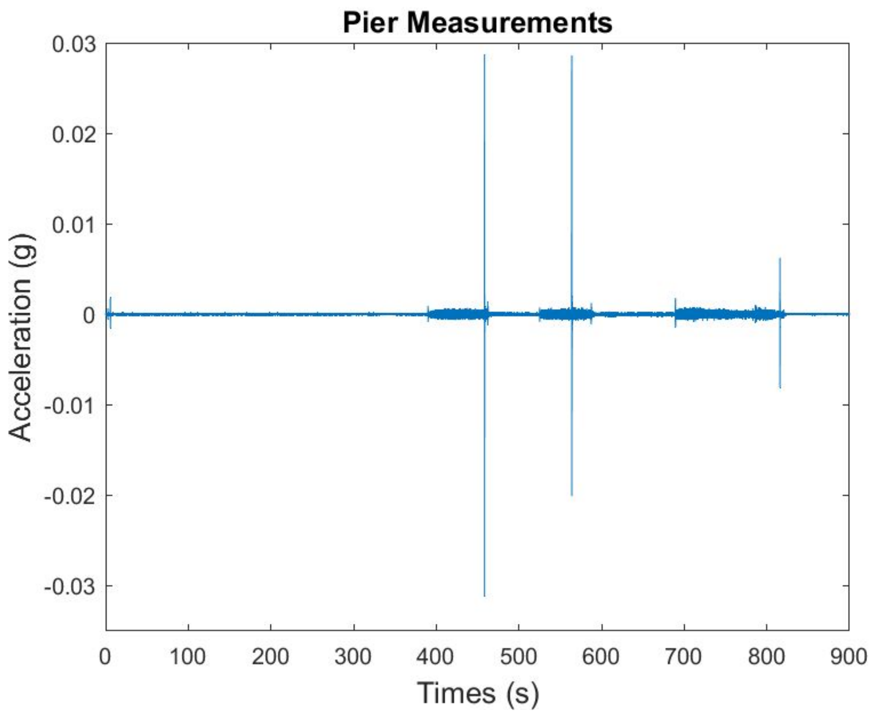
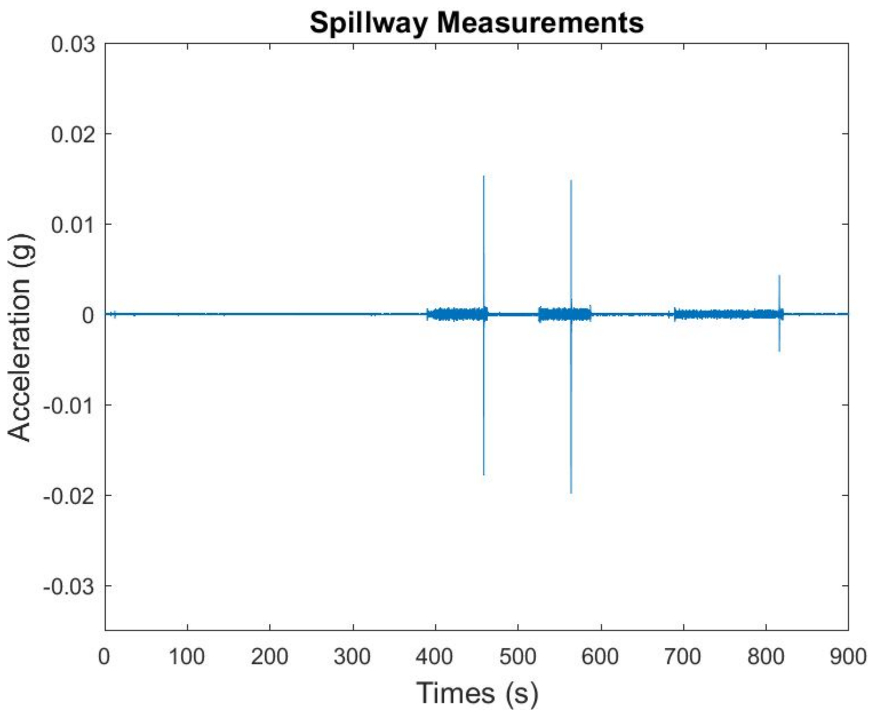
The Problem

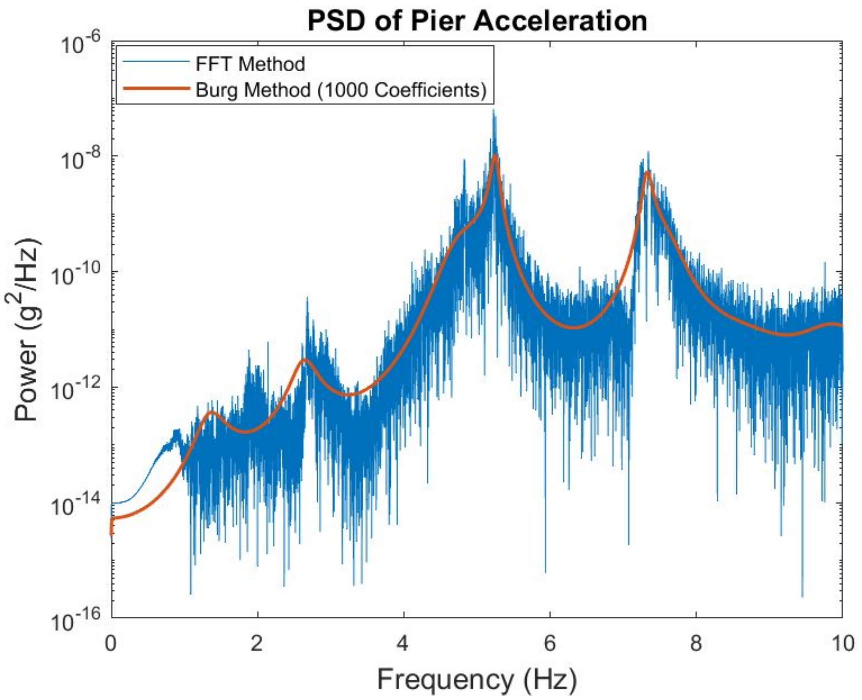
Formulate and validate a model that correctly characterizes the behavior of Mossyrock Dam, taking the pier-spillway interactions into account.

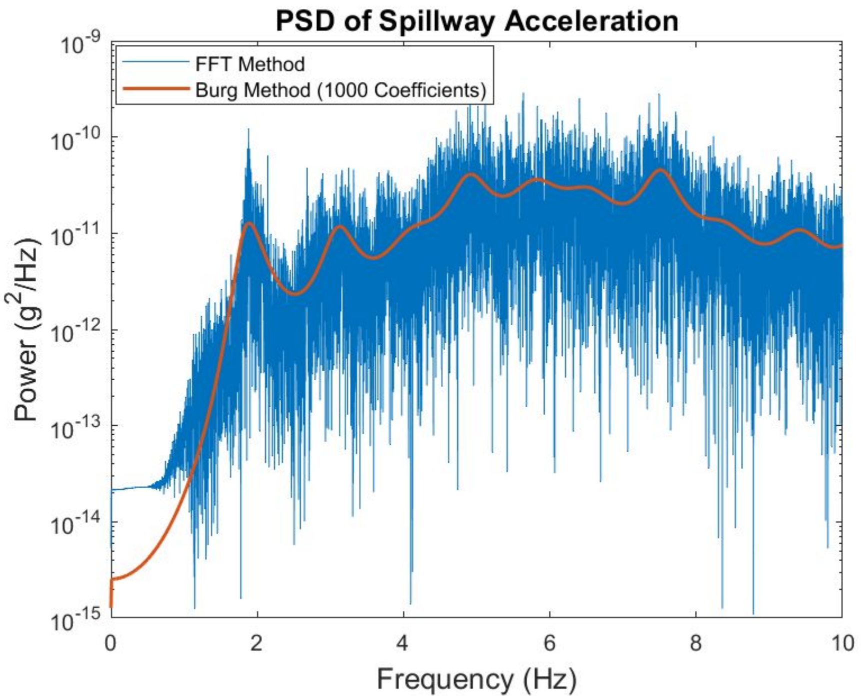
Measured Responses from the Dam





Power Spectral Density





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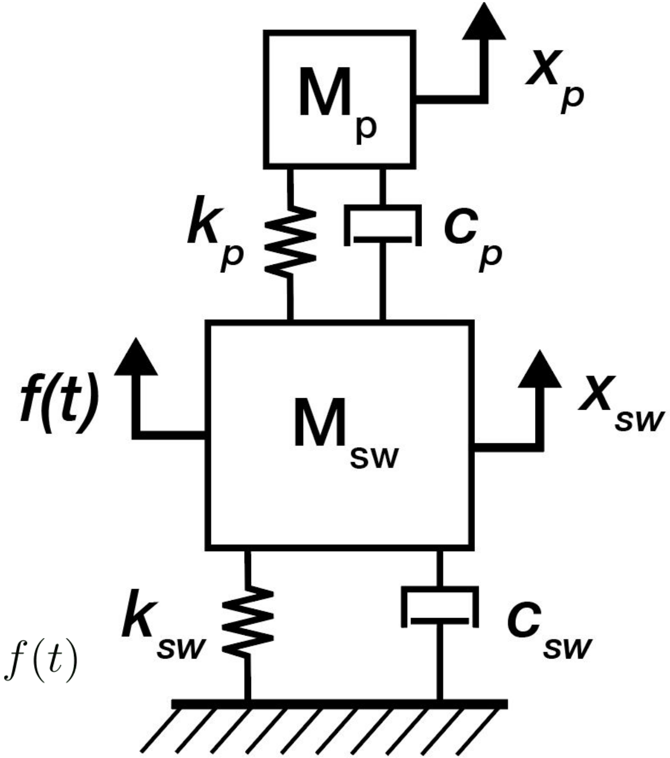
Model

- We used a 2 degree of freedom model with an input force acting on the spillway
- The governing equations of the models are in terms of unknown physical parameters M_p, M_{sw}, k_p, k_{sw}, c_p, and c_{sw}
- Governing equations:

$$M_p\ddot{x}_p + c_p(\dot{x}_p - \dot{x}_{sw}) + k_p(x_p - x_{sw}) = 0$$

$$M_{sw}\ddot{x}_{sw} + c_p(\dot{x}_{sw} - \dot{x}_p) + c_{sw}\dot{x}_{sw} + k_p(x_{sw} - x_p) + k_{sw}x_{sw} = f(t)$$

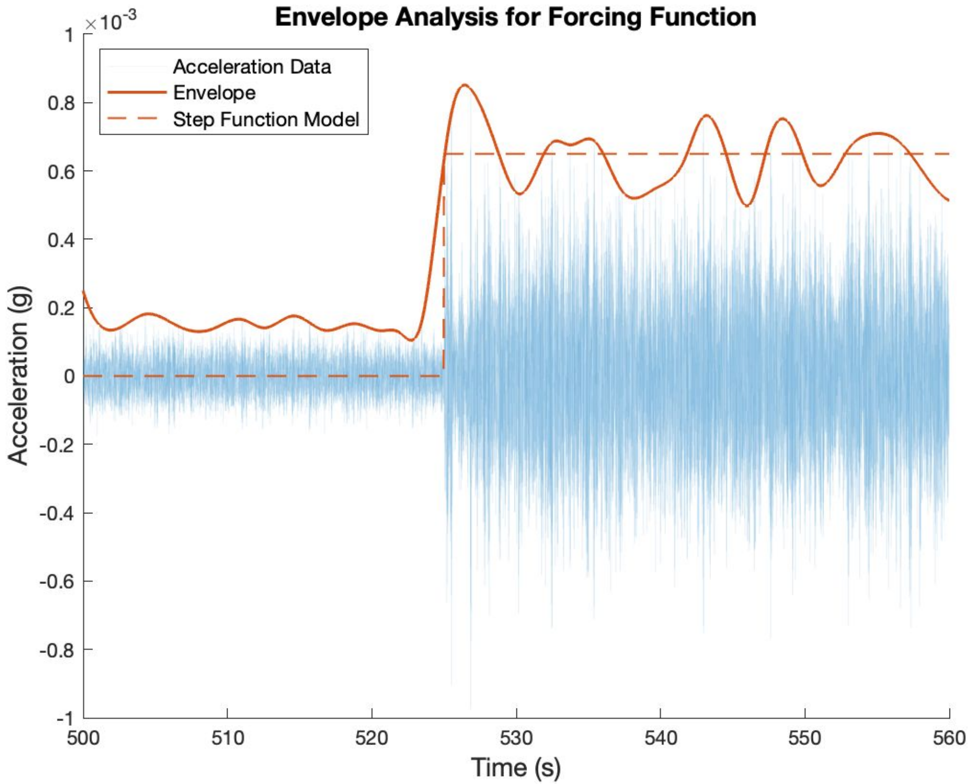


System parameters

- We want our equations in terms of known system parameters w_p, w_{sw}, ζ_p, and ζ_{sw}
- The resonant frequencies, w_p and w_{sw}, of the pier and spillway can be found using the PSD, in samples per second:
  - w_p: [5.24 7.33]
  - w_{sw}: [1.89 3.14 4.91 5.83 6.55 7.53]
- The damping factors ζ_p and ζ_{sw} can be found using the ½ power approach from the PSDs:
  - $\zeta_p = 0.0097$
  - $\zeta_{sw} = 0.0345$

Forcing Function

- The forcing function is the equivalent forcing function the system undergoes when the gates to the spillway are opened and water flows
- From intuition, this would resemble a step function
- Envelope analysis confirms our practical hypothesis
- Thus, $f(t) = \beta u(t)$ and $f(jw) = \beta/(jw)$



Derivation of Frequency Response Function

$$\begin{bmatrix} M_P & 0 \\ 0 & M_{SW} \end{bmatrix} \begin{bmatrix} \ddot{x}_P \\ \ddot{x}_{SW} \end{bmatrix} + \begin{bmatrix} c_P & -c_P \\ -c_P & c_P + c_{SW} \end{bmatrix} \begin{bmatrix} \dot{x}_P \\ \dot{x}_{SW} \end{bmatrix} + \begin{bmatrix} k_P & -k_P \\ -k_P & k_P + k_{SW} \end{bmatrix} \begin{bmatrix} x_P \\ x_{SW} \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

Let

$$\mathbf{x} = \begin{bmatrix} x_P \\ x_{SW} \end{bmatrix} = \bar{\mathbf{x}} e^{jwt} \quad \text{and} \quad \mathbf{f} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix} = \bar{\mathbf{f}} e^{jwt} \quad \text{and} \quad \alpha = \frac{M_{SW}}{M_P}$$

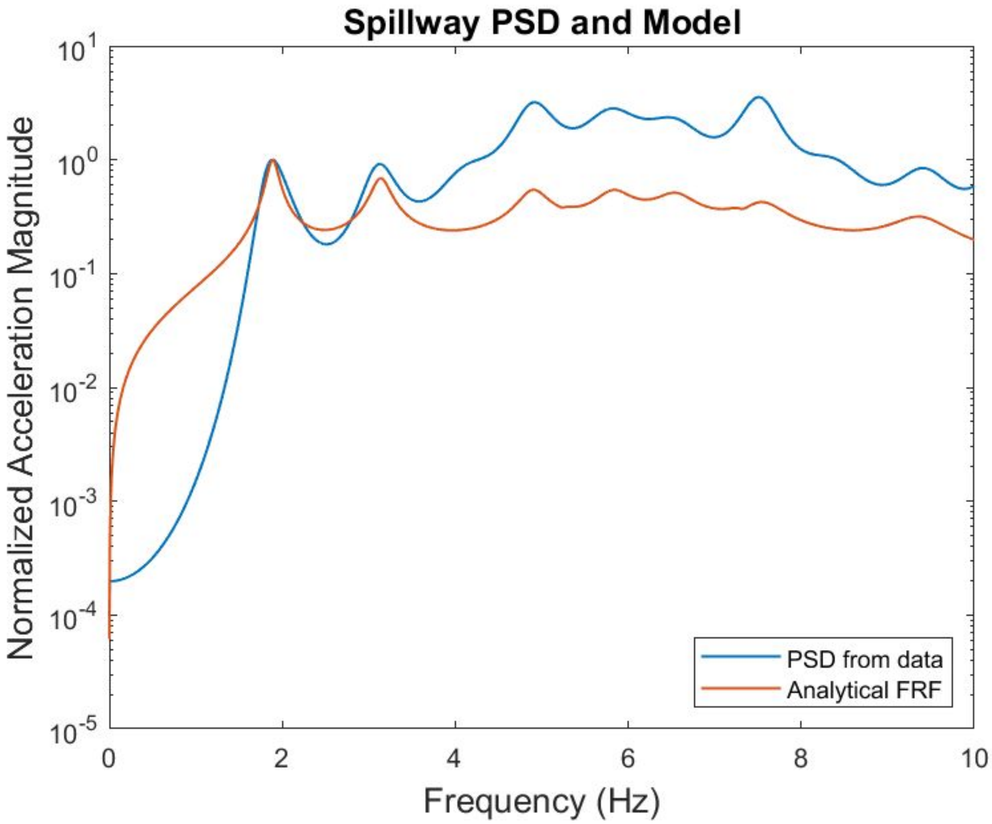
Then

$$\begin{bmatrix} -\omega^2 + 2\zeta_P \omega_P j\omega + \omega_P^2 & -2\zeta_P \omega_P j\omega - \omega_P^2 \\ -2\zeta_P \omega_P j\omega - \omega_P^2 & -\alpha\omega^2 + 2j\omega\left(\zeta_p \omega_p + \alpha \zeta_{SW} \omega_{SW}\right) + \omega_P^2 + \alpha\omega_{SW}^2 \end{bmatrix} \bar{\mathbf{x}} = \begin{bmatrix} 0 \\ \beta \bar{f} \end{bmatrix}.$$

To find $X_{SW}(j\omega)$ in terms of acceleration, we need to multiply by $-\omega^2$ and the Fourier Transform of the forcing term. Altogether, we are left with:

$$X_{acc,SW} = \frac{j\omega\left(-\omega^2 + 2\zeta_P \omega_P j\omega + \omega_P^2\right)}{\left(-\omega^2 + 2\zeta_P \omega_P j\omega + \omega_P^2\right)\left(-\alpha\omega^2 + 2j\omega\left(\zeta_p \omega_p + \alpha\zeta_{SW}\omega_{SW}\right) + \omega_P^2 + \alpha\omega_{SW}^2\right) - \left(-2\zeta_P \omega_P j\omega - \omega_P^2\right)^2}$$

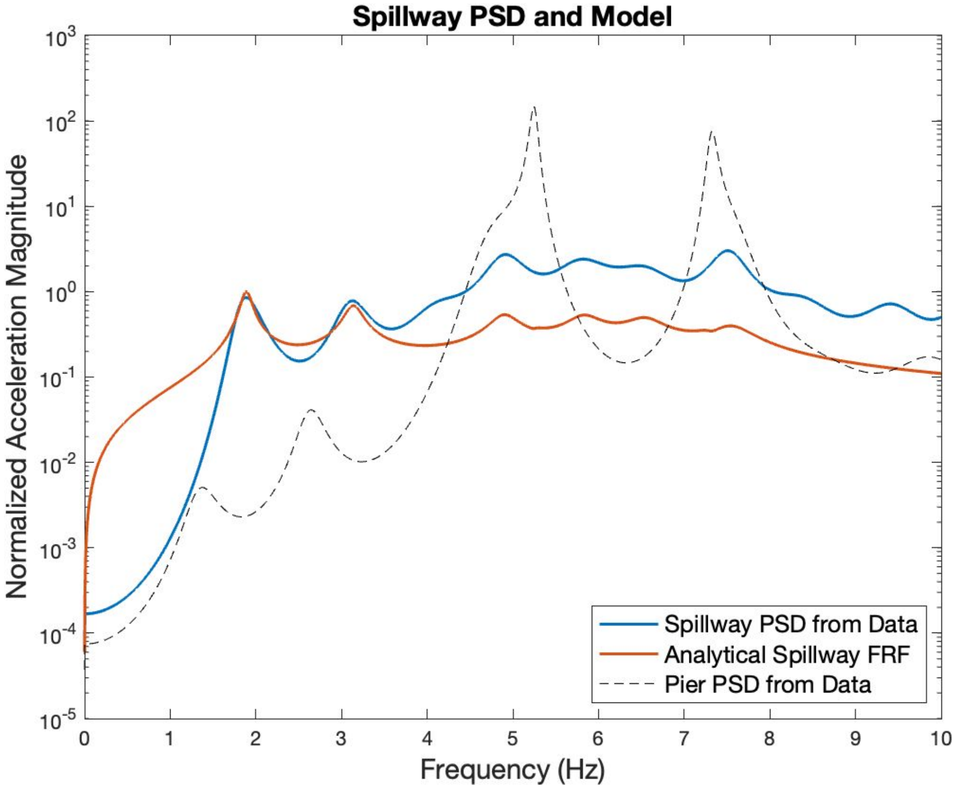
Frequency Response Function



- Computed using data-derived resonances, damping constants, and superposition
- Model matches low frequency behavior well, but there's a gap in the higher frequencies

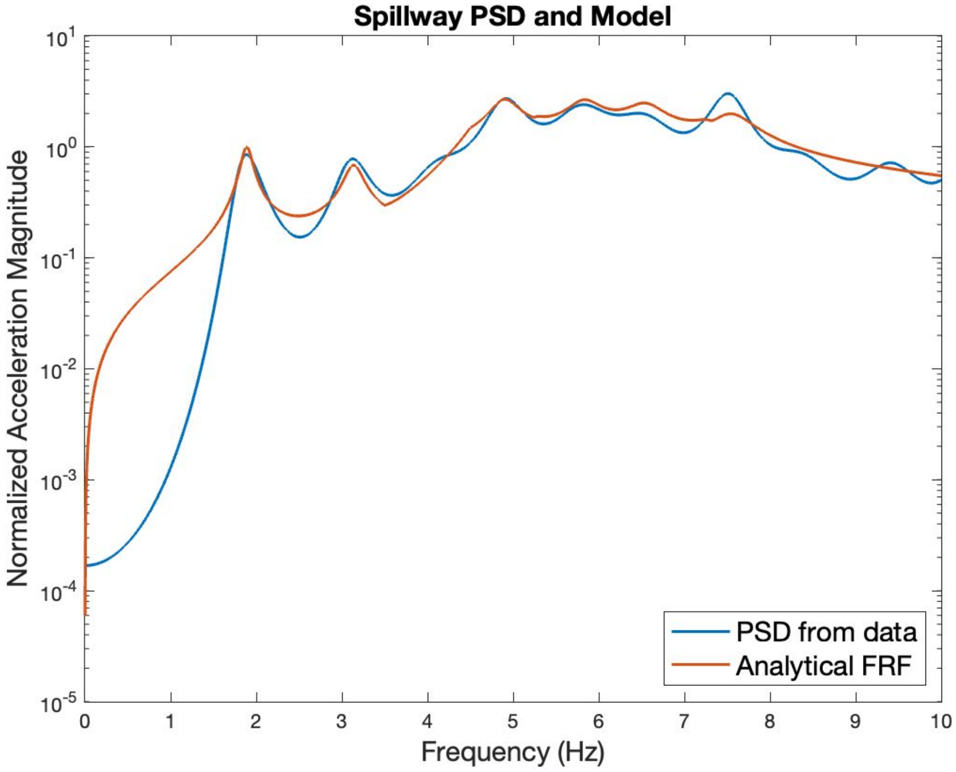
Model Analysis

- At low frequencies, pier and spillway show similar spectral characteristics, indicating a rigid body mode
- At higher frequencies, pier has high resonances but spillway displays broadband behavior and anti-resonances, indicating damping effect from the pier.



Revised Model

- To compensate for the gap at higher frequencies, we considered the effects of all five piers acting on the spillway



Conclusion and Analysis

- At lower frequencies, the piers oscillate with the dam
- At higher frequencies, the dam exhibits broadband behavior, indicating a damping effect from the piers
- Original engineer’s model does not take this interaction into consideration

- Future work:
  - More pier measurements and 6-DoF model
  - Integrating a seismic displacement term to determine maximum loading